

# Belief Distortions, Asset Prices, and Unemployment Fluctuations\*

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January 2, 2026

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## Abstract

This paper studies the dynamics of asset prices and unemployment when expectations deviate from a rational benchmark. Using machine learning forecasts as a benchmark for objective beliefs, I quantify distortions in survey forecasts of firms' future cash flows. Survey forecasts overreact to cash flow news, while machine forecasts do not. These belief distortions explain over 60% of the variation in hiring at both the aggregate and firm levels. Following positive idiosyncratic shocks to their cash flows, firms with distorted beliefs overhire relative to firms with objective beliefs, while their stock returns initially overshoot and subsequently reverse. A search model in which firms learn about cash flows with fading memory reproduces not only the overreaction in beliefs but also the resulting volatility in asset valuations and unemployment. Distorted beliefs that raise asset valuations also raise the value firms attach to new hires, leading stock prices and hiring to move together.

JEL Classification: E71, E24, E27, G41, J64

Keywords: Behavioral, Beliefs, Risk premia, Business Cycles, Unemployment

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\*I am grateful to Jaroslav Borovička, Sophia Chen, Daniel Greenwald, Sydney Ludvigson, Virgiliu Midrigan, and participants at seminars and conferences for their valuable comments. All errors remain my own.

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# 1 Introduction

Why are asset prices and unemployment so volatile? Unemployment is a key business-cycle indicator and a target of macroeconomic policy. Yet the standard search-and-matching (DMP) model generates unemployment volatility an order of magnitude smaller than observed in the data, a disconnect known as the “unemployment volatility puzzle” (Shimer, 2005). Recent rational models have proposed explaining this disconnect with time-varying discount rates inferred from asset prices, with the assumption that recessions bring high risk premia that depress the present value of the long-term cash flows generated from firms’ hiring decisions (Hall, 2017; Borovičková and Borovička, 2017; Kehoe et al., 2023). At the same time, a growing body of evidence finds that stock market investors and firm managers are neither perfectly rational nor efficient at processing information. Despite this evidence, little research has attempted to quantify the extent to which distortions in subjective beliefs (if any) can explain the unemployment volatility puzzle.

This paper addresses this gap in the literature. I measure firms’ subjective expectations using consensus survey forecasts from equity research analysts and CFOs, which proxy for the beliefs of corporate managers who make hiring decisions.<sup>1</sup> I then construct a real-time measure of non-distorted and efficient expectations using the machine learning methodology developed in Bianchi et al. (2025) (BLLM, hereafter) to quantify distortions and inefficiencies in subjective beliefs. For brevity, I refer to such deviation, whether due to nonrational expectation or informational inefficiency, as a belief distortion of subjective beliefs from the objective machine benchmark, and use the BLLM methodology to quantify the extent to which these belief distortions explain fluctuations in the labor market. While BLLM apply this approach to explain distortions in subjective expectations of earnings growth and stock market returns, I extend it to show that these same belief distortions also generate large fluctuations in unemployment and hiring.

I document several results showing that belief distortions in expected cash flows, measured by the deviation between the survey and machine forecast of corporate earnings, can quantitatively account for much of the observed variation in aggregate unemployment and firm-level hiring, as well as predictable patterns in stock returns. First, I find that subjective beliefs implied by survey forecasts systematically overreact to cash flow news while machine forecasts do not, confirming the presence of belief distortions. Second, this distortionary component explains a large share of observed variation in both labor markets and asset prices. In the aggregate time

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<sup>1</sup>Analyst and managerial earnings forecasts are highly correlated when both are available (Gennaioli et al., 2016), and managers routinely incorporate analyst projections into corporate decisions (Kothari et al., 2016). Analysts also communicate directly with managers and share similar information sets (Brown et al., 2015), making their forecasts a useful measure of the belief environment relevant for hiring decisions.

series, it accounts for 90% of vacancy filling rates and 68% of unemployment. In the cross-section across firms, it explains 66% of the dispersion in hiring rates, 50% of profits per worker, and 62% of stock returns. Third, following positive idiosyncratic shocks, firms with distorted beliefs overreact to the shock and overhire relative to firms whose beliefs are less distorted, leading to lower subsequent profits per worker. These same firms experience stock returns that initially overshoot, followed by a predictable reversal. Finally, a calibrated search-and-matching model consistent with these distortions accounts for 60% of U.S. unemployment volatility and 66% of hiring dispersion, compared with 16% and 30% under an otherwise equivalent rational model.

This paper makes two contributions to understanding the link between asset prices and unemployment fluctuations. First, I go beyond documenting violations of full information rational expectations (FIRE) to quantifying their importance for labor market outcomes and asset valuations. While previous work has investigated whether forecast errors made by survey respondents deviate from FIRE, knowing whether a deviation exists does not necessarily quantify its importance for hiring behavior. Second, belief overreaction offers an alternative to models with time-varying discount rates that generates testable predictions at the firm level linking employment decisions to return predictability. Existing models explain employment fluctuations with rational time-varying discount rates in which idiosyncratic shocks are not priced. This would be inconsistent with the evidence documented here that such shocks have important implications for both hiring decisions and stock returns when beliefs are distorted.

The evidence for survey overreaction comes from comparing survey forecasts against machine-based predictions. The machine learning algorithm trains a neural network on the same historical information available to survey respondents and deviates from survey forecasts only when respondents made predictable mistakes.<sup>2</sup> This design ensures that the gap between survey and machine forecasts isolates inefficiencies in belief formation rather than differences in information or hindsight. Survey overreaction manifests in a specific pattern: positive forecast revisions systematically predict negative forecast errors, indicating that upward adjustments are too large and subsequently reversed. Machine forecasts exhibit no such pattern, confirming they avoid the predictable mistakes present in survey expectations. Therefore, the wedge between these two measures, the belief distortion, arises specifically from survey overreaction to news rather than from any systematic errors in the machine forecasts. This pattern holds consistently both in the aggregate time series and across individual firms, providing evidence that the distortions reflect

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<sup>2</sup>The machine learning forecast approximates Bayesian updating in a high-dimensional learning problem, with regularization acting as a prior that excludes economically implausible extremes (Martin and Nagel, 2022). This prior reduces the variance of the forecasts, which limits their overreaction and improves their out-of-sample accuracy.

genuine biases in expectation formation rather than measurement error.

Using the firm’s hiring condition from a search-and-matching model, I link this belief distortion to hiring behavior by decomposing the value of hiring into expected cash flows, measured by expected earnings, and discount rates, measured by expected stock returns. The structure of this decomposition is analogous to the present-value identity of Campbell and Shiller (1988), which attributes fluctuations in valuation ratios in asset markets to revisions in expected cash flows versus discount rates. Comparing the decomposition under survey and machine forecasts quantifies how much of the variation in hiring arises from biased beliefs.

The decomposition reveals that hiring is substantially more sensitive to cash flow news under subjective beliefs than under objective beliefs. In models that rely on rational expectations with time-varying discount rates, it is perhaps unsurprising that objective measures of discount rates dominate, explaining 69% of aggregate variation and 60% of cross-sectional dispersion in hiring. By contrast, under subjective beliefs, distorted cash flow expectations account for 97% of aggregate variation and 79% of cross-sectional dispersion. A key result is that it is not the subjective belief *per se*, but rather the identified distortion in these beliefs that drives the greatest part of this high sensitivity. The distortionary component of subjective expectations defined as the gap between survey and machine forecasts dominates the explanatory power for both aggregate fluctuations and cross-sectional dispersion. Within the aggregate decomposition, building on evidence from Shimer (2005) that the vacancy filling rate accounts for roughly three quarters of unemployment fluctuations, this implies that belief distortions explain approximately  $90\% \times 3/4 = 68\%$  of aggregate unemployment fluctuations. The results are unlikely to arise from random noise or analyst bias, since such factors would weaken, not strengthen, the observed link between belief distortions and hiring, which operates through firms’ subjective asset valuations.

Firm-level responses to idiosyncratic shocks to firms’ cash flows, measured using firms’ earnings growth, differ sharply under objective versus subjective expectations. For firms holding close to objective beliefs, hiring should respond only weakly to idiosyncratic earnings shocks, which are transitory and unpriced. While such shocks are not priced into risk premia, rational models do not preclude hiring responses to idiosyncratic shocks more generally. Rather, because these particular shocks I show are highly transitory and hiring involves costly adjustment, a firm with close to objective beliefs correctly anticipates limited benefits from expanding employment before the shock dissipates. Consistent with this prediction, when earnings expectations align closely with the machine-learning benchmark, profits per worker remain stable after idiosyncratic shocks, reflecting efficient employment adjustments without overreaction. Stock returns also show minimal response to these shocks under objective beliefs, consistent with rational asset

pricing models in which diversifiable risk is not priced.

By contrast, firms with distorted beliefs overreact to both aggregate and idiosyncratic shocks, generating a strong response in their hiring behavior and stock returns. Firms whose analyst forecasts are overly optimistic relative to the objective benchmark exhibit clear signs of over-hiring. Inflated beliefs about the expected value of hiring new workers lead firms to expand employment aggressively, raising labor costs faster than revenues and causing both profits and profits per worker to decline. Simultaneously, their stock prices initially overshoot as optimistic beliefs get embedded in valuations, but subsequently reverse as earnings disappoint, generating predictable negative abnormal returns. Quantitatively, belief distortions account for 82% of the explanatory power of subjective forecast revisions for contemporaneous profits per worker. In contrast, objective forecast revisions explain only about 10% of the same variation. This finding demonstrates that belief distortions provide a new mechanism through which firm-level hiring responds to non-priced idiosyncratic shocks and creates predictable patterns in stock returns.

Finally, I interpret this empirical evidence quantitatively through the lens of a structural model. I calibrate a search-and-matching model with constant-gain (fading-memory) learning that matches the documented over-reaction. The model allows me to construct a counterfactual economy where firms form full information rational expectations when hiring, which can be compared against the economy we observe in the data where firms hold distorted beliefs. In the model, firms learn about the long-run mean of their cash flow process with fading memory. Firms with fading memory overreact to observed cash flow news because they place excessive weight on recent observations when updating beliefs, interpreting transitory shocks as more persistent than they truly are. Overoptimism about future cash flows in transitory booms inflates the perceived value of hiring and leads firms to post more vacancies than warranted, tightening labor markets. In recessions, pessimism overshoots, reducing the perceived value and leading to a collapse in hiring. This distorted belief about expected cash flows then drives excessive fluctuations in the value of hiring, which allows the model to generate the large subjective cash flow component in the variance decompositions.

**Related Literature** This paper contributes to the literature on unemployment fluctuations, labor markets, asset prices, and expectation formation by quantifying the extent to which distorted beliefs relevant to asset prices can explain labor market fluctuations.

First, it relates to the literature on the unemployment volatility puzzle in search-and-matching models under the Diamond, Mortensen, and Pissarides (DMP) framework (Shimer, 2005).<sup>3</sup> The

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<sup>3</sup>See also Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Pissarides (2009), Elsby and Michaels

search model struggles to generate sufficient volatility in unemployment unless firms’ responses to shocks are amplified through mechanisms such as time-varying discount rates (Hall, 2017; Borovičková and Borovička, 2017; Kehoe et al., 2023).<sup>4</sup> Another strand of the labor search literature attributes high unemployment volatility to wage rigidity arising from credible bargaining (Hall, 2005; Hall and Milgrom, 2008), staggered negotiations (Gertler and Trigari, 2009), or incentive contracts (Gaur et al., 2023). Existing work in this literature generally assumes that firms rationally process information about cash flows and discount rates. My paper complements this approach by highlighting distortions in subjective beliefs as a distinct mechanism generating large unemployment fluctuations. My contribution is to show that firms’ overreaction to cash flow news can generate aggregate unemployment volatility and the cross-sectional dispersion in hiring, a dimension that existing aggregate models with time-varying discount rates miss.

A growing literature embeds nonrational expectations in models of labor market frictions (Venkateswaran, 2014; Acharya and Wee, 2020; Mueller et al., 2021; Balleer et al., 2021; Menzio, 2023; Faberman et al., 2022; Bhandari et al., 2024; Jäger et al., 2024; Du et al., 2025; Bigio et al., 2025; Ding, 2025; Mitra, 2025).<sup>5</sup> Notably, Bhandari et al. (2024) show that pessimism in households and firms can explain the volatility of unemployment fluctuations. Bigio et al. (2025) develop a theoretical framework with heterogeneous beliefs and operational leverage to link the volatility of asset prices to the volatility in the labor market. Du et al. (2025) document stickiness in workers’ job finding and separation expectations, showing that belief stickiness attenuates precautionary saving and leads workers to under-insure during recessions. My paper provides two pieces of complementary evidence. First, I directly measure expectational distortions by comparing survey forecasts with a real-time machine learning benchmark, quantifying their importance for asset prices and labor market fluctuations. Second, I document firm-level overreaction to idiosyncratic shocks in both hiring and stock returns, providing cross-sectional evidence that such shocks shape hiring decisions through belief distortions.

The empirical analysis builds on survey-based evidence on subjective expectations. On the firm side, a large literature documents systematic forecasting biases.<sup>6</sup> Ben-David et al. (2013),

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(2013), Kudlyak (2014), Chodorow-Reich and Karabarbounis (2016), and Ljungqvist and Sargent (2017).

<sup>4</sup>Related work that takes an asset pricing perspective on the labor market under rational expectations includes Merz and Yashiv (2007), Donangelo (2014), Belo et al. (2014), Favilukis and Lin (2015), Kuehn et al. (2017), Kilic and Wachter (2018), Mitra and Xu (2019), Donangelo et al. (2019), Kehoe et al. (2019), Liu (2021), Belo et al. (2023), and Meeuwis et al. (2023).

<sup>5</sup>Related work links belief distortions to credit cycles (Bordalo et al., 2021; Gulen et al., 2024) and shows loose credit generates boom-bust dynamics in employment (Blank and Maghzian, 2023). My paper complements this work by showing that distorted cash flow expectations affect hiring directly, over and above their effects through credit conditions.

<sup>6</sup>On the worker side, early work shows that workers’ beliefs including job loss expectations forecast their labor

Alti and Tetlock (2014), and Gennaioli et al. (2016), and Barrero (2022) show that managers extrapolate recent performance, generating predictable forecast errors that influence managerial decisions. Greenwood and Hanson (2014) provide evidence of similar overextrapolation in the dry bulk shipping industry, where firms overinvest during booms because they expect elevated earnings to persist, leading to excess volatility in investment. More recently, Bloom et al. (2025) use incentivized forecast data to show that firms exhibit significant overoptimism, predictable errors, and overprecision, and that these biases can meaningfully distort firm responsiveness to changes in fundamentals. My paper builds on this work by showing how distortions in survey expectations translate into labor market outcomes. Relatedly, Ma et al. (2020) link managerial forecast biases to distortions in investment, and Coibion et al. (2018) and Candia et al. (2020) document sluggish and dispersed inflation expectations among managers. My analysis is complementary: I show that distortions in expected earnings influence hiring decisions and unemployment fluctuations. While Gormsen and Huber (2025) emphasize the role of internal hurdle rates in shaping investment dynamics, I highlight how distorted beliefs about future cash flows can move labor demand even when discount rate expectations remain stable.

The variance decomposition and learning model in this paper builds on recent work using survey-based expectations to reassess the drivers of asset prices.<sup>7</sup> The approach adapts the present-value identity framework from Campbell and Shiller (1988) and Cochrane (2007), which attributes variation in valuation ratios to expected cash flows and discount rates. Recent survey-based applications challenge the traditional view that discount-rate variation dominates asset price fluctuations. Bordalo et al. (2024a) find that overreaction in long-term earnings expectations drives return predictability. De La O and Myers (2021) show that subjective expectations of cash flow growth explain most of the variation in valuation ratios, but remain agnostic about whether these expectations systematically over- or under-react to information. My contribution is to construct an explicit measure of objective beliefs that quantifies the magnitude and direction of distortions in subjective beliefs about firms' cash flows. This allows me to extend the variance decomposition results by demonstrating that belief distortions not only drive asset prices but also shape real decisions such as hiring, a channel not examined in prior work.

I adopt the machine learning approach of BLLM to measure the magnitude of distortions in subjective beliefs. As in Bianchi et al. (2022) and BLLM, I structure the algorithm so that

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market outcomes (Stephens, 2004).

<sup>7</sup>Related work includes Timmermann (1993), Barberis et al. (1998), Chen et al. (2013), Greenwood and Shleifer (2014), Collin-Dufresne et al. (2016), Adam et al. (2016), De La O and Myers (2021), Giglio et al. (2021), Nagel and Xu (2022), Jin and Sui (2022), van Binsbergen et al. (2022), Bordalo et al. (2024a), De La O et al. (2024), Adam and Nagel (2023), Décaire and Graham (2024), Bastianello et al. (2024), and Chaudhry (2025).

the machine’s forecasts can differ from the survey forecasts only if the machine finds evidence of predictable mistakes in the survey responses immediately prior to the machine making a true out-of-sample forecast. The method uses tools from machine learning by training Long Short-Term Memory (LSTM) neural networks with recursive re-estimation and hyperparameter tuning (Gu et al., 2020, Cong et al., 2020, Nagel, 2021, van Binsbergen et al., 2022, Bastianello, 2022, Bybee et al., 2024). The resulting forecasts are fully ex-ante and provide high-dimensional empirical counterparts to rational expectations for evaluating belief distortions. While BLLM focus on using identified belief distortions to explain stock market outcomes, my contribution is to extend their methodology to study labor market outcomes and show that distorted beliefs have real economic consequences beyond financial markets.

The rest of the paper proceeds as follows. Section 2 describes the data used in the empirical analysis. Section 3 documents overreaction in survey expectations and compares the predictive performance of machine and survey forecasts. Section 4 presents a search and matching model with belief distortions and derives a decomposition of the vacancy filling rate. Section 5 presents the estimated variance decomposition of the aggregate vacancy filling rate. Section 6 presents cross-sectional evidence motivated by a firm-level extension of the baseline model. Section 7 introduces a model of constant-gain learning about future earnings that could match the decompositions estimated from the data. Section 8 discusses model extensions and robustness checks. Finally, section 9 concludes. Appendix A contains additional results referenced in the main text. A separate Online Appendix (Sections OA through OE) provides supplementary results and technical details supporting the main analysis.<sup>8</sup>

## 2 Data

This section describes the data used to estimate the time-series and cross-sectional variance decompositions. For each outcome, I measure subjective expectations with survey forecasts  $\mathbb{F}_t[\cdot]$  and objective expectations with machine learning forecasts  $\mathbb{E}_t[\cdot]$ . The estimation sample is quarterly, 2005Q1-2023Q4. Throughout,  $t$  indexes quarters and  $i$  indexes firms.<sup>9</sup>

**Labor Market Outcomes** *Aggregate level.* The vacancy filling rate  $q_t$  measures the probability that a posted vacancy is filled in period  $t$ :

$$q_t \equiv \frac{\text{Total Hires}}{\text{Total Job Vacancies}} = \frac{f_t U_t}{V_t},$$

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<sup>8</sup>The Online Appendix is available at <https://doqlee.github.io/files/onlineappendix.pdf>

<sup>9</sup>Appendix OC provides additional details. Figure A.1 and Table OA.1 report stylized facts.



where  $V_t$  is total job vacancies from JOLTS job openings (available since 2000:12; earlier periods use the help-wanted index, Barnichon, 2010), and  $U_t$  is the unemployment level (BLS). The job finding rate  $f_t$  measures the probability that an unemployed worker finds a job within the month. Following Shimer (2005), I infer how many people must have exited unemployment each period from the decline in total unemployment after removing very short-term unemployed workers:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}},$$

where  $U_t^s$  is short-term unemployment less than 5 weeks (UEMPLT5). The job separation rate  $\delta_t$  is from JOLTS. Variables are constructed monthly, aggregated to quarterly averages, and detrended with an HP filter ( $\lambda = 10^5$ ) following Shimer (2005).

*Firm level.* Since firm-level vacancies are not readily observable, I proxy hiring at the firm level by the hiring rate, defined as net employment growth after accounting for job separations

$$hl_{i,t} \equiv \log\left(\frac{L_{i,t+1} - (1 - \delta_{i,t})L_{i,t}}{L_{i,t}}\right) = \log\left(\frac{\text{Total Hires}}{\text{Total Employment}}\right),$$

where  $L_{i,t}$  is employment for firm  $i$ , recorded at fiscal year-end and carried forward to quarterly frequency.  $\delta_{i,t}$  is the separation rate of firm  $i$ 's NAICS2 industry (from JOLTS). Including job separations ensures the hiring rate captures total hires needed to both replace departing workers and expand employment. The firm-level sample includes publicly listed firms with common stocks (share codes 10, 11) on NYSE/AMEX/NASDAQ with IBES analyst coverage of expected earnings and stock price targets.

**Earnings (Realized Cash Flows)** *Firm level.* Firm-level earnings  $E_{i,t}^*$  are constructed from IBES street earnings per share (EPS), converted to total earnings by multiplying by each firm's shares outstanding (1983Q4-2023Q4). Street earnings exclude one-off items that are not informative for ongoing operations and better proxy expected cash flows (Hillenbrand and McCarthy, 2024). To handle instances where firm earnings may be zero or negative, which would make log transformations undefined, I follow the approach of Vuolteenaho (2002) and define a transformed measure of earnings that is strictly positive. Specifically, each firm is treated as a portfolio consisting of a fraction  $(1 - \lambda)$  of equity and a small fraction  $\lambda$  of risk-free assets, such that

$$E_{i,t} = (1 - \lambda)E_{i,t}^* + \lambda r_t^f P_{i,t-1}, \quad \lambda = 0.10,$$

where  $P_{i,t-1}$  is lagged equity value and  $r_t^f$  is the one-year Treasury bill rate. The risk-free component ensures  $E_{i,t} > 0$  for all observations while preserving the cross-sectional variation in

earnings. This transformation allows log earnings  $e_{i,t} \equiv \log E_{i,t}$  to be well defined even in periods when reported earnings  $E_{i,t}^*$  are negative.

*Aggregate level.* I aggregate firm-level earnings to the S&P 500 level to obtain a time-series for aggregate earnings:

$$E_t = \Omega_t \sum_{i \in x_t} \frac{E_{i,t}^*}{Divisor_t},$$

where  $x_t$  is the set of S&P 500 firms with IBES data,  $\Omega_t$  is a scaling factor that adjusts for occasionally incomplete IBES coverage of the S&P 500, and  $Divisor_t$  is the S&P 500 divisor.

**Cash Flow Expectations (Surveys)** *Firm level.* Subjective earnings expectations  $\mathbb{F}_t[\Delta e_{i,t+h}]$  are obtained from the Institutional Brokers' Estimate System (IBES) database, which reports the median consensus forecasts of equity research analysts (1983Q4-2023Q4).

I use analyst forecasts as a proxy for managerial expectations. This approach is supported by three considerations. First, analyst and CFO forecasts of earnings growth are highly correlated (approximately 0.60 at the one-year horizon), indicating broadly consistent beliefs across groups (Gennaioli et al., 2016).<sup>10</sup> Second, prior research shows that analyst forecasts are widely followed by managers and investors and influence corporate decisions (Kothari et al., 2016). Third, analysts have access to similar information sets as managers and often communicate directly with them (Brown et al., 2015). Although analyst forecasts are not produced by firm managers themselves, they provide a market-based measure of expectations that reflects the informational environment in which managers form beliefs. Throughout, I interpret optimism or pessimism in analyst forecasts as indicative of the informational environment and belief distortions surrounding the firm's decision-making.

IBES analysts provide forecasts of "street" earnings per share (EPS) for each firm's upcoming fiscal years, typically over horizons of one to four years ahead, as well as a long-term growth (LTG) forecast representing the expected average annual growth in operating earnings over the next three to five years. These forecasts are released monthly and are widely followed by market participants as professional assessments of firms' expected performance. Because they are tied to specific firms, the IBES forecasts capture firm-level subjective expectations, which can be aggregated to the market level.

For each firm  $i$ , I compute expected earnings growth as the forward annual log difference between adjacent horizon level forecasts. Specifically, for  $h = 1, 2$ , the forecasted annual growth rate  $\mathbb{F}_t[\Delta e_{i,t+h}]$  is the log change between the  $h$ -year-ahead and  $(h-1)$ -year-ahead level forecasts.

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<sup>10</sup>Appendix Table A.1 reports robustness checks using managerial earnings forecasts from the quarterly CFO survey. The correlation between analyst and managerial expectations is 0.60 at the one-year horizon.

For horizons  $h = 3, 4, 5$ , I interpret the LTG forecast as the expected annualized growth rate over the next three to five years, following De La O and Myers (2021).

*Aggregate level.* Firm-level forecasts are aggregated to the S&P 500 level by value weights based on market capitalization, while long-term growth forecasts are aggregated using value-weighted averages. This aggregation produces an estimate of aggregate expected earnings growth that reflects the beliefs of analysts tracking the firms that constitute the S&P 500. These forecasts therefore serve as proxies for subjective expectations of future corporate cash flows at both the firm and aggregate levels.

**Discount Rates (Stock Returns)** Subjective expectations of stock returns  $\mathbb{F}_t[r_{t+h}]$  are drawn from two survey sources, depending on whether the analysis is at the aggregate or firm level.

*Aggregate level.* For the S&P 500, I use the quarterly *CFO Global Business Outlook Survey* (2001Q4-2023Q4), which reports the mean expected nominal return on the S&P 500 over horizons of one and ten years. Respondents include chief financial officers, vice presidents of finance, and other senior financial executives from a broad set of U.S. firms ranging from small private companies to large publicly listed corporations (approximately 1,600 members as of 2022). These expectations represent forward-looking assessments of aggregate equity returns held by decision makers directly involved in firms' investment and financing decisions. For intermediate horizons between one and ten years, I interpolate linearly between the reported forecasts.

*Firm level.* For individual firms, I use stock price target data from the *IBES* and *Value Line* databases, which report the median forecasted stock price over 12 month and 3 to 5 year horizons, respectively. These forecasts are provided by sell-side equity research analysts who specialize in evaluating firm fundamentals and valuation prospects. I convert these price targets into implied expected total returns as

$$\mathbb{F}_t[r_{i,t+h}] \approx \log \left( \frac{\mathbb{F}_t[P_{i,t+h}]}{P_{i,t}} + \frac{D_{i,t}}{P_{i,t}} \frac{\mathbb{F}_t[D_{i,t+h}]}{D_{i,t}} \right),$$

where  $P_{i,t}$  is the firm's current stock price (CRSP),  $D_{i,t}$  is its dividend (Compustat), and the expected dividend growth ratio  $\mathbb{F}_t[D_{i,t+h}]/D_{i,t}$  is set to 1.064, the postwar average for U.S. equities (Nagel and Xu, 2022). I interpret Value Line targets as five-year-ahead forecasts and interpolate intermediate horizons linearly between the IBES one-year and Value Line five-year projections.

**Price-Earnings Ratio** The current log price-earnings ratio is  $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$ . I construct  $\mathbb{F}_t[pe_{i,t+h}]$  using the (Campbell and Shiller, 1988) present-value identity:

$$\mathbb{F}_t[pe_{i,t+h}] = \frac{1}{\rho^h} pe_{i,t} - \frac{1}{\rho^h} \sum_{j=1}^h \rho^{j-1} (c_{pe} + \mathbb{F}_t[\Delta e_{i,t+j}] - \mathbb{F}_t[r_{i,t+j}]),$$

where  $c_{pe}$  is a constant and  $\rho = \frac{\exp(\overline{pe})}{(1+\exp(\overline{pe}))}$  is the discount factor from the log-linearization. I construct time-series data on actual and expected log price-earnings similarly by using the corresponding aggregate time series data at the S&P 500 level.

**Machine Learning Forecasts** For each survey forecast, I construct the corresponding machine learning forecast using Long Short-Term Memory (LSTM) neural networks following Bianchi et al. (2022, 2024, 2025). The LSTM model predicts stock returns or earnings growth  $y \in \{r, \Delta e\}$  at horizons  $h = 1, \dots, 5$  years using a data-rich set of real-time predictors:<sup>11</sup>

$$\mathbb{E}_t[y_{t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_t^{TS}; \boldsymbol{\lambda}_t^{TS}), \quad \mathbb{E}_t[y_{i,t+h}] = G(\mathcal{X}_{i,t}, \boldsymbol{\beta}_t^{CS}; \boldsymbol{\lambda}_t^{CS}),$$

where  $G(\cdot)$  is the nonlinear mapping learned by the LSTM network. The time-series predictor set  $\mathcal{X}_t$  includes real-time macroeconomic, financial, and textual variables (LDA-based news sentiment from the *Wall Street Journal*), as well as survey forecasts. Including the survey forecast allows the machine to combine public information with intangible private information embedded in the survey responses.<sup>12</sup> The cross-sectional set  $\mathcal{X}_{i,t} = \mathcal{X}_t \otimes \mathcal{C}_{i,t}$  augments  $\mathcal{X}_t$  with firm characteristics  $\mathcal{C}_{i,t}$  including valuation, profitability, size, momentum, volatility, and industry dummies (Chen and Zimmermann, 2022).

The parameters  $\boldsymbol{\beta}_t^{TS}$  and  $\boldsymbol{\beta}_t^{CS}$  are re-estimated dynamically over rolling samples (quarterly for the time-series model and annually for the cross-sectional model) to allow for evolving relationships between predictors and outcomes. In the cross-sectional specification, all firms share the same estimated parameters  $\boldsymbol{\beta}_t^{CS}$ , so differences in predicted outcomes arise only from firm-specific inputs  $\mathcal{X}_{i,t}$ . Regularization hyperparameters  $\boldsymbol{\lambda}_t^{TS}$  and  $\boldsymbol{\lambda}_t^{CS}$  control model complexity using  $L_1$  and  $L_2$  penalties, dropout layers, early stopping, and ensemble averaging. These techniques prevent overfitting and ensure smooth updates in the presence of structural change or regime shifts. The model's out-of-sample testing period spans 2005Q1-2023Q4.

<sup>11</sup>Appendix OE provides additional details about the machine learning algorithm and the list of predictors.

