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Applications of the Direct Method in Sequential Analysis

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This paper treats the application of the direct method in sequential analysis. The theory, with new results, is reviewed. This includes sequential estimation after a sequential test. Next, general applications are given. Finally, a selected list of papers in sequential life testing using the direct method is reviewed. Actually, sequential life tests preceded the direct method, but life testing is easily treated as a special case of the direct method. Applications in maintainability theory, in medical trials, and quality control are covered.

KEY WORDS
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1. THEORY OF DIRECT METHOD

The direct method in sequential analysis includes the Wald theory, curtailed sampling, and fixed size sample tests as special cases. It is exact, unrestricted by the shape or dimensionality of the test region, has no difficulty with truncation or with tests in time, applies to composite as well as simple hypotheses, and may be followed by estimation and the determination of confidence limits for the parameter under test. The direct method is a step by step procedure providing exact results at each step.

What are the elements of the direct method? First a statistic T must be given (a statistic is a function of sample values), and preferably a sufficient statistic. A test region consisting of an acceptance region, a continuation test region, and a rejection region must be determined by Wald theory, or by some optimization procedure. A hypothesis must be stated for testing H_0 versus an alternative H_1. Now to test the hypothesis H_0, we proceed by taking an item from the population under test; if the statistic T has a value placing it in the acceptance region, we accept H_0, if it falls in the continuation region we take another item from the population, and if it falls in the rejection region, we accept H_1. Now the region is chosen in such a manner that if H_0 is true, the probability that the test accepts H_0 is $1 - \alpha$, and that it rejects H_0 is α; and if H_1 is true, the probability that the test accepts H_0 is β, and that it accepts H_1 is $1 - \beta$, $\alpha + \beta \leq 1$. The sequential test may be characterized by the DSN, decisive sample number, the OC, the operating characteristic function (which is defined as the probability of accepting H_0), the ASN, the average sample number or the mean of the DSN distribution, the MSN, the median of the DSN distribution, and m_0, the trial at which the sequential test is terminated.

At each trial the probability of acceptance H_0, $P(A_i)$, the probability of continuation of the test, $P(C_i)$, and the probability of rejection of H_0, $P(R_i)$, is determined. Clearly,

$$P(A_i) + P(C_i) + P(R_i) = P(C_{i-1}),$$

$$i = 1, 2, 3, \cdots m_0 \text{ , with } P(C_0) = 1$$

The DSN density is given by

$$(i, P(A_i) + P(R_i)) \quad \text{and} \quad \text{ASN} = \sum_{i=1}^{m_0} i\{P(A_i) + P(R_i)\},$$

and it follows (Aroian [7])

$$\text{ASN} = \sum_{i=1}^{m_0-1} P(C_i) - m_0 P(C_{m_0}), \quad \text{and for } m_0 \text{ finite}$$

$$\text{ASN} = 1 + P(C_1) + P(C_2) + \cdots + P(C_{m_0-1}) \leq m_0 \qquad (1.1)$$

This enlightening result shows exactly how the ASN is built up at each step. If m_0 is chosen as the fixed size sample number N with a presumed α and β,

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then the ASN will always be less than N, though the actual α and β may be somewhat larger than the presumed α and β. Moreover, it strongly indicates there is very little point in going much past $1.2N$ for m_0, and for most problems m_0 should first be chosen as N. If the resulting OC is not satisfactory, m_0 may be increased. Now Wald and Wolfowitz (Ghosh [30], showed that of all tests of a simple hypothesis H_0 versus H_1, then the Wald SPRT, sequential probability ratio test, has a minimum ASN at H_0 and H_1 for specified (α, β). However, other criteria such as a min max ASN over all values of H would be preferable even at the loss of a minimum ASN at H_0 and H_1. The direct method has been generalized to cover composite as well as simple hypotheses and for more than two decisions. The Wald theory proceeds by approximations and provides very general approximate formulas for simple hypotheses. But such generality means that many specific and exact results are unattainable, and for composite hypotheses, it leaves one almost entirely in the dark. Briefly then, the elements of the direct method are: the Markovian character of the sequential test from trial i to $i + 1$, the random walk, and the definition of the DSN, OC, and the ASN. D. H. Bhate in his thesis [18] and later in [19], [20], [21] has shown how the DSN distribution may be approximated. Some of his ideas are related to the direct method.

