The Photoelectric Effect: Determination of Planck's Constant

Javier M. G. Duarte

Massachusetts Institute of Technology, MA 02142∗

(Dated: September 25, 2008)

We test Einstein's theory of the photoelectric effect, which predicts a linear relation between the energy of light incident on a metal surface and the energy of the emitted electrons. We determine the electron stopping voltage as a function of frequency of incident radiation by illuminating a potassium cathode and constraining the wavelength of incident radiation using several optical band-pass filters. From this, we calculate the constant of proportionality h/e as well as the constant offset, known as the work function. Our analysis included two different selection criteria to determine the stopping voltage, V_{cutoff}, each of which led to a different estimate of Planck's constant and the effective work function, ϕ_{eff}. Our final results for both methods are $h = (7.12 \pm 0.88) \times 10^{-34}$ J·s and $\phi_{\text{eff}} = 2.20 \pm 0.32$ eV and $h = (4.08 \pm 0.88) \times 10^{-34}$ J·s and $\phi_{\text{eff}} = 1.28 \pm 0.32$ eV .

INTRODUCTION

In 1887, Hertz was the first to discover that a metallic surface, when illuminated by light of sufficient frequency, may emit electricity [1]. This 'photoelectric effect' was unexplained until Einstein connected this experimental curiosity with Planck's idea that radiation comes in small packets, or *quanta* [2]. He proposed that the energy of the ejected electrons is proportional to the energy of the incident light with a constant offset that is unique to the metal, referred to as the work function [3]. This phenomenon was a crucial precursor to the formulation of quantum mechanics as it was one of the first to show the wave-particle duality of light. We will examine this effect, test the hypothesized linear relation, and extract values for Planck's constant and the effective work function.

THEORY

There are several main features of the photoelectric effect that cannot be explained by the classical wave description of light [4]:

(1) According to Maxwell's equations, the magnitude of the electric field vector of a light wave goes like the square root of the intensity of the light, $|\vec{E}| \propto \sqrt{I}$. Thus, as the intensity increases , the electric field magnitude increases. Since the force on an electron is proportional to the electric field vector, it is expected that the kinetic energy of a photoelectron should increase with the intensity of incident light. However, it is observed that the maximum kinetic energy of the photoelectrons does not depend on intensity.

(2) Classical wave theory predicts that photoemission should occur at all wavelengths of incident light, given that the incident light has sufficient intensity. On the contrary, there exists a longest wavelength, specific to the metal, such that light of longer wavelength cannot liberate electrons from the metallic surface.

(3) Classically, an electron would not be emitted from the surface until the incident light transferred enough of its energy to the illuminated metal. This would require a delay from when the light first hit the surface to when the first electron was ejected. But, experimentally it is seen that the emission of electrons occurs very shortly after the arrival of the radiation.

Motivated by these, Einstein produced one of his seminal papers in 1905, "A heuristic point of view concerning the production and transformation of light," which put forth his law of the photoelectric effect. He extended Planck's notion that the energy of radiation comes in chunks of the frequency multiplied by a constant, $h\nu$, to light. In effect, he treated light as if it were composed of particles, or photons, each carrying an energy $h\nu$.

To explain the problem of (2) above, Einstein reasoned that is a work threshold required for an electron to escape the metal. The particular amount of work is a complicated combination of the energy required for optical excitation of a valence band electron into the conduction band as well as the energy required in the diffusion of the photo-excited electron through the solid. Thus, the work is related to the electronic structure and lattice configuration of the metal among other things. As a result, this work function is extremely dependent on the metal.

