Belief Distortions, Asset Prices, and Unemployment Fluctuations

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Motivation: Unemployment Volatility Puzzle (Shimer 2005)

Macro puzzle: Standard search model generates low unemployment volatility

Hiring is a forward-looking investment:

- Firms hire based on their beliefs about the expected value of a new worker

Expected value of hiring: Depends on two components

- Expected cash flows: Future earnings generated by the worker
- Discount rate: Risk-adjusted present value of future earnings

Standard model: Limited source of volatility

- Risk neutral $\Rightarrow$ Constant discount rates
- Rational expectations $\Rightarrow$ True cash flows not volatile enough

Rational vs. Behavioral Perspectives on the Puzzle ▸ Literature

Rational finance view: Hall 2017, Borovičková-Borovička 2017, Kehoe et al. 2023, ...

- Time-varying discount rates under full info rational expectations
- Recessions bring high discount rates that depress present value of hiring

$\Rightarrow$ Aggregate: **Rational discount rate news** drives unemployment

Cross section: Hard to explain why diversifiable idiosyncratic shocks affect hiring

Behavioral finance view: This paper

- Subjective beliefs from survey forecasts overreact to cash flow news
- Good news about cash flow leads to overoptimism, inflating the value of hiring

$\Rightarrow$ Aggregate: **Belief overreaction to cash flow news** drives unemployment

Cross section: Overreaction explains hiring response to idiosyncratic shock

Roadmap

1. **Measure belief distortions:** Compare survey vs. machine learning forecasts
2. **Document overreaction:** Belief distortion quantifies survey overreaction
3. **Aggregate evidence:** Decompose U.S. vacancy filling rate variation
4. **Firm-level evidence:** Decompose cross-sectional hiring rate variation
5. **Dynamics:** Impulse response to idiosyncratic shocks
6. **Model:** Constant-gain learning explains empirical patterns

Approach: Quantify Importance of Belief Distortions

Measure belief distortions as predictable mistakes: $\mathbb{F}_t - \mathbb{E}_t$

- Subjective expectation $\mathbb{F}_t$: Survey forecast from equity research analysts
  - Proxy for investor beliefs, consistent with manager beliefs (Gennaioli et al. 2016)
- Objective expectation $\mathbb{E}_t$: Machine learning forecast (Bianchi et al. 2025)
  - Can efficiently process large info set to produce accurate out-of-sample forecast
- This paper: Focus on belief distortions in cash flow (earnings) expectations

Why machine learning?

- Gives us explicit measure of efficient belief formation
- Comparison with survey quantifies magnitude of predictable mistakes ($\mathbb{F}_t - \mathbb{E}_t$)
- Predictable mistakes arise from inefficient use of available information

Main Finding: Belief Distortion Drives Labor Market Fluctuations

1. **Overreaction:** Belief distortion $\mathbb{F}_t - \mathbb{E}_t$ captures overreaction to cash flow news
   - Survey $\mathbb{F}_t$: $(+)$ revision to good news predicts disappointing (-) forecast error
   - Machine $\mathbb{E}_t$ does not, consistent with an objective benchmark
2. **Variance decomposition:** Belief distortion explains labor market volatility
   - 90% of U.S. vacancy filling rate variation $\Rightarrow$ 68% of unemployment rate variation
3. **Firm-level response to idiosyncratic cash flow shocks:**
   - Belief distortion drives predictable booms and busts in hiring and stock returns
4. **Learning model with fading memory** explains these findings:
   - Fading memory $\Rightarrow$ overweight recent cash flows $\Rightarrow$ overreaction amplifies hiring
   - Generates over 60% of observed aggregate unemployment volatility & hiring dispersion

Data Overview

- Labor market outcomes

	Time-Series	**Cross-Section**
Labor market	Vacancy filling rate (JOLTS)	Firm-level hiring rate (Compustat)

- Belief distortion: $\mathbb{F}_t - \mathbb{E}_t$ (predictable mistakes in survey)

	Subjective $\mathbb{F}_t$	**Objective** $\mathbb{E}_t$
Cash flows	IBES analyst forecasts	LSTM neural network
Discount rates	IBES analyst forecasts	LSTM neural network

Data: Labor Market Outcomes ▸ Details

Time-series: q_t = U.S. Vacancy filling rate for quarter t (Source: JOLTS, BLS)

$$q_t = \frac{f_t U_t}{V_t} = \frac{\text{Total Hires}}{\text{Total Job Vacancies}}$$

- V_t job vacancies, U_t unemployment, f_t job finding rate (= total hires/unemployment)
- Countercyclical: High U_t relative to V_t during recession $\Rightarrow$ High q_t

Cross-section: $hl_{i,t}$ = Hiring rate for firm i quarter t (Source: Compustat, JOLTS)

$$hl_{i,t} = \frac{L_{i,t+1} - (1 - \delta_{i,t}) L_{i,t}}{L_{i,t}} = \frac{\text{Total Hires}}{\text{Total Employment}}$$

- $L_{i,t}$ employees at fiscal year-end, carried forward to quarterly
- $\delta_{i,t}$ job separation rate of firm i's 2-digit NAICS industry
- **Firm-level sample:** Firms with common stocks (share codes 10, 11) on NYSE/AMEX/NASDAQ with IBES analyst coverage of expected earnings and stock price targets

Data: Cash Flows (Realized Earnings) ▸ Details

Cross-section: $E_{i,t}$ = firm-level "Street" earnings (IBES, 1983Q4-2023Q4):

- "Street": Exclude one-off items not relevant to firm's future operation
- Transformation to ensure positive values (Vuolteenaho 2002):

$$E_{i,t} = (1-\lambda)E^*_{i,t} + \lambda r^f_t P_{i,t-1} > 0, \quad \lambda = 0.10$$

  - Interpret as portfolio of 90% equity and 10% T-bills
  - Allows $\log(E_{i,t})$ to be well-defined when reported earnings negative $E^*_{i,t} \leq 0$
  - Apply similar transformation to firm-level stock returns

Time-series: E_t = Aggregate firm-level street earnings to S&P 500 level

$$E_t = \Omega_t \sum_{i \in x_t} E^*_{i,t} / Divisor_t$$

- x_t S&P 500 firms with IBES data, Ω_t adjust for IBES coverage, $Divisor_t$ S&P 500 divisor

Data: Cash Flows (Survey Expectation) ▸ Details

Cross-section: $\mathbb{F}_t[\Delta e_{i,t+h}]$ = Firm-level IBES median consensus forecast of earnings growth

- Respondents: Equity research analysts (1983Q4-2023Q4)
- Years $h = 1, 2$: Construct annual log growth forecast $\mathbb{F}_t[\Delta e_{i,t+h}]$ from level forecast
  - Prediction target: Street earnings level $\mathbb{F}_t[E^*_{i,t+h}]$
  - To ensure positive earnings: $\mathbb{F}_t[E_{i,t+h}] = (1-\lambda)\mathbb{F}_t[E^*_{i,t+h}] + \lambda r^f_t \mathbb{F}_t[P_{i,t+h-1}]$
  - Growth forecast: $\mathbb{F}_t[\Delta e_{i,t+h}] \approx \log(\mathbb{F}_t[E_{i,t+h}]/\mathbb{F}_t[E_{i,t+h-1}])$
- Years $h = 3, 4, 5$: Interpret long-term growth (LTG) forecast as $\mathbb{F}_t[\Delta e_{i,t+h}]$
  - LTG: Annualized growth forecast over next "three-to-five years"

Time-series: $\mathbb{F}_t[\Delta e_{t+h}]$ = Aggregate firm-level forecasts to S&P 500 level

- Years $h = 1, 2$: Aggregate using $\mathbb{F}_t[E_{t+h}] = \Omega_t \sum_{i \in x_t} \mathbb{F}_t[E^*_{i,t+h}] \cdot S_{i,t}/Divisor_t$
- Years $h = 3, 4, 5$: Value-weighted average of LTG forecasts

Data: Discount Rates (Stock Returns) ▸ Details

Time-series: r_t = Log annual return on the S&P 500 with dividends

- Expected: $\mathbb{F}_t[r_{t+h}]$ = CFO survey median consensus forecast (2001Q4-2023Q4)
  - Respondents: CFOs, VPs of finance, directors (~1,600 members as of 2022)
  - Horizon h: 1 and 10 years ahead; interpolate intermediate horizons linearly

Cross-section: $r_{i,t}$ = Log annual return on firm i's stock with dividends

- Expected: $\mathbb{F}_t[r_{i,t+h}]$ from IBES & Value Line median consensus price target $\mathbb{F}_t[P_{i,t+h}]$

$$\mathbb{F}_t[r_{i,t+h}] \approx \log\left(\frac{\mathbb{F}_t[P_{i,t+h}]}{P_{i,t}} + \frac{D_{i,t}}{P_{i,t}}\frac{\mathbb{F}_t[D_{i,t+h}]}{D_{i,t}}\right)$$

  - Respondents: Equity research analysts (1999Q4-2023Q4)
  - Horizon h: 1 year (IBES) and 5 years (Value Line), interpolate intermediate horizons
  - Price $P_{i,t}$ (CRSP), dividend $D_{i,t}$ (Compustat), $\mathbb{F}_t[D_{i,t+h}]/D_{i,t} \approx 1.064$ (postwar avg)

Data: Machine Learning Forecasts ▸ Details

Long Short-Term Memory (LSTM) neural network (Bianchi, Lee, Ludvigson, Ma 2025):

$$\mathbb{E}_t[y_{t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_t^{TS}; \boldsymbol{\lambda}_t^{TS}) \qquad \text{(Time-Series)}$$
$$\mathbb{E}_t[y_{i,t+h}] = G(\mathcal{X}_{i,t}, \boldsymbol{\beta}_t^{CS}; \boldsymbol{\lambda}_t^{CS}) \qquad \text{(Cross-Section)}$$

- Forecast target: $y \in \{r, \Delta e\}$ at horizons $h = 1, \ldots, 5$ years
- Parameter $\boldsymbol{\beta}_t$: Re-estimate quarterly (TS) or annually (CS) over rolling sample
- Regularization $\boldsymbol{\lambda}_t$: L_1/L_2 penalty, dropout, early stopping, ensemble average

Input data:

- Time-series: $\mathcal{X}_t$ = Real-time macro/financial, text (LDA from WSJ), survey data
- Cross-section: $\mathcal{X}_{i,t} = [\mathcal{X}_t, \mathcal{C}_{i,t}]$ where $\mathcal{C}_{i,t}$ includes firm characteristics (e.g. valuation, profitability, size, momentum, volatility), industry dummies (Chen-Zimmermann 2022)

$\Rightarrow$ Proxy for rational agent's real-time forecast without knowing true data generating process

Survey Overreacts, Machine Does Not: Time-Series

$$\overbrace{\Delta e_{t,t+h} - \mathbb{F}_t[\Delta e_{t,t+h}]}^{\text{Forecast Error}} = \beta \cdot \overbrace{[\mathbb{F}_t[\Delta e_{t,t+h}] - \mathbb{F}_{t-1}[\Delta e_{t,t+h}]]}^{\text{Forecast Revision}} + \varepsilon_t$$

- Survey:

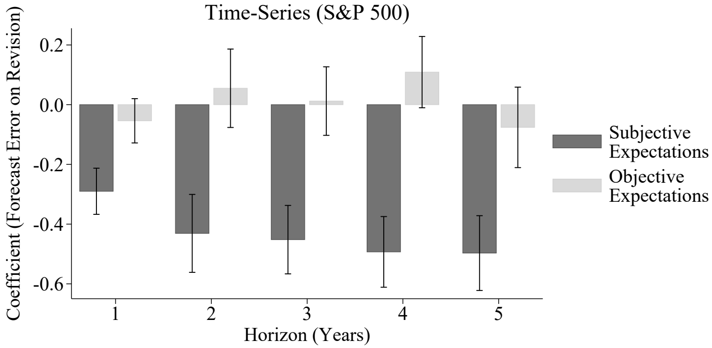
  (+) forecast revision predicts (-) forecast error

  $\Rightarrow$ Overreaction

- Machine:

  Forecast errors not predictable

  $\Rightarrow$ No overreaction



Notes: Figure reports β from time-series Coibion and Gorodnichenko 2015 regressions of survey and machine forecast errors on forecast revisions for aggregate S&P 500 cash flows. Sample: 2005Q1-2023Q4.

▸ Discount Rates ▸ State Dependence

Survey Overreacts, Machine Does Not: Cross-Section

$$\overbrace{\Delta e_{i,t,t+h} - \mathbb{F}_t[\Delta e_{i,t,t+h}]}^{\text{Forecast Error}} = \beta \cdot \overbrace{[\mathbb{F}_t[\Delta e_{i,t,t+h}] - \mathbb{F}_{t-1}[\Delta e_{i,t,t+h}]]}^{\text{Forecast Revision}} + \overbrace{\alpha_i + \alpha_t}^{\text{Firm \& Time FE}} + \varepsilon_{i,t}$$

- Survey:

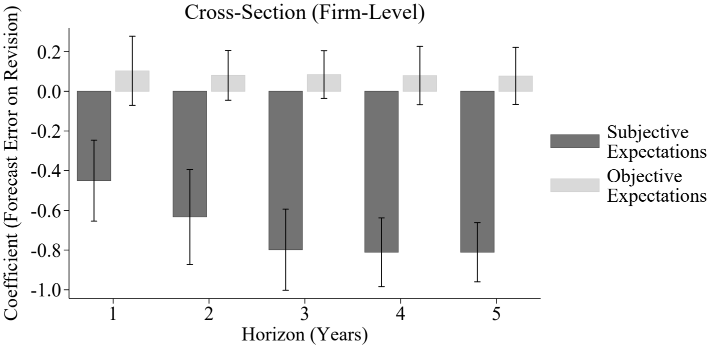
  (+) forecast revision predicts (-) forecast error

  $\Rightarrow$ Overreaction

- Machine:

  Forecast errors not predictable

  $\Rightarrow$ No overreaction

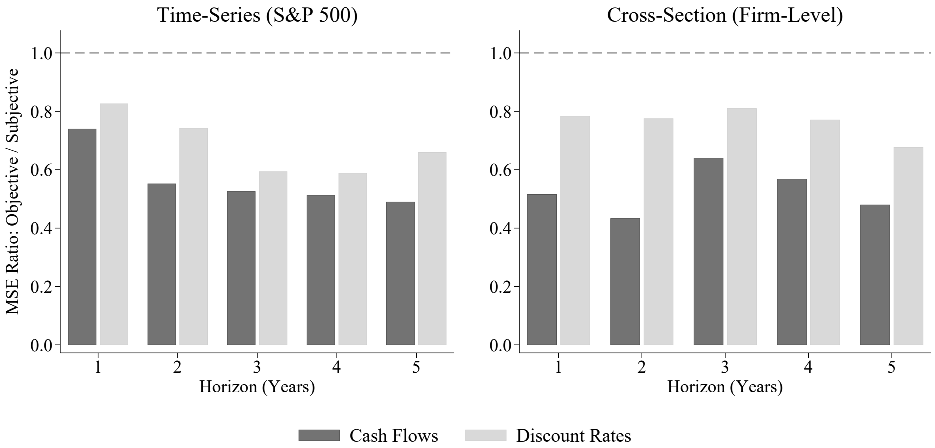


Notes: Figure reports β from firm-level Coibion and Gorodnichenko 2015 regressions of survey and machine forecast errors on forecast revisions with firm & time fixed effects. Sample: 2005Q1-2023Q4.

