**ST3009 - Statistical Methods for Computer Science**

**Week 5 Questions**

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***Question 1 –*** *A box contains 5 red and 5 blue marbles. Two marbles are withdrawn*

*randomly. If they are the same color, then you win $1.10; if they are different colors, then*

*you lose $1.00. Calculate:*

1. Let the event Y be the event that the marbles are the same colour. The probability of Y occurring is the probability of pulling one colour, and then the same colour again on the next selection which is as follows:

Using this we can then calculate the expected value of winnings W:

1. For a discrete random variable an expression for variance is as follows:

Let X be our winnings i.e X = W. Therefore using what we calculated as the expected value in part (a) the variance on the amount you win can be calculated as follows:

Hence, we can calculate:

And using this value we can then calculate Var(W):

***Question 2 –*** *Suppose you carry out a poll following an election. You do this by selecting n*

*people uniformly at random and asking whether they voted or not, letting Xi = 1 if person*

*i voted and Xi = 0 otherwise. Suppose the probability that a person voted is 0.6.*

1. is the expected value of asking person I whether they voted or not. This can be calculated as follows. is an indicator variable for event E (person has voted) i.e if event E occurs and otherwise. Therefore we can calculate the expected value as:

Similarly to question 1 (b) we can calculate the variance of using the equation:

First we must calculate

Using this we can then calculate :

1. is the expected value of the number of people that did vote when n people are uniformly selected at random. This can also be written in terms of as follows (using the linearity of the expected value to move the E[] inside the sum).

Using the fact that all people are sampled independently we can derive for all i:

Thus is clearly not the same as as shown and also because as you were to survey more than one person the expected value would change.

1. is the expected value for one nth of the survey sample. Using the equation derived in part (a) and the linearity of the expectation this can be written as:

From this we can infer that as it is the same as selecting one person out of a sample of n.

1. We can calculate the variance of using the equation:

Since people are sampled independently we can infer:

Hence:

***Question 3 –*** *Suppose that 2 balls are chosen without replacement from an urn consisting*

*of 5 white and 8 red balls. Let Xi equal 1 if the i’th ball selected is white, and let it equal*

*0 otherwise.*

1. Since can only take the values of 0 and one the joint probability mass function of and would look like the following:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | P(Y=y) |
|  |  |  |  |
|  |  |  |  |
| P(X=x) |  |  |  |

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1. For the events to be independent they must satisfy the following formula:

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Therefore we can determine that the events and are not independent as they do not satisfy the formal definition of independence.

1. The expected value of can be calculated as follows:
2. For two discrete random variables X and Y the conditional expectation of X given. Y = y is as follows:

Therefore, given that