When it's all just too much: Outsourcing MPC Pre-processing

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Outline

- MPC
  - Overview
  - Pre-processing Model
- SPDZ Family of MPC
  - Linear Secret-sharing
  - Opening secrets
  - Multiplication of secrets
- Generating Pre-processed Data
- Protocol

Outline

What is Multi-Party Computation?

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A set of parties computing a function

on their secret input

so that all parties learn the output

and nothing else about other parties' input that can't be deduced from the output alone.

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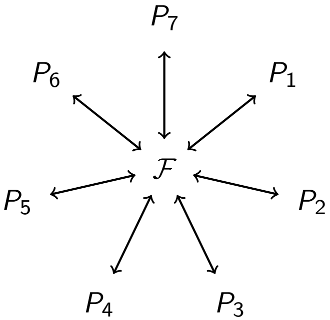

Ideal World

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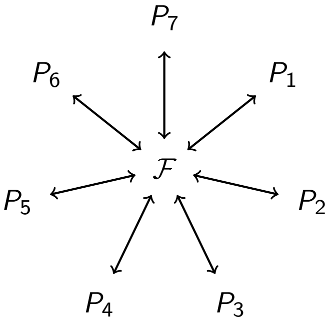

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$\rightsquigarrow$

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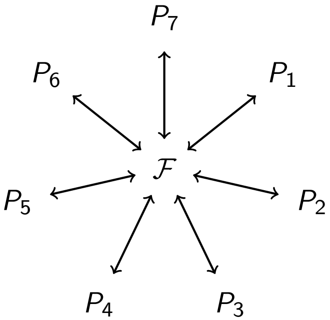

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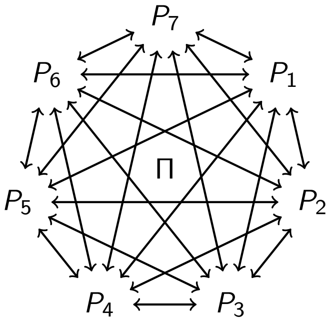

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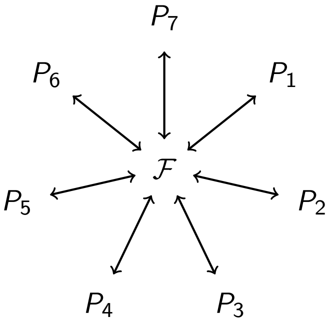

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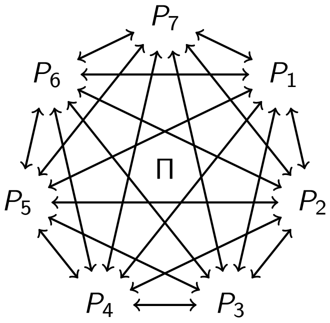

Real World

Possible guarantees:
Privacy/Secrecy
Correctness
Fairness
(etc.)

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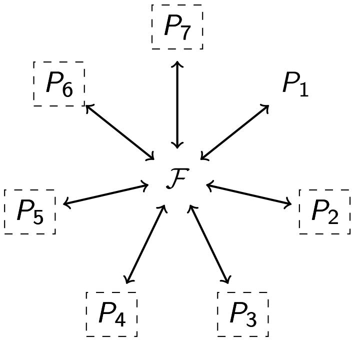

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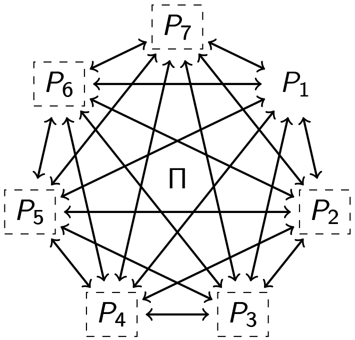

Real World

Corruption models:
Active/Passive
Access structure
(etc.)

Pre-processing Model

$$\boxed{\Pi_{\text{MPC}}}$$

- Offline phase: computed whenever, uses public-key crypto
- Online phase: only uses information-theoretic primitives

Pre-processing Model

Π_{Prep}	Π_{Online}

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Π_{Prep} Π_{Online}

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Goal

Take SPDZ, find simple method to outsource Π_{Prep} to different set of parties.

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Linear Secret-sharing

Secret x is shared via $\sum_{i=1}^{n} x_i = x$, where P_i holds x_i. Want to compute an arithmetic circuit.