‣ Discount Rates ‣ State Dependence

Machine Learning $\mathbb{E}_t$ More Accurate than Survey Forecast $\mathbb{F}_t$

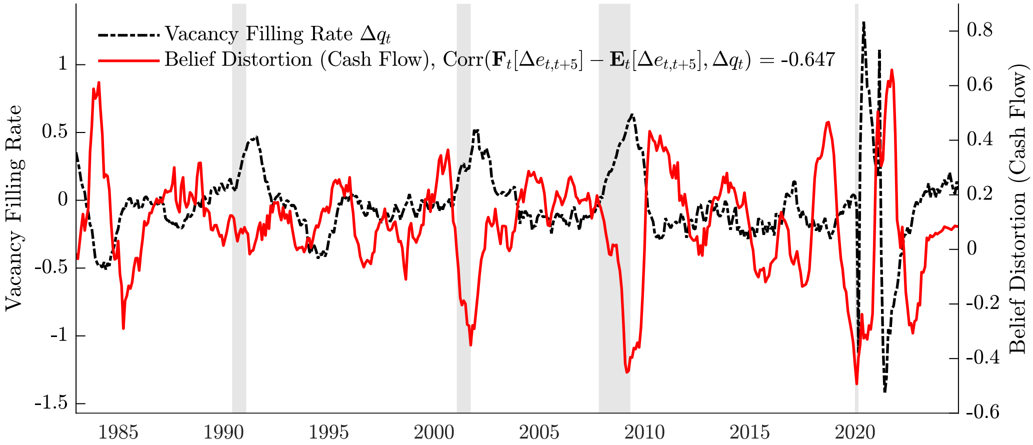
- Consistent with ex-ante distortions in subjective beliefs



Notes: Figure shows relative forecast errors $MSE_{\mathbb{E}} / MSE_{\mathbb{F}}$ comparing machine learning to survey forecasts. Left panel: aggregate S&P 500 time-series forecasts. Right panel: cross-sectional forecasts across listed firms. Dark bars: cash flows (earnings growth); light bars: discount rates (stock returns). Out-of-sample testing period: 2005Q1–2023Q4.

Vacancy Filling Rate Tracks Belief Distortions in Cash Flows ▸ Details

- Recession: Beliefs over-pessimistic, vacancy filling rate rise (few vacancies available)



Notes: Left axis: Annual log change in the U.S. vacancy filling rate. Right axis: Belief distortion measured as expectational errors $\mathbb{F}_t[\Delta e_{t,t+5}] - \mathbb{E}_t[\Delta e_{t,t+5}]$ in 5-year IBES survey forecasts of annualized S&P 500 earnings growth. Gray shaded areas indicate NBER recessions.

Search and Matching Model (Diamond-Mortensen-Pissarides)

- Firm posts job vacancies V_t to attract unemployed workers U_t
  - Matches formed at vacancy filling rate q_t, separated at rate δ_t
  - Posting a job vacancy costs κ per period
- Firm value $\mathcal{V}$ satisfies Bellman equation:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \{E_t + \mathbb{F}_t[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})]\}$$
$$s.t. \quad L_{t+1} = (1-\delta_t)L_t + q_t V_t$$

  - Cash flows (earnings): $E_t = A_t L_t - W_t L_t - \kappa V_t$
  - Productivity A_t, labor input L_t, wage rate W_t
  - Subjective expectation $\mathbb{F}_t[\cdot]$, stochastic discount factor M_{t+1}

Firm's Hiring Condition

- First-order condition: Firm equates cost of hiring with its expected discounted value

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \frac{\mathcal{V}(A_{t+1}, L_{t+1})}{L_{t+1}} \right] = \frac{P_t}{L_{t+1}}$$

where $P_t \equiv \mathbb{F}_t \left[M_{t+1} \mathcal{V}(A_{t+1}, L_{t+1}) \right]$ ex-dividend market value (stock price)

- Take logarithms, rearrange terms, split the price-employment ratio $\frac{P_t}{L_{t+1}}$:

$$\log q_t = \log \kappa - \underbrace{\log \left(\frac{P_t}{E_t} \right)}_{\equiv\ pe_t} - \underbrace{\log \left(\frac{E_t}{L_{t+1}} \right)}_{\equiv\ el_t}$$

Present Value Identity for Price-Earnings (Campbell-Shiller 1988) ▸ Details

- Log-linearize price-earnings ratio $pe_t \equiv \ln(P_t/E_t)$ around long-run mean $\overline{pe}$

$$pe_t = c_{pe} - \mathbb{F}_t[r_{t+1}] + \mathbb{F}_t[\Delta e_{t+1}] + \rho \mathbb{F}_t[pe_{t+1}]$$

  - c_{pe} constant, $\rho = \frac{\exp(\overline{pe})}{1+\exp(\overline{pe})} \approx 0.98$ time discount factor from log-linearization
  - $r_{t+1} = \log(\frac{P_{t+1}+E_{t+1}}{P_t})$ stock return, assuming firm pays out dividends $=$ earnings
  - Same identity holds approximately if dividends $\neq$ earnings e.g., $D_t \approx \frac{1}{2}E_t$

- Substitute recursively for the next h periods to obtain present value identity:

$$\underbrace{pe_t}_{\text{Price-Earnings}} = \underbrace{\sum_{j=1}^{h} \rho^{j-1} c_{pe}}_{\text{Constant}} - \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\text{Discount Rate}} + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t[pe_{t+h}]}_{\text{Future Price-Earnings}}$$

- Combine log-linearized pe_t with firm's hiring equation ($c_q = \log \kappa - \sum_{j=1}^{h} \rho^{j-1} c_{pe}$):

$$\underbrace{\log q_t}_{\substack{\text{Vacancy Filling} \\ \text{Rate}}} = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\substack{\text{Discount Rate} \\ \equiv\ \mathbb{F}_t[r_{t,t+h}]}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\substack{\text{Cash Flow} \\ \equiv\ \mathbb{F}_t[e_{t,t+h}]}} - \underbrace{\rho^h \mathbb{F}_t[pe_{t+h}]}_{\substack{\text{Future Price-Earnings} \\ \equiv\ \mathbb{F}_t[pe_{t,t+h}]}}$$

- Vacancy filling rate q_t is high (recession) either because of:
  - High discount rates (return required to justify hiring)
  - Low expected cash flows (profit from hiring)
  - Or low expected price-earnings (terminal value)

Variance Decomposition: Subjective vs. Objective Expectations

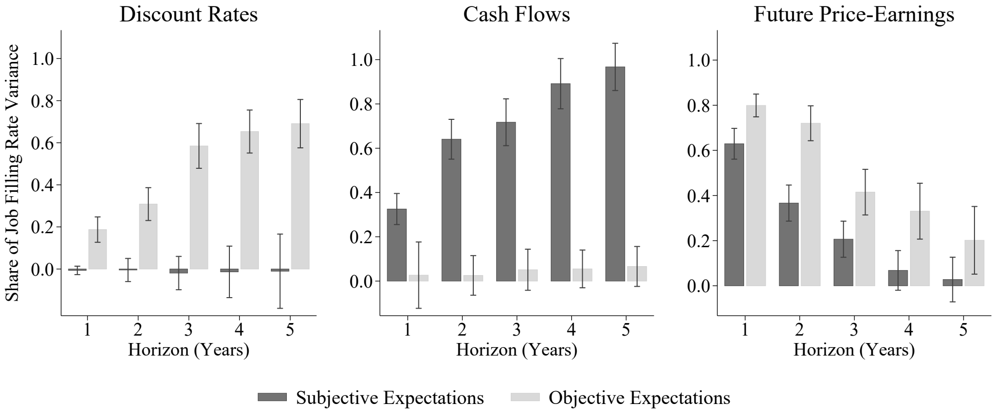
- Decomposition of $Var\left[\log q_t\right]$ under subjective belief: Use survey forecast $\mathbb{F}_t[\cdot]$

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings}}$$

- Estimate using OLS regression coefficients
  - Regress survey forecast $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, $\mathbb{F}_t[pe_{t,t+h}]$ on vacancy filling rate $\log q_t$
  - Forecast horizons: $h = 1, \ldots, 5$ years
- Decomposition under objective belief:
  - Replace survey with machine forecast $\mathbb{E}_t[\cdot]$
  - $\mathbb{F}_t - \mathbb{E}_t$ captures share that can be explained by belief distortion

Variance Decomposition of the Vacancy Filling Rate

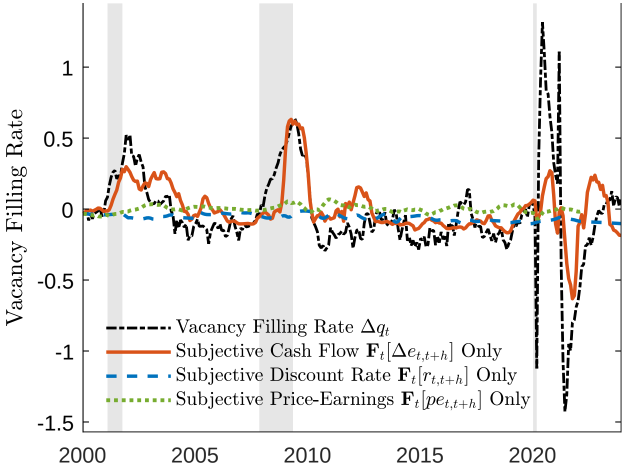
- Belief distortion $\mathbb{F}_t - \mathbb{E}_t$ makes hiring sensitive to cash flow news



Notes: Sample: 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

Role of Components in the Vacancy Filling Rate ▸ Details

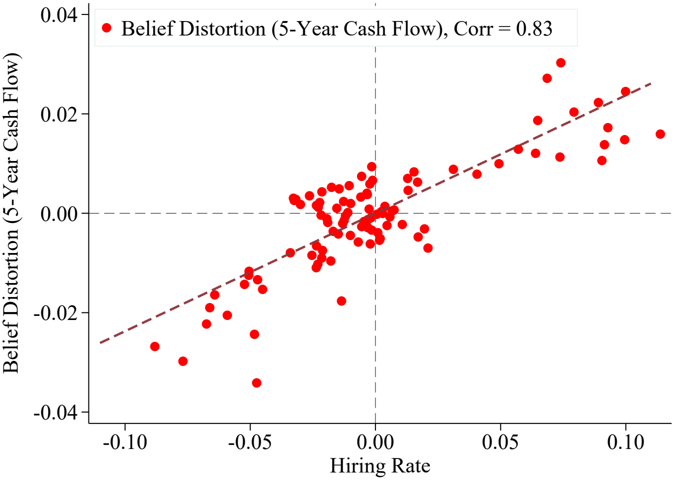
- Counterfactual series allowing one component to vary while holding others fixed
- Subjective cash flows play largest role in explaining vacancy filling rate variation



Notes: Counterfactual series are constructed by accumulating fitted values from regressions of vacancy filling rate growth on individual expectation measures at the $h = 5$ year horizon, with all series initialized to the actual vacancy filling rate growth in 2001Q4.

Firms with Pessimistic Belief Distortions Hire Less ▸ Details

- Cross-sectional dispersion after removing time & firm fixed effect
- Hiring rate = New hires / employment
- Belief distortion: Expectational errors $\mathbb{F}_t[\Delta e_{i,t,t+5}] - \mathbb{E}_t[\Delta e_{i,t,t+5}]$



Notes: y-axis: belief distortion measured as expectational errors $\mathbb{F}_t[\Delta e_{i,t,t+5}] - \mathbb{E}_t[\Delta e_{i,t,t+5}]$ in 5-year IBES survey forecasts of annualized S&P 500 earnings growth. x-axis: hiring rate. Each dot is a bin scatter representing one percentile across all observations in the sample. Sample: 2005Q1 to 2023Q4.

Why Do Some Firms Hire More Than Others?

- Define hiring rate at firm level:

$$hl_{i,t} \equiv \log\left(\frac{L_{i,t+1}}{L_{i,t}} - (1-\delta_{i,t})\right)$$

  - $L_{i,t}$ employment (Compustat), $\delta_{i,t}$ job separation rate from firm i's industry (JOLTS)

- Decompose cross-sectional variance of hiring rate $\text{Var}(\widetilde{hl}_{i,t})$:

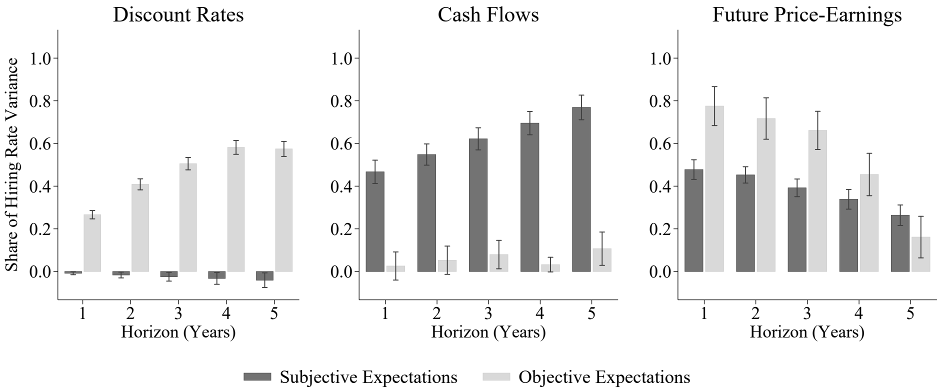
$$1 \approx -\underbrace{\frac{\text{Cov}\left(\mathbb{F}_t[\widetilde{r}_{i,t,t+h}], \widetilde{hl}_{i,t}\right)}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{Discount Rate}} + \underbrace{\frac{\text{Cov}\left(\mathbb{F}_t[\widetilde{e}_{i,t,t+h}], \widetilde{hl}_{i,t}\right)}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{Cash Flow}} + \underbrace{\frac{\text{Cov}(\mathbb{F}_t[\widetilde{pe}_{i,t,t+h}], \widetilde{hl}_{i,t})}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{Future Price-Earnings}}$$

  - $\widetilde{x}_{i,t} = x_{i,t} - \sum_{i \in I} x_{i,t}$ cross-sectionally demeans variable $x_{i,t}$

- Estimate as coefficients from panel regressions with firm & time fixed effect

Cross-Sectional Decomposition of Hiring Rate

- Belief distortion $\mathbb{F}_t - \mathbb{E}_t$ makes hiring sensitive to cash flow news
  - Implies distortions can operate at firm level where actual hiring decisions are made



Notes: Sample: 2005Q1 to 2023Q4. Each bar shows 95% confidence intervals two-way clustered by firm and time.

‣ Book-to-Market Sort ‣ Idiosyncratic Shock Sort ‣ Industry Sort ‣ Financial Constraints ‣ Shift-Share Instrument ‣ Capital Investment

How Do Profits per Worker Respond to Idiosyncratic Shocks? ▸ Decomposition

- Firm-level (i) local projection of profits per worker on forecast revisions:

$$\log\left(\frac{E_{i,t+h}}{L_{i,t+h}}\right) = \beta_h Revision_{i,t} + \alpha_i + \alpha_{s(i),t} + \varepsilon_{i,t+h}, \qquad s(i) \in SIC2$$

$\Rightarrow$ Dispersion in $Revision_{i,t}$ captures belief response to idiosyncratic shock

- Subjective: Survey cash flow forecast revision $Revision_{i,t} = \mathbb{F}_t[\Delta e_{i,t+1}] - \mathbb{F}_{t-1}[\Delta e_{i,t+1}]$
  $\Rightarrow$ Overreact to idiosyncratic shock, affect profits per worker by over- or under-hiring

- Objective: Machine cash flow forecast revision $Revision_{i,t} = \mathbb{E}_t[\Delta e_{i,t+1}] - \mathbb{E}_{t-1}[\Delta e_{i,t+1}]$
  $\Rightarrow$ No overreaction, stable profits per worker

Profits per Worker Fall Only Under Subjective Beliefs ▸ E and L ▸ Stock Returns

- Objective forecast revision:

  ⇒ No overreaction

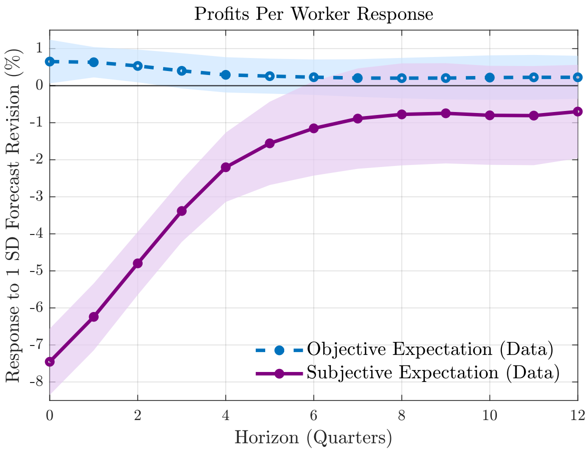
  ⇒ Stable profit per worker

- Subjective forecast revision:

  ⇒ Overreact (forecast revision)

  ⇒ Overhire

  ⇒ Profit per worker drop



Notes: Blue (purple) line: IRF under objective (subjective) expectations proxied by machine (survey) forecasts. Shaded area: 90% confidence intervals two-way clustered by firm and time. Sample: 1984Q1-2023Q4.

