

Problem 1.

The temperature extremes in Seoul for each month, starting in January, are given by (in degrees Celsius):

max: 17, 19, 21, 28, 33, 38, 37, 37, 31, 23, 19, 18

min: -62, -59, -56, -46, -32, -18, -9, -13, -25, -46, -52, -58

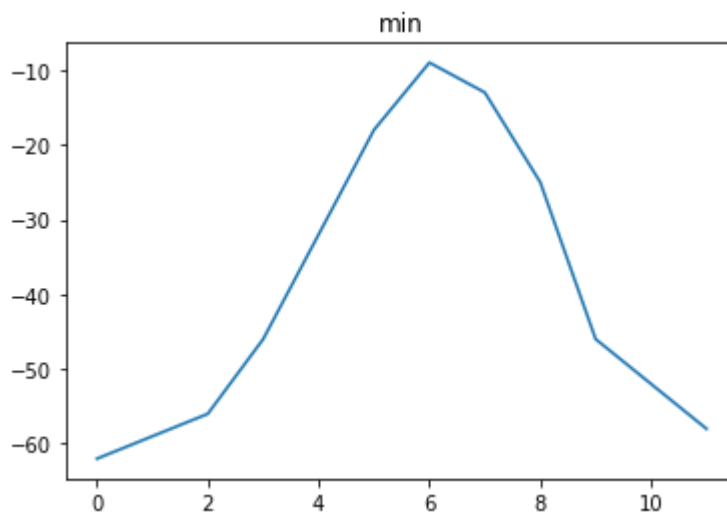
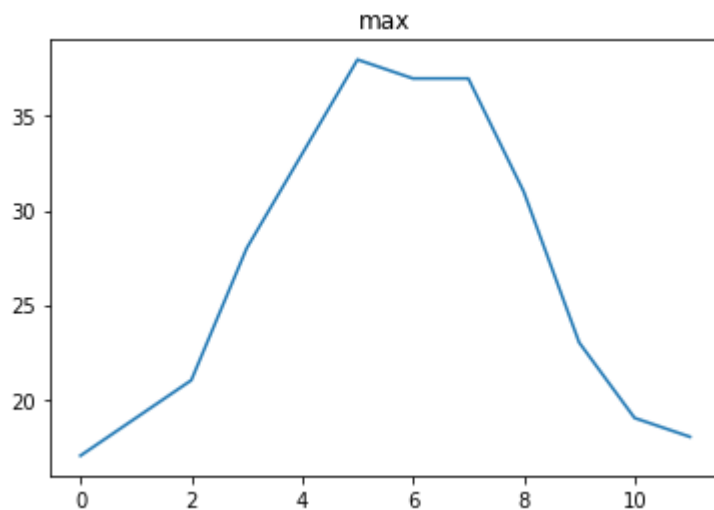
- Plot these temperature extremes.
- Define a function that can describe min and max temperatures. Hint: this function has to have a period of 1 year. Hint: include a time offset.
- Fit this function to the data with `scipy.optimize.curve_fit()`.
- Plot the result. Is the fit reasonable? If not, why?
- Is the time offset for min and max temperatures the same within the fit accuracy?

<https://datascienceschool.net/view-notebook/d0b1637803754bb083b5722c9f2209d0/>

max = {17, 19, 21, 28, 33, 38, 37, 37, 31, 23, 19, 18}

```
In [86]: max = [17, 19, 21, 28, 33, 38, 37, 37, 31, 23, 19, 18]
min = [-62, -59, -56, -46, -32, -18, -9, -13, -25, -46, -52, -58]

plt.title("max")
plt.plot(max)
plt.show()
plt.title("min")
plt.plot(min)
plt.show()
```



Problem 2.

The six-hump camelback function

$$f(x, y) = (4 - 2.1x$$

$2 +$

x

4

3

$)x$

$2 + xy + (4y$

$2 - 4)y$

2

(1)

has multiple global and local minima. Find the global minima of this function.

Hints:

Variables can be restricted to $-2 < x < 2$ and $-1 < y < 1$.

- Use `numpy.meshgrid()` and `pylab.imshow()` to find visually the regions.
- Use `scipy.optimize.minimize()`, optionally trying out several of its methods.
- How many global minima are there, and what is the function value at those points?

