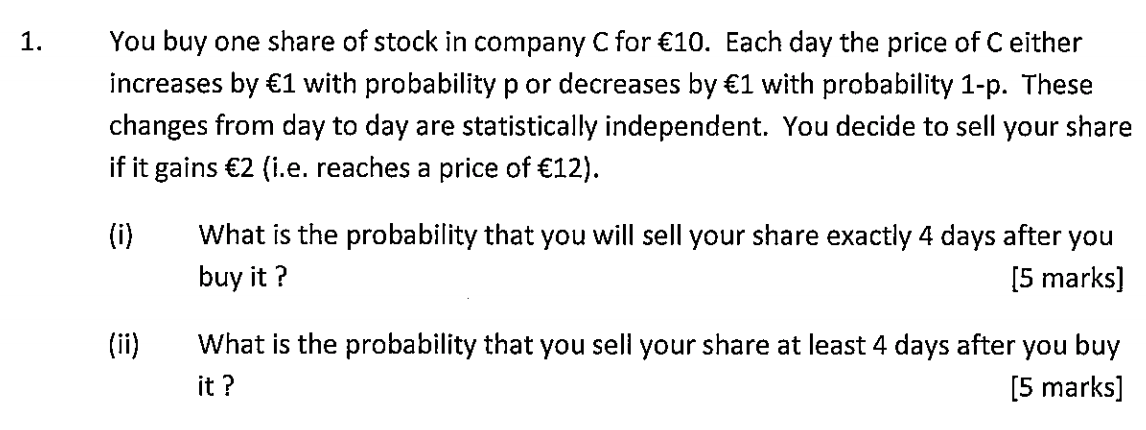
**ST3009 – Statistics**

**2018 Exam Solutions**

**Question 1**



Increase = p

Decrease = 1-p

Sell if gain €2

i) In order to satisfy criteria to sell the share price must reach €12. For this to happen exactly 4 days after purchasing it, the following must occur:

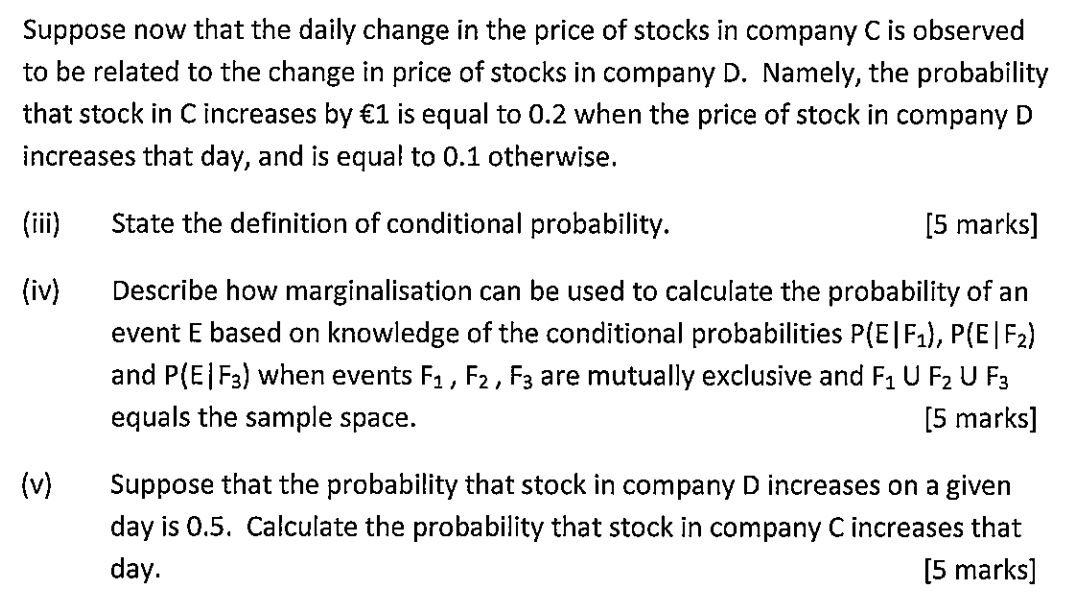
* Decrease -> Increase -> Increase -> Increase
* Increase -> Decrease -> Increase -> Increase

The probability of either of these scenarios occurring is as follows:

ii) In order to sell our share at least 4 days after you buy it we can do 1-(Prob Sell on Day 1 + Prob Sell on Day 2 + Prob Sell on Day 3).

