# Lab 1: As the electron turns—
# An investigation of relativistic momentum and energy

## Introduction

In this lab, we will analyze a photograph of a track left behind by a fast electron moving through a bubble chamber (see Figure 1). As the electron moves through liquid hydrogen in the chamber, it collides with and transfers energy to the hydrogen molecules, producing hydrogen gas and showing the electron's path. The electron is moving in an intense magnetic field of strength $B = 1.3T$ that we will assume is perpendicular to the velocity of the electron. We would expect this magnetic field to make the electron move with a circular path, but the motion is instead a spiral because the electrons are constantly losing energy due to their collisions with the hydrogen molecules.

We are going to determine the kinetic energy and momentum of the electron at various points along its path. Einstein's theory of special relativity gives a relationship between the kinetic energy and momentum for a relativistic particle that is much different than the classical formulation. In this lab, we will find out which description best applies to these bubble track electrons.

For this particular lab, instead of using SI units, we will use the cgs (centimeter-gram-second) unit system, which is also known as the Gaussian system. This system of units is often helpful when studying electrodynamics. The unit of energy used in the cgs system is called the *erg*, where 1 *joule* = $10^7$ *ergs*.

## Experiment

In order to determine if the momentum and energy of electrons is better described by classical or special relativity, you will pick out at least six well-spaced points along the path of the electron in the photograph, and determine the momentum and energy of the electron at each location. Then you will determine whether the classical relationship between momentum and energy or the relativistic relationship between momentum and energy best fit your data.

*Kinetic energy*

The distance traveled by the electron in the liquid hydrogen depends on how much kinetic energy the electron has initially. We have available to us results from experiments where electrons with known kinetic energy have been shot into liquid hydrogen, and the distance they traveled, *s*, before they stopped was measured (Figure 2). Measure the distance the electron travels from each marked point to the point where it stops. (We have some special map rollers to help us do this, but **note that you should use the scale on the picture, which is not the same as the scale of the roller or the ruler**.) Use the graph in Figure 2 to determine the kinetic energy corresponding to that travel distance.

*Momentum*

We can determine the electron's momentum at various points on its path by determining the radius of curvature for small sections of the path along the way. The electron in the bubble track pictures is constantly losing energy to the liquid hydrogen, and therefore the radius of curvatures of its path gets smaller and smaller. The momentum of an electron in a magnetic field depends on the local radius of curvature of its path and on the strength of the magnetic field in the following way:

$$p = eBR. \qquad (1)$$

In cgs units, $eB = 2.08\times10^{-16}\ g/s$, so we can rewrite equation (5) as

$$p = (2.08 \times 10^{-16}\ g/s) \cdot R\,, \tag{2}$$

where $R$ is in centimeters.

One way to determine the radius of curvature of a section of path is to draw tangents to the path on either side of the point of interest as well as a tangent line at the point (Figure 3). Then lines drawn perpendicular to each of these tangents will cross at the center of the circle that fits the path at the point. (Again remember that the scale on the paper and the measurements with your ruler are not the same.)

**Theory**

According to classical Newtonian mechanics, the momentum of the electron is given by

$$p = m_e u\,. \tag{2}$$

The kinetic energy can be written in terms of this momentum as

$$KE_{classsical} = \frac{1}{2} m_e u^2 = \frac{p^2}{2m_e}\,. \tag{3}$$

According to special relativity, the kinetic energy of an electron is given by

$$KE_{relativistic} = \sqrt{p^2c^2 + m_e{}^2c^4} - m_e c^2\,, \tag{4}$$

where $c$ is the speed of light ($c = 3.00 \times 10^{10}\ cm/s$ in cgs units).

**MATLAB analysis**

1. After gathering your measurements of path radius and distance traveled to the electron's stopping point, enter your data into KGraph, in separate labeled columns for "Radius (cm)", "Unc in Radius (cm)", "Distance traveled (cm)", and "Distance traveled (cm)". Your column titles don't have to be identical to these titles but make sure those titles are descriptive and easy to understand.

2. Save your data in an Excel file, giving it a specific title that will clearly identify that data. Note that using names like "Data1" or "Lab2" is not as useful as a title like "BubbleTrackData".

3. Download "BubbleTrackDataAnalysis.m" from KATIE, open the script in MATLAB, and add your name and the date to the header of the script. Go through the script and try to get the sense of what the code is doing. This script has the framework you need, but some key parts are missing so the script will not run as is. The first 44 lines of the script bring in the data from your Excel document and name the variables appropriately.

4. The first line you'll need to change is line 49. The values for the radius and distance traveled that you measured need to be rescaled so that the distances you measured on the paper correspond with distances in the actual experiment. On line 49 of BubbleTrackDataAnalysis.m, enter the appropriate conversion factor which will convert your measured distances to actual distances.