<sup>12</sup>While the machine uses survey forecasts as one input among many predictive variables, a rational agent would efficiently extract signals from all available public information, including potentially biased survey data, and debias them optimally. The machine learning model approximates this process by finding the optimal forecast given an information set that includes survey responses alongside other predictive variables.

I interpret the machine forecast as the ex-ante belief for an agent who can process large quantities of information efficiently given a prior that introduces shrinkage. Regularized training methods controlled by hyperparameters  $\lambda_t^{TS}$  and  $\lambda_t^{CS}$  introduce a prior for smoothness and parsimony. These regularization techniques correspond to Bayesian estimation under a complexity prior, where the forecast represents the posterior mean that minimizes expected squared loss. Importantly, the machine must make its forecast using only real-time information, without look-ahead bias. All inputs are restricted to historical, timestamped data that we verify were accessible to market participants at the time, ensuring the predictions reflect only information a firm manager could have used when deciding whether to hire.

### 3 Evidence of Belief Distortions

**Predictability of Survey vs. Machine Forecast Errors** To assess whether survey expectations deviate from objective expectations, Figure 1 reports the coefficient  $\beta$  from Coibion and Gorodnichenko (2015) regressions of survey forecast errors on survey forecast revisions at both the aggregate and firm levels:

$$\Delta e_{t+h} - \mathbb{F}_t[\Delta e_{t+h}] = \beta[\mathbb{F}_t[\Delta e_{t+h}] - \mathbb{F}_{t-1}[\Delta e_{t+h}]] + \varepsilon_t \quad (1)$$

$$\Delta e_{i,t+h} - \mathbb{F}_t[\Delta e_{i,t+h}] = \beta[\mathbb{F}_t[\Delta e_{i,t+h}] - \mathbb{F}_{t-1}[\Delta e_{i,t+h}]] + \alpha_i + \alpha_t + \varepsilon_{i,t} \quad (2)$$

where  $\Delta e_{t+h}$  denotes aggregate earnings growth (cash flows) in the time-series regression, and  $\Delta e_{i,t+h}$  denotes firm-level earnings growth in the cross-sectional regression, as defined in Section 2. The cross-sectional regressions include firm  $\alpha_i$  and time  $\alpha_t$  fixed effects, implying that the identifying variation comes from revisions in expectations that are idiosyncratic to each firm.

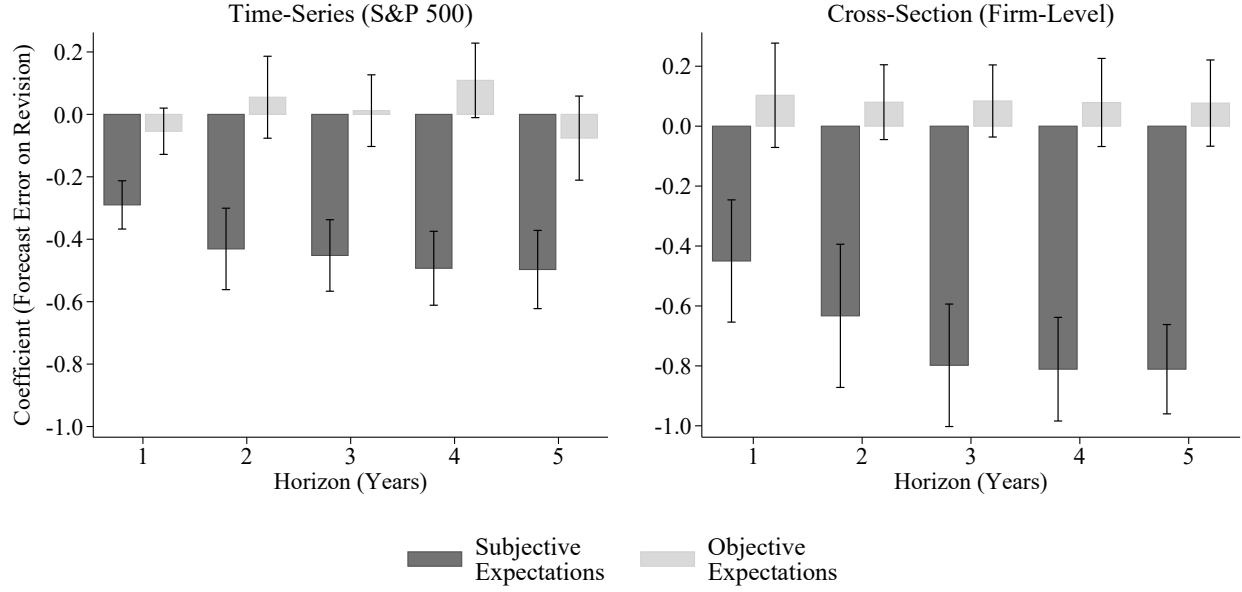
The left panel of Figure 1 shows time-series results for the S&P 500, while the right panel shows cross-sectional results for individual firms. In both cases, the coefficients on survey forecast revisions are negative, indicating overreaction. In particular, at the firm level, coefficients range from  $-0.433$  at the one-year horizon to  $-0.818$  at five years.<sup>13</sup> Upward revisions in survey forecasts are followed by negative forecast errors, suggesting that survey respondents respond too strongly to positive earnings news and generate overly optimistic forecasts.<sup>14</sup> In contrast, the

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<sup>13</sup>Appendix Figure A.2 reports analogous results for discount rate expectations (stock returns). Survey forecasts of discount rates also overreact to news, though the magnitude is smaller than for cash flows. Through the Campbell-Shiller present-value identity, this suggests cash flow overreaction dominates discount rate overreaction in driving price-earnings forecasts. For future price-earnings expectations, the regression coefficient is also negative at  $-0.709$ , showing that price-earnings forecasts overreact to news.

<sup>14</sup>Appendix Figure A.3 shows that overreaction in cash flow expectations occurs in both NBER expansions and recessions, with slightly stronger overreaction during recessions. This suggests that belief distortions are a robust feature of expectation formation that persists across economic states.

Figure 1: Predictability of Survey vs. Machine Forecast Errors of Cash Flows



*Notes:* Figure reports regression coefficients  $\beta$  from regressions of forecast errors on forecast revisions. Left panel: time-series regressions for the S&P 500. Right panel: cross-sectional regressions for a quarterly panel of listed firms with firm and time fixed effects. The forecast target is earnings growth  $\Delta e_{t+h}$  (cash flows). Time-series survey forecasts  $\mathbb{F}_t$  come from IBES. Cross-sectional survey forecasts  $\mathbb{F}_t$  come from IBES. Machine learning expectations  $\mathbb{E}_t$  are generated using a Long Short-Term Memory (LSTM) model trained in real time on macroeconomic, financial, textual, and firm-level data. The sample covers quarterly data from 2005Q1 to 2023Q4. Whiskers show 95% confidence intervals (Newey-West with 4 lags for time-series; two-way clustered by firm and time for cross-section).

coefficients on machine forecast revisions are small and statistically insignificant at all horizons, with values near zero. The absence of a correlation between forecast errors and prior forecast revisions is consistent with the behavior of objective expectations, under which forecast errors should be unpredictable.

One possible interpretation is that survey forecasts incorporate private information unavailable to the machine learning model, in which case apparent belief distortions might reflect rational responses to superior information. Under this interpretation, forecast revisions reflect analysts updating their beliefs based on new public and private signals. If analysts efficiently incorporated these signals, their revisions should not predict subsequent forecast errors. However, the evidence shows that positive revisions systematically predict negative errors, indicating overreaction rather than efficient information use. Moreover, distortions can arise even when agents possess private information if they overweight the precision of their private signals or inefficiently combine them with public data (Bianchi et al., 2022).

**Accuracy of Machine Learning vs. Survey Forecasts** To assess whether survey respondents misweight relevant information, Figure 2 evaluates the out-of-sample accuracy of machine

learning relative to survey forecasts for cash flows  $e_{t+h}$  and discount rates  $r_{t+h}$ . These variables are factors that can influence the value of hiring through the firm’s optimal hiring decision in the search model. I measure the relative predictive performance using the ratio  $MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$  of mean-squared-forecast-error of the machine ( $MSE_{\mathbb{E}}$ ) over that of the survey ( $MSE_{\mathbb{F}}$ ):

$$\text{Machine MSE} = \frac{1}{T} \sum_{t=1}^T (y_{t+h} - \mathbb{E}_t[y_{t+h}])^2, \quad \text{Survey MSE} = \frac{1}{T} \sum_{t=1}^T (y_{t+h} - \mathbb{F}_t[y_{t+h}])^2$$

where  $y_{t+h}$  denotes either cash flows or discount rates, and  $T$  is the length of the out-of-sample testing period, which spans 2005Q1 to 2023Q4. The cross-sectional forecasts are evaluated similarly based on the mean-squared-forecast-error across all firms over the testing period.

Figure 2 shows that machine learning forecasts consistently outperform survey forecasts across all variables and horizons, with MSE ratios well below one. The performance gap widens with forecast horizon, indicating that belief distortions are larger at longer horizons. The machine outperforms the survey for both time-series and cross-sectional forecasts, suggesting that belief distortions affect not only aggregate expectations but also the cross-sectional dispersion of beliefs across firms. If survey respondents were efficient in forming their beliefs, their forecasts would have performed at least on par with the machine.<sup>15</sup> The superior performance of the machine also highlights its ability to process a large amount of real-time data efficiently and objectively, supporting its use as a reliable benchmark of undistorted beliefs.

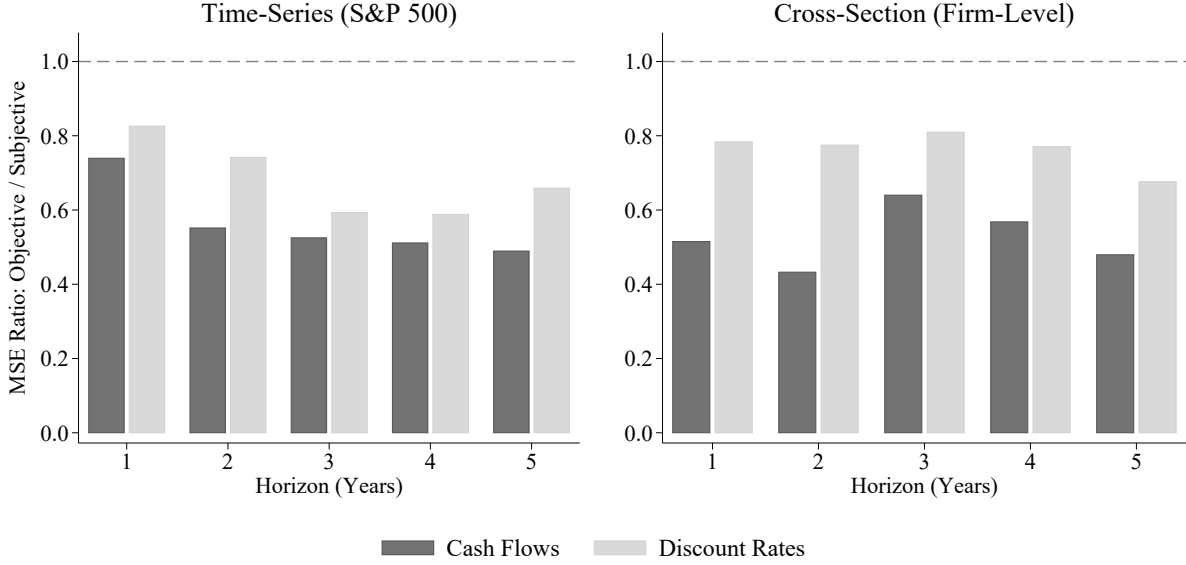
Survey forecasts could in principle contain private information that is not available to the machine. However, the evidence from the Coibion and Gorodnichenko (2015) regressions from Table 1 above show that respondents do not use their information efficiently, since forecast revisions systematically predict future errors. The machine addresses this by conditioning on the survey forecasts as predictors, which allows it to incorporate any information embedded in the surveys while correcting for their inefficient use. The fact that the machine still outperforms indicates that the performance gap reflects systematic biases in belief formation rather than efficient use of superior private information.

**Hiring Outcomes and Belief Distortions** Given the evidence of overreaction in subjective cash flow forecasts, this section examines the relationship between the vacancy filling rate and the belief distortion in subjective 5-year cash flow expectations. In search-and-matching models of the labor market, the vacancy filling rate reflects the marginal value of job creation. Hiring

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<sup>15</sup>The magnitude of the MSE ratio is more consistent with truly subjective beliefs rather than objective but risk-neutral beliefs. Estimates of risk premia typically range 5-10% annually (Adam et al., 2021), which is insufficient to explain the 15-30% deterioration in MSE ratios observed in Figure 2. The magnitude and persistence of forecast errors across horizons instead also point to behavioral biases rather than risk compensation.

Figure 2: Accuracy of Machine Learning vs. Survey Forecasts



*Notes:* Figure plots  $MSE_E/MSE_F$ , the ratio of mean squared forecast errors between machine learning and survey forecasts. Lower values indicate greater accuracy of the machine learning forecast.  $MSE_E$  and  $MSE_F$  refer to out-of-sample forecast errors from machine and survey forecasts, respectively. The out-of-sample testing period is 2005Q1–2023Q4. Left panel: aggregate S&P 500 time-series results. Right panel: cross-sectional forecasts across listed firms. Dark bars show cash flows (earnings growth); light bars show discount rates (stock returns). The forecast target  $y_{t+h}$  is the present value of discount rates  $r_{t,t+h}$  and cash flows  $e_{t,t+h}$ , as defined in equation (17). Time-series survey forecasts  $F_t$  come from the CFO survey (discount rates) and IBES (cash flows). Cross-sectional survey forecasts  $F_t$  come from IBES (discount rates and cash flows). Time-series and cross-sectional machine learning expectations  $E_t$  are generated using a Long Short-Term Memory (LSTM) model trained in real time on macroeconomic, financial, and textual data.

creates a match with expected duration  $1/\delta$  years given a job separation rate  $\delta$ . A 5-year forecast horizon captures the lifetime of a typical match since it aligns with the median job tenure observed in the data, which ranged between 3.4 to 4.6 years over the 1983–2023 sample period Bureau of Labor Statistics, 2024. Using a horizon slightly above the median ensures that the forecast covers the duration of most employment relationships above the median, and the expected duration likely exceeds the median given the right-skewed tenure distribution.

Figure 3 plots the growth rate of the U.S. vacancy filling rate against belief distortions in cash flow growth expectations, measured as the gap between subjective and objective 5-year forecasts of annualized S&P 500 earnings growth ( $F_t[\Delta e_{t,t+5}] - E_t[\Delta e_{t,t+5}]$ ). The vacancy filling rate, defined as total hires divided by total vacancies, measures the rate at which posted job openings are successfully filled. Total hires are constructed as the job finding rate multiplied by the unemployment level, where the job finding rate captures the probability an unemployed worker finds a job in a given quarter. The subjective forecasts come from survey data, while the objective benchmark uses machine-learning predictions.

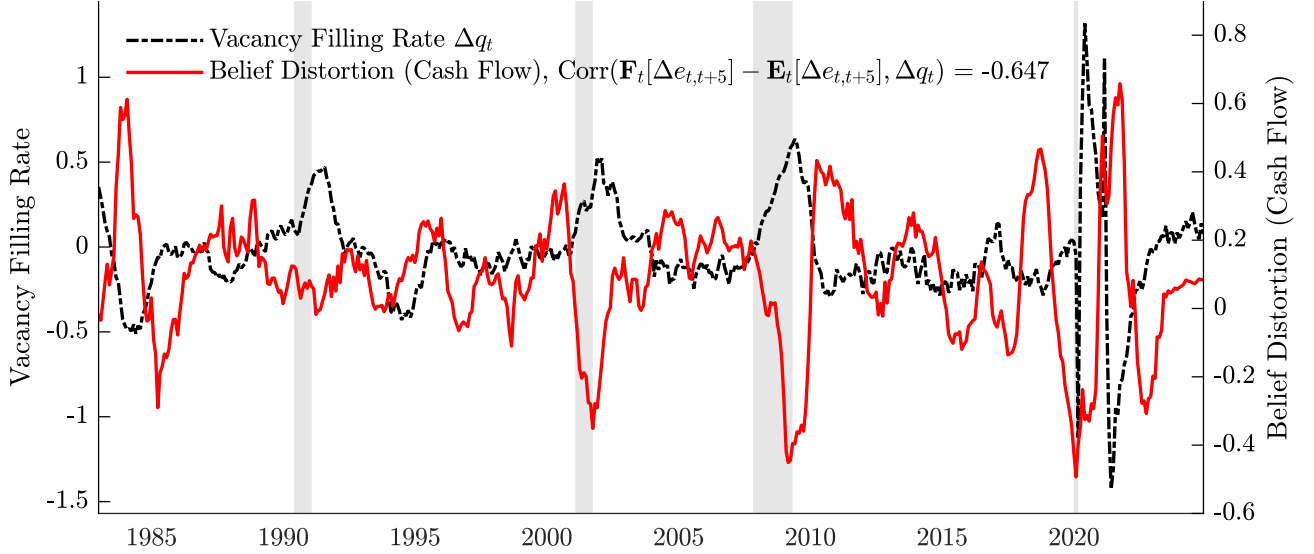
The figure shows that belief distortions exhibit strong cyclical patterns that closely track labor market fluctuations, with a negative correlation of -0.647. During expansions, positive be-



belief distortions emerge as subjective forecasts become overly optimistic relative to the objective benchmark. These periods of optimism coincide with declines in the vacancy-filling rate. The negative correlation reflects a congestion effect: when firms become optimistic, they simultaneously surge in vacancy posting, which tightens the labor market and makes it harder for any given vacancy to be filled. Conversely, when belief distortions turn negative and firms scale back vacancy posting, the vacancy-filling rate rises as the market becomes less congested.

This pattern is particularly evident during the large positive belief distortions of the late 1990s and the pre-2008 period, both followed by dramatic reversals to negative distortions that coincided with sharp contractions in vacancy filling rate growth and NBER-dated recessions. The tight co-movement between these expectational errors in cash flow forecasts and vacancy filling rate growth suggests that distortions in firms' earnings expectations are a powerful driver of labor market fluctuations, operating independently of changes in discount rates.

Figure 3: Vacancy Filling Rate and Belief Distortions in Subjective Cash Flows

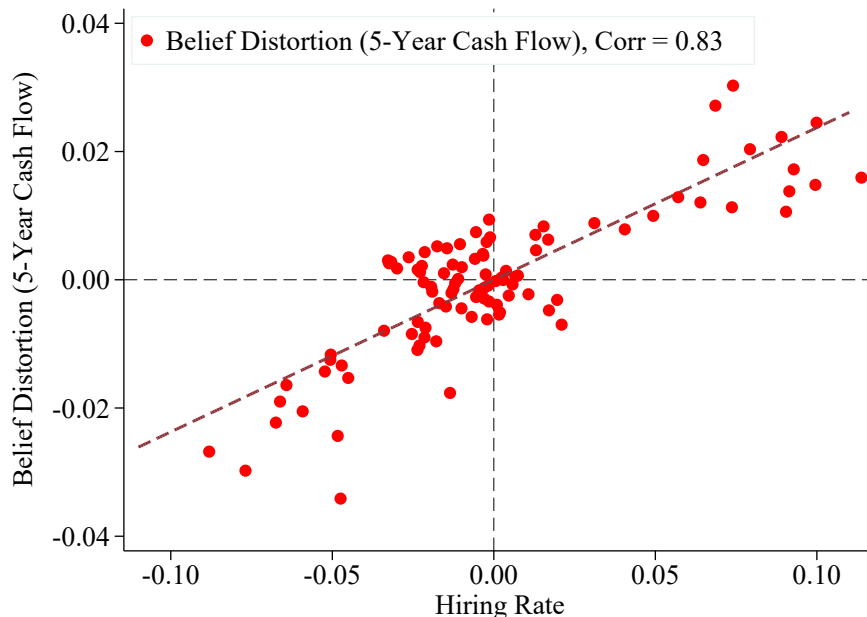


*Notes:* Figure plots the annual log growth of the U.S. vacancy filling rate  $\Delta q_t$  (left axis) against the belief distortion in cash flows, which is measured as expectational errors  $\mathbb{F}_t[\Delta e_{t,t+5}] - \mathbb{E}_t[\Delta e_{t,t+5}]$  in 5-year forecasts of annualized S&P 500 earnings growth (right axis). Survey expectations  $\mathbb{F}_t[\Delta e_{t,t+5}]$ : IBES median analyst forecasts for the next four fiscal years and long-term growth (LTG). Objective expectations  $\mathbb{E}_t[\Delta e_{t,t+5}]$ : Machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The vacancy filling rate, defined as total hires divided by total vacancies, measures the rate at which posted job openings are filled. Sample is quarterly from 1983Q1 to 2023Q4. Gray shaded areas indicate NBER recessions.

Figure 4 illustrates this key relationship in the cross-section by plotting the cross-sectional correlation between actual hiring rates and belief distortions in cash flow expectations across listed firms. The hiring rate, defined as total hires divided by existing employment, captures firm-level hiring behavior. Unlike the vacancy filling rate, which measures how quickly vacancies are filled, the hiring rate directly measures employment growth at the firm level. The scatter plot reveals a clear positive relationship with a correlation of 0.830 across firms. The figure

suggests that firms with equity analyst coverage that is excessively optimistic compared to the machine about future cash flow growth at the 5-year forecast horizon exhibit systematically higher hiring rates. Each point in the binned scatter represents a percentile of the joint distribution in hiring rates and belief distortions across the firms, and the strong positive slope confirms that belief distortions in expected cash flows translate directly into observable differences in hiring behavior across firms. Intuitively, the positive firm-level correlation in the hiring rate complements with the negative aggregate relationship for the vacancy filling rate. When firms become more optimistic, they expand employment and raise hiring rates, but collectively this tightens the labor market and lowers the probability that each vacancy is filled.

Figure 4: Cross-Sectional Hiring Rates and Belief Distortions



*Notes:* Figure plots the relationship between hiring rates and cash flow expectations across listed firms.  $y$ -axis reports the corresponding cross-sectionally demeaned log hiring rate,  $hl_{i,t}$ .  $x$ -axis reports the cross-sectionally demeaned belief distortion measured as expectational errors  $\mathbb{F}_t[\Delta e_{t,t+5}] - \mathbb{E}_t[\Delta e_{t,t+5}]$  in 5-year forecasts of annualized S&P 500 earnings growth. Subjective expectations  $\mathbb{F}_t$ : IBES forecasts. Objective expectations  $\mathbb{E}_t$ : Machine learning (LSTM) forecasts. The hiring rate, defined as total hires divided by existing employment, captures firm-level hiring behavior. Each dot is a bin scatter representing one percentile of the pooled distribution across all observations in the sample. The sample is quarterly from 2005Q1 to 2023Q4.

## 4 Theoretical Framework

The reduced-form link between fluctuations in the vacancy filling rate and belief distortions about cash flows motivate a structural interpretation. This section develops a search and matching model of the labor market in which firms' expectations about future cash flows and discount rates may be distorted, leading to fluctuations in vacancy filling rates and unemployment. The

model builds on the Diamond (1982), Mortensen (1982), and Pissarides (2009) framework but departs from the standard rational expectations assumption, allowing firms' hiring decisions to be influenced by biased subjective beliefs.

**Environment** Consider a discrete time economy populated by a representative household and a mass of firms  $I$ , normalized to one, that hires workers in a frictional labor market. The firm uses labor as a single input to production. The household's population is normalized to one and has a continuum of members, where a fraction  $L_t$  are employed and the rest are unemployed  $U_t = 1 - L_t$ . The household's intertemporal consumption decision gives rise to a stochastic discount factor  $M_{t+1}$ .

**Labor Market** Each period, each firm posts job vacancies at a cost  $\kappa > 0$  to maximize its cum-dividend value of equity. Employment  $L_{i,t}$  reflects the number of workers employed in firm  $i$  at the beginning of period  $t$  before any separations or new hires.<sup>16</sup> During the period, a fraction  $\delta_{i,t}$  of employed workers separate, while the firm posts vacancies  $V_{i,t}$  to search from a pool of unemployed workers  $U_t$ . Let  $L_t = \sum_{i \in I} L_{i,t}$  and  $V_t = \sum_{i \in I} V_{i,t}$  denote the aggregate number of employed workers and vacancies posted by all firms. Matches are formed at the end of period  $t$  according to a Cobb-Douglas matching function  $\mathcal{M}(U_t, V_t)$ :

$$\mathcal{M}(U_t, V_t) = BU_t^\eta V_t^{1-\eta} \quad (3)$$

where  $B$  is the matching efficiency parameter, and  $\eta \in (0, 1)$  governs the elasticity of matches with respect to unemployment. The probability that a firm fills a posted vacancy, the vacancy filling rate, is then given by:

$$q_t = \frac{\mathcal{M}(U_t, V_t)}{V_t} = B \left( \frac{U_t}{V_t} \right)^\eta = B\theta_t^{-\eta} \quad (4)$$

and the job finding rate is given by  $f_t \equiv \mathcal{M}(U_t, V_t)/U_t$ . These new hires enter employment at the start of period  $t + 1$ , so employment  $L_{i,t}$  and unemployment  $U_t = 1 - L_t$  evolve according to the law of motion:

$$L_{i,t+1} = (1 - \delta_{i,t})L_{i,t} + q_t V_{i,t} \quad (5)$$

Aggregate unemployment  $U_t = 1 - L_t$  evolves according to:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \quad (6)$$

where  $\theta_t = V_t/U_t$  denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

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<sup>16</sup>I adopt an end-of-period matching convention following Petrosky-Nadeau et al. (2018). See Hansen et al. (2005) and Kogan and Papanikolaou (2012) for similar conventions applied for the  $q$  theory of investment.

**Firm’s Technology and Cash Flow** Each firm  $i$  uses labor  $L_{i,t}$  to produce output  $Y_{i,t}$  via a Cobb-Douglas production function at constant returns to scale:

$$Y_{i,t} = A_{i,t}L_{i,t} \quad (7)$$

where  $A_{i,t}$  is firm-level productivity,  $L_{i,t}$  is the level of employment. The firm pays wages  $W_{i,t}$ , incurs hiring costs  $\kappa V_{i,t}$ , and generates cash flows defined as the firm’s period earnings:

$$E_{i,t} = Y_{i,t} - W_t L_{i,t} - \kappa V_{i,t} \quad (8)$$

Earnings represent the net flow profits from operating the firm, which is the output net of the wage bill and vacancy posting costs. I assume that the household owns the equity of the firm and the firm pays out all of its earnings  $E_{i,t}$  as dividends (Petrosky-Nadeau et al., 2018). I also assume that the firm’s manager has access to complete markets so that the return to hiring equals the stock market return in equilibrium (Cochrane, 1991).

**Firm’s Problem** The firm chooses vacancy postings  $V_{i,t}$  to maximize the present discounted value of future cash flows. The firm’s value function  $\mathcal{V}$  satisfies the Bellman equation:

$$\mathcal{V}(A_{i,t}, L_{i,t}) = \max_{V_{i,t}, L_{i,t+1}} \{E_{i,t} + \mathbb{E}_t[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})]\} \quad (9)$$

subject to the employment accumulation equation (5).  $\mathbb{E}_t[\cdot]$  is the firm’s subjective expectations conditional on information available at the beginning of period  $t$ .<sup>17</sup> These beliefs may depart from objective expectations  $\mathbb{E}_t[\cdot]$ , where the nature and magnitude of the deviation will be disciplined using survey data.  $M_{t+1}$  is the stochastic discount factor that prices the firm’s cash flows. The firm does not observe this discount factor directly and needs to form expectations about it by forecasting the household’s marginal utility of consumption (Venkateswaran, 2014).

**Hiring Condition** Under search frictions, hiring is forward-looking investment. The firm’s optimal hiring decision equates the marginal cost of posting a vacancy with the expected discounted marginal value of employment:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \mathbb{E}_t \left[ \underbrace{M_{t+1} \frac{\partial \mathcal{V}(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}}}_{\text{Expected discounted value of hiring}} \right] \quad (10)$$

The left side represents the expected cost of hiring an additional worker while accounting for the probability  $q_t$  that a posted vacancy will be filled. The right side captures the expected discounted

<sup>17</sup>I use the term “firm’s beliefs” as shorthand to refer to the expectations held by decision makers within firms (Coibion et al., 2018; Candia et al., 2020).

value of the marginal worker, incorporating both the firm's subjective beliefs about the future state and the discount rate for valuing risky cash flows.<sup>18</sup> Subjective distortions in beliefs can thus shift the perceived value of hiring through  $\mathbb{E}_t[\cdot]$  and affect equilibrium vacancy filling rates, which in turn affect unemployment through its law of motion in equation (6). Assuming constant returns to scale, the marginal value of hiring equals its average value:

$$\frac{\partial \mathcal{V}(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}} = \frac{\mathcal{V}(A_{i,t+1}, L_{i,t+1})}{L_{i,t+1}} \quad (11)$$

Define the firm's ex-dividend market value as  $P_{i,t} \equiv \mathbb{E}_t[M_{t+1} \mathcal{V}(A_{i,t+1}, L_{i,t+1})]$  to derive a direct link between the vacancy filling rate and the firm's market value per worker:

$$\frac{\kappa}{q_t} = \frac{P_{i,t}}{L_{i,t+1}} \quad (12)$$

where employment  $L_{i,t+1}$  is determined at the end of date  $t$  under the timing convention from equation (5). Take logarithms, rearrange terms, and expand the price-employment ratio  $P_{i,t}/L_{i,t+1}$ :

$$\log q_t = \log \kappa - \log \left( \frac{P_{i,t}}{E_{i,t}} \right) - \log \left( \frac{E_{i,t}}{L_{i,t+1}} \right) \quad (13)$$

Define log price-earnings  $pe_{i,t} = \log(P_{i,t}/E_{i,t})$  and earnings-employment  $el_{i,t} = \log(E_{i,t}/L_{i,t+1})$ :

$$\log q_t = \log \kappa - pe_{i,t} - el_{i,t} \quad (14)$$

**Log-linear Approximation of Price-Earnings Ratio** To decompose the vacancy filling rate into economically meaningful components, I apply the Campbell and Shiller (1988) present value identity to the price-earnings ratio. Log-linearize the price-earnings ratio  $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$  around its long-run mean  $\overline{pe}$  to obtain the approximate relationship:

$$pe_{i,t} = c_{pe} - r_{i,t+1} + \Delta e_{i,t+1} + \rho pe_{i,t+1} \quad (15)$$

where  $c_{pe}$  is a linearization constant,  $\rho = \exp(\overline{pe})/(1 + \exp(\overline{pe}))$  is the time discount factor from the log-linearization,  $r_{i,t+1} = \log((P_{i,t+1} + E_{i,t+1})/P_{i,t})$  is the stock return assuming that the firm pays out its earnings as dividends, and  $\Delta e_{i,t+1}$  denotes earnings growth.<sup>19</sup> This equation is an

<sup>18</sup>The hiring equation is the labor market analogue of the optimality condition for physical capital in the  $q$  theory of investment (Hayashi, 1982), where the upfront cost of hiring  $\kappa/q_t$  is analogous to Tobin's marginal  $q$  and the separation rate  $\delta_{t+1}$  is analogous to the depreciation rate (Borovičková and Borovička, 2017). See Lettau and Ludvigson (2002) and Kogan and Papanikolaou (2012) for a similar log-linearization applied for the  $q$  theory of physical capital investment.