Stock Returns React Only Under Subjective Beliefs

- Objective forecast revision:

  $\Rightarrow$ Diversifiable shock not priced

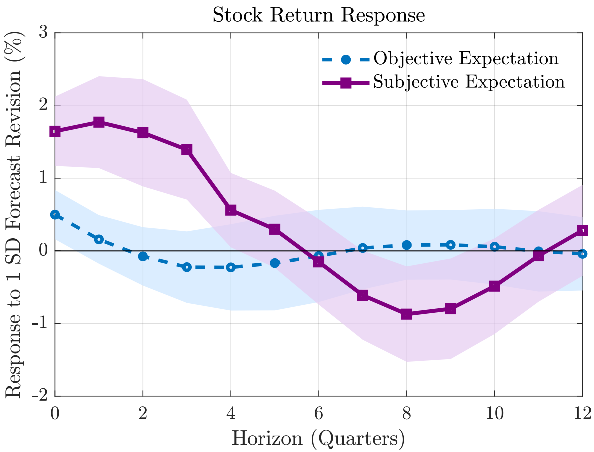
  $\Rightarrow$ No return response

- Subjective forecast revision:

  $\Rightarrow$ Overreact (forecast revision)

  $\Rightarrow$ Initial overshoot (overoptimism)

  $\Rightarrow$ Subsequent reversal (disappointment)



Notes: Blue (purple) line: IRF under objective (subjective) expectations proxied by machine (survey) forecasts. Shaded area: 90% confidence intervals two-way clustered by firm and time. Sample: 1984Q1-2023Q4.

Model of Constant-Gain Learning: Environment

- True cash flow process has aggregate E_t and idiosyncratic $\widetilde{E}_{i,t}$ components

$$E_{i,t} = E_t \cdot \widetilde{E}_{i,t} = \exp(e_t + \widetilde{e}_{i,t})$$

- Log of each component follows an AR(1) process:

$$\begin{aligned} e_t &= \mu + \phi e_{t-1} + u_t, \quad u_t \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_u^2) \\ \widetilde{e}_{i,t} &= \widetilde{\mu}_i + \widetilde{\phi}\widetilde{e}_{i,t-1} + v_{i,t}, \quad v_{i,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_v^2) \end{aligned}$$

- Log stochastic discount factor m_{t+1}:

$$m_{t+1} = -r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1}$$

with risk-free rate r_f, constant relative risk aversion γ

Model of Constant-Gain Learning: Subjective Beliefs ▸ Data

- Objectively, long-run mean identical across firms: $\mu = \tilde{\mu}_i = 0$
  - But firm does not know true μ and $\widetilde{\mu}_i$ in cash flow process
- Update beliefs using constant-gain learning rule: Learn at rate ν

$$\mathbb{F}_t[\mu] = \mathbb{F}_{t-1}[\mu] + \nu \left(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]\right)$$
$$\mathbb{F}_t[\widetilde{\mu}_i] = \mathbb{F}_{t-1}[\widetilde{\mu}_i] + \nu \left(\Delta \widetilde{e}_{i,t} - \mathbb{F}_{t-1}[\Delta \widetilde{e}_{i,t}]\right)$$

  - Initial beliefs objective $\mathbb{F}_0[\mu] = \mathbb{F}_0[\widetilde{\mu}_i] = 0$ so that $\nu = 0$ case rational expectation
- Fading memory: Assigns smaller decaying weight on older observations
  - Generates over-reaction as beliefs over-extrapolate from recent observations

Hiring Under Subjective Firm Valuations ▸ Details (Asset Pricing) ▸ Details (Labor Market)

- Firm valuation: Equilibrium stock price $P_{i,t}$ under subjective beliefs

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)} = \sum_{h=1}^{\infty} \exp\left\{A_i^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \widetilde{B}^{(h)}\mathbb{F}_t[\widetilde{\mu}_i] + \phi^h e_t + \widetilde{\phi}^h \widetilde{e}_{i,t}\right\}$$

  - $P_{i,t}$ driven by belief distortions in cash flows $\mathbb{F}_t[\mu]$, $\mathbb{F}_t[\widetilde{\mu}_i]$ (from fading memory)
  - $P_{i,t}^{(h)}$ strip prices with recursively defined coefficients $A_i^{(h)}, B^{(h)}, \widetilde{B}^{(h)}$

- Hiring condition: Firms post vacancies until marginal cost equals marginal value

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of Hiring}} = \underbrace{\frac{P_{i,t}}{L_{i,t+1}}}_{\text{Value of Hiring}}$$

  - Partial equilibrium: Exogenous cash flows absorb wage setting and worker belief

- Subjective valuation $P_{i,t}$ drives both stock returns $R_{i,t} = \frac{P_{i,t}+E_{i,t}}{P_{i,t-1}}$ and hiring $L_{i,t+1}$

Model Parameters ▸ Details

Parameter	Value	Description/Moments
ν	0.018	Constant-gain learning (Malmendier and Nagel 2015)
ϕ	0.856	Autocorrelation aggregate earnings growth
σ_u	0.268	S.D. aggregate earnings growth
$\widetilde{\phi}$	0.698	Autocorrelation firm-level earnings growth
σ_v	0.194	S.D. firm-level earnings growth
r_f	0.046	Risk-free rate (De La O, Han, and Myers 2022)
γ	1.586	Average aggregate return (De La O, Han, and Myers 2022)
ρ	0.980	Average price-earnings ratio
B	0.562	Matching function efficiency (Kehoe et al. 2023)
η	0.500	Matching function elasticity (Kehoe et al. 2023)
δ	0.286	Separation rate (Kehoe et al. 2023)
κ	0.314	Per worker hiring cost (Elsby and Michaels 2013)

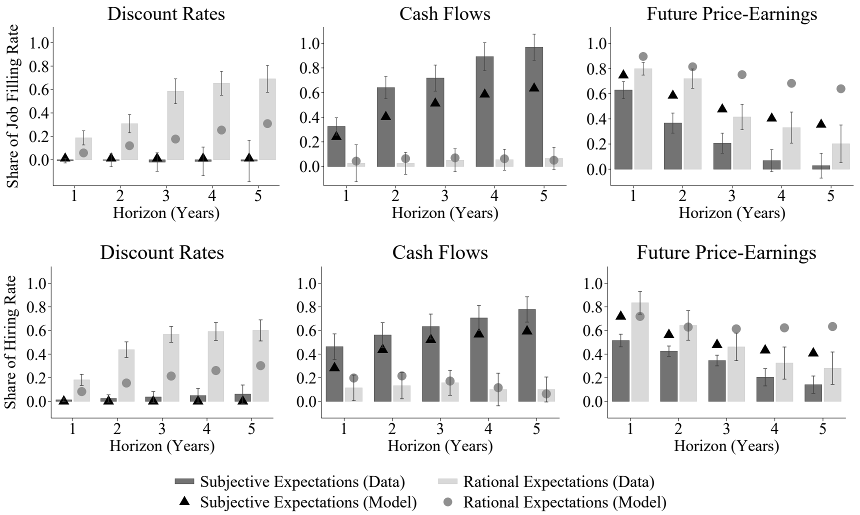
Notes: Table reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The model is calibrated at an annual frequency.

Model vs. Data: Variance Decompositions

▸ Details (Labor Market) ▸ Model Parameters

(a) Time-Series Decomposition (Vacancy Filling Rate)

(b) Cross-Sectional Decomposition (Hiring Rate)



Notes: Circle and triangle dots show the values of rational and subjective expectations implied by the model, respectively, derived from a simulation of 300 firms over 500 periods, with the first 150 periods discarded as a burn-in.

Model vs. Data: Moments

- Learning improves model's ability to match asset market and labor market moments

(a) Asset Market

Moment	Data	Learning Model	Rational Model
SD$(pe_t) \times 100$	**47.0**	**43.5**	**13.0**
SD$_i(pe_{i,t}) \times 100$	**22.6**	**21.1**	**4.2**
SD$(r_t) \times 100$	16.0	12.3	3.0
SD$(\mathbb{F}_t[r_{t+1}]) \times 100$	1.1	1.4	0.5
SD$(\mathbb{F}_t[\Delta e_{t+1}]) \times 100$	26.8	24.3	7.2
SD$_i(r_{i,t}) \times 100$	5.7	3.1	1.2
SD$_i(\mathbb{F}_t[r_{i,t+1}]) \times 100$	2.6	0.2	0.2
SD$_i(\mathbb{F}_t[\Delta e_{i,t+1}]) \times 100$	14.0	16.6	3.9

(b) Labor Market

Moment	Data	Learning Model	Rational Model
SD$(u_t) \times 100$	**2.10**	**1.28**	**0.34**
SD$_i(hl_{i,t}) \times 100$	**15.70**	**10.39**	**4.65**
SD$(q_t) \times 100$	8.70	6.16	0.91
Corr(u_t, q_t)	-0.82	-0.86	-0.99

Notes: SD$(\cdot)$ = time-series standard deviation. SD$_i(\cdot)$ = cross-sectional standard deviation. pe_t = log price-earnings ratio, r_t = log stock return, Δe_t = log earnings growth, q_t = job-filling rate, u_t = unemployment rate, $hl_{i,t}$ = firm-level hiring rate. $\mathbb{F}_t[\cdot]$ = subjective expectations formed at time t. Data = empirical moments. Model (Learning) = constant-gain learning model. Model (Rational) = rational expectations benchmark.

Conclusion

- Subjective beliefs are distorted as they over-react to news
  - Comparing survey vs. machine learning forecasts uncovers these distortions
- Over-reaction to cash flow news drives asset prices and hiring
  - Both in the time-series and the cross-section
  - Results consistent with learning about cash flow with fading memory
- Offers new perspective on unemployment volatility puzzle
  - Belief distortions drive value of hiring, driving unemployment fluctuations

Appendix

Related Literature ▸ Return

- **Unemployment Volatility Puzzle:** Shimer 2005; Hagedorn and Manovskii 2008; Hall and Milgrom 2008; Pissarides 2009; Elsby and Michaels 2013; Kudlyak 2014; Chodorow-Reich and Karabarbounis 2016; Ljungqvist and Sargent 2017; Hall 2017; Borovičková and Borovička 2017; Kilic and Wachter 2018; Mitra and Xu 2019; Kehoe, Midrigan, and Pastorino 2019; Kehoe et al. 2023; Meeuwis et al. 2023
  - This paper: Reframes unemployment volatility as belief-driven
- **Labor Market Frictions and Asset Prices:** Merz and Yashiv 2007; Donangelo 2014; Belo, Lin, and Bazdresch 2014; Favilukis and Lin 2015; Kuehn, Simutin, and Wang 2017; Petrosky-Nadeau, Zhang, and Kuehn 2018; Donangelo et al. 2019; Liu 2021; Belo et al. 2023
  - This paper: Introduce subjective beliefs to explain differences in hiring across firms
- **Non-Rational Expectations and Business Cycles:** Marcet and Sargent 1989; Evans and Honkapohja 2001; Woodford 2001; Mankiw and Reis 2002; Sims 2003; Venkateswaran 2014; Coibion and Gorodnichenko 2015; Gabaix 2019; Ma et al. 2020; Acharya and Wee 2020; Bordalo et al. 2021; Bianchi, Ludvigson, and Ma 2022; Bianchi, Ilut, and Saijo 2023; Menzio 2023; Bhandari, Borovička, and Ho 2024; Fukui, Gormsen, and Huber 2024; Du et al. 2025; Bigio, Silva, and Zilberman 2025; Bloom, Codreanu, and Fletcher 2025
  - This paper: Clarify that bias in expected cash flow drive unemployment fluctuations
- **Non-Rational Expectations and Asset Prices:** Timmermann 1993; Barberis, Shleifer, and Vishny 1998; Chen, Da, and Zhao 2013; Greenwood and Shleifer 2014; Greenwood and Hanson 2014; Adam, Marcet, and Nicolini 2016; Giglio et al. 2021; De La O and Myers 2021; Nagel and Xu 2022; Barrero 2022; Jin and Sui 2022; De La O, Han, and Myers 2022; Binsbergen, Han, and Lopez-Lira 2022; Adam and Nagel 2023; Bianchi, Ludvigson, and Ma 2024; Bordalo et al. 2024; Décaire and Graham 2024; Chaudhry 2025
  - This paper: Show that biases drive both asset prices and real hiring decisions

Data: Labor Market Outcomes ▸ Return

Time-series: Vacancy filling rate q_t for quarter t (Source: JOLTS, BLS)

$$q_t = \frac{f_t U_t}{V_t}$$

- V_t and U_t: U.S. job openings and unemployment level
- $f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}}$: Job finding rate, where U_t^s short-term unemployed

Cross-section: Hiring rate $hl_{i,t}$ for firm i quarter t (Source: Compustat, JOLTS)

$$hl_{i,t} = \log\left(\frac{L_{i,t+1}}{L_{i,t}} - (1 - \delta_{i,t})\right)$$

- $L_{i,t}$: Annual employees, interpolate to quarterly using latest value
- $\delta_{i,t}$: Job separation rate of firm i's NAICS2 industry
- Sample: All common stocks (share codes 10, 11) on NYSE/AMEX/NASDAQ with IBES analyst coverage of earnings and stock price targets

Data: Discount Rates (Time-Series) ▸ Return

- Realized return r_t: Annual log return on CRSP value-weighted index with dividend
- Expected return $\mathbb{F}_t[r_{t+h}]$:
  - Source: CFO survey median consensus forecast (2001Q4-2023Q4)
  - Respondents: CFOs, VPs of finance, directors (1,600 members as of 2022)
  - Prediction target: Aggregate S&P 500 stock returns
  - Horizon h: 1 and 10 years ahead; interpolate intermediate horizons linearly

Data: Discount Rates (Cross-Section) ▸ Return

- Realized return $r_{i,t}$: Annual log return on firm i's stock with dividend
- Expected return $\mathbb{F}_t[r_{i,t+h}]$:
  - Source: IBES (1-year) and Value Line (3-5 years) median consensus price target
  - Respondents: Equity research analysts
  - Prediction target: Firm i's stock price level $\mathbb{F}_t[P_{i,t+h}]$
  - Horizon h: 1 year (IBES) and 5 years (Value Line), interpolate intermediate horizons
  - Construct return forecasts (with dividends) from price level forecasts using

$$\mathbb{F}_t[r_{i,t+h}] \approx \log\left(\frac{\mathbb{F}_t[P_{i,t+h}]}{P_{i,t}} + \frac{D_{i,t}}{P_{i,t}}\frac{\mathbb{F}_t[D_{i,t+h}]}{D_{i,t}}\right)$$

    - Expected dividend growth $\frac{\mathbb{F}_t[D_{i,t+h}]}{D_{i,t}} \approx 1.064$ equal post war average (Nagel and Xu 2022)
    - Dividend-price ratio $\frac{D_{i,t}}{P_{i,t}}$ from Compustat/CRSP

Data: Cash Flows (Cross-Section) ▸ Return

- Firm-level earnings $E^*_{i,t}$: Street earnings for firm i (Source: IBES)
  - "Street": Exclude discontinued operations, extraordinary charges, non-operating item
  - Construct from earnings per share: $E^*_{i,t} = EPS^*_{i,t} \cdot S_{i,t}$ where $S_{i,t}$ shares outstanding
  - Transformation to ensure positive earnings (Vuolteenaho 2002):

$$E_{i,t} = (1-\lambda)E^*_{i,t} + \lambda r^f_t P_{i,t-1} > 0, \quad \lambda = 0.10$$

    - Define firm as a portfolio of 90% equity & 10% 1-year T-bills (with rate r^f_t)
    - Allows $\log(E_{i,t})$ to be well-defined when reported earnings negative $E^*_{i,t} \leq 0$
- Firm-level expected log earnings growth $\mathbb{F}_t[\Delta e_{i,t+h}]$:
  - Source: IBES median consensus forecast (1983Q4-2023Q4)
  - Respondents: Equity research analysts

Data: Cash Flows (Construction by horizon h) ▸ Return

- Years $h = 1, 2$ from level forecasts
  - Prediction target: Street earnings per share ($EPS^*_{i,t}$) over next 1, 2, 3 fiscal years
  - Interpolate 1, 2, 3 fiscal year horizons to 1, 2 calendar year horizons
  - Construct earnings level forecast using $\mathbb{F}_t[E^*_{i,t+h}] = \mathbb{F}_t[EPS^*_{i,t+h}] \cdot S_{i,t}$
  - Transformation to ensure positive earnings (Vuolteenaho 2002):

$$\mathbb{F}_t[E_{i,t+h}] = (1-\lambda)\mathbb{F}_t[E^*_{i,t+h}] + \lambda r^f_t \mathbb{F}_t[P_{i,t+h-1}] > 0, \quad \lambda = 0.10$$