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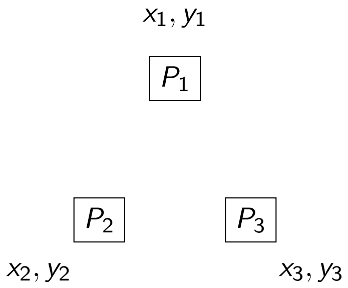
Parties can locally:

Add secrets



Linear Secret-sharing

Secret x is shared via $\sum_{i=1}^{n} x_i = x$, where P_i holds x_i. Want to compute an arithmetic circuit.

Parties can locally:

Add secrets

$\hat{x}_1 := x_1 + y_1$

$\boxed{P_1}$

$\boxed{\sum_{i=1}^{3} \hat{x}_i = x + y}$

$\boxed{P_2}$ $\boxed{P_3}$

$\hat{x}_2 := x_2 + y_2$ $\hat{x}_3 := x_3 + y_3$

Linear Secret-sharing

Secret x is shared via $\sum_{i=1}^{n} x_i = x$, where P_i holds x_i. Want to compute an arithmetic circuit.

Parties can locally:

Multiply a secret by a public constant

k, x_1

$\boxed{P_1}$

$\boxed{P_2}$ $\boxed{P_3}$

k, x_2 k, x_3

Linear Secret-sharing

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$\hat{x}_1 := k \cdot x_1$

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$\boxed{\sum_{i=1}^{3} \hat{x}_i = k \cdot x}$

$\boxed{P_2}$ $\boxed{P_3}$

$\hat{x}_{:} = k \cdot x_2$ $\hat{x}_3 := k \cdot x_3$

Linear Secret-sharing

Secret x is shared via $\sum_{i=1}^{n} x_i = x$, where P_i holds x_i. Want to compute an arithmetic circuit.

Parties can locally:

Add a public constant to a secret

k, x_1

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k, x_2 k, x_3

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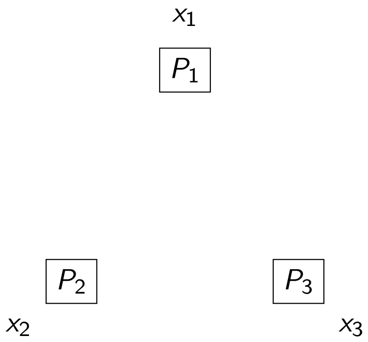
$\boxed{\sum_{i=1}^{3} \hat{x}_i = x + k}$

$\boxed{P_2}$ $\boxed{P_3}$

$\hat{x}_2 = x_2$ $\hat{x}_3 = x_3$

Opening a secret

To reveal or *open* a secret, the parties broadcast their share:



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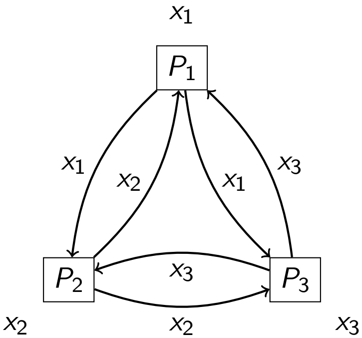

Opening a secret

To reveal or *open* a secret, the parties broadcast their share:

x_1, x_2, x_3

$\boxed{P_1}$

$$\boxed{x = x_1 + x_2 + x_3}$$

$\boxed{P_2}$ $\boxed{P_3}$

x_2, x_3, x_1 x_3, x_1, x_2

Multiplication?

Want $(z_i)_{i=1}^{3}$ s.t. $\sum_{i=1}^{3} z_i = xy$.

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Observe that:

$$\begin{aligned} xy &= (x-a+a)(y-b+b) \\ &= (x-a)(y-b) + (x-a)b + a(y-b) + ab \end{aligned}$$

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so if the parties *already* have

shares of two random secrets, $a = \sum_{i=1}^3 a_i$ and $b = \sum_{i=1}^3 b_i$...

...and a share of their product $ab = \sum_{i=1}^3 c_i$,

then define

$$\begin{aligned} z_1 &:= (x-a)(y-b) + (x-a)b_1 + a_1(y-b) + c_1 \\ z_2 &:= (x-a)b_2 + a_2(y-b) + c_2 \\ z_3 &:= (x-a)b_3 + a_3(y-b) + c_3 \end{aligned}$$

Multiplication?

