Belief Distortions and Unemployment Fluctuations*

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Abstract

This paper studies unemployment fluctuations when expectations deviate from a rational benchmark. Using survey forecasts, I decompose variation in the aggregate job filling rate, a key determinant of unemployment, into news about future discount rates and expected cash flows. Under subjective beliefs, the job filling rate is driven by predictable errors in expected cash flows, while discount rates play a limited role. In contrast, rational expectations assign a dominant role to discount rates. A cross-sectional decomposition shows that dispersion in hiring rates across firms can be largely explained by distorted beliefs about future cash flows. A search and matching model with constant-gain learning about long-run earnings growth can reproduce these findings.

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1 Introduction

Unemployment surges during recessions, often more sharply than what standard models predict. This disconnect lies at the heart of the "unemployment volatility puzzle" identified by Shimer (2005). Canonical search-and-matching models, even when calibrated to match observed separation rates and vacancy posting behavior, generate far less volatility in unemployment than observed in the data. To reconcile this gap, recent approaches emphasize the link between hiring decisions and fluctuations in rational risk premia inferred from financial markets (Hall, 2017; Kehoe et al., 2022). However, these models rely on the rational expectations assumption, which implies that firms correctly perceive the relative importance of discount rates and expected cash flows when posting job vacancies.

This paper offers an alternative explanation: distorted beliefs about future cash flows are a key source of fluctuations in labor markets. The main idea is that firms make hiring decisions based on their subjective expectations about the discounted value of a newly hired worker. If deviations from rational expectations lead firms to overreact to news about cash flows, the distortion could provide a source of fluctuation to the expected value of hiring. For example, overly pessimistic beliefs about future cash flows during downturns can lead firms to cut back on hiring more sharply than warranted—driving deeper and more persistent unemployment spikes—even if perceived discount rates remain unchanged.

I quantify the importance of these distorted beliefs using a two-step strategy. First, I develop a time-series decomposition of the aggregate job filling rate, which is a key determinant of aggregate fluctuations in unemployment (Shimer, 2012). The decomposition attributes fluctuations in the job filling rate to either expectations of future cash flows or discount rates from the aggregate stock market. The variance decomposition I derive from the search model mirrors Campbell-Shiller decompositions of price-dividend or price-earnings ratios in asset pricing, providing a common framework to analyze how subjective expectations distort both financial and real decisions. Second, I extend this framework to the cross-section, decomposing variation in firm-level hiring using measures of firms' subjective expectations for future discount rates and cash flows. I estimate the decompositions using survey-based measures of subjective expectations and compare them to machine learning-based proxies for rational expectations.

I use out-of-sample forecasts from a Long Short-Term Memory (LSTM) neural network to proxy for rational expectations. Rational expectations require agents to form beliefs by efficiently processing all available information. A high-dimensional neural network trained on a rich set of macroeconomic and financial variables can approximate this benchmark by learning complex nonlinear relationships without imposing strong parametric assumptions about the underlying data-generating process. To avoid look-ahead bias, the machine forecasts are constructed in real time using only information available at each date.

The results reveal a clear divergence between subjective and rational expectations. In the time series, subjective cash flow expectations account for the majority of variation in hiring, while subjective discount rates play only a limited role. In the cross-section, firms with greater exposure to fluctuations in subjective cash flow expectations exhibit significantly larger fluctuations in hiring. Under rational expectations, the pattern reverses: discount rates dominate both time-series and cross-sectional variation. This reversal mirrors recent asset pricing findings where subjective expectations flip the classical Campbell-Shiller decomposition, with cash flow news rather than discount rate variation explaining most stock price volatility (De La O and Myers, 2021; Nagel and Xu, 2021; Bordalo et al., 2024a; De La O et al., 2022). My paper extends these insights from asset pricing to real decisions, showing that the same belief distortions shape both financial markets and labor market outcomes.

These belief distortions can have important implications for aggregate unemployment fluctuations. Incorporating subjective cash flow expectations into predictive models of the unemployment rate significantly improves their explanatory power compared to models based solely on rational discount rates. Subjective beliefs can better explain the sharp and persistent spikes in unemployment during downturns that rational models often miss. Pessimistic beliefs depress labor demand in recessions, while excessive optimism can drive unsustainable hiring in booms. This finding suggests that misperceptions about future cash flows can substantially influence hiring behavior and drive fluctuations in unemployment over the business cycle.

To interpret these findings, I develop a structural model in which firms engage in constant-gain learning about the long-run mean of earnings growth. The model embeds subjective beliefs into the firm's problem in a search-and-matching model of the labor market. Firms update their earnings expectations using a constant-gain filter and choose hiring optimally based on their beliefs. Simulations from the model reproduce the empirical decomposition results: under subjective expectations, firms over-attribute hiring fluctuations to persistent shifts in expected earnings, and understate the role of discount rates. The model also captures the cross-sectional pattern in which belief-driven dispersion in cash flow expectations drives firm-level hiring.

Taken together, the results show that labor market fluctuations reflect not only rational changes in discount rates, but also systematic distortions in belief formation. Rational models that emphasize time-varying discount rates may understate the role of subjective cash flow expectations in driving labor market volatility. These findings highlight the need for macroeconomic models to incorporate subjective beliefs more explicitly, and for policy tools to address the behavioral biases they generate.

Related Literature This paper contributes to several strands of literature on unemployment fluctuations, labor markets, asset prices, and expectation formation.

First, it relates to the literature on the unemployment volatility puzzle in search-and-matching models. A central challenge in macroeconomics is to explain why unemployment is highly volatile relative to productivity (Shimer, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Pissarides, 2009; Elsby and Michaels, 2013; Kudlyak, 2014; Chodorow-Reich and Karabarbounis, 2016; Ljungqvist and Sargent, 2017). Traditional search and matching models struggle to generate sufficient volatility in unemployment unless firms' responses to shocks are amplified through mechanisms such as rational expectations of time-varying discount rates (Merz and Yashiv, 2007; Donangelo, 2014; Belo et al., 2014; Favilukis and Lin, 2015; Hall, 2017; Borovickova and Borovička, 2017; Kuehn et al., 2017; Kilic and Wachter, 2018; Mitra and Xu, 2019; Donangelo et al., 2019; Kehoe et al., 2019; Liu, 2021; Kehoe et al., 2022; Belo et al., 2023; Meeuwis et al., 2023). These models assume that firms rationally process information about cash flows and discount rates. My approach of using survey forecasts complements these rational models by introducing subjective expectations. Belief distortions—particularly in cash flows—better explain variation in job filling rates, offering an alternative resolution to the unemployment volatility puzzle.

A growing literature embeds non-rational expectations in macro models with labor market frictions (Acharya and Wee, 2020; Mueller et al., 2021; Menzio, 2023; Faberman et al., 2022; Bhandari et al., 2024). Notably, Bhandari et al. (2024) show that systematic pessimism in households and firms can explain the volatility of unemployment fluctuations. My paper complements their findings by providing direct survey evidence on the content and cyclicality of firm expectations, showing that firms over-react to news about earnings expectations while under-reacting to discount rates and labor costs. Cross-sectional analysis in this paper adds another dimension to belief-driven volatility by showing that firms with more distorted beliefs experience larger swings in hiring.

The empirical analysis of this paper builds on existing survey-based evidence on the empirical properties of firm expectations. Ben-David et al. (2013) document persistent over-optimism in CFO forecasts. Gennaioli et al. (2016) document that extrapolative CFO expectations of earnings growth predict corporate investment. Ma et al. (2020) link systematic biases in managerial forecasts to distortions in firm investment. Coibion et al. (2018) and Candia et al. (2020) find that firm managers' inflation expectations adjust slowly and display substantial dispersion. My paper builds on this work by showing how distortions in survey expectations shape labor markets.

The variance decomposition in this paper builds on recent work using survey-based expectations to reassess the drivers of asset prices (Greenwood and Shleifer, 2014; Giglio et al., 2021; De La O and Myers, 2021; Nagel and Xu, 2022; De La O et al., 2022; Adam and Nagel, 2023; Bordalo et al., 2024a; Décaire and Graham, 2024). The variance decomposition method adapts the Campbell-Shiller framework (Campbell and Shiller, 1988; Cochrane, 2007), which attributes price-dividend and price-earnings ratio variation to expected cash flows and discount rates. Recent applications of this framework using survey-based expectations have challenged traditional views about the sources of asset price volatility. De La O and Myers (2021) show that subjective expectations of cash flow growth, rather than discount rates, explain most of the variation in price-dividend and price-earnings ratios—challenging standard decompositions that assume rational expectations. Bordalo et al. (2024a) find that over-reaction in long-term earnings growth expectations—not time-varying discount rates—accounts for a substantial share of aggregate and cross-sectional return predictability. My contribution is to show that the Campbell-Shiller methodology can be adapted to study real decisions by linking asset valuation to hiring through the firm's optimality condition, revealing that the same belief distortions operate in both financial markets and labor markets.

Informed by this literature, I adopt a machine learning approach to measure rational expectations using a dynamic real-time forecasting framework developed in Bianchi et al. (2022) and Bianchi et al. (2024b). It is based on the principle that rational expectations require agents to efficiently use the full set of real-time information available to them. The algorithm uses high-dimensional prediction models estimated on rolling samples of real-time data to produce a benchmark that is free from human cognitive biases and look-ahead bias, while also addressing overfitting and structural change. The method uses tools from machine learning by training LSTM networks with recursive re-estimation and hyperparameter tuning (Gu et al., 2020, Cong et al., 2020, Bybee et al., 2024). The resulting forecasts are fully ex-ante and provide high-dimensional empirical counterparts to rational expectations for evaluating belief distortions.

The rest of the paper proceeds as follows. Section 2 presents a search and matching model with belief distortions and derives a decomposition of the job filling rate. Section 3 describes the data used in the empirical analysis. Section 4 compares the predictive performance of machine and survey forecasts. Section 5 presents the estimated variance decomposition of the aggregate job filling rate. Section 6 presents evidence that subjective cash flow expectations predict unemployment. Section 7 presents cross-sectional evidence motivated by a firm-level extension of the baseline model. Section 8 introduces a model of constant-gain learning about future earnings that could match the decompositions estimated from the data. Section 9 discusses model extensions and robustness checks. Finally, section 10 concludes.

2 Theoretical Framework

This section develops a search and matching model of the labor market in which firms' expectations about future cash flows and discount rates may be distorted, leading to fluctuations in job filling rates and unemployment. The model builds on the Diamond (1982), Mortensen (1982), and Pissarides (2009) framework but departs from the standard rational expectations assumption, allowing firms' hiring decisions to be influenced by biased subjective beliefs.

Environment Consider a discrete time economy populated by a representative household and a representative firm that hires workers in a frictional labor market. The firm uses labor as a single input to production. The household's population is normalized to one and has a continuum of members, where a fraction L_t are employed and the rest are unemployed $U_t = 1 - L_t$. The household's intertemporal consumption decision gives rise to a stochastic discount factor M_{t+1}.

Each period, the firm posts job vacancies at a cost $\kappa > 0$ to maximize its cum-dividend value of equity. I adopt a standard end-of-period matching convention (Petrosky-Nadeau et al., 2018).[1] At the beginning of period t, the stock of employment L_t reflects the total number of workers carried over from the previous period, before any separations or new hires occur during period t. During the period, a fraction δ_t of employed workers separate, while the firm posts vacancies V_t to search for unemployed workers U_t. Matches are formed at the end of period t according to a matching function $m(U_t, V_t)$, with job filling rate $q_t \equiv m(U_t, V_t)/V_t$ and job finding rate $f_t \equiv m(U_t, V_t)/U_t$. These new hires enter employment at the start of period $t+1$, so employment L_t evolves according to a law of motion:

$$L_{t+1} = (1 - \delta_t)L_t + q_t V_t \tag{1}$$

Unemployment $U_t = 1 - L_t$ evolves according to:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{2}$$

where $\theta_t = V_t/U_t$ denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

Firm Technology and Cash Flows The firm produces output using a Cobb-Douglas production function with labor L_t as it input. Period cash flows (earnings) E_t are given by:

$$E_t = A_t L_t^\alpha - W_t L_t - \kappa V_t \tag{3}$$

where $A_t L_t^\alpha$ is gross output with total factor productivity A_t and returns to scale α. $W_t L_t$ total wage payments, and κV_t vacancy posting costs. I assume that the household owns the equity of the firm and the firm pays out all of its earnings E_t as dividends (Petrosky-Nadeau et al., 2018), and that the firm's manager has access to complete markets so that the return to hiring equals the stock market return in equilibrium (Cochrane, 1991).

Firm's Problem The firm chooses vacancy postings V_t to maximize the present discounted value of future cash flows. The firm's value function $\mathcal{V}$ satisfies the Bellman equation:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \{E_t + \mathbb{F}_t[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})]\} \tag{4}$$

subject to the employment accumulation equation (1). M_{t+1} denotes the stochastic discount factor that prices the firm's cash flows. $\mathbb{F}_t[\cdot]$ denotes expectations conditional on information

[1] See Hansen et al. (2005) and Kogan and Papanikolaou (2012) for similar conventions applied for the q theory of investment.

available at the beginning of period t, formed under the firm's potentially distorted beliefs.[2] These beliefs may depart from rational expectations $\mathbb{E}_t[\cdot]$, with the nature and magnitude of the deviation disciplined using survey data (e.g., Bhandari et al. (2024)). Subjective distortions in beliefs can thus shift the perceived returns to hiring through $\mathbb{F}_t[\cdot]$ and affect equilibrium job filling rates, which in turn affects unemployment through its law of motion in equation (2).

Hiring Equation Under search frictions, hiring is forward-looking investment. The firm's optimal hiring decision equates the marginal cost of posting a vacancy with the expected discounted marginal value of employment:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \underbrace{\mathbb{F}_t\left[M_{t+1}\frac{\partial\mathcal{V}(A_{t+1},L_{t+1})}{\partial L_{t+1}}\right]}_{\text{Expected discounted value of hiring}} \tag{5}$$

The left side represents the expected cost of hiring one additional worker, accounting for the probability q_t that a posted vacancy will be filled. The right side captures the expected discounted value of the marginal worker, incorporating both the firm's subjective beliefs about future conditions and the appropriate discount rate for valuing risky cash flows.[3] Under constant returns to scale ($\alpha = 1$), the firm's marginal value of hiring coincides with its average value:

$$\frac{\partial\mathcal{V}(A_{t+1},L_{t+1})}{\partial L_{t+1}} = \frac{\mathcal{V}(A_{t+1},L_{t+1})}{L_{t+1}} \tag{6}$$

Define the firm's ex-dividend market value as $P_t \equiv \mathbb{F}_t[M_{t+1}\mathcal{V}(A_{t+1},L_{t+1})]$ to write:

$$\frac{\kappa}{q_t} = \frac{P_t}{L_{t+1}} \tag{7}$$

where employment L_{t+1} is determined at the end of date t under our timing convention from equation (1). This equation establishes a direct link between the job filling rate and the firm's market value per worker, providing the foundation for connecting labor market dynamics to asset pricing. Take logarithms, rearrange terms, and expand the price-employment ratio P_t/L_{t+1}:

$$\log q_t = \log\kappa - \log\left(\frac{P_t}{E_t}\right) - \log\left(\frac{E_t}{L_{t+1}}\right) \tag{8}$$

Defining log price-earnings $pe_t = \log(P_t/E_t)$ and earnings-employment $el_t = \log(E_t/L_{t+1})$:

$$\log q_t = \log\kappa - pe_t - el_t \tag{9}$$

[2] I use the term "firms' beliefs" as shorthand to refer to the expectations held by decision makers within firms (Coibion et al., 2018; Candia et al., 2020).

[3] The hiring equation is the labor market analogue of the optimality condition for physical capital in the q theory of investment (Hayashi, 1982), where the upfront cost of hiring κ/q_t is analogous to Tobin's marginal q and the separation rate δ_{t+1} is analogous to the depreciation rate. See Lettau and Ludvigson (2002) and Kogan and Papanikolaou (2012) for a similar log-linearization applied for the q theory of physical capital investment.

