Problem 1 - insertion sort

Recurrence relation: The time to sort an array of N elements is equal to the time to sort an array of N-1 elements plus N-1 comparisons. Initial condition: the time to sort an array of 1 element is constant:

T(1) = 1

T(N) = T(N-1) + N-1

Next we perform telescoping: re-writing the recurrence relation for N-1, N-2, ..., 2

T(N) = T(N-1) + N-1

T(N-1) = T(N-2) + N-2

T(N-2) = T(N-3) + N-3

......

T(2) = T(1) + 1

Next we sum up the left and the right sides of the equations above:

T(N) + T(N-1) + T(N-2) + T(N-3) + ... T(3) + T(2) =

T(N-1) + T(N-2) + T(N-3) + ... T(3) + T(2) + T(1) + 1 + 2 + 3 + ... + N-2 + N-1

Finally, we cross the equal terms on the opposite sides and simplify the remaining sum on the right side:

T(N) = T(1) + 1 + 2 + 3 + ... + N-2 + N-1 (Open form)

T(N) = 1 + $\frac{N(N-1)}{2}$(Closed form)

Therefore, the running time of insertion sort is:

T(N) = $O(N^2)$. (big O)

Problem 2

T(1) = 1

T(N) = T(N–1) + 2 // 2 is a constant like c

Telescoping:

T(N) = T(N-1) + 2

T(N-1) = T(N-2) + 2

T(N-2) = T(N-3) + 2

......

T(2) = T(1) + 2

Next we sum up the left and the right sides of the equations above:

T(N) + T(N-1) + T(N-2) + T(N-3) + ... + T(2) =

T(N-1) + T(N-2) + T(N-3) + ... + T(1) + 2+2+2 ... +2

Finally, we cross the equal terms on the opposite sides and simplify the remaining sum on the right side: T(N) = T(1) + 2 + 2 + 2 + ... + 2 (open form)

T(N) = 1 + 2(N-1) (closed form)

Therefore, the running time of reversing a queue is:

T(N) = O(N) (Big O)

Problem 3 - Power()

```
long power(long x, long n) {
 if (n == 0) return 1;
 else return x * power(x, n-1);
}
```

T(n) = Time required to solve a problem of size n

Recurrence relations are used to determine the running time of recursive programs–recurrence relations themselves are recursive

T(0) = time to solve problem of size 0 : Base Case

T(n) = time to solve problem of size n : Recursive Case

T(0) = 1

T(n)=T(n–1)+1 // +1 is a constant

Solution by telescoping:

If we knew T(n−1), we could solve T(n).

T(n) = T(n–1) +1

T(n–1) = T(n-2) + 1

T(n–2) = T(n-3) + 1

....

T(2) = T(1) + 1

T(1) = T(0) + 1

Next we sum up the left and the right sides of the equations above:

T(n) + T(n-1) + T(n-2) + ... + T(1)

= T(n-1) + T(n-2) + ... T(0) + 1 + 1 + 1 + ... + 1

Finally, we cross the equal terms on the opposite sides and simplify the remaining sum on the right side: T(n) = T(0) + 1 + 1 + 1 + ... + 1 (Open form)

T(n) = 1 + n-1 (Closed form)

T(n) = O(n) (Big O)

Problem 4 - Power()

```
long power(long x, long n) {
 if (n == 0) return 1;
 if (n == 1) return x;

 if ((n % 2) == 0) return power(x * x, n/2);
 else return power(x * x, n/2) * x;
}
```

T(0) = 1

T(1) = 1

T(n) = T(n / 2) + 1 // Assume n is power of 2, +1 is a constant

Solution by unfolding:

T(0) = 1

T(1) = 1

T(n) = T(n/2) + 1

= T(n/4) + 1 + 1

= T(n/8) + 1 + 1 + 1

= T(n/16) + 1 +1 + 1 + 1

....

= $T\left(\frac{n}{2^k}\right) + k$

We want to get rid of $T(n/2^k)$.

We solve directly when we reach T(1)

$n/2^k = 1$

$n = 2^k$

log n = k

T(n) = T(1) + k (Open form)

= T(1) + log n (Open form)

$= \underline{1 + \log n}$ (Closed form)

Therefore, $T(n) = \underline{\log n}$ (Big O)