Key points:

- Need a triple: shared random independent a and b and sharing of ab.
- Triple (a, b, c) is *independent* of x and y
- Requires 2 openings
- None of the secrets x, y, or xy are revealed

Outline

Generating Pre-processing in Π_{Prep}

Need to generate Beaver Triples.

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Generated via:

- SHE

$a_1, b_1 \leftarrow \mathbb{F}$

$\boxed{P_1}$

$\boxed{P_2}$ $\boxed{P_3}$

$a_2, b_2 \leftarrow \mathbb{F}$ $a_3, b_3 \leftarrow \mathbb{F}$

Generating Pre-processing in Π_{Prep}

Need to generate Beaver Triples.
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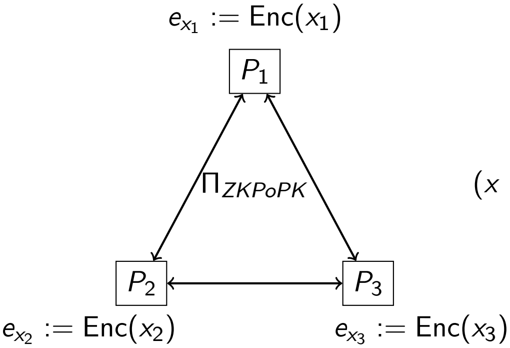

$(x \in \{a, b\})$

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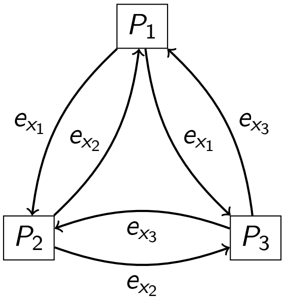

$(x \in \{a, b\})$

Generating Pre-processing in Π_{Prep}

Need to generate Beaver Triples.
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$$e_x := e_{x_1} + e_{x_2} + e_{x_3}$$

$$\boxed{P_1}$$

$$(x \in \{a, b\})$$

$$\boxed{P_2} \qquad \boxed{P_3}$$

$$e_x := e_{x_1} + e_{x_2} + e_{x_3} \qquad e_x := e_{x_1} + e_{x_2} + e_{x_3}$$

Generating Pre-processing in Π_{Prep}

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$$e_c := e_a \times e_b$$

$$\boxed{P_1}$$

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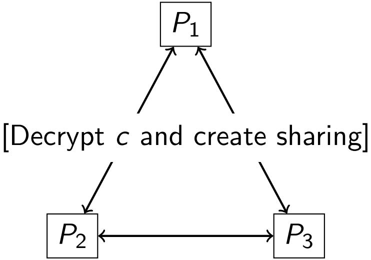

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Need to generate Beaver Triples.
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a_1, b_1, c_1

$\boxed{P_1}$

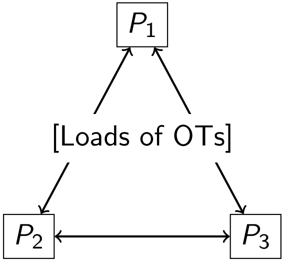
$\boxed{P_2}$ $\boxed{P_3}$

a_2, b_2, c_2 a_3, b_3, c_3

Generating Pre-processing in Π_{Prep}

Need to generate Beaver Triples.
Generated via:

- SHE (+ ZKPs), or
- OT



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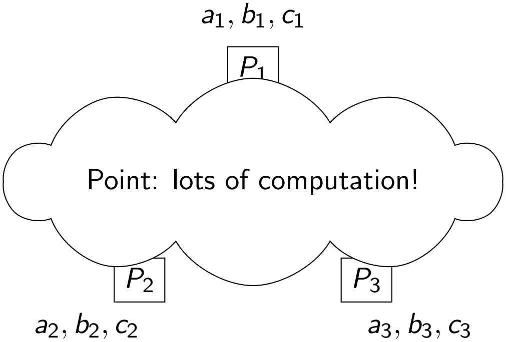

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Problem: in pre-processing,

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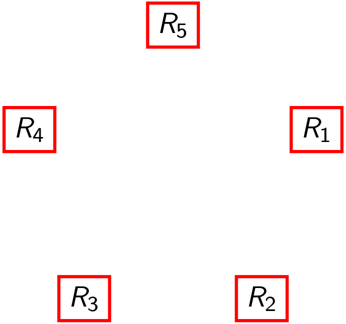
Solution:

- Outsource to some set of parties R.
- Make R do all pre-processing.
- When computing parties Q need pre-processed data, request from parties in R.