<sup>19</sup>This identity also holds approximately when dividends differ from earnings. Following Lintner (1956), dividends can be approximated as a stable fraction of earnings (with a long-run payout ratio near 50%). The resulting payout ratio term  $(1 - \rho)de_{t+1}$  becomes negligible after log-linearization since  $1 - \rho \approx 0.02$ , allowing this term to be absorbed into the constant  $c_{pe}$  (De La O et al., 2024). See Appendix Section OB for a derivation.

accounting identity that links current valuation ratios to future cash flows and discount rates. Substituting recursively for the next  $h$  periods yields the present value identity:

$$pe_{i,t} = \sum_{j=1}^h \rho^{j-1} c_{pe} - \sum_{j=1}^h \rho^{j-1} r_{i,t+j} + \sum_{j=1}^h \rho^{j-1} \Delta e_{i,t+j} + \rho^h pe_{i,t+h} \quad (16)$$

**Decomposition of Vacancy Filling Rate** Substitute log-linearized price-earnings (16) into the hiring equation (14) to obtain a decomposition of the vacancy filling rate  $q_t$ :

$$\log q_t = c_q + \underbrace{\sum_{j=1}^h \rho^{j-1} r_{i,t+j}}_{\equiv r_{i,t,t+h}} - \underbrace{\left[ el_{i,t} + \sum_{j=1}^h \rho^{j-1} \Delta e_{i,t+j} \right]}_{\equiv e_{i,t,t+h}} - \underbrace{\rho^h pe_{i,t+h}}_{\equiv pe_{i,t,t+h}} \quad (17)$$

where  $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$  is a constant. The vacancy filling rate has been decomposed into three forward-looking components: the present value of future discount rates  $r_{i,t,t+h} \equiv \sum_{j=1}^h \rho^{j-1} r_{i,t+j}$ , cash flows  $e_{i,t,t+h} \equiv el_{i,t} + \sum_{j=1}^h \rho^{j-1} \Delta e_{i,t+j}$ , and price-earnings ratio  $pe_{i,t,t+h} \equiv \rho^h pe_{i,t+h}$ . The cash flow component consists of the current earnings-employment ratio  $el_t$ , which captures short-term fluctuations in cash flows, and  $j = 1, \dots, h$  period ahead earnings growth  $\Delta e_{t+j}$ , which captures news about future cash flows. The persistence of labor matches enters through the continuation value term  $\rho^h F_t[pe_{t+h}]$ . In the firm's Bellman equation, match persistence determines how much of the job's value extends beyond the  $h$ -year horizon. A lower separation rate (higher match persistence) raises the continuation value, increasing the contribution of the terminal value component in the decomposition.

To decompose time-series variation in the vacancy filling rate  $q_t$ , the right-hand side of equation (17) can be aggregated across firms:

$$\underbrace{\log q_t}_{\text{Vacancy Filling Rate}} = c_q + \underbrace{\mathbb{F}_t[r_{t,t+h}]}_{\text{Discount Rate}} - \underbrace{\mathbb{F}_t[e_{t,t+h}]}_{\text{Cash Flow}} - \underbrace{\mathbb{F}_t[pe_{t,t+h}]}_{\text{Future Price-Earnings}} \quad (18)$$

where  $x_t = \sum_{i \in I} w_{i,t} x_{i,t}$  aggregates firm-level variable  $x_{i,t}$  using employment weights  $w_{i,t} = L_{i,t} / \sum_{j \in I} L_{j,t}$  for  $x \in \{r, e, pe\}$ .<sup>20</sup> Intuitively, the vacancy filling rate rises when firms expect high future cash flows (making hiring valuable) or low future discount rates (making future profits more valuable today). The Campbell-Shiller formula allows us to separate these two channels by recursively decomposing the price-earnings ratio into expected returns and earnings growth.

<sup>20</sup>The aggregate decomposition follows from the firm-level decomposition by summing across individual firms' hiring conditions under two assumptions: (i) vacancy posting costs are linear in the number of vacancies, and (ii) the labor market for matches is competitive, so that all firms face a common vacancy filling rate  $q_t$ . Under these conditions, the aggregate hiring equation is obtained by weighting firm-level equations by employment, which ensures that the decomposition holds exactly at the aggregate level. For the cross-sectional analysis, the same logic applies to portfolios of firms.

Since equation (17) holds both ex-ante and ex-post, it can be evaluated under either subjective or objective expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their ex-ante subjective expectation  $\mathbb{F}_t[\cdot]$ :

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] \quad (19)$$

The equation implies that the vacancy filling rate is high when firms subjectively expect future returns to be high, expected cash flows to be low, or both. Alternatively, the *objective decomposition* replaces each subjective expectation  $\mathbb{F}_t[\cdot]$  with its objective expectation  $\mathbb{E}_t[\cdot]$ :

$$\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}] \quad (20)$$

Comparing these decompositions can quantify how belief distortions affect the vacancy filling rate. The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current vacancy filling rate. For the subjective decomposition, demean each variable in equation (19), multiply both sides by the current log vacancy filling rate  $\log q_t$ , and take the sample average:

$$Var[\log q_t] = Cov[\mathbb{F}_t[r_{t,t+h}], \log q_t] - Cov[\mathbb{F}_t[e_{t,t+h}], \log q_t] - Cov[\mathbb{F}_t[pe_{t,t+h}], \log q_t] \quad (21)$$

where  $Var[\cdot]$  and  $Cov[\cdot]$  are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by  $Var[\log q_t]$  to decompose its variance:

$$1 = \underbrace{\frac{Cov[\mathbb{F}_t[r_{t,t+h}], \log q_t]}{Var[\log q_t]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov[\mathbb{F}_t[e_{t,t+h}], \log q_t]}{Var[\log q_t]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov[\mathbb{F}_t[pe_{t,t+h}], \log q_t]}{Var[\log q_t]}}_{\text{Future Price-Earnings News}} \quad (22)$$

The left-hand side represents the full variability in vacancy filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or future price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing  $\mathbb{F}_t[r_{t,t+h}]$ ,  $\mathbb{F}_t[e_{t,t+h}]$ , and  $\mathbb{F}_t[pe_{t,t+h}]$  on the current log vacancy filling rate  $\log q_t$ , respectively. Finally, the decomposition under objective expectations can be estimated similarly based on equation (22) by replacing the subjective expectation  $\mathbb{F}_t[\cdot]$  with its objective counterpart  $\mathbb{E}_t[\cdot]$ .

Comparing the decompositions implied by subjective and objective expectations can highlight the role of belief distortions, which I define as the gap between the survey and machine forecasts:  $\mathbb{F}_t - \mathbb{E}_t$ . This comparison allows us to assess the role of belief distortions in explaining labor market dynamics and determine whether firms systematically mis-perceive economic conditions

when making hiring decisions. Although the variance decomposition does not necessarily capture causal relationships, it has the advantage of not requiring the researcher to take a stand on the deep determinants of vacancy filling rates because the evolution of discount rates and cash flows summarize the combined effects of these deep determinants.

## 5 Time-Series Decomposition of the Vacancy Filling Rate

The evidence of overreaction in survey expectations and the superior forecasting performance of machine learning suggests the presence of distortions in subjective expectations. This section quantifies how those distortions affect hiring behavior by estimating the contributions of discount rate and cash flow expectations to fluctuations in the aggregate vacancy filling rate.

**Objective Expectations** Figure 5 presents the variance decomposition of the vacancy filling rate. The figure shows that, under objective beliefs, discount rate news is the dominant driver of variation in vacancy filling rates. At the five-year horizon, discount rates explain 69.1% of the variation in vacancy filling rates, while cash flow news accounts for 6.6%.<sup>21</sup> Consistent with the predictions of the search and matching model, higher vacancy filling rates are associated with higher discount rates and lower expected cash flows because a lower value of hiring leads firms to post fewer vacancies, reducing tightness and raising  $q_t$ . The contribution from terminal price-earnings ratios still remains sizable at the five-year horizon, accounting for 20.1%. The combined contribution from the three components sums to 95.8% at the five-year horizon, a value reasonably close to 100.0% suggesting that the decomposition is empirically accurate despite being estimated freely without imposing this constraint.

These findings are consistent with predictions from search-and-matching models that emphasize time-varying risk premia while maintaining rational expectations. The large contribution from discount rate news is consistent with models that introduce rational time-varying discount rates to explain unemployment fluctuations (Hall, 2017).<sup>22</sup> The increasing importance of discount rate news at longer horizons is consistent with rational models that match observed fluctuations in unemployment by modeling hiring as a risky investment with long-duration returns (Kehoe

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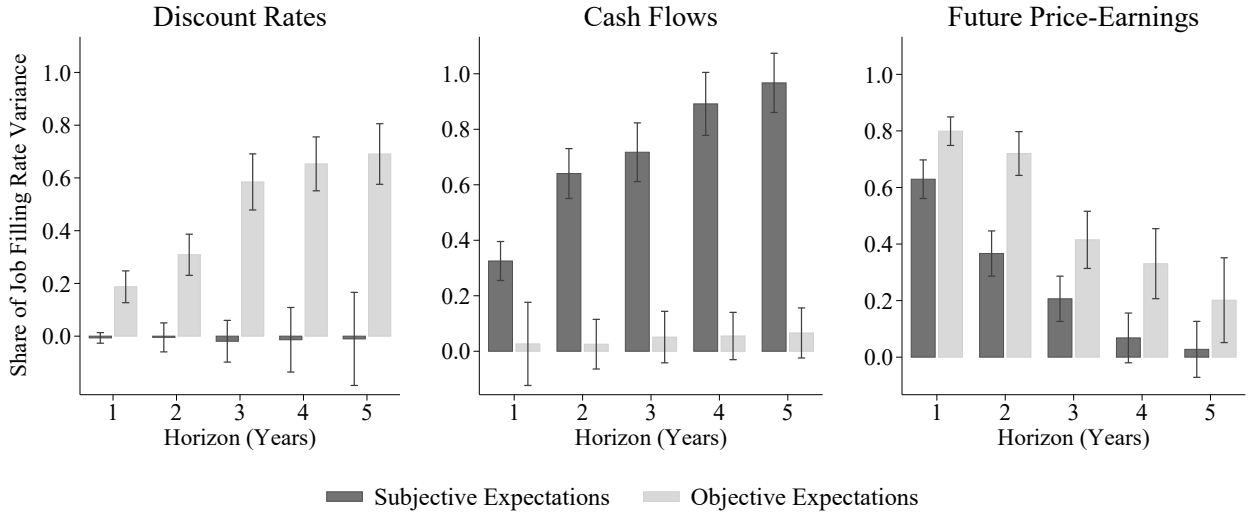
<sup>21</sup>Table OA.2 reports more detailed statistics on the variance decomposition. First-differenced estimates in Figure OA.1 show similar patterns under objective beliefs, with discount rates explaining 58.7% and cash flows explaining only 10.0% of vacancy filling rate variation. Figure OA.2 uses a VAR to extend the decomposition to the infinite horizon where discount rates explain 78.1% of vacancy filling rate variation.

<sup>22</sup>On the relative importance of risk-free rates and risk premia, which are two components of discount rates, Figure OA.5 shows that objective risk-free rate expectations explain less than 5% of the variation in the vacancy filling rate. This implies that the explanatory power of discount rate news is driven primarily by risk premia, consistent with rational models of labor markets that introduce time-varying risk premia (Borovičková and Borovička, 2017).



et al., 2023). Finally, the small objective cash flow component aligns with the unemployment volatility puzzle, as Shimer (2005) showed that, without time-varying discount rates, standard search models cannot generate enough unemployment volatility from productivity shocks, which would mainly be reflected in the cash flow component.

Figure 5: Time-Series Decomposition of the Vacancy Filling Rate



*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components from the time-series decomposition of the U.S. aggregate vacancy filling rate. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. Subjective expectations  $\mathbb{F}_t$  are constructed from CFO survey forecasts (discount rates) and IBES analyst forecasts (cash flows). Objective expectations  $\mathbb{E}_t$  are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks.  $x$ -axis denotes the forecast horizon  $h$ . The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

**Subjective Expectations** On the other hand, Figure 5 reveals a strikingly different result under subjective expectations. At the five-year horizon, subjective cash flow news explains 96.7% of the variation in vacancy filling rates, while subjective discount rate news accounts for only -1.0%.<sup>23</sup> These results suggest that, under subjective beliefs, firms' hiring behavior is highly sensitive to news about cash flows. Since vacancy filling rates are countercyclical (declining during expansions as labor markets tighten), the positive cash flow coefficient indicates that firms expect high cash flows during periods when  $q_t$  is low, consistent with optimism during economic booms. The negative contribution on the discount rate component indicates that survey respondents predict lower future returns during recessions, contrary to what a rational forecast would imply.

The terminal price-earnings ratio's contribution falls with horizon and is negligible by year five (2.8%), compared with 20.1% under objective expectations. This implies that subjective beliefs

<sup>23</sup>First-differenced estimates in Figure OA.1 show similar results, with subjective cash flows explaining 90.6% and discount rates explaining only -1.3% of the vacancy filling rate. Figure OA.2 uses a VAR to extend the decomposition to the infinite horizon where subjective cash flows explain 95.4% of vacancy filling rate variation.

place larger weight on the firm’s near-term cash flows relative to its long-run valuations. Finally, the three components sum to 98.5% at the five-year horizon, showing that survey expectations are internally consistent and the model’s approximation is reasonably accurate, with any remaining gap likely attributable to measurement error in the survey data (e.g., Ma et al., 2020).<sup>24</sup>

Compared to the objective benchmark, the implied overreaction to cash flow news is substantial. Low vacancy filling rates during expansions are associated with a significant disappointment in future cash flows. Defining the belief distortion as the difference between subjective and objective expectations  $\mathbb{F}_t - \mathbb{E}_t$ , the estimates imply that, at the five-year horizon,  $96.7\% - 6.6\% = 90.1\%$  of variation in vacancy filling rates can be attributed to the fact that the vacancy filling rate predicts distortions in cash flow expectations with a significant positive relationship (Table OA.4). These distortions capture inefficiencies or behavioral biases in survey respondents’ subjective beliefs that the machine learning model could have identified ex-ante.

These findings are inconsistent with explanations based on measurement error or analyst conflicts of interest, since random noise would bias results toward zero rather than generate explanatory power. Conflicts of interest could add persistent bias or idiosyncratic noise but not systematic overreaction. The strong predictive power of belief distortions and the fact that forecast revisions predict subsequent errors indicate that the variation reflects genuine overreaction in expectations, not noise.

**Sources of Vacancy Filling Rate Variation** While Table 1 has shown subjective discount rate forecasts to exhibit overreaction, their contribution to the variance decomposition of vacancy filling rates remains small in Figure 5. This can be reconciled by the fact that subjective discount rate expectations display relatively little time-series variation, so even overreactive revisions have limited impact on hiring decisions. In contrast, subjective cash flow expectations vary much more over time, making them the primary driver of belief-driven fluctuations in hiring.<sup>25</sup>

To visualize the relative importance of each component in the decomposition, Figure 6 constructs counterfactual time series for the annual growth in the vacancy filling rate. These series represent the component of vacancy filling rate growth explained by a single driver, based on the fitted values from the variance decomposition from Figure 5. As shown in the figures, each counterfactual series is initialized to match the actual vacancy filling rate in 2005Q1 to provide

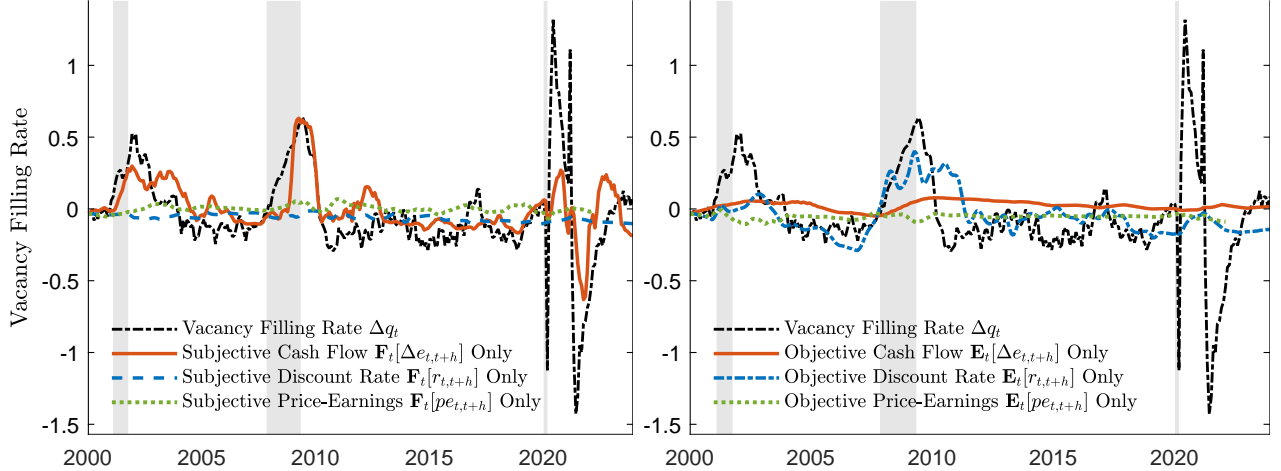
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<sup>24</sup>This residual term captures any model misspecification or approximation errors in the decomposition, including potential misspecification of the stochastic discount factor or approximation errors in the Campbell-Shiller decomposition under subjective beliefs. As shown in Section OA.2.2, the residual is quantitatively negligible and approximately orthogonal to the model’s main components of discount rates and cash flow expectations.

<sup>25</sup>Figure A.1 illustrates this point visually by showing that subjective expectations exhibit strong cyclicalities in cash flow forecasts and muted responses in discount rate forecasts compared to their machine-based counterparts.

a common baseline for comparison. The results are consistent with the variance decomposition. Under subjective beliefs, the counterfactual based on cash flow expectations tracks the realized path of vacancy filling rate growth remarkably well, explaining a substantial portion of its cyclical fluctuations. In contrast, while the discount rate component under objective beliefs accounts for a significant share of fluctuations, its explanatory power is visibly less pronounced than that of subjective cash flows.

Figure 6: Role of Components in the Vacancy Filling Rate



*Notes:* Figure presents counterfactual time series showing the evolution of vacancy filling rate growth if driven solely by each expectation component. Left panel shows subjective expectations; right panel shows objective expectations. Counterfactual series are constructed by accumulating fitted values from regressions of vacancy filling rate growth on individual expectation measures, with all series initialized to the actual vacancy filling rate growth in 2005Q1. Black line shows actual vacancy filling rate growth for comparison. Subjective expectations  $\mathbb{F}_t$  are based on CFO survey forecasts (discount rates) and IBES analyst forecasts (cash flows, price-earnings). Objective expectations  $\mathbb{E}_t$  are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. Gray shaded areas indicate NBER recessions. Sample period: 2000Q4 to 2023Q4.

**Discussion** Although the decomposition does not necessarily estimate causal relationships, it can account for possible sources of variation in the vacancy filling rate. A large estimate for subjective cash flow news means that, whatever shocks drive the vacancy filling rate, they must have a larger impact on subjective cash flow expectations than subjective discount rates. Under objective expectations, by contrast, firms correctly interpret those same shocks as signals about future risk compensation embedded in discount rates. This divergence points to belief distortions as a key source of vacancy filling rate fluctuations. By overreacting to perceived changes in future cash flows, firms may cut hiring and vacancies too sharply during downturns, amplifying unemployment volatility beyond what rational models predict.

The large contribution of subjective cash flows in shaping hiring decisions is consistent with models that introduce nonrational expectations about earnings growth to account for fluctuations in asset prices (Bordalo et al., 2024a; Bianchi et al., 2024) and the business cycle (Bordalo et al.,

2024b). This parallel implies that the belief distortions known to influence asset valuations can also extend to real economic behavior through the labor market.

On the other hand, the small and negative contribution of subjective discount rates (although not statistically different from zero) is consistent with existing survey evidence showing that subjective return expectations are acyclical (Nagel and Xu, 2022) or even procyclical (Greenwood and Shleifer, 2014; Adam et al., 2016), contrary to the countercyclical discount rate variation implied by rational models (Cochrane, 2017). In standard asset pricing models, discount rates reflect the firm’s market-based cost of capital, such as the weighted average cost of debt and cost of equity (WACC). In contrast, survey evidence suggests that CFOs likely rely on internal discount rates that are persistent and often unresponsive to market conditions, even when firms are not financially constrained (Gormsen and Huber, 2025). My findings extend this evidence to labor markets, where hiring decisions appear similarly detached from subjective beliefs about risk premia or financial constraints.

## 6 Cross-Sectional Decomposition of the Hiring Rate

To analyze the sources of dispersion in hiring across firms, I implement a cross-sectional decomposition of the log hiring rate based on the same theoretical framework developed for the time-series decomposition. The log hiring rate for each firm can be constructed using the employment accumulation equation:

$$hl_{i,t} = \log \left( \frac{q_t V_{i,t}}{L_{i,t}} \right) = \log \left( \frac{L_{i,t+1}}{L_{i,t}} - (1 - \delta_{i,t}) \right) \quad (23)$$

where  $L_{i,t}$  uses data from Compustat number of employees (EMP) and  $\delta_{i,t}$  uses JOLTS industry-level job separation rate. The hiring rate captures the fraction of new hires per existing employee, conditional on vacancies being filled at rate  $q_t$ . This demeaned hiring rate is then decomposed into three components:<sup>26</sup>

$$\tilde{hl}_{i,t} = \underbrace{- \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\tilde{r}_{i,t+j}]}_{\text{Discount Rate} \equiv \mathbb{F}_t[\tilde{r}_{i,t,t+h}]} + \underbrace{\left[ \tilde{el}_{i,t} + \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\Delta \tilde{e}_{i,t+j}] \right]}_{\text{Cash Flow} \equiv \mathbb{F}_t[\tilde{e}_{i,t,t+h}]} + \underbrace{\rho^h \mathbb{F}_t[\tilde{pe}_{i,t+h}]}_{\text{Future Price-Earnings} \equiv \mathbb{F}_t[\tilde{pe}_{i,t,t+h}]} \quad (24)$$

where  $\rho = \exp(\overline{pe}) / (1 + \exp(\overline{pe}))$  is the time discount factor from the log-linearization. The first term represents cross-sectional dispersion in expected returns, which affect the discount rate

<sup>26</sup>See Appendix Section OB.2 for a derivation. Note that the signs on each component are reversed compared to the vacancy filling rate decomposition in equation (22). The flipped sign reflects the distinction between how easy it is to fill vacancies (vacancy filling rate) versus how much hiring actually occurs (hiring rate). Good economic conditions make vacancies harder to fill but increase overall hiring activity.

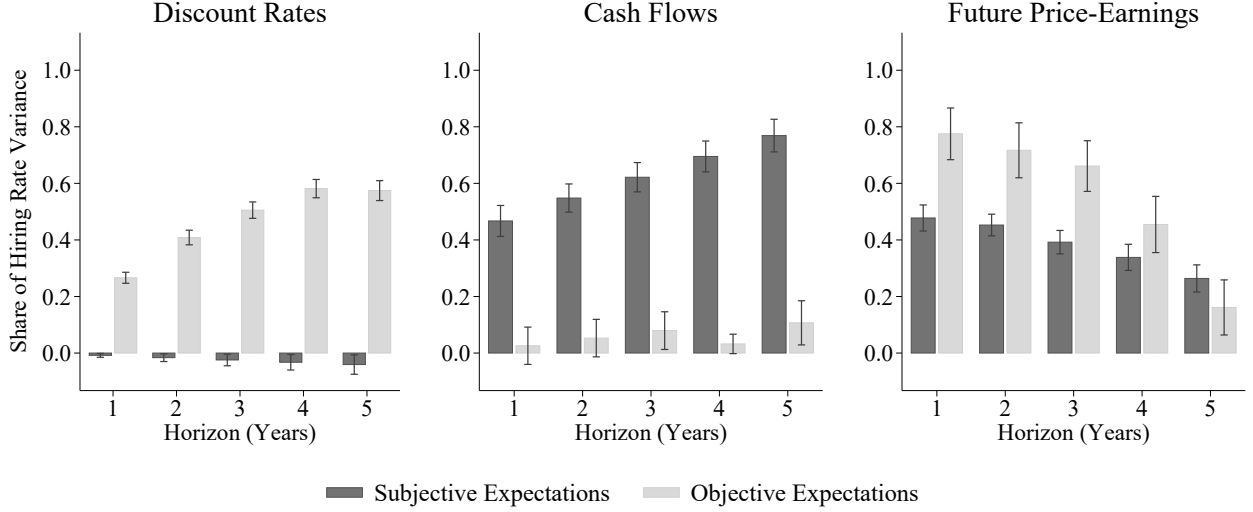
at which future expected cash flows are converted to present value. The second term captures dispersion in the current earnings per worker,  $\tilde{e}_{i,t}$ , and the sum of expected earnings growth over the forecast horizon  $h$ . The third term is the dispersion in expected future price-earnings ratios, which is a terminal value that captures longer-run influences not already captured in discount rates and expected cash flows by horizon  $h$ . All expectations are formed using either survey (subjective expectation) or machine learning forecasts (objective expectation benchmark). To isolate cross-sectional variation, I demean each variable across firms indexed by  $I$ , defining  $\tilde{hl}_{i,t} = hl_{i,t} - \frac{1}{I} \sum_{j \in I} hl_{j,t}$ , so that the decomposition isolates the extent to which deviations from the average hiring rate can be traced to each component.

Under stationarity, the econometrician can estimate these shares using OLS coefficients from regressing  $\mathbb{F}_t[\tilde{r}_{i,t,t+h}]$ ,  $\mathbb{F}_t[\tilde{e}_{i,t,t+h}]$ , and  $\mathbb{F}_t[\tilde{pe}_{i,t,t+h}]$  on the current log hiring rate  $\tilde{hl}_{i,t}$ , respectively. The sample covers all common stocks (share code 10 and 11) listed on NYSE, AMEX, and NASDAQ, restricted to firms that have sufficient data to construct total employee counts (EMP) from Compustat and the median analyst stock return and earnings growth forecasts at the five-year horizon from IBES, as described in Section 2.

Figure 7 shows that under subjective expectations, cross-sectional dispersion in the hiring rate is dominated by differences in expected cash flows. At the five-year horizon, 79.2% of the cross-sectional variance is explained by the expected cash flows. In contrast, only -5.2% of the variation is explained by discount rates, and 29.8% is attributed to differences in the terminal future price-earnings expectation. Under subjective beliefs, a higher discount rate is associated with a higher hiring rate, a direction that is not consistent with the predictions of the search model. These results indicate that firms sorted into different idiosyncratic shock deciles have sharply different expectations about future cash flows when expectations are subjective, and these differences in beliefs translate into differences in perceived hiring incentives. The combined contribution of the three components sums to 103.8% at the five-year horizon, a value close to 100.0% suggesting that the approximations used in the decomposition are reasonably accurate despite being freely estimated without imposing this constraint.

Under objective expectations, 60.2% of cross-sectional variation in hiring is explained by differences in expected discount rates at horizon five, while only 13.1% is explained by expected cash flows. This pattern is consistent with existing estimates showing that, under objective expectations, much of the variation across firms in asset valuations comes from dispersion in risk premia rather than expected cash flows (De La O et al., 2024). Finally, the contribution of the terminal price-earnings component is 17.8%, which is sizeable and is larger than the same component estimated under subjective beliefs.

Figure 7: Cross-Sectional Decomposition of the Hiring Rate



*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components from the cross-sectional decomposition of the hiring rate. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. Subjective expectations  $\mathbb{F}_t$  are constructed from IBES analyst forecasts (discount rates and cash flows). Objective expectations  $\mathbb{E}_t$  are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks.  $x$ -axis denotes the forecast horizon  $h$ . The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows 95% confidence intervals under two-way clustering by firm and time.