2. Applications

Aroian [5] applied the direct method to tests of the binomial distribution $p = p_0$ versus $p = p_1$, a two-decision test. A one-decision test may be constructed for rejection of $p = p_1$. Cumulative sum charts may be evaluated exactly by the direct method for the binomial using either the two-decision Wald regions or V-shaped three-decision regions, with either right-angle or wedge-type truncations. D. Öksoy [45] applied the direct method to find the exact OC and ASN for the Method One maintainability demonstration test which I understand is no longer recommended. This is a nonparametric combined test of the mean and the variance using the binomial twice. Goss [33] has studied in depth the use of the median as a nonparametric test for maintainability demonstration. He gives various plans and computer programs, shows how to truncate, how to estimate p after the sequential test, and how to find confidence limits. This is made possible since the direct method finds the probability of continuation of the test at each step i. This same method may be used for a nonparametric sequential test for any quantile. Three-decision sequential tests have been constructed for testing $p = p_0$ versus $p_1 > p_0$ or $p_2 < p_0$ with exact DSN, OC, and ASN in my classes at Union. For further results on the binomial, see Aroian [6].

Meeker [39] in his thesis has completed exact sequential tests for the hypergeometric distribution with either two or three decisions [40], and has also determined for the first time sequential tests for the two-by-two contingency tables [41]. The hypergeometric may be used for acceptance sampling from finite size lots without assumption of the binomial as an approximation to the hypergeometric. The contingency sequential test has very broad applicability to problems of independence and association, in sociology, education, industry, medical trials and health delivery systems.

Aroian [5] and Aroian and Robison [11] showed how to test $\mu = \mu_0$ versus $\mu = \mu_1$, the mean in the normal distribution with variance known. Goss [35], [37] extended this to three decisions, $\mu = \mu_0$, $\mu = \mu_1 > \mu_0$, $\mu = \mu_2 < \mu_0$. These tests may be used for cumulative sum charts as well as for maintainability tests. In addition, rewards and penalties may be given: rewards for achieving $\mu_1 > \mu_0$, $\mu = \mu_0$, standard no rewards, and penalties for $\mu_2 < \mu_0$. Monahan [44] has considered a Bayesian approach to the problem of testing $\mu = \mu_0$ versus $\mu = \mu_1$ for the normal, the variance known, considering the prior normal distribution, the costs of sampling, and the cost of incorrect decisions.

Goss [36] and Schmee [50] have originated a direct method approach to estimation and confidence limits after a sequential test of any kind. Schmee [49] has solved exactly the sequential t test, for testing $\mu = \mu_0$ versus $\mu = \mu_1$, when the variance in the normal case is unknown. Aroian, Goss, and Schmee [14] have shown how the sequential test for the mean, variance known, and for the variance, mean known or unknown, may be used for maintainability demonstration tests, drastically reducing the test time. Schmee's results for the sequential t will provide results even if the variance is unknown.

Goss [34] has extended the work of Aroian, Gorge, and Robison [12] and has obtained exact truncated sequential tests for the variance of the normal distribution when the mean is known or unknown. His work includes the probability of rejection, acceptance, and continuation for a wide variety of cases, for which he also provides a computer program. Let H_0 be $\sigma = \sigma_0$ versus H_1, $\sigma = \sigma_1 = k\sigma_0$, $k > 1$. He lists results for $k = 1.5, 2$, for $\sigma_0 = 1$, with α and β varying from .025 through .10. These tests are used always for testing the reliability of high reliability gyroscopes as well as in maintainability.

