Heat Generation by Ion Friction in Water under an AC Electric Field

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In this study we investigate the theoretical background for the heat generation by ion friction for an electrode boiler by using sodium triphosphate and sodium chloride salt water solutions with different concentrations under AC electric fields. For comparisons, distilled, mineral, tap and sea water were investigated, as well. The input power and conductivity were calculated from the currents and the temperatures measured by applying different AC voltages. We compared the experimentally obtained conductivity data with the theoretical prediction. The energy efficiency was determined by calculating the enthalpy of the input heating power and the output power due to the temperature rise of the water. Applications of this electrode boiler and a heating system with a very high energy efficiency of nearly 100% at the initial stage was also proposed. This work can also be used for physics education, because its subjects are strongly related to undergraduate and graduate courses such as mechanics and electrodynamics.

I. INTRODUCTION

Heating boilers are widely known to the public. In most residence areas, small boilers are installed except for some areas that provide district heating and hot water. Boilers can be categorized into combustion and electric boilers while in its turn, electric ones are classified into resistance and electrode boilers. Most household boilers are gas boilers [1–4] because they are supplied with city gas in most areas. In areas where the city gas supply is limited, other types of boilers, such as electric boilers, oil boilers and boiler using late night electricity, are used. Moreover, household boilers are not safe when water is heated above the boiling point. Although simplified boilers are composed of burners and ports, actual boilers are equipped with a variety of devices to maximize thermal efficiency, and the shape also varies greatly

due to the need for additional equipment.

Various kinds of boilers, such as direct- and circulation-type boilers, have functions, depending on the purpose of use, supplying hot water or heat. In addition, due to efficiency and safety issues, the water heating boiler uses various methods to circulate the heated water to the motor instead of steam. Various fuels such as wood, coal, petroleum or natural gas are used as heat sources for combustion boilers, and combustion of solid fuels leads to serious ash-related issues, such as corrosion, erosion, fouling and slagging, which eventually cause the efficiency of the boiler to be reduced and the possibility of an unexpected shutdown [1,2]. Combustion has a damaging effect on Earth's environment and human health due to the emission of toxic gases, as well [3–5]. For example, coal combustion not only provides energy but also produces hazardous substances and their mixtures [6]: anthropogenic emissions (COx, NOx, SOx); ash and slag waste; fly ash with high contents of heavy metals (As, Cr, Ba, Sr, Zn, Pb, Mo, *etc.*); and toxic radionuclide-contaminated water. The hazardous sub-

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stances also have an adverse effect on human health [7]. Solar heat boilers [8–10] use heated water generated by using a mirror to collect light, and solar power is used through solar cell development. Electric heating boilers [11–13] use resistive or immersive heating elements.

The history and use of electric power in heating boilers is wider. The main part of the electric boiler is a large water tank with an inlet and an outlet. An array of electrical heating elements exists in a large tank. The electric boiler can either use an immersion heater resistance heating element to heat the water or use an electric current through the water directly as the electrode boiler [11–13]. Two common categories of electric boilers [11–13] are hot water and steam boilers. Most small commercial buildings, manufacturers and some food processors use hot water boilers. Water is usually heated to the appropriate dispensing temperature of ∼50 °C and is usually returned at room temperature for reheating. These systems are often closed without fresh water replenishment. Where possible, hot water boilers are generally preferred because they do not require operators or special water chemistry. Because hot water boilers operate at low temperatures, they can operate at higher fuel conversion efficiencies than steam boilers.

In popular electric boilers, heat is usually generated by electrical resistance using the heat of the line. Electrode boilers [14–16] are very different in characters compared to other boilers and are used for special purposes. The advantages of the electrode boiler are clean combustion (no combustion product discharge or no stacking), high efficiency with minimal losses, fast response time, small size (volume and area are less than those of fossil fuel boiler), availability in various sizes, and easiness to control and maintain. They are typically used in industrial applications that requires high heat output and fast recovery. The conventional electric boiler relies on immersion heater resistance heating elements to heat the water. The heater generates the Joule heat due to the friction between the material and electrons. This heating system always experiences energy dissipation, such as overheating of the heater itself, additional energy consumption, or energy loss through the boiler barrier to the cold environment. On the other hand, an electrode boiler converts kinetic energy into heat energy due to the friction caused by collisions between ions and fluid molecules as ions move in the fluid.

Electric resistance boilers typically use resistive heating elements to convert electrical energy into thermal energy. The resulting heat is transferred to water in order to reach the desired temperature. The thermostat controls the flow of current to control the heating.