Taken together, the results reveal that under subjective beliefs, cross-sectional variation in hiring is driven primarily by firms overreacting to news about their future cash flows. This provides a micro-foundation for the aggregate results by showing that the same type of belief distortion that drives fluctuations in aggregate unemployment also operates at the firm level, where hiring decisions are actually made.<sup>27</sup>

**Employment Response to Idiosyncratic Shocks** Objective and subjective beliefs have qualitatively different implications for how firms respond to idiosyncratic shocks. Under rational expectations with hiring frictions, positive idiosyncratic shocks generate stable or temporary increases in profits per worker as firms face adjustment costs. Under subjective expectations with systematic overreaction, firms overhire relative to the non-distorted benchmark, depressing profits per worker. Figure 8 tests these competing predictions by estimating local projections under two specifications to isolate the role of belief distortions. For the impulse response under subjective beliefs, I estimate:

$$y_{i,t+h} = \alpha_i^{\mathbb{F}} + \tau_{s(i),t}^{\mathbb{F}} + \beta^{\mathbb{F}}(\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]) + \gamma^{\mathbb{F}'} \mathbf{X}_{i,t} + \varepsilon_{i,t+h}^{\mathbb{F}} \quad (25)$$

<sup>27</sup>The cross-sectional results suggest that much of the dispersion in hiring rates reflects belief-driven forecast errors. Such distorted expectations can act as a wedge that misallocates labor by inducing overhiring at optimistic firms and underhiring at pessimistic firms (Ma et al., 2020; David et al., 2022; Ropele et al., 2024).

where  $y_{i,t}$  denotes the outcome variable ( $\log$  profits per worker  $\log(E_{i,t+h}/L_{i,t+h})$ , employment  $\log(L_{i,t+h})$ , or profits  $\log(E_{i,t+h})$ ), and  $\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]$  captures subjective forecast revisions for earnings growth. For the impulse response implied by objective beliefs, I estimate a similar local projection while replacing the survey forecast revision with the corresponding machine forecast revision. To directly test the role of belief distortions, I estimate:

$$y_{i,t+h} = \alpha_i^{\mathbb{D}} + \tau_{s(i),t}^{\mathbb{D}} + \beta^{\mathbb{D}}(\mathbb{D}_{i,t}[\Delta e_{i,t+1}] - \mathbb{D}_{i,t-1}[\Delta e_{i,t+1}]) + \gamma^{\mathbb{D}'}\mathbf{X}_{i,t} + \varepsilon_{i,t+h}^{\mathbb{D}} \quad (26)$$

where  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] \equiv \mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{E}_t[\Delta e_{i,t+1}]$  denotes the belief distortion in earnings growth forecasts, defined as the wedge between subjective expectations and objective machine learning expectations. The regressor  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] - \mathbb{D}_{i,t-1}[\Delta e_{i,t+1}]$  captures the revision in belief distortion, measuring changes in how much firms over- or under-estimate earnings growth relative to the objective benchmark. As documented in Table 1, these belief distortions exhibit overreaction to idiosyncratic shocks. Both specifications include firm fixed effects  $\alpha_i$ , industry-time fixed effects  $\tau_{s(i),t}$  where  $s(i)$  is firm  $i$ 's 2-digit SIC industry, allowing firms in different industries to have different exposures to aggregate risk. Controls  $\mathbf{X}_{i,t}$  contain lags of the dependent variable and forecast revisions. The comparison of  $\beta^{\mathbb{F}}$  versus  $\beta^{\mathbb{D}}$  provides a direct test of belief distortions: if systematic overreaction drives the response, then  $\beta^{\mathbb{F}}$  and  $\beta^{\mathbb{D}}$  should be similar in magnitude.

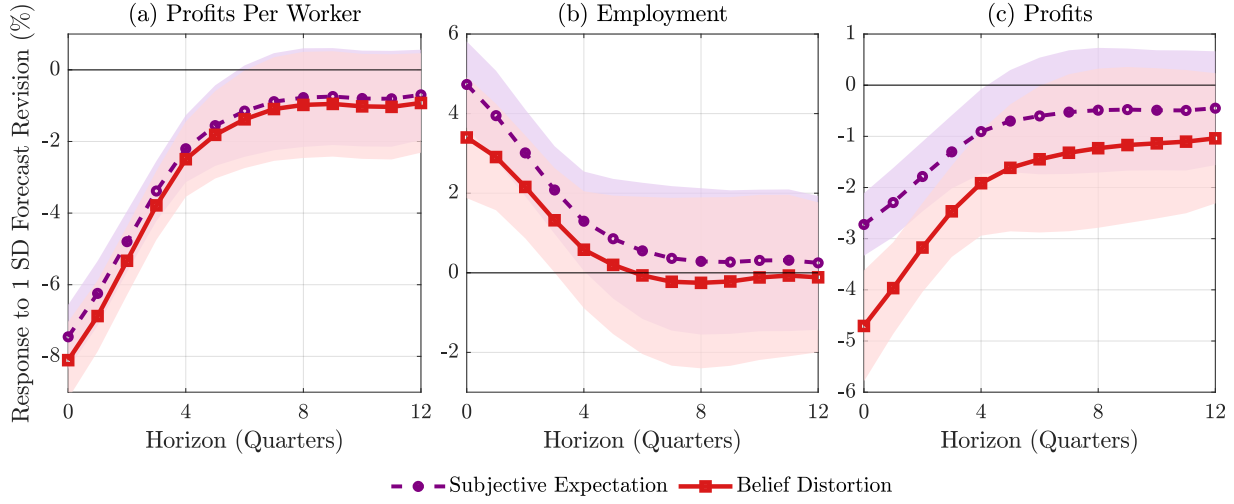
Figure 8 reports the impulse responses estimated from (25) and (26). Panel (a) shows the response of profits per worker, Panel (b) shows the response of employment, and Panel (c) shows the response of total profits. In each panel, the violet dashed line corresponds to the survey forecast revision, and the red solid line corresponds to the revision in the belief distortion. Across all three outcomes, the responses under subjective beliefs and belief distortions are nearly identical: profits per worker decline, employment rises sharply, and total profits fall slightly. The close alignment between the two responses indicates that the hiring response to idiosyncratic shocks is driven almost entirely by the distortion component of subjective beliefs.<sup>28</sup> The belief distortion revision alone explains 50% of the variation in contemporaneous profits per worker ( $R^2 = 0.50$ ), accounting for 82% of the total variation explained by subjective forecast revisions ( $R^2 = 0.61$ ). This quantitative decomposition confirms that the distortionary component of subjective beliefs drives the observed dynamics in profits per worker.

Following a positive idiosyncratic shock, firms with distorted beliefs become overly optimistic, post more vacancies, and expand employment aggressively. The rapid increase in the wage bill

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<sup>28</sup>The corresponding impulse response under objective expectations (machine learning forecasts) shows profits per worker remaining stable following idiosyncratic shocks, as shown in Appendix Figure A.4. This stability is consistent with rational expectations models where firms correctly identify idiosyncratic shocks as transitory and diversifiable, and therefore do not overreact to the shock.

Figure 8: Employment Response to Idiosyncratic Forecast Revision



*Notes:* Figure displays impulse responses to earnings growth forecast revisions, estimated via local projections with firm and time fixed effects. Panel (a): log profits per worker  $\log(E_{i,t+h}/L_{i,t+h})$ . Panel (b): log employment  $\log(L_{i,t+h})$ . Panel (c): log profits  $\log(E_{i,t+h})$ . Violet line: impulse response under subjective expectations, where survey forecast revision is the earnings growth revision  $\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]$ . Red line: impulse response to belief distortion revision  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] - \mathbb{D}_{i,t-1}[\Delta e_{i,t+1}]$ , where  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] \equiv \mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{E}_t[\Delta e_{i,t+1}]$ . Observations weighted by each firm's market value. Shaded areas: two-way clustered 95% confidence intervals by firm and time. Data sample: 1999Q1–2023Q4.

and hiring costs (vacancy posting, recruiting, training) outpaces any revenue gains from the shock, leading to a decline in total profits. Because idiosyncratic shocks are transitory, the productivity improvement fades quickly while employment remains persistently elevated due to labor market frictions. By the time new workers are hired and on the payroll, the productivity benefit has dissipated, leaving firms with elevated labor costs but no offsetting revenue increase. The result is both declining total profits and sharply declining profits per worker, demonstrating that belief distortions drive fluctuations in hiring decisions.<sup>29</sup> The magnitude of these responses is economically meaningful. The peak increase in employment represents roughly ten percent of the cross-sectional dispersion in firm-level employment growth, placing the response well within the range of differences observed across firms.

Under objective expectations, however, firms recognize that idiosyncratic earnings shocks are typically short-lived, with limited persistence. Given labor market frictions, including vacancy posting costs, recruiting and training expenses, and separation costs, hiring represents a costly investment whose payoff materializes only gradually. A firm with objective beliefs would therefore anticipate that by the time new workers are hired and fully productive, a transitory shock may

<sup>29</sup>The profit measure is IBES street earnings, which corresponds to adjusted net income excluding one-off charges but including depreciation. Since capital expenditures are capitalized rather than expensed, they affect earnings gradually through depreciation. In the short run, investment does not directly depress IBES earnings, so the observed decline in profits per worker reflects a strong response in hiring rather than capital investment patterns.



have already dissipated, leaving little benefit from expanding employment. Moreover, the option value of waiting to observe whether a shock persists further dampens the incentive to respond immediately. As a result, while some hiring response to idiosyncratic shocks remains optimal, objective firms should exhibit substantially muted responses relative to permanent shocks, with the optimal response declining as adjustment costs increase. In contrast, the empirical evidence shows that firms with distorted beliefs exhibit large hiring responses to idiosyncratic shocks that subsequently reduce profits per worker, consistent with belief-driven overreaction rather than rational adjustment.

**Stock Return Response to Idiosyncratic Shocks** If the same distortions that lead firms to overhire also get priced into stock valuations, then subjective cash flow revisions should predict returns with an initial overshoot followed by disappointment, while objective discount rate revisions should not predict returns since idiosyncratic shocks are diversifiable. To test this prediction, I estimate local projections of annual stock returns on forecast revisions. For the impulse response under subjective beliefs, I estimate:

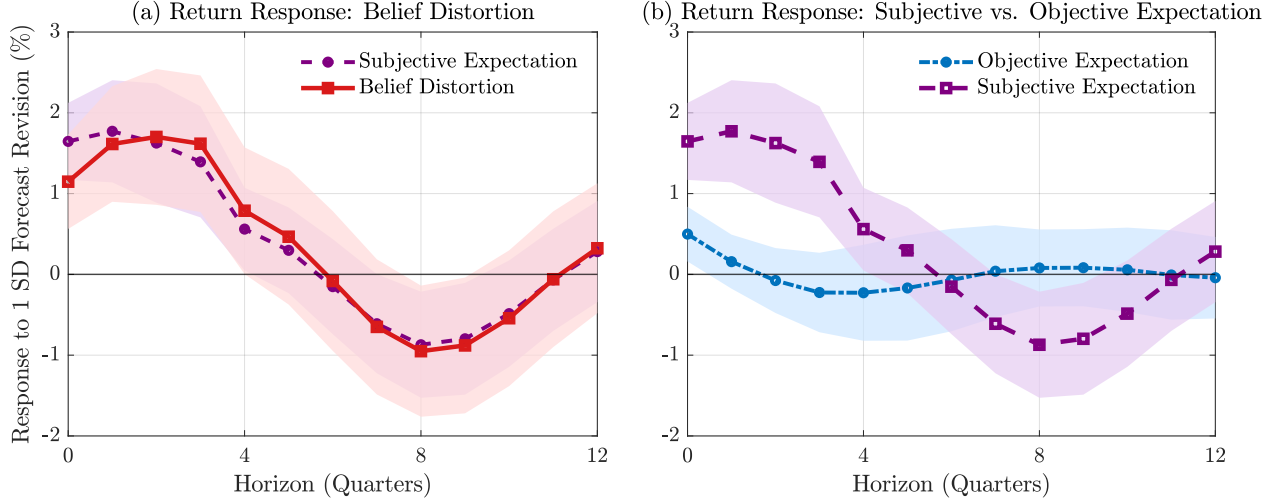
$$r_{i,t+h} = \alpha_i^{\mathbb{F}} + \tau_{s(i),t}^{\mathbb{F}} + \beta^{\mathbb{F}}(\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]) + \gamma^{\mathbb{F}'}\mathbf{X}_{i,t} + \varepsilon_{i,t+h}^{\mathbb{F}} \quad (27)$$

where  $r_{i,t+h}$  denotes the one-year stock return from  $t$  to  $t+h$ . I use the revision in the machine forecast of stock returns to estimate similar impulse responses under objective beliefs. To isolate the role of belief distortions, I estimate:

$$r_{i,t+h} = \alpha_i^{\mathbb{D}} + \tau_{s(i),t}^{\mathbb{D}} + \beta^{\mathbb{D}}(\mathbb{D}_{i,t}[\Delta e_{i,t+1}] - \mathbb{D}_{i,t-1}[\Delta e_{i,t+1}]) + \gamma^{\mathbb{D}'}\mathbf{X}_{i,t} + \varepsilon_{i,t+h}^{\mathbb{D}} \quad (28)$$

where  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] \equiv \mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{E}_t[\Delta e_{i,t+1}]$  denotes the belief distortion. Figure 9 reports the resulting impulse responses. Under subjective beliefs, positive forecast revisions generate a short-run increase in realized returns as optimistic investors bid up prices, followed by a reversal as the initial overreaction is corrected. In contrast, under objective (machine) expectations, the response of returns to idiosyncratic forecast revisions is statistically insignificant at all horizons, consistent with rational models in which firm-specific shocks are transitory and diversifiable. The implied return response is large in economic terms and accounts for close to ten percent of the cross-sectional dispersion in stock returns, highlighting that belief distortions amplify movements in both employment and asset prices.

Figure 9: Stock Return Response to Idiosyncratic Forecast Revision



*Notes:* Figure displays impulse responses to earnings growth forecast revisions, estimated via local projections with firm and time fixed effects. Panel (a): Impulse response comparing subjective beliefs and belief distortions. Panel (b): Impulse response comparing subjective beliefs and belief distortions. Blue line: impulse response under objective expectations, where machine forecast revision is the earnings growth revision  $\mathbb{E}_t[\Delta e_{i,t+1}] - \mathbb{E}_{t-1}[\Delta e_{i,t+1}]$ . Violet line: impulse response under subjective expectations, where survey forecast revision is the earnings growth revision  $\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]$ . Red line: impulse response to belief distortion revision  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] - \mathbb{D}_{i,t-1}[\Delta e_{i,t+1}]$ , where  $\mathbb{D}_{i,t}[\Delta e_{i,t+1}] \equiv \mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{E}_t[\Delta e_{i,t+1}]$ . Observations weighted by each firm's market value. Shaded areas: two-way clustered 95% confidence intervals by firm and time. Data sample: 1999Q1–2023Q4.

## 7 Model of Constant-Gain Learning

In this section, I introduce a model of hiring in which firms form subjective beliefs about cash flows and prices using a constant-gain learning rule.<sup>30</sup> The model embeds belief distortions in a search-and-matching framework from Section 4. The distortions shape firms' vacancy posting decisions and drive variation in hiring and vacancy filling rates. Simulations from the model generate decompositions that can match those estimated from the data in Sections 5 and 6.

**Cash Flow Process** Firms do not have full knowledge of the stochastic processes governing their cash flows. Instead, they form beliefs about their long-run mean using constant-gain learning. Assume that the firm's cash flow process consists of aggregate and idiosyncratic components. Firm  $i$ 's earnings at time  $t$  are given by:

$$E_{i,t} = E_t \cdot \tilde{E}_{i,t} = \exp(e_t + \tilde{e}_{i,t}) \quad (29)$$

The aggregate component follows an AR(1) process:

$$e_t = \mu + \phi e_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_u^2) \quad (30)$$

<sup>30</sup>For applications of constant-gain learning in macroeconomics, see Evans and Honkapohja (2001) and Marcat and Sargent (1989). For applications in asset pricing, see Adam et al. (2016), Nagel and Xu (2021), and De La O et al. (2024).

where  $\mu$  is the unknown long-run mean,  $\phi < 1$  is the known persistence parameter, and  $u_t$  is an i.i.d. Gaussian innovation. This specification captures both persistent variation in aggregate earnings and stochastic fluctuations. The idiosyncratic component also follows an AR(1) process:

$$\tilde{e}_{i,t} = \tilde{\mu}_i + \tilde{\phi}\tilde{e}_{i,t-1} + v_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0, \sigma_v^2) \quad (31)$$

where  $\tilde{\mu}_i$  is a firm-specific long-run mean in earnings (unknown to the firm),  $\tilde{\phi} < 1$  is a known persistence parameter, and  $v_{i,t}$  is an i.i.d. idiosyncratic shock.

**Subjective Expectations Under Learning** Objectively, mean growth is identical across firms:  $\mu = \tilde{\mu}_i = 0$ . Under subjective beliefs, however, agents do not observe the true long-run mean  $\mu$  and  $\tilde{\mu}_i$ . Instead, they form beliefs and update these beliefs recursively as new information arrives. I assume firms employ constant-gain learning, which places greater weight on recent forecast errors. The updating rules are:

$$\mathbb{F}_t[\mu] = \mathbb{F}_{t-1}[\mu] + \nu (\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]) \quad (32)$$

$$\mathbb{F}_t[\tilde{\mu}_i] = \mathbb{F}_{t-1}[\tilde{\mu}_i] + \nu (\Delta \tilde{e}_{i,t} - \mathbb{F}_{t-1}[\Delta \tilde{e}_{i,t}]) \quad (33)$$

where  $\nu$  is the constant gain parameter and governs the speed of learning.<sup>31</sup> A higher  $\nu$  implies greater responsiveness to new data. The term  $\mathbb{F}_t[\mu]$  denotes the firm's time- $t$  belief about the aggregate long-run mean, and  $\Delta e_t \equiv e_t - e_{t-1}$  is the realized growth in aggregate earnings, which the firm observes at time  $t$ . I assume that the learning rule starts with an initial value equal to the true values  $\mathbb{F}_0[\mu] = \mathbb{F}_0[\tilde{\mu}_i] = 0$  so that the model nests the full information rational expectations case at  $\nu = 0$  by having the beliefs remain fixed at the true values. Both updating rules use the same learning rate  $\nu$  for parsimony. Existing estimates of the learning rate are deliberately small, ensuring slow learning that allows subjective beliefs to remain persistently distorted even after large forecast errors (Malmendier and Nagel, 2015; Adam et al., 2016).<sup>32</sup> This persistence can generate the sustained belief distortions needed to explain fluctuations in hiring.

Constant-gain learning assigns exponentially decreasing weights to past observations, causing memory to fade over time. Beliefs never fully converge to rational expectations, even in stationary environments. This avoids the unrealistic declining volatility implied by other learning schemes

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<sup>31</sup>To isolate the importance of expected cash flows, the baseline model assumes agents learn only about cash flows. One can obtain qualitatively similar results when agents employ constant-gain learning to update beliefs about both earnings growth and stock price growth (e.g., Adam et al., 2016).

<sup>32</sup>This constant-gain learning specification is supported by empirical evidence presented in Appendix OA.5, which shows that survey respondents update their long-run earnings expectations only gradually following short-term earnings surprises. Specifically, the response of 5-year-ahead forecasts to earnings news is small and often statistically insignificant, consistent with the slow, partial updating implied by constant-gain learning.

such as ordinary least squares (OLS) learning, while allowing for beliefs to adapt to potential regime shifts.<sup>33</sup> The mechanism generates overreaction to economic news because agents cannot distinguish temporary shocks  $(u_t, v_{i,t})$  from information about the long-run mean  $(\mu, \tilde{\mu}_i)$ . They misattribute recent surprises to persistent shifts in underlying growth rates, over-extrapolating short-run fluctuations and creating persistent belief distortions that amplify valuation and hiring responses. Given these beliefs, firms forecast future aggregate earnings growth using:

$$\mathbb{F}_t[\Delta e_{t+h}] = \phi^{h-1}(\mathbb{F}_t[\mu] + (\phi - 1)e_t) \quad (34)$$

$$\mathbb{F}_t[\Delta \tilde{e}_{i,t+h}] = \tilde{\phi}^{h-1}(\mathbb{F}_t[\tilde{\mu}_i] + (\tilde{\phi} - 1)\tilde{e}_{i,t}) \quad (35)$$

This reflects the fact that agents know the process is AR(1) with known persistence  $\phi$  and  $\tilde{\phi}$  but uncertain  $\mu$  and  $\tilde{\mu}_i$ , and project the process forward using current levels of earnings and the estimated perceived long-run mean. The forecast of firm-level earnings growth is then:

$$\mathbb{F}_t[\Delta e_{i,t+h}] = \mathbb{F}_t[\Delta e_{t+h}] + \mathbb{F}_t[\Delta \tilde{e}_{i,t+h}] \quad (36)$$

These expectations will feed directly into the firm's market value, which in turn influences their vacancy posting decisions through the hiring equation (10).

**Aggregate Stock Price and Returns** Firms use their beliefs about future earnings to form expectations about asset returns and valuations. I assume that the economy is governed by a representative household such that the log stochastic discount factor (SDF) is:

$$m_{t+1} = -r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \quad (37)$$

where  $r_f$  is the risk-free rate,  $\gamma$  is the coefficient of relative risk aversion, and  $u_{t+1}$  is the aggregate earnings shock from the earnings process. The stochastic discount factor determines how firms value future cash flows.  $-r_f$  reflects time discounting, where future payoffs are worth less than current payoffs because investors can always earn the risk-free rate.  $-\frac{1}{2}\gamma^2\sigma_u^2$  is the price of uncertainty, where uncertainty about the future aggregate shock  $u_{t+1}$  lowers the discounted value of risky payoffs.  $-\gamma u_{t+1}$  is the price of risk, making the SDF high in bad times and low in good times depending on the aggregate shock  $u_{t+1}$ . During bad economic shocks ( $u_{t+1} < 0$ ), the SDF is high because an extra dollar is highly valued. Risk aversion  $\gamma$  amplifies these effects, making the SDF more sensitive to aggregate shocks. With constant relative risk aversion and

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<sup>33</sup>Constant-gain learning can be micro-founded using an overlapping generations model where agents learn from recent experience across generations, where the average expectation closely approximates a constant-gain learning rule (Nagel and Xu, 2021).

log-normal shocks, this SDF implies a constant risk premia under rational beliefs. The exercise here is to ask how far we can explain asset prices and labor market fluctuations by relying only on distortions in subjective beliefs, without relying on rational time-varying risk premia.

Let  $P_t^{(h)}$  denote the time  $t$  price for an aggregate strip of a one-dollar payoff received  $h$  periods in the future. I guess and verify a log-linear solution:<sup>34</sup>

$$P_t^{(h)} = \mathbb{E}_t[M_{t+1}P_{t+1}^{(h-1)}] = \exp \{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\} \quad (38)$$

The strip price reflects the discounted value of a dollar paid at horizon  $h$ . This expression implies that strip prices depend on current earnings  $e_t$ , beliefs about the aggregate long-run mean  $\mathbb{F}_t[\mu]$ , and constants  $A^{(h)}$ ,  $B^{(h)}$  that evolve recursively (see Appendix OB.3 for derivations):

$$A^{(h)} = A^{(h-1)} - r_f + \frac{1}{2}C^{(h)} [C^{(h)} - 2\gamma] \sigma_u^2 \quad (39)$$

$$B^{(h)} = B^{(h-1)} + \phi^{h-1} = \frac{1 - \phi^h}{1 - \phi} \quad (40)$$

$$C^{(h)} = \nu B^{(h-1)} + \phi^{h-1} \quad (41)$$

where  $A^{(0)} = B^{(0)} = C^{(0)} = 0$ .  $A^{(h)}$  captures discounting and risk premia, pushing down long-horizon strip prices.  $B^{(h)}$  measures sensitivity to beliefs about long-run cash flow growth  $\mu$ , with its influence rising in the horizon.  $C^{(h)}$  reflects the effect of constant-gain learning, where recent forecast errors are overweighted, distorting valuations relative to rational expectations. The aggregate stock price is the sum of strip prices across all future periods:

$$P_t = \sum_{h=1}^{\infty} P_t^{(h)} \quad (42)$$

The aggregate stock return realized between  $t$  and  $t+1$  is defined as the value-weighted average return across all strips:

$$R_{t+1} = \frac{\sum_{h=1}^{\infty} P_{t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_t^{(h)}} = \sum_{h=1}^{\infty} w_{t,h} R_{t+1}^{(h)}, \quad w_{t,h} = \frac{P_t^{(h)}}{\sum_{k=1}^{\infty} P_t^{(k)}} \quad (43)$$

where  $R_{t+1}^{(h)} = P_{t+1}^{(h-1)}/P_t^{(h)}$  is the return on the  $h$ -period strip, and  $w_{t,h}$  is the share of total market value accounted for by strip  $h$ . Under the assumption that expected strip weights are

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<sup>34</sup>In constant-gain learning models, fading memory breaks the law of iterated expectations, making the buy-and-hold and resale valuation methods non-equivalent. While the former values long-run payoffs using only today's beliefs, the resale method prices assets iteratively using updated, one-period-ahead expectations. Following Nagel and Xu (2021), I adopt the resale method because it is time-consistent and better reflects trading among agents with evolving beliefs. For consistency between firm decisions and asset pricing, I assume both the manager and the representative investor share the same beliefs and use this valuation approach.

approximately constant ( $w_{t+j-1,h} \approx w_{t,h}$ ), the expected aggregate return for horizon  $j \geq 1$  is:<sup>35</sup>

$$\mathbb{F}_t[R_{t+j}] \approx \sum_{h=1}^{\infty} w_{t,h} \mathbb{F}_t[\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+j-1}[R_{t+j}^{(h)}]]] = \sum_{h=1}^{\infty} w_{t,h} \exp \{r_f + C^{(h)} \gamma \sigma_u^2\} \quad (44)$$

which shows that expected strip returns decline with horizon  $h$  when agents believe earnings growth is persistent ( $\phi < 1$ ) and a small learning rate  $\nu$  slows perceived mean reversion. Since  $C^{(h)} = \nu B^{(h-1)} + \phi^{h-1}$ , a smaller learning rate  $\nu$  makes  $C^{(h)}$  decline more quickly, muting the influence of long-horizon strip returns. Finally, to obtain the expected log return, I apply the approximation:  $\mathbb{F}_t[r_{t+j}] \approx \log(\mathbb{F}_t[R_{t+j}])$ .

**Firm Stock Price and Returns** Each firm's total value is the sum of expected discounted future cash flows. Let  $P_{i,t}$  be the ex-dividend value of firm  $i$ , which is the sum of the firm's strip prices  $P_{i,t}^{(h)}$  across all future horizons  $h$ :

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)}, \quad P_{i,t}^{(h)} = \mathbb{F}_t[M_{t+1} P_{i,t+1}^{(h-1)}] \quad (45)$$

Assuming independence between aggregate discounting and idiosyncratic earnings:

$$P_{i,t}^{(h)} = P_t^{(h)} \cdot \mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]] \quad (46)$$

Following similar logic as in the aggregate case (see Appendix OB.3 for derivations), the firm-level stock return is the value-weighted average return across all strips maturing from  $h = 1$  onwards:

$$R_{i,t+1} = \frac{\sum_{h=1}^{\infty} P_{i,t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_{i,t}^{(h)}} = \sum_{h=1}^{\infty} w_{i,t,h} R_{i,t+1}^{(h)}, \quad w_{i,t,h} = \frac{P_{i,t}^{(h)}}{\sum_{k=1}^{\infty} P_{i,t}^{(k)}} \quad (47)$$

where  $R_{i,t+1}^{(h)} = P_{i,t+1}^{(h-1)} / P_{i,t}^{(h)}$  is the return on the  $h$ -period strip, and  $w_{i,t,h}$  is the share of total market value accounted for by strip  $h$ . Assuming that expected strip weights are approximately constant under subjective beliefs  $w_{i,t+j-1,h} \approx w_{i,t,h}$ , the expected firm-level return is:

$$\mathbb{F}_t[R_{i,t+j}] \approx \sum_{h=1}^{\infty} w_{i,t,h} \exp \left\{ r_f + C^{(h)} \gamma \sigma_u^2 + \frac{1}{2} ((\tilde{C}^{(h)})^2 - \tilde{\phi}^{2(h-1)}) \sigma_v^2 \right\} \quad (48)$$

where  $\tilde{C}^{(h)} \equiv \tilde{\phi}^{h-1} + \nu \frac{1 - \tilde{\phi}^{h-1}}{1 - \tilde{\phi}}$  captures the effect of learning from the idiosyncratic shock.

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<sup>35</sup>It can be shown that the assumption holds approximately in the small-gain limit. As  $\nu \rightarrow 0$ , strip prices move proportionally to changes in cash flows:  $P_{t+1}^{(h)} / P_t^{(h)} \approx \exp\{\phi^h(e_{t+1} - e_t)\}$ . Since the strip price of all maturities shift by the same factor raised to different powers  $\phi^h$ , relative strip weights  $w_{t,h} = P_t^{(h)} / \sum_k P_t^{(k)}$  remain approximately constant over time.

**Subjective Firm Valuation** The firm's equilibrium stock price under subjective beliefs is the sum of its strip prices:

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)} = \sum_{h=1}^{\infty} \exp \left\{ A_i^{(h)} + B^{(h)} \mathbb{F}_t[\mu] + \tilde{B}^{(h)} \mathbb{F}_t[\tilde{\mu}_i] + \phi^h e_t + \tilde{\phi}^h \tilde{e}_{i,t} \right\} \quad (49)$$

where the coefficients are defined as  $A_i^{(h)} = A^{(h)} + \frac{1}{2} \sigma_v^2 \frac{1 - \tilde{\phi}^{2h}}{1 - \phi^2}$  and  $\tilde{B}^{(h)} = \frac{1 - \tilde{\phi}^h}{1 - \phi}$ . The equation shows that the firm's value rises with expected cash flow intercepts  $\mathbb{F}_t[\mu]$  and  $\mathbb{F}_t[\tilde{\mu}_i]$ . The belief distortions captured in these expectation terms will affect the firm's hiring decisions through its valuation.

**Hiring Condition** I close the model by connecting asset valuations to firm hiring behavior. The connection to labor markets operates through the hiring condition. As shown in Section 4, firms post vacancies until the marginal cost of hiring equals its marginal value:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of Hiring}} = \underbrace{\frac{P_{i,t}}{L_{i,t+1}}}_{\text{Value of Hiring}} \quad (50)$$

where  $\kappa$  is the cost per vacancy posting,  $q_t$  is the vacancy filling rate, and  $L_{i,t+1}$  denotes employment. Overly pessimistic beliefs about expected cash flows (low  $\mathbb{F}_t[\mu_i]$ ) lower the firm value  $P_{i,t}$ , which reduces the value of hiring and leads to fewer job postings. The resulting decrease in vacancy creation drives up the vacancy filling rate  $q_t$  and unemployment  $U_t$ .

Belief distortions in the model affect both financial and real outcomes. When optimistic beliefs inflate  $P_{i,t}$ , the realized stock return  $R_{i,t} = (P_{i,t} + E_{i,t})/P_{i,t-1}$  increases and the perceived value of hiring increases, which leads to higher employment  $L_{i,t+1}$ . In contrast, when pessimism depresses  $P_{i,t}$ , realized returns and hiring both fall. The model therefore jointly predicts that stock returns and hiring will move together as belief distortions fluctuate over time.