Schultz and Elfring [51] have contributed greatly to new exact results. They treated the case of $p_1 = p_2$, already considered by Öksoy [46] but chose

items in groups of two or more from each of the two population under consideration. This is most useful in medical research where it is more economical to take observations in groups. They give the OC and the ASN. Of course, this is a fundamental problem in clinical trials testing a drug against a a placebo. Further, they treated the very difficult problem of closed group sequential rank sum tests for two samples [53]. They have also investigated the sequential sign rank test [52]. These nonparametric tests work in very general circumstances since only the form of the distribution is assumed. Öksoy [46] in his thesis considered the sequential test for the binomial $p_1 = p_2$. This problem occurs if one wants to know whether one industrial process is better than another, whether two drugs differ in medical efficacy, and in any case where one method is compared with another in any field whatever.

Ramesh Chandra in his thesis [26] has extended the theory of sequential analysis by explicitly showing how the OC and ASN are changed by changing the region boundaries. Frequently for the binomial it is important to test $H_0 : p = p_0$ versus $H_1 : p = p_1 > p_0$ and $H_2 : p = p_2 < p_0$. Chandra [22] shows how to do this sequentially, and how to discriminate among $k \geq 2$ binomial hypotheses. He also compares Anderson type triangular regions versus Wald truncated regions after minimizing the maximum ASN.

In a related set of problems Chandra [24] shows how to find exact sequential tests for complete ranking of k binomials using Bechhofer-Kiefer-Sobel procedures [16]. These problems arise in determining which of $k \geq 2$ drugs are most effective, which of k economic measures in a poll are considered best by the public, or which of k production methods is best.

Of great interest this year is the question asked by pollsters: which of k candidates has the largest support of the public. This sequential procedure is done exactly by Chandra [23] following the theory originated by Bechhofer and Sobel [17], Alam [1], and Paulson [40]. This problem is the selection of the most probable category in a multinomial population.

He has extended the methods of Goss [33] for sequential estimation and confidence limits from the binomial to the multinomial [25], basing his results on those of Good [31] and Good and Crook [32]. Chandra includes computer programs in his reports and thesis.

3. Life Testing

Exact sequential life tests depend on the Poisson process in time, $p_x = (e^{-\lambda t}(\lambda t)^x)/x!, x = 0, 1, 2, \cdots$ where p_x is the probability of exactly x successes in time t, and λ is the failure rate. The first approximate sequential life test was given by Epstein and Sobel [29]. The first paper on exact methods was by Houghton [38], followed by Aroian [2], and Epstein, Patterson and Qualls [28]. Of course MIL-STD 781-A and 781B [42] owe much to these. Apparently Bhate [19] has done work along similar lines.

The OC, the approximate ATT, the average time to terminate the test, the DST, decisive sample time density, and its distribution, the probability of acceptance, rejection, or continuation at any point in time are all readily determined. The exact ATT is a little harder [2] [28]. The MET, median time to decision, may be found by interpolation in the DST distribution. In Aroian's paper [2] exact Dvoretzky-Kiefer-Wolfowitz [27] regions were truncated and evaluated. These regions provide exact values of α and β. Epstein et al. [28] used Agree type regions truncated at the equivalent fixed size sample. Later Aroian et al [9] used Agree regions with different truncations.

Parenthetically, the Poisson discrete probability function

$$p_x = (e^{-\mu}\mu^x)/x!, x = 0, 1, 2, \cdots.$$

may be done exactly by the direct method for testing μ_0 versus μ_1 . In this case instead of time, the x axis will be trials and the y axis either successes or failures. The ASN is of course exact since it is not possible in the discrete Poisson process to have a value between two integer trials. Wald regions truncated at right angles should be used throughout. This has done in classes at Union College.

The analogue to (1.1) Aroian [7] for sequential time processes is $\int_0^{t_0} p(c, t)\, dt$, where $p(c, t)$ is the probability of continuation at time t. The result is ATT, the average time to terminate a test for continuous time processes.