Outline

1. Pre-processing



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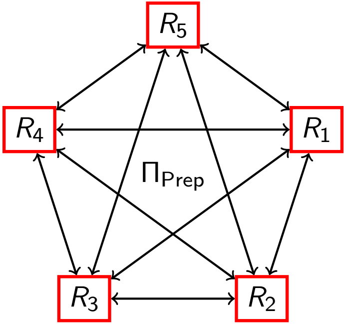

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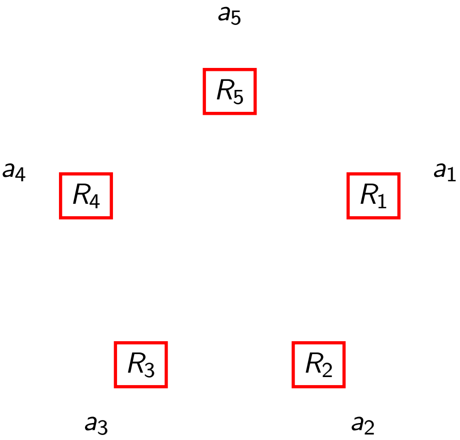
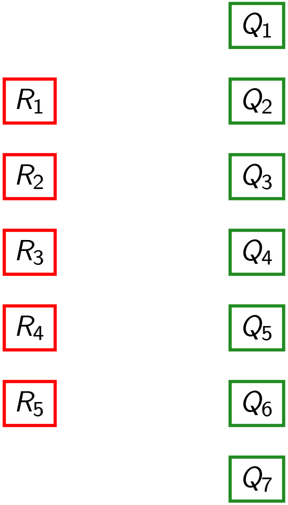


2. Feeding

Π_{Feed}



2. Feeding

Π_{Feed}

$\boxed{Q_1}$

$a_1 = a_1^1 + a_1^2$ $\boxed{R_1}$ $\boxed{Q_2}$

$\boxed{R_2}$ $\boxed{Q_3}$

$\boxed{R_3}$ $\boxed{Q_4}$

$\boxed{R_4}$ $\boxed{Q_5}$

$\boxed{R_5}$ $\boxed{Q_6}$

$\boxed{Q_7}$

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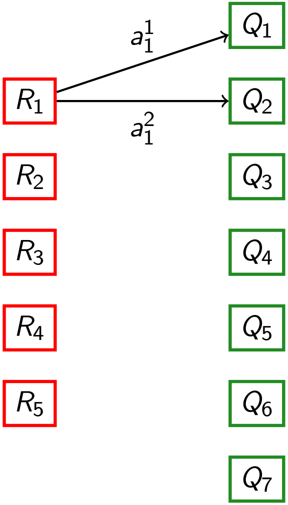

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$\boxed{Q_1}$ a_1^1

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$\boxed{R_2}$ $\boxed{Q_3}$

$\boxed{R_3}$ $\boxed{Q_4}$

$\boxed{R_4}$ $\boxed{Q_5}$

$\boxed{R_5}$ $\boxed{Q_6}$

$\boxed{Q_7}$

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Π_{Feed}

$\boxed{Q_1}$ a_1^1

$a_1 = a_1^1 + a_1^2$ $\boxed{R_1}$ $\boxed{Q_2}$ a_1^2

$a_2 = a_2^4 + a_2^7$ $\boxed{R_2}$ $\boxed{Q_3}$

$\boxed{R_3}$ $\boxed{Q_4}$

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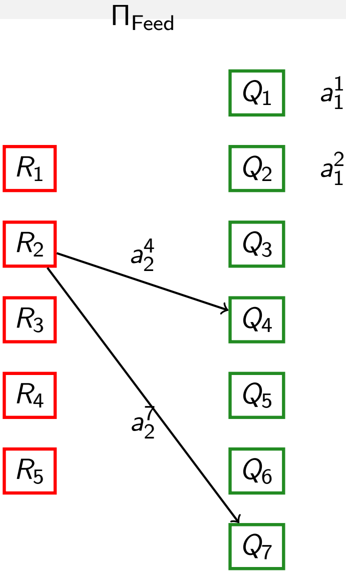

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$\boxed{R_3}$ $\boxed{Q_4}$ a_2^4