Given values for  $\kappa, \delta, B, \eta, P_{i,t}$  and initial values for employment  $L_{i,0}$ , one can construct the sequence of vacancies  $V_{i,t}$ , employment  $L_{i,t+1}$ , labor market tightness  $\theta_t$ , vacancy filling rates  $q_t$ , and unemployment rate  $U_t$  by solving for the employment accumulation (5), firm valuation (49), and optimal hiring (50) equations under a Cobb-Douglas matching function (4).

1. Initialize labor market tightness:  $\theta_t^{(0)} = 1$
2. At iteration  $s$ , use labor market tightness  $\theta_t^{(s)}$  to construct vacancy filling rate by using the Cobb-Douglas matching function in equation (4):

$$q_t^{(s)} = B(\theta_t^{(s)})^{-\eta} \quad (51)$$

3. Update each firm's employment policy using the hiring equation (50):

$$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa} \quad (52)$$

where  $P_{i,t}$  is determined by the firm valuation equation (49) under the constant-gain learning rules in equations (32) and (33).

4. Update each firm's vacancy posting using the employment accumulation equation (5):

$$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}} (L_{i,t+1}^{(s)} - (1 - \delta) L_{i,t}) \quad (53)$$

5. Aggregate firm-level variables over the set of firms  $I$ :

$$V_t^{(s)} = \sum_{i \in I} V_{i,t}^{(s)}, \quad L_{t+1}^{(s)} = \sum_{i \in I} L_{i,t+1}^{(s)}, \quad U_t^{(s)} = 1 - \sum_{i \in I} L_{i,t} \quad (54)$$

6. Update labor market tightness:  $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$ . Check convergence:  $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$  for some small tolerance  $\varepsilon > 0$ . If not, return to step 2 with the updated values.

In this simplified framework, I abstract from wage determination and workers' beliefs to isolate the role of firms' expectations. Wages and worker-side beliefs are thus treated as residual objects consistent with the assumed cash flow process. The cash flow dynamics themselves are disciplined using data on firms' realized and expected earnings, allowing the model to capture belief-driven fluctuations in hiring without imposing additional structure on wage setting or worker expectations. This simplification highlights that firms' belief distortions alone can generate large fluctuations in vacancy creation and employment. In a richer model, if workers' beliefs differ from firms' beliefs, such disagreement could introduce further frictions in wage bargaining and amplify the effects of belief distortions on labor market dynamics.

**Model-Implied Decompositions** I use simulated data implied by the model to decompose the vacancy filling rate at the aggregate level (Section 5) and hiring rates at the firm level (Section 6). The time-series decomposition of the aggregate vacancy filling rate  $q_t$  is given by:

$$\log q_t = \underbrace{\sum_{j=1}^h \rho^{j-1} \mathbb{E}_t[r_{t+j}]}_{\text{Discount Rate}} - \underbrace{\left[ el_t + \sum_{j=1}^h \rho^{j-1} \mathbb{E}_t[\Delta e_{t+j}] \right]}_{\text{Cash Flow}} - \underbrace{\rho^h \mathbb{E}_t[pe_{t+h}]}_{\text{Future Price-Earnings}} \quad (55)$$



where  $x_t = \sum_{i \in I} x_{i,t}$  aggregates firm-level variable  $x_{i,t}$ .  $el_{i,t} \equiv e_{i,t} - l_{i,t+1} = \log E_{i,t} - \log L_{i,t+1}$  denotes log earnings per worker. To analyze heterogeneity across firms, I estimate a cross-sectional decomposition of hiring rates using simulated firm-level data:

$$\tilde{h}l_{i,t} = \underbrace{-\sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\tilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[ \tilde{e}l_{i,t} + \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\Delta \tilde{e}_{i,t+j}] \right]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t[\tilde{p}e_{i,t+h}]}_{\text{Future Price-Earnings}} \quad (56)$$

where  $\tilde{x}_{i,t} = x_{i,t} - \frac{1}{I} \sum_i x_{i,t}$  denotes a cross-sectional deviation from the mean at time  $t$ .<sup>36</sup>

Subjective expectations will over-weight the role of the cash flow channel relative to the discount rate channel, in contrast to the pattern observed under rational expectations. Under rational expectations, agents know the true long-run mean, making these belief distortions zero. Any distortions in beliefs about the intercept terms  $\mu - \mathbb{F}_t[\mu]$  (aggregate) and  $\tilde{\mu}_i - \mathbb{F}_t[\tilde{\mu}_i]$  (idiosyncratic) could serve as a common driving force behind the cash flow component and the vacancy filling rate  $q_t$ . The distortion then drives fluctuations in expected cash flow growth  $\mathbb{F}_t[\Delta e_{t+j}]$  and  $\mathbb{F}_t[\Delta \tilde{e}_{i,t+j}]$  through equations (34) and (35), respectively, with persistent effects on hiring decisions over time through the firm's hiring condition in equation (50).

The expected return at both the aggregate (44) and firm level (48) is driven by the term  $C(h)\gamma\sigma_u^2$ . This component is constant over time for fixed parameters and horizon  $h$ , so the implied risk premium is effectively time-invariant. Hence the subjective discount rate exhibits only minor variation, arising only through changes in portfolio weights  $w_{t,h}$  if those are not treated as constant. This supports the conclusion that belief distortions primarily affect the cash flow channel rather than the discount-rate channel.

**Simulation Details** To evaluate the model's quantitative performance, I simulate a panel of 300 firms over 500 periods, where the first 150 periods are discarded as a burn-in to eliminate the influence of initial conditions. Each firm updates its beliefs using constant-gain learning based on the updating rules in equations (32) and (33). All expectations, returns, and decompositions are computed at an annual frequency using the model equations derived above. At each horizon  $h$ , I compute the model-implied time-series decomposition of the aggregate vacancy filling rate based on equation (18) and the cross-sectional decomposition of the firm-level hiring rates (56). I then compare these model-implied decompositions to those estimated from the observed data from Figures 5 and 7.

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<sup>36</sup>Note that the decomposition consists of expectations of the future values of the three components, not the contemporaneous values. The free-entry condition of the search model pins down the current value of  $pe_{i,t} + el_{i,t}$ , but it does not necessarily eliminate cross-sectional variation in forward-looking expectations. These subjective beliefs can differ across firms even when the current sum is identical.

**Model Estimation** Table 1 reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The model is calibrated at an annual frequency. The persistence  $\phi$  and volatility  $\sigma_u$  of aggregate earnings growth is set to match the autocorrelation and standard deviation of aggregate S&P 500 earnings growth for the period between 1983 to 2022 (De La O et al., 2024).<sup>37</sup> The persistence  $\tilde{\phi}$  and volatility  $\sigma_v$  of idiosyncratic earnings growth is set to match the autocorrelation and standard deviation of earnings growth across publicly listed firms over the same period, after cross-sectionally demeaning the variable. The risk-free rate  $r_f$  and risk aversion  $\gamma$  match the average level and volatility of aggregate S&P 500 stock returns (De La O et al., 2024). The time discount rate  $\rho = \exp(\overline{pe}) / (1 + \exp(\overline{pe})) = 0.98$  is chosen to be consistent with a steady-state price-earnings ratio from the Campbell and Shiller (1988) present value identity, where  $\overline{pe}$  is the long-run average of the log price-earnings ratio.

Table 1: Model Parameters

Parameter	Value	Moments
$\nu$	0.018	Constant-gain learning (Malmendier and Nagel, 2015)
$\phi$	0.856	Autocorrelation aggregate earnings growth
$\sigma_u$	0.268	S.D. aggregate earnings growth
$\tilde{\phi}$	0.698	Autocorrelation firm-level earnings growth
$\sigma_v$	0.194	S.D. firm-level earnings growth
$r_f$	0.046	Average risk-free rate
$\gamma$	1.586	Average and S.D. aggregate return
$\rho$	0.980	Average price-earnings ratio
$B$	0.562	Matching function efficiency (Kehoe et al., 2023)
$\eta$	0.500	Matching function elasticity (Kehoe et al., 2023)
$\delta$	0.286	Separation rate (Kehoe et al., 2023)
$\kappa$	0.314	Per worker hiring cost (Elsby and Michaels, 2013)

*Notes:* Table reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The model is calibrated at an annual frequency.

The speed at which agents discount past observations of realized cash flow growth depends on the constant gain parameter  $\nu$  in the learning rule. This parameter shapes the persistence and volatility of the price-earnings ratio and the extent of return predictability. I take the value directly from survey-based estimates in Malmendier and Nagel (2015), setting it to  $\nu = 0.018$  at the quarterly frequency.<sup>38</sup> This implies that in forming expectations, agents assign a weight of

<sup>37</sup>See Section OD for a mapping from the AR(1) level parameters to the implied moments of earnings growth.

<sup>38</sup>Malmendier and Nagel (2015) estimate  $\nu = 0.018$  at the quarterly frequency, while the model is simulated annually. Using this value unchanged does not materially alter the implied speed of learning: for small gains, the difference between quarterly and annual updating is second order (De La O et al., 2024). Thus, applying the quarterly estimate at the annual frequency still yields an effective half-life of roughly a decade, consistent with the survey evidence.

0.018 to the most recent growth surprise and  $1 - \nu = 0.982$  to their previous estimate, making the perceived growth rate evolve slowly over time.<sup>39</sup>

Labor market parameters are mainly from Kehoe et al. (2023). Following Shimer (2005), I normalize the value of labor market tightness  $\theta$  to one in the deterministic steady state, which implies an efficiency of the matching function  $B = 0.562$  by noting from the matching function that  $q = B\theta^{-\eta}$ . I set the elasticity of the matching function to  $\eta = 0.5$  following Ljungqvist and Sargent (2017). I use an annual job separation rate of  $\delta = 0.286$ , which is the annualized value of the Abowd-Zellner corrected estimate by Krusell et al. (2017) based on data from the Current Population Survey (CPS). Following Elsby and Michaels (2013), per-worker vacancy posting cost 0.314 is targeted to match a per-worker hiring cost  $\kappa/q$  equal to 14 percent of the quarterly worker compensation. In the context of the annual calibration of this model, this implies a value approximately equal  $\kappa = 4 \times 0.14 \times q = 0.314$ , where  $4 \times 0.14$  is the annualized percent of worker compensation, while  $q = 0.562$  is the long-run average of the vacancy filling rate in the historical sample from 1983 to 2023.

**Model vs. Data: Variance Decompositions** The model successfully replicates the empirical variance decompositions from the data. Figure 10 shows that the model can reproduce the finding that belief distortions drive the high sensitivity to cash flow news in explaining labor market fluctuations under subjective beliefs, both in the time series and the cross section.

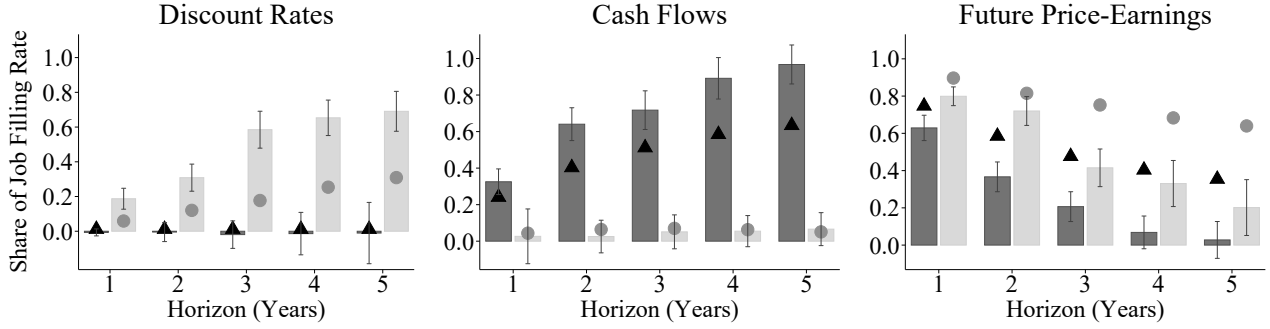
Panel (a) presents the time-series decomposition of the vacancy filling rate, comparing contributions under subjective and rational expectations. The model captures the empirical pattern where subjective expectations (dark bars) assign a larger role to cash flows compared to objective expectations (light bars). The model-implied values (circles and triangles) align closely with the empirical estimates, demonstrating the model’s ability to match the data. The large estimated discount rate component under objective beliefs is consistent with existing search models formulated under rational expectations, which have emphasized time-varying discount rates to match the volatility of unemployment fluctuations. Panel (b) presents the cross-sectional decomposition of hiring rates across firms. Again, the model captures the empirical pattern that subjective belief distortions drive the high sensitivity of hiring to cash flow news. This cross-sectional fit is important as it shows that the model can explain not just aggregate patterns but also the heterogeneity in hiring behavior across different firms.

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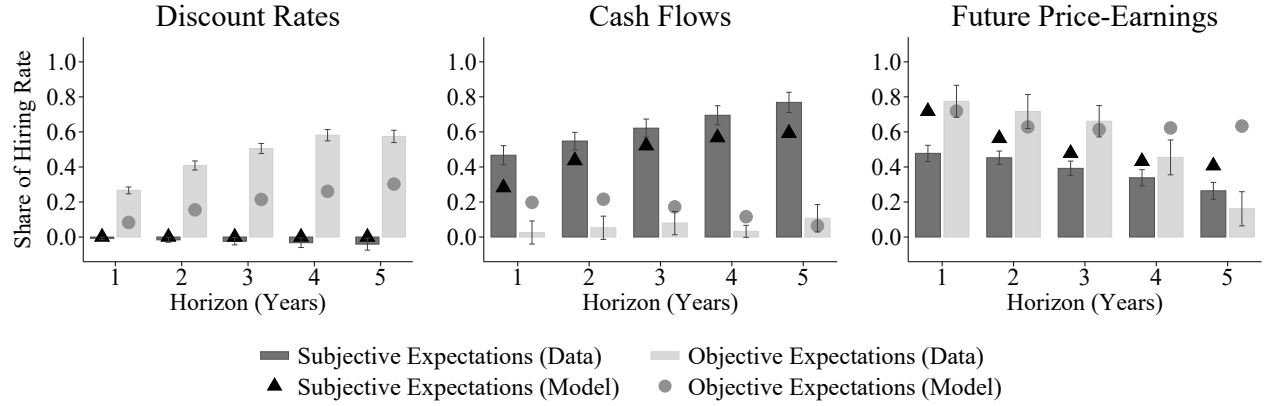
<sup>39</sup>Appendix OD describes an alternative estimation using the Method of Simulated Moments (MSM), where  $\nu$  is disciplined jointly with other structural parameters by matching model-implied and empirical moments. The MSM results yield learning rate estimates that are close in magnitude to the calibrated survey-based value ( $\hat{\nu}^{MSM} = 0.013$ ), providing independent support for the baseline choice of  $\nu$ .

Figure 10: Model vs. Data: Variance Decompositions

(a) Time-Series Decomposition of the Vacancy Filling Rate



(b) Cross-Sectional Decomposition of the Hiring Rate



*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate vacancy filling rate (panel (a)) and cross-sectional decomposition of the hiring rate (panel (b)). Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4. Circle and triangle dots show the values of rational and subjective expectations implied by the model, respectively.

**Model vs. Data: Moments** Table 2 demonstrates that the constant-gain learning model successfully matches both asset market and labor market moments. The table compares moments generated by the learning model against those generated from a rational model under no learning and constant discount rates, where all agents have full information rational expectations. Since the constant-gain learning rule in equations (32) and (33) start from the true values, a zero learning rate  $\nu = 0$  nests the rational expectations case. To generate simulations under the rational model, I employ the same sequence of shocks as in the baseline learning specification but set the learning rate parameter to zero. This eliminates belief updating and, conditional on the true initial values, reduces the model exactly to its rational expectations counterpart.

Panel (a) reports time-series and cross-sectional moments for asset prices. The learning model broadly matches the mean and volatility of price-earnings ratios, the persistence in valuations,

and the volatility of returns and expected returns. In contrast, the rational expectations model severely understates price-earnings volatility and generates virtually no variation in expected returns, confirming that belief distortions are essential for matching observed financial market behavior (Adam et al., 2016). For the cross-sectional moments, the learning model captures the dispersion in price-earnings ratios, expected earnings growth, and returns. These moments confirm that the firm-specific beliefs  $\mathbb{F}_t[\tilde{\mu}_i]$  can generate realistic heterogeneity in firm valuations and expectations. The rational expectations model, by construction, produces substantially smaller cross-sectional variation in expectations.

Panel (b) reports moments related to the labor market. The constant-gain learning model matches the volatility and persistence of the vacancy filling rate  $q_t$  and the unemployment rate  $u_t$  better than the rational benchmark. The model explains about 60% of observed U.S. unemployment volatility and cross-sectional hiring dispersion, compared with less than 30% explained by the standard rational search model without time-varying discount rates (Shimer, 2005).<sup>40</sup> The learning model’s ability to match these moments demonstrates that the constant-gain learning mechanism provides an explanation for both asset market and labor market fluctuations.

**Response to 1 Std. Dev. Shock to Cash Flow Growth Expectation** To examine the dynamic implications of the model and compare them with the data, Figure 11 estimates a four-variable VAR where the observation vector includes expected cash flow growth, expected returns, expected price-earnings, and the job-filling rate. The VAR is estimated using both the actual survey data and the simulated series generated from the model. For identification, I apply a recursive (Cholesky) scheme in which expected cash flow growth is ordered first, so that the estimated impulse responses trace out the effect of a one standard deviation shock to cash flow growth expectations.

The impulse response functions in Figure 11 reveal several notable patterns. Expectations of cash flow growth jump immediately on impact and then gradually decay back toward zero. Subjective expected returns exhibit a flat response consistent with a near constant subjective discount rate. The subjective price-earnings ratio rises initially before decaying back to zero. Finally, the job-filling rate falls immediately after the shock and then slowly converges back to its baseline level.

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<sup>40</sup>This improvement isolates the contribution of belief distortions in expected cash flows while abstracting from variation in discount rates. Models that incorporate rational expectations of time-varying discount rates, such as Kehoe et al. (2023), can explain up to 95% of unemployment volatility once longer-duration cash flows are introduced, indicating that belief distortions and discount-rate fluctuations are complementary channels in explaining observed labor market dynamics.

Table 2: Model vs. Data: Asset Market and Labor Market Moments

Moment	Data	Learning Model	Rational Model
(a) Asset Market			
$SD(pe_t) \times 100$	47.0	43.5	13.0
$AC(pe_t)$	0.75	0.84	0.92
$SD(r_t) \times 100$	16.0	12.3	3.0
$SD(\mathbb{E}_t[r_{t+1}]) \times 100$	1.1	1.4	0.5
$SD(\mathbb{E}_t[\Delta e_{t+1}]) \times 100$	26.8	24.3	7.2
$SD_i(pe_{i,t}) \times 100$	22.6	21.1	4.2
$SD_i(r_{i,t}) \times 100$	5.7	3.1	1.2
$SD_i(\mathbb{E}_t[r_{i,t+1}]) \times 100$	2.6	0.2	0.2
$SD_i(\mathbb{E}_t[\Delta e_{i,t+1}]) \times 100$	14.0	16.6	3.9
(b) Labor Market			
$SD(u_t) \times 100$	2.10	1.28	0.34
$AC(u_t)$	0.91	0.95	0.99
$SD(q_t) \times 100$	8.70	6.16	0.91
$AC(q_t)$	0.94	0.83	0.99
$\text{Corr}(u_t, q_t)$	-0.82	-0.86	-0.99
$SD_i(hl_{i,t}) \times 100$	15.70	10.39	4.65

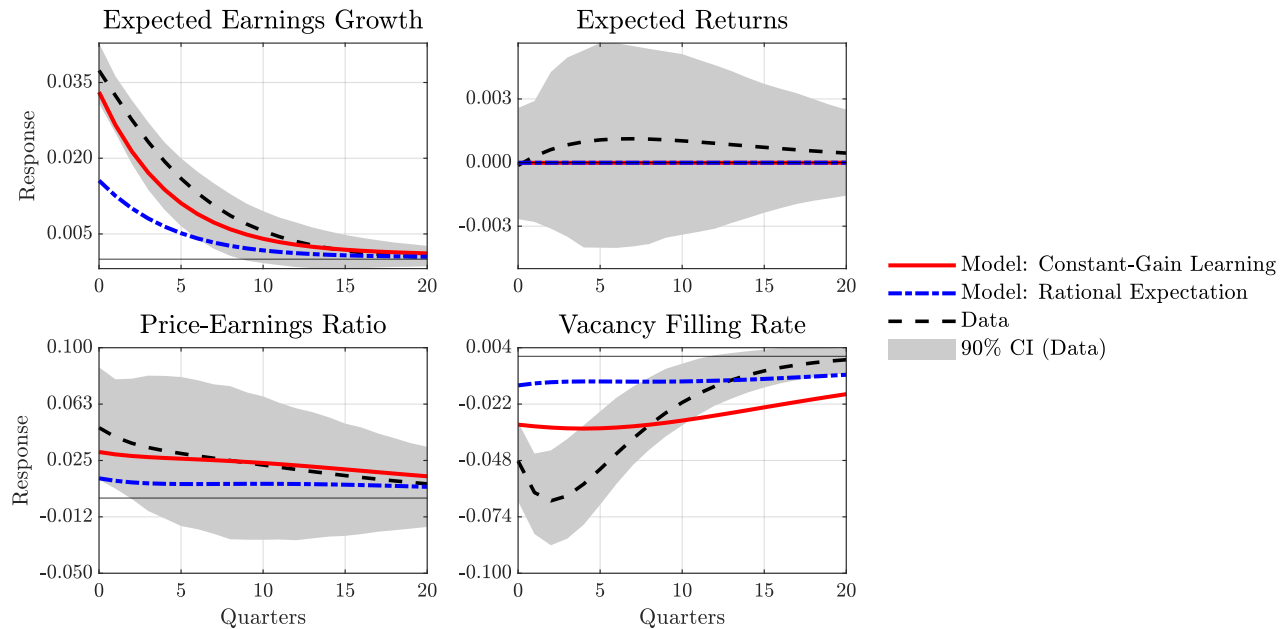
*Notes:* This table compares empirical moments with model-generated moments with and without constant-gain learning.  $SD(\cdot)$  denotes the time-series standard deviation of aggregate variables.  $SD_i(\cdot)$  denotes the cross-sectional standard deviation across firms at each point in time, averaged over time.  $AC(\cdot)$  denotes the first-order autocorrelation coefficient.  $\text{Corr}(\cdot)$  denotes the correlation between two time series.  $pe_t$  is the log price-earnings ratio,  $r_t$  is the log stock return,  $\Delta e_t$  is log earnings growth,  $q_t$  is the job-filling rate,  $u_t$  is the unemployment rate, and  $hl_{i,t}$  is the firm-level hiring rate.  $\mathbb{E}_t[\cdot]$  denotes subjective expectations formed at time  $t$ . Data column reports empirical moments estimated from historical data. Learning model reports moments from simulations of the constant-gain learning model. Rational model reports moments from the rational expectations benchmark where agents have perfect knowledge of the earnings process.

## 8 Robustness Checks and Extensions

This section presents additional results that reinforce the main finding by showing that belief distortions about expected cash flows explain a large share of not only firms’ hiring decisions but also their asset valuations and capital investment.

**Managerial vs. Analyst Expectations** To capture subjective expectations of earnings growth, this paper primarily use survey forecasts from IBES analysts. As a robustness check, this section also incorporates one-year-ahead earnings forecasts from the CFO Survey, which reflects managerial expectations (Gennaioli et al., 2016; Hillenbrand and McCarthy, 2024). While the precise accounting measure that CFOs use in forming their expectations is unclear, many CFOs focus on pro-forma earnings, which exclude special items and are conceptually similar to the “Street earnings” forecasted by analysts in IBES. This comparability suggests that CFO forecasts and IBES forecasts likely target similar earnings concepts. Consistent with this inter-

Figure 11: Impulse responses to a one standard deviation innovation in expected cash flow growth



Notes: Red solid line: model-based IRFs from simulated series under constant-gain learning. Blue solid line: model-based IRFs from simulated series under rational expectations. Black dashed line: data-based IRFs. Shaded area: 95% bootstrap confidence interval for the data VAR. Sample: 1984Q1-2023Q4.

pretation, the two survey measures have a positive correlation of 0.60 at the one-year horizon. Moreover, Appendix Table A.1 shows that replacing IBES expectations with CFO expectations yields similar variance decomposition results: cash flow expectations remain the dominant driver of hiring fluctuations under subjective managerial beliefs implied by a survey of CFOs.

**Subjective vs. Risk-Neutral Expectations** A natural question is whether survey expectations reflect risk-neutral pricing rather than true belief distortions (Cochrane, 2017). Evidence from risk-neutral expectations implied by futures prices indicates otherwise. Subjective expectations attribute even more variation to cash flow news relative to risk-neutral benchmarks (Figure A.5), which is more consistent with the presence of behavioral biases rather than only a risk-neutral change of measure. Moreover, survey forecasts of stock returns exceed risk-free rates and vary predictably over time, revealing predictable optimism inconsistent with risk-neutral beliefs (Adam et al., 2021).

**Financial Constraints** Figure A.6 shows that controlling for five firm-level financial constraint proxies (firm size, payout ratio, SA index, expected free cash flow, and Whited-Wu index) only modestly reduces the contribution of expected earnings under subjective expectations but leaves

most of the decomposition intact. Under objective expectations, however, discount rate contributions drop substantially, consistent with rational models in which tightening financial constraints can influence the discount rate.

**Predictability of Unemployment and Hiring** Section A.8 extends the baseline analysis to unemployment and firm-level employment growth using predictive regressions derived from unemployment and employment accumulation equations. Distortions in subjective cash flow expectations emerge as the strongest predictor in both time series and cross section, improving in-sample fit and out-of-sample performance over predictive models that only use objective measures of discount rates. These results are not consistent with subjective beliefs being observationally equivalent to objective expectations, and instead point to heterogeneous, distorted forecasts that crowd out objective discount rate variation as the main driver of hiring fluctuations.

**Regional Model using Shift-Share Instrument** Appendix A.9 strengthens the causal interpretation of belief distortions using a Bartik shift-share instrument that isolates exogenous variation in subjective expectations. The instrument interacts national industry-level beliefs about future cash flows and discount rates with historical state-industry employment shares, affecting local unemployment only through perceived expectations. State-level regressions show that local unemployment responds strongly to changes in subjective earnings forecasts, even after controlling for state and time fixed effects (Table A.3). The results indicate that belief distortions that arise from overreaction to perceived cash flow news can causally influence hiring across regions rather than merely correlating with labor demand.

**Decomposition of Price-Earnings Ratios** Following De La O and Myers (2021) and De La O et al. (2024), Figure A.8 applies the Campbell and Shiller (1988) present-value identity to the aggregate price-earnings ratio. The decomposition yields results parallel to those for the vacancy filling rate: objective expectations attribute most variation to discount rates, while subjective expectations overweight cash flows. Cash flows priced in equity have much longer duration than those tied to employment, so the price-earnings decomposition assigns more variation to the terminal value component. The hiring analysis uses a five-year forecast horizon, close to the upper end of typical job tenure (Bureau of Labor Statistics, 2024), which captures most employment relationships. These results confirm that the same belief distortions that drive asset valuations also drive firms' hiring behavior.



**Capital Investment** Appendix A.11 extends the framework to firm investment, distinguishing between tangible and intangible capital. Firms jointly choose hiring and investment facing convex adjustment costs and forming expectations over future productivity and returns. Decompositions of investment rates in Figures A.9 and A.10 show that distortions in subjective expectations highly sensitive to news about cash flows, as in the case for hiring.

## 9 Conclusion

This paper develops a framework linking asset valuations and hiring behavior to distortions in firms' beliefs about future cash flows. The framework explains both aggregate fluctuations and cross-sectional dispersion across firms. Using machine learning forecasts as an objective benchmark for rational and efficient belief formation, I document that survey forecasts systematically overreact to cash flow news, while machine forecasts do not. These belief distortions make hiring highly sensitive to perceived cash flow news, explaining over 60% of variation in both aggregate and firm-level hiring. Firms with distorted beliefs overhire following positive idiosyncratic shocks, reducing realized profits per worker, while objective firms do not.

A search-and-matching model in which firms learn about the long-run mean of their cash flows with fading memory can reproduce these empirical patterns. The learning process generates persistent overreaction to transitory shocks, producing fluctuations in hiring and unemployment that rational models cannot explain. The model accounts for about 60% of observed U.S. unemployment volatility and matches the cross-sectional dispersion in hiring rates across firms. By overreacting to recent news, firms interpret temporary improvements in cash flows as lasting booms and downturns as protracted slumps, leading to amplified swings in vacancy posting. These findings suggest that the same belief distortions that drive asset price fluctuations also drive volatility in labor demand.

Belief distortions offer a common framework for understanding business cycle fluctuations by linking asset valuations and real activity. Because hiring is a forward-looking decision valued like any other asset, the same behavioral biases that drive asset valuations can also drive fluctuations in hiring, capital investment, R&D spending, and broader business cycle dynamics. Incorporating belief formation into macroeconomic models can therefore improve our understanding of how expectations shape both asset valuations and real economic activity. Exploring these connections across different types of investment decisions remains an important avenue for future research.

These findings have implications for macroeconomic stabilization policy, as firms overhire during booms and underhire during recessions. Because firms overreact to economic conditions,

monetary policy can lean against the wind more forcefully to temper excessive optimism, but this same overreaction also calls for cautious state-contingent calibration of interest rate changes to avoid amplifying volatility. Fiscal policy through unemployment insurance may need to be more generous during recessions, as firms' pessimistic overreaction can prolong unemployment spells beyond what true fundamentals justify. Forward guidance and policy communication can help correct distorted beliefs by providing relevant information in a clear and accessible format, anchoring expectations when uncertainty is high. Furthermore, because firms overreact to idiosyncratic earnings shocks, belief distortions generate excess cross-sectional dispersion in hiring relative to what objective beliefs would imply, increasing the variance of marginal products of labor across firms and reducing aggregate productivity through misallocation. Finally, the measured belief distortions themselves can serve as a practical early-warning indicator for shifts in unemployment and labor demand.