  - Approximate log growth forecast using $\mathbb{F}_t[\Delta e_{i,t+h}] \approx \log(\mathbb{F}_t[E_{i,t+h}]/\mathbb{F}_t[E_{i,t+h-1}])$
- Years $h = 3, 4, 5$ from long-term growth (LTG): Interpret as $\mathbb{F}_t[\Delta e_{i,t+h}]$
  - LTG: Forecast of annualized growth over the next "three-to-five years"

Data: Cash Flows (Time-Series) ▸ Return

- Aggregate S&P 500 earnings: x_t set of S&P 500 firms with IBES forecasts, Ω_t adjusts for incomplete IBES coverage, $Divisor_t$ S&P 500 divisor

$$E_t = \Omega_t \sum_{i \in x_t} EPS^*_{i,t} \cdot S_{i,t} / Divisor_t$$

- Aggregate S&P 500 expected log earnings growth $\mathbb{F}_t[\Delta e_{t+h}]$:
  - Years $h = 1, 2$: $\mathbb{F}_t[\Delta e_{t+h}] \approx \log(\mathbb{F}_t[E_{t+h}] / \mathbb{F}_t[E_{t+h-1}])$

$$\mathbb{F}_t[E_{t+h}] = \Omega_t \sum_{i \in x_t} \mathbb{F}_t[EPS^*_{i,t+h}] \cdot S_{i,t} / Divisor_t$$

  - Years $h = 3, 4, 5$: Value-weighted aggregate of firm-level LTG forecasts

$$\mathbb{F}_t[\Delta e_{t+h}] = LTG_t = \sum_{i=1}^{S} LTG_{i,t} \frac{P_{i,t} S_{i,t}}{\sum_{i=1}^{S} P_{i,t} S_{i,t}}$$

where S is the number of firms in the S&P 500 index

Data: Price-Earnings Ratio ▸ Return

Time-series: Log price-earnings ratio for the aggregate S&P 500

- Realized values: $pe_t = \log(P_t/E_t)$
- Expected values: Use Campbell and Shiller 1988 present value identity

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} \big(c_{pe} + \underbrace{\mathbb{F}_t[\Delta e_{t+j}]}_{\text{IBES}} - \underbrace{\mathbb{F}_t[r_{t+j}]}_{\text{CFO}}\big)$$

  – c_{pe} constant, $\rho = \frac{\exp(\overline{pe})}{(1+\exp(\overline{pe}))}$ time discount factor from log-linearization

Cross-section: Log price-earnings ratio at firm level

- Realized values: $pe_{i,t} = \log(P_{i,t}/E_{i,t})$
- Expected values: Use Campbell and Shiller 1988 present value identity

$$\mathbb{F}_t[pe_{i,t+h}] = \frac{1}{\rho^h} pe_{i,t} - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} \big(c_{pe} + \underbrace{\mathbb{F}_t[\Delta e_{i,t+j}]}_{\text{IBES}} - \underbrace{\mathbb{F}_t[r_{i,t+j}]}_{\text{IBES}}\big)$$

Data: Machine Learning Forecast (Time-Series) ▸ Return

Long Short-Term Memory (LSTM) neural network (Bianchi et al. 2022, 2024, 2025)

$$\mathbb{E}_t[y_{t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_t; \boldsymbol{\lambda}_t)$$

- Target: $y_{t+h} \in \{r_{t+h}, \Delta e_{t+h}\}$ predicted $h = 1, 2, \ldots, 5$ years ahead
- Estimation:
  - Training: Rolling samples, parameters $\boldsymbol{\beta}_t$ re-estimated quarterly in real-time
  - Validation: Pseudo out-of-sample for hyperparameters $\boldsymbol{\lambda}_t$
  - Regularization: L_1/L_2 penalties, dropout, early stopping, ensemble averaging
  - Architecture selection: Layers, neurons, training/validation window lengths
- Input data $\mathcal{X}_t$:
  - Macro/financial data, text (LDA factors from WSJ), macro data & FOMC surprises
  - Lagged survey forecast $\mathbb{F}_{t-1}[y_{t+h-1}]$

Data: Machine Learning Forecast (Cross-Section) ▸ Return

Estimate using pooled panel firm level data

$$\mathbb{E}_t[y_{i,t+h}] = G(\mathcal{X}_{i,t}, \boldsymbol{\beta}_t; \boldsymbol{\lambda}_t)$$

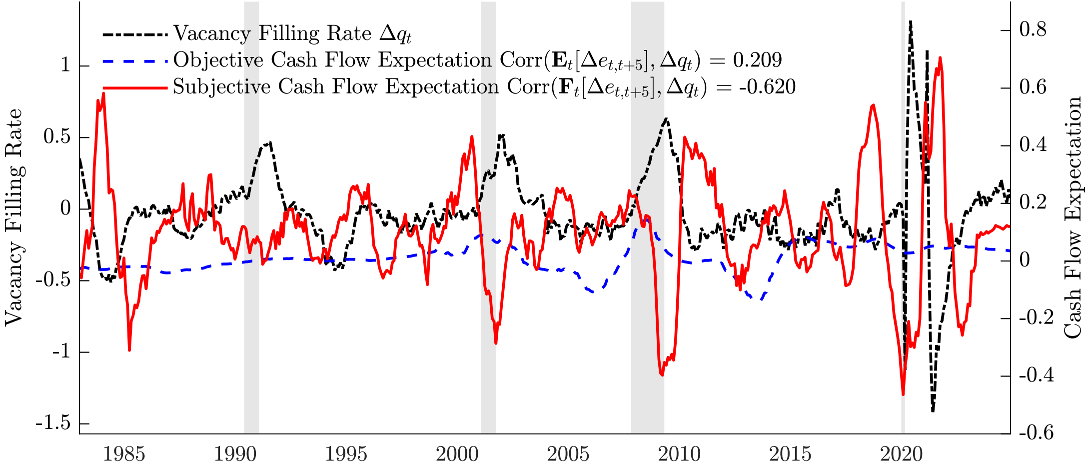
- Input data: Interaction of macro and firm characteristics

$$\mathcal{X}_{i,t} = \mathcal{X}_t \otimes \mathcal{C}_{i,t}$$

  - $\mathcal{X}_t$: Aggregate macro/financial variables (same as time-series)
  - $\mathcal{C}_{i,t}$: Firm characteristics (94 variables)
    - Valuation, profitability, size, momentum, volatility
    - Industry dummies (74 industries, 2-digit SIC codes)
- Re-estimation: Parameters and hyperparameters updated every 4 quarters

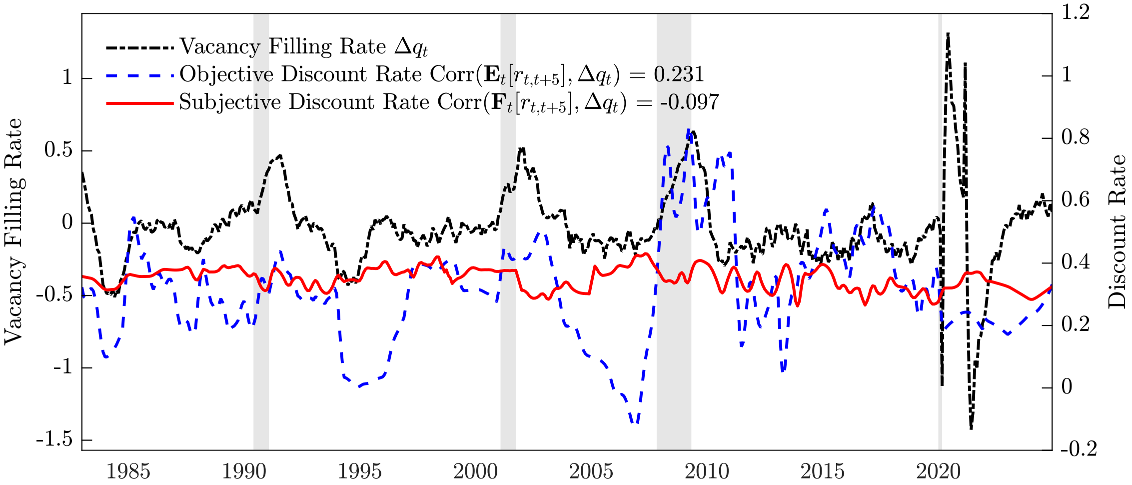
Vacancy Filling Rate Tracks Belief Distortions in Cash Flows ▸ Return

- Belief distortion: Expectational errors $\mathbb{F}_t[\Delta e_{t,t+h}] - \mathbb{E}_t[\Delta e_{t,t+h}]$



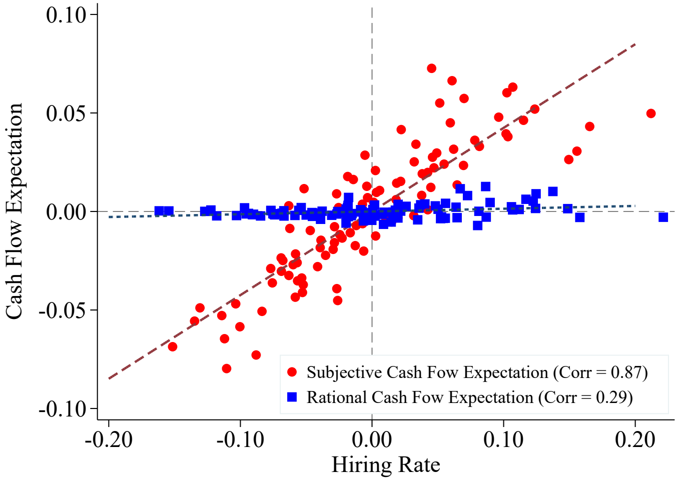
Notes: Left axis: Annual log change in the U.S. vacancy filling rate. Right axis: 5-year forecasts of annualized S&P 500 earnings growth. Subjective expectation: IBES median analyst projections for the next four fiscal years and long-term growth (LTG). Objective expectation: Machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. Gray shaded areas indicate NBER recessions.

Vacancy Filling Rate Tracks Objective Discount Rates ▸ Return



Firms with Pessimistic Belief Distortions Hire Less ▸ Return

- Cross-sectional dispersion net of time & portfolio fixed effect
- Hiring rate = New hires / employment
- Belief distortion: Expectational errors $\mathbb{F}_t[\Delta \widetilde{e}_{i,t,t+h}] - \mathbb{E}_t[\Delta \widetilde{e}_{i,t,t+h}]$



Notes: x-axis: Cross-sectionally demeaned real-time expectation of future earnings. Subjective Expectation: $\mathbb{F}_t[\widetilde{e}_{i,t,t+j}]$ based on IBES survey forecasts. Objective Expectation: $\mathbb{E}_t[\widetilde{e}_{i,t,t+j}]$ based on machine learning forecasts. y-axis: Cross-sectionally demeaned hiring rate. Each dot is a bin scatter representing one percentile across all observations in the sample. Sample: 2005Q1 to 2023Q4.

Survey Overreacts to Discount Rate News: Time-Series ▸ Return

$$\overbrace{r_{t,t+h} - \mathbb{F}_t[r_{t,t+h}]}^{\text{Forecast Error}} = \beta \cdot \overbrace{[\mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_{t-1}[r_{t,t+h}]]}^{\text{Forecast Revision}} + \varepsilon_t$$

- Survey:

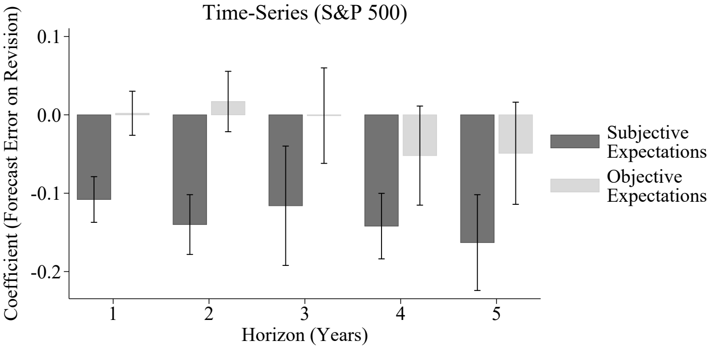
  (+) forecast revision predicts (-) forecast error

  $\Rightarrow$ Overreaction

- Machine:

  Forecast errors not predictable

  $\Rightarrow$ No overreaction



Notes: Figure reports β from time-series Coibion and Gorodnichenko 2015 regressions of survey and machine forecast errors on forecast revisions for aggregate S&P 500 cash flows. Sample: 2005Q1-2023Q4. Whiskers: 95% confidence intervals (Newey-West, 4 lags).

Survey Overreacts to Discount Rate News: Cross-Section ▸ Return

$$\overbrace{r_{i,t,t+h} - \mathbb{F}_t[r_{i,t,t+h}]}^{\text{Forecast Error}} = \beta \cdot \overbrace{[\mathbb{F}_t[r_{i,t,t+h}] - \mathbb{F}_{t-1}[r_{i,t,t+h}]]}^{\text{Forecast Revision}} + \overbrace{\alpha_i + \alpha_t}^{\text{Firm \& Time FE}} + \varepsilon_{i,t}$$

- Survey:

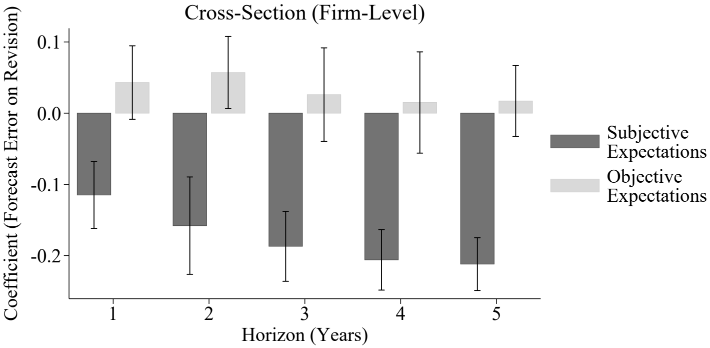
  (+) forecast revision predicts (-) forecast error

  $\Rightarrow$ Overreaction

- Machine:

  Forecast errors not predictable

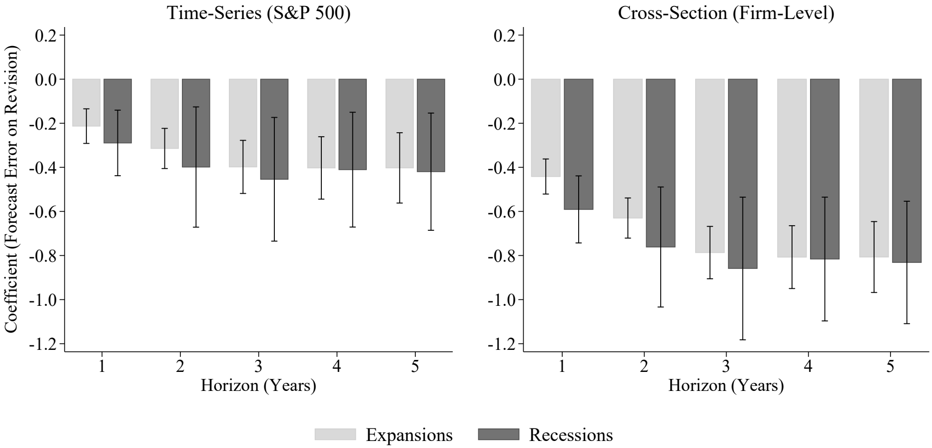
  $\Rightarrow$ No overreaction



Notes: Figure reports β from firm-level Coibion and Gorodnichenko 2015 regressions of survey and machine forecast errors on forecast revisions with firm & time fixed effects. Sample: 2005Q1-2023Q4. Whiskers: 95% confidence intervals (two-way clustered by firm & time).

Overreaction in Cash Flow Expectations: State Dependence ▸ Return

- Overreaction present in both during and outside of NBER recessions



Notes: Figure reports β from Coibion and Gorodnichenko 2015 regressions of survey forecast errors on forecast revisions, estimated separately for NBER expansions and recessions. Left: S&P 500 time-series. Right: firm-level with firm & time FE. Sample: 2001Q1-2020Q4. Whiskers: 95% CI (Newey-West for TS; clustered by firm for CS).