Conventional electrode boilers are connected to an AC source at medium voltage (1–35 kV). The electrode boiler can operate on both single-phase and three-phase electricity. Using DC voltage electrolysis of water causes water to decompose at the cathode to H_2 and the anode to O_2. Electrode boilers consume almost all the energy to produce steam with an efficiency of 99.9%. Energy
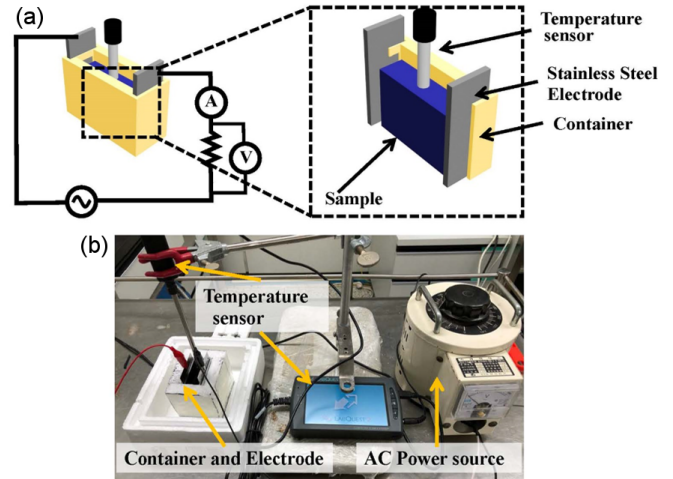

Fig. 1. (Color online) (a) Schematic diagram of the circuit and device and (b) Photograph of the experimental setup.

loss is due the radiant heat and occurs only in the vessel. Because pure water is very low in conductivity, water-containing conductive materials such as salts are supplied for efficient boiler operation. The efficiency and the rate of temperature increase are determined by the conductivity of the water and the applied voltage.

In this research, we investigated the theoretical background for the electrode boiler by using a polymethyl methacrylate (PMMA) sample cell in which two stainless steel plates are fixed 2 cm apart, with different salt solutions of varying concentrations in between. We examined how ionic friction in aqueous solutions of salts, such as NaCl and sodium trisphosphate (STP), is related to the temperature increase at different concentrations. For comparison, tap water, mineral water, sea water and distilled water were investigated as well. Data are presented for the current, power and temperature as functions of time, and for the conductivity and energy efficiency as functions of time and temperature. We also compare the experimentally obtained conductivity with the theoretical value. The energy efficiency of the heating mechanism based on ion friction and drag theory for various salt-water systems was determined. We showed that we can apply an electrode boiler using household electricity to a real system, instead of a conventional electrode boiler using an AC source at relatively high voltage (1–35 kV). This work can also be used as a physical educational aid because its contents are governed by the subjects of undergraduate and graduate courses, such as mechanics and electrodynamics.

II. EXPERIMENTS

The solutions were prepared by dissolving STP (weighed by using an electronic balance) in 400 ml of distilled water to have concentrations of 0.025, 0.05, and

0.075 wt%. Also we prepared water with NaCl dissolved at 0.025 to 0.2 wt%, tap water, distilled water, sea water and samdasu (mineral water). The samples were then stirred for 1 hour with a magnetic stirrer with the speed of 70 rpm.

Figure 1(a) shows the experiment setup and circuits, and Fig. 1(b) presents a picture of the setup picture with the its components labelled. The device consists of a PMMA polymer box containing two stainless steel electrodes at a 2-cm separation. Before the experiment, the PMMA box was placed in a few-centimeter thick styrofoam box to reduce heat loss to the environment. Forty ml of the solution was placed in the PMMA box, and the temperature sensor was installed in it. In this case, the dimension of the salted water sample was $2 \times 4 \times 5$ cm^3. For applying the voltage, we used a device with a slidax and household electricity. A 10-Ohm resistor was connected in series to the sample cell for safety. The amplitude of the applied voltage was measured using an oscilloscope. Data on the variation in the AC current with time every 0.5 seconds was collected using a Keysight 34465A digital multimeter for 1 hour as a standard in the sample.

III. THEORETICAL BACKGROUND

When a particle moves through a fluid, a drag force is generated. In a viscid fluid, the drag may generate heat due to the friction between the fluid and the surface of the object. In fluids with low viscosity, the energized fluid, rather than the object is slowly heated due to the friction. Ultimately, as in the first law of thermodynamics, all energy transferred from a dragged body to a fluid is changed into heat. In the case of low viscosity, the drag force is often described in the form of the Stokes equation as follows:

$$F = -b\nu, \qquad (1)$$

where b is a constant and ν is the velocity of the object undergoing the drag. This can be written as the Stokes drag force $F_{drag} = 6\pi\eta a\nu$, where η is the solution's viscosity and a is the Stokes or hydrated radius.

Similarly, ions in salted aqueous solutions experience a drag under an electric field. The considered force is a type of frictional force that can be described as a fluid resistance and is basically a force acting in a direction opposite to the relative motion of a moving object with respect to the surrounding fluid. The ultimate cause of the drag force is the viscous friction. The electric mobility of the solvated ions moving in a liquid at a constant drift velocity ν is determined by two equal and opposite forces, electric force ZeE and $F_{drag} = 6\pi\eta a\nu$. In a collisional ionic medium, $b = 6\pi\eta a \rightarrow mf$, where m is the mass of the object, and f is the characteristic frequency, which is the average ionic momentum transfer collision

frequency in this case. Then the Newton equation can be written as

$$m_i \frac{d\nu}{dt} = F - m_i f\nu, \qquad (2)$$

where F is the applied force.