Log-linear Approximation of Price-Earnings Ratio To decompose the job filling rate into economically meaningful components, I apply the Campbell and Shiller (1988) present value identity to the price-earnings ratio. I log-linearize the price-earnings ratio $pe_t \equiv \ln(P_t/E_t)$ around its long-run mean $\overline{pe}$ to obtain the approximate relationship:

$$pe_t = c_{pe} - r_{t+1} + \Delta e_{t+1} + \rho pe_{t+1} \tag{10}$$

where c_{pe} is a linearization constant, $\rho = \frac{\exp(\overline{pe})}{1+\exp(\overline{pe})} \approx 0.98$ is the time discount factor from the log-linearization, $r_{t+1} = \log(\frac{P_{t+1}+E_{t+1}}{P_t})$ represents the stock return (assuming the firm pays out earnings as dividends), and Δe_{t+1} denotes earnings growth. This identity holds approximately even when earnings differ from dividends, as the resulting payout ratio term is quantitatively small and can be treated as constant (De La O et al., 2024).[4] Substituting recursively for the next h periods yields the Campbell-Shiller present value identity:

$$pe_t = \sum_{j=1}^{h} \rho^{j-1} c_{pe} - \sum_{j=1}^{h} \rho^{j-1} r_{t+j} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j} + \rho^h pe_{t+h} \tag{11}$$

Decomposition of Job Filling Rate Substitute log-linearized price-earnings from (11) into the hiring equation from (9) to obtain a decomposition of the log job filling rate:

$$\underbrace{\log q_t}_{\text{Job Filling Rate}} = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{\substack{\text{Discount Rate} \\ \equiv\ r_{t,t+h}}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j} \right]}_{\substack{\text{Cash Flow} \\ \equiv\ e_{t,t+h}}} - \underbrace{\rho^h pe_{t+h}}_{\substack{\text{Future Price-Earnings} \\ \equiv\ pe_{t,t+h}}} \tag{12}$$

where $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The equation decomposes the job filling rate into future discount rates $r_{t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$, cash flows $e_{t,t+h} \equiv el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$, and price-earnings $pe_{t,t+h} \equiv \rho^h pe_{t+h}$. The cash flow component consists of currentl log earnings-employment el_t, which captures short-term fluctuations in cash flows, and $j = 1, \ldots, h$ period ahead log earnings growth Δe_{t+j}, which captures news about future cash flows.[5] $pe_{t,t+h}$ is a terminal value that captures other long-term influences beyond h periods into the future not already captured in discount rates and cash flows.

Since equation (12) holds both ex-ante and ex-post, it can be evaluated under either subjective or rational expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their subjective expectations $\mathbb{F}_t[\cdot]$:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] \tag{13}$$

[4]See Appendix Section B for a derivation.

[5]The earnings-employment ratio can be interpreted as a measure of the marginal product of labor under constant returns to scale (David et al., 2022).

Alternatively, the *rational decomposition* replaces ex-post realizations of future outcomes with their rational expectations $\mathbb{E}_t[\cdot]$:

$$\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}] \tag{14}$$

Comparing these decompositions can quantify how belief distortions affect the job filling rate.

Estimation The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current job filling rate. For the subjective decomposition, demean each variable in equation (13), multiply both sides by the current log job filling rate $\log q_t$, and take the sample average:

$$Var\left[\log q_t\right] = Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right] \tag{15}$$

where $Var[\cdot]$ and $Cov[\cdot]$ are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by $Var\left[\log q_t\right]$ to decompose its variance:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{16}$$

The left-hand side represents the full variability in job filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, and $\mathbb{F}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively.

Finally, the decomposition under rational expectations can be estimated similarly based on equation (14) by replacing the subjective expectation $\mathbb{F}_t[\cdot]$ with its rational counterpart $\mathbb{E}_t[\cdot]$. This comparison allows us to assess the role of belief distortions in labor market dynamics and determine whether firms systematically mis-perceive economic conditions when making hiring decisions. The goal is to quantify how biases in subjective beliefs about discount rates and cash flows contribute to fluctuations in the job filling rate.

Although the variance decomposition does not necessarily capture causal relationships, it has the advantage of not requiring the researcher to take a stand on the deep determinants of job filling rates in any given period. The evolution of discount rates and cash flows summarize the consequences of the combination of these deep determinants. Not needing to link fluctuations in the job filling rate to a primitive shock makes this approach useful for studying business cycle frequency outcomes that span multiple cycles, each driven by its own unique deep determinants.

3 Data

This section describes the data used to estimate the variance decomposition. For each outcome variable, I use survey forecasts to measure subjective expectations $\mathbb{F}_t[\cdot]$ and machine learning forecasts to measure rational expectations $\mathbb{F}_t[\cdot]$. The final estimation sample is quarterly and spans 2005Q1 to 2021Q4.[6]

Employment Employment L_t is measured using annual total employee counts (EMP) for S&P 500 firms from the CRSP/Compustat Merged Annual Industrial Files. I interpolate this data to a quarterly frequency by using quarterly averages of the fitted values from regressing annual S&P 500 employment on the monthly BLS nonfarm payrolls.

Job Filling Rate Vacancies V_t is measured using JOLTS job openings starting 2000:12 and help-wanted index for earlier periods (Barnichon, 2010). Unemployment U_t is sourced from the BLS unemployment series (UNEMPLOY). The job filling rate q_t is defined as the share of filled vacancies out of unemployment:

$$q_t = \frac{f_t V_t}{U_t} \tag{17}$$

The job finding rate f_t is the share of unemployed workers that find jobs within the period:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}} \tag{18}$$

where U_t^s is short-term unemployment less than 5 weeks (UEMPLT5). I construct the variables at a monthly frequency, time-aggregate to quarterly averages, and detrend using an HP filter with a smoothing parameter of 10^5 to ensure stationarity (Shimer, 2005).

Stock Returns Stock returns are measured using monthly Center for Research in Security Prices (CRSP) value-weighted returns with dividends (VWRETD). Annualized cumulative h-year log stock returns are compounded from monthly returns: $r_{t,t+h} = (1/h)\sum_{j=1}^{12h} \log(1 + VWRETD_{t+j/12})$. I use the quarterly CFO survey from 2001Q4 to 2021Q4 to construct 1 and 10 year ahead survey forecasts of annualized cumulative log S&P 500 returns. For intermediate horizons between 1 and 10 years, I interpolate linearly between the 1 and 10 year ahead forecasts.

Earnings To construct subjective expectations of future cash flows, I follow the approach of De La O and Myers (2021) and Bordalo et al. (2019), using analyst forecasts to construct forecasts of earnings from the I/B/E/S database. The forecasts reflect the views of financial analysts who actively track firms for their investment research. Analysts have a strong incentive to report their forecasts accurately because these forecasts are not anonymous (Cooper et al., 2001; De La O et al., 2024). Prior research shows that these forecasts are widely followed by

[6]See Appendix Section C for details about data sources, and Appendix Section D for details about the machine learning forecasts.

market participants and are priced in by investors, lending credibility to their use as proxies for subjective expectations (Kothari et al., 2016).[7]

I/B/E/S provides monthly median analyst forecasts for earnings per share (EPS) at one through four year horizons, as well as long-term growth (LTG) forecasts.[8] The LTG forecast represents the median expected average annual growth rate in operating earnings over the next three to five years. One- through four-year-ahead forward annual log earnings growth forecasts $\mathbb{F}_t[\Delta e_{t+h}]$ for $h = 1, 2, 3, 4$ are log differences between adjacent forecast horizons. For the five-year horizon $\mathbb{F}_t[\Delta e_{t+5}]$, I interpret the LTG forecast as the expected log growth in earnings from year four to five (Bianchi et al., 2024b). The sample spans 1982 to 2021 at a monthly frequency, and the forecasts cover approximately 80% of total market capitalization, providing broad coverage of U.S. public firms.

Quarterly earnings for the S&P 500 are sourced from IBES street earnings per share (EPS) data that starts in 1983:Q4 from Hillenbrand and McCarthy (2024). Street earnings differ from GAAP earnings by excluding discontinued operations, extraordinary charges, and other non-operating items. According to the IBES user guide, analysts submit forecasts after backing out these transitory components, and IBES constructs the realized series to align with those forecasts. As noted in Hillenbrand and McCarthy (2024), this series closely aligns with Compustat earnings before special items, indicating it accurately reflects analysts' forecast targets.

Price-Earnings Ratio The current price-earnings ratio $PE_t \equiv P_t/E_t$ is calculated using the end-of-quarter S&P 500 stock price index P_t and total earnings E_t. Following De La O and Myers (2021), I derive subjective expectations of log price-earnings $\mathbb{F}_t[pe_{t+h}]$ from the log-linear approximation in equation (11):

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \mathbb{F}_t[\Delta e_{t+j}] - \mathbb{F}_t[r_{t+j}]) \tag{19}$$

where expected returns and earnings growth come from the CFO survey and IBES, respectively.

Earnings-Employment Ratio The current earnings-employment ratio is defined as $EL_t \equiv E_t/L_{t+1}$, where E_t denotes quarterly total earnings for the S&P 500 and L_{t+1} is the employment stock at the beginning of period $t+1$. I measure L_{t+1} using end-of-period employment levels within each quarter. This timing assumption ensures that the measures are consistent with the

[7]As a robustness check, Appendix Table A.11 also use aggregate earnings forecasts from the CFO survey, which reflect managerial expectations. The two series are positively correlated (correlation coefficient = 0.60 at the 1-year horizon), suggesting alignment between analyst and managerial beliefs.

[8]IBES forecasts generally reflect pro forma earnings that exclude stock-based compensation. While this could raise concerns about comparability to comprehensive earnings measures, prior research finds that market participants primarily price firms based on these adjusted earnings figures (Bradshaw and Sloan, 2002). Because our analysis focuses on how forecasted earnings influence firm decisions and valuations, using the earnings concept most relevant for investor behavior is appropriate.

timing conventions from Section B while still remaining known to firms by the end of period t. Let $el_t = \log(EL_t)$ denote the log earnings-employment ratio.

Machine Learning Forecasts For each survey forecast, I construct the corresponding machine learning forecast using a Long Short-Term Memory (LSTM) neural network:

$$\mathbb{E}_t[y_{t,t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t}) \tag{20}$$

where $y_{t,t+h}$ denotes the outcome variable y to be predicted h periods ahead of time t. $\mathcal{X}_t$ is a large input dataset of macroeconomic, financial, and textual predictors. $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$ denotes predicted values from Long Short-Term Memory (LSTM) neural networks that can be represented by a potentially high dimensional set of finite-valued parameters $\boldsymbol{\beta}_{h,t}$. The parameters are estimated using an algorithm that takes into account the data-rich environment in which firms operate in (Bianchi et al., 2022 and Bianchi et al., 2024b). Machine learning forecasts of log price-earnings $\mathbb{E}_t[pe_{t+1}]$ is constructed similarly to the survey counterpart by replacing the survey forecasts on the right-hand side of equation (19) with the corresponding machine learning forecasts.

4 Stylized Facts

Forecasting Performance To assess whether survey respondents systematically misweight relevant information, Table 1 evaluates the out-of-sample accuracy of survey forecasts relative to machine learning forecasts. I measure the relative predictive performance using the ratio of mean-square-forecast-error (MSE) of the machine ($MSE_{\mathbb{E}}$) over that of the survey ($MSE_{\mathbb{F}}$). The out-of-sample R-squared ($OOS\ R^2$) is the inverse of the MSE ratio, where a positive value indicates better performance of the machine: $1 - MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$. To minimize the influence of random errors in each forecast, the forecast errors are averaged over a sufficiently long out-of-sample testing period spanning from 2005Q1 to 2021Q4. The variables I consider are discount rates $r_{t,t+h}$, cash flows $e_{t,t+h}$, and future price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). These variables influence the job filling rate through the firm's optimal hiring decision.

Table 1 shows that machine learning forecasts consistently outperform survey forecasts across all variables and horizons. MSE ratios are well below one, and out-of-sample R^2 values are positive, confirming that survey expectations systematically deviate from an unbiased machine learning benchmark. The discrepancy grows with horizon length, indicating that belief distortions in long-term forecasts are larger than those at shorter horizons. These results suggest that subjective expectations used in hiring decisions may be biased in persistent and predictable ways—biases that machine learning models can correct using real-time data. If survey respondents were rational in forming their beliefs, their forecasts would have performed on par with machine learning forecasts.[9] The superior performance of the machine also highlights its abil-

[9]As supplementary evidence, Table A.2 estiamtes Coibion and Gorodnichenko (2015) regressions of forecast

Table 1: Accuracy of Machine Learning and Survey Forecasts

	Out-of-sample Testing Period: 2005Q1-2021Q4				
Horizon h (Years)	1	2	3	4	5
			(a) Discount Rates $r_{t,t+h}$		
$MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$	0.826	0.742	0.593	0.588	0.659
OOS R^2	0.174	0.258	0.407	0.412	0.341
			(b) Cash Flows $e_{t,t+h}$		
$MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$	0.739	0.552	0.520	0.509	0.485
OOS R^2	0.261	0.448	0.480	0.491	0.515
			(c) Price-Earnings $pe_{t,t+h}$		
$MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$	0.676	0.466	0.434	0.395	0.416
OOS R^2	0.324	0.534	0.566	0.605	0.584

Notes: This table shows the relative mean squared forecast errors (MSE) between machine learning forecasts and survey forecasts. $MSE_{\mathbb{E}}$ and $MSE_{\mathbb{F}}$ denote machine learning and survey mean squared forecast errors, respectively. The out-of-sample R-squared, OOS R^2, is defined as $1 - MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$, where $MSE_{\mathbb{F}}$ always denotes the MSE of the benchmark forecast. $y_{t,t+h}$ denotes the variable y to be predicted h years ahead of time t. The variables to be predicted are h-year present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine learning forecasts $\mathbb{E}_t$ are based on a Long Short-Term Memory (LSTM) neural network $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The out-of-sample testing period is 2005Q1 to 2021Q4.

ity to process a large amount of real-time data efficiently and objectively, making it a reliable benchmark of undistorted beliefs.

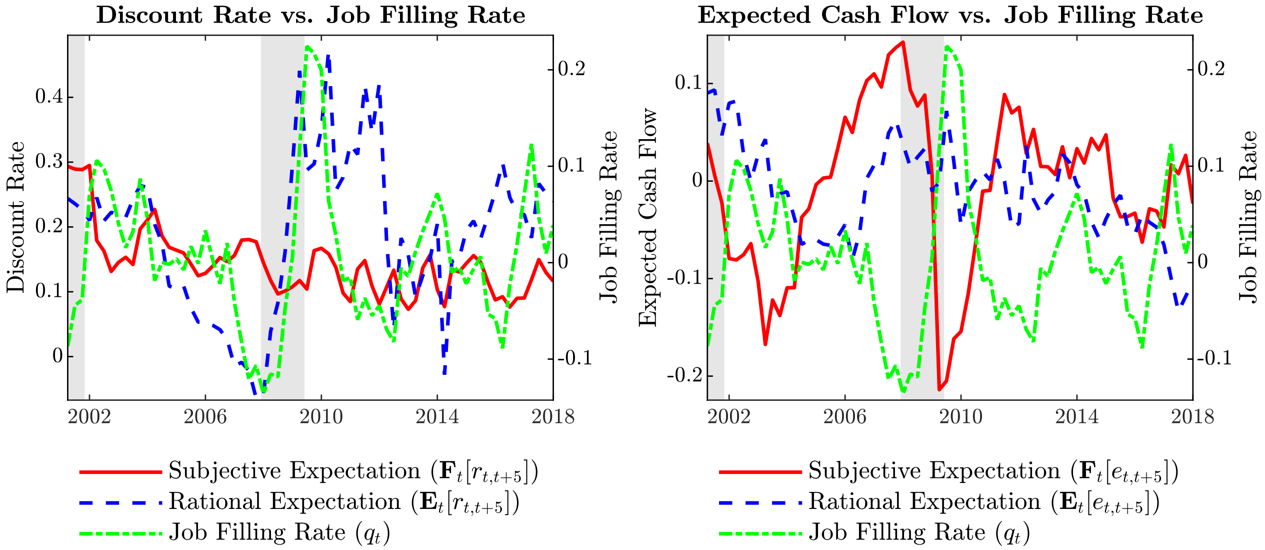
One natural question is whether survey respondents report risk-neutral rather than subjective expectations. However, risk premia typically range from 5-10% annually, insufficient to explain the 15-30% deterioration in MSE ratios observed in Table 1. The magnitude and persistence of forecast errors, along with their correlation with forecast revisions, instead point to behavioral biases (e.g., extrapolation) rather than rational risk compensation.