5. On lines 57 and 58 of the script, insert the equation to compute momentum and the uncertainty in momentum.

6. One way to determine the relationship between the range of an electron and its kinetic energy is to use Figure 2 from this handout to create a conversion equation between range and kinetic energy. Assuming that the curve shown in Figure 2 is linear, determine the equation for that curve. Put your equation in the form: KE = slope × range + intercept. What is the range of validity of that equation? If your data fall outside that range of validity, what must you do in order to analyze that data?

7. Using the equation that you determined in step 6, add to lines 62 and 63 of the script the values for your slope and intercept.

8. On lines 87 and 93 you will enter the classical and relativistic formulas that relate momentum to kinetic energy. First let's enter some constants in lines 71 and 73. That way we don't have to type in the value of these constants every time they appear later in the script. Enter on line 71 the mass of the electron in grams. Enter on line 73 the speed of light in cm/s.

9. Now enter the equations to calculate the classical and relativistic kinetic energy on lines 87 and 93. Enter these equations just like you would enter a regular equation, except because p is a vector, we need to use the special operator for squaring an element of a vector: `.^2` . We want to use this operator instead of the ^2 operator because ^2 multiplies the vector times itself. We just want to square each element, so use the .^2 operator.

10. Run your script and check for any errors. If you do have errors, use your resources (MATLAB help, Dr. Flater, Google search, etc) to debug your code. If your code runs without error, check your processed data. Is the measured distance to actual distance converted correctly? Is the momentum calculated correctly from the radius? Is the kinetic energy calculated correctly from the range? If you find any discrepancies, determine how you need to modify your script so that the calculations are performed correctly.

11. If you are satisfied with your graph, save the graph as a .mat file and as a .jpg file, so you have the plot available to you for your lab write-up (more on the lab write-up next week).

**Interpretation**

12. Can you determine how fast are the electrons moving in the bubble chamber? If so, try to determine the velocity of the electron a couple points on its path.

13. In your lab notebook (along with everything else you are documenting), determine the theoretical slope of the kinetic energy *vs.* momentum curve when a particle is traveling at almost the speed of light. Show your derivation in the lab report, and comment on how this slope compares to the actual slope of your data at high speeds.

    When you write you lab report (we'll talk more about this next week), make sure you integrate your answers to 12 and 13 into your lab report, not just as separate unconnected answers.