We apply this situation to the ionic motion of the fluid under an alternating electric field. Many textbooks discuss the energy transfer from an oscillating field E to ions through collisions. Let us consider an ionic liquid coupled to neutral water through collisions. Suppose that the thermal motion of the electrons is negligible compared to the motion resulting from the electromagnetic field E ($\nu_{th} \gg \nu_i$, cold plasma approximation). Then, the corresponding hydrodynamic equations for momentum transfer can be written as

$$m_i \frac{d\nu}{dt} = qE_0 e^{i\omega t} - m_i f\nu, \qquad (3)$$

where ν is the (macroscopic) velocity of ions, and f is the average ion-neutron momentum transfer collision frequency. Here, the value of $m_i f$ corresponds to the friction coefficient, b, in the previous case.

In the cold plasma approximation, the ion velocity is purely periodic, such as $\nu = \nu_0 e^{i\omega t}$, and substituting ν into the hydrodynamic equation above gives

$$\nu = \frac{qE(t)}{m_i(f + i\omega)}. \qquad (4)$$

The movement of the charged particles in the field E creates a current. For a charge density $n_i q$, the current density can be written as $J = n_i q\nu$. In complex notation,

$$J = \frac{n_i q^2}{m_i} \frac{E}{(f + i\omega)}. \qquad (5)$$

Because $J = \sigma E$, where σ is the (scalar) electrical conductivity of ions, we find

$$\sigma = \frac{n_i q^2}{m_i} \frac{1}{(f + i\omega)}. \qquad (6)$$

For $f/\omega \gg 1$ (low-frequency discharge approximation), we obtain

$$\sigma = \frac{n_i q^2}{m_i f}. \qquad (7)$$

Using $b = 6\pi\eta a \rightarrow mf$, we obtain

$$\sigma = \frac{n_i q^2}{m_i f} = \frac{n_i q^2}{6\pi\eta a}. \qquad (8)$$

A salted water solution such as a NaCl-water solution, has are more than one ion that can contribute to the conductivity. Then, the ionic conductivity can be written as

$$\sigma_{tot} = \sigma_1 + \sigma_2 = \frac{n_1 q^2}{m_1 f} + \frac{n_2 q^2}{m_2 f} = n_1 \lambda_1 + n_2 \lambda_2, \qquad (9)$$
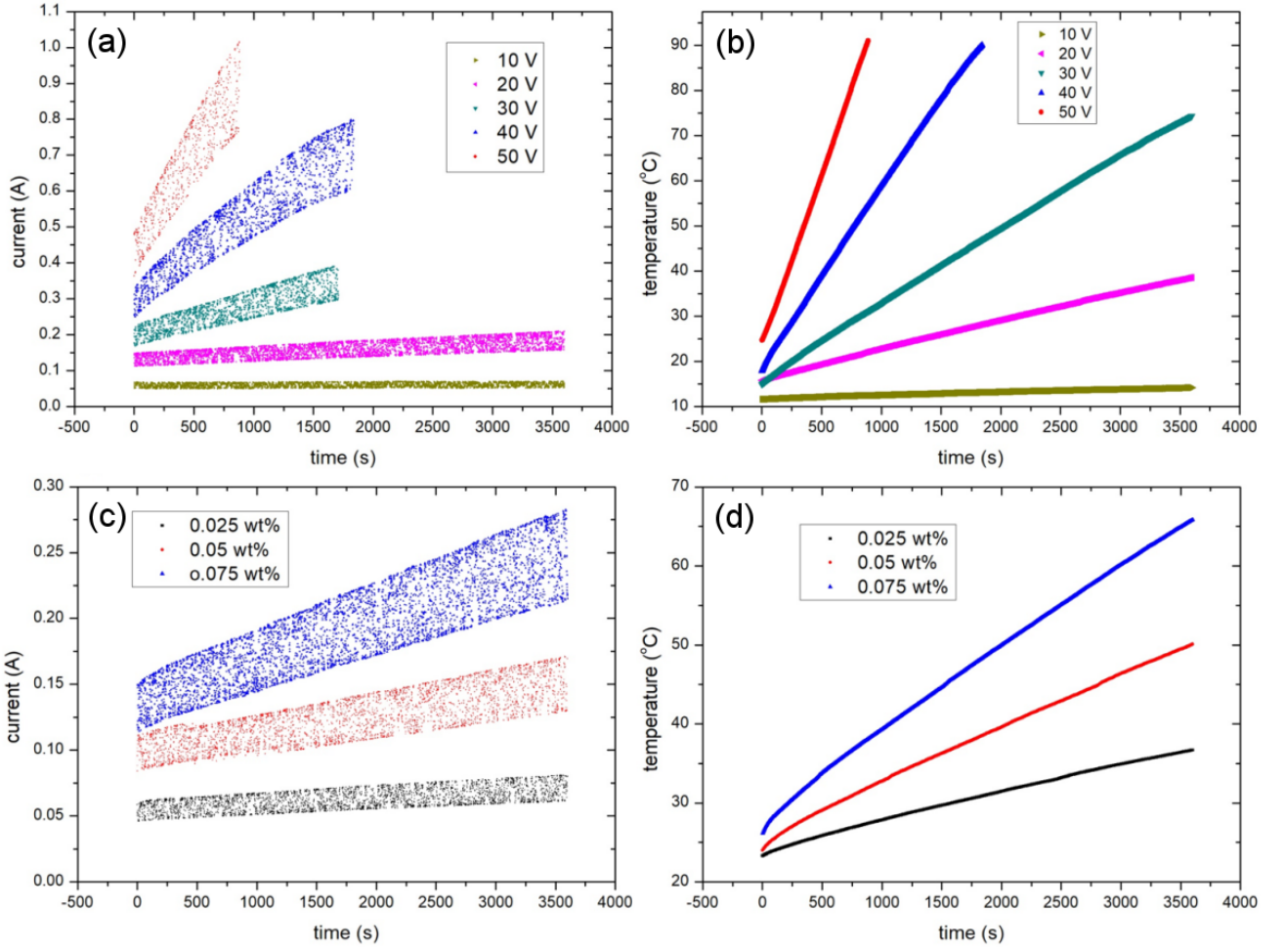
Fig. 2. (Color online) (a) Measured AC current versus time for a 0.1 wt% solution of NaCl for 10, 20, 30, 40, and 50 V. (b) Temperature measurements performed simultaneously with the current measurement for a 0.1 wt% solution of NaCl every 0.5 s for 10, 20, 30, 40, and 50 V. (c) Measured amplitude of AC current as functions of time for 0.025, 0.05, and 0.075 wt% solutions of sodium triphosphate at 30 V, and (d) corresponding temperature measurements made at the same time as the current measurements every 0.5 s for 0.025, 0.05, and 0.075 wt% sodium triphosphate solutions at 30 V.