Descriptive Statistics Figure 1 compares subjective and machine expectations for discount rates and cash flows, plotted against the job filling rate. These series represent the specific theoretical components that drive hiring decisions in the DMP framework: discount rates capture the firm's intertemporal trade-offs when evaluating the present value of a new hire, while cash flows reflect expected future productivity gains from employment. Through the lens of the decomposition in equation (12), these components should move systematically with job filling rates if firms correctly interpret economic conditions.

Machine expectations of discount rates exhibit a strong positive relationship with job filling rates, particularly around the Global Financial Crisis. This pattern aligns with the theoretical

errors on forecast revisions to show that survey forecast errors are predictable: upward revisions in survey forecasts consistently lead to negative forecast errors, indicating overreaction. Regressions of the survey bias—defined as deviations of survey forecasts relative to the machine learning benchmark—on forecast revisions suggest that 72.4% of the long-horizon forecast error is attributable to ex-ante biases in beliefs.

Figure 1: Job Filling Rates, Discount Rates, and Cash Flows



Notes: Figure plots $h = 5$ year ahead survey forecasts $\mathbb{F}_t[\cdot]$ and machine learning forecasts $\mathbb{E}_t[\cdot]$ of discount rates $r_{t,t+h}$ and cash flows $e_{t,t+h}$ (left axis) against the current detrended job filling rate q_t (right axis). x axis denotes the date on which each forecast has been made and the job filling rate was realized. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The out-of-sample forecast testing period is quarterly and spans 2005Q1 to 2021Q4. Cash flow forecasts and job filling rates have been detrend using an HP filter with a smoothing parameter of 10^5 (Shimer, 2005). NBER recessions are shown with gray shaded bars.

prediction that higher discount rates (reflecting greater compensation for risk) should coincide with increased hiring as firms perceive better risk-adjusted returns to employment. Survey expectations of discount rates, by contrast, are relatively flat and display little sensitivity to the business cycle, consistent with studies that find acyclical subjective risk premia (Nagel and Xu, 2022). This disconnect suggests that firms fail to internalize how macroeconomic conditions affect the risk-adjusted value of hiring decisions.

For cash flows, the pattern reverses. Survey expectations show exaggerated cyclical variation, becoming sharply pessimistic during downturns, such as the Global Financial Crisis, when job filling rates are high. Machine forecasts also vary cyclically but to a much lesser extent, indicating that survey respondents tend to over-react to macroeconomic conditions when forming cash flow expectations. This over-reaction manifests in the decomposition as an outsized role for subjective cash flow news in explaining job filling rate variation, even when underlying fundamentals suggest that discount rate changes should be the primary driver of hiring fluctuations.

Appendix Table A.1 confirms this visual comparison by summarizing the distributions of survey forecasts and machine learning forecasts for the key components of the variance decomposition. The most striking pattern is the contrast in volatility between survey and machine forecasts. At the five-year horizon, survey-based expectations of discount rates $\mathbb{F}_t[r_{t,t+5}]$ are substantially less volatile than machine-based expectations $\mathbb{E}_t[r_{t,t+5}]$, with standard deviations of 0.050 and 0.119, respectively. In contrast, subjective expectations of cash flows $\mathbb{F}_t[e_{t,t+5}]$ are more volatile than their machine counterparts. At the 5-year horizon, survey expectations of cash flows have a standard deviation of 0.147, compared to 0.070 for machine expectations.

5 Time-Series Decomposition of the Job Filling Rate

The superior forecasting performance of machine learning over survey forecasts suggests the presence of systematic distortions in subjective expectations. This section quantifies how those distortions affect hiring behavior by estimating the contribution of discount rate and cash flow expectations to fluctuations in the aggregate job filling rate.

Decomposition Framework Building on the search model from Section 2, the log job filling rate can be written as:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] \tag{21}$$

where c_q is a constant and $\mathbb{F}_t[\cdot]$ represents subjective expectations. $r_{t,t+h}$, $e_{t,t+h}$, and $pe_{t,t+h}$ are h-year present discounted values of future discount rates, cash flows, and price-earnings ratios (terminal value), respectively, as defined in equation (12). To quantify the contribution of each term, I estimate variance decompositions by regressing each forecasted component on $\log q_t$:[10]

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{22}$$

The decomposition under rational expectations $\mathbb{E}_t[\cdot]$ can be estimated similarly while replacing the survey forecast $\mathbb{F}_t[\cdot]$ with its corresponding machine learning counterpart $\mathbb{E}_t[\cdot]$.

Rational Expectations Panel (a) of Table 2 shows that under rational expectations, discount rate news is the dominant driver of variation in job filling rates. At the five-year horizon, rational discount rates explain 69.1% of the variation in job filling rates, while rational cash flow news accounts for 6.6%.[11] Consistent with the predictions of the search and matching model, higher job filling rates predict either higher future discount rates or lower cash flows. The contribution from terminal price-earnings ratios at the five-year horizon is 20.1%, which is statistically insignificant.[12] Note that the three components are estimated freely without

[10] The decomposition can also be estimated as a multivariate system by estimating a Vector Autoregression (VAR) of the job filling rate and its key determinants (Table A.4). Since the time series in levels can be serially correlated or nonstationary, I also consider a first differenced decomposition (Table A.5).

[11] First-difference estimates in Table A.5 show broadly similar patterns, with rational discount rate news explaining 58.7% while rational cash flow news explaining only 10.0% of job filling rate variation.

[12] Hyatt and Spletzer (2016) document that about half of U.S. workers have job tenure exceeding five years, reflecting the prevalence of long-term employment relationships. Despite relatively long job tenures, discounting could diminish the weight of earnings far into the future (Kehoe et al., 2022). Under rational expectations, mean-reversion in earnings growth could also limit the contribution of variance from long-term earnings.

requiring them to sum to one. Nevertheless, the residual component remains close to zero across horizons, suggesting that the approximations applied to derive the decompositions are reasonably accurate.

These findings align with predictions from rational search-and-matching models that emphasize time-varying risk premia. The large contribution from discount rate news is consistent with rational models that introduce time-varying discount rates to match unemployment fluctuations (Hall, 2017). The increasing importance of discount rate news at longer horizons is consistent with models that match observed fluctuations in unemployment by modeling hiring as a risky investment with long-duration returns (Kehoe et al., 2022). On the relative importance of risk-free rates and risk premia, Figure A.2 shows that rational risk-free rate expectations explain little of the variation in the job filling rate. This implies that the explanatory power of discount rate news is driven primarily by the risk premium component, consistent with rational models of labor markets with time-varying risk premia (Borovickova and Borovička, 2017).

Subjective Expectations Panel (b) of Table 2 reveals a striking reversal under subjective expectations. At the five-year horizon, subjective cash flow news explains 96.4% of the variation in job filling rates, while subjective discount rate news accounts for only 1.6%.[13] These results suggest that firms systematically overweight news about cash flows and underweight news about discount rates. The direction of the relationship is still consistent with the rational case from panel (a), suggesting that mistakes in subjective expectations are about the magnitudes, not about directions. The contribution from subjective price-earnings ratios declines with horizon and is negligible by year five, suggesting that firms mainly overweight near-term cash flows.

Compared to the rational benchmark, the implied over-reaction to cash flow news is substantial. Low job filling rates during expansions are associated with a significant disappointment in future cash flows. Defining the survey bias as the difference between subjective and rational expectations, the estimates imply that $96.4\% - 6.6\% = 89.8\%$ of variation in job filling rates can be attributed to the fact that the job filling rate predicts biases in cash flows with a significant positive relationship (Table A.9). These biases capture systematic distortions in survey respondents' subjective beliefs that the machine learning model could have identified ex-ante.

The residual component remains small under subjective expectations as well, suggesting that any deviations from the model's approximation are not significantly related to the job filling rate. Thus survey respondents' expectations appear internally consistent, but they systematically misperceive which fundamentals matter most for hiring decisions. Any remaining deviations in the residual could be attributed to measurement errors in the survey responses (e.g., Ma et al., 2020). The residual term is generally smaller when the decomposition is estimated under

[13] First-difference estimates in Table A.5 show similar results, with subjective cash flow news explaining 83.5% and subjective discount rates explaining only 5.4% of the job filling rate.

Table 2: Time-Series Decomposition of the Job Filling Rate

Horizon h (Years)	1	2	3	4	5
			(a) Rational Expectations $\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}]$		
Discount Rate	0.187*	0.309**	0.585***	0.653***	0.691***
t-stat	(1.655)	(2.354)	(2.988)	(3.487)	(3.329)
Cash Flow	−0.078	0.026	0.051	0.055	0.066
t-stat	(−0.233)	(0.181)	(0.364)	(0.459)	(0.472)
Price-Earnings	0.895***	0.720**	0.415	0.331	0.201
t-stat	(2.810)	(2.161)	(0.666)	(0.406)	(0.245)
Residual	−0.004	−0.054	−0.051	−0.039	0.042
t-stat	(−0.009)	(−0.141)	(−0.076)	(−0.046)	(0.049)
N	68	68	68	68	68
			(b) Subjective Expectations $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$		
Discount Rate	0.018	0.018	0.042	0.050	0.016
t-stat	(1.538)	(0.512)	(0.819)	(0.826)	(0.180)
Cash Flow	0.325**	0.641***	0.738***	0.856***	0.964***
t-stat	(1.970)	(4.500)	(4.192)	(4.797)	(7.016)
Price-Earnings	0.629***	0.366	0.207	0.072	0.027
t-stat	(2.794)	(1.410)	(0.765)	(0.187)	(0.077)
Residual	0.027	−0.025	0.014	0.022	−0.007
t-stat	(0.096)	(−0.084)	(0.043)	(0.051)	(−0.017)
N	68	68	68	68	68

Notes: This table reports variance decompositions of the aggregate job filling rate under rational expectations (panel (a)) or subjective expectations (panel (b)). Each row reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Residual term represents the variation in job filling rates that are not captured by the other components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

subjective expectations, suggesting that incorporating non-rational beliefs into the estimation could improve the ability of the model to explain fluctuations in job filling rates.

Discussion Although the decomposition does not necessarily measure causal relationships, it is useful for quantifying possible sources of variation in the job filling rate. A large estimate for subjective cash flow news means that, whatever shock drives variation in the job filling rate must have a larger impact on subjective cash flow growth expectations than subjective return expectations. Under rational expectations, by contrast, firms correctly interpret those same fluctuations as signals about future risk compensation or discount rates.

This divergence points to a distortion in belief formation: firms over-react to perceived shifts in cash flows while largely ignoring the intertemporal pricing of risk. The large contribution of subjective cash flow news in shaping hiring decisions is consistent with models that introduce non-rational expectations about earnings growth to account for fluctuations in asset prices (Bordalo et al., 2024a; Bianchi et al., 2024b) and the business cycle (Bordalo et al., 2024b). My results suggest that firms exhibit similar behavior when hiring: they respond primarily to exaggerated expectations about future earnings, not shifts in discount rates. My results show this same empirical pattern extends beyond asset pricing to real economic decisions: the same psychological biases that distort asset valuations also distort firms' hiring choices. The increasing importance of subjective cash flow news at longer horizons is consistent with behavioral models in which over-reaction increases with the horizon of the survey forecast (Bordalo et al., 2020; Bianchi et al., 2024a; Augenblick et al., 2024). Importantly, these patterns persist even when using risk-neutral expectations extracted from futures prices rather than subjective survey forecasts, confirming that the documented distortions reflect genuine departures from rational belief formation rather than differences in risk compensation (Figure A.1).

On the other hand, the result also aligns with survey evidence showing that subjective return expectations are acyclical (Nagel and Xu, 2022) or even procyclical (Greenwood and Shleifer, 2014), contrary to the countercyclical discount rate variation implied by rational models (Cochrane, 2017). In standard asset pricing models, the discount rate is often equated with the firm's cost of capital e.g., the weighted average cost of debt and cost of equity (WACC). In contrast, subjective expectations about discount rates, as reported by CFOs, may reflect internal hurdle rates rather than market-based discount rates. This aligns with survey evidence showing that firms' internally applied discount rates are persistent, upward-biased, and often unresponsive to market conditions, even when firms are not financially constrained. (Gormsen and Huber, 2025). This divergence suggests that belief distortions in discounting may be a first-order force in real firm decisions, independently of financial frictions. My findings extend this evidence to labor markets, where hiring decisions appear similarly detached from rational responses to financial constraints or risk-based incentives.

These belief distortions offer a new explanation for labor market volatility. Firms that misinterpret hiring conditions as news about cash flows—rather than changes in risk—may cut vacancies excessively in downturns, explaining the volatility of unemployment fluctuations beyond what rational models predict. The next section quantifies this mechanism by directly decomposing the unemployment rate.

6 Implications for Unemployment Fluctuations

To complement the decomposition of the job filling rate, this section analyzes the unemployment rate directly. While the job filling rate captures the proximate drivers of employment dynamics in search models, the unemployment rate is the key macroeconomic outcome of interest and the direct target of policy. Start from the unemployment accumulation equation of the search model in Section 2:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{23}$$

which states that the number of unemployed workers at the beginning of next period U_{t+1} equals the number of unemployed worker who fail to find a job in the current period $(1 - q_t\theta_t)U_t$ plus the number of employed workers who lose their jobs due to separations $\delta_t(1 - U_t)$. Log-linearize around the steady state and substitute in equation (12), which is a decomposition of the job filling rate q_t into discount rate, cash flow, and future price-earnings components. Then the log unemployment rate u_{t+1} satisfies the following predictive relationship:[14]

$$u_{t+1} = \alpha + \beta_r \mathbb{F}_t[r_{t,t+h}] + \beta_e \mathbb{F}_t[e_{t,t+h}] + \gamma' X_t + \varepsilon_{s,t+1} \tag{24}$$

where $X_t \equiv [u_t, \log\theta_t, \log\delta_t]'$ collects standard labor market controls: the lagged log unemployment rate u_t, the log vacancy-to-unemployment ratio $\log\theta_t$, and the log separation rate $\log\delta_t$. The coefficients of interest, β_r and β_e, quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

I estimate the decomposition using multivariate OLS regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations.[15] The regression is designed to test whether perceived shocks to discount rates or earnings forecasts help explain variation in unemployment rates. If firms form biased beliefs about future returns or earnings, those belief distortions should manifest in hiring behavior and thus influence unemployment. A counterpart regression can be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with machine learning-based forecasts $\mathbb{E}_t[\cdot]$.

Table 3 reports estimates from regressions that decompose the predictability of annual unemployment growth under both rational and subjective expectations. The fit of the model in terms of Adjusted R^2 is notably better under subjective expectations than under rational expectations, indicating that incorporating belief distortions improves the model's explanatory power.

Incorporating machine forecasts of discount rates in column (1) results in a large and statistically significant (0.664). Adding the machine cash flow forecast yields no meaningful improvement, as its coefficient is insignificant (0.147). This suggests that under rational expectations, discount rates capture much of the information content relevant for unemployment fluctuations. Column (2) replaces machine forecasts with survey-based subjective expectations. Survey discount rate forecasts remain insignificant, while subjective cash flow expectations emerge as the dominant predictor, with a large, negative, and highly significant coefficient of -0.904 and a substantially higher adjusted R^2 of 0.755.

[14] See Appendix Section B.2 for a derivation.

[15] The future price-earnings ratio term $\mathbb{F}_t[pe_{t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\mathbb{F}_t[r_{t,t+h}]$ and cash flows $\mathbb{F}_t[e_{t,t+h}]$ through the Campbell and Shiller (1988) present value identity in equation (11).