Present Value Identity for Price-Earnings: Details ▸ Return

- Log-linearize log price-dividend $pd_t \equiv \ln(P_t/D_t)$ around long-term average $\overline{pd}$

$$pd_t = c_{pd} + \Delta d_{t+1} - r_{t+1} + \rho pd_{t+1}$$

where c_{pd} constant, $r_{t+1} \equiv \log(\frac{P_{t+1}+D_{t+1}}{P_t})$ stock returns, $\rho \equiv \frac{\exp(\overline{pd})}{1+\exp(\overline{pd})} = 0.98$

- Substitute in log price-earnings $pe_t = pd_t + de_t$ where de_t log payout ratio

$$pe_t = c_{pd} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} + (1-\rho)de_{t+1}$$

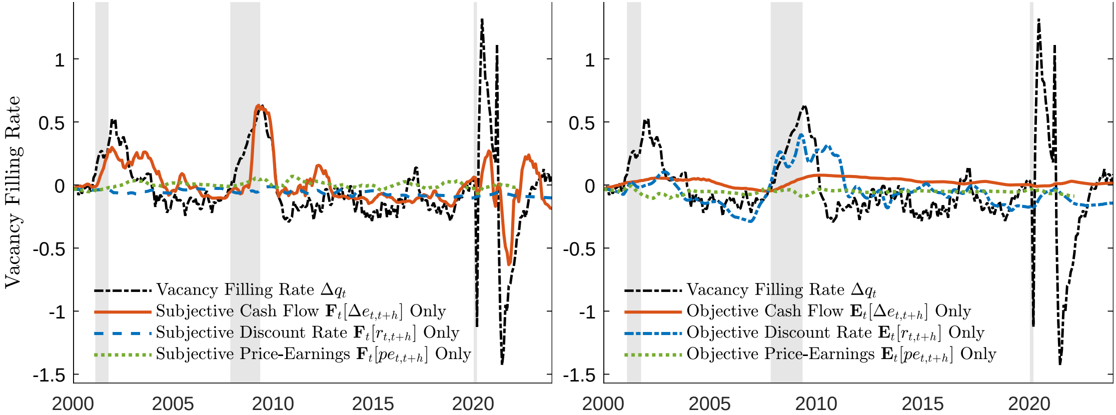
- Since $1-\rho \approx 0$ and de_t bounded, approximate $(1-\rho)de_{t+1}$ as a constant

$$pe_t \approx c_{pe} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1}, \qquad c_{pe} \approx c_{pd} + (1-\rho)de_{t+1}$$

- Recursively substitute for the next h periods

$$pe_t = \sum_{j=1}^{h} \rho^{j-1}(c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h}$$

Role of Components in the Vacancy Filling Rate: $\mathbb{F}_t$ vs. $\mathbb{E}_t$ Return



Notes: Counterfactual series are constructed by accumulating fitted values from regressions of vacancy filling rate growth on individual expectation measures at the $h = 5$ year horizon, with all series initialized to the actual vacancy filling rate growth in 2001Q4. Gray shaded areas indicate NBER recessions. Sample period: 2000Q4 to 2023Q4.

Correlation with Residual Term ▸ Return

- Allow for a residual $\upsilon_{t,h}$ in the decomposition:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[\Delta e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] + \upsilon_{t,h}$$

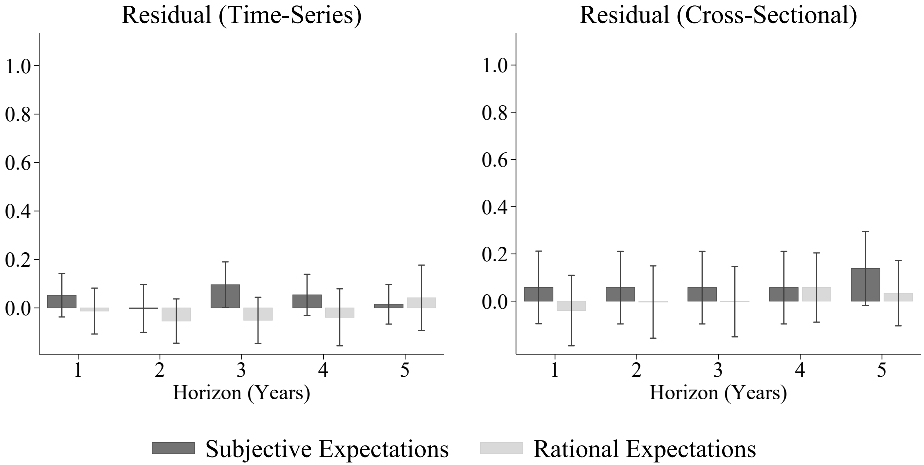
- Residual $\upsilon_{t,h}$ approximately unrelated to dependent variables and components

Component	**Time-Series**	**Cross-Sectional**
Dependent Variables		
Vacancy Filling Rate	0.015	—
Hiring Rate	—	0.015
Decomposition Components		
Discount Rate $\mathbb{F}_t[r_{t,t+5}]$	0.026	0.032
Cash Flow $\mathbb{F}_t[\Delta e_{t,t+5}]$	0.078	0.089
Future Price-Earnings $\mathbb{F}_t[pe_{t,t+5}]$	−0.001	−0.033

Notes: This table reports correlations between residuals and each component. Cross-sectional correlations use firm-level deviations from the corresponding time t means. Sample: 2005Q1 to 2023Q4.

Contribution of Residual Term ▸ Return

- Residual $\upsilon_{t,h}$ contributes up to 13.8% in variance decompositions



Notes: Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters. Sample: 2005Q1 to 2023Q4.

Belief Distortions and Vacancy Filling Rates ▸ Return

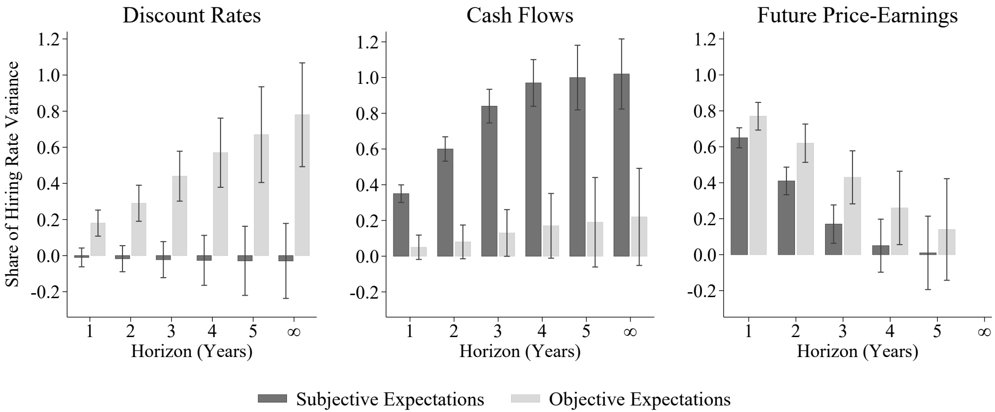
- At the 5-year horizon:
  - Cash flow bias leads to over-attribution of hiring variation to earnings
  - Discount rate bias offsets this by under-attributing to discount rates

Horizon h (Years)	1	2	3	4	5
			Biases in Subjective Expectations $\mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \quad y = r, e, pe$		
Discount Rate	-0.194	-0.313^{**}	-0.604^{***}	-0.667^{***}	-0.701^{***}
t-stat	(-1.574)	(-2.167)	(-2.896)	(-2.918)	(-2.740)
(-) Cash Flow	0.299	0.615^{***}	0.666^{***}	0.837^{***}	0.901^{***}
t-stat	(1.421)	(5.476)	(5.703)	(7.365)	(6.665)
(-) Price-Earnings	-0.170	-0.354^{**}	-0.209	-0.262	-0.174
t-stat	(-0.464)	(-2.373)	(-0.503)	(-0.479)	(-0.292)
Residual	-0.065	-0.052	-0.147	-0.093	0.026
t-stat	(-0.148)	(-0.219)	(-0.306)	(-0.154)	(0.040)

Notes: Newey-West t-statistics with lags = 4 in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

Time-Series Decomposition of Vacancy Filling Rate: VAR(1) Return

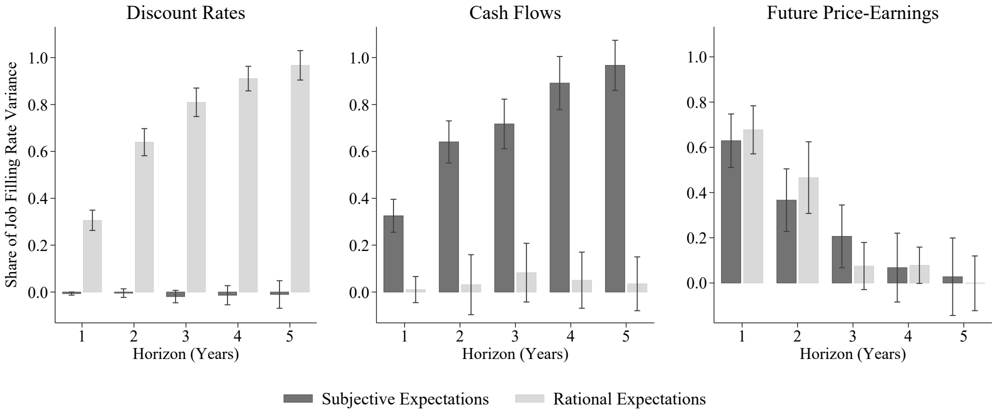
$$X_{t+1} = AX_t + \varepsilon_{t+1}, \quad X_t = \begin{bmatrix} \mathbb{F}_t[r_{t,t+1}] & \mathbb{F}_t[e_{t,t+1}] & \mathbb{F}_t[pe_{t,t+1}] & \log q_t \end{bmatrix}'.$$



Notes: Figure reports variance decompositions of the aggregate vacancy filling rate based on a Vector Autoregression (VAR). Light (dark) bars show the contribution under objective (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows bootstrapped 95% confidence intervals.

Time-Series Decomposition of Vacancy Filling Rate: Ex-Post ▸ Return

- Replace machine learning forecast $\mathbb{E}_t[x_{t,t+h}]$ with ex-post realized value $x_{t,t+h}$



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate vacancy filling rate. Light bars show the contribution under objective expectations. Dark bars show the contribution under subjective expectations. The Sample: 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Risk-Neutral Measure: Decomposition ▸ Return

- Start with ex-post decomposition of vacancy filling rate:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{r_{t,t+h}} - \underbrace{\left[dl_t + \sum_{j=1}^{h} \rho^{j-1} \Delta d_{t+j} \right]}_{d_{t,t+h}} - \underbrace{\rho^h pd_{t+h}}_{pd_{t,t+h}}$$

  where d_t denotes log S&P 500 dividends

- Evaluate under risk-neutral measure $\mathbb{E}_t^Q[\cdot]$:

$$\log q_t = c_q + \mathbb{E}_t^Q[r_{t,t+h}] - \mathbb{E}_t^Q[d_{t,t+h}] - \mathbb{E}_t^Q[pd_{t,t+h}]$$

- Under no arbitrage, futures price = expected future spot price under $\mathbb{E}_t^Q[\cdot]$ (Ait-Sahalia, Wang, and Yared 2001)

Risk-Neutral Measure: Futures Prices ▸ Return

- Risk-neutral discount rates:

$$\mathbb{E}_t^Q[r_{t,t+h}] = \sum_{j=1}^{h} \rho^{j-1}(f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500}), \qquad f_{t,t}^{sp500} \equiv p_t$$

  - Measure $\mathbb{E}_t^Q[r_{t+j}] = f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500}$ assuming $f_{t,t+j}^{sp500} \approx \log \mathbb{E}_t^Q[P_{t+j}]$
  - $f_{t,t+j}^{sp500}$ time t log S&P 500 futures price (CME E-mini) for maturity $t+j$
- Risk-neutral cash flow expectations:

$$\mathbb{E}_t^Q[d_{t,t+h}] = dl_t + \sum_{j=1}^{h} \rho^{j-1}(f_{t,t+j}^{div} - f_{t,t+j-1}^{div}), \qquad f_{t,t}^{div} \equiv d_t$$

  - Measure $\mathbb{E}_t^Q[\Delta d_{t+j}] = f_{t,t+j}^{div} - f_{t,t+j-1}^{div}$ assuming $f_{t,t+j}^{div} \approx \log \mathbb{E}_t^Q[D_{t+j}]$
  - $f_{t,t+j}^{div}$ time t log S&P 500 dividend futures price (Bloomberg) for maturity $t+j$
- Risk-neutral $\mathbb{E}_t^Q[pd_{t+h}]$: Use Campbell and Shiller 1988 identity

Risk-Neutral Measure: Long-horizon Imputation ▸ Return

- Futures price growth for $j > 1$ years ahead: Use estimates from AR(1) model

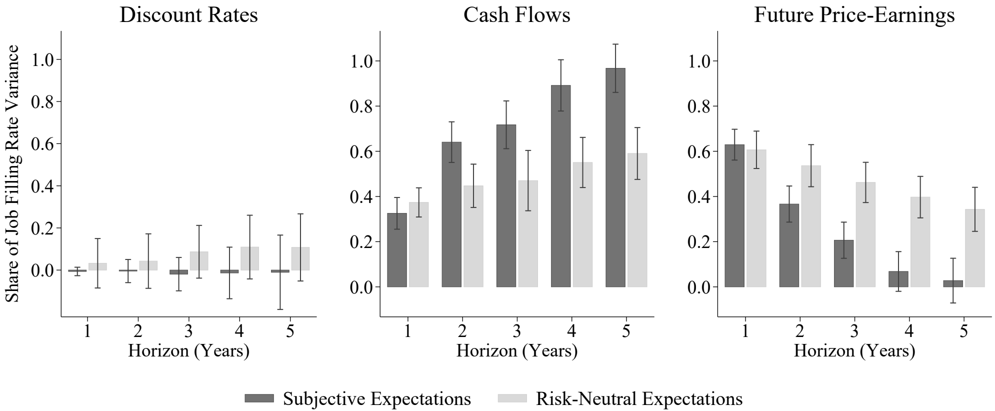
$$f^{sp500}_{t,t+1} - p_t = \mu_{sp500} + \rho_{sp500}(p_t - p_{t-1}) + \varepsilon_{sp500,t}$$
$$f^{div}_{t,t+1} - d_t = \mu_{div} + \rho_{div}(d_t - d_{t-1}) + \varepsilon_{div,t}$$

- Implies predicted values

$$f^{sp500}_{t,t+j} - f^{sp500}_{t,t+j-1} = \frac{\mu_{sp500}(1 - \rho^{j-1}_{sp500})}{1 - \rho_{sp500}} + \rho^{j-1}_{sp500}(f^{sp500}_{t,t+1} - p_t)$$
$$f^{div}_{t,t+j} - f^{div}_{t,t+j-1} = \frac{\mu_{div}(1 - \rho^{j-1}_{div})}{1 - \rho_{div}} + \rho^{j-1}_{div}(f^{div}_{t,t+1} - d_t)$$

Risk-Neutral Measure: Decomposition of Vacancy Filling Rate ▸ Return

- Subjective beliefs more sensitive to cash flow news than risk-neutral beliefs



Notes: Light bars show the contribution under risk-neutral expectations implied by S&P 500 and dividend futures. Dark bars show the contribution under subjective expectations. The Sample: 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

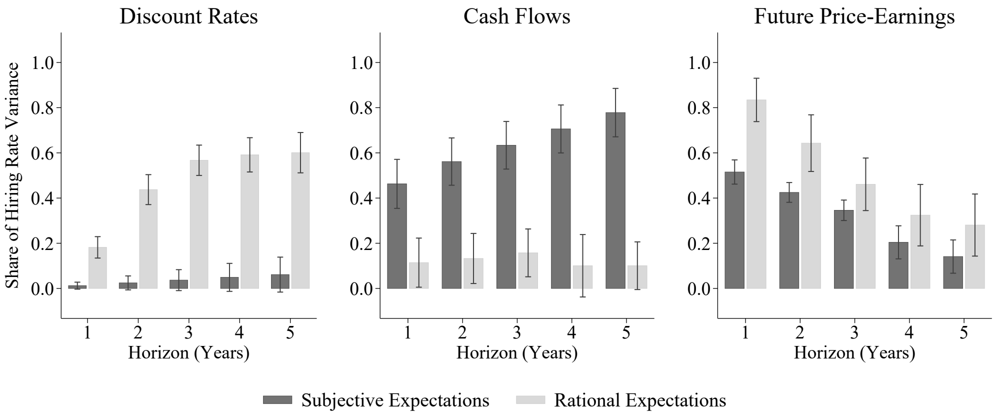
Alternative Measures of Subjective Cash Flow Expectations ▸ Return