where λ_i is the conductivity of ion i.

The average HF power absorbed per ion is given by the work per unit time and per electron in the field E : $qE \cdot \nu$ which represents the instantaneous power taken in the field. The average power, the power taken from the electric field over a period of time, per ion is

$$P \equiv \mathrm{Re}\left(\frac{qE \cdot \nu^*}{2}\right) = \mathrm{Re}\left(\frac{q^2 E^2}{2m_i}\frac{1}{f - i\omega}\right)$$
$$= \frac{q^2}{m_i}\frac{f}{f^2 + \omega^2}\overline{E^2}, \qquad (10)$$

where $\sqrt{\overline{E^2}} = E_0/\sqrt{2}$ is the mean square value of the electric field. For $f/\omega \gg 1$ (low-frequency discharge approximation), we get

$$P \approx \frac{q^2}{m_i}\frac{\overline{E^2}}{f}. \qquad (11)$$

From Eq. (8) and $\sigma = 1/\rho$, where ρ is the resistivity, the average power can be written as

$$P \approx \frac{q^2}{m_i}\frac{\overline{E^2}}{f} = \frac{q^2\overline{E^2}}{n_i q^2 \rho} = \frac{\overline{E^2}}{n_i \rho}. \qquad (12)$$

For s system of two parallel-plate electrodes, the voltage is given by $E = V/L$ and the resistance is given by $R = L/\sigma A = \rho L/A$, with L being the distance between the two electrodes and A being the area of the electrode. Then, the average power per ion, in terms of applied voltage V and resistance R, is as follows:

$$P = \frac{\overline{V^2}}{R}, \qquad (13)$$

where $\sqrt{\overline{V^2}} = V_0/\sqrt{2}$.

In this study, we assume that the heat is generated due to the translational motion of the charged ions in salt solutions with respect to the applied electric field only,

while the contribution of the rotational motion of the dipolar water molecules to heat generation is neglected.

IV. RESULTS AND DISCUSSION

AC current measurements were performed every 0.5 second for 1 hour. When the temperature reached 90 °C, the power supply was cut off for safety reasons. Also, when the smell of burning was emanating from the resistance, which usually happened at higher concentrations due to the high ion concentration, the power supply was turned off.

Figure 2(a) shows the measured current versus time for a water solution with 0.1 wt% of NaCl at 10, 20, 30, 40, and 50 V. The higher the voltage, the larger current generated is. For 50 V, the temperature reaches 90 °C quickly, so the current measurement was completed in less than 1 hour and the power supply was turned off due to safety reasons. As shown in the figure, the amplitude varies widely, even though a smoothly changing value is expected. We attribute that this result to numerous scattering that occur between different ions, because the salted-water sample has many different ions. Figure 2(b) shows the temperature measurement performed every 0.5 seconds simultaneously with the current measurement. Clearly, a positive correlation between the current and temperature. Therefore, the higher the voltage, the faster the temperature rises. Figure 2(c) shows the measured current versus time for water solutions with 0.025, 0.05, and 0.075 wt% sodium triphosphate at 30 V. The higher the concentration is, the larger current that is generated. The reason for this is as follows: For a more highly concentrated solution, more species are available to collide and produce the higher current. Figure 2(d) shows the corresponding temperature measurements taken every 0.5 seconds at the same time as the current measurements. As in Fig. 2(b), clearly, a positive correlation exists between the current and temperature: *i.e.* the higher the concentration is, the faster the temperature rises.

