csc410-a3

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1 Question 1

1.1 a

4 paths in total
(3,4,5,7,8,9,15)
(3,4,5,7,11,12,13,15)
(3,7,8,9,15)
(3,7,11,12,13,15)

1.2 b

Not feasible path: (3,4,5,7,8,9,15)

Line No.	Assignment	Path Condition
3, 4, 5	$x \leftarrow -x, y \leftarrow -y$	$x < y$
7,8,9		$-x \leq -y$ AND $x < y$
15		$-x \leq -y$ AND $x < y$

No Solution, infeasible path.
Because we're not able to satisfy $-x <= -y \wedge x < y$ at the same time when reaching 7,8,9

1.3 c

Line No.	Assignment	Path Condition
3		$x \geq y$
7,8,9	$x \leftarrow -x, y \leftarrow -y$	$x \leq y$ AND $x \geq y$
15		$x \leq y$ AND $x \geq y$

Since at line 7,8,9, we need a condition satisfy $x \leq y \wedge x \geq y$ to make it a feasible path, which is $x = y$, and line 15 asserts $x < y$, then in order to not violate the path we need $x = y \wedge x < y$, and it's impossible. Thus, it's a violation

2 Question 2

2.1 a

$\Box(r \Rightarrow \bigcirc(r \bigcup \Box(\neg r)))$

2.2 b

$\Box(\neg o \wedge (r \Rightarrow \bigcirc(w)) \wedge (w \Rightarrow \bigcirc(r)) \wedge (r \vee w))$

2.3 c

$\Box(\neg o \wedge (r \vee b \vee w) \wedge (r \Rightarrow \bigcirc(r \bigcup b)) \wedge (b \Rightarrow \bigcirc(b \bigcup w)) \wedge (w \Rightarrow \bigcirc(w \bigcup r)$

2.4 d

3.2 b

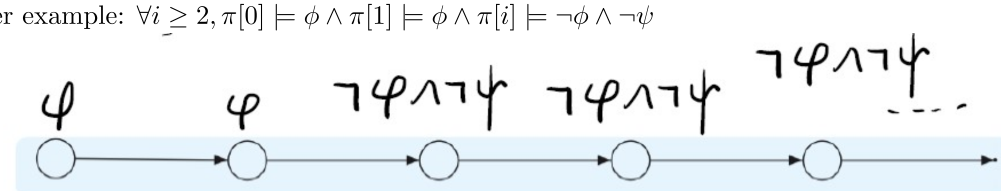
$(\Diamond\Box\phi_1) \wedge (\Diamond\Box\phi_2)$
$\equiv \exists j_1 \geq 0, \pi[j...] \models \Box\phi 1 \wedge \exists j_2 \geq 0, \pi[j...] \models \Box\phi_2$
$\equiv \exists j_1 \geq 0, \forall k \geq j_1, \pi[k...] \models \phi_1 \wedge \exists j_2 \geq 0, \forall i \geq j_2, \pi[i...] \models \phi_2$
$\Rightarrow \exists j_3 = max\{j_1, j_2\}, \forall i \geq j_3, \pi[i..] \models \phi_1 \wedge \pi[i..] \models \phi_2$
$\equiv \Diamond(\Box\phi_1 \wedge \Box\phi_2)$

$\Diamond(\Box\phi_1 \wedge \Box\phi_2) \equiv \exists j_3, \forall i \geq j_3, \pi[i..] \models \phi_1 \wedge \pi[i..] \models \phi_2$
$\Rightarrow \exists j_1 = j_3 \geq 0, \forall i \geq j_1, \pi[i..] \models \phi_1 \wedge \exists j_2 = j_1 = j3 \geq 0, \forall i \geq j_2, \pi[i..] \models \phi_2$
$\equiv \Diamond\Box\phi_1) \wedge (\Diamond\Box\phi_2)$, thus

$(\Diamond\Box\phi_1) \wedge (\Diamond\Box\phi_2) \equiv \Diamond(\Box\phi_1 \wedge \Box\phi_2)$

3.3 c

counter example: $\forall i \geq 2, \pi[0] \models \phi \wedge \pi[1] \models \phi \wedge \pi[i] \models \neg\phi \wedge \neg\psi$



3.4 d

$\phi \bigcup (\psi \vee \neg\phi)$
$\equiv \exists j \geq 0, \forall i, 0 \leq i < j, \pi[j...] \models (\psi \vee \neg\phi) \wedge \pi[i...] \models \phi$
$\equiv \pi[j] \models (\psi \vee \neg\phi)$
$\equiv \pi[j] \models \psi \vee \pi[j] \models \neg\phi$
$\equiv \exists j_1 = j, \pi[j_1] = \psi \vee \exists j_1 = j, \pi[j_1] = \neg\phi$
$\equiv \Diamond\psi \vee \neg\forall j \geq 0, \pi[j] \models \phi$
$\equiv \Diamond\psi \vee \neg\Box\phi$
$\equiv \Box\phi \implies \Diamond\psi$

3.5 e

$\bigcirc\Diamond\phi \equiv \pi[1..] \models \Diamond\phi$
$\equiv \exists j \geq 1, \pi[j] \models \phi$
$\equiv \exists j \geq 1, i = j - 1 \geq 0, \pi[j] \models \phi$
$\equiv \exists i \geq 0, \exists j = i + 1, \pi[j] \models \phi$
$\equiv \exists i \geq 0, \pi[i] \models \bigcirc(\phi)$
$\equiv \Diamond \bigcirc \phi$