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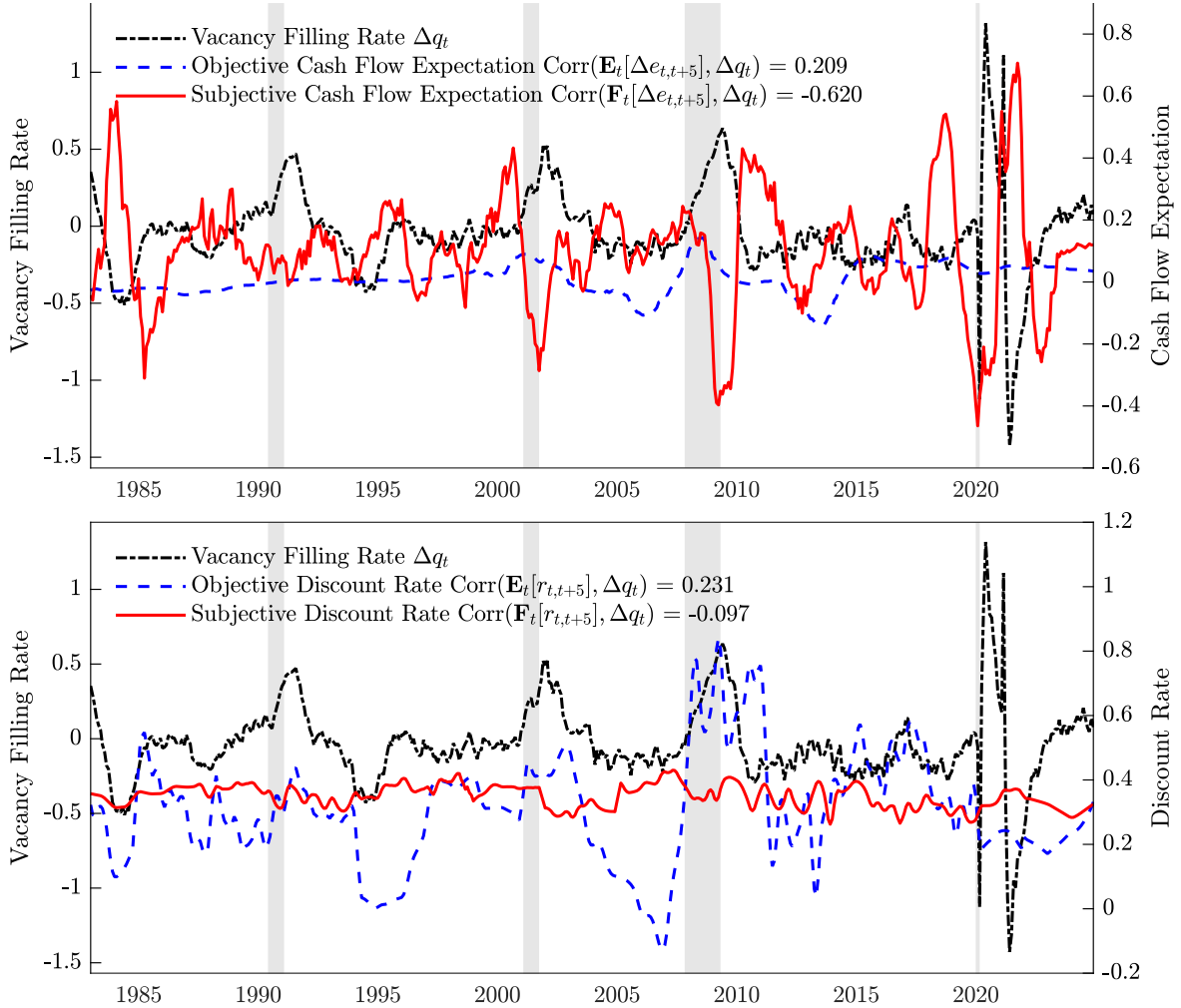


# A Appendix: Additional Results

## A.1 Stylized Facts

**Vacancy filling rate, discount rate, and expected cash flows** Figure A.1 compares subjective and machine expectations for discount rates and cash flows, plotted against the vacancy filling rate. These series represent the components that drive hiring decisions in the search model. Machine expectations of discount rates exhibit a strong positive relationship with vacancy filling rates, particularly around recessions. This pattern aligns with the theoretical prediction that higher discount rates (reflecting greater compensation for risk) should coincide with lower hiring as firms perceive a lower present discounted value of employment. Survey expectations of discount rates, by contrast, are relatively flat and display little sensitivity to the business cycle, consistent with studies that find acyclical subjective risk premia (Nagel and Xu, 2022). For cash flows, however, survey expectations show exaggerated cyclical variation, becoming sharply pessimistic during downturns when vacancy filling rates are high. Machine forecasts also vary cyclically but to a much lesser extent, consistent with survey respondents overreacting to new information when forming cash flow expectations.

Figure A.1: Vacancy Filling Rates, Discount Rates, and Expected Cash Flows

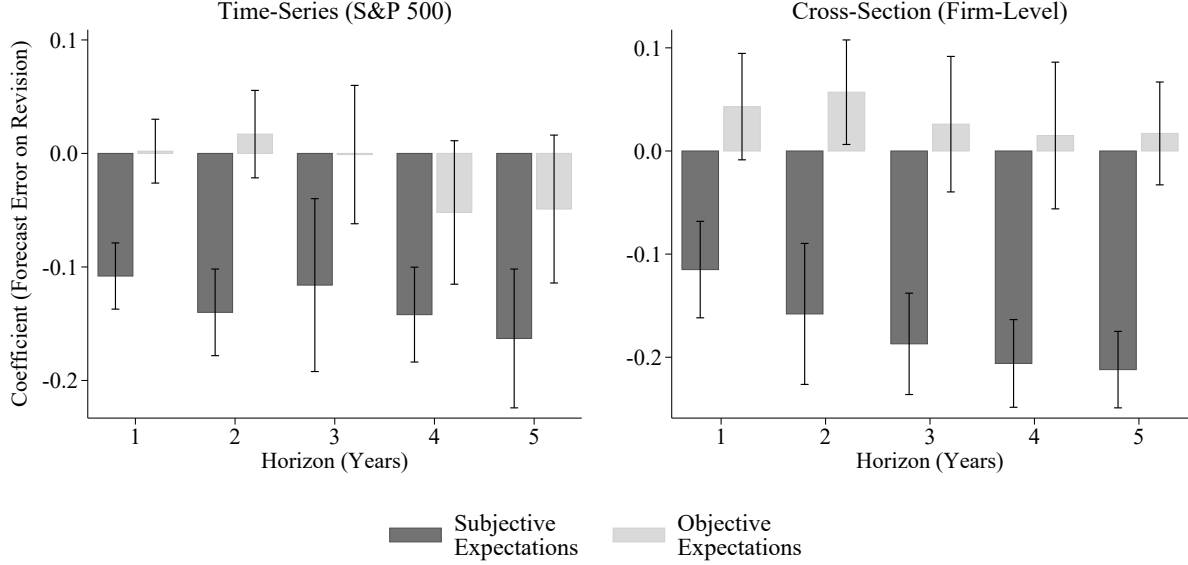


*Notes:* Figure plots  $h = 5$  year ahead survey forecasts  $\mathbb{F}_t[\cdot]$  and machine learning forecasts  $\mathbb{E}_t[\cdot]$  of discount rates  $r_{t,t+h}$  and annual cash flow growth  $\Delta e_{t,t+h}$  (left axis) against the annual log growth in the vacancy filling rate  $q_t$  (right axis).  $x$  axis denotes the date on which each forecast has been made and the vacancy filling rate was realized. Subjective expectations  $\mathbb{F}_t$  are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts  $\mathbb{E}_t$  from Long Short-Term Memory (LSTM) neural networks  $G(\mathcal{X}_t, \beta_{h,t})$ , whose parameters  $\beta_{h,t}$  are estimated in real time using  $\mathcal{X}_t$ , a large scale dataset of macroeconomic, financial, and textual data. The out-of-sample forecast testing period is quarterly and spans 2005Q1 to 2023Q4. NBER recessions are shown with gray shaded bars.

## A.2 Predictability of Survey Forecast Errors in Discount Rates

Figure A.2 reports Coibion-Gorodnichenko regression coefficients for discount rate expectations (stock returns) at both the aggregate and firm levels. The left panel shows time-series results for the S&P 500, while the right panel shows cross-sectional results for individual firms. Survey forecasts of discount rates overreact to news at both levels, with negative coefficients indicating that upward forecast revisions predict negative forecast errors. However, the magnitude of overreaction is smaller than for cash flows, consistent with the view that cash flow expectations are the primary driver of belief distortions in firm valuations. Machine learning forecasts show no significant predictability in forecast errors, consistent with rational expectations.

Figure A.2: Predictability of Survey Forecast Errors: Discount Rates

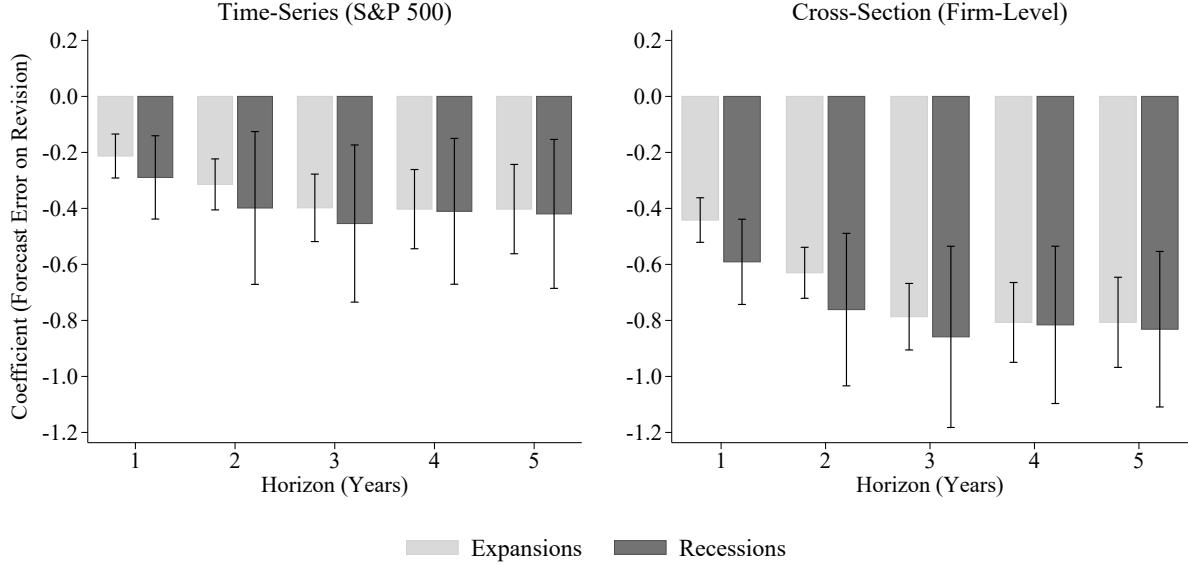


*Notes:* Figure reports regression coefficients  $\beta$  from regressions of forecast errors on forecast revisions. Left panel: time-series regressions for the S&P 500. Right panel: cross-sectional regressions for a quarterly panel of listed firms with firm and time fixed effects. The forecast target is stock returns  $r_{t+h}$  (discount rates). Time-series survey forecasts  $\mathbb{F}_t$  come from the CFO survey. Cross-sectional survey forecasts  $\mathbb{F}_t$  come from IBES. Machine learning expectations  $\mathbb{E}_t$  are generated using a Long Short-Term Memory (LSTM) model trained in real time on macroeconomic, financial, textual, and firm-level data. The sample covers quarterly data from 2005Q1 to 2023Q4. Whiskers show 95% confidence intervals (Newey-West with 4 lags for time-series; two-way clustered by firm and time for cross-section).

### A.3 Predictability of Survey Forecast Errors in Cash Flows: State Dependence

Figure A.3 examines whether the predictability of survey forecast errors in cash-flow expectations varies across the business cycle. The left panel shows time-series results for the S&P 500, while the right panel shows cross-sectional results for individual firms. In both panels, Coibion-Gorodnichenko regression coefficients are estimated separately during NBER expansions and recessions. The estimates are similar across states at all horizons and at both levels of aggregation, indicating that overreaction in survey cash-flow expectations is not concentrated in downturns but instead reflects a stable feature of expectation formation over time.

Figure A.3: Predictability of Survey Forecast Errors in Cash Flows: State Dependence

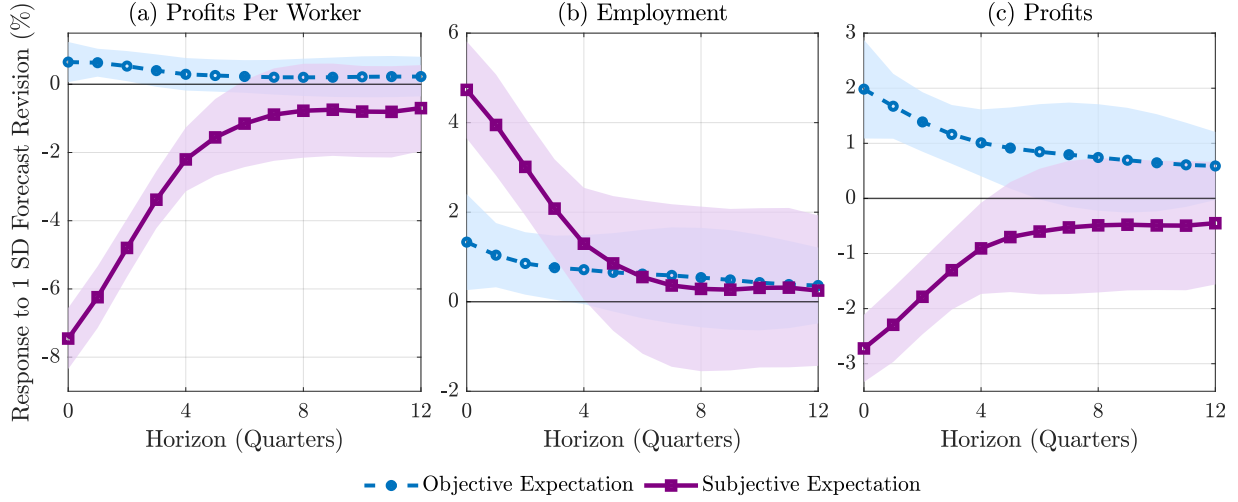


*Notes:* Figure reports regression coefficients  $\beta$  from regressions of forecast errors on forecast revisions, estimated separately during NBER expansions and recessions. Left panel: time-series regressions for the S&P 500. Right panel: cross-sectional regressions for a quarterly panel of listed firms with firm and time fixed effects. The forecast target is earnings growth  $\Delta e_{t+h}$  (cash flows). Time-series and cross-sectional survey forecasts  $\mathbb{F}_t$  come from IBES. The sample covers quarterly data from 2001Q1 to 2020Q4. Whiskers show 95% confidence intervals (Newey-West with 4 lags for time-series; clustered by firm for cross-section).

## A.4 Employment Response to Forecast Revisions: Objective vs. Subjective Beliefs

Figure A.4 compares impulse responses under objective expectations (machine learning forecasts, blue dashed lines) versus subjective expectations (analyst forecasts, violet solid lines) across three related outcomes. Panel (a) shows that profits per worker remain stable under objective expectations following positive idiosyncratic shocks, while they decline significantly under subjective expectations. This contrast reveals that the decline in profits per worker documented in the main text is driven by belief distortions rather than rational responses to fundamental shocks.

Figure A.4: Employment Response to Idiosyncratic Forecast Revisions: Objective vs. Subjective Beliefs



*Notes:* Figure displays impulse responses to earnings growth forecast revisions, estimated via local projections:  $y_{i,t+h} = \beta \text{ForecastRevision}_{i,t} + \alpha_i + \tau_t + \varepsilon_{i,t+h}$ , where  $\alpha_i$  and  $\tau_t$  denote firm and time fixed effects. Panel (a): log profits per worker  $\log(E_{i,t+h}/L_{i,t+h})$ . Panel (b): log employment  $\log(L_{i,t+h})$ . Panel (c): log profits  $\log(E_{i,t+h})$ . Blue dashed line: impulse response under objective expectations, where forecast revision is the machine learning earnings growth revision  $\mathbb{E}_t[\Delta e_{i,t+1}] - \mathbb{E}_{t-1}[\Delta e_{i,t+1}]$ . Violet solid line: impulse response under subjective expectations, where forecast revision is the analyst earnings growth revision  $\mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]$ . Observations weighted by each firm's market value. Shaded areas: two-way clustered 95% confidence intervals by firm and time. Data sample: 1999Q1–2023Q4.

## A.5 Alternative Survey Measures of Subjective Cash Flow Expectations

The large role played by subjective cash flow expectations in explaining the vacancy filling rate holds more generally across alternative survey forecasts of earnings growth. Table A.1 re-estimates the subjective variance decomposition while replacing IBES survey forecasts of earnings growth with the corresponding forecast from the Bloomberg (BBG) and CFO surveys. The forecast horizon for the CFO survey has been limited to  $h = 1$  year ahead and the sample covers a shorter period over 2002Q1 to 2019Q3 due to missing earnings growth forecasts in the CFO survey.

To summarize the alternative survey measures into a single series, I construct Filtered Investor (FI) expectations by extracting the common component of subjective cash flow beliefs using a Kalman filter. The latent state variable is defined as the  $h$ -month-ahead expected earnings growth,  $S_t \equiv \mathbb{F}_t[\Delta e_{t+h}]$ , which captures investors' subjective beliefs about future cash flows. The observation vector  $X_t$  contains survey measures of expected earnings growth over the next  $h$  periods from IBES, Bloomberg, and CFO surveys. The Kalman filter then estimates the latent  $S_t$  as the optimal linear combination of these noisy survey indicators  $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$ , where  $C, T, R$  are matrices of the model's primitive parameters  $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$ .  $\varepsilon_t$  is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster.  $\alpha$  is the intercept,  $\rho$  is the persistence, and  $\sigma_\varepsilon$  is the standard deviation of the latent innovation error. The Observation equation takes the form  $X_t = D + ZS_t + Uv_t$ , where  $h$  is a fixed forecast horizon. The observation vector  $X_t$  contains measures of survey expected cash flows from IBES, BBG, and CFO surveys over the next  $h$  periods.  $v_t$  is a vector of observation errors with standard deviations in the diagonal matrix  $U$ .  $Z$  and  $D$  are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters  $\alpha, \rho, \sigma_\varepsilon, U$ . Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table A.1: Variance Decomposition of Vacancy Filling Rate: Alternative Subjective Cash Flow Expectations

Horizon $h$ (Years)	1	2	3	4	5
Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$					
(a) Filtered Investor (FI) Expectations					
(-) Cash Flow	0.578***	0.625***	0.684***	0.887***	0.933***
$t$ -stat	(3.046)	(4.275)	(4.894)	(6.019)	(7.612)
$N$	76	76	76	76	76
(b) Bloomberg (BBG) Survey					
(-) Cash Flow	0.586***	0.830***	0.851***	0.896***	0.949***
$t$ -stat	(8.476)	(8.317)	(7.213)	(5.288)	(4.541)
$N$	76	76	76	76	76
(c) CFO Survey					
(-) Cash Flow	0.637*				
$t$ -stat	(1.934)				
$N$	71				

*Notes:* Table reports variance decompositions of the vacancy filling rate while replacing IBES earnings growth forecast with alternative surveys as measures of subjective cash flows. FI summarizes the alternative survey measures into a single series using a Kalman filter. The sample for BBG and FI is quarterly from 2005Q1 to 2023Q4. The sample for CFO is quarterly from 2002Q1 to 2019Q3. Newey-West corrected  $t$ -statistics with lags = 4 are reported in parentheses: \*sig. at 10%. \*\*sig. at 5%. \*\*\*sig. at 1%.

## A.6 Risk-Neutral Measure Implied by Futures Prices

To address whether forecast errors simply reflect risk compensation rather than belief distortions, I re-evaluate the decomposition using risk-neutral expectations extracted from futures prices. Under risk-neutral pricing, forecast errors should equal risk premia plus noise, with no patterns beyond those explained by time-varying risk compensation. In contrast to subjective survey forecasts, which may reflect belief distortions, risk-neutral expectations are extracted directly from financial market prices and reflect the valuations of marginal investors in the economy. The decomposition parallels the earlier analysis based on subjective beliefs but replaces the expectations operator  $\mathbb{F}_t[\cdot]$  with the risk-neutral operator  $\mathbb{E}_t^Q[\cdot]$ , where  $Q$  denotes the risk-neutral probability measure. I begin with the ex-post decomposition of the vacancy filling rate  $\log q_t$ , which can be expressed as:

$$\log q_t = c_q + \sum_{j=1}^h \rho^{j-1} r_{t+j} - \left( dl_t + \sum_{j=1}^h \rho^{j-1} \Delta d_{t+j} \right) - \rho^h pd_{t+h}$$

where  $r_{t+j}$  denotes the return on the S&P 500 index,  $\Delta d_{t+j}$  denotes the change in log dividends, and  $pd_{t+h}$  is the terminal log price-dividend ratio. Since market-based risk-neutral expectations are available for dividends but not for earnings, I re-write the decomposition in terms of dividend growth. To evaluate this decomposition under the risk-neutral measure, I replace each future variable with its risk-neutral expectation. Using the standard no-arbitrage pricing result that the futures price equals the risk-neutral expectation of the future spot price (Ait-Sahalia et al., 2001), I compute the expected return over horizon  $h$  using log differences of S&P 500 futures prices:

$$\mathbb{E}_t^Q[r_{t,t+h}] = \sum_{j=1}^h \rho^{j-1} (f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500})$$

where  $f_{t,t+j}^{sp500}$  denotes the log futures price of the S&P 500 at time  $t$  for delivery at  $t+j$ , and  $f_{t,t}^{sp500} \equiv p_t$  is the log spot price. This expression captures the risk-neutral expectation of the capital-gain component of returns. In principle, total returns also include the dividend yield. However, since dividends represent only a small fraction of S&P 500 total returns over this sample period, and reliable dividend futures are limited in maturity and liquidity, I abstract from this component and focus on the capital gain for tractability. Similarly, I measure expected dividend growth using dividend futures:

$$\mathbb{E}_t^Q[d_{t,t+h}] = dl_t + \sum_{j=1}^h \rho^{j-1} (f_{t,t+j}^{div} - f_{t,t+j-1}^{div})$$

where  $f_{t,t+j}^{div}$  is the log price of the dividend future for maturity  $t+j$ , and  $f_{t,t}^{div} \equiv d_t$  is the log of current dividends. To compute the terminal price-dividend ratio  $\mathbb{E}_t^Q[pd_{t+h}]$ , I apply a forward iteration of the log-linear price-dividend identity:

$$\mathbb{E}_t^Q[pd_{t+h}] = \frac{1}{\rho^h} pd_t - \frac{1}{\rho^h} \sum_{j=1}^h \rho^{j-1} (c_{pd} + \mathbb{E}_t^Q[\Delta d_{t+j}] - \mathbb{E}_t^Q[r_{t+j}])$$

where  $c_{pd}$  is a constant from the log-linearization. Since market data on futures prices is typically limited to near-term maturities (e.g., 1-year ahead), I extrapolate longer-horizon expectations using fitted values from autoregressive models. Specifically, I estimate first-order predictive regressions of the 1-year forward S&P 500 return and dividend growth on the lagged spot value:

$$\begin{aligned} f_{t,t+1}^{sp500} - p_t &= \mu_{sp500} + \rho_{sp500}(p_t - p_{t-1}) + \varepsilon_t \\ f_{t,t+1}^{div} - d_t &= \mu_{div} + \rho_{div}(d_t - d_{t-1}) + \varepsilon_t \end{aligned}$$

and then forecast growth at horizons  $j > 1$  recursively using the standard multi-step formula for an AR(1) process:

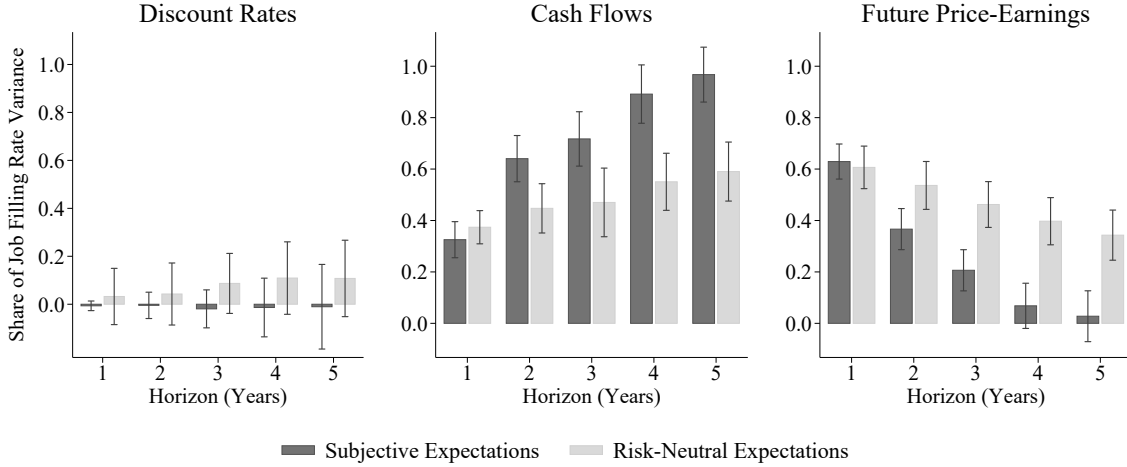
$$f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500} = \frac{\mu_{sp500}(1 - \rho_{sp500}^{j-1})}{1 - \rho_{sp500}} + \rho_{sp500}^{j-1}(f_{t,t+1}^{sp500} - p_t)$$

$$f_{t,t+j}^{div} - f_{t,t+j-1}^{div} = \frac{\mu_{div}(1 - \rho_{div}^{j-1})}{1 - \rho_{div}} + \rho_{div}^{j-1}(f_{t,t+1}^{div} - d_t)$$

Using these forward-imputed values, I compute the full set of risk-neutral expectations required for the decomposition.

The results of this exercise are shown in Figure A.5. Compared to subjective expectations, risk-neutral expectations attribute a smaller role to future cash flows and a greater role to discount rates in explaining the variation in the vacancy filling rate. This contrast suggests that belief distortions in survey forecasts may overweight the informational content of short-term earnings outlooks and underweight changes in risk premia, leading to distorted hiring incentives.

Figure A.5: Variance Decomposition of Vacancy Filling Rate: Risk-Neutral Expectations



*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate vacancy filling rate. Light bars show the contribution under risk-neutral expectations implied by S&P 500 and dividend futures. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

## A.7 Financial Constraints

A natural concern is that variation in hiring may reflect differences in financial constraints rather than distortions in beliefs. In a rational expectations model, financial constraints appear as a Lagrange multiplier that tightens the firm’s stochastic discount factor (SDF), raising internal hurdle rates and suppressing hiring (Kehoe et al., 2019). In this setting, constraint-induced fluctuations in hiring would be objectively attributed to higher discount rates. By contrast, under subjective expectations, survey respondents may misattribute the effect of constraints to lower future cash flows, especially if internal hurdle rates are persistent, upward-biased, and unresponsive to market conditions (Gormsen and Huber, 2025). Financial constraints could also allow the effects of belief distortions to persist by limiting arbitrage that would otherwise correct them (De La O et al., 2024).

**Measures of Financial Constraints** To test these hypotheses, I incorporate firm-level financial constraint measures into the decomposition framework:

- **Firm Size (Total Assets):** Firms in the bottom tertile of the asset size distribution are classified as financially constrained, while those in the top tertile are unconstrained (Erickson and Whited, 2000).
- **Payout Ratio:** Defined as dividends plus stock repurchases scaled by total assets. Firms with the lowest (highest) payout ratios are classified as constrained (unconstrained), consistent with the idea that constrained firms conserve internal funds (Fazzari et al., 1988).
- **SA Index:** The size-age index developed by Hadlock and Pierce (2010), constructed as  $SA = -0.737 \cdot \text{Size} + 0.043 \cdot \text{Size}^2 - 0.040 \cdot \text{Age}$ , where Size is log real assets and Age is years since listing. Higher SA values indicate tighter constraints.
- **Expected Free Cash Flow:** Based on Lewellen and Lewellen (2016), firms are sorted into constraint groups using predicted free cash flow, estimated from cross-sectional regressions on lagged characteristics. Low expected FCF implies tighter constraints.
- **WW Index:** The Whited-Wu index (Whited and Wu, 2006), a linear combination of cash flow, dividend status, leverage, size, and sales growth, where higher index values imply greater constraints.

Each measure is updated annually and firms are classified based on terciles or continuous index values. Each measure is aggregated to the portfolio level and standardized before entering the regression as controls.