Further, he derived a beautiful relation from conservation of energy. In short, an ejected electron can have at most the incident energy minus the total energy needed to escape the metal.

$$K_{\max} = h\nu - \phi_{\text{metal}} \qquad (1)$$

In the actual experiment, we an expect the ejected electrons to have energy less than this due to the random processes involved in the photoemission: thermal

───────────

∗Electronic address: woodson@mit.edu

vibrations, diffusion, multiple photon interactions, and scatterings.
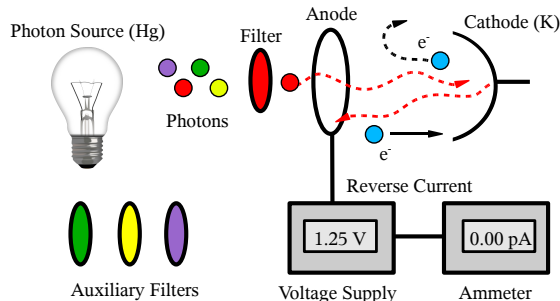
EXPERIMENTAL SETUP

Design

FIG. 1: The experimental setup including the high power Mercury lamp

Our actual experimental setup had many of the same features as the schematic shown in figure 1. For the first component, the photon source, we used a high power mercury lamp with well documented transition lines [5]. We aimed this source through an optical narrow band-pass filter, which isolates a specific emission line. The light continues through an aperture and then an anode ring until finally striking the potassium coated cathode, which acts as a photoemission surface. If electrons are ejected, this may complete the electrical circuit and allows us to detect a current with an ultra-sensitive ammeter. A voltage power supply was used to apply a known potential between the anode and the cathode, which may manipulate the trajectory of the ejected electrons from the cathode.

By adjusting the applied retarding voltage and recording the measured current, we may determine the potential difference required to stop the photoelectrons from crossing the gap. This potential corresponds to the maximum kinetic energy of the electrons divided by the elementary charge, displayed in equation **??**. Utilizing five different filters that select distinct mercury transition lines, we can take several data point pairs of (ν, V_{cutoff}) and test the linear relation among incident photon energy and maximum electron kinetic energy.

Execution

The apparatus was grounded properly and the path of light was protected from ambient light background. For each band-pass filter, i.e. wavelength of incident light, we first recorded the voltage corresponding to a zero **net**

current reading on the ammeter. Next, we incremented the applied voltage starting from 0.00 V over an appropriate range and recorded the current to achieve at least seven data point pairs. We repeated this procedure five times for each filter, for a total of 25 runs. All the data was taken on the same day over a two hour period.

DATA AND ANALYSIS

Systematic Errors

Two sources of systematic error that may confound the data and affect our analysis include the spread in frequencies permitted to pass through the filter and the negative current. The former is a result of the reported error on the central wavelength of the band-pass filter: $\pm 2.0 nm$. Originally, we considered this as a main contributer in the nonlinearity of our data. However, further investigation, revealed that the spectral output of the mercury lamp is far too peaked for this to be a significant source of error.

Over time, potassium residue builds up on the anode and light reflected from the cathode back onto the anode may cause photoemission at the anode, resulting in electrons being accelerated toward the cathode [6]. In effect, this means that a zero current reading doesn't correspond to no photoelectrons reaching the anode from the cathode, but it indicates where the photocurrents from the cathode and from the anode cancel out. Therefore the true stopping voltage at which electrons with maximum kinetic energy are being stopped is detected as a negative current. This complicates our analysis greatly, as we now need a V_{cutoff} selection method that takes this effect into account.

Since the reverse current decreases monotonically with increasing positive retarding voltage, we expect that the data should have a characteristic linear tail.