Table 3: Time-Series Predictability of the Unemployment Rate

	Dependent Variable: Log Unemployment Rate u_{t+1}		
	(1)	(2)	(3)
$\mathbb{E}_t[r_{t,t+h}]$	0.664***		0.182
t-stat	(4.641)		(1.534)
$\mathbb{E}_t[e_{t,t+h}]$	0.147		−0.101
t-stat	(0.421)		(−0.374)
$\mathbb{F}_t[r_{t,t+h}]$		0.041	−0.015
t-stat		(1.622)	(−1.199)
$\mathbb{F}_t[e_{t,t+h}]$		−0.904***	−0.746***
t-stat		(−10.142)	(−6.231)
Labor Market Factors	Yes	Yes	Yes
N	68	68	68
Adj. R^2	0.619	0.755	0.771
OOS R^2	0.167	0.460	0.473
$\text{MSE}_{\text{Model}}/\text{MSE}_{\text{SPF}}$	0.833	0.540	0.527

Notes: This table reports decompositions of log annual growth in the unemployment rate from equation (24), under subjective or rational expectations. Labor market factors X_t include the log annual growth of lagged log unemployment rate u_t, log labor market tightness $\log\theta_t$ and log job separation rate $\log\delta_t$. The sample is quarterly from 2005Q1 to 2021Q4. $MSE_{\text{Model}}/MSE_{\text{SPF}}$ denotes the ratio of each model's out-of-sample mean squared forecast error to that of the Survey of Professional Forecasters (SPF) consensus forecast. OOS R^2 is defined as $1 - MSE_{\text{Model}}/MSE_{\text{SPF}}$. Out-of-sample forecasts are constructed as 1-year-ahead predictions using model parameters estimated over a rolling 10-year window. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

Finally, column (3) includes both machine and survey forecasts, revealing that subjective cash flow expectations subsume the predictive power of machine-based discount rate forecasts. This result suggests that behavioral factors can crowd out rational forces in driving labor market fluctuations, consistent with models of behavioral overreaction where salient signals dominate decision-making (Bordalo et al., 2020). Since machine forecasts incorporate a high-dimensional set of real-time predictors, this displacement implies that unemployment fluctuations stem less from omitted variables, but more from systematic misperceptions of underlying shocks.

Out-of-sample performance varies markedly across specifications, highlighting the importance of incorporating subjective expectations. Column (1), which includes rational discount rates an cash flow expectations, has a modest forecasting performance with an OOS R^2 of 0.167 and an MSE ratio of 0.833. Notably, Columns (2) and (3), which incorporate subjective expectations of discount rates and cash flows, deliver substantial gains in forecast accuracy—achieving OOS R^2 values of 0.460 and 0.473, respectively, and cutting MSE nearly in half relative to the SPF.[16] These results suggest that forward-looking survey expectations, contain valuable information not captured by the consensus forecast alone.

[16]Traditional labor market factors—lagged unemployment, labor market tightness, and separations—explain only a modest portion of unemployment fluctuations, with an in-sample adjusted R^2 of 0.260. In terms of out-of-sample performance, a model that excludes expectations entirely performs worse than the Survey of Professional Forecasters (SPF) benchmark with a negative OOS R^2 of −0.094.

Figure A.4 examines the fit of the predictive model more closely. I use the estimates from columns (1) and (2) of Table 3 to plot the actual annual change in unemployment against its model-implied decomposition using either rational and subjective expectations, respectively. Under subjective beliefs, fluctuations in unemployment closely track the component attributed to expected cash flow news. The model captures the sharp rise and fall in unemployment during and after recessions with considerable precision.

This finding underscores that belief distortions can have macroeconomically meaningful consequences. Systematic misperceptions about future cash flows—rather than discount rates or job separations—can significantly influence firms' hiring behavior and, in turn, aggregate unemployment dynamics. The resulting gap between unemployment predicted under rational versus subjective expectations can be interpreted as a reduced-form belief-driven unemployment wedge. This wedge provides a novel empirical counterpart to theoretical mechanisms in existing behavioral models where non-rational beliefs generate business cycle volatility (Bordalo et al., 2021; Bianchi et al., 2023). By propagating through forward-looking hiring behavior, belief distortions lead not just to forecast errors, but to real fluctuations in labor market outcomes.[17]

7 Cross-Sectional Decomposition of the Hiring Rate

To analyze the sources of dispersion in hiring across firms, I implement a cross-sectional decomposition of the log hiring rate based on the same theoretical framework developed for the time-series decomposition. I construct a panel of five value-weighted portfolios sorted by book-to-market ratio. At each point in time, firms are assigned to one of five bins based on their book-to-market ranking, and portfolio-level variables are computed using value weights. This approach smooths out firm-level measurement errors and negative values for earnings, allowing us to systematically evaluate how hiring rates vary across the distribution of valuation ratios. The log hiring rate for each portfolio is defined using the employment accumulation equation:

$$hl_{i,t} = \log\left(\frac{q_t V_{i,t}}{L_{i,t}}\right) = \log\left(\frac{L_{i,t+1}}{L_{i,t}} - (1-\delta^l_{i,t})\right) \tag{25}$$

where $L_{i,t}$ uses data from Compustat number of employees (EMP) and $\delta^l_{i,t}$ uses JOLTS industry-level job separation rate. The hiring rate captures the fraction of workers hired per existing employee, conditional on vacancies filled at rate q_t. To isolate cross-sectional variation, I demean this hiring rate across the five portfolios, defining $\widetilde{hl}_{i,t} = hl_{i,t} - \frac{1}{5}\sum_{j=1}^{5} hl_{j,t}$. This demeaned

[17] While this paper focuses on publicly listed firms, preliminary evidence suggest that similar patterns likely emerge among smaller private businesses. A 2010 report from the National Federation of Independent Business (NFIB) on small business credit during the recession shows that 51% of small employers cited weak sales expectations as their top concern, compared to just 8% who cited access to credit Dennis, 2010. To the extent that access to credit capture financial frictions that would show up in discount rates, this survey suggests that subjective beliefs about future cash flows—not discount rates—also shape employment decisions in the small business sector.

hiring rate is then decomposed into three components: a discount rate term, a cash flow term, and a terminal price-earnings term. The decomposition equation is given by:

$$\widetilde{hl}_{i,t} = \underbrace{-\sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\Delta\widetilde{e}_{i,t+j}]\right]}_{\text{Cash Flow}} + \underbrace{\rho^h\mathbb{F}_t\left[\widetilde{pe}_{i,t+h}\right]}_{\text{Future Price-Earnings}} \tag{26}$$

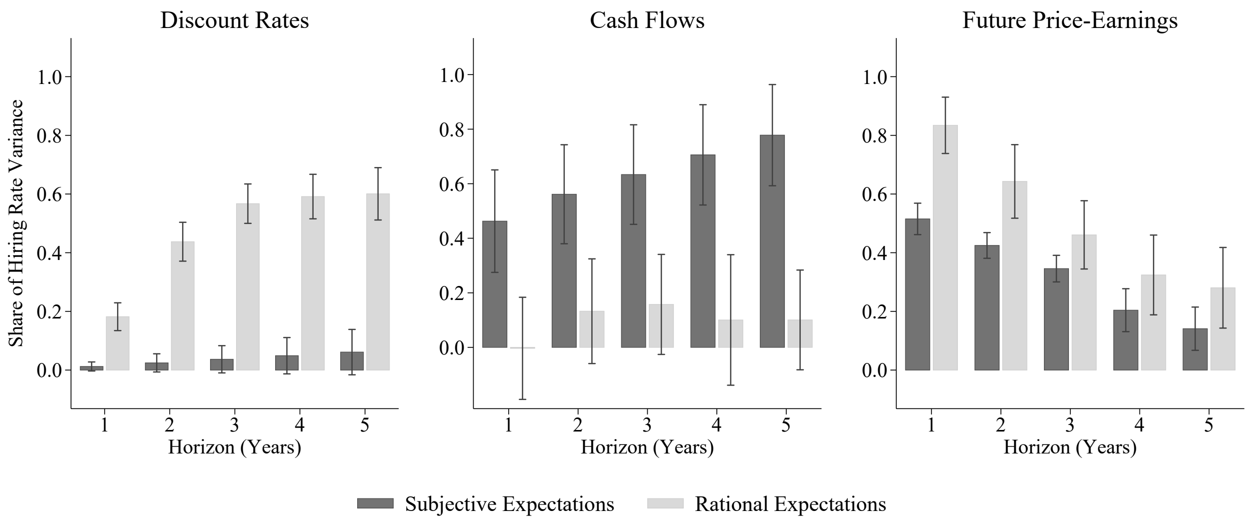
where each term corresponds to a distinct source of hiring dispersion. The first term represents cross-sectional deviations in expected returns, which affect the value firms place on future cash flows. The second term aggregates the deviation in current earnings per worker, $\widetilde{el}_{i,t}$, and the sum of expected earnings growth differentials across the forecast horizon. The third term captures long-run differences in valuation multiples, measured by cross-sectional variation in expected price-earnings ratios at horizon h. All expectations are formed using either survey (subjective beliefs) or machine learning forecasts (rational expectations benchmark). Variables are cross-sectionally demeaned at each time t so that the decomposition isolates the extent to which deviations from the average hiring rate can be traced to each channel.

Figure 2 shows that under subjective expectations, cross-sectional hiring variation is dominated by differences in expected earnings growth. At the five-year horizon, 83.3% of the cross-sectional variance in log hiring rates is explained by the cash flow component. In contrast, only 4.4% of the variation is explained by the discount rate component, and the remainder is attributed to differences in long-run price-earnings expectations. These results indicate that firms sorted into different book-to-market portfolios have sharply different expectations about future cash flows when expectations are subjective, and these differences in beliefs translate into differences in perceived hiring incentives.

Under rational expectations, the decomposition reverses. At horizon five, 71.6% of cross-sectional variation in hiring is explained by differences in expected discount rates, while only 22.6% is explained by expected earnings. This pattern reflects the fact that under rational expectations, much of the variation across firms in asset valuations—and therefore hiring incentives—comes from variation in risk premia rather than cash flow forecasts. Finally, the contribution of the terminal price-earnings component is limited, more so for subjective expectations. By the 5-year horizon, this channel accounts for 28.0% and 14.1% of the cross-sectional hiring variance for rational and subjective expectations, respectively.

Taken together, the results reveal that under subjective beliefs, cross-sectional variation in hiring is driven primarily by firms over-weighting news about their cash flows. The result is consistent with a model in which beliefs about cash flows are updated sluggishly, since firms that experience recent positive earnings shocks continue to expect strong growth, leading to persistent differences in hiring rates across firms. As shown in Figure A.3, these belief distortions are especially pronounced among growth firms with low book-to-market ratios. Under rational

Figure 2: Cross-Sectional Decomposition of the Hiring Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the cross-sectional decomposition to the dispersion of the current hiring rate. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

expectations, such distortions are absent, and the primary driver of hiring differences is variation in required returns.

8 Model of Constant-Gain Learning

In this section, I introduce a model of hiring in which firms form subjective beliefs about future earnings using a constant-gain learning rule. These evolving expectations shape firms' vacancy posting decisions and drive variation in hiring and job filling rates. The model embeds belief distortions in a standard search-and-matching framework and generates decompositions that can match those estimated from the data in Sections 5 and 7.

Environment and Firm Problem The model features a frictional labor market in which unemployed workers are matched with job vacancies using a Cobb-Douglas matching function:

$$\mathcal{M}(U_t, V_t) = BU_t^{\eta} V_t^{1-\eta} \tag{27}$$

where $\mathcal{M}(U_t, V_t)$ denotes the total number of matches in period t and is a function of aggregate unemployment U_t and job vacancies V_t. B is the matching efficiency parameter, and $\eta \in (0, 1)$ governs the elasticity of matches with respect to unemployment. The probability that a firm fills a posted vacancy, the job filling rate, is then given by:

$$q_t = \frac{\mathcal{M}(U_t, V_t)}{V_t} = B\left(\frac{U_t}{V_t}\right)^{\eta} = B\theta_t^{-\eta} \tag{28}$$

where $\theta_t \equiv V_t/U_t$ denotes labor market tightness. A firm that posts a vacancy incurs a cost $\kappa > 0$ per period. Matches dissolve at an exogenous separation rate δ, and each firm hires new workers by posting vacancies in anticipation of future returns. Each firm i uses labor to produce output via a constant returns to scale (CRS) production function:

$$Y_{i,t} = A_{i,t}L_{i,t} \tag{29}$$

where $A_{i,t}$ is firm-level productivity and $L_{i,t}$ is the level of employment. The firm pays wages $W_{i,t}$, incurs hiring costs $\kappa V_{i,t}$, and generates earnings:

$$E_{i,t} = Y_{i,t} - W_t L_{i,t} - \kappa V_{i,t} \tag{30}$$

Earnings represent the net flow profits from operating the firm: output net of the wage bill and the costs associated with posting vacancies. Firms maximize the expected present discounted value of earnings. Let $\mathcal{V}(A_{i,t}, L_{i,t})$ denote the value of the firm as a function of current productivity and employment. The Bellman equation for the firm's dynamic problem is:

$$\mathcal{V}(A_{i,t}, L_{i,t}) = \max_{V_{i,t}, L_{i,t+1}} \{E_{i,t} + \mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]\} \tag{31}$$

The firm chooses the number of vacancies $V_{i,t}$ to post and the resulting employment $L_{i,t+1}$ to maximize the sum of current earnings and the discounted continuation value, formed under subjective expectations $\mathbb{F}_t[\cdot]$ and a stochastic discount factor M_{t+1}. Employment evolves according to the accumulation equation:

$$L_{i,t+1} = (1-\delta)L_{i,t} + q_t V_{i,t} \tag{32}$$

which states that next period's employment depends on retained workers $(1-\delta)L_{i,t}$ and new hires $q_t V_{i,t}$ from current vacancies. Under constant returns to scale, the firm's marginal value of labor equals average value, and the first-order condition with respect to $V_{i,t}$ simplifies to:

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \frac{\partial \mathcal{V}(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}}\right] = \frac{\mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]}{L_{i,t+1}} \equiv \frac{P_{i,t}}{L_{i,t+1}} \tag{33}$$

This condition equates the marginal cost of hiring a worker today, κ/q_t, to the expected marginal benefit of that hire, defined as the expected continuation value per worker. The term $P_{i,t} \equiv \mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]$ denotes the firm's ex-dividend market value. Rewriting in logs:

$$\log q_t = \log \kappa - \log \left(\frac{P_{i,t}}{L_{i,t+1}}\right) = \log \kappa - \ln \left(\frac{P_{i,t}}{E_{i,t}}\right) - \ln \left(\frac{E_{i,t}}{L_{i,t+1}}\right) \tag{34}$$

$$\equiv \log \kappa - pe_{i,t} - el_{i,t} \tag{35}$$

where $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$ is the log price-earnings ratio and $el_{i,t} \equiv \log(E_{i,t}/L_{i,t+1})$ is log earnings per worker.

Expectations and Learning Firms do not have full knowledge of the stochastic processes governing future earnings. Instead, they form beliefs about the long-run drift in earnings using constant-gain learning. The log of firm-level earnings is modeled as the sum of an aggregate and an idiosyncratic component:[18]

$$e_{i,t} = e_t + \tilde{e}_{i,t} \tag{36}$$

where e_t is the aggregate earnings component common to all firms and $\tilde{e}_{i,t}$ is a firm-specific deviation. The aggregate component follows an AR(1) process:

$$e_t = \mu + \phi e_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_u^2) \tag{37}$$

where μ is the unknown drift term, ϕ is the known persistence parameter (with $\phi < 1$), and u_t is an i.i.d. Gaussian innovation. This specification captures both persistent variation in aggregate earnings and stochastic fluctuations. The idiosyncratic component is modeled as a deterministic linear trend plus white noise:

$$\tilde{e}_{i,t} = \tilde{\mu}_i \cdot t + v_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0, \sigma_v^2) \tag{38}$$

where $\tilde{\mu}_i$ is a firm-specific drift in earnings growth, also unknown to the firm, and $v_{i,t}$ is i.i.d. noise. Objectively, mean growth is identical across firms: $\mu = \tilde{\mu}_i = 0$. Under subjective beliefs, however, agents do not observe the true drift terms μ and $\tilde{\mu}_i$. Instead, they form beliefs and update these beliefs recursively as new information arrives. I assume firms employ constant-gain learning, where more weight is placed on recent forecast errors. The updating rules are:

$$\mathbb{F}_t[\mu] = \mathbb{F}_{t-1}[\mu] + \nu \left(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]\right) \tag{39}$$

$$\mathbb{F}_t[\tilde{\mu}_i] = \mathbb{F}_{t-1}[\tilde{\mu}_i] + \nu \left(\Delta \tilde{e}_{i,t} - \mathbb{F}_{t-1}[\Delta \tilde{e}_{i,t}]\right) \tag{40}$$

where ν is the constant gain parameter and governs the speed of learning. A higher ν implies greater responsiveness to new data. The term $\mathbb{F}_t[\mu]$ denotes the firm's time-t belief about the aggregate drift, and $\Delta e_t \equiv e_t - e_{t-1}$ is the realized growth in aggregate earnings, which the firm observes at time t.