**Decomposition with Financial Constraints** I modify the baseline decomposition regression as follows:

$$\mathbb{F}_t[y_{i,t,t+h}] = \beta \cdot hl_{i,t} + \Gamma \cdot \text{FC}_{i,t} + \alpha_i + \alpha_t + \varepsilon_{i,t}$$

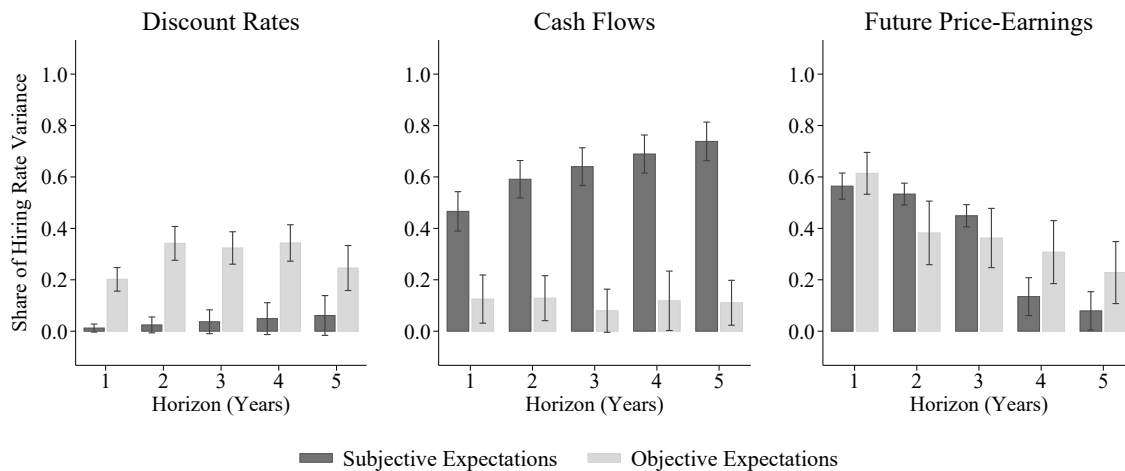
where  $y = r, e, pe$  denotes either discount rates, cash flows, or price-earnings (the components of the decompositions), and  $\text{FC}_{i,t}$  is a vector of standardized financial constraint measures for firm  $i$  at time  $t$ .  $\alpha_i$  and  $\alpha_t$  are firm and time fixed effects. As before, the parameter of interest is  $\beta$ , which captures the share of variation in the hiring rate  $hl_{i,t}$  explained by subjective expectations, but this time conditional on financial constraints. I run analogous regressions to estimate the contributions of discount rate expectations and future price-earnings ratios. I also replace survey forecasts with machine learning forecasts to estimate the decomposition under objective expectations, again controlling for financial constraints using the same specification.

**Results** Figure A.6 presents the decomposition estimates with and without financial constraint controls, under both subjective and objective expectations. Under subjective expectations, the contribution of expected earnings to hiring variation remains large and significant, with only a modest reduction in explanatory power after controlling for financial constraints. This suggests that distorted beliefs about cash flows persist even after adjusting for observable constraint-related fundamentals. These findings are consistent with the view that constrained firms overreact to cash flow news or internalize persistent pessimism about earnings. Under objective expectations,



however, the contribution of discount rate expectations drops substantially once constraint controls are included. This is consistent with a rational model in which financial constraints tighten the SDF and raise internal hurdle rates. When this variation is accounted for, the rational model assigns less importance to discount rate news in explaining hiring variation. The results supports the interpretation that financial constraints can explain a nontrivial share, but do not fully explain, variation in hiring. While objective forecasts attribute constraint effects to discount rates, subjective expectations appear to reflect persistent pessimism about cash flows.

Figure A.6: Cross-Sectional Decomposition of Hiring Rate: Control for Financial Constraints



*Notes:* Figure estimates cross-sectional decomposition of hiring rate, controlling for measures of financial constraints. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. Financial constraint controls include firm size, payout ratio, SA index, expected free cash flow, and the Whited-Wu index. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

## A.8 Predictability of Unemployment and Hiring Rates

**Time-Series Predictability of Aggregate Unemployment Rate** To complement the decomposition of the vacancy filling rate, this section analyzes the unemployment rate directly. While the vacancy filling rate captures the main driver of unemployment dynamics in search models, the unemployment rate is the key macroeconomic outcome of interest and the direct target of policy. Start from the unemployment accumulation equation of the search model in Section 4:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \quad (\text{A.1})$$

which states that the number of unemployed workers at the beginning of next period  $U_{t+1}$  equals the number of unemployed worker who fail to find a job in the current period  $(1 - q_t\theta_t)U_t$  plus the number of employed workers who lose their jobs due to separations  $\delta_t(1 - U_t)$ . Log-linearize around the steady state and substitute in equation (17), which is a decomposition of the vacancy filling rate  $q_t$  into discount rate, cash flow, and future price-earnings components. As shown in Section A.8.1, the log unemployment rate  $u_{t+1}$  satisfies the following predictive relationship:

$$u_{t+1} = \alpha + \beta_r \mathbb{F}_t[r_{t,t+h}] + \beta_e \mathbb{F}_t[e_{t,t+h}] + \gamma' X_t + \varepsilon_{s,t+1} \quad (\text{A.2})$$

where  $X_t \equiv [u_t, \log \theta_t, \log \delta_t]'$  collects labor market factors including the lagged log unemployment rate  $u_t$ , vacancy-to-unemployment ratio  $\log \theta_t$ , and job separation rate  $\log \delta_t$ . The coefficients of interest,  $\beta_r$  and  $\beta_e$ , quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

To isolate the contribution of belief distortions, I further decompose each subjective expectation  $\mathbb{F}_t$  into its objective expectation  $\mathbb{E}_t$  and its distortion  $\mathbb{F}_t - \mathbb{E}_t$ :

$$\begin{aligned} u_{t+1} = & \alpha + \beta_r \mathbb{E}_t[r_{t,t+h}] + \beta_r (\mathbb{F}_t[r_{t,t+h}] - \mathbb{E}_t[r_{t,t+h}]) \\ & + \beta_e \mathbb{E}_t[e_{t,t+h}] + \beta_e (\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]) + \gamma' X_t + \varepsilon_{s,t+1} \end{aligned} \quad (\text{A.3})$$

I estimate equation (A.3) using multivariate OLS regressions, allowing the data to inform the relative importance of each component. The future price-earnings ratio term  $\mathbb{F}_t[pe_{t,t+h}]$  has been omitted in the multivariate regression because it is nearly collinear with future discount rates  $\mathbb{F}_t[r_{t,t+h}]$  and cash flows  $\mathbb{F}_t[e_{t,t+h}]$  as long as the Campbell and Shiller (1988) present value identity holds in equation (16). To ensure stationarity and remove seasonal effects, I estimate the regression in log growth rates relative to the same quarter of the previous year. The regression is designed to test whether perceived shocks to discount rates or earnings forecasts help predict fluctuations in unemployment rates. If firms form distorted beliefs about future returns or earnings, they should manifest in hiring behavior and thus influence unemployment.

Table A.2 reports the results. Column (1) predicts the unemployment rate based on a benchmark model using only machine-based forecasts of discount rates and cash flows. Objective discount rates  $\mathbb{E}_t[r_{t,t+h}]$  significantly predict unemployment (coefficient 0.551), consistent with objective models that introduce time-varying discount rates to generate realistic fluctuations in unemployment. The objective cash flow expectation  $\mathbb{E}_t[e_{t,t+h}]$  is not a significant predictor (-0.041), consistent with the unemployment volatility puzzle where productivity shocks on its own struggle to generate sufficient unemployment fluctuations. Overall, the sign of the estimated coefficients are consistent with the implications of the search model, since higher discount rates or low expected cash flows depress the expected discounted value of job creation, leading to reduced hiring and higher future unemployment.

Column (2) extends the baseline model by incorporating belief distortions in subjective discount rate and cash flow expectations. The distortion in subjective cash flow expectation  $\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]$  emerges as the strongest predictor of future unemployment, with a large statistically significant coefficient of -0.701. The inclusion of belief distortions improves model performance substantially. The adjusted  $R^2$  increases from 0.528 to 0.745 in-sample and the out-of-sample  $R^2$  increases from 0.149 to 0.254, where the out-of-sample  $R^2$  implies an improvement in the MSE ratio relative to the Survey of Professional Forecasters (SPF) by  $0.254 - 0.149 = 0.105$ . Traditional labor market factors including lagged unemployment, labor market tightness, and separations explain only a modest portion of unemployment fluctuations, with an in-sample adjusted  $R^2$  of 0.260. In terms of out-of-sample performance, a model that excludes expectations entirely performs worse than the Survey of Professional Forecasters (SPF) benchmark with a negative OOS  $R^2$  of -0.094. These results suggest that the distortions

Table A.2: Time-Series and Cross-Sectional Predictability

Forecast Target: Unemployment Growth $\Delta u_{t+1}$			Forecast Target: Employment Growth $\Delta \tilde{l}_{i,t+1}$		
	(1)	(2)		(3)	(4)
$\mathbb{E}_t[r_{t,t+h}]$	0.551***	0.236	$\mathbb{E}_t[\tilde{r}_{i,t,t+h}]$	-0.498***	-0.119
$t$ -stat	(5.046)	(0.893)	$t$ -stat	(-3.058)	(-0.734)
$\mathbb{E}_t[e_{t,t+h}]$	-0.041	-0.018	$\mathbb{E}_t[\tilde{e}_{i,t,t+h}]$	0.154	0.053
$t$ -stat	(-0.108)	(-0.050)	$t$ -stat	(1.304)	(0.754)
$\mathbb{F}_t[r_{t,t+h}] - \mathbb{E}_t[r_{t,t+h}]$		-0.006	$\mathbb{F}_t[\tilde{r}_{i,t,t+h}] - \mathbb{E}_t[\tilde{r}_{i,t,t+h}]$		-0.043
$t$ -stat		(-0.033)	$t$ -stat		(-0.410)
$\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]$		-0.701***	$\mathbb{F}_t[\tilde{e}_{i,t,t+h}] - \mathbb{E}_t[\tilde{e}_{i,t,t+h}]$		0.759***
$t$ -stat		(-5.584)	$t$ -stat		(6.412)
Labor Market Factors	Yes	Yes	Labor Market Factors	Yes	Yes
$N$	76	76	$N$	380	380
Adj. $R^2$	0.528	0.745	Adj. $R^2$	0.135	0.253
OOS $R^2$	0.149	0.254	OOS $R^2$	0.207	0.447

*Notes:* This table reports predictive regressions of log annual growth in the unemployment rate (time-series) and employment growth (cross-section) from equation (A.3), under subjective or objective expectations. Labor market factors in the time-series regression  $X_t$  include the log annual growth of lagged log unemployment rate  $u_t$ , log labor market tightness  $\log \theta_t$  and log job separation rate  $\log \delta_t$ ; cross-sectional regressions include the same set of controls at the portfolio level. The sample is quarterly from 2005Q1 to 2023Q4. OOS  $R^2$  is defined as  $1 - MSE_{\text{Model}}/MSE_{\text{Benchmark}}$ . Out-of-sample forecasts are constructed as 1-year-ahead predictions using model parameters estimated over a rolling 10-year window.  $MSE_{\text{Model}}/MSE_{\text{Benchmark}}$  denotes the ratio of each model's out-of-sample mean squared forecast error to that of a benchmark, which is the Survey of Professional Forecasters (SPF) consensus for time-series predictions and an AR(1) model for cross-sectional predictions. Newey-West corrected (time-series) and two-way clustering by portfolio and quarter (cross-sectional)  $t$ -statistics with lags = 4 are reported in parentheses: \*sig. at 10%. \*\*sig. at 5%. \*\*\*sig. at 1%.

embedded in survey expectations contain valuable information not captured by other objective forecasts and pre-existing labor market factors.

Strikingly, distortions in subjective cash flow expectations drive out the predictive power of the machine-based discount rate forecast, whose coefficient has been reduced to 0.236 and is no longer statistically significant. This result suggests that behavioral factors can crowd out objective forces in driving labor market fluctuations, consistent with models of behavioral overreaction where salient signals can dominate decision making (Bordalo et al., 2020). Since machine forecasts already incorporate a high-dimensional set of real-time predictors, this displacement likely reflect misperceptions of underlying economic shocks rather than statistical bias due to omitted variables.

Figure A.7 illustrates the result by plotting the actual annual change in unemployment against its model-implied decomposition using both objective expectations and belief distortions based on equation (A.3). Fluctuations in unemployment closely track the component attributed to the distortion in expected cash flows. In particular, the cash flow distortion component captures the sharp rise and fall in unemployment during the global financial crisis and COVID-19 recessions with considerable precision.

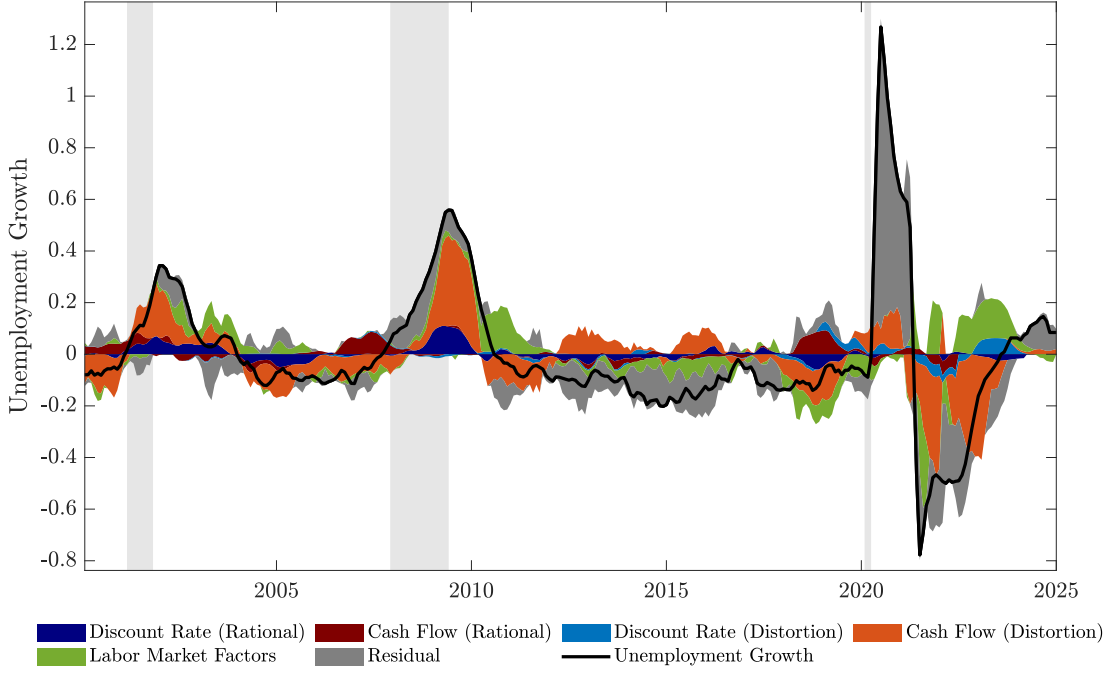
**Cross-Sectional Predictability of Employment Growth** To complement the aggregate analysis, I examine whether belief distortions also explain cross-sectional differences in hiring behavior across firms. Start from the employment accumulation equation:

$$L_{i,t+1} = (1 - \delta_{i,t})L_{i,t} + H_{i,t} \quad (\text{A.4})$$

for firm  $i$ , where  $\delta_{i,t}$  is the job separation rate and  $H_{i,t}$  denotes hires. Then we can approximate employment growth  $\Delta l_{i,t+1} \equiv \Delta \log L_{i,t+1}$  as:

$$\Delta l_{i,t+1} \approx h l_{i,t} - \delta_{i,t} \quad (\text{A.5})$$

Figure A.7: Time-Series Decomposition of the U.S. Unemployment Rate



*Notes:* Figure plots decompositions of log annual growth in the unemployment rate from equation (A.17), using objective expectations  $\mathbb{E}_t$  and belief distortions  $\mathbb{F}_t - \mathbb{E}_t$  of expected cash flows and discount rates. Labor market factors include the log annual growth of lagged unemployment  $\Delta u_t$ , labor market tightness  $\Delta \theta_t$  and job separations  $\Delta \delta_t$ . Residual (dark gray) represents the variation in vacancy filling rates that are not captured by the other components. Subjective expectations  $\mathbb{F}_t$  are based on survey forecasts from CFOs and IBES financial analysts. Objective expectations  $\mathbb{E}_t$  are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. NBER recessions are shown with light gray shaded bars.

where  $h_{i,t} = H_{i,t}/L_{i,t}$  is the hiring rate. As shown in Section 4, the hiring rate reflects the firm's valuation of a job match and embeds forward-looking expectations of return, cash flow, and terminal value:

$$hl_{i,t} = -\mathbb{F}_t[r_{i,t,t+j}] + \mathbb{F}_t[e_{i,t,t+j}] + \mathbb{F}_t[pe_{i,t,t+j}], \quad (\text{A.6})$$

where expectations are formed under the firm's subjective belief measure  $\mathbb{F}_t$ . Substituting into the employment growth approximation yields a predictive regression:

$$\Delta \tilde{l}_{i,t+1} = \alpha_i - \beta_1 \mathbb{F}_t[\tilde{r}_{i,t,t+j}] + \beta_2 \mathbb{F}_t[\tilde{e}_{i,t,t+j}] + \beta_3 \tilde{\delta}_{i,t} + \varepsilon_{i,t+1}, \quad (\text{A.7})$$

where  $\alpha_i$  denotes a firm fixed effect, and  $\delta_{i,t}$  is included directly as a control for firm-level separations. The terminal price-earnings term  $\mathbb{F}_t[\tilde{p}_{i,t,t+j}]$  has been dropped due to its near collinearity with expected returns and expected earnings growth under the Campbell and Shiller (1988) present value identity. To isolate cross-sectional variation, I demean each variable across the firms, defining  $\tilde{x}_{i,t} = x_{i,t} - \frac{1}{5} \sum_{j=1}^5 x_{j,t}$  for variable  $x$ . This specification can be estimated using panel methods with firm and time fixed effects. To isolate the contribution of belief distortions, I further decompose each subjective expectation  $\mathbb{F}_t$  into its objective expectation  $\mathbb{E}_t$  and its distortion  $\mathbb{F}_t - \mathbb{E}_t$ :

$$\begin{aligned} \Delta \tilde{l}_{i,t+1} = & \alpha_i - \beta_{1,\mathbb{E}} \mathbb{E}_t[\tilde{r}_{i,t,t+j}] - \beta_{1,\mathbb{F}} (\mathbb{F}_t[\tilde{r}_{i,t,t+j}] - \mathbb{E}_t[\tilde{r}_{i,t,t+j}]) \\ & + \beta_{2,\mathbb{E}} \mathbb{E}_t[\tilde{e}_{i,t,t+j}] + \beta_{2,\mathbb{F}} (\mathbb{F}_t[\tilde{e}_{i,t,t+j}] - \mathbb{E}_t[\tilde{e}_{i,t,t+j}]) + \beta_3 \tilde{\delta}_{i,t} + \varepsilon_{i,t+1}. \end{aligned} \quad (\text{A.8})$$

If firms overreact to news about cash flows, we expect significant positive coefficients on  $\beta_{2,\mathbb{F}}$ , reflecting inflated expectations of future cash flows that induce increased hiring. Similarly, if firms overreact to news about discount rates, we may observe large distortions in  $\beta_{1,\mathbb{F}}$ .

Table A.2 column (3) predicts portfolio-level employment growth using only machine forecasts of future returns and earnings growth. Objective return expectations  $\mathbb{E}_t[\tilde{r}_{i,t,t+j}]$  predict future employment growth (coefficient - 0.498), consistent with the search model's implication that firms hire more when the expected value of a match rises due to lower discounting. In contrast, the objective cash flow expectation  $\mathbb{E}_t[\tilde{e}_{i,t,t+j}]$  is not a significant predictor, although the size of the estimate remains nontrivial (coefficient 0.154).

Column (4) extends the baseline model by incorporating belief distortions in subjective return and cash flow expectations. Strikingly, distortions in subjective cash flow expectations  $\mathbb{F}_t[\tilde{e}_{i,t,t+j}] - \mathbb{E}_t[\tilde{e}_{i,t,t+j}]$  emerge as the dominant predictor of future employment growth, with a large and statistically significant coefficient of 0.759. At the same time, the coefficient on the machine return forecast falls to -0.119 and becomes statistically insignificant. The inclusion of belief distortions substantially improves the model's predictive accuracy. The adjusted  $R^2$  rises from 0.135 to 0.253 in-sample, and the out-of-sample  $R^2$  rises from 0.207 to 0.443, indicating that distorted expectations provide explanatory power beyond what is captured by objective benchmarks. These cross-sectional findings reinforce the aggregate evidence that survey expectations embed economically meaningful belief distortions driven by boom-bust cycles that help explain differences in hiring across firms.

**Discussion** The results can be informative about whether the survey-based subjective expectation is observationally equivalent to objective expectations. If subjective beliefs differ from objective beliefs only through a change of measure based on a Radon–Nikodym derivative that preserves its pricing implications, then subjective and objective forecasts should have equal predictive power for unemployment and hiring. While the gap  $\mathbb{F}_t - \mathbb{E}_t$  might initially appear to represent a risk premium that should affect hiring, the crucial constraint is that pricing implications are preserved. This requires both return and cash flow expectations to move in perfect lock-step, canceling out their individual effects on the hiring decision. In that case, the difference between the two expectations should be pure noise and should not improve predictions.

However, the predictive regressions show that the belief distortion component  $\mathbb{F}_t - \mathbb{E}_t$  has a highly significant explanatory power for both aggregate unemployment and cross-sectional employment growth. These results reject the null of observational equivalence and suggest that the implied stochastic discount factor under subjective beliefs is distinct from the one used under rational expectations. This difference implies that deviations from rational expectations can meaningfully influence real decisions.

In particular, the cross-sectional predictability results point to a meaningful departure from standard search models that assume a common, rational stochastic discount factor across firms. Rather than rational variation in discount rates, the evidence indicates that distorted beliefs about future cash flows are the main driver of both aggregate unemployment fluctuations and cross-sectional differences in hiring. If subjective and rational beliefs differed only by a change of measure, they would have similar predictive power. The result that belief distortions in cash flows predict cross-sectional differences in hiring better than rational discount rate forecasts suggests that the distortion term varies substantially across firms. Firm-specific differences in the distortion term implies that subjective beliefs influence the perceived value of job creation in firm-specific ways, possibly reflecting differences in perceived patience or risk even when fundamentals are held constant. These findings suggest the need for models that allow for heterogeneous and biased beliefs, rather than relying on a uniform stochastic discount factor with no distortions.

### A.8.1 Decomposition of Unemployment Rates

The unemployment rate can be decomposed into components similar to the decomposition for vacancy filling rates from equation (OA.35). Log linearize the unemployment accumulation equation from equation (6):

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \quad (\text{A.9})$$

Denote the steady state values without time subscripts:  $U$ ,  $\delta$ ,  $q$ , and  $\theta$ . Define log deviations from steady state as  $\hat{x}_t = \log(X_t) - \log(X)$  for some variable  $X$ . Log-linearizing the accumulation equation around the steady state involves taking a first-order Taylor approximation:

$$Ue^{\hat{u}_{t+1}} \approx \delta e^{\hat{\delta}_t}(1 - Ue^{\hat{u}_t}) + (1 - q\theta e^{\hat{q}_t + \hat{\theta}_t})Ue^{\hat{u}_t} \quad (\text{A.10})$$

Use the approximation  $Xe^{x_t} \approx X(1 + x_t)$ , expand, and simplify:

$$U(1 + \hat{u}_{t+1}) \approx \delta(1 + \hat{\delta}_t)(1 - U(1 + \hat{u}_t)) + (1 - q\theta(1 + \hat{q}_t + \hat{\theta}_t))U(1 + \hat{u}_t) \quad (\text{A.11})$$

Use the steady state equation and collect terms with log deviations:

$$U\hat{u}_{t+1} \approx \delta(1 - U)\hat{\delta}_t - \delta U\hat{u}_t - q\theta U\hat{q}_t - q\theta U\hat{\theta}_t + U(1 - q\theta)\hat{u}_t \quad (\text{A.12})$$

Divide both sides by  $U$ :

$$\hat{u}_{t+1} \approx \frac{\delta(1 - U)}{U}\hat{\delta}_t - \delta\hat{u}_t - q\theta\hat{q}_t - q\theta\hat{\theta}_t + (1 - q\theta)\hat{u}_t \quad (\text{A.13})$$

The steady state relationship  $\delta(1 - U) = q\theta U$  implies:  $\frac{\delta(1 - U)}{U} = q\theta$ . Substitute this back into our equation:

$$\hat{u}_{t+1} \approx -q\theta\hat{q}_t + (1 - \delta - q\theta)\hat{u}_t - q\theta\hat{\theta}_t + q\theta\hat{\delta}_t \quad (\text{A.14})$$

Finally, substitute in equation (OA.35), which is a decomposition of the vacancy filling rate  $\hat{q}_t$  into discount rate, cash flow, and future price-earnings under subjective expectations:

$$\hat{u}_{t+1} \approx \underbrace{-q\theta \cdot \mathbb{F}_t[\hat{r}_{t,t+h}]}_{\text{Discount Rate}} + \underbrace{q\theta \cdot \mathbb{F}_t[\hat{e}_{t,t+h}]}_{\text{Cash Flow}} + \underbrace{q\theta \cdot \mathbb{F}_t[\hat{p}e_{t,t+h}]}_{\text{Future Price-Earning}} + \underbrace{(1 - \delta - q\theta) \cdot \hat{u}_t - q\theta \cdot \hat{\theta}_t + q\theta \cdot \hat{\delta}_t}_{\text{Lag Unemployment, Tightness, Separations}} \quad (\text{A.15})$$

The equation holds both ex-ante and ex-post. Therefore, I compare results from evaluating the equation under subjective  $\mathbb{F}_t[\cdot]$  or objective  $\mathbb{E}_t[\cdot]$  expectations. The decomposition can be estimated using regressions of the log unemployment rate on each of the components shown in the equation:

$$u_{t+1} = \beta_0 + \beta_1 u_t + \beta_2 \log \theta_t + \beta_3 \log \delta_t + \beta_4 \mathbb{F}_t[r_{t,t+h}] + \beta_5 \mathbb{F}_t[e_{t,t+h}] + \varepsilon_{t+1} \quad (\text{A.16})$$

where lowercase variables denote log deviations from steady state. I estimate the decomposition using multivariate OLS regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations. To ensure stationarity and remove seasonal effects, I estimate the regression in log annual growth rates relative to the same quarter of the previous year.

$$\Delta u_{t+1} = \beta_1 \Delta u_t + \beta_2 \Delta \log \theta_t + \beta_3 \Delta \log \delta_t + \beta_4 \Delta \mathbb{F}_t[r_{t,t+h}] + \beta_5 \Delta \mathbb{F}_t[e_{t,t+h}] + v_{t+1} \quad (\text{A.17})$$

The future price-earnings ratio term  $\Delta \mathbb{F}_t[p e_{t,t+h}]$  has been omitted in the multivariate regression because it is nearly collinear with future discount rates  $\Delta \mathbb{F}_t[r_{t,t+h}]$  and cash flows  $\Delta \mathbb{F}_t[e_{t,t+h}]$  through the Campbell and Shiller (1988) present value identity in equation (16). Similarly, the equation can also be estimated under objective expectations by replacing  $\mathbb{F}_t[\cdot]$  with its objective expectations counterpart  $\mathbb{E}_t[\cdot]$  based on machine learning forecasts.

## A.9 Regional Model and Shift-Share Instrument

The aggregate analysis in Section 5 shows that belief distortions in subjective expectations play an important role in explaining hiring fluctuations. This section extends that analysis by exploiting cross-sectional variation in state-level data to strengthen identification and test whether the theoretical mechanism generalizes beyond aggregate dynamics.

**Overview** While the aggregate-level variance decompositions are informative, they cannot establish causality. The limited number of business cycles in the time series also restricts inference. This section addresses these challenges by extending the aggregate model to a regional framework. In estimating the regional model, I introduce a Bartik shift-share instrument for survey expectations to address endogeneity challenges in identifying the relative importance of subjective discount rate and cash flow expectations. Specifically, I investigate whether regional labor markets characterized by more distorted subjective cash flow expectations experience larger swings in vacancy filling rates. This analysis is motivated by empirical evidence of substantial geographic variation in unemployment dynamics, especially during crises (Beraja et al., 2019, Kehoe et al., 2019; Chodorow-Reich and Wieland, 2020). While existing work studies these regional differences under a rational expectations framework, differences in subjective beliefs may also be an important explanatory factor.

**Regional Model** To guide the empirical strategy, I extend the baseline search model to a multi-region, multi-sector environment, building from the models in Kehoe et al. (2019) and Chodorow-Reich and Wieland (2020). The economy consists of a continuum of islands indexed by  $s$ . Each island produces a differentiated variety of tradable goods that is consumed everywhere and a nontradable good. Both of these goods are produced using intermediate goods. Each consumer is endowed with one of two types of skills which are used in different intensities in the nontradable and tradable goods sectors. Labor is immobile across islands but can switch sectors. This assumption aligns with empirical evidence indicating that labor markets are predominantly local in nature (Manning and Petrongolo, 2017). Consumers receive utility from a composite consumption good that is either purchased in the market or produced at home. Consumers and firms are ex-ante homogeneous and share the same subjective belief measure  $\mathbb{F}_t[\cdot]$ . The islands only differ in the shocks that hit them.

**Predictability of Regional Unemployment Rates** In this environment, the log unemployment rate  $u_{s,t+1}$  in region  $s$  approximately satisfies the following predictive relationship:

$$u_{s,t+1} = \beta_r \mathbb{F}_t[r_{s,t,t+h}] + \beta_e \mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \quad (\text{A.18})$$

where  $X_{s,t} \equiv [u_{s,t}, \log \theta_{s,t}, \log \delta_{s,t}]'$  collects standard labor market controls: the lagged unemployment rate  $u_{s,t}$ , the log vacancy-to-unemployment ratio  $\log \theta_{s,t}$ , and the log separation rate  $\log \delta_{s,t}$ . The cross-sectional unit  $s$  corresponds to U.S. states, and time  $t$  is measured at the monthly frequency. Following Korniotis (2008), each firm is assigned to the state in which it is headquartered. The regression includes state fixed effects  $\alpha_s$  to absorb time-invariant regional heterogeneity and time fixed effects  $\alpha_t$  to capture national shocks. The coefficients of interest,  $\beta_r$  and  $\beta_e$ , quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

This regional equation extends the aggregate specification in equation (A.3), and is designed to test whether perceived shocks to discount rates or earnings forecasts help explain variation in unemployment across local labor markets. If firms form biased beliefs about future returns or earnings, those belief distortions should manifest in regional hiring behavior and thus influence unemployment at the state level. A counterpart regression can be estimated under objective expectations by replacing  $\mathbb{F}_t[\cdot]$  with machine learning-based forecasts  $\mathbb{E}_t[\cdot]$ .