Data Reduction

Data reduction and analysis was done using MATLAB and linear fit scrips provided by the Junior Lab staff. Raw data of current versus voltage looked similar to that of figure 2, with a characteristic linear tail below the zero current level as well as a departure from linearity. However, as seen in figure 3, the 577.0 nm data never actually crosses the zero current mark. In addition, the departure from linearity is much less pronounced in this data set (keeping in mind the relative scales). Finally, the cutoff voltage, as determined by method II because method I does not apply (explained later in this section), is completely off the linear structure that the other four data sets follow so closely.
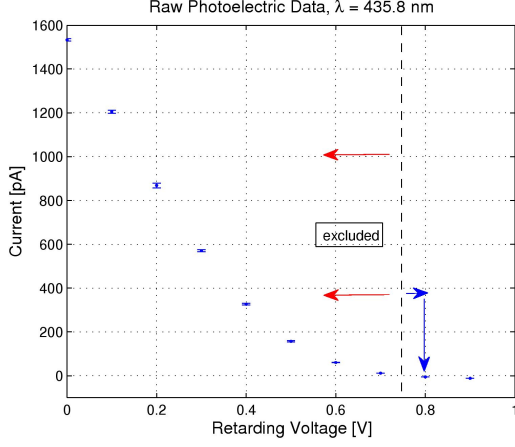
FIG. 2: Method I: the simple selection criteria for the stopping voltage. The stopping voltages chosen in this way will be denoted $V_{\text{cutoff}}^{\text{I}}$.
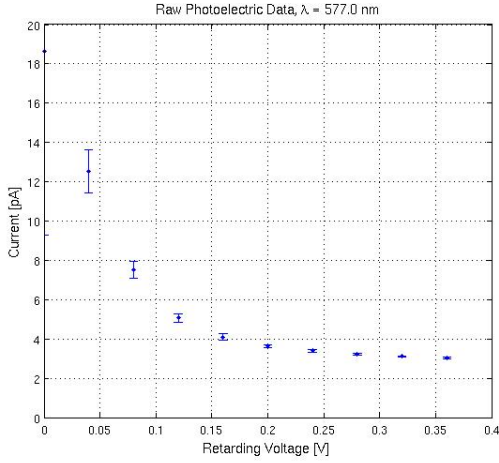

FIG. 3: The data for the 577.0 nm transition is not in agreement with the characteristic photoemission line.

We determined two distinct V_{cutoff} selection methods based on the systematic errors we were trying to overcome as well as the photoelectric hypothesis. The first, hereafter referred to as method I, consisted of recalling the voltage which gave a zero net current reading during the experiment and selecting the first voltage bin to the right of it (the next greatest voltage measured at regular interval). The method is portrayed in figure 2. We justify this method by again noting that the true stopping voltage is actually detected as a negative current due to the reverse current tail.

For the second selection technique, method II, we fit the first three bins and the last three bins of data to separate lines by a χ^2 minimizing procedure, and extrapolated. Then, we chose the voltage bin closest to the x-value (Voltage) of the intersection of the two lines. Figure 4 displays method II on 435 nm data. You can
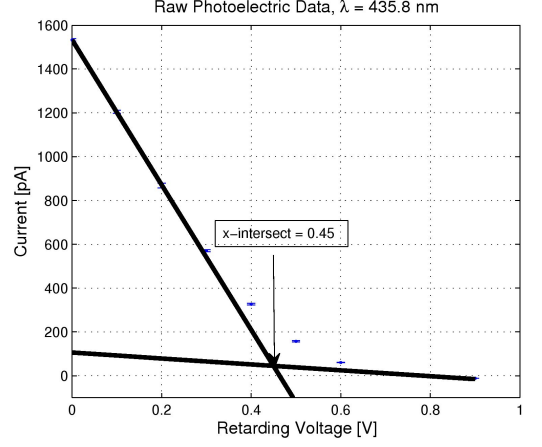

FIG. 4: Method II: the double linear fit selection criteria for the stopping voltage. The stopping voltages chosen in this way will be denoted $V_{\text{cutoff}}^{\text{II}}$

λ	med-errorbar	$V_{\text{cutoff}}^{\text{I}}$	$V_{\text{cutoff}}^{\text{II}}$
365.0 nm	$\pm$ 14.4 pA	0.24 $\pm$ 0.04 V	0.12 $\pm$ 0.04 V
404.7 nm	$\pm$ 6.54 pA	0.80 $\pm$ 0.10 V	0.50 $\pm$ 0.10 V
435.8 nm	$\pm$ 9.21 pA	1.20 $\pm$ 0.20 V	0.60 $\pm$ 0.20 V
546.1 nm	$\pm$ 1.35 pA	1.50 $\pm$ 0.25 V	0.75 $\pm$ 0.25 V
577.0 nm	$\pm$ 0.11 pA	N/A	N/A

TABLE I: These are the results of the data, the med-errorbar is the median error bar for each plot of current versus retarding voltage, taken at the 95 % confidence level.

compare this to figure 2. We know that the asymptotic behavior of the curved data on each side of the range should approximate a line. So we are merely, 'flattening out' the curve to find where the departure from linearity takes place.