**List of Figures**

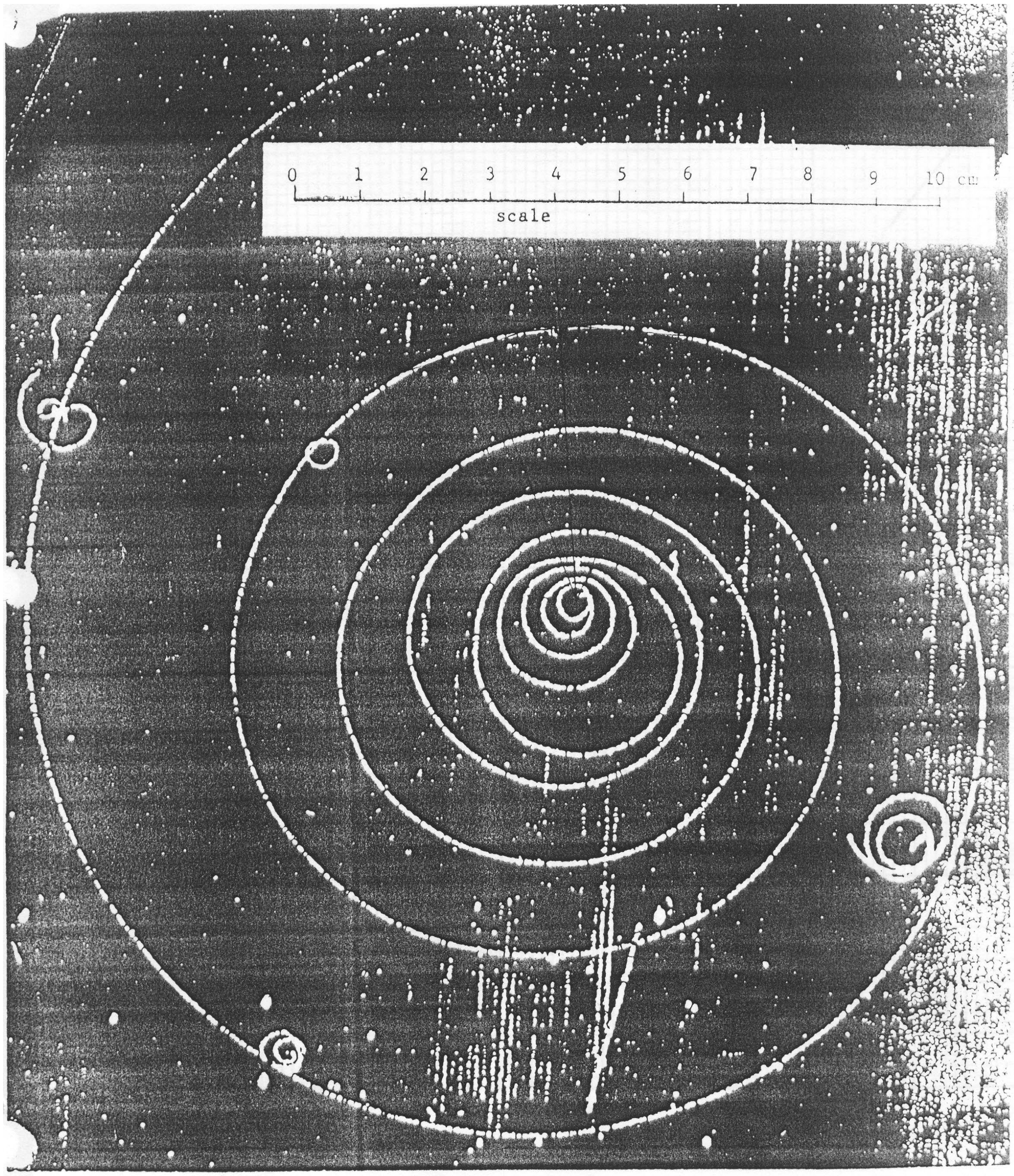


**Figure 1. A photograph of a path of an electron in a magnetic field of 1.3 T in a liquid hydrogen bubble chamber. The electron starts at a large velocity and slows down as it collides with hydrogen molecules in the chamber. The radius of curvature of the electrons path decreases as the electron loses energy and th electrons spirals into the cetner until it finally stops.**