This learning specification is supported by empirical evidence presented in Appendix A.3, which shows that survey respondents update their long-run earnings expectations only gradually following short-term earnings surprises. Specifically, the response of 5-year-ahead forecasts to earnings news is small and often statistically insignificant, consistent with the slow, partial

[18] The assumed cash flow process is motivated by empirical estimates using annual Compustat data from 1983 to 2021. We decompose firm-level log cash flows $e_{i,t}$ into aggregate and firm-specific components: $e_{i,t} = e_t + \tilde{e}_{i,t}$, where $e_t = \mu + \phi e_{t-1} + u_t$ and $\tilde{e}_{i,t} = \tilde{\mu}_i + \tilde{\phi}\tilde{e}_{i,t-1} + v_{i,t}$. The empirical estimates yield $\mu = 0.016$, $\phi = 0.833$, $\tilde{\mu}_i = 0.000$, and $\tilde{\phi} = 0.976$. Given that the estimated drift parameters are close to zero ($\mu \approx 0, \tilde{\mu}_i \approx 0$), we simplify the firm-specific component to a trend specification $\tilde{e}_{i,t} = \tilde{\mu}_i \cdot t + v_{i,t}$ while maintaining the autoregressive structure for the aggregate component.

updating implied by constant-gain learning. This gradual adjustment breaks the law of iterated expectations and introduces persistence in belief distortions, reinforcing the model's assumption that agents form expectations using a fading memory of past information.

Given these beliefs, firms forecast future aggregate earnings growth using:

$$\mathbb{F}_t[\Delta e_{t+h}] = \phi^{h-1}\left((\phi-1)e_t + \mathbb{F}_t[\mu]\right) \tag{41}$$

This reflects the fact that agents know the process is AR(1) with known ϕ but uncertain μ, and project the process forward using current levels of earnings and the estimated perceived drift.

Idiosyncratic earnings growth expectations under subjective beliefs follow:

$$\mathbb{F}_t[\Delta \tilde{e}_{i,t+h}] = \mathbb{F}_t[\tilde{\mu}_i] \tag{42}$$

The forecast of firm-level earnings growth is then:

$$\mathbb{F}_t[\Delta e_{i,t+h}] = \mathbb{F}_t[\Delta e_{t+h}] + \mathbb{F}_t[\Delta \tilde{e}_{i,t+h}] \tag{43}$$

These expectations feed directly into the firm's forecasts of future cash flows and asset valuations, which in turn influence their hiring and investment decisions.

Aggregate Stock Price and Returns Firms use their beliefs about future earnings to form expectations about asset returns and valuations. I assume that the economy is governed by a representative household such that the log stochastic discount factor (SDF) is:

$$m_{t+1} = -r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \tag{44}$$

where r_f is the risk-free rate, γ is the coefficient of relative risk aversion, and u_{t+1} is the aggregate earnings shock from the earnings process.

In models with constant-gain learning, beliefs evolve with fading memory, breaking the law of iterated expectations and making resale and buy-and-hold valuation methods non-equivalent. The buy-and-hold approach evaluates long-run payoffs under today's beliefs, while the resale method prices assets through a sequence of one-period-ahead valuations, each using updated beliefs. Following Nagel and Xu (2021), I adopt the resale valuation approach because it ensures time consistency under belief updating and reflects the idea that assets are effectively resold across agents with evolving expectations. I assume that the manager and the representative investor share the same beliefs and both apply the resale method, ensuring consistency between decision-making and valuation.

Let $P_t^{(h)}$ denote the time t price for an aggregate strip from of a one-dollar payoff received h periods in the future. I guess and verify a log-linear solution:

$$P_t^{(h)} = \mathbb{F}_t[M_{t+1}P_{t+1}^{(h-1)}] = \exp\left\{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\} \tag{45}$$

This expression implies that strip prices depend on current earnings e_t, beliefs about the aggregate drift $\mathbb{F}_t[\mu]$, and constants $A^{(h)}$, $B^{(h)}$ that evolve recursively. These recursive coefficients are computed using the method of undetermined coefficients:

$$A^{(h)} = A^{(h-1)} - r_f + \frac{1}{2}C^{(h)}\left[(1+\nu^2)C^{(h)} - 2\gamma\right]\sigma_u^2 \tag{46}$$

$$B^{(h)} = B^{(h-1)} + \phi^{h-1} = \frac{1-\phi^h}{1-\phi} \tag{47}$$

$$C^{(h)} = \nu B^{(h-1)} + \phi^{h-1} \tag{48}$$

where $B^{(h)}$ captures the sensitivity of strip prices to beliefs about μ, and $C^{(h)}$ captures how beliefs update over time due to learning. The term $A^{(h)}$ captures the impact of risk premia and discounting. The aggregate strip price implies the realized return on the strip:

$$R_{t+1}^{(h)} = \frac{P_{t+1}^{(h-1)}}{P_t^{(h)}} = \exp\{[A^{(h-1)} - A^{(h)}] + C^{(h)}(\mu - \mathbb{F}_t[\mu] + u_{t+1})\} \tag{49}$$

The expected return on a strip of horizon h is then:

$$\mathbb{F}_t[R_{t+1}^{(h)}] = \exp\left\{r_f + C^{(h)}\gamma\sigma_u^2\right\} \tag{50}$$

which shows that expected returns decline with horizon h when agents believe earnings growth is persistent (i.e., $\phi < 1$), and when learning slows the perceived mean reversion. The aggregate stock price is the sum of strip prices across all future periods:

$$P_t = \sum_{h=1}^{\infty} P_t^{(h)} \tag{51}$$

This equation interprets the stock price as a sum of discounted future payoffs, consistent with the notion that equity is a long-duration claim on future earnings. The aggregate stock return realized between t and $t+1$ is defined as the value-weighted average return across all strips maturing from $h = 1$ onwards:

$$R_{t+1} = \frac{\sum_{h=1}^{\infty} P_{t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_t^{(h)}} = \sum_{h=1}^{\infty} w_{t,h} R_{t+1}^{(h)}, \qquad w_{t,h} = \frac{P_t^{(h)}}{\sum_{h=1}^{\infty} P_t^{(h)}} \tag{52}$$

where $R_{t+1}^{(h)} = P_{t+1}^{(h-1)}/P_t^{(h)}$ is the return on the h-period strip, and $w_{t,h}$ is the share of total market value accounted for by strip h. I assume that, under subjective beliefs, the expected strip weights are approximately constant: $w_{t+j-1,h} \approx w_{t,h}$. This assumption reflects the idea that firms simplify their expectations by projecting a constant term structure of asset values forward. Then the expected aggregate return at time t for horizon $j \geq 1$ is:

$$\mathbb{F}_t[R_{t+j}] = \sum_{h=1}^{\infty} \mathbb{F}_t[\mathbb{F}_{t+1}[\ldots\mathbb{F}_{t+j-1}[w_{t+j-1,h}R_{t+j}^{(h)}]]] \approx \sum_{h=1}^{\infty} w_{t,h}\mathbb{F}_t[\mathbb{F}_{t+1}[\ldots\mathbb{F}_{t+j-1}[R_{t+j}^{(h)}]]] \tag{53}$$

$$= \sum_{h=1}^{\infty} w_{t,h} \exp\left\{r_f + C^{(h)}\gamma\sigma_u^2\right\} \tag{54}$$

The final equality follows from the assumption that $R_{t+j}^{(h)}$ is conditionally homoskedastic and forecastable using time-t information due to the recursive structure of beliefs. To obtain the expected log return, I apply the log approximation: $\mathbb{F}_t[r_{t+j}] \approx \log(\mathbb{F}_t[R_{t+j}])$.

Firm Stock Price and Returns Each firm's total value is the sum of expected discounted future earnings. Let $P_{i,t}$ be the ex-dividend value of firm i, which is the sum of strip prices across all future periods:

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)}, \qquad P_{i,t}^{(h)} = \mathbb{F}_t[M_{t+1} P_{i,t+1}^{(h-1)}] = \mathbb{F}_t[M_{t+1} \dots \mathbb{F}_{t+h-1}[M_{t+h} E_{t+h} \tilde{E}_{i,t+h}]] \tag{55}$$

Assuming independence between aggregate discounting and idiosyncratic earnings:

$$P_{i,t}^{(h)} = P_t^{(h)} \cdot \mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]] \tag{56}$$

Firm-level strip price implies realized next-period return on the strip:

$$R_{i,t+1}^{(h)} = \frac{P_{i,t+1}^{(h-1)}}{P_{i,t}^{(h)}} = \frac{P_{t+1}^{(h-1)} \mathbb{F}_{t+1}[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]]}{P_t^{(h)} \mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\tilde{E}_{i,t+h}]]} \tag{57}$$

$$= R_{t+1}^{(h)} \exp\left\{\nu(\Delta \tilde{e}_{i,t+1} - \mathbb{F}_t[\tilde{\mu}_i])h - \frac{1}{2}\nu^2 h^2 \sigma_v^2\right\} \tag{58}$$

Expected return on the firm-level strip:

$$\mathbb{F}_t[R_{i,t+1}^{(h)}] = \mathbb{F}_t[R_{t+1}^{(h)}]\mathbb{F}_t[\exp\{\nu(\Delta \tilde{e}_{i,t+1} - \mathbb{F}_t[\tilde{\mu}_i])h - \frac{1}{2}\nu^2 h^2 \sigma_v^2\}] \tag{59}$$

$$= \mathbb{F}_t[R_{t+1}^{(h)}]\mathbb{F}_t[\exp\{\nu(\tilde{\mu}_i + v_{i,t+1} - \mathbb{F}_t[\tilde{\mu}_i])h - \frac{1}{2}\nu^2 h^2 \sigma_v^2\}] \tag{60}$$

$$= \mathbb{F}_t[R_{t+1}^{(h)}] \tag{61}$$

The firm-level stock return realized between t and $t+1$ is defined as the value-weighted average return across all strips maturing from $h = 1$ onwards:

$$R_{i,t+1} = \frac{\sum_{h=1}^{\infty} P_{i,t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_{i,t}^{(h)}} = \sum_{h=1}^{\infty} w_{i,t,h} R_{i,t+1}^{(h)}, \qquad w_{i,t,h} = \frac{P_{i,t}^{(h)}}{\sum_{h=1}^{\infty} P_{i,t}^{(h)}} \tag{62}$$

where $R_{i,t+1}^{(h)} = P_{i,t+1}^{(h-1)}/P_{i,t}^{(h)}$ is the return on the h-period strip, and $w_{i,t,h}$ is the share of total market value accounted for by strip h. Assuming that expected strip weights are approximately constant under subjective beliefs $w_{i,t+j-1,h} \approx w_{i,t,h}$, the expected firm-level return is:

$$\mathbb{F}_t[R_{i,t+j}] = \sum_{h=1}^{\infty} \mathbb{F}_t[\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+j-1}[w_{i,t+j-1,h} R_{i,t+j}^{(h)}]]] \approx \sum_{h=1}^{\infty} w_{i,t,h} \mathbb{F}_t[\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+j-1}[R_{i,t+j}^{(h)}]]] \tag{63}$$

$$= \sum_{h=1}^{\infty} w_{i,t,h} \exp\left\{r_f + C^{(h)} \gamma \sigma_u^2\right\} \tag{64}$$

Price-Earnings Ratio Expectations To evaluate firm valuations, firms also form expectations about future price-earnings ratios. I apply a log-linear Campbell-Shiller approximation from equation (11) to express the expected price-earnings ratio $pe_{i,t+h}$ as:

$$\mathbb{F}_t[pe_{i,t+h}] = \frac{1}{\rho^h} pe_{i,t} - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} \left(c_{pe} + \mathbb{F}_t[\Delta e_{i,t+j}] - \mathbb{F}_t[r_{i,t+j}] \right) \tag{65}$$

This equation shows that expectations of future price-earnings ratios depend on the current $pe_{i,t}$ and the difference between expected earnings growth and expected returns.

Employment, Job Filling Rates, and Hiring Rates To connect the theoretical model to simulated firm behavior, I track the joint evolution of log earnings, employment, and hiring. Log earnings per worker at the firm level is defined as $el_{i,t} \equiv e_{i,t} - l_{i,t+1}$, where $e_{i,t} = \log E_{i,t}$ is log earnings and $l_{i,t+1} = \log L_{i,t+1}$ is log employment. Firm-level hiring rate is defined as $hl_{i,t} \equiv \log(\frac{q_t V_{i,t}}{L_{i,t}})$. Employment evolves according to the accumulation equation:

$$L_{i,t+1} = (1-\delta) L_{i,t} + q_t V_{i,t} \tag{66}$$

The firm's optimality condition equates the marginal cost and marginal value of a new hire:

$$\frac{\kappa}{q_t} = \frac{P_{i,t}}{L_{i,t+1}} \tag{67}$$

Given values for $\kappa, \delta, B, \eta, P_{i,t}$ and initial values for employment $L_{i,0}$, the system can be solved by iterating over the two equations above:

1. Initialize labor market tightness: $\theta_t^{(0)} = 1$
2. At iteration s, construct job filling rate under Cobb-Douglas matching: $q_t^{(s)} = B(\theta_t^{(s)})^{-\eta}$
3. Update each firm's employment policy using the hiring equation:
$$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa} \tag{68}$$
4. Update each firm's vacancy posting policy using the employment accumulation equation:
$$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}} (L_{i,t+1}^{(s)} - (1-\delta) L_{i,t}) \tag{69}$$
5. Aggregate firm-level variables:
$$P_t = \sum_{i \in I} P_{i,t}, \quad V_t^{(s)} = \sum_{i \in I} V_{i,t}^{(s)}, \quad L_{t+1}^{(s)} = \sum_{i \in I} L_{i,t+1}^{(s)}, \quad U_t^{(s)} = \overline{L} - \sum_{i \in I} L_{i,t} \tag{70}$$
6. Update labor market tightness: $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$
7. Check convergence: $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$ for some small tolerance $\varepsilon > 0$. If not, return to step 2 with the updated values.

Model-Implied Decompositions I use simulated data implied by the model to decompose the job filling rate at the aggregate level and hiring rates at the firm level. The decompositions mirror those applied to the data in Sections 5 and 7. The time-series decomposition of the aggregate job filling rate q_t is given by:

$$\log q_t = \underbrace{\sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[r_{t+j}]}_{\text{Discount Rate}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\Delta e_{t+j}]\right]}_{\text{Cash Flow}} - \underbrace{\rho^h \mathbb{F}_t\left[pe_{t+h}\right]}_{\text{Future Price-Earnings}} \tag{71}$$

where $x_t = \sum_{i \in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. To analyze heterogeneity across firms, I compute a cross-sectional decomposition of hiring rates:

$$\widetilde{hl}_{i,t} = \underbrace{-\sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}]\right]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t\left[\widetilde{pe}_{i,t+h}\right]}_{\text{Future Price-Earnings}} \tag{72}$$

where $hl_{i,t} = \log(q_t V_{i,t}/L_{i,t})$ is the hiring rate and $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{N}\sum_i x_{i,t}$ denotes a cross-sectional deviation from the mean at time t.