For each wavelength, the mean and standard deviation was calculated for each voltage bin; error bars were taken at 95 % confidence levels, or 2σ. Furthermore, for both V_{cutoff} selection methods, the standard error associated with the cut off voltage was approximated to be the resolution of the current vs. voltage plot for that wavelength. So, for example, for the 435.8 nm data, we used ± 0.20 V. This data is summarized in table I.

Finally, the magnitude of the cut off voltage points and their associated error bars were plotted against frequency, figure 5, for both methods, and fitted to a line by least squares. The intercept, slope and respective error bars correspond to ϕ_{eff}/e and h/e, respectively.
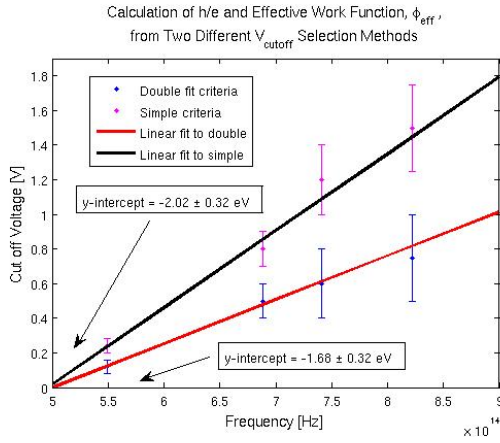
FIG. 5: The linear fits to the cut off voltage points for each frequency for both methods of selection. The chi-square values of the fit to the data from method I and method II was $\chi_{\mathrm{I}}^2/dof = 0.6523/2$ and $\chi_{\mathrm{II}}^2/dof = 0.1325/2$, respectively.

Results

Assuming the accepted value for the elementary charge ($e = 1.602 \times 10^{-19}$ C), we find for method I:

$$h = (7.12 \pm 0.88) \times 10^{-34} \text{ J·s}$$

$$\phi_{\mathrm{eff}} = 2.20 \pm 0.32 \text{ eV}$$

and for method II:

$$h = (4.08 \pm 0.88) \times 10^{-34} \text{ J·s}$$

$$\phi_{\mathrm{eff}} = 1.28 \pm 0.32 \text{ eV}$$

We see that only one of the methods seems to agree with the accepted value of $h = 6.626 \times 10^{-34}$ J·s.

CONCLUSIONS

Only one of the methods managed to be in agreement with nature. This may be a result of the arbitrary nature of the criteria: it seems there are many reasonable criteria and they give you a wide range of final results. Method I not only provides a better approximation to Planck's constant, but to the effective work function as well. Assuming that our analysis of the 577.0 nm data is correct, it is safe to assume that no photoemission took place during the trials with this wavelength. We note that these photons have an energy of $E = hc/\lambda = 2.15$ eV. We see that within error, method I yields a work function greater than the energy of the incident photons in this case, thus explaining the lack of photoemission. However, method II does not yield parameters that agree with this interpretation within standard error.

We conclude that these issues might have been resolved had we backed out the anode, which could have highly reduced the reverse current and limited the ambiguity in selecting a cut off voltage. Also, we could have improved the accuracy of our measurement by taking more filter cycles, with better resolution, and out to further voltages. However, time constraints prevented us from doing so.

The properties of the photoelectric effect that we observed reflect the wave-particle duality of light as we see that there is a defined minimum packet, or quanta, of light energy, which is proportional to frequency. This seeming contradiction within the framework of Maxwellian theory is resolved in the much richer context of quantum mechanics, a branch of physics which this experimental observation helped to develop and eventually flourish.

---

[1] Harnell and Livingood. *Experimental Atomic Physics*. McGraw-Hill, 1933.
[2] MIT Department of Physics. The photoelectric effect. Lab Guide, 2007.
[3] A. Einstein. *Annalen der Physik*, 132, 1905.
[4] R. Eisberg and R. Resnick. *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*. John Wiley & Sons, Inc., 1985.
[5] Thermo-Oriel. Oriel high power mercury lamp specifications. Specs.
[6] Y. Kraftmakher. *Am. J. Phys.*, 2, 2006.