- Results are robust to using alternative surveys for earnings growth forecasts:
  - IBES, Bloomberg (BBG), CFO survey
  - Kalman-filtered (KF) composite of IBES, BBG, and CFO

Horizon h (Years)	1	2	3	4	5
Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$					
(-) Cash Flow (KF)	0.578***	0.625***	0.684***	0.887***	0.933***
t-stat	(3.046)	(4.275)	(4.894)	(6.019)	(7.612)
(-) Cash Flow (BBG)	0.586***	0.830***	0.851***	0.896***	0.949***
t-stat	(8.476)	(8.317)	(7.213)	(5.288)	(4.541)
(-) Cash Flow (CFO)	0.637*				
t-stat	(1.934)				

Notes: KF summarizes the alternative survey measures into a single series using a Kalman filter. The sample for BBG and KF is quarterly from 2006Q1 to 2023Q4. The sample for CFO is quarterly from 2005Q1 to 2019Q3. Newey-West t-statistics with lags = 4 in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

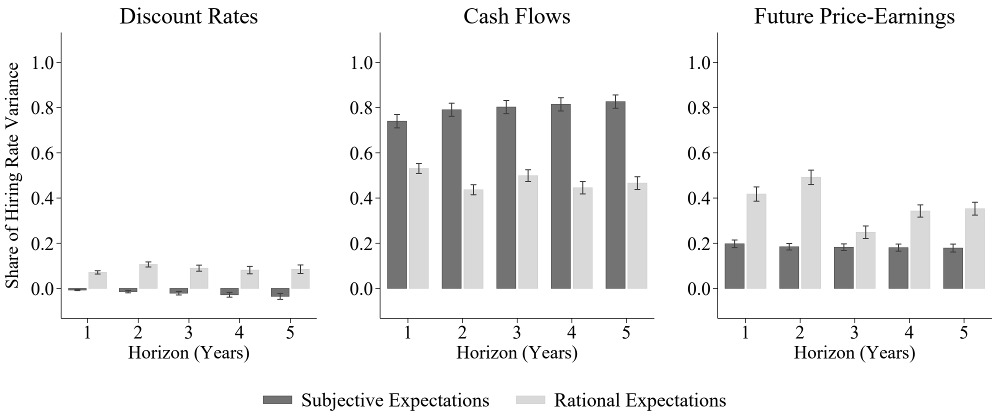
- Cash flow belief distortion $\mathbb{F}_t - \mathbb{E}_t$ accounts for most of hiring variation
  - Implies distortions can operate at firm level where actual hiring decisions are made



Notes: Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light (dark) bars show the contribution under objective (subjective) expectations. Sample: 2005Q1 to 2023Q4. Each bar shows two-way clustered 95% confidence intervals by portfolio and time.

Cross-Sectional Decomposition of Hiring Rate: Sort by Idiosyncratic Shock ▸ Return

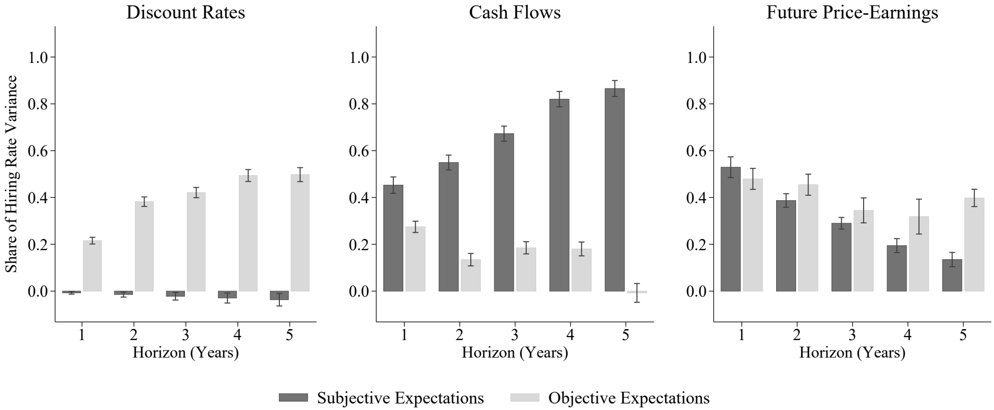
- Idiosyncratic shock: Earnings AR(1) residual (firm & time fixed effects)



Notes: Firms have been sorted into 10 value-weighted portfolios by idiosyncratic shock. Light (dark) bars show the contribution under objective (subjective) expectations. Sample: 2005Q1 to 2023Q4. Each bar shows two-way clustered 95% confidence intervals by portfolio and time.

Cross-Sectional Decomposition of Hiring Rate: Industry Portfolio ▸ Return

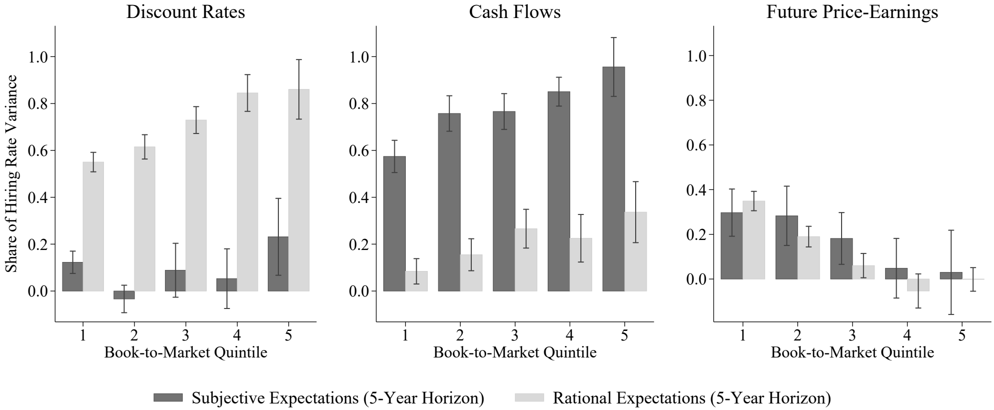
- Subjective beliefs over-weight cash flows



Notes: Firms have been sorted into Fama-French 49 industry portfolios. Light bars show the contribution under objective expectations. Dark bars show the contribution under subjective expectations. The Sample: 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Time-Series Decomposition of Hiring Rate: By Book-to-Market Portfolio ▸ Return

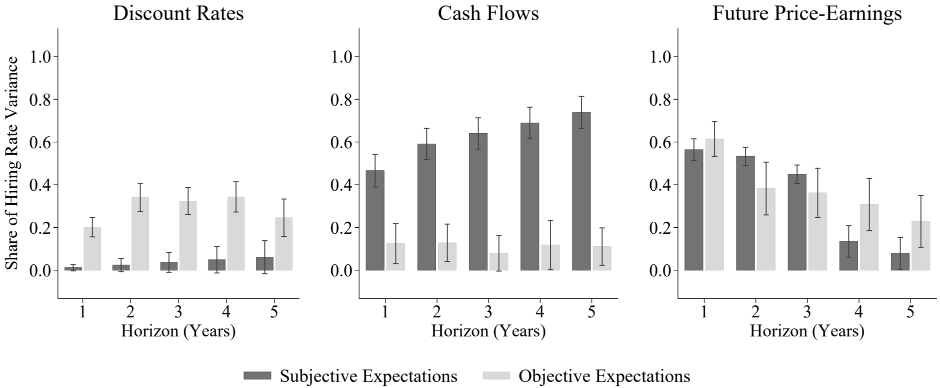
- Subjective beliefs over-weight cash flows across all portfolios
- Terminal value (future price-earnings) more important for low B/M (growth)



Notes: Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under objective expectations. Dark bars show the contribution under subjective expectations. The Sample: 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Role of Financial Constraints: Subjective Cash Flows Remain Dominant ▸ Return

- Financial constraint proxies:
  - Size, payout ratio, SA index (size and age), expected free cash flow (size, leverage, profitability, growth), Whited-Wu index (leverage, payout, size, growth)

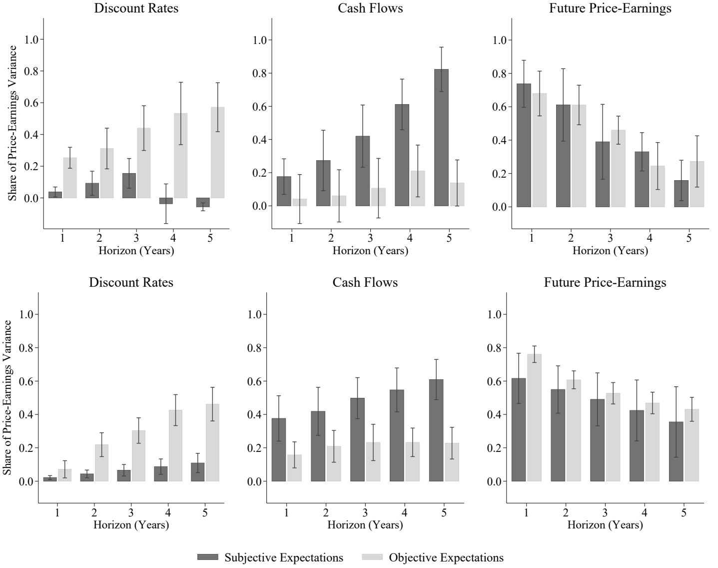


Notes: Figure estimates cross-sectional decomposition of hiring rate, controlling for measures of financial constraints. Financial constraint controls include firm size, payout ratio, SA index, expected free cash flow, and the Whited-Wu index. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows 95% confidence intervals clustered by portfolio and time.

Variance Decomposition of the Price-Earnings Ratio ▸ Return

(a) Time-Series Decomposition (Price-Earnings Ratio)

(b) Cross-Sectional Decomposition (Price-Earnings Ratio)



Predictability of Unemployment and Hiring

▸ Return

- Distortion in cash flow expectation strongest predictor, raises Adj. R^2 and OOS R^2

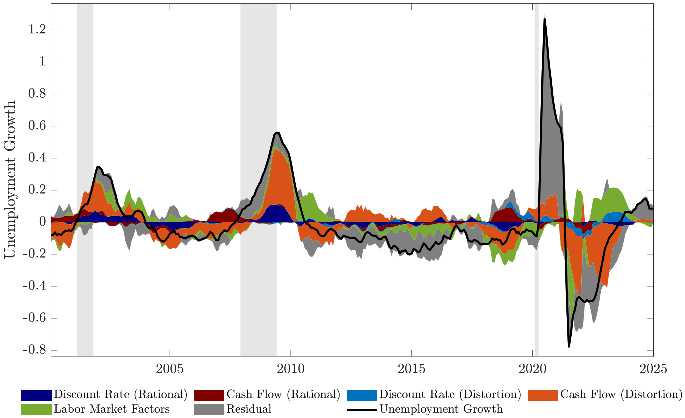
Forecast Target: Unemployment Growth Δu_{t+1}	(1)	(2)
$\mathbb{E}_t[r_{t,t+h}]$	0.551^{***}	0.236
t-stat	(5.046)	(0.893)
$\mathbb{E}_t[e_{t,t+h}]$	-0.041	-0.018
t-stat	(-0.108)	(-0.050)
$\mathbb{F}_t[r_{t,t+h}] - \mathbb{E}_t[r_{t,t+h}]$		-0.006
t-stat		(-0.033)
$\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]$		-0.701^{***}
t-stat		(-5.584)
Adj. R^2	0.528	0.745
OOS R^2	0.149	0.254

Forecast Target: Employment Growth $\Delta \widetilde{l}_{i,t+1}$	(3)	(4)
$\mathbb{E}_t[\widetilde{r}_{i,t,t+h}]$	-0.498^{***}	-0.119
t-stat	(-3.058)	(-0.734)
$\mathbb{E}_t[\widetilde{e}_{i,t,t+h}]$	0.154	0.053
t-stat	(1.304)	(0.754)
$\mathbb{F}_t[\widetilde{r}_{i,t,t+h}] - \mathbb{E}_t[\widetilde{r}_{i,t,t+h}]$		-0.043
t-stat		(-0.410)
$\mathbb{F}_t[\widetilde{e}_{i,t,t+h}] - \mathbb{E}_t[\widetilde{e}_{i,t,t+h}]$		0.759^{***}
t-stat		(6.412)
Adj. R^2	0.135	0.253
OOS R^2	0.207	0.447

Notes: The sample is quarterly from 2005Q1 to 2023Q4. OOS R^2 is defined as $1 - MSE_{\text{Model}}/MSE_{\text{Benchmark}}$. Out-of-sample forecasts are constructed as 1-year-ahead predictions using model parameters estimated over a rolling 10-year window. $MSE_{\text{Model}}/MSE_{\text{Benchmark}}$ denotes the ratio of each model's out-of-sample mean squared forecast error to that of a benchmark, which is the Survey of Professional Forecasters (SPF) consensus for time-series predictions and an AR(1) model for cross-sectional predictions. Newey-West corrected (time-series) and two-way clustering by portfolio and quarter (cross-sectional) t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

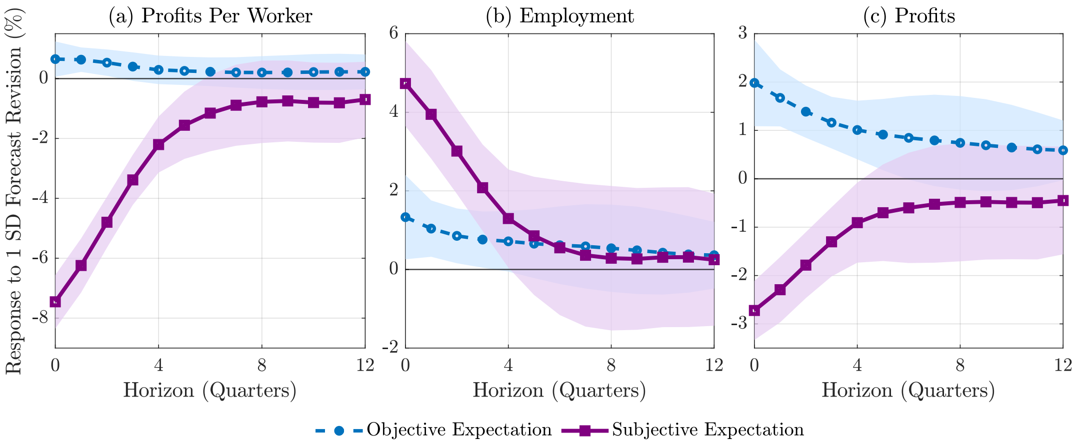
Time-Series Decomposition of the U.S. Unemployment Rate ▸ Return

- Unemployment tracks the cash flow distortion component closely



Notes: Figure plots decompositions of log annual growth in the unemployment rate, using objective expectations $\mathbb{E}_t$ and belief distortions $\mathbb{F}_t - \mathbb{E}_t$ of expected cash flows and discount rates. Labor market factors include the log annual growth of lagged unemployment Δu_t, labor market tightness $\Delta\theta_t$ and job separations $\Delta\delta_t$. Residual (dark gray) represents the variation in vacancy filling rates that are not captured by the other components. NBER recessions are shown with light gray shaded bars.

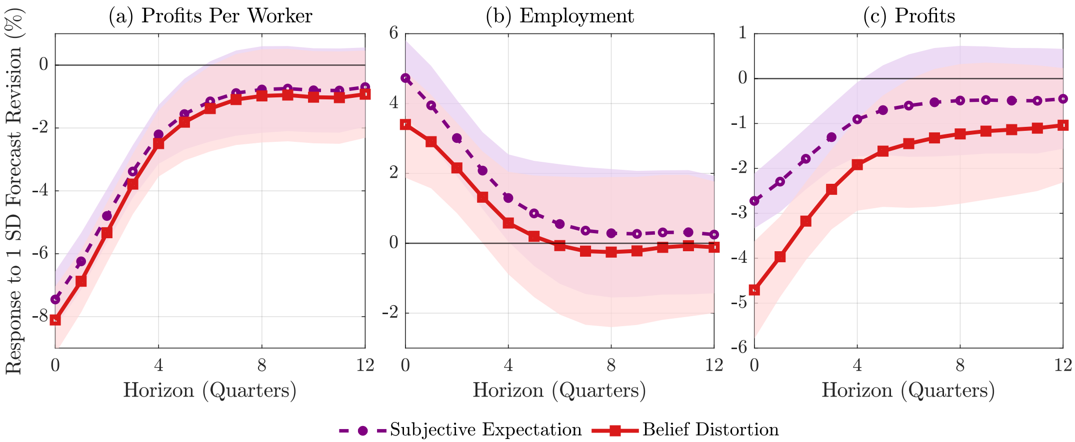
Employment Rise, Profits Fall Under Subjective Beliefs ▸ Return



Notes: Blue (violet) line: IRF from revisions in objective (subjective) expectation. Shaded area: 90% confidence intervals two-way clustered by firm and time. Sample: 1984Q1-2023Q4.