4 Question 4

4.1 a

satisfiable
path satisfied example:
$\pi \models \forall j \geq 1, \exists 0 \leq i < j, \pi[j] = b \wedge \pi[i..] = a \wedge \pi[0] \neq b$

path not satisfied example:
$\pi \models \exists j \geq 1, \pi[j-1] = \neg a \wedge \pi[j] = b$

Thus, satisfiable

4.2 b

valid
$\bigcirc(a \vee \Diamond a) \Rightarrow \Diamond a \equiv \neg(\bigcirc(a \vee \Diamond a)) \vee \Diamond a \equiv \bigcirc(\neg a \wedge \neg \Diamond a) \vee \Diamond a \equiv \bigcirc(\neg a \wedge \Box \neg a) \vee \Diamond a$

$$\equiv (\forall j \geq 1, \pi[1] \neq a \wedge \pi[j] \neq a) \vee (\exists j \geq 0, \pi[j] = a) \equiv$$

$$\forall j \geq 1, \pi[j] \neq a \vee \exists j \geq 0, \pi[j] = a \equiv$$

$$\forall j \geq 1, \pi[j] \neq a \vee \pi[0] = a \vee (\pi[0] \neq a \wedge (\exists j \geq 1, \pi[j] = a)) \equiv$$

$$\forall j \geq 1, \pi[j] \neq a \vee \pi[0] = a \vee ((\forall j \geq 1, \pi[j] \neq a \vee \pi[0] \neq a) \wedge (\forall j \geq 1, \pi[j] \neq a \vee \exists j \geq 1, \pi[j] = a)) \equiv$$

$$\forall j \geq 1, \pi[j] \neq a \vee \pi[0] = a \vee ((\forall j \geq 1, \pi[j] \neq a \vee \pi[0] \neq a) \wedge true) \equiv$$

$$(\forall j \geq 1, \pi[j] \neq a \vee \pi[0] = a) \vee (\forall j \geq 1, \pi[j] \neq a \vee \pi[0] \neq a) \equiv$$

$$(\forall j \geq 1, \pi[j] \neq a \vee (\pi[0] = a \vee \pi[0] \neq a)) \equiv$$

$$\forall j \geq 1, \pi[j] \neq a \vee true \equiv true$$

5 Question 5

5.1 a

$\forall \Box(\forall \neg g \bigcup g \wedge \forall \bigcirc (\forall g \bigcup \neg g \wedge \forall \bigcirc (\forall \neg g \bigcup g \wedge \forall \bigcirc (\forall g \bigcup \forall \Box \neg g))))$

5.2 b

$\forall \Box(\exists \Box(r \Rightarrow \exists \bigcirc (\forall r \bigcup b) \vee w \Rightarrow \exists \bigcirc (\forall \Diamond b)) \wedge \forall \Box(b \Rightarrow \forall \bigcirc (\forall \Box \neg r)))$

5.3 c

$\forall \Box($
$r \Rightarrow \forall \bigcirc (\forall \Diamond \neg r)$
$\wedge g \Rightarrow \forall \bigcirc (\forall \Diamond \neg g))$
$\wedge w \Rightarrow \forall \bigcirc (\forall \Diamond \neg w)$
$\wedge b \Rightarrow \forall \bigcirc (\forall \Diamond \neg b)$
$)$

5.4 d

$\forall\Box($
$w \wedge \forall \bigcirc (\forall w \bigcup b) \Rightarrow \forall \bigcirc (b \Rightarrow \forall \bigcirc (\neg b \vee \forall \bigcirc \neg w))$
$)$

6 Question 6

$TS' \iff TS \wedge \forall\Box(\Phi \wedge \neg\Psi \vee \exists\Diamond(\Psi \vee \neg\Phi))$
$\models \exists(\Phi \bigcup \Psi) \wedge \forall\Box(\Phi \wedge \neg\Psi \vee \exists\Diamond(\Psi \vee \neg\Phi))$
$\iff \exists\pi \in Paths(TS') : \pi \models (\Phi \bigcup \Psi) \wedge \forall\Box((\Phi \wedge \neg\Psi) \vee \exists\Diamond(\Psi \vee \neg\Phi))$
$\iff \exists\pi \in Paths(TS') : \pi \models (\exists j \geq 0, \forall 0 \leq i < j, \pi[i..] \models \Phi \wedge \pi[j..] \models \Psi) \wedge (\forall j \geq 0, \pi[j] \models (\Phi \wedge \neg\Psi) \vee \exists k \geq j, \pi[k] \models (\Psi \vee \neg\Phi))$
$\iff \exists\pi \in Paths(TS') : \pi \models (\exists j \geq 0, \forall 0 \leq i < j, \pi[i..] \models (\Phi \wedge \neg\Psi) \wedge \pi[j] \models \Psi \vee \pi[j] \models \Phi \vee (\Psi \vee \neg\Phi))$
$\Rightarrow \exists\pi \in Path(TS') : \pi \models (\exists j \geq 0, \pi[j] = \Psi)$
$\models \exists\Diamond\Psi$

7 Question 7

see source.zip