What happens for an initial guess of $(x, y) = (0, 0)$?

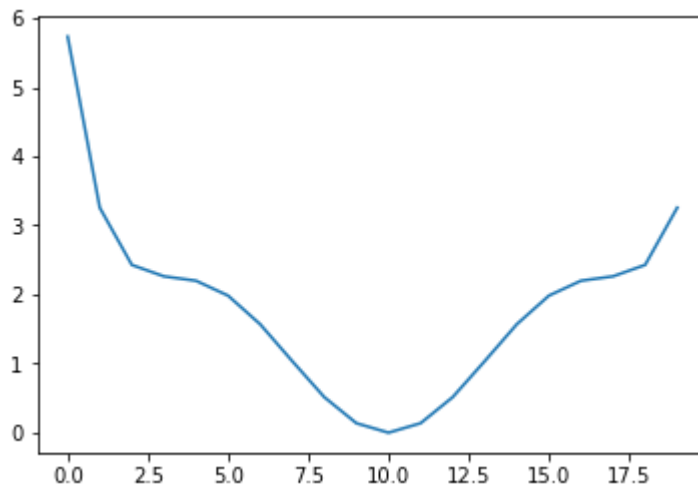
```
In [65]: from scipy import optimize
import numpy as np
import matplotlib.pyplot as plt

def f(x,y) :
    return (4 - 2.1*x*x + x*x*x*x/3)*x*x + x*y + (4*y*y-4)*y*y

x = np.arange(-2, 2, 0.2)
y = np.arange(-1, 1, 0.1)

plt.plot(f(x,y))
plt.show
```

Out[65]: <function matplotlib.pyplot.show(*args, **kw)>



Problem 3.

Using SymPy, confirm that the stationary points in the previous question are minima.

Remember that:

Let (a, b) be a stationary point, so that $f_x = 0$ and $f_y = 0$ at (a, b) . Then:

- if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a, b) then (a, b) is a saddle point.

2

• if $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a, b) then (a, b) is either a maximum or a minimum.

- if $f_{xx} < 0$ and $f_{yy} < 0$ at (a, b) then (a, b) is a maximum point.

2

• if $f_{xx} > 0$ and $f_{yy} > 0$ at (a, b) then (a, b) is a minimum point.

Distinguish between these as follows:

- if $f_{xx} < 0$ and $f_{yy} < 0$ at (a, b) then (a, b) is a maximum point.

- if $f_{xx} > 0$ and $f_{yy} > 0$ at (a, b) then (a, b) is a minimum point.

If $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a, b) then the test is inconclusive.

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 $xy = 0$ then anything is possible. More advanced methods are required to classify the stationary point properly.

```
In [59]: from scipy import optimize
import numpy as np
import matplotlib.pyplot as plt
import sympy as sym

def sixhump(x,y) :
    return ((4 - 2.1*x*x + x**4/3)*x**2 + x*y + (-4+4*y**2)*y**2)
```

```
In [64]: print(len(x),len(y))
```

40 20

```
In [76]: import sympy as sym
x = sym.Symbol('x')
y = sym.Symbol('y')
dx = sym.diff((4 - 2.1*x*x + x**4/3)*x**2 + x*y + (-4+4*y**2)*y**2,x)
dxx = sym.diff((dx),x)
dy = sym.diff((4 - 2.1*x*x + x**4/3)*x**2 + x*y + (-4+4*y**2)*y**2,y)
dyy = sym.diff((dy),y)
dxy = sym.diff(dx,y)
print(dx)
print(dxx)
print(dy)
print(dyy)
print(dxy)

x**2*(4*x**3/3 - 4.2*x) + 2*x*(x**4/3 - 2.1*x**2 + 4) + y
2*x**4/3 + x**2*(4*x**2 - 4.2) - 4.2*x**2 + 4*x*(4*x**3/3 - 4.2*x) + 8
x + 8*y**3 + 2*y*(4*y**2 - 4)
48*y**2 - 8
1
```

```
In [78]: sym.simplify(dxx-dyy)
```

```
Out[78]: 10.0*x**4 - 25.2*x**2 - 48.0*y**2 + 16.0
```

```
In [ ]:
```