$\boxed{R_4}$ $\boxed{Q_5}$

$\boxed{R_5}$ $\boxed{Q_6}$

$\boxed{Q_7}$ a_2^7

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Π_{Feed}

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$a_3 = a_3^2 + a_3^3$	$\boxed{R_3}$	$\boxed{Q_4}$	a_2^4
	$\boxed{R_4}$	$\boxed{Q_5}$	
	$\boxed{R_5}$	$\boxed{Q_6}$	
		$\boxed{Q_7}$	a_2^7

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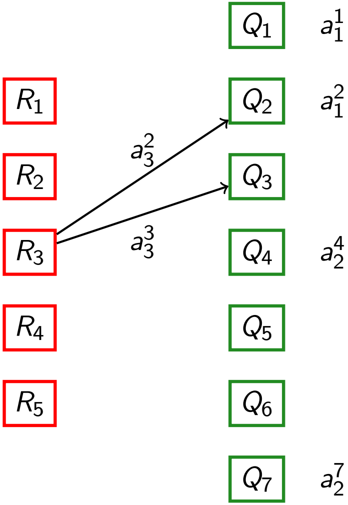

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	$\boxed{R_5}$	$\boxed{Q_6}$	
		$\boxed{Q_7}$	a_2^7

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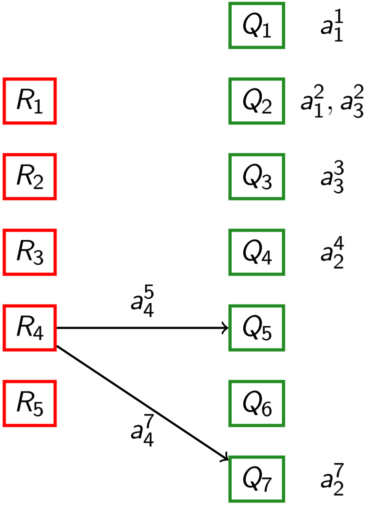

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	$\boxed{R_5}$	$\boxed{Q_6}$	
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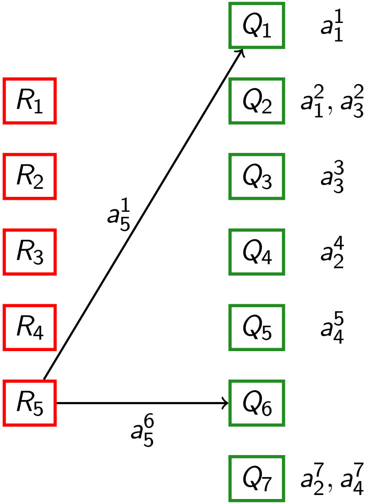

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Π_{Feed}

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$a_2 = a_2^4 + a_2^7$	$\boxed{R_2}$	$\boxed{Q_3}$	a_3^3
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$a_5 = a_5^1 + a_5^6$	$\boxed{R_5}$	$\boxed{Q_6}$	a_5^6
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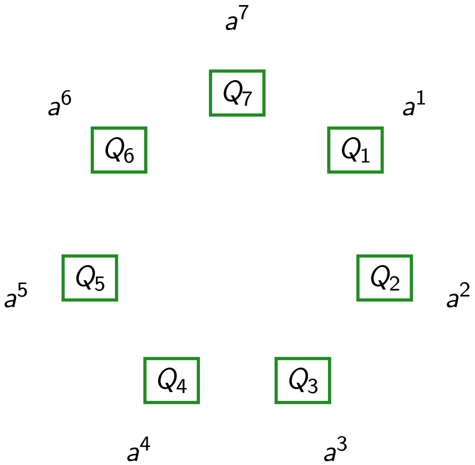
2. Feeding