* Prob Sell on Day 1 = 0
* Prob Sell on Day 2 = (Increase -> Increase) =
* Prob Sell on Day 3 = 0

Therefore the probability of us selling our share at least 4 days after we buy it is:

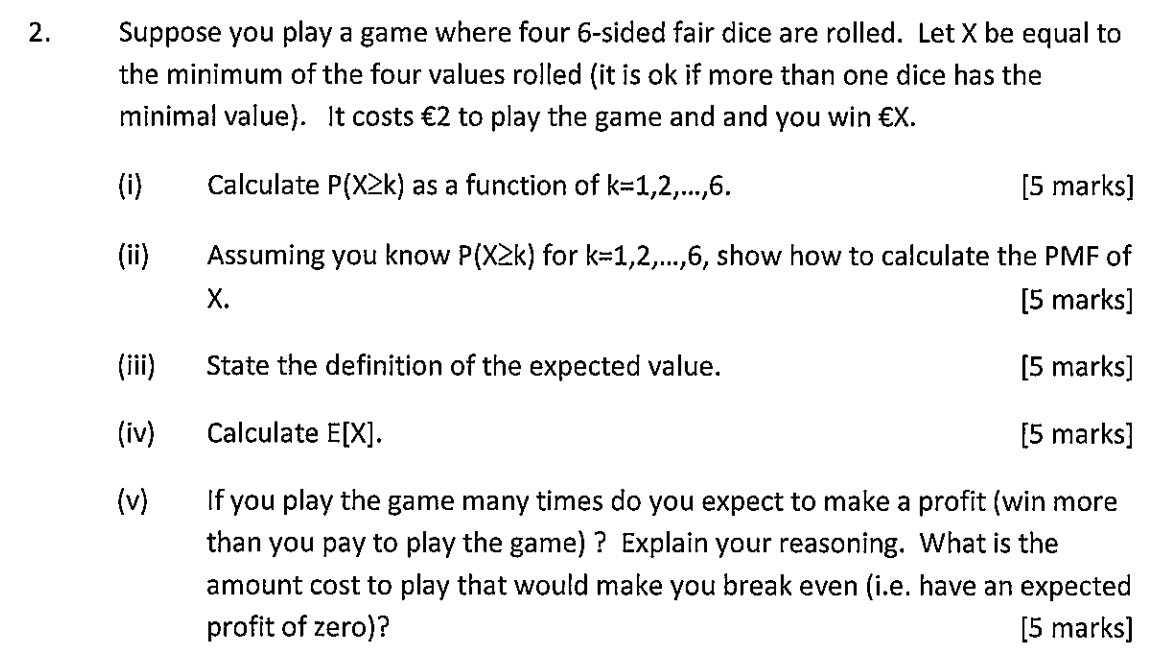


iii) **Conditional probability** is the probability that event E will occur given that event F has already been observed.

iv) Suppose we have mutually exclusive events F1 , F2  and F3 which together equal the entire sample space S, then:

v) Probability of D increasing on a given day is 0.5. Let C be the probability that company C’s stock increases:

**Question 2**



i)



ii) We can use the above values to calculate the PMF as follows:

iii) The definition of the expected value of a discrete random variable X taking values in {x1, x2, …, xn} is defined to be:

iv) For our given dice game we can calculate the expected value as follows:

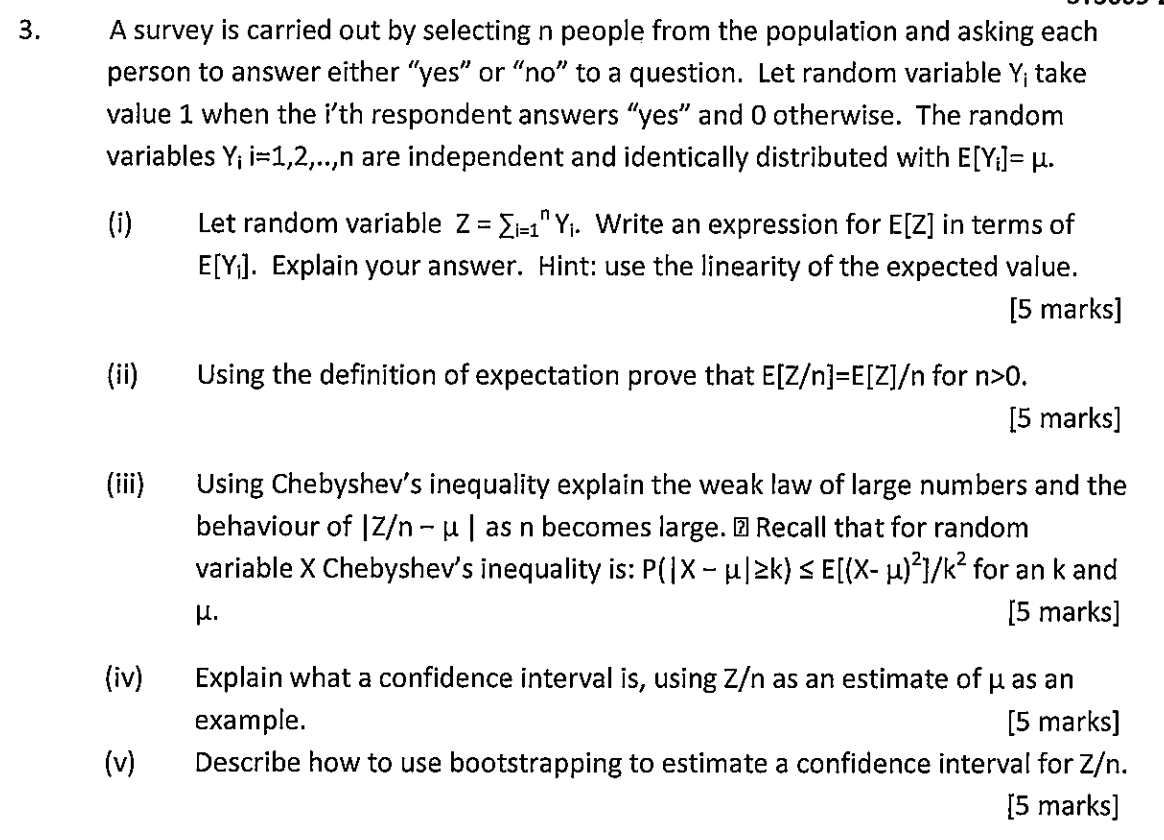
v) The expected profit for the game is as follows:

If we were to play the game N times, with N being substantially large we could expect a profit of:

In order for the cost to play to enable us to break even:

Therefore, the cost to play must equal the expected value in order for us to break even.

**Question 3**

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i) Using the linearity of the expected value we can write an expression for E[Z] as follows:

ii) Using the definition of expectation we can prove the above by:

iii)

Using the above formulae we can then plug then into Chebyshev’s inequality:

From this, we can infer that for any value of k>0, as n goes to infinity then the RHS goes to 0.

iv) A confidence interval, is an interval [a, b] within which a random variable X lies with a specified probability e.g with a probability of at least 0.95. This can be written as:

In the case of Z/n, as an estimate of we might consider the interval:

From part iii) we know that the above probability tends to 1 for any as n grows large. Thus, as we increase the number of samples our confidence in stating the that X lies within any given interval tends to 1.

v) From the observed data, and i=1, 2, …, n, draw a sample of m points uniformly at random with replacement. Using this sample calculate an estimate for Z/n. Repeat to obtain a multiple number of estimates. From the distribution of these estimates we can then estimate a confidence interval for Z/n.