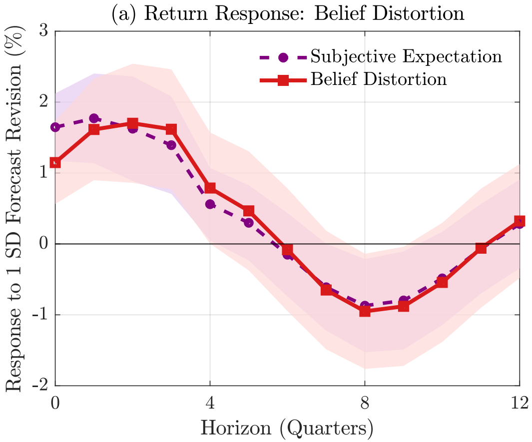
Subjective Belief Response Driven by Belief Distortion: Hiring

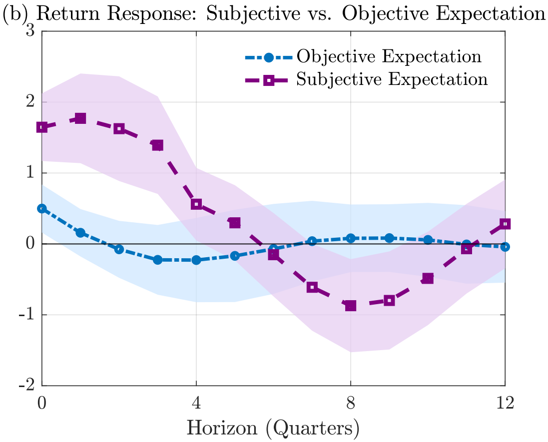
▸ Return



Notes: Red (violet) line: IRF from revisions in belief distortion (subjective expectation). Shaded area: 90% confidence intervals two-way clustered by firm and time. Sample: 1984Q1-2023Q4.

Subjective Belief Response Driven by Belief Distortion: Returns ▸ Return





Notes: Red (violet) line: IRF from revisions in belief distortion (subjective expectation). Shaded area: 90% confidence intervals two-way clustered by firm and time. Sample: 1984Q1-2023Q4.

Gradual Adjustment of Expectations: Constant-Gain Learning ▸ Return

- Earnings surprises only lead to small revisions in subjective cash flow expectation
  - Consistent with constant gain learning (Nagel Xu 2021, De La O, Han, Myers 2024)

Regression: $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j}$				
Target Horizon h (Years)	5	5	5	5
Revision Horizon j (Years)	1	2	3	4
(a) Earnings Growth	0.0929	0.0934	0.1121	0.1245
	(0.0734)	(0.0455)	(0.0776)	(0.0743)
(b) Earnings to Employment	0.0600	0.0508	0.0697	0.0745
	(0.1281)	(0.0725)	(0.0321)	(0.0419)

Notes: Table shows the gradual adjustment of expectations about future earnings $\widetilde{x}_{i,t+h}$ after an earnings surprise at $t+1$. The sample period is 1999 to 2023. Newey-West t-statistics with lags 12 in parentheses. *sig. at 10%. **sig. at 5%. ***sig. at 1%.

Model of Constant-Gain Learning: Aggregate Strip Price ▸ Return

- Aggregate strip price (h-period): Guess and verify recursive form

$$P_t^{(h)} = \mathbb{F}_t[M_{t+1} P_{t+1}^{(h-1)}] = \exp\{A^{(h)} + B^{(h)} \mathbb{F}_t[\mu] + \phi^h e_t\}$$

with coefficients:

$$A^{(h)} = A^{(h-1)} - r_f + \frac{1}{2} C^{(h)} [(1+\nu^2) C^{(h)} - 2\gamma] \sigma_u^2$$

$$B^{(h)} = B^{(h-1)} + \phi^{h-1}, \qquad C^{(h)} = \nu B^{(h-1)} + \phi^{h-1}, \qquad A^{(0)} = B^{(0)} = C^{(0)} = 0$$

- Realized return on strip:

$$R_{t+1}^{(h)} = \frac{P_{t+1}^{(h-1)}}{P_t^{(h)}} = \exp\{[A^{(h-1)} - A^{(h)}] + C^{(h)}(\mu - \mathbb{F}_t[\mu] + u_{t+1})\}$$

- Subjective expected return on strip:

$$\mathbb{F}_t\left[R_{t+1}^{(h)}\right] = \exp\left\{r_f + C^{(h)} \gamma \sigma_u^2\right\}$$

Model of Constant-Gain Learning: Aggregate Return ▸ Return

- Realized stock price: Sum of strip prices

$$P_t = \sum_{h=1}^{\infty} P_t^{(h)}$$

- Realized return: Weighted average of strip returns

$$R_{t+j} = \frac{\sum_{h=1}^{\infty} P_{t+j}^{(h-1)}}{\sum_{h=1}^{\infty} P_{t+j-1}^{(h)}} = \sum_{h=1}^{\infty} w_{t+j-1,h} R_{t+j}^{(h)}, \quad w_{t+j-1,h} = \frac{P_{t+j-1}^{(h)}}{\sum_{h=1}^{\infty} P_{t+j-1}^{(h)}}, \quad j \geq 1$$

- Subjective expected return: Assume agents use current weights $w_{t+j-1,h} \approx w_{t,h}$

$$\mathbb{F}_t[R_{t+j}] \approx \sum_{h=1}^{\infty} w_{t+j-1,h} \mathbb{F}_t[\mathbb{F}_{t+1}[\ldots \mathbb{F}_{t+j-1}[R_{t+j}^{(h)}]]] = \sum_{h=1}^{\infty} w_{t,h} \exp\left\{r_f + C^{(h)} \gamma \sigma_u^2\right\}$$

Model of Constant-Gain Learning: Firm-Level Strip Price ▸ Return

- Realized strip price: Apply independence of aggregate and idiosyncratic shocks:

$$\begin{aligned} P_{i,t}^{(h)} &= \mathbb{F}_t[M_{t+1} \dots \mathbb{F}_{t+h-1}[M_{t+h} E_{t+h} \tilde{E}_{i,t+h}]] \\ &= P_t^{(h)} \cdot \mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]] \end{aligned}$$

- Realized return on strip:

$$R_{i,t+1}^{(h)} = \frac{P_{i,t+1}^{(h-1)}}{P_{i,t}^{(h)}} = R_{t+1}^{(h)} \frac{\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]]}{\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]]}$$

- Subjective expected return on strip (where $\widetilde{C}^{(h)} \equiv \widetilde{\phi}^{h-1} + \nu \frac{1-\widetilde{\phi}^{h-1}}{1-\widetilde{\phi}}$):

$$\mathbb{F}_t[R_{i,t+1}^{(h)}] = \exp\left\{ r_f + C^{(h)} \gamma \sigma_u^2 + \frac{1}{2}((\widetilde{C}^{(h)})^2 - \widetilde{\phi}^{2(h-1)}) \sigma_v^2 \right\}$$

Model of Constant-Gain Learning: Firm-Level Return ▸ Return

- Realized stock price: Sum of strip prices

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)} = \sum_{h=1}^{\infty} \exp\left\{ A_i^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \widetilde{B}^{(h)}\mathbb{F}_t[\widetilde{\mu}_i] + \phi^h e_t + \widetilde{\phi}^h \widetilde{e}_{i,t} \right\}$$

- Realized return: Weighted average of strip returns

$$R_{i,t+1} = \frac{\sum_{h=1}^{\infty} P_{i,t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_{i,t}^{(h)}} = \sum_{h=1}^{\infty} w_{i,t,h} R_{i,t+1}^{(h)}, \qquad w_{i,t,h} = \frac{P_{i,t}^{(h)}}{\sum_{h=1}^{\infty} P_{i,t}^{(h)}}$$

- Subjective expected return: Assume agents use current weights $w_{i,t+j-1,h} \approx w_{i,t,h}$

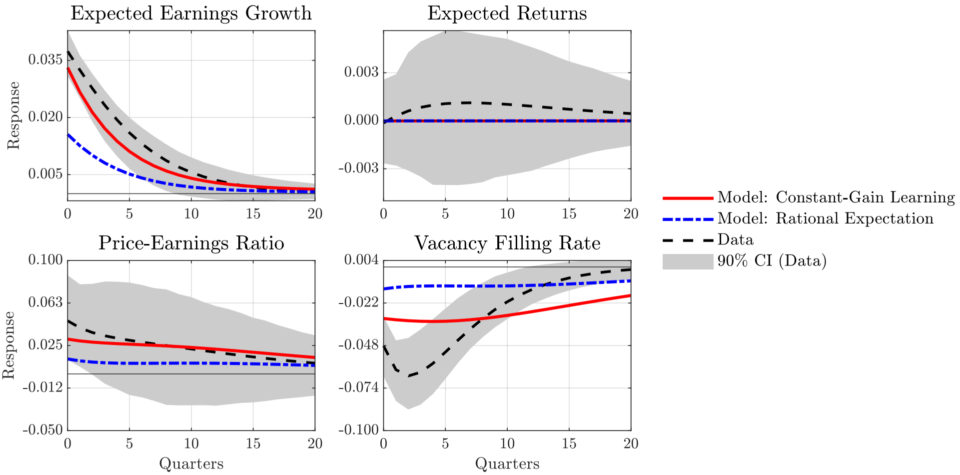
$$\begin{aligned}
\mathbb{F}_t[R_{i,t+j}] &\approx \sum_{h=1}^{\infty} w_{i,t+j-1,h} \mathbb{F}_t[\mathbb{F}_{t+1}[\ldots \mathbb{F}_{t+j-1}[R_{i,t+j}^{(h)}]]] \\
&= \sum_{h=1}^{\infty} w_{i,t,h} \exp\left\{ r_f + C^{(h)}\gamma\sigma_u^2 + \frac{1}{2}((\widetilde{C}^{(h)})^2 - \widetilde{\phi}^{2(h-1)})\sigma_v^2 \right\}
\end{aligned}$$

Model of Constant-Gain Learning: Labor Market ▸ Return

- Solve for q_t and $L_{i,t+1}$ by iterating on labor market tightness θ_t until convergence:
  1. Initialize labor market tightness: $\theta_t^{(0)} = 1$
  2. Construct vacancy filling rate using Cobb-Douglas matching: $q_t^{(s)} = B(\theta_t^{(s)})^{-\eta}$
  3. Update each firm's employment policy using the hiring equation:
  $$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa}$$
  4. Update each firm's vacancy posting using the employment accumulation equation:
  $$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}} (L_{i,t+1}^{(s)} - (1-\delta) L_{i,t})$$
  5. Aggregate firm-level variables over the set of firms I:
  $$V_t^{(s)} = \sum_{i \in I} V_{i,t}^{(s)}, \qquad L_{t+1}^{(s)} = \sum_{i \in I} L_{i,t+1}^{(s)}, \qquad U_t^{(s)} = 1 - \sum_{i \in I} L_{i,t}$$
  6. Update labor market tightness: $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$. Check convergence: $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$

Response to 1 Std. Dev. Shock to Cash Flow Growth Expectation ▸ Return

- VAR(1): $X_t = [\mathbb{F}_t[\Delta e_{t+1}], \mathbb{F}_t[r_{t+1}], pe_t, \log q_t]'$, Cholesky identification



Notes: Red solid line: model-based IRFs from simulated series under constant-gain learning. Blue solid line: model-based IRFs from simulated series under rational expectations. Black dashed line: data-based IRFs. Shaded area: 90% bootstrap confidence interval for the data VAR. Sample: 1984Q1-2023Q4.

Method of Simulated Moments (MSM): Estimation Setup ▸ Return

Estimated parameters: $\theta = (\nu, \phi, \sigma_u, r_f, \gamma)$

- ν: Constant gain in belief updating
- (ϕ, σ_u): Persistence and volatility of earnings process
- (r_f, γ): Risk-free rate and risk aversion

Key empirical moments to match:

- Earnings: Variance and autocorrelation of Δe_t
- Returns: Mean, equity premium, volatility of stock returns
- Valuations: Volatility and persistence of price-earnings ratio
- Learning: Coibion-Gorodnichenko regression slopes at multiple horizons

MSM criterion: $\hat{\theta}_N = \arg\min_\theta \, (S_N - S(\theta))' W_N^{-1} (S_N - S(\theta))$

CG Coefficient under Constant-Gain Learning ▸ Return

- Earnings follow AR(1): $e_t = \mu + \phi e_{t-1} + u_t$, $\Delta e_t = (\phi - 1)e_{t-1} + u_t$
- Belief updating: $\mathbb{F}_t[\mu] - \mathbb{F}_{t-1}[\mu] = \nu(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t])$
- Forecast error and revision:

$$\mathrm{FE}_{t,1} = u_t - \mathbb{F}_{t-1}[\mu], \quad \mathrm{Rev}_{t,h} = \phi^{h-1}\nu\,\mathrm{FE}_{t,1} + \phi^{h-1}(\phi - 1)\Delta e_t.$$

- Coibion-Gorodnichenko regression slope:

$$\beta^{CG}(h) = \phi^{h-1}\left[\nu + (\phi - 1)\frac{\mathrm{Cov}(\Delta e_t, \mathrm{FE}_{t,1})}{\mathrm{Var}(\mathrm{FE}_{t,1})}\right] = \phi^{h-1}\left[\nu + (\phi - 1)\,\tfrac{2-\nu}{2}\cdot\tfrac{1-\phi+\nu}{1-\phi+\phi\nu}\right]$$

- $\beta^{CG}(h)$ increases with ν; negative at low ν, positive at high ν

Method of Simulated Moments (MSM): Estimation Results ▸ Return

Moment or parameter	Data	Model	t statistic
Panel A: Moments			
Mean log stock return	0.072	0.088	-0.510
SD log stock return	0.160	0.118	0.568
Mean log risk free rate	0.046	0.045	0.144
Mean of log price earnings	2.980	2.392	0.424
SD of log price earnings	0.285	0.293	-0.084
AC of log price earnings	0.750	0.798	-0.457
SD of aggregate earnings growth	0.268	0.294	-0.455
AC of aggregate earnings growth	-0.144	-0.142	-0.045
CG slope h equals 4 aggregate	-0.263	-0.266	0.063
CG slope h equals 8 aggregate	-0.463	-0.454	-0.040
Panel B: Estimated Parameters			
Gain coefficient ν		0.013	
AR coefficient aggregate ϕ		0.854	
Aggregate shock standard deviation σ_u		0.271	
Risk free rate r_f		0.045	
Risk aversion γ		1.647	

Regional Model ▸ Return

- Aggregate results suggest belief distortions matter, but causality is unclear
- Use state-level variation to test whether distorted beliefs drive unemployment:

$$u_{s,t+1} = \beta_r \mathbb{F}_t[r_{s,t,t+h}] + \beta_e \mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1}$$

  - Subjective expectations: $\mathbb{F}_t[r_{s,t,t+h}]$ discount rate, $\mathbb{F}_t[e_{s,t,t+h}]$ cash flow
    - Aggregate across firms with headquarters in state s
  - $X_{s,t} = [u_{s,t}, \theta_{s,t}, \delta_{s,t}]'$ labor market factors, α_s state fixed effect, α_t time fixed effect
- Bartik shift-share instrument: Replace state-level forecast $\mathbb{F}_t[y_{s,t,t+h}]$ with

$$\widehat{\mathbb{F}}_t[y_{s,t,t+h}] = \sum_{i \in I} s_{s,i,t-1} \mathbb{F}_t[y_{i,t,t+h}], \qquad y = r, e$$

  - $s_{s,i,t-1}$: Industry i's (2-digit NAICS) employment share in state s
  - Instrument isolates variation in beliefs from national trends, not local conditions