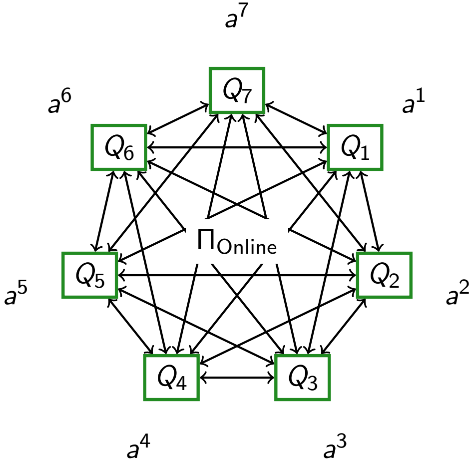
Π_{Feed}

$$\boxed{\sum_{i=1}^{5} a_i = \sum_{j=1}^{7} a^j}$$

		$\boxed{Q_1}$	a_1^1, a_5^1	$a^1 := a_1^1 + a_5^1$
$a_1 = a_1^1 + a_1^2$	$\boxed{R_1}$	$\boxed{Q_2}$	a_1^2, a_3^2	$a^2 := a_1^2 + a_3^2$
$a_2 = a_2^4 + a_2^7$	$\boxed{R_2}$	$\boxed{Q_3}$	a_3^3	$a^3 := a_3^3$
$a_3 = a_3^2 + a_3^3$	$\boxed{R_3}$	$\boxed{Q_4}$	a_2^4	$a^4 := a_2^4$
$a_4 = a_4^5 + a_4^7$	$\boxed{R_4}$	$\boxed{Q_5}$	a_4^5	$a^5 := a_4^5$
$a_5 = a_5^1 + a_5^6$	$\boxed{R_5}$	$\boxed{Q_6}$	a_5^6	$a^6 := a_5^6$
		$\boxed{Q_7}$	a_2^7, a_4^7	$a^7 := a_2^7 + a_4^7$


a^7
Q_7
a^6
a^1
Q_6
Q_1
Q_5
Q_2
a^5
a^2
Q_4
Q_3
a^4
a^3

3. Online



Correctness and Privacy

Correctness

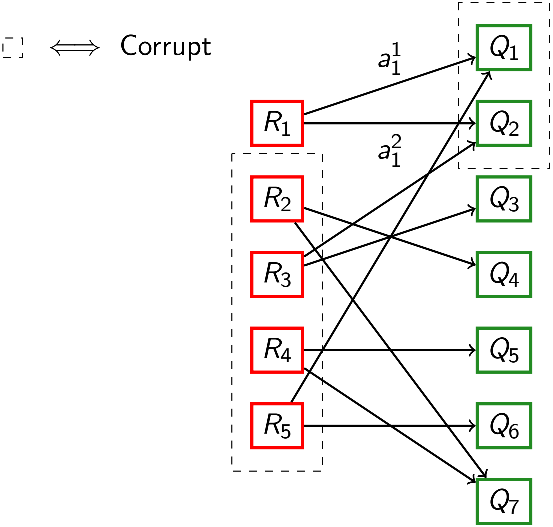
Must ensure all parties receive at least one share.

Privacy

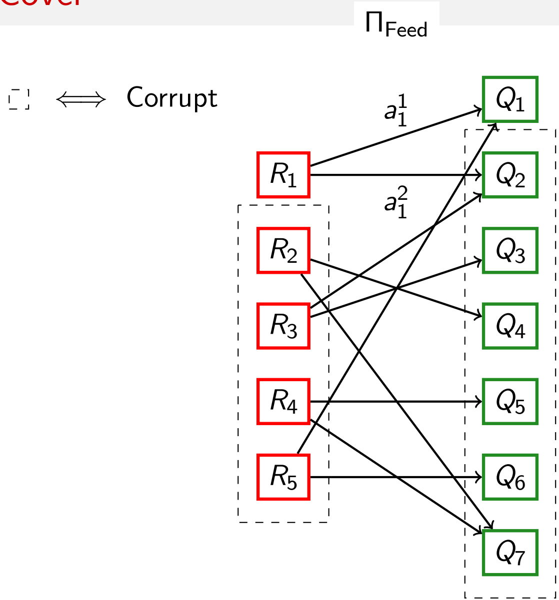
?

Q: When does $\mathcal{A}$ *not* learn the secret?

Π_{Feed}



A: Secure Cover



Ensuring a secure cover

Guaranteed…

…statistically?

- if cover assigned probabilistically; possible only for large numbers of parties

…perfectly?

- if $R \subset Q$ (but then a different access structure – why use SPDZ?); or
- if each party in R reshares to *all* parties in Q

Active Security?

SPDZ uses linear MACS:

$$\mathsf{MAC}_\alpha(a) = \alpha \cdot a$$

Theorem

If adversary corrupts at most all but one party in each of R and Q, and the cover is secure, then the MAC can be forged with probability at most $1/|\mathbb{F}|$.

N.B. in SPDZ, $|\mathbb{F}|$ is $O(2^\lambda)$.

Potential use-cases

- Client-server system of delivering pre-processing
  - a few servers compute pre-processing and deliver to a set of ‘weaker’ clients
- Dynamically adjusting pre-processing during Π_{Online} if other parties want to join a computation
  - requires randomness!

Thanks!

Questions?