Because the resistance of the sample is known, $1/R = I_0/V_0$, where I_0 and V_0 are the amplitude of current and voltage in the sample, respectively, the conductivity of the sample can be calculated by inserting physical dimensions into the formula $\sigma = L/(AR)$, where L is the length of the box, A is the surface area of the electrode plate and R is the resistance of the sample. Figure 3 shows the calculated conductivity as a function of temperature at various concentrations and applied voltages for the water solutions of NaCl and sodium triphosphate. From Fig. 3(a), the conductivities for solutions of the same concentration are obviously are identical, and no voltage dependence is observed. This is reasonable because solutions with the same concentration, have approximately the same number of conductive species. Moreover, for the highest concentration of NaCl (0.8 gram per 400 ml,
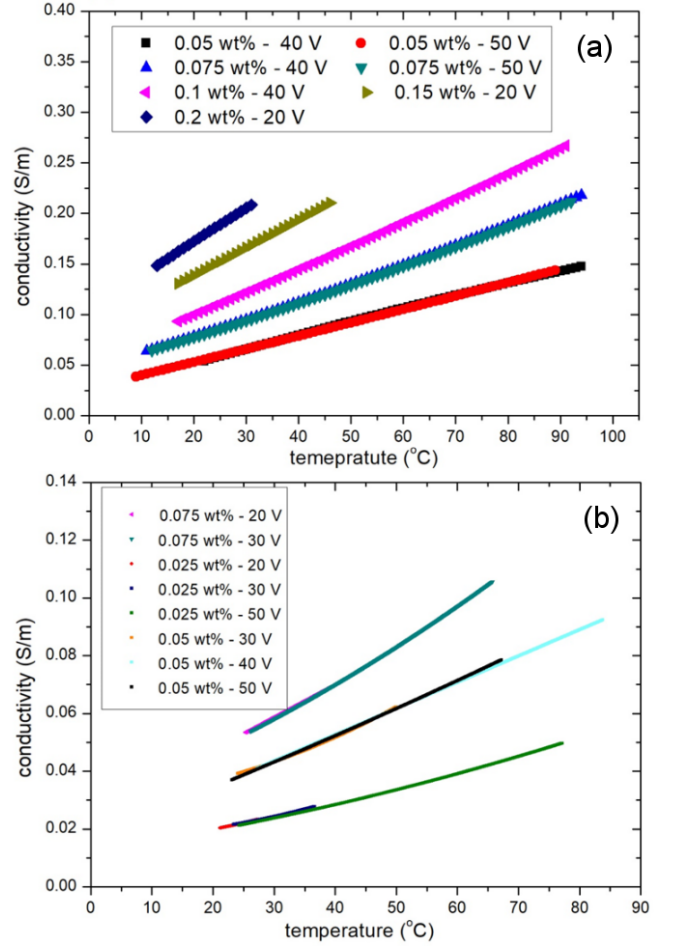

Fig. 3. (Color online) Calculated conductivities of (a) NaCl and (b) sodium triphosphate as functions of temperature for various concentrations and applied voltages.

0.2 wt%), the conductivity is clearly the largest, which is quite reasonable. The same is true for the conductivity of the STP solutions as depicted in the Fig. 3(b).

Now, we can compare the experimental value of the conductivity to the theoretical value given by Eq. (8). Let us calculate the conductivity of the 0.025-wt% sodium triphosphate ($Na_5P_3O_{10}$) water solution at 30 °C. Firstly, because the molecular weight of STP is 367.864 g with 5 sodium ions per molecule, the density of sodium ions can be calculated as follows:

$$
\begin{aligned}
n \ = \ & 0.025 \cdot 10^{-2} \frac{\text{g}}{\text{cm}^3} \times \frac{5 \text{ charges}}{\text{molecule}} \\
& \times \frac{6.023 \cdot 10^{23} \text{ molecules mole}^{-1}}{367.864 \text{ g mole}^{-1}} \\
= \ & 2.05 \cdot 10^{18} \text{ charges cm}^{-3}.
\end{aligned}
$$

Therefore, for this condition, the parameters are given

as follows:

Viscosity of water at 30 °C [17]
$\eta = 0.8 \times 10^{-3}$ kgm^{-1}s^{-1}
Hydrated radius of the sodium ion [18]
$a = 1.78 \times 10^{-10} \sim 3.60 \times 10^{-10}$ m
Electric charge of the sodium ion
$q = 1.6 \times 10^{-19}$ C
Density of sodium ions
$n = 2.05 \times 10^{24}$ m^{-3}

Then, if these parameters are inserted into Eq. (8), the theoretical conductivity of Na ions is given by

$$
\begin{aligned}
\sigma_{\text{Na}} &= \frac{n_i q^2}{6\pi \eta a} \\
&= \frac{(2.05 \times 10^{24} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2}{6\pi(0.8 \times 10^{-3} \text{ Kgm}^{-1}\text{s}^{-1})(2.7 \times 10^{-10} \text{ m})} \\
&= 1.3 \times 10^{-2} \text{ S} \cdot \text{m}^{-1}.
\end{aligned}
$$

In this calculation, we use the average values of the reported hydrated radius of Na ions.