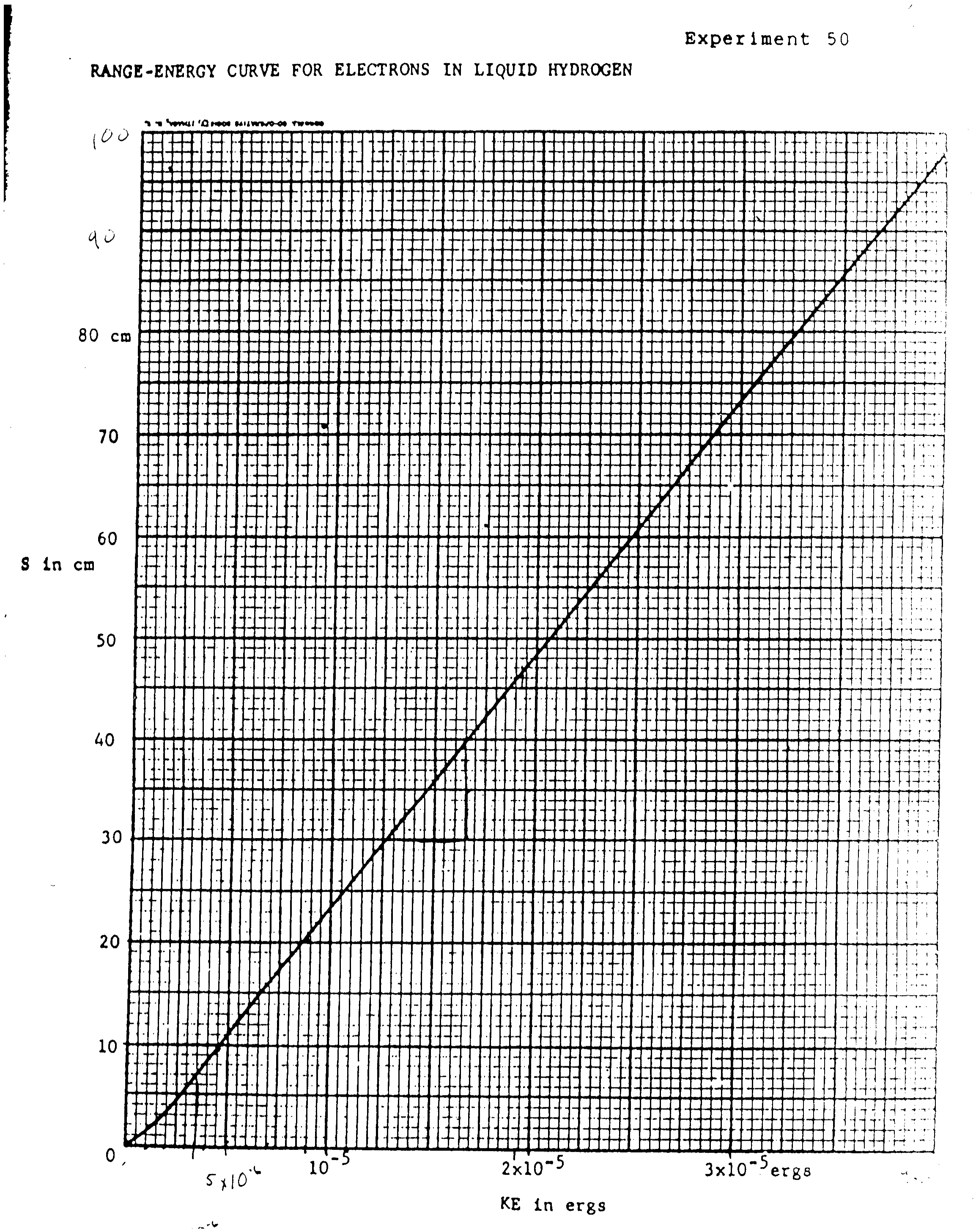


**Figure 2. The range of an electron vs. its kinetic energy.**

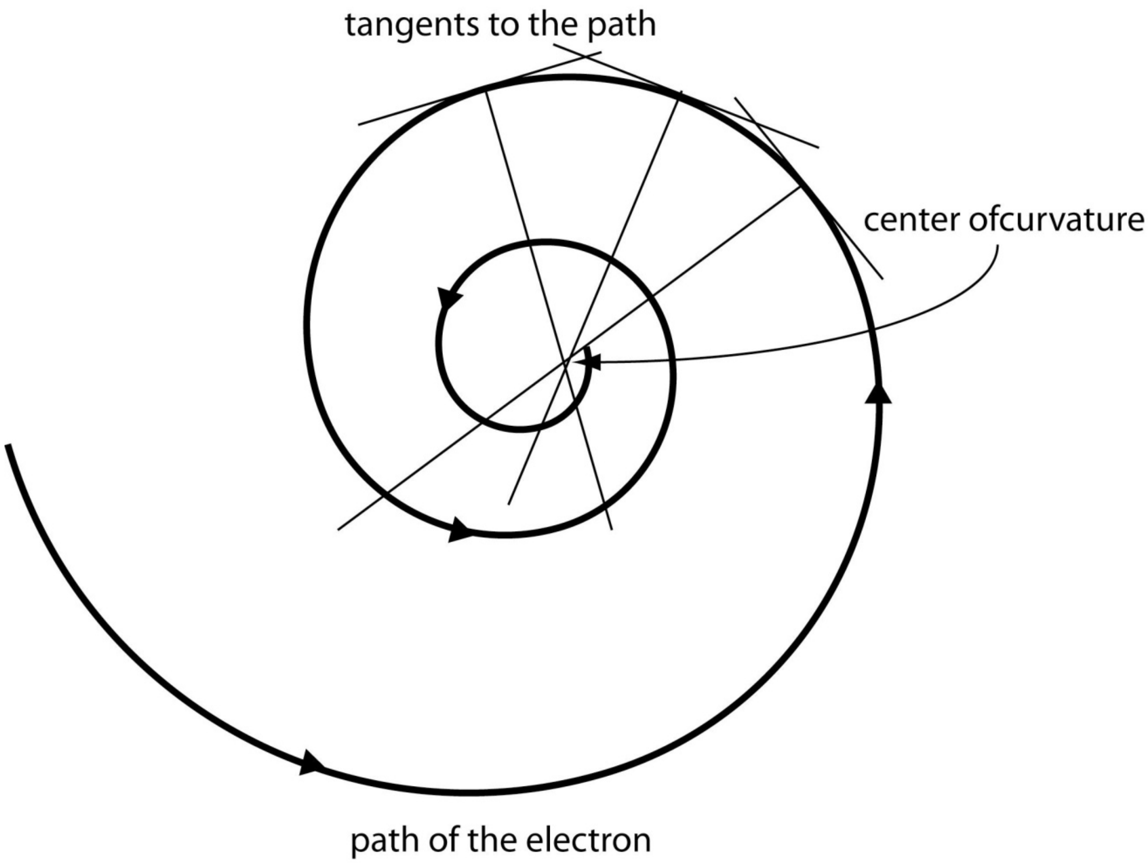


**Figure 3. An example of a bubble track and how to determine the radius of curvature of the path. If one draws lines that are tangent to the electron's path, the radial lines are perpendicular to these tangent lines. The lines should intersect to show the center of the electron's circular path.**