Subjective Cash Flow Expectations Predict Regional Unemployment ▸ Return

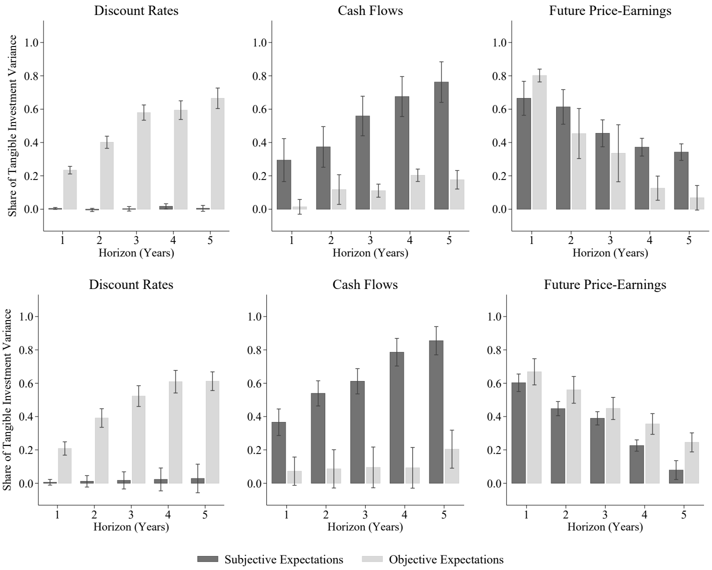
- Shift-share estimates confirm causal impact of belief distortions

	OLS			Shift-Share Instrument		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[r_{s,t,t+h}]$	0.725***		0.470	0.572***		0.207
	(0.235)		(0.780)	(0.222)		(0.240)
$\mathbb{E}_t[e_{s,t,t+h}]$	-0.247		-0.065	-0.064		0.005
	(0.499)		(0.182)	(0.075)		(0.168)
$\mathbb{F}_t[r_{s,t,t+h}]$		0.248	0.233		0.052	0.052
		(0.297)	(0.300)		(0.228)	(0.228)
$\mathbb{F}_t[e_{s,t,t+h}]$		-0.817***	-0.791***		-0.690***	-0.708***
		(0.236)	(0.242)		(0.160)	(0.200)
R^2	0.414	0.558	0.558	0.414	0.549	0.549
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Labor Market Factors	Yes	Yes	Yes	Yes	Yes	Yes

Variance Decomposition of the Tangible Investment Rate ▸ Return

(a) Time-Series Decomposition

(b) Cross-Sectional Decomposition



Variance Decomposition of the Intangible Investment Rate ▸ Return

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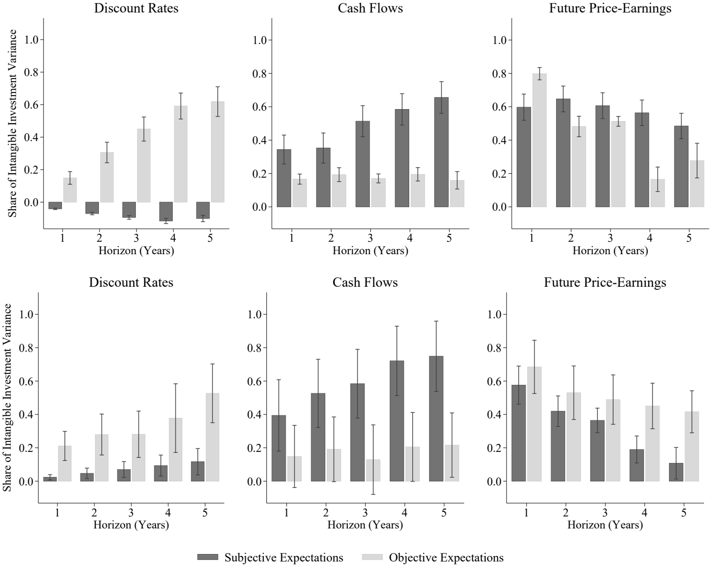


Extension with On-the-Job Search: Environment ▸ Return

- Measured earnings E_t reflect contribution from all new hires
  - Introduce model accounting for job-to-job transitions
  - Kuhn, Manovskii, and Qiu 2021; Faberman et al. 2022
- Assume fraction ϕ of employed workers search for a job each period
  - Total searchers: $S_t \equiv U_t + \phi L_t = U_t + \phi(1 - U_t)$
  - Matches formed under CRS matching function $\mathcal{M}(S_t, V_t)$
- Assume only fraction χ of on-the-job searchers accept offers
  - Hiring efficiency: $\varphi_t \equiv \frac{U_t + \chi\phi(1-U_t)}{U_t + \phi(1-U_t)}$
- Firm's value satisfies Bellman equation:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \{E_t + (1 - \phi\chi f_t)\mathbb{F}_t [M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})]\}$$

$$\text{s.t. } L_{t+1} = (1 - \delta_t)L_t + q_t\varphi_t V_t$$

  - Employee retention rate $1 - \phi\chi f_t$, efficiency-adjusted Vacancy Filling Rate $q_t\varphi_t$

Extension with On-the-Job Search: Model ▸ Return

- First-order condition under constant returns to scale:

$$\frac{\kappa}{q_t \varphi_t} = (1 - \phi\chi f_t)\frac{P_t}{L_{t+1}}$$

  - Ex-dividend firm value: $P_t \equiv \mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})\right]$

- Take logs, combine with Campbell and Shiller 1988 identity for price-earnings:

$$\log q_t = c_q - \underbrace{\log(1 - \phi\chi f_t)}_{\substack{\text{Job-to-Job}\\ \text{Transitions}}} + \underbrace{\mathbb{F}_t[r_{t,t+h}]}_{\substack{\text{Discount}\\ \text{Rate}}} - \underbrace{\mathbb{F}_t[e_{t,t+h}]}_{\substack{\text{Cash}\\ \text{Flow}}} - \underbrace{\mathbb{F}_t[pe_{t,t+h}]}_{\substack{\text{Future}\\ \text{Price-Earnings}}}$$

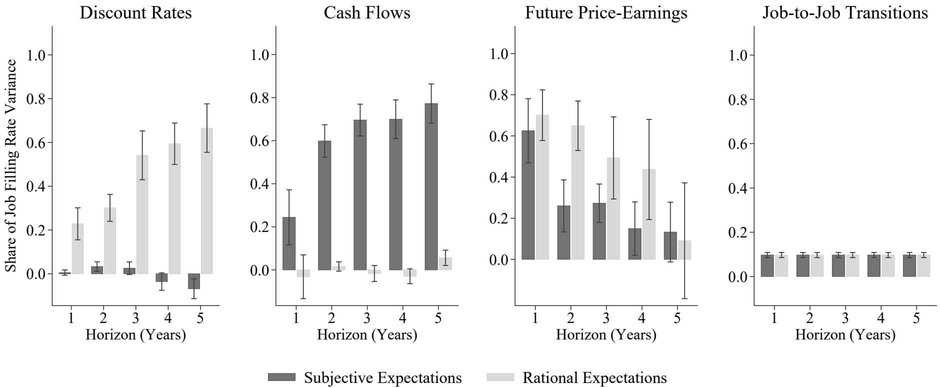
  - Constant $c_q = \log\kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho} - \log\varphi_t$ with $\log\varphi_t \approx \log\varphi$
  - Estimate by regressing each component on $\log q_t$

Extension with On-the-Job Search: Data ▸ Return

- Parameters based on Kuhn, Manovskii, and Qiu 2021; Faberman et al. 2022
  - Fraction of on-the-job searchers $\phi = 0.12$
  - Fraction of on-the-job searchers accepting offered job $\chi = 0.75$
- Subjective expectations: IBES earnings forecasts
  - Analysts forecast total earnings, which pools contribution from all new hires
- Vacancy Filling Rate: Hires over vacancies $q_t = \frac{H_t}{V_t}$
  - JOLTS hires H_t (includes UE and J2J) / JOLTS job openings V_t
- Labor market tightness: Vacancies over job searchers $\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_t + \phi(1 - U_t)}$
  - JOLTS job openings V_t, BLS unemployment level U_t
- Job finding rate: Infer from CRS matching function $f_t = \frac{\mathcal{M}(S_t, V_t)}{S_t} = q_t \theta_t$

Extension with On-the-Job Search: Results ▸ Return

- Cash flow belief distortion $\mathbb{F}_t - \mathbb{E}_t$ accounts for most of hiring variation
- Job-to-job transitions $\log(1 - \phi\chi f_t)$ account for 8.9% of $\log q_t$ variation



Notes: Light bars show the contribution under objective expectations. Dark bars show the contribution under subjective expectations. The Sample: 2005Q1 to 2024Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Model of Constant-Gain Learning: Decreasing Returns to Scale ▸ Return

- Assume decreasing returns to scale production: $Y_{i,t} = A_{i,t} L_{i,t}^{\alpha}$, $0 < \alpha < 1$
- Earnings $E_{i,t}$ defined as profits $\Pi_{i,t}$ after vacancy posting cost:

$$E_{i,t} = \Pi_{i,t} - \kappa V_{i,t}, \qquad \Pi_{i,t} = Y_{i,t} - W_{i,t} L_{i,t}$$

$A_{i,t}$ productivity, $L_{i,t}$ employment, $W_{i,t}$ wages, $\kappa V_{i,t}$ vacancy posting cost

- After recursive substitution, first-order condition implies:

$$\frac{\kappa}{q_t} = \sum_{j=1}^{\infty} \mathbb{F}_t \left[\frac{1}{R_{i,t,t+j}} \frac{(\pi_{i,t+j} L_{i,t+j} - \kappa V_{i,t+j})}{L_{i,t+1}} \right]$$

$\pi_{i,t+j} = \frac{\partial \Pi_{i,t+j}}{\partial L_{i,t+j}}$ marginal profit, $\frac{1}{R_{i,t,t+j}} = \prod_{k=1}^{j} \frac{1}{R_{i,t+j}}$ cumulative discount rate

Model of Constant-Gain Learning: Decreasing Returns to Scale ▸ Return

- DRS introduces wedge between marginal vs. average profit: $(1-\alpha)Y_{i,t}$

$$\pi_{i,t}L_{i,t} - \kappa V_{i,t} = \alpha A_{i,t}L_{i,t}^{\alpha} - W_{i,t}L_{i,t} - \kappa V_{i,t} = E_{i,t} - (1-\alpha)Y_{i,t}$$

- Substitute DRS wedge into the hiring equation
- Aggregate into averages weighted by employment share $S_{i,t+1} \equiv \frac{L_{i,t+1}}{\sum_{i\in I} L_{i,t+1}}$

$$\frac{\kappa}{q_t} = \sum_{i\in I}\sum_{j=1}^{\infty}\mathbb{F}_t\left[\frac{1}{R_{i,t,t+j}}\left(\underbrace{\frac{E_{i,t+j}}{L_{i,t+1}}}_{EL_{i,t+j}} - (1-\alpha)\underbrace{\frac{Y_{i,t+j}}{L_{i,t+1}}}_{YL_{i,t+j}}\right)\right]\underbrace{\frac{L_{i,t+1}}{L_{t+1}}}_{S_{i,t+1}}$$

- Log linearize around steady state

$$\log q_t = \sum_{i\in I}\sum_{j=1}^{\infty}\left[\mathbb{F}_t\left[\rho_{i,j}^{r} r_{i,t,t+j}\right] - \mathbb{F}_t\left[\rho_{i,j}^{el} el_{i,t+j}\right] + (1-\alpha)\mathbb{F}_t\left[\rho_{i,j}^{yl} yl_{i,t+j}\right] - \rho_{i,j}^{s} s_{i,t+1}\right]$$

  - $\rho_{i,j}^{el} = \frac{\bar{q}}{\kappa}\cdot\frac{\overline{EL}_i\cdot\bar{S}_i}{\bar{R}^j}$, $\rho_{i,j}^{yl} = \frac{\bar{q}}{\kappa}\cdot\frac{\overline{YL}_i\cdot\bar{S}_i}{\bar{R}^j}$, $\rho_{i,j}^{s} = \rho_{i,j}^{r} = \frac{\bar{q}}{\kappa}\cdot\frac{(\overline{EL}_i+(1-\alpha)\overline{YL}_i)\cdot\bar{S}_i}{\bar{R}^j}$
  - $s_{i,t+1}$ captures shifts in firms size distribution (composition effect)

Model of Constant-Gain Learning: Decreasing Returns to Scale ▸ Return

- Time-series decomposition of aggregate Vacancy Filling Rate:

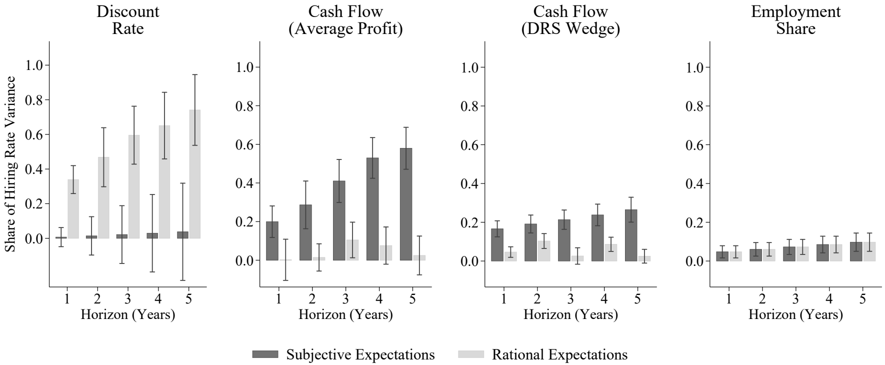
$$\log q_t = \sum_{j=1}^{\infty} \left[\underbrace{\mathbb{F}_t\left[r_{t,t+j}\right]}_{\text{Discount Rate}} - \underbrace{\mathbb{F}_t\left[el_{t+j}\right]}_{\substack{\text{Cash Flow}\\ \text{(Earnings)}}} + \underbrace{(1-\alpha)\mathbb{F}_t\left[yl_{t+j}\right]}_{\substack{\text{Cash Flow}\\ \text{(DRS Wedge)}}} - \underbrace{s_{t+1}}_{\substack{\text{Composition}\\ \text{Effect}}} \right]$$

  - $x_t = \sum_{i \in I} \rho^x_{i,j} x_{i,t}$ aggregates firm-level variable $x_{i,t}$

- Data and measurement
  - Sort firms into 5 value-weighted portfolios by employee count
  - Measure expected output $\mathbb{F}_t[yl_{i,t+j}]$ using IBES sales forecasts
  - $\alpha = 0.72$ labor share, $\kappa = 0.133$ flow vacancy cost (Elsby and Michaels 2013)
  - $\overline{q}$, $\overline{R}$, $\overline{EL}_i$, $\overline{YL}_i$, $\overline{S}_i$ long-run sample averages
  - Approximate the infinite sum by truncating up to $h \leq 5$ years

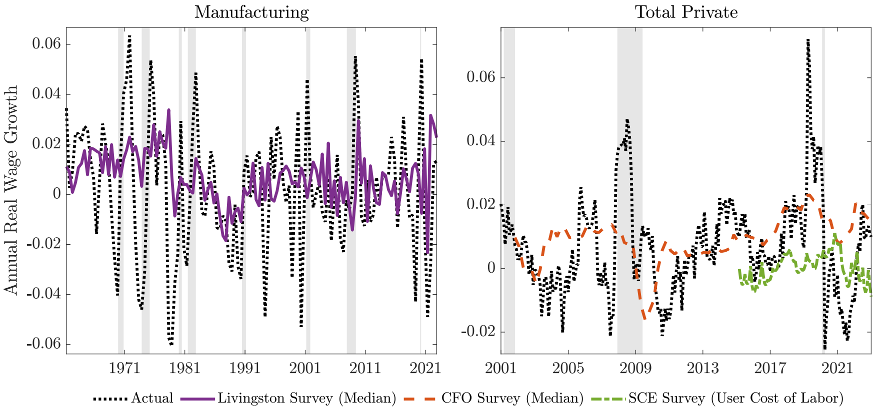
Time-Series Decomposition of the Vacancy Filling Rate: DRS ▸ Return

- Cash flow belief distortion $\mathbb{F}_t - \mathbb{E}_t$ accounts for most of hiring variation
  - DRS wedge (output-employment) about 1/3 of total weight on cash flow
  - Composition effect (employment share) accounts for less than 10%



Notes: The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Real Wage Growth: Actual vs. Subjective Expectations



Notes: This figure plots ex-post realized outcomes (Actual) and 1-year ahead subjective expectations (Survey) of real wage growth. x axis denotes the date on which actual values were realized and the period on which the survey forecast is made, making the vertical distance between the actual and survey lines the forecast error. Actual values are deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2023Q4, SCE is monthly spanning 2015M5 to 2022M12. NBER recessions are shown with gray shaded bars.