**ST3009 - Statistical Methods for Computer Science**

**Week 3 Questions**

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***Question 1 –*** *Say we roll a fair 6-sided die six times. Using the fact that each roll is an*

*independent random event, what is the probability that we roll:*

1. Since each roll is an independent event we consider the probability of each event individually for each occurrence. The probability of rolling a specific number on a six sided die is:

As a result, the probability of rolling the sequence 1,1,2,2,3,3 – P(E) is:

1. The probability of exactly four occurrences of 3 can be calculated by calculating all possible outcomes of exactly four 3’s and divide this by the total sample space. This can be done as follows:

The total number of outcomes containing exactly k occurrences of 3 in n rolls - |E| - can be calculated as follows:

The total number of outcomes over 6 rolls of 6-sided die can be calculated as follows:

Thus, the probability of exactly four occurrences of 3 over 6 rolls can be calculated as:

1. The probability of rolling a single one is the probability of a one occurring in one trial and in no others. Similar to above this can be described as:

= 0.4019

1. The probability of one or more 1’s occurring is the probability that at least one 1 is rolled. This can be calculated as:

The probability of no ones being rolled is the probability that on each roll any number besides one occurs. This can be calculated as:

We can then use this probability to calculate the probability that at least one 1 is rolled:

***Question 2 –*** *Suppose one 6-sided and one 20-sided die are rolled. Let A be the event that*

*the first die comes up 1 and B that the sum of the dice is 2. Are these events independent*

*? Explain using the formal definition of independence.*

First we must calculate P(A) and P(B). The probability of A, P(A) is the probability of a 1 occurring on the first roll (we don’t know whether this is a 6-sided or 20-sided die. Thus:

To calculate P(B) investigate the possible outcomes that can produce a sum of 2. This occurs when both die roll a 1 so there are two possible ways in which this can occur:

Two events are independent if the order in which they occur does not matter. Therefore two events E and F are independent if they satisfy:

P(A B) is the probability that the first die comes up a 1 and that the sum of both dies is 2. This can be calculated as follows:

If the two events are independent the above result for P(A B) should be equal to P(A) \* P(B):

Therefore the two events A and B are not independent of each other and are in fact dependent.

***Question 3 –*** *. Say a hacker has a list of n distinct password candidates, only one of which*

*will successfully log her into a secure system.*

1. The probability of the hacker choosing a successful password is:

However, as the hacker unsuccessfully chooses a password candidate from n she removes this candidate from n this reducing n by one on every attempt.

The probability that her first successful login will be (exactly) on her k-th attempt is the probability that she chooses the wrong option every time until the kth time. Let the random variable X describe the k-th attempt and P(X = k) be the probability that the hacker is successful on her k-th attempt. Therefore:

Therefore the probability of the hacker being successful on her k-th attempt is 1/n if she is deleting the passwords as she progresses.

1. With n = 6 and k = 3 the value of this probability is as follows:
2. If the hacker was to stop deleting the candidates that she has already tried from the potential options n the probability of her succeeding on her k-th attempt would be:
3. Now, with n = 6 and k = 3, the probability of the hacker succeeding on her k-th attempt is:

***Question 4 –*** *A website wants to detect if a visitor is a robot. They decide to deploy three*

*CAPTCHA tests that are hard for robots and if the visitor fails in one of the tests, they*

*are flagged as a possible robot. The probability that a human succeeds at a single test is*

*0.95, while a robot only succeeds with probability 0.3. Assume all tests are independent.*

* **P(Human Succeeds Single Test) =** 0.95
* **P(Robot Succeeds Single Test) =** 0.3

1. Let P(RF) be the probability that a robot is flagged and P(RF’) be the probability that a robot is not flagged. The probability that a visitor gets flagged given that they are a robot can be calculated as follows:

Given that there are three CAPTCHA tests the probability that a robot is not flagged can be calculated by:

Therefore, using this we can calculate the probability that a visitor is flagged that it is actually a robot:

From this we can determine that there is a 97.3% chance of a visitor being flagged if it is a robot.

1. Let P(HF) be the probability that a human is flagged and P(HF’) be the probability that a human is not flagged. The probability that a visitor gets flagged given that they are a human can be calculated as follows:

Given that there are three CAPTCHA tests the probability that a robot is not flagged can be calculated by:

Therefore, using this we can calculate the probability that a visitor is flagged that it is actually a human:

From this we can determine that there is a 14.26% chance of a visitor being flagged if it is a human.

1. The probability that a visitor is a robot given that it has been flagged can be calculated using Bayes Rule as follows:

**- P(R|F):** Probability that visitor is a robot given flagged = unknown

**- P(F|R):** Probability that visitor is a flagged given robot = 0.973 (part a)

**- P(R):** Probability of visitor being a robot = 0.1 (given)

**- P(F):** Probability of a visitor being flagged (regardless if robot or human)

We can calculate P(F) as follows:

Using this value for P(F) we can then solve for P(R|F) using Bayes Rule:

Therefore, from this we can determine that the probability of a visitor being a robot given they have been flagged is 0.4323 or there is a chance of 43.23%.