It is now possible to estimate λ, the failure rate and to find confidence limits for λ after a sequential test. If we use the results of Goss [36] and Schmee [50], we find $f(\lambda, m)$ (normalized such that $\int_0^\infty f(\lambda, m)\, d\lambda = 1$), the probability of acceptance or rejection of H_0 along the line beyond each acceptance and rejection point, then the estimate of λ is the mean of $f(\lambda, m)$ and the confidence limits for λ are found by solving for λ, $F(\lambda, m) = \alpha$ and $F(\lambda, m) = 1 - \alpha$, assuming a constant λ where m is the number of failures. This is possible since $f(\lambda, m)$ may be found exactly at each decision point in a sequential life test. Of course, as β gets smaller, the confidence bands for λ become more narrow. For exact details, see Goss [33], [36] and Schmee [50].

Summerlin [54] exhibited the probability of arriving at any point in a sequential test plan, whether interior, exterior, rejection or acceptance

point. He showed how to find confidence limits for the failure rate, and also how to use Bayes prior distributions of the gamma type. Aroian and Öksoy [13] essentially follow the same procedure and further estimate λ as the mean of the posterior distribution. Summerlin has a complete analysis of MIL-STD 781 test plans I, III, IV, and the lower confidence limit with confidence $1 - \beta$ versus the method of calculation (assumption of prior distribution) for Test Plans I through IX. The newer method of Goss [36] and Schmee [50] extend the distribution over the whole line instead of stopping at a continuation or rejection point. However, the methods of Summerlin, and Aroian and Öksoy are adequate for most work. Summerlin's [46] paper should be required reading for anyone interested in sequential life tests.

Mogg [43] shows how to find the optimum number of units to place on test based upon a hypothetical cost model. The cost of testing is considered to be a composite of those costs which depend upon the number of units employed and of those depending upon the duration of the test facility participation. A cost model of these fixed and variable costs is illustrated for Test Plan I of 781B under five assumed levels of mean time between failures and for four unit cost-to-facility cost ratios.

When the failure rate is not constant, one may use the piecewise exponential distribution, or simply break up the hazard rate into small segments such that the hazard rate is essentially constant or use an upper or lower bound at each point. This is the idea in Aroian [2] and implemented in Aroian and Robison [10]. However, in their TRW Systems report [8], they included additionally life tests for the Weibull and normal distribution. This does take care of cases such as the Weibull distribution if the shape parameter is known, but not if it is unknown. In such a case, one could need to do an integration over the shape parameter at each step as well as the location parameter. Another method is to transform each distribution to an exponential [4]. Aroian and Öksoy [15] have used the fundamental Poisson process to obtain three-decision regions for testing $\lambda = \lambda_0$ versus $\lambda = \lambda_1$, $\lambda_1 > \lambda_0$ and $\lambda = \lambda_2$, $\lambda_2 < \lambda_0$. Barnard regions were put together there but more general Wald-Sobel regions could also have been used. Further, Aroian and Öksoy [13] showed how to estimate the failure rate, and confidence limits for the three-decision problem using the same procedure as Summerlin [46] in the case of the confidence limits. The more general results of Goss and Schmee, of course, are better theoretically for these purposes but considerably more time-consuming.

Aroian [3] warns against the error of using fixed size sample limits at each step instead of the proper sequential region.

4. Future Prospects

Three-decision problems will be solved for the sequential t test, for the difference in two means when the variances are known but different and the corresponding case when the variances are unknown but different. Exact analysis of variance, multivariate, nonparametric, and multinomial sequential tests appear to be possible. These would involve problems in goodness of fit, correlation, regression, and the analysis of variance. The application of sequential methods to problems in market survey, public and management surveys, polls of public opinion have been investigated by Ramesh Chandra. Schafer and Takenaga [48] have formulated a sequential test to which the direct method may be applied to obtain exact results. Other applications are listed in Aroian [7]. Certainly the way is clear for many applications of the direct method in sequential analysis, a catalyst for exact results. It should be in the arsenal of every reliability engineer. The author appreciates the many helpful comments of the referees.

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