Each component is computed using the beliefs formed via constant-gain learning, implying that subjective expectations may overweight certain sources of variation. In particular, revisions in expected earnings growth—especially persistent changes to beliefs about mean earnings growth $\mathbb{F}_t[\mu]$ and $\mathbb{F}_t[\tilde{\mu}_i]$—could serve as a common source of variation in the cash flow component and the job filling rate q_t. In both decompositions, the importance of each term depends on the forecast horizon h and the learning gain ν. Revisions to beliefs about cash flow growth (both aggregate and idiosyncratic) feed into $\mathbb{F}_t[\Delta e_{t+j}]$ and $\mathbb{F}_t[\Delta \tilde{e}_{i,t+j}]$, with persistent effects on hiring decisions over time. Subjective expectations thus over-weight the role of the cash flow channel relative to the discount rate channel, in contrast to the pattern observed under rational expectations.

Simulation Details To evaluate the model's quantitative performance, I simulate a panel of firms over a 500-period horizon. Each firm updates its beliefs using constant-gain learning based on its own realized earnings and the aggregate state. The number of firms is set to 300, and I discard the first 150 periods as a burn-in to eliminate the influence of initial conditions. All expectations, returns, and decompositions are computed at an annual frequency using the model equations derived above. At each horizon h, I compute the model-implied variance decomposition of the job filling rate and the cross-sectional dispersion in firm hiring rates. I then compare the simulated decompositions of the job filling rate and hiring rate to those estimated from the data using both survey expectations and machine learning forecasts.

Model Estimation Table 4 reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The first panel, Cash Flow

Process, governs the dynamics of firm earnings: the persistence of aggregate earnings growth is set to $\phi = 0.83$, with aggregate and firm-level volatility calibrated to match the standard deviations of earnings growth in the data (De La O et al., 2022). The second panel describes the Stochastic Discount Factor (SDF) parameters. The risk-free rate $r_f = 0.046$ and risk aversion $\gamma = 1.61$ match the average level and volatility of aggregate returns (De La O et al., 2022). The time discount rate $\rho = 0.98$ is chosen to be consistent with a steady-state price-earnings ratio from the Campbell and Shiller (1988) present value identity. The third panel covers the Learning block, where the constant-gain learning parameter $\nu = 0.018$ is taken from Malmendier and Nagel (2015), reflecting how agents update beliefs about earnings growth. The final panel includes Labor Market parameters adopted from Kehoe et al. (2022): $B = 0.46$ governs matching efficiency, $\eta = 0.5$ the elasticity of matching, $\kappa = 1.0$ the per-worker hiring cost, and $\delta = 0.286$ the quarterly separation rate.

Table 4: Model Estimation

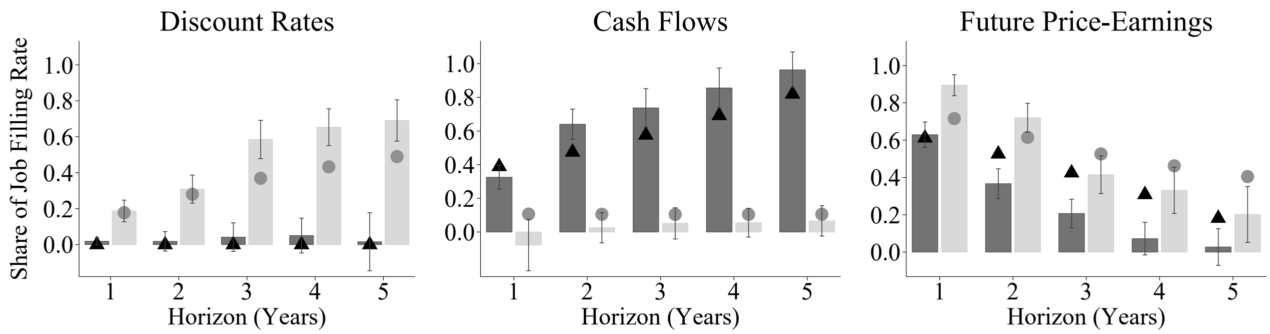
Parameter	Value	Moments
		Cash Flow Process
ϕ	0.83	Autocorrelation aggregate earnings growth
σ_u	0.34	S.D. aggregate earnings growth
σ_v	0.10	S.D. firm-level earnings growth
		SDF
r_f	0.046	Risk-free rate
γ	1.61	Average aggregate return
ρ	0.98	$\exp(\overline{pe})/(1+\exp(\overline{pe}))$
		Learning
ν	0.018	Constant-gain learning (Malmendier and Nagel (2015))
		Labor Market
$\overline{L}$	18.65	Labor force size (Elsby and Michaels (2013))
B	0.46	Matching function efficiency (Kehoe et al. (2022))
η	0.5	Matching function elasticity (Kehoe et al. (2022))
κ	1.0	Per worker hiring cost (Kehoe et al. (2022))
δ	0.286	Separation rate (Kehoe et al. (2022))

Notes: Table reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from.

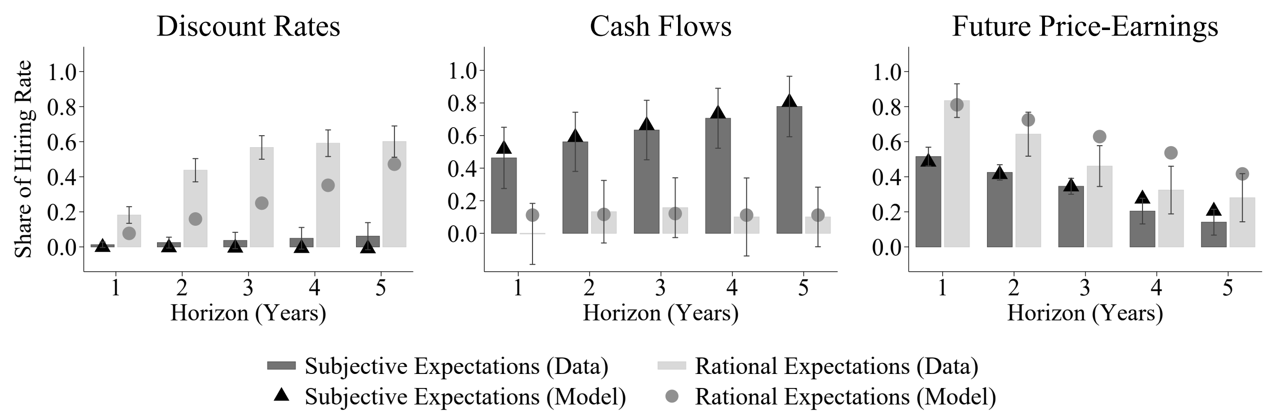
Model vs. Data Figure 3 panel (a) shows the time-series decomposition of the job filling rate under subjective and rational expectations. The model reproduces the finding that subjective beliefs over-attribute fluctuations in hiring to expected cash flows, while underestimating the role of discount rates. Figure 3 panel (b) presents the cross-sectional decomposition of hiring rates across firms. Again, the model captures the empirical pattern that subjective beliefs overstate the contribution of earnings expectations and understate the variation in firm-level discount rates.

Figure 3: Model vs. Data

(1) Time-Series Decomposition of the Job Filling Rate



(2) Cross-Sectional Decomposition of the Hiring Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate (panel (a)) and cross-sectional decomposition of the hiring rate (panel (b)). Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4. Circle and triangle dots show the values of rational and subjective expectations implied by the model, respectively.

Table 5 demonstrates that the constant-gain learning model successfully replicates the empirical relationship between unemployment, discount rates, and expected cash flows observed in Table 3. A comparison between columns (2) and (3) reveals that the model achieves a substantially higher R^2 when expectations are modeled subjectively rather than under the rational expectations benchmark. This improvement in explanatory power highlights the importance of incorporating subjective beliefs into the model. Furthermore, column (4) shows that once subjective cash flow expectations are included, the rational discount rate loses its predictive power. This suggests that belief distortions in cash flow expectations play a central role in explaining unemployment fluctuations within the model. These findings confirm that the constant-gain learning model can quantitatively match key features of the data.

Table 5: Time-Series Predictability of the Unemployment Rate: Model vs. Data

	Dependent Variable: Log Unemployment Rate u_{t+1}					
	(1)	(2)	(3)	(4)	(5)	(6)
	Data			Model		
$\mathbb{E}_t[r_{t,t+h}]$	0.664***		0.182	0.403***		0.059
t-stat	(4.641)		(1.534)	(3.378)		(0.694)
$\mathbb{E}_t[e_{t,t+h}]$	0.147		−0.101	0.013		−0.000
t-stat	(0.421)		(−0.374)	(0.282)		(−0.007)
$\mathbb{F}_t[r_{t,t+h}]$		0.041	−0.015		0.033	0.045
t-stat		(1.622)	(−1.199)		(0.134)	(0.179)
$\mathbb{F}_t[e_{t,t+h}]$		−0.904***	−0.746***		−0.674***	−0.617***
t-stat		(−10.142)	(−6.231)		(−5.168)	(−4.698)
Labor Market Factors	Yes	Yes	Yes	Yes	Yes	Yes
N	68	68	68	350	350	350
Adj. R^2	0.618	0.755	0.772	0.523	0.803	0.808
OOS R^2	0.158	0.460	0.478			
$\text{MSE}_{\text{Model}}/\text{MSE}_{\text{SPF}}$	0.842	0.540	0.522			

Notes: Labor market factors include the log annual growth of lagged unemployment u_t, log labor market tightness $\log\theta_t$ and log job separation rate $\log\delta_t$. The sample is quarterly from 2005Q1 to 2021Q4. $MSE_{\text{Model}}/MSE_{\text{SPF}}$ denotes the ratio of each model's out-of-sample mean squared forecast error to that of the Survey of Professional Forecasters (SPF) consensus forecast. OOS R^2 is defined as $1 - MSE_{\text{Model}}/MSE_{\text{SPF}}$. Out-of-sample forecasts are constructed as 1-year-ahead predictions using model parameters estimated over a rolling 10-year window. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

The large role of subjective cash flow news poses a quantitative challenge for existing search models formulated under rational expectations, which have emphasized time-varying discount rates to match the volatility of unemployment fluctuations. Table A.8 compares the empirical variance decomposition from Table 2 with those implied by benchmark search-and-matching models.[19] The Diamond-Mortensen-Pisarides and Hall (2017) models predict that discount rate fluctuations should explain more than 75% of the variance of job filling rates. The Kehoe et al. (2022) (KLMP) model predicts a more balanced decomposition, attributing 54% to discount rates and 32% to cash flows, consistent with the model's amplification mechanism based on human capital accumulation. In contrast, the empirical decomposition using survey data shows that subjective expectations assign just 1.6% of the variation to discount rates, and 96.4% to cash flows. These results highlight a discrepancy between the predictions of rational models and observed survey expectations, underscoring the importance of belief distortions.

[19]For each model, I set a sample length of 500 periods, produce 300 simulations, and discard the first 150 periods as a burn-in sample. I use the simulated data from each model to estimate a variance decomposition of the job filling rate according to equation (14), and report the average across the simulated runs. All parameter values in the calibration use estimates from the original papers.

9 Robustness Checks and Extensions

This section presents additional results that reinforce the main finding. Across multiple robustness checks, the evidence consistently shows that firms overweight expected cash flows and underweight discount rates under subjective expectations.

Predictability of Survey Forecast Errors In Appendix Section A.2, Table A.2 provides independent validation of the paper's main findings using Coibion and Gorodnichenko (2015) regressions that test for predictable forecast errors. The results confirm asymmetric belief distortions: survey forecasts over-react to cash flow news (negative coefficients ranging from -0.317 to -0.965) but under-react to discount rate news (positive coefficients ranging from 2.453 to 3.731). When the same regressions are estimated using machine learning forecasts as a rational expectations benchmark, the coefficients become dramatically smaller in magnitude and statistically insignificant. This stark contrast between survey and machine learning forecast errors supports the use of machine learning as a rational expectations benchmark and confirms that the systematic patterns in survey forecasts represent genuine belief distortions rather than inherent limitations of the forecasting exercise.

Managerial vs. Analyst Expectations To capture subjective expectations of earnings growth, this paper primarily use survey forecasts from IBES analysts. As a robustness check, this section also incorporates one-year-ahead earnings forecasts from the CFO Survey, which reflects managerial expectations (Gennaioli et al., 2016; Hillenbrand and McCarthy, 2024). While the precise accounting measure that CFOs use in forming their expectations is unclear, many CFOs focus on pro-forma earnings, which exclude special items and are conceptually similar to the "Street earnings" forecasted by analysts in IBES. This comparability suggests that CFO forecasts and IBES forecasts likely target similar earnings concepts. Consistent with this interpretation, the two survey measures have a positive correlation of 0.60 at the one-year horizon. Moreover, Appendix Table A.11 shows that replacing IBES expectations with CFO expectations yields similar variance decomposition results: cash flow expectations remain the dominant driver of hiring fluctuations under subjective managerial beliefs implied by a survey of CFOs.

Subjective Beliefs vs. Risk-Neutral Expectations A natural question is whether subjective beliefs implied by survey expectations reflect a risk-neutral measure rather than genuine belief distortions. While one might argue that these survey errors reflect a risk premium, i.e., that survey forecasts are risk-neutral, the magnitude of the errors far exceeds what standard risk premia can explain. Typical annual equity risk premia are on the order of 5–10%, yet the survey forecasts exhibit mean squared errors 15–30% larger than machine forecasts (Table 1). Moreover, under a risk-neutral measure, forecast errors would approximate the risk premium plus noise.

Such errors should be mostly unpredictable, as they reflect compensation for bearing systematic risk. However, the forecast errors under survey expectations display systematic extrapolation, not randomness (Table A.2). Survey respondents tend to project past trends forward, consistent with the constant-gain learning model where agents fail to mean-revert. This behavior is inconsistent with rational or risk-neutral pricing and suggests that subjective beliefs reflect genuine behavioral distortions rather than a rational risk-neutral measure.

Capital Investment Appendix Section A.9 extends the baseline framework to include firm investment decisions, distinguishing between tangible and intangible capital. Firms choose investment and hiring jointly to maximize value, facing convex adjustment costs and forming expectations over future productivity, returns, and earnings. A decomposition of investment rates in Figures A.5 (time-series) and A.6 (cross-section) reveals that distortions in subjective beliefs play a central role in driving capital allocation, mirroring results for hiring. Using IBES and Compustat data, the decomposition shows that subjective expectations substantially overstate the role of expected earnings and understate the importance of discount rates.

Regional Model using Shift-Share Instrument Appendix Section A.8 strengthens the causal interpretation of belief distortions by using a Bartik shift-share instrument to isolate exogenous variation in regional hiring conditions. By leveraging national industry-level hiring shocks weighted by historical state industry shares, the instrument generates plausibly exogenous shifts in local job filling rates. State-level regressions reveal that subjective forecasts of earnings growth respond strongly to these local shocks, even after controlling for state and time fixed effects, while discount rates respond less (Table A.14). The results confirm that belief distortions—especially overreaction to perceived cash flow opportunities—are not merely correlated with labor demand, but causally influence hiring decisions across regions.

Decreasing Returns to Scale and Composition Effects Appendix Section A.10 extends the baseline decomposition by allowing for decreasing returns to scale (DRS), which introduces a wedge between marginal and average profits and adds composition effects due to shifts in firm size. The resulting hiring condition incorporates not only expected earnings-employment and discount rate components, but also an output-employment term capturing the DRS wedge and an employment share term capturing compositional dynamics. Using survey-based forecasts and IBES data, Figure A.7 shows that under subjective expectations, the output-employment ratio accounts for about 30% of the variation in the job filling rate, while the earnings-employment ratio explains just under 60%. The compositional term plays a minor role, confirming that even after accounting for DRS and firm heterogeneity, belief distortions in expected cash flows remain the dominant driver of hiring fluctuations.

On-the-Job Search Appendix Section A.11 extends the baseline model to incorporate on-the-job search, recognizing that measured earnings and hires include both unemployed-to-employed and job-to-job transitions. By allowing a fraction of employed workers to search and selectively switch jobs, the model introduces an endogenous retention rate and adjusts the hiring efficiency accordingly. The decomposition shows that, even after accounting for job-to-job transitions, subjective expectations about future cash flows remain the primary driver of hiring fluctuations. Job-to-job transitions explain a modest share—about 9%—of the variation in the job filling rate, confirming that distortions in expected earnings continue to dominate firms' hiring behavior.