**Empirical Specification: OLS** As a baseline, I estimate the regression above using multivariate OLS applied to a panel of state-level data. This allows for a direct assessment of whether variation in firm-level beliefs, aggregated to the state level, predicts changes in unemployment. The future price-earnings ratio term  $\mathbb{F}_t[pe_{s,t,t+h}]$  is omitted from the regression due to its near collinearity with forecasted discount rates and cash flows via the present-value identity of Campbell and Shiller (1988). State-level forecasts of discount rates  $\mathbb{F}_t[r_{s,t,t+h}]$  are

constructed from IBES price target forecasts. These targets are used to infer expected returns by back-solving from analysts' price projections. Forecasts are assigned to states based on firm headquarters and then aggregated using value-weighted averages. Expected cash flows  $\mathbb{F}_t[e_{s,t,t+h}]$  are constructed analogously from IBES analyst forecasts of earnings per share.

Regional labor market variables are constructed from publicly available BLS datasets. Unemployment rates  $u_{s,t}$  are sourced from the Local Area Unemployment Statistics (LAUS). The vacancy-to-unemployment ratio  $\theta_{s,t}$  is computed using job openings from the state-level Job Openings and Labor Turnover Survey (JOLTS) combined with unemployment counts from LAUS. Separation rates  $\delta_{s,t}$  are also taken from JOLTS. Monthly series are time-aggregated to the quarterly frequency by averaging values within each quarter.

**Empirical Specification: Bartik Shift-Share Instrument** A key challenge in estimating the regional decomposition is that regional labor market conditions and subjective expectations may be jointly determined, potentially leading to biased estimates. For example, firms might revise their beliefs in response to local shocks in unemployment or hiring, making it difficult to separate cause from effect. Additionally, state-level aggregates of firm-level forecasts may suffer from measurement error if the geographic scope of a firm's operations does not align with the location of its headquarters.

To address these concerns, I construct a leave-one-out Bartik-style shift-share instrument  $\widehat{\mathbb{F}}_t[y_{s,t,t+h}]$  that isolates plausibly exogenous variation in subjective expectations at the regional level, while avoiding mechanical feedback between local shocks and the national forecast component:

$$\widehat{\mathbb{F}}_t[y_{s,t,t+h}] = \sum_{i \in I} \phi_{s,i,t-1} \cdot \mathbb{F}_t^{-s}[y_{i,t+h}], \quad \phi_{s,i,t} = \frac{L_{s,i,t}}{\sum_{i' \in I} L_{s,i',t}}, \quad y \in \{r, e\} \quad (\text{A.19})$$

Here,  $\phi_{s,i,t}$  denotes the lagged employment share of industry  $i$  in state  $s$ , sourced from the Quarterly Census of Employment and Wages (QCEW).  $\mathbb{F}_t^{-s}[y_{i,t+h}]$  is the national IBES forecast for industry  $i$  constructed by excluding all firms headquartered in state  $s$ . The leave-one-out structure ensures that local shocks in state  $s$  do not mechanically influence the national industry-level forecasts used to construct the instrument, strengthening the validity of the exogeneity assumption. Using the leave-one-out Bartik instrument, I estimate the following predictive regression:

$$u_{s,t+1} = \beta_r \widehat{\mathbb{F}}_t[r_{s,t,t+h}] + \beta_e \widehat{\mathbb{F}}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \quad (\text{A.20})$$

The coefficients  $\beta_r$  and  $\beta_e$  now reflect the causal effect of variation in subjective discount rate and earnings expectations that is exogenous to state-specific labor market conditions.

**Identification Assumptions** Compared to the OLS specification, the Bartik approach offers stronger identification by addressing both measurement error and endogeneity concerns. First, it reduces measurement error by replacing noisy state-level aggregates of firm-level forecasts with industry-level forecasts weighted by predetermined employment shares. Second, it mitigates endogeneity by exploiting the fact that national industry trends in expectations are unlikely to respond to contemporaneous state-level labor market shocks.

For example, consider a scenario where national energy sector earnings expectations surge due to geopolitical developments. The shift-share instrument would assign Texas (with high energy employment shares) a much larger increase in instrumented expectations than Vermont (with minimal energy exposure). Crucially, this variation stems from predetermined industrial composition interacted with national sectoral trends, rather than from endogenous responses to Texas-specific labor market conditions or measurement error in aggregating individual firm forecasts within Texas.

The identifying assumption is that, conditional on fixed effects and controls, there are no omitted factors that simultaneously affect both national industry-level expectations and local hiring behavior in states more exposed to those industries. While many shift-share designs rely on the exogenous shocks assumption, in our setting the exogenous shares assumption is likely more appropriate. In sectors where specific regions have large exposures to (e.g., Texas in oil energy), national energy industry-level expectations  $\mathbb{F}_t[e_{i,t,t+h}]$  may be influenced by news from



firms headquartered in those regions. Even with the leave-one-out construction, regional developments can create spillover effects that contaminate the national industry shock. For example, a slowdown in hiring or disappointing earnings guidance from large Texas energy firms could cause IBES analysts to revise downward their national energy sector earnings forecasts. If so, the national shock would be endogenous to Texas-specific developments, violating the exogenous shock assumption. In contrast, the state-level industry shares  $\phi_{s,i,t-1}$ , measured using lagged QCEW employment data, reflect slow-moving industrial structure and are plausibly predetermined. We therefore treat industry shares as conditionally exogenous and interpret our identification through the lens of the exogenous shares assumption following Borusyak et al. (2025).

This assumption would be violated, for example, if pre-existing trends in local demand systematically coincided with national shocks. To mitigate this concern, I include a rich set of controls and fixed effects. Specifically, state fixed effects  $\alpha_s$  absorb time-invariant differences in labor market characteristics across states. Time fixed effects  $\alpha_t$  account for common national shocks such as business cycles or federal policy changes. By leveraging only the cross-sectional variation in state exposure to national shocks, the Bartik specification helps isolate the exogenous component of belief-driven hiring fluctuations.

**Predictability of the State-Level Unemployment Rate** Table A.3 reports regression estimates that evaluate the predictive power of state-level expectations for future unemployment. Each column adds different combinations of objective or subjective forecasts for discount rates and cash flows, with all specifications controlling for standard labor market factors and including both state and time fixed effects.

Table A.3: Predictability of the State-Level Unemployment Rate

	Dependent Variable: Log Unemployment Rate $u_{t+1}$					
	OLS			Shift-Share Instrument		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[r_{s,t,t+h}]$	0.725*** (0.235)		0.470 (0.780)	0.572*** (0.222)		0.207 (0.240)
$\mathbb{E}_t[e_{s,t,t+h}]$	-0.247 (0.499)		-0.065 (0.182)	-0.064 (0.075)		0.005 (0.168)
$\mathbb{F}_t[r_{s,t,t+h}]$		0.248 (0.297)	0.233 (0.300)		0.052 (0.228)	0.052 (0.228)
$\mathbb{F}_t[e_{s,t,t+h}]$		-0.817*** (0.236)	-0.791*** (0.242)		-0.690*** (0.160)	-0.708*** (0.200)
$R^2$	0.414	0.558	0.558	0.414	0.549	0.549
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Labor Market Factors	Yes	Yes	Yes	Yes	Yes	Yes
$N$	4,358	4,358	4,358	4,358	4,358	4,358

*Notes:* Labor market factors include the log annual growth of lagged log unemployment rate  $u_{s,t}$ , log labor market tightness  $\log \theta_{s,t}$  and log job separation rate  $\log \delta_{s,t}$ . The sample is quarterly from 2005Q1 to 2023Q4. Forecasts use a horizon of  $h = 5$  years. Newey-West corrected  $t$ -statistics with lags = 4 are reported in parentheses: \*sig. at 10%. \*\*sig. at 5%. \*\*\*sig. at 1%.

The estimates demonstrate that subjective earnings expectations are not only informative about regional unemployment but crowd out the predictive power of objective components. Column (1) shows that objective discount rate expectations  $\mathbb{E}_t[r_{s,t,t+5}]$  significantly predict unemployment, with a coefficient of 0.725 and  $R^2$  of 0.414. This implies that a one standard deviation increase in objective discount rate expectations predicts a 0.240 percentage point increase in the unemployment rate. Column (2) shows that among subjective forecasts, only expected earnings  $\mathbb{F}_t[e_{s,t,t+5}]$  matter, with a large negative coefficient ( $-0.817$ ) and higher explanatory power ( $R^2 = 0.558$ ). A one standard deviation increase in expected earnings predicts a 0.129 percentage point decrease in the unemployment rate. Column (3) includes both sets of expectations. Subjective earnings dominate: their coefficient remains significant ( $-0.791$ ), while objective expectations become insignificant.

Column (4) repeats the objective belief regression using Bartik instruments; the discount rate remains significant (0.572), implying a 0.181 percentage point increase in unemployment per standard deviation increase in instrumented discount rate expectations. In Column (5), only instrumented subjective earnings are significant ( $-0.690$ ), with a standard deviation of 0.168 implying a 0.116 percentage point decrease in unemployment. Column (6) confirms that instrumented subjective earnings expectations ( $-0.708$ ) continue to drive out all other predictors, implying a 0.119 percentage point decline in unemployment for a one standard deviation increase.

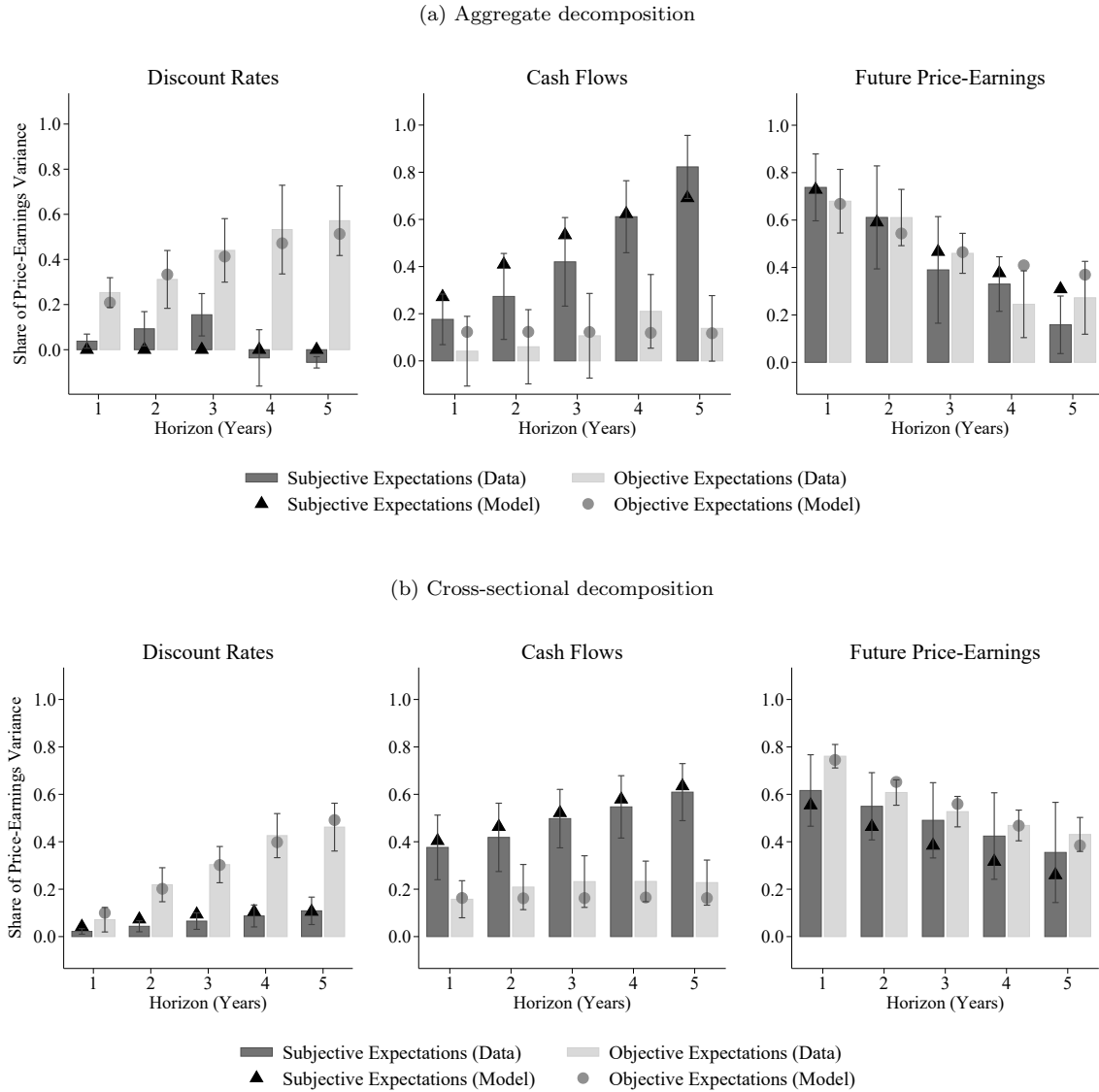
The shift-share estimates are generally smaller in magnitude than their OLS counterparts, as expected, since the shift-share instrument isolates only variation that is plausibly exogenous to regional labor market conditions. The attenuation suggests that some of the OLS signal reflects endogenous responses to regional shocks, such as changes in local labor supply, that amplify belief-driven dynamics. Nevertheless, the fact that the earnings coefficient remains large and significant under instrumentation supports a causal interpretation: belief distortions about cash flows play a central role in driving unemployment fluctuations across regions.

Taken together, the results provide robust evidence that distorted beliefs about future earnings are a key driver of regional labor market volatility. The strong and consistent link between subjective earnings expectations and unemployment, even when instrumented, suggests that firms' hiring decisions are shaped not only by fundamentals but also by biased beliefs. Regions where firms overreact to cash flow news experience deeper hiring cuts during downturns and more aggressive expansions during booms, thereby driving business cycle volatility. These findings indicate that persistent regional differences in unemployment may arise not only from structural characteristics such as industry mix or demographics, but also from variation in how firms perceive and respond to economic signals.

## A.10 Decomposition of Price-Earnings Ratios

The decomposition of the price-earnings ratio developed by De La O and Myers (2021) and De La O et al. (2024) provide a useful benchmark for thinking about the role of distorted beliefs in financial markets. Their analysis applies the Campbell and Shiller (1988) identity to the aggregate price-earnings ratio and shows that subjective expectations systematically understate the role of discount rates while overstating the importance of cash flows. Figure A.8 shows that the decompositions for price-earnings ratios and vacancy filling rates yield broadly similar magnitudes. In both cases, objective expectations attribute most variation to discount-rate news, while subjective expectations shift the weight strongly toward cash flows. The parallel magnitudes underscore that the two decompositions are consistent, but the economic implications differ. Whereas the price-earnings decomposition highlights distortions in asset valuations, the vacancy filling rate decomposition shows how these same distortions translate into fluctuations in job creation and unemployment.

Figure A.8: Variance Decomposition of the Price-Earnings Ratio



*Notes:* Panels (a) and (b) illustrate the discount rate, cash flow, and future price-earnings components of the time-series and cross-sectional decompositions of the price-earnings ratio. Firms are sorted into five value-weighted portfolios by book-to-market ratio in the cross-sectional case. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is 2005Q1–2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4. Circle and triangle dots show the values of objective and subjective expectations implied by the model, respectively.

## A.11 Capital Investment

This section extends the baseline model by incorporating firm investment decisions and distinguishing between tangible and intangible capital. I show how belief distortions about future returns and earnings influence not only hiring decisions, but also capital investment behavior. I then decompose the investment rate into components associated with discount rates and cash flows.

**Model Setup** I assume firms produce output using a Cobb-Douglas production function that depends on both capital and labor inputs:

$$Y_{i,t} = A_{i,t} K_{i,t}^\alpha L_{i,t}^{1-\alpha}$$

where  $A_{i,t}$  denotes total factor productivity,  $K_{i,t} = K_{i,t}^{\text{phy}} + K_{i,t}^{\text{int}}$  is total capital input composed of tangible and intangible capital, and  $L_{i,t}$  is labor input. Following Hall (2001) and Hansen et al. (2005), I treat tangible and intangible capital as perfect substitutes. Earnings are defined as:

$$E_{i,t} = Y_{i,t} - W_{i,t}L_{i,t} - \kappa V_{i,t} - I_{i,t} - \phi\left(\frac{I_{i,t}}{K_{i,t}}\right) K_{i,t}$$

where  $W_{i,t}$  is the wage rate,  $\kappa V_{i,t}$  is the vacancy posting cost,  $I_{i,t} = I_{i,t}^{\text{phy}} + I_{i,t}^{\text{int}}$  is total investment, and  $\phi(\cdot)$  denotes convex adjustment costs. I adopt a piecewise-quadratic specification for  $\phi(\cdot)$  with different coefficients for expansion and contraction:

$$\phi\left(\frac{I_{i,t}}{K_{i,t}}\right) = \begin{cases} \frac{c_k^+}{2} \left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} \geq 0 \\ \frac{c_k^-}{2} \left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} < 0 \end{cases}$$

Firms choose investment  $I_{i,t}$  and vacancies  $V_{i,t}$  to maximize firm value:

$$V(A_{i,t}, K_{i,t}, L_{i,t}) = \max_{I_{i,t}, V_{i,t}} \{E_{i,t} + \mathbb{F}_t [M_{t+1} V(A_{i,t+1}, K_{i,t+1}, L_{i,t+1})]\}$$

subject to both capital and employment accumulation equations:

$$\begin{aligned} K_{i,t+1} &= (1 - \delta_{i,t}^k) K_{i,t} + I_{i,t} \\ L_{i,t+1} &= (1 - \delta_{i,t}^l) L_{i,t} + q_t V_{i,t} \end{aligned}$$

The first order condition with respect to investment implies:

$$1 + \phi'\left(\frac{K_{i,t+1} - (1 - \delta_{i,t}^k) K_{i,t}}{K_{i,t}}\right) = \frac{P_{i,t}}{K_{i,t+1}}$$

where  $P_{i,t} = \mathbb{F}_t [M_{t+1} V(A_{i,t+1}, K_{i,t+1})]$  is the ex-dividend firm value.

**Recovering Intangible Capital** For each firm, I measure realized data on physical capital  $K_{i,t}^{\text{phy}}$ , tangible investment  $I_{i,t}^{\text{phy}}$ , depreciation rates  $\delta_{i,t}^k$ , and market value  $P_{i,t}$ . The physical capital stock  $K_{i,t}^{\text{phy}}$  is measured using Compustat's PPEGT item, and tangible investment  $I_{i,t}^{\text{phy}}$  is measured using capital expenditures (CAPX). The depreciation rate  $\delta_{i,t}^k$  is calculated as depreciations (DP) as a share of physical capital stock (PPEGT), and applied to both tangible and intangible capital (Hall, 2001). I construct the firm's total market value  $P_{i,t}$  as the sum of the market value of equity, the book value of debt, minus current assets. Starting from an initial value  $K_{i,1970Q1} = P_{i,1970Q1}$ , I recursively solve the first order condition for  $K_{i,t+1}$ , using observed investment, depreciation, and market value. Intangible capital is then recovered as the residual:

$$K_{i,t}^{\text{int}} = K_{i,t} - K_{i,t}^{\text{phy}}$$

**Decomposition of Investment Rates** Taking logs and linearizing the first order condition:

$$\log \left( 1 + c_k \frac{I_{i,t}}{K_{i,t}} \right) \approx \log c_k + \log \left( \frac{I_{i,t}}{K_{i,t}} \right) = \log \left( \frac{P_{i,t}}{K_{i,t+1}} \right)$$

I decompose the right-hand side into price-to-earnings and earnings-to-capital terms:

$$\underbrace{\log \left( \frac{I_{i,t}}{K_{i,t}} \right)}_{ik_{i,t}} = -\log c_k + \underbrace{\log \left( \frac{P_{i,t}}{E_{i,t}} \right)}_{pe_{i,t}} + \underbrace{\log \left( \frac{E_{i,t}}{K_{i,t+1}} \right)}_{ek_{i,t}}$$

Using a Campbell and Shiller (1988) log-linear approximation for the price-earnings ratio:

$$pe_{i,t} = \sum_{j=1}^h \rho^{j-1} (c_{pe} + \Delta e_{i,t+j} - r_{i,t+j}) + \rho^h pe_{i,t+h}$$

Substituting yields the final decomposition:

$$ik_{i,t} = c_{ik} - \sum_{j=1}^h \rho^{j-1} r_{i,t+j} + \left( ek_{i,t} + \sum_{j=1}^h \rho^{j-1} \Delta e_{i,t+j} \right) + \rho^h pe_{i,t+h}$$

where  $c_{ik} \equiv \frac{c_{pe}(1-\rho^h)}{1-\rho} - \log c_k$ . To separately analyze tangible and intangible investment, I define  $ik_{i,t}^m \equiv \log \left( \frac{I_{i,t}^m}{K_{i,t}} \right)$  and  $s_{i,t}^m \equiv \log \left( \frac{I_{i,t}^m}{I_{i,t}} \right)$  so that:

$$ik_{i,t}^m = s_{i,t}^m + ik_{i,t}, \quad m = phy, int$$

implying the decomposition structure remains unchanged up to an additive shift  $s_{i,t}^m$ . I estimate the decomposition separately for tangible and intangible investment. The time-series decomposition of the aggregate investment rate is:

$$ik_t^m \approx - \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[r_{t+j}] + \left( ek_t + \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right) + \rho^h \mathbb{F}_t[pe_{t+h}]$$

where  $x_t = \sum_{i \in I} x_{i,t}$  aggregates firm-level variable  $x_{i,t}$ . For the cross-section, demeaned variables yield:

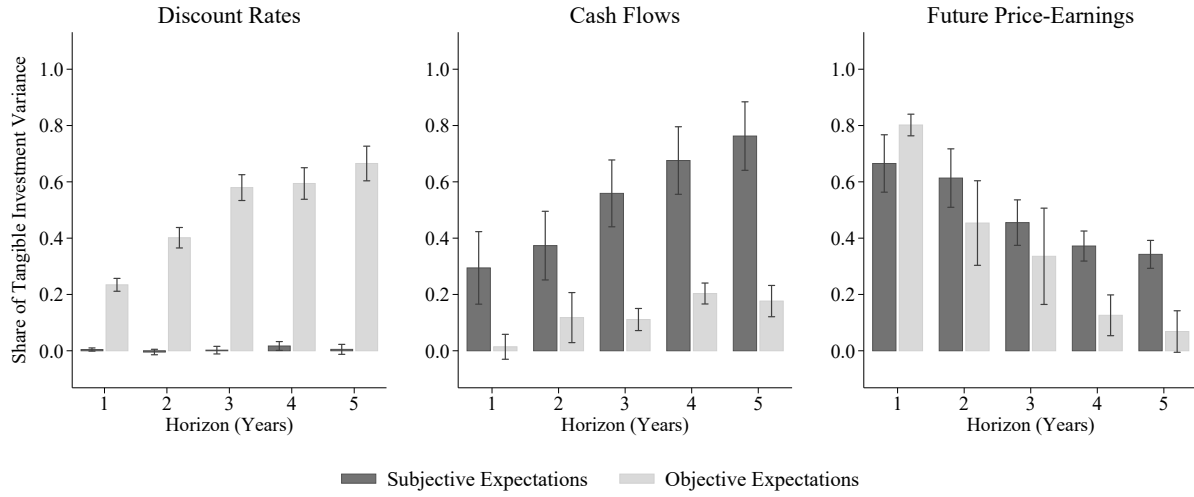
$$\tilde{ik}_{i,t}^m \approx - \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\tilde{r}_{i,t+j}] + \left( \tilde{ek}_{i,t} + \sum_{j=1}^h \rho^{j-1} \mathbb{F}_t[\Delta \tilde{e}_{i,t+j}] \right) + \rho^h \mathbb{F}_t[\tilde{pe}_{i,t+h}]$$

where  $\tilde{x}_{i,t} = x_{i,t} - \sum_{i \in I} x_{i,t}$  cross-sectionally demeans variable  $x_{i,t}$ .

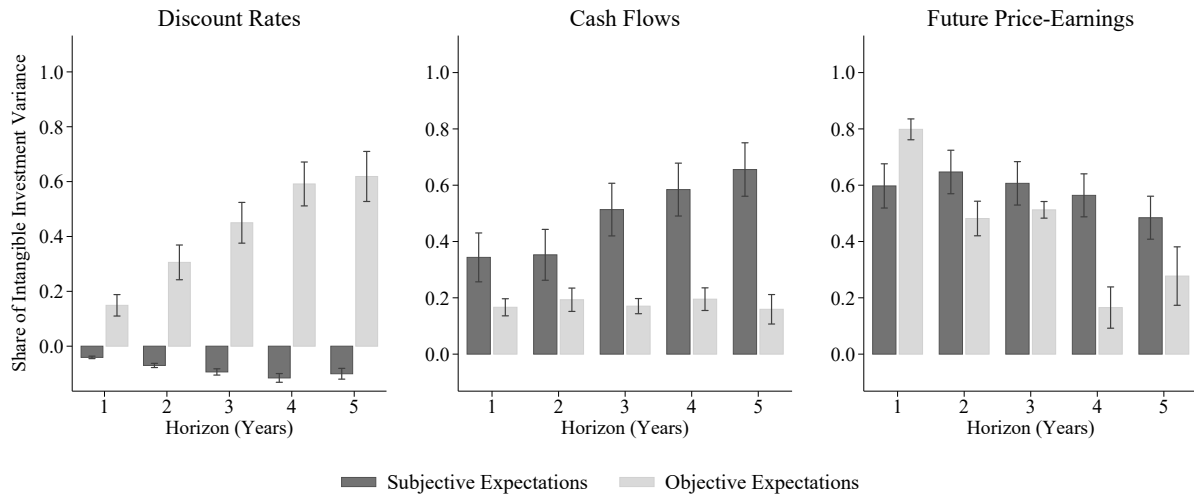
**Results** The empirical results mirror those for hiring rates, both in the time series (Figure A.9) and the cross-section (Figure A.10). Subjective expectations substantially overstate the contribution of cash flows and understate that of discount rates, both for tangible and intangible investment. Notably, the distortions are stronger for intangible investment, consistent with greater uncertainty and measurement error in expectations about intangible value creation. These findings highlight how belief distortions affect not only labor demand but also capital allocation decisions across asset types.

Figure A.9: Time-Series Decomposition of Capital Investment

(a) Tangible Capital Investment Rate



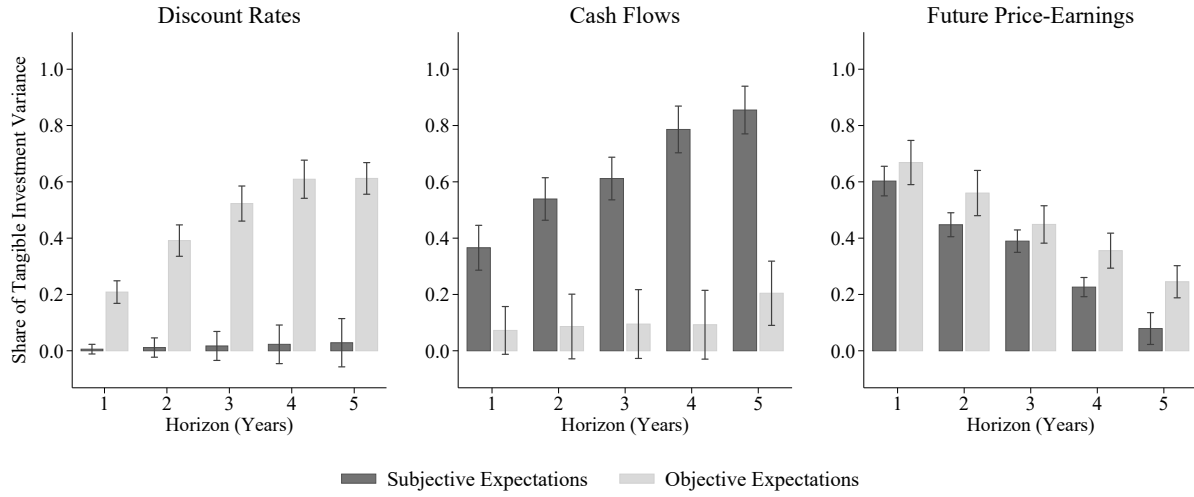
(b) Intangible Capital Investment



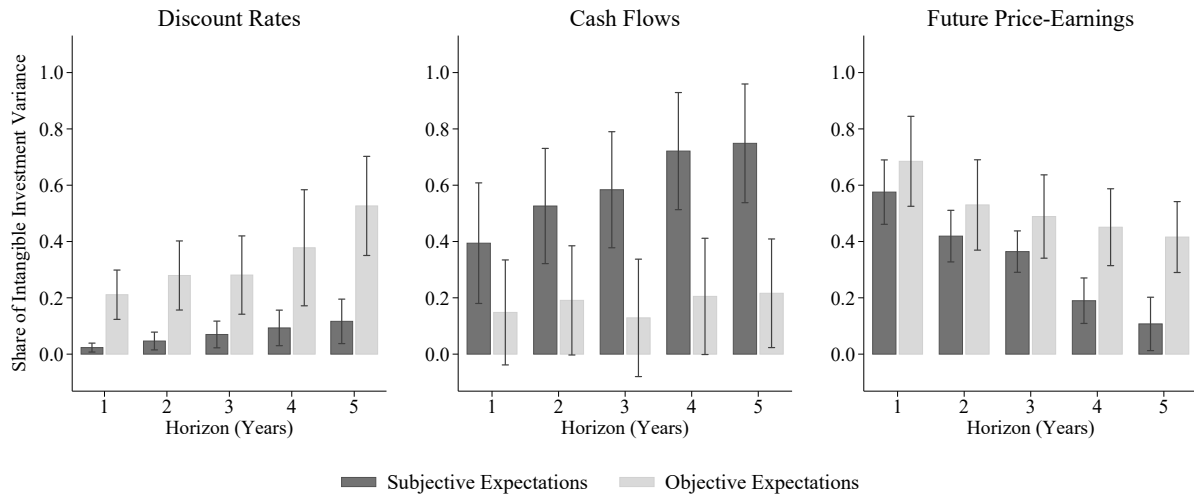
*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate tangible and intangible capital investment rate. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Figure A.10: Cross-Sectional Decomposition of Capital Investment

(a) Tangible Capital Investment Rate



(b) Intangible Capital Investment



*Notes:* Figure illustrates the discount rate, cash flow, and future price-earnings components of the cross-sectional decomposition to the dispersion of the current tangible and intangible capital investment rate. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.