Next, because the number of triphosphate ions per molecule is one, the number density is given by 0.41×10^{24} m^{-3}, and the electric charge of the triphosphate is $5 \times 1.6 \times 10^{-19}$ C. When the same calculation is repeated for the triphosphate ions, we have

$$
\begin{aligned}
\sigma_{\text{PO}} &= \frac{n_i q^2}{6\pi \eta a} \\
&= \frac{(0.41 \times 10^{24} \text{ m}^{-3})(5 \times 1.6 \times 10^{-19} \text{ C})^2}{6\pi(0.8 \times 10^{-3} \text{ Kgm}^{-1}\text{s}^{-1})(2.7 \times 10^{-9} \text{ m})} \\
&= 6.5 \times 10^{-3} \text{ S} \cdot \text{m}^{-1} .
\end{aligned}
$$

In this calculation, we use approximated values of the reported ionic radius of P_3O_{10} ions. Therefore, the theoretical value of the conductivity for the 0.025-wt% STP water solution is given as $\sigma = \sigma_{Na} + \sigma_{PO} \approx 0.02$ S/m. This value is very comparable to the experimental value of 0.0245 S/m. The discrepancy between two the values results from our choices for the hydrated radii of Na and P_3O_{10} ions.

Again, similarly, let us repeat the calculation for the conductivity of the 0.05-wt% NaCl water solution at 20 °C. Firstly, because the molecular weight of NaCl is 58.45 g with 1 sodium and chloride ions per molecule, the density of sodium and chloride ions can be calculated as follows:

$$
\begin{aligned}
n &= 0.05 \cdot 10^{-2} \frac{\text{g}}{\text{cm}^3} \times \frac{1 \text{ charges}}{\text{molecule}} \\
&\quad \times \frac{6.023 \cdot 10^{23} \text{ molecules moles}^{-1}}{58.45 \text{ g mole}^{-1}} \\
&= 5.15 \cdot 10^{18} \text{ charges cm}^{-3}.
\end{aligned}
$$

For this condition, the parameters are given as follows:

Viscosity of water at 30 °C [17]
$\eta = 10^{-3}$ kg m^{-1} s^{-1}
Density of sodium ions and density of chloride ions
$n = 5.15 \times 10^{24}$ m^{-3}
Hydrated radius of a sodium ion [18]
$a = 1.78 \times 10^{-10} \sim 3.60 \times 10^{-10}$ m
Hydrated radius of a chloride ion [18]
$a = 3.24 \times 10^{-10} \sim 3.32 \times 10^{-10}$ m
Electric charge of sodium and chloride ions
$q = 1.6 \times 10^{-19}$ C

Then, if these parameters are inserted into Eq. (8), the theoretical conductivities of Na and Cl ions are given by

$$
\begin{aligned}
\sigma_{\text{Na}} &= \frac{n_i q^2}{6\pi \eta a} \\
&= \frac{(5.15 \times 10^{24} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2}{6\pi(1 \times 10^{-3} \text{ Kgm}^{-1}\text{s}^{-1})(2.7 \times 10^{-10} \text{ m})} \\
&= 2.6 \times 10^{-2} \text{ Sm}^{-1}. \\
\sigma_{\text{Cl}} &= \frac{n_i q^2}{6\pi \eta a} \\
&= \frac{(5.15 \times 10^{24} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2}{6\pi(1 \times 10^{-3} \text{ Kgm}^{-1}\text{s}^{-1})(3.3 \times 10^{-10} \text{ m})} \\
&= 2.1 \times 10^{-2} \text{ Sm}^{-1}.
\end{aligned}
$$

In this calculation, we use the average values of reported hydrated radii of Na and Cl ions. Therefore, the theoretical conductivity of the 0.05-wt% NaCl water solution is given as $\sigma = \sigma_{Na} + \sigma_{Cl} = 0.047$ Sm^{-1}. This value is somewhat smaller than the experimental value of 0.0528 S/m. The discrepancy between the two values again results from our choice for the hydrated radii of Na and Cl ions. We can also estimate the average value of the hydrated radii of Na and Cl ions from our experimental data by rewriting Eq. (8) and using Eq. (9) as follows:

$$
\begin{aligned}
a &= \frac{n_i q^2}{6\pi \eta \sigma} \\
&= \frac{(2 \times 5.15 \times 10^{24} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})^2}{6\pi(1 \times 10^{-3} \text{ Kgm}^{-1}\text{s}^{-1})(5.28 \times 10^{-2} \text{ S/m})} \\
&= 2.7 \times 10^{-10} \text{ m}.
\end{aligned}
$$

This value is very comparable to the reported value [18].