Cyclicality of the Subjective User Cost of Labor Appendix Section A.12 shows that subjective wage expectations are far less cyclical than realized wages, leading firms to perceive the user cost of labor as relatively rigid over the business cycle. Using survey data, both aggregate and individual-level analyses reveal that workers and firms consistently underestimate the extent of wage fluctuations over the business cycle (Figure A.9 and Table A.15). As a result, firms may fail to anticipate declines in wages during downturns, which keeps the perceived user cost of labor high and discourages job creation.

Additional Results Appendix Section A presents the following supplementary results. Table A.1 reports summary statistics for each variable used in the empirical analysis. Table A.2 estimates Coibion and Gorodnichenko (2015) regressions of forecast errors on forecast revisions. The variance decomposition of the job filling rate from Table 2 is robust to alternative specifications: using Vector Autoregressions (Table A.4), taking first-differences (Table A.5), replacing machine learning forecasts with their ex-post realized values (Table A.6), conditioning on control variables including lagged job filling rates and survey forecasts (Table A.7), and allowing for time-varying vacancy posting costs (Section A.4.5). Table A.8 compares the variance decomposition obtained from survey and machine learning forecasts against the theoretical predictions from existing search-and-matching models.

Predictable deviations from rational expectations can explain a substantial share of job filling rate variation since the job filling rate strongly predicts survey biases, defined as the deviation of subjective expectations from a rational expectations benchmark (Table A.9). Decompositions under risk-neutral expectations show that subjective expectations overweight long-horizon cash flows even against risk-neutral expectations (Figure A.1). The significant contribution of rational discount rates is driven by fluctuations in risk premia instead of risk-free rates (Figure A.2). Figure A.3 estimates time-series decompositions of the hiring rate separately for each of the five book-to-market portfolios and find that subjective beliefs strongly overstate cash flow effects for low book-to-market (growth) firms.

Job filling rates are closely linked to the forward-looking price-employment ratio, consistent with the baseline results (Table A.10). The baseline result is robust to using alternative survey sources such as the Bloomberg and CFO surveys (Tables A.11 and A.12) Estimating a similar decomposition for labor market tightness assuming a constant returns to scale matching function (Table A.13) shows that subjective earnings expectations account for nearly all variation, while rational models emphasize discount rates, echoing the findings for job filling rates.

10 Conclusion

This paper examines how belief distortions drive unemployment fluctuations by comparing survey-based subjective expectations with machine learning forecasts that proxy for rational expectations. Using a decomposition of the job filling rate grounded in a search-and-matching model, I show that firms' hiring decisions under subjective expectations are driven almost entirely by expected future cash flows, with little role for discount rates. In contrast, rational expectations assign a dominant role to discount rate variation. Cross-sectional evidence using firm level data reinforces this mechanism: firms exhibiting stronger distortions in cash flow expectations experience larger swings in their hiring rates. To interpret these findings, I develop a structural model in which firms engage in constant-gain learning about earnings growth and base hiring decisions on evolving subjective beliefs. The model reproduces both the time-series and cross-sectional patterns observed in the data.

Together, the results suggest that incorporating subjective expectations into existing models can enhance our understanding of labor market dynamics. While standard rational models imply that agents correctly understand the relative importance of discount rates and cash flows, belief distortions provide a complementary channel through which non-rational beliefs can influence hiring and unemployment volatility.

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A Appendix: Additional Results

A.1 Additional Stylized Facts

Table A.1: Summary statistics

Panel (a): Aggregate U.S.	Obs	Mean	St. Dev.	Min	p25	Median	p75	Max
$r_{t,t+5}$	68	0.300	0.294	-0.279	0.095	0.363	0.541	0.789
$\mathbb{F}_t[r_{t,t+5}]$	68	0.143	0.050	0.073	0.107	0.139	0.161	0.295
$\mathbb{E}_t[r_{t,t+5}]$	68	0.188	0.119	-0.064	0.107	0.209	0.256	0.472
$e_{t,t+5}$	68	1.461	0.476	-0.053	1.216	1.513	1.683	2.785
$\mathbb{F}_t[e_{t,t+5}]$	68	1.859	0.147	1.548	1.807	1.859	1.952	2.124
$\mathbb{E}_t[e_{t,t+5}]$	68	0.851	0.070	0.692	0.804	0.861	0.897	0.978
pe_{t+5}	68	2.227	0.314	1.784	1.995	2.240	2.347	3.294
$\mathbb{F}_t[pe_{t,t+5}]$	68	1.675	0.192	1.322	1.534	1.678	1.762	2.132
$\mathbb{F}_t[pe_{t,t+5}]$	68	3.740	0.371	2.864	3.476	3.681	3.955	4.741
q_t	68	0.616	0.228	0.211	0.436	0.603	0.738	1.202
U_t	68	0.062	0.021	0.036	0.046	0.054	0.079	0.130
θ_t	68	0.573	0.298	0.160	0.328	0.542	0.735	1.233
δ_t	68	0.018	0.004	0.013	0.016	0.018	0.019	0.047

Panel (b): State-Level	Obs	Mean	St. Dev.	Min	p25	Median	p75	Max
$\mathbb{F}_t[r_{s,t,t+5}]$	4,358	0.085	0.041	-0.076	0.063	0.089	0.104	0.228
$\mathbb{E}_t[r_{s,t,t+5}]$	4,358	0.191	0.331	-0.992	-0.005	0.198	0.343	1.174
$\mathbb{F}_t[e_{s,t,t+5}]$	4,358	3.630	0.158	2.876	3.548	3.644	3.726	4.034
$\mathbb{E}_t[e_{s,t,t+5}]$	4,358	3.389	0.124	2.711	3.329	3.411	3.462	3.652
$\widehat{\mathbb{F}}_t[r_{s,t,t+5}]$	4,358	0.084	0.040	-0.090	0.064	0.088	0.107	0.240
$\widehat{\mathbb{E}}_t[r_{s,t,t+5}]$	4,358	0.207	0.317	-0.972	0.044	0.188	0.365	1.175
$\widehat{\mathbb{F}}_t[e_{s,t,t+5}]$	4,358	3.044	0.168	1.868	2.981	3.054	3.129	6.112
$\widehat{\mathbb{E}}_t[e_{s,t,t+5}]$	4,358	2.827	0.155	1.753	2.766	2.829	2.892	5.652
$q_{s,t}$	4,358	0.298	0.066	0.145	0.247	0.298	0.344	0.530
$U_{s,t}$	4,358	0.057	0.024	0.016	0.041	0.052	0.068	0.248
$\theta_{s,t}$	4,358	0.668	0.373	0.084	0.398	0.607	0.845	3.202
$\delta_{s,t}$	4,358	0.977	1.001	0.093	0.300	0.762	1.143	10.523

Notes: This table reports summary statistics for ex-post realized outcomes (Actual), subjective expectations (Survey), and machine expectations (Machine) of key variables used in the variance decomposition. Panel (a) reports aggregate U.S. statistics, and Panel (b) shows state-level statistics. The variables to be forecast are $h = 5$ year present discounted values of discount rates $r_{t,t+h}$, cash flows $e_{t,t+h}$, and price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Aggregate labor market variables include the job filling rate q_t, unemployment rate U_t, vacancy-to-unemployment θ_t, and job separation rate δ_t. The corresponding variables for state s are defined similarly. $\widehat{\mathbb{F}}_t[x_{s,t,t+5}]$ denotes the Bartik shift-share instrument for state-level expectation $\mathbb{F}_t[x_{s,t,t+5}]$ for some variable x. The instrument interacts state-level employment shares in 2-digit NAICS industries with national industry-specific fluctuations in survey or machine forecasts. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The sample is quarterly and spans 2005Q1 to 2021Q4.

A.2 Predictability of Survey Forecast Errors

To assess whether survey expectations systematically deviate from rational expectations, panel (a) of Table A.2 estimates Coibion and Gorodnichenko (2015) regressions of forecast errors on forecast revisions:

$$y_{t,t+h} - \mathbb{F}_t[y_{t,t+h}] = \beta_{FE,0} + \beta_{FE,1}[\mathbb{F}_t[y_{t,t+h}] - \mathbb{F}_{t-1}[y_{t,t+h}]] + \beta_{FE,2}\mathbb{F}_{t-1}[y_{t,t+h}] + \varepsilon_{FE,t} \tag{A.1}$$

Cash Flow (Earnings) Forecasts: The negative coefficients ($\beta_{FE,1}$ ranging from -0.317 to -0.965) confirm over-reaction to earnings news. Upward forecast revisions predict negative forecast errors, indicating that survey respondents update too aggressively to cash flow information and produce overly optimistic forecasts.

Discount Rate (Return) Forecasts: The positive coefficients ($\beta_{FE,1}$ ranging from 2.453 to 3.731) indicate under-reaction to return news. Upward forecast revisions predict positive forecast errors, suggesting that survey respondents update insufficiently to discount rate information, leading to persistently pessimistic forecasts.

These asymmetric responses—over-reaction to cash flows and under-reaction to discount rates—provide validation of the main variance decomposition results using a different empirical methodology.

To assess how much of these systematic patterns are driven by genuinely irrational expectations versus limitations of survey-based forecasting, panel (b) of Table A.2 estimates the same regression but replaces survey forecasts with machine learning forecasts. The results show coefficients with the same signs as in panel (a), but they are dramatically smaller in magnitude and none are statistically significant. This stark contrast suggests that machine learning forecasts exhibit minimal predictable forecast errors, supporting their use as a rational expectations benchmark. The absence of significant predictability in machine learning forecast errors indicates they more closely approximate the behavior expected under rational expectations, where forecast errors should be unpredictable.

Table A.2: Predictability of Survey Forecast Errors

Horizon h (Years)	1	2	3	4	5
(a) Survey Forecast Errors: $y_{t,t+h} - \mathbb{F}_t[y_{t,t+h}] = \beta_{FE,0} + \beta_{FE,1}[\mathbb{F}_t[y_{t,t+h}] - \mathbb{F}_{t-1}[y_{t,t+h}]] + \beta_{FE,2}\mathbb{F}_{t-1}[y_{t,t+h}] + \varepsilon_{FE,t}$					
Discount Rate	2.453^{***}	3.448^{***}	3.731^{***}	3.422^{**}	3.056^{**}
t-stat	(3.135)	(4.053)	(3.377)	(2.495)	(2.010)
Cash Flow	-0.317^{**}	-0.476^{***}	-0.799^{***}	-0.813^{***}	-0.965^{***}
t-stat	(−1.969)	(−4.337)	(−5.490)	(−5.523)	(−7.896)
(b) Machine Forecast Errors: $y_{t,t+h} - \mathbb{E}_t[y_{t,t+h}] = \beta_{FE,0} + \beta_{FE,1}[\mathbb{E}_t[y_{t,t+h}] - \mathbb{E}_{t-1}[y_{t,t+h}]] + \beta_{FE,2}\mathbb{E}_{t-1}[y_{t,t+h}] + \varepsilon_{FE,t}$					
Discount Rate	0.234	−0.005	0.221	0.010	0.096
t-stat	(1.224)	(−0.036)	(1.638)	(0.114)	(0.973)
Cash Flow	−0.255	−0.454	−0.223	−0.531	−0.280
t-stat	(−0.507)	(−1.387)	(−1.255)	(−1.563)	(−1.594)

Notes: Table reports slope coefficients $\beta_{FE,1}$ and $\beta_{B,1}$ from regressions of the survey forecast error and its bias on the survey forecast revisions. $y_{t,t+h}$ denotes the variable y to be predicted h years ahead of time t: h year present discounted values of discount rates ($r_{t,t+h}$), cash flows ($e_{t,t+h}$), and log price-earnings ratios ($pe_{t,t+h}$). The survey bias is defined as ex-ante deviations of the survey forecast from its machine learning benchmark: $\mathbb{E}_t[y_{t,t+h}] - \mathbb{F}_t[y_{t,t+h}]$. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3 Gradual Adjustment of Expectations: Constant-Gain Learning

To provide evidence on the dynamics of belief formation, this section examines how survey respondents revise their expectations about future earnings following an earnings surprise. The following regression estimates the responsiveness of long-horizon forecasts to short-term earnings news:

$$\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j},$$

where $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}]$ denotes the expectation formed at time $t+j$ for earnings-related variable $\widetilde{x}$ at horizon h, and $\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]$ captures the earnings surprise. The coefficient $\gamma_{h,j}$ measures how much of the surprise is incorporated into expectations for long-run outcomes.

Table A.3 reports estimates for two forward-looking variables: (a) long-run earnings growth, and (b) the long-run ratio of earnings to employment. The target horizon is fixed at $h = 5$ years, while the revision horizon j ranges from 1 to 4 years. The estimated $\gamma_{h,j}$ coefficients are uniformly small and often statistically insignificant, indicating that respondents only partially incorporate short-term earnings news into their long-run expectations. This pattern is consistent with models of belief formation under constant-gain learning, in which agents update expectations gradually and exhibit fading memory. In such models, a fixed updating gain leads to persistent deviations from rational expectations and a breakdown of the law of iterated expectations.

These results echo findings in recent studies such as Nagel and Xu (2022) and De La O et al. (2024), which document slow-moving expectations in both financial and macroeconomic contexts. The evidence presented here supports the interpretation that belief distortions in my model—particularly exaggerated subjective cash flow expectations—are not merely the result of transitory shocks, but instead reflect persistent learning dynamics grounded in empirically observed behavior.

Table A.3: Gradual adjustment of expectations

Regression: $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j}$				
Target Horizon h (Years)	5	5	5	5
Revision Horizon j (Years)	1	2	3	4
(a) Earnings Growth	0.0929	0.0934	0.1121	0.1245
	(0.0734)	(0.0455)	(0.0776)	(0.0743)
(b) Earnings to Employment	0.0600	0.0508	0.0697	0.0745
	(0.1281)	(0.0725)	(0.0321)	(0.0419)

Notes: Table shows the gradual adjustment of expectations about future earnings $\widetilde{x}_{i,t+h}$ after an earnings surprise at $t+1$. The sample period is 1999 to 2000. Newey-West corrected t-statistics with lags 12 are reported in parentheses. *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4 Variance Decomposition of Job Filling Rate: Additional Results

A.4.1 VAR Estimates

To further validate the robustness of the variance decomposition in Table 2, I estimate a Vector Autoregression (VAR) model of the job filling rate and its key determinants under both subjective and rational expectations. The estimated system follows a VAR(1) specification:

$$X_{t+1} = AX_t + \varepsilon_{t+1} \tag{A.2}$$

where the state vector X_t consists of the present discounted values of discount rates, cash flows, log price-earnings ratios, and the log job filling rate.

Table A.4 reports the estimates across forecast horizons of $h = 1$ to $h = 5$ years, along with the full (long-run) horizon $h = \infty$. Under rational expectations, discount rate fluctuations explain a moderate but significant share of the variance in job filling rates, rising from 16% at the one-year horizon to 71% at the 5-year horizon, and 85% for the full horizon. Under subjective expectations, cash flow expectations overwhelmingly dominate, accounting for 34% at the one-year horizon and rising to 94% at the 5-year horizon, and 95% for the full horizon. Subjective discount rate fluctuations play an insignificant role, remaining close to zero across all horizons. These findings confirm the main results in Table 2: Firms using subjective expectations place disproportionate weight on cash flow expectations, whereas models with rational expectations emphasize the role of discount rate fluctuations.

Table A.4: Variance Decomposition of Job Filling Rate: VAR Estimates

Horizon h (Years)	1	2	3	4	5	∞
Rational Expectations $X_t = [\mathbb{E}_t[r_{t,t+1}], \mathbb{E}_t[e_{t,t+1}], \mathbb{E}_t[pe_{t,t+1}], \log q_t]'$						
Discount Rate	0.16	0.28	0.44	0.59	0.71	0.84
(-) Cash Flow	0.03	0.05	0.08	0.11	0.13	0.16
(-) Price-Earnings	0.81	0.67	0.48	0.30	0.16	0.00
Subjective Expectations $X_t = [\mathbb{F}_t[r_{t,t+1}], \mathbb{F}_t[e_{t,t+1}], \mathbb{F}_t[pe_{t,t+1}], \log q_t]'$						
Discount Rate	0.02	0.03	0.05	0.05	0.05	0.05
(-) Cash Flow	0.34	0.58	0.79	0.90	0.94	0.95
(-) Price-Earnings	0.64	0.39	0.16	0.05	0.01	0.00

Notes: Table reports variance decompositions based on a Vector Autoregression (VAR) model of the job filling rate and its key determinants under both subjective and rational expectations, for horizons of $h = 1$ to $h = 5$ years, along with the full (long-run) horizon $h = \infty$. The estimated system follows a VAR(1) specification $X_{t+1} = AX_t + \varepsilon_{t+1}$, where the state vector X_t consists of the present discounted values of discount rates, cash flows, log price-earnings ratios, and the log job filling rate. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2021Q4.

