Given $\alpha_p = 3\,dB$; $\omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/s

$\alpha_s = 10\,dB$; $\omega_s = 2 \times \pi \times 350 = 700\pi$ rad/s

$$T = 1/f = \frac{1}{5000} = 2 \times 10^{-4} \text{ sec}$$

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{2}{2 \times 10^{-4}} \tan\left(\frac{2000\pi \times 2 \times 10^{-4}}{2}\right)$$

$$= 7265 \text{ rad/s}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2} = 10^4 \tan(0.07\pi) = 2235 \text{ rad/s}$$

Order of filter

$$N = \frac{\log \sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \frac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log \sqrt{9}}{\log 3.25}$$

$$= 0.932$$

$\therefore N = 1.$

The 1^{st} order butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

$$\Omega_c = \Omega_p = 7265 \text{ rad/s}$$

Transfer function →

$$H(s) = \left.\frac{1}{s+1}\right|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{s}{s+7265}\right|_{s = \frac{2}{10^{-4} \times 2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.5792\,(1-z^{-1})}{1-0.1584\,z^{-1}}$$