The AC power output of the sample was calculated. To do this, the voltage across the 10-Ohm resistor was subtracted from the total applied voltage, and calculations were done using the Ohm's Law: first the voltages across 10-Ohm resistor and across the sample were found; then, the resistance of the sample was found, and the power of the sample was calculated by using $P = I_0 V_0/2$. Figure 4(a) shows the calculated powers for different waters such as seawater, tap water and water doped with NaCl and sodium triphosphate. Figure 4(b) shows the conductivities of various water samples as functions of
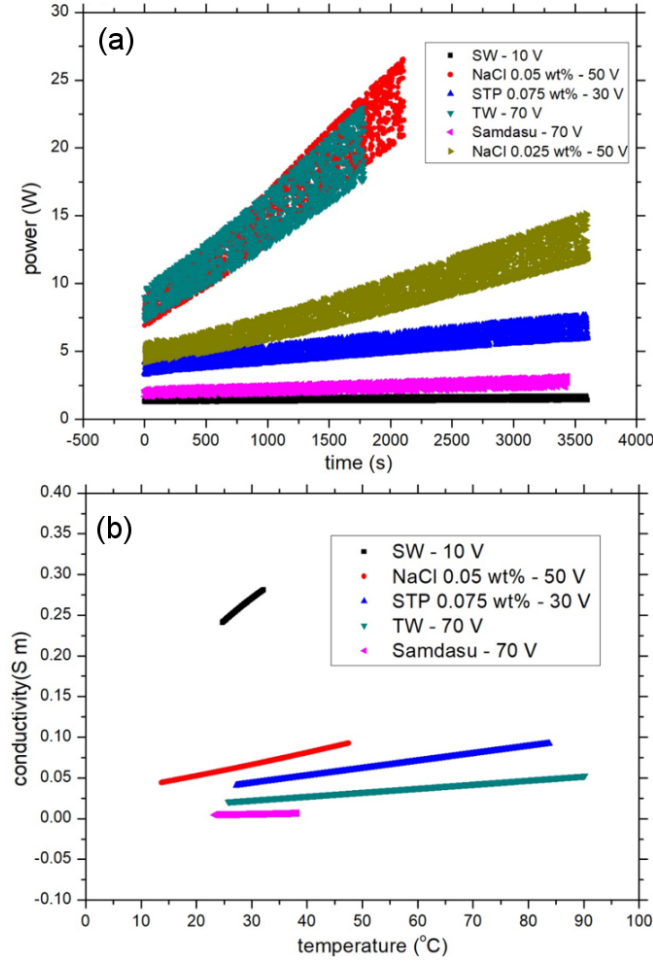
Fig. 4. (Color online) (a) Calculated power as functions of time and (b) calculated conductivities as functions of temperature for seawater, tap water, mineral water (samdasu) and NaCl and sodium triphosphate water solutions.
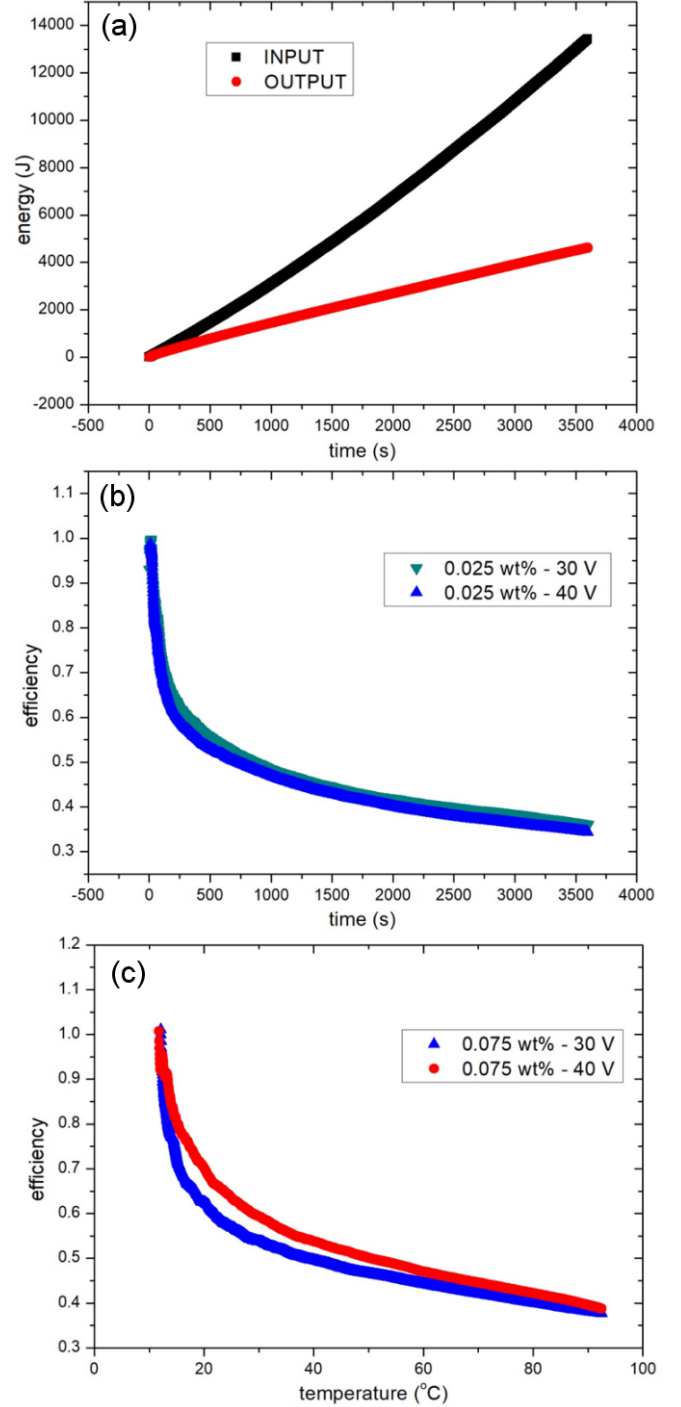

