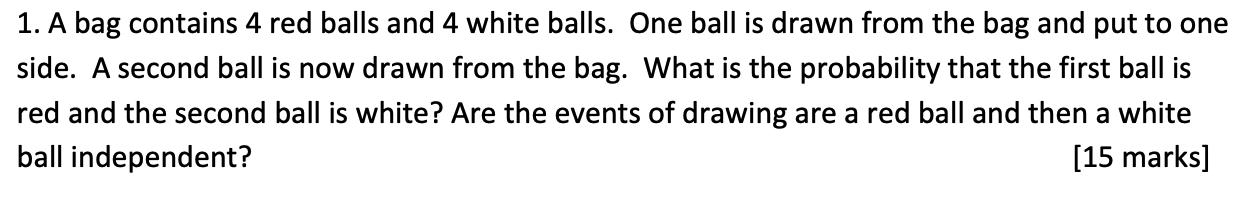
**ST3009 – Statistics**

**2016 Exam Solutions**

**Question 1**



Sample Space = { R, R, R, R, W, W, W, W }

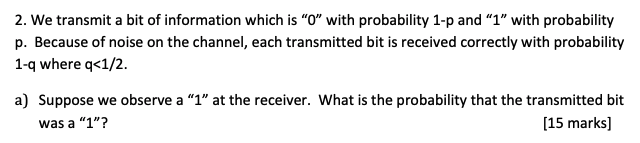
Balls removed without replacement.

For two events to be independent:

But:

Therefore, the events are not independent.

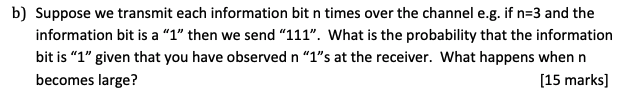
**Question 2**

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Probability of receiving a 0 🡪 1-p

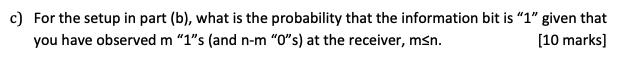
Probability of receiving a 1 🡪 p

Probability of correct result 🡪 1-q



Let F be the event that n 1’s are observed at the receiver and E be the event that a 1 is sent:

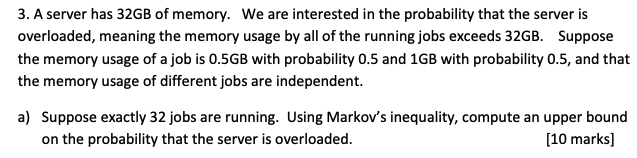
As n becomes large, goes to 0 since q < 0.5 and so, as a result P(E|F) tends to 1.



Let F be the event that m 1’s are observed at the receiver and E be the event the information bit is a 1:

And so:

**Question 3**

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Overloaded 🡪 Memory of all running jobs > 32GB

Memory of a job 🡪 0.5GB w/ probability of 0.5

🡪 1GB w/ probability of 0.5

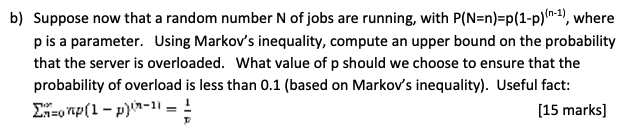
Let be the memory used by job i:

The total memory usage is:

Therefore, the expected memory usage is:

By Markov’s inequality:

Therefore, the upper bound on the probability that the server is overloaded is 0.75 or 75%.



Random number N of jobs running at any one time with:

Now the memory usage, U can be defined as:

The new expected value is then:

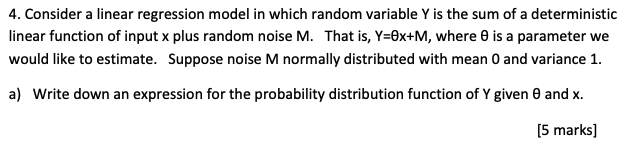
We can also calculate E[N] as follows:

Therefore:

Using this we can then calculate an upper bound of Markov’s inequality as follows:

In order to ensure the probability of overloading the server has an upper bound of 0.1 solve:

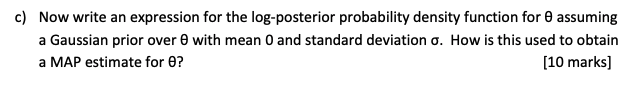
**Question 4**

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Noise is a number M normally distributed with mean 0 and variance of 1:

So we can then express the probability distribution function f of y given and x as:





By Bayes rule the posterior is:

We know from part (b):

The prior () is as follows (Gaussian with mean 0 and standard deviation :

Putting these together and taking logs we get:

The MAP estimate for is the value that maximises this.