Fig. 5. (Color online) (a) Input and output energies as functions of time for a 0.025wt% sodium triphosphate water solution.at applied voltage of 30 V. (b) Energy efficiencies of a 0.025 wt% sodium triphosphate water solution at 30 and 40 V as functions of time and (c) energy efficiencies of a 0.075 wt% NaCl water solution at 30 and 40 V as functions of temperature.

temperature. Even in the case of tap water, the conductivity is high enough to increase the temperature. Figure 4(a) shows that sea water produces the smallest power amongst all solutions. This is due its being subjected to the smallest applied voltage of 10V, even though a large number of ions that may allow a large current are present. The next higher power was found to be for mineral water measured at 70 V, followed by the 0.075-wt% STP-water solution at 30 V, then the 0.025-wt% NaCl water solution at 50 V, and tap water at 70 V. The highest power was generated by the 0.05-wt% NaCl water solution at 50 V. The result shows, of course, that the power depends on the applied voltage as well as the ion concentration (or current). Figure 4(b) shows the calculated conductivities as functions of temperature for the same solutions for which the power was calculated and shown in Fig. 4(a). Clearly, the conductivity of sea water is the highest; therefore, for sea waters the temperature can be increased very easily with the least applied voltage. The mineral water turned out to have the least

conductivity of all the sample solutions. A higher conductivity than that of mineral water was a property of

tap water which is also reasonable because the mineral water is for drinking; therefore, it is of a higher purity than tap water. For the higher purity water, the concentration of ions that will collide under an AC filed is less; therefore, the conductivity is less, and the generated current will be less. Moreover, the figure, shows that the conductivity of the 0.075-wt% STP water solution is less than that of the 0.05-wt% NaCl solution.

Finally, the efficiency of the system is considered. The first law of thermodynamics states $E_{int} = W + Q$. In other words, any work done by (or on) an object results in changing its internal energy (temperature). You need to have a well-insulated object (when $Q = 0$) and you'll definitely know that work will result in a temperature change. However, in a real system, heat loss due to the temperature difference between the boiler and atmosphere always occurs.

The efficiency, η, of the system is given by the ratio of the energy output to the energy input. The input energy is given as an integral of the power graph with time from initial time to a certain time, $\int P dt$, where P is the input power from the applied voltage, $P = \frac{V^2}{R}$. The output energy is the heat of the sample, that is given as the integral of $M c_p$ over initial temperature to the corresponding temperature for a certain time, $\int M c_p dT$, where M is the mass of the water sample and c_p is its specific heat capacity. Here, the contribution of the salted water solution was only considered in the output energy calculation because we focused on the heating efficiency of the water.

Figure 5(a) shows the input and the output energies as functions of time for the STP water solution of 0.025 wt% at an applied voltage of 30 V. Figs. 5(b) and (c) show the energy efficiencies of the sodium phosphate and the NaCl water solutions as functions of time and temperature, respectively. As seen from the Fig. 5(a), the value of the input energy is higher than the value of the output energy which is fair because some of the energy is lost to the electrode, sample cell, and thermometer as only the water solution was considered when the output energy was calculated. Moreover, the discrepancies between the input and the output values becomes bigger with increasing time because the temperature of the sample rises with time, which means that the temperature difference between the environment and sample becomes larger and creates a stronger gradient for heat to be lost to the environment. In the Figs. 5(b) and 5(c) the energy efficiencies of the STP and the NaCl water solutions as functions of time and temperature are depicted, respectively. The efficiency is almost 100% at an early stage because the temperature difference between water and the environment is small, so input power provides heat to the water as a whole. As the time or the corresponding temperature increases, the energy loss to the electrode, sample cell, thermometer and the environment increases significantly because the sample cell is not isolated. We believe that a higher efficiency can be observed when a larger amount of water solution is used with higher thermal insulation against environment and geometry reduced the effects of the electrode, thermometer and sample cell is used.

V. CONCLUSION

In conclusion, we measured the current and the corresponding temperature of various water solutions including sodium triphosphate and NaCl as a function of time. The temperature of the sample rose with respect to the voltage applied due to ionic friction or drag. We compared the experimentally obtained conductivity with the theoretical value, and found two values to be relatively consistent with each other. We were also able to determine the hydrated radius of the ions. This system enables us to investigate the effectiveness of these heating mechanisms and it can be applied to boiling and heating systems with low power consumption. If the efficiency is to be increased, reducing the size, material, and heat loss of boiler containers is important. These are things to consider regardless of the type of boiler. In this paper, we discussed the maximization of the efficiency of basic heat production. An important advantage of the electrode boiler is that the water has the highest temperature among the components of boiler; therefore, the energy efficiency can be maximized when an adiabatic boundary is set up and when the effects of the electrode, the boiler's wall material and thermometer are reduced. A possible next experiment will be to check the energy efficiency when the boiler is controlled at a desired temperature for a state nearly in equilibrium. This work can also be used as a physics educational aid because its subject matters are included in undergraduate and graduate courses such as mechanics and electrodynamics.

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