A.4.2 First Differences

Table A.5: Variance Decomposition of Job Filling Rate: First Differences

Horizon h (Years)	1	2	3	4	5
(a) Rational Expectations: First Differences $\Delta \log q_t = \Delta\mathbb{E}_t[r_{t,t+h}] - \Delta\mathbb{E}_t[e_{t,t+h}] - \Delta\mathbb{E}_t[pe_{t,t+h}]$					
Discount Rate	0.343*	0.479**	0.521**	0.551***	0.587***
t-stat	(1.877)	(2.251)	(2.493)	(2.856)	(3.013)
(-) Cash Flow	0.008	0.028	0.121	0.072	0.100
t-stat	(0.430)	(0.268)	(1.226)	(1.077)	(1.107)
(-) Price-Earnings	0.888**	0.777**	0.745*	0.398	0.362
t-stat	(2.398)	(2.165)	(1.918)	(0.959)	(0.893)
Residual	−0.239	−0.284	−0.387	−0.021	−0.049
t-stat	(−0.578)	(−0.660)	(−0.856)	(−0.045)	(−0.106)
N	68	68	68	68	68
(b) Subjective Expectations: First Differences $\Delta \log q_t = \Delta\mathbb{F}_t[r_{t,t+h}] - \Delta\mathbb{F}_t[e_{t,t+h}] - \Delta\mathbb{F}_t[pe_{t,t+h}]$					
Discount Rate	0.028	0.033	0.045	0.074	0.054
t-stat	(1.068)	(0.592)	(0.620)	(0.833)	(0.568)
(-) Cash Flow	0.286*	0.420**	0.601***	0.746***	0.835***
t-stat	(1.779)	(2.517)	(3.443)	(3.884)	(4.346)
(-) Price-Earnings	0.710***	0.529***	0.371	0.204	0.133
t-stat	(2.789)	(2.680)	(1.640)	(0.872)	(0.536)
Residual	−0.023	0.019	−0.016	−0.024	−0.023
t-stat	(−0.076)	(0.071)	(−0.054)	(−0.077)	(−0.070)
N	68	68	68	68	68

Notes: Table reports variance decompositions of the aggregate job filling rate under subjective expectations (panel (a)) or rational expectations (panel (b)). Each row reports the share of the variation in job filling rates that can be explained by h-year present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$. Residual represents the variation in job filling rates that are not captured by the other components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.3 Ex-post Decomposition

Since the log-linear decomposition of the job filling rate holds both ex-ante and ex-post, a variance decomposition of the job filling rate can also be estimated using ex-post realized data, under the assumption of the firm's perfect foresight:

$$1 \approx \underbrace{\frac{Cov\left[r_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate news}} - \underbrace{\frac{Cov\left[e_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[pe_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{A.3}$$

Table A.6 reports the estimates. Ex-post discount rate news is the main driver of fluctuations in job filling rates, with ex-post cash flow news playing only a minor role. At the 5 year horizon, 79.4% of the variation in the job filling rate is driven by discount rate news. In contrast, cash flow news has a smaller effect, contributing only 10.3% over the same period. The ex-post decomposition differs from both rational and subjective decompositions, suggesting that survey respondents not only need to learn about the true data generating process of the economy (when compared against rational expectations), but also hold significant distortions in their beliefs about them (when compared against subjective expectations).

Table A.6: Variance Decomposition of Job Filling Rate: Ex-Post Measure

Horizon h (Years)	1	2	3	4	5
Ex-Post Decomposition (Full Information Rational Expectations) $\log q_t = c_q + r_{t,t+h} - e_{t,t+h} - pe_{t,t+h}$					
Discount Rate	0.445	0.675**	0.705***	0.780***	0.794***
t-stat	(1.220)	(2.429)	(2.816)	(3.850)	(3.439)
(-) Cash Flow	−0.017	0.047	0.083	0.080	0.103
t-stat	(−0.191)	(0.567)	(0.996)	(1.026)	(1.206)
(-) Price-Earnings	0.572***	0.273***	0.208***	0.123	0.074
t-stat	(7.423)	(5.965)	(4.037)	(1.468)	(0.671)
Residual	−0.000	0.005	0.003	0.016	0.028
t-stat	(−0.001)	(0.031)	(0.017)	(0.089)	(0.137)
N	68	68	68	68	68

Notes: Table reports variance decompositions of the job filling rate from equation using ex-post realized outcomes. The components of the decomposition are present discounted values of discount rates $r_{t,t+h}$, cash flows $e_{t,t+h}$, and log price-earnings ratios $pe_{t,t+h}$. Residual represents the variation in job filling rates that are not captured by the other components. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.4 Additional Controls

The large contribution from subjective long-term cash flow expectations in explaining the job filling rate is robust to conditioning on additional variables that could distort the relationship between long-term cash flow expectations and job filling rates. Table A.7 re-estimates the subjective variance decomposition at the 5 year horizon with additional control variables on the right-hand side of the regression: 1 year lag of the log job filling rate, 1 year lag of the dependent variable, and the 1 year ahead survey forecast of the same variable. Controlling for the short-term expectation $\mathbb{F}_t[y_{t+1}]$ accounts for the possibility that survey respondents' long-term forecasts could be influenced by the short-term component of cash flows (Nagel and Xu, 2021).

Table A.7: Variance Decomposition of Job Filling Rate: Additional Controls

Regression: $\mathbb{F}_t[y_{t,t+h}] = \beta_{0,\mathbb{F}} + \beta_{1,\mathbb{F}} \log q_t + \beta_{2,\mathbb{F}} \log q_{t-1} + \beta_{3,\mathbb{F}}\mathbb{F}_{t-1}[y_{t+h-1}] + \beta_{4,\mathbb{F}}\mathbb{F}_t[y_{t+1}] + \varepsilon_{t,\mathbb{F}}$			
Dep Var $y_{t,t+h}$	Discount Rate	Cash Flow	Price-Earnings
Horizon h (Years)	5	5	5
Share of job filling rate variation (β_1)	-0.007	0.734***	0.057
t-stat	(-0.108)	(3.942)	(0.556)
Adj. R^2	0.456	0.550	0.462
N	68	68	68
Controls	Yes	Yes	Yes

Notes: Table reports variance decompositions of the job filling rate under subjective expectations $\mathbb{F}_t$ implied by survey forecasts. $y_{t,t+h}$ denotes the dependent variable of type j to be predicted $h = 5$ years ahead of time t: h year present discounted values of discount rates ($r_{t,t+h} = \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$), cash flows ($e_{t,t+h} = el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$), and log price-earnings ratios ($pe_{t,t+h} = \rho^h pe_{t+h}$). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. The sample is quarterly over 2005Q1 to 2019Q3. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.5 Time-Varying Vacancy Posting Cost

This section extends the baseline decomposition to allow for vacancy posting costs that are *linear* in the job filling rate q_t, as in Petrosky-Nadeau et al. (2018). Specifically, assume that the cost of posting vacancies is given by

$$\kappa(q_t) = \kappa_0 + \kappa_1 q_t, \tag{A.4}$$

where $\kappa_0 = 0.5$ is a fixed component and $\kappa_1 = 0.5$ captures the sensitivity of vacancy posting costs to the job filling rate. The firm's optimality condition equates the marginal cost of filling a vacancy to the marginal value of a filled job:

$$\kappa_1 + \frac{\kappa_0}{q_t} = \frac{P_t}{L_{t+1}}. \tag{A.5}$$

Solving for q_t and taking logs:

$$\log q_t = \log \kappa_0 - \log\left(\frac{P_t}{L_{t+1}} - \kappa_1\right) \equiv \log \kappa_0 - \log\left(pl_t - \kappa_1\right) \tag{A.6}$$

If $\kappa_1 \ll \exp(pl_t)$, a first-order Taylor expansion gives:

$$\log q_t \approx \log \kappa_0 - pl_t + \frac{\kappa_1}{\exp(pl_t)} \tag{A.7}$$

Combine with the Campbell and Shiller (1988) present value identity in equation (11) and collect terms:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{r_{t,t+h}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}\right]}_{e_{t,t+h}} - \underbrace{\rho^h pe_{t+h}}_{pe_{t,t+h}} + \underbrace{\frac{\kappa_1}{\exp(pl_t)}}_{\text{Vacancy Cost}} \tag{A.8}$$

where $c_q \equiv \log \kappa_0 - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The contribution of the vacancy cost term can be estimated by regressing it on $\log q_t$. Since pl_t and q_t are both current values, this term is identical under either subjective or rational expectations. Using time-series data from 2005Q1–2021Q4, the estimated contribution is:

$$\underbrace{\frac{Cov\left[\frac{\kappa_1}{\exp(pl_t)}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Vacancy Cost}} = 0.016 \tag{A.9}$$

indicating that time variation in vacancy posting costs plays a minor role in explaining fluctuations in the job filling rate.

A.4.6 Model vs. Data

Table A.8 compares the variance decomposition obtained using survey-based subjective expectations and machine learning-based rational expectations against the theoretical predictions from prominent search-and-matching models in the literature.

Hall (2017) and DMP (Diamond-Mortensen-Pisarides) models predict that discount rate fluctuations should explain more than 75% of the variance of job filling rates. Kehoe et al. (2022) (KLMP model) predict a more balanced role for both discount rates and cash flows, attributing 54% to discount rates and 32% to cash flows. Empirical results using data on subjective expectations differ from these models, showing that firms place almost no weight on discount rates (1.6%) and instead attribute 96.4% of the variance to cash flows. These differences suggest that belief distortions play a substantial role in shaping labor market fluctuations and challenge the standard rational expectations assumption in existing search models.

Table A.8: Variance Decomposition of Job Filling Rate: Model vs. Data

Dep. Var. Horizon h (Years)	Discount Rate 5	(-) Cash Flow 5	(-) Price-Earnings 5	Residual 5
		Subjective Expectations $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$		
Data (Survey)	0.016	0.964***	0.027	−0.007
t-stat	(0.180)	(7.016)	(0.077)	
Model (Learning)	−0.000	0.819***	0.181	0.000
t-stat	(−0.005)	(8.271)	(1.758)	
		Rational Expectations $\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}]$		
Data (Machine)	0.691***	0.066	0.201	0.042
t-stat	(3.329)	(0.472)	(0.245)	
Model (DMP)	0.782***	0.017**	0.201***	0.000
t-stat	(12.334)	(1.992)	(47.883)	
Model (Hall)	0.838***	0.073	0.088	0.000
t-stat	(12.000)	(1.387)	(1.074)	
Model (KLMP)	0.543***	0.319	0.138***	0.000
t-stat	(4.484)	(0.937)	(16.392)	
Model (Learning)	0.490***	0.105	0.405	0.000
t-stat	(2.806)	(1.442)	(1.212)	

Notes: Table compares the variance decomposition estimated from the data (Table 2) against the implied decomposition from simulations of alternative search-and-matching models. The models are simulated annually over 500 periods and 300 firms, discarding the first 150 periods as a burn-in, All parameter values in the calibration use estimates from the original papers. Learning: Constant-gain learning model from Section 8; DMP: Diamond-Mortensen-Pissarides Model; Hall: Hall (2017); KLMP: Kehoe et al. (2022). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.7 Biases in Subjective Beliefs

To directly quantify the importance of biases in subjective beliefs, I consider predictive regressions of biases in subjective expectations of discount rates, cash flows, and price-earnings ratios on the job filling rate. I define the bias as the difference between subjective and machine expectations. Table A.9 reports estimates $\beta_{1,B}$ from regressing biases in subjective discount rate, cash flow, and log price-earnings expectations on the job filling rate:

$$Bias_t[y_{t,t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \qquad y = r, e, pe \tag{A.10}$$

where the $Bias_t[y_{t,t+h}] \equiv \mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}]$ is defined as the difference between subjective and machine expectations of the same variable.

The results indicate that biases in survey forecasts are important contributors to fluctuations in job filling rates, especially at longer horizons. At the 5-year horizon, biases in cash flow expectations lead survey respondents to over-weight 70.5% of the variation in job filling rates to the cash flow component. This mis-perception is counteracted by biases in subjective discount rate expectations, which leads survey respondents to under-weight 67.5% of the variation in the job filling rate. These findings emphasize the importance of belief distortions in driving labor market fluctuations. The profile of the response across forecast horizons is broadly consistent with the profile of the MSE ratios across horizons in Table 1. For discount rate and cash flow expectations, the machine outperformed the survey by a wider margin over longer horizons, suggesting that the bias in survey responses likely play a bigger role over these longer horizons.

Table A.9: Biases in Subjective Beliefs and the Job Filling Rate

Horizon h (Years)	1	2	3	4	5
	Biases in Subjective Expectations $\mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \quad y = r, e, pe$				
Discount Rate	−0.169	−0.291**	−0.543**	−0.603***	−0.675***
t-stat	(−1.474)	(−1.992)	(−2.505)	(−2.734)	(−2.876)
(-) Cash Flow	0.403*	0.615***	0.686***	0.801***	0.898***
t-stat	(1.679)	(5.476)	(5.467)	(6.245)	(6.599)
(-) Price-Earnings	−0.265	−0.354**	−0.208	−0.259	−0.174
t-stat	(−0.675)	(−2.373)	(−0.484)	(−0.469)	(−0.293)
Residual	−0.031	−0.029	−0.065	−0.061	0.048
t-stat	(−0.066)	(−0.123)	(−0.130)	(−0.100)	(0.074)
N	68	68	68	68	68

Notes: This table reports estimates $\beta_{1,B}$ from regressing the survey bias $\mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}]$ on the job filling rate q_t. $y_{t,t+h}$ denotes the dependent variable of type j to be predicted h years ahead of time t. The components of the decomposition are h-year present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$. The residual term captures variation in the bias that cannot be explained by the three components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The bias is defined as the difference between subjective and machine expectations: $Bias_t = \mathbb{F}_t - \mathbb{E}_t$. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.8 Risk-Neutral Measure Implied by Futures Prices

To address whether systematic forecast errors simply reflect risk compensation rather than belief distortions, I re-evaluate the decomposition using risk-neutral expectations extracted from futures prices. Under risk-neutral pricing, forecast errors should equal risk premia plus noise, with no systematic patterns beyond those explained by time-varying risk compensation. In contrast to subjective survey forecasts, which may reflect belief distortions, risk-neutral expectations are extracted directly from financial market prices and reflect the valuations of marginal investors in the economy. The decomposition parallels the earlier analysis based on subjective beliefs but replaces the expectations operator $F_t[\cdot]$ with the risk-neutral operator $\mathbb{E}_t^Q[\cdot]$, where Q denotes the risk-neutral probability measure. I begin with the ex-post decomposition of the job filling rate $\log q_t$, which can be expressed as:

$$\log q_t = c_q + \sum_{j=1}^{h} \rho^{j-1} r_{t+j} - \left(dl_t + \sum_{j=1}^{h} \rho^{j-1} \Delta d_{t+j} \right) - \rho^h pd_{t+h} \tag{A.11}$$

where r_{t+j} denotes the return on the S&P 500 index, Δd_{t+j} denotes the change in log dividends, and pd_{t+h} is the terminal log price-dividend ratio.

To evaluate this decomposition under the risk-neutral measure, I replace each future variable with its risk-neutral expectation. Using the standard no-arbitrage pricing result that the futures price equals the risk-neutral expectation of the future spot price (Ait-Sahalia et al., 2001), I compute the expected return over horizon h using log differences of S&P 500 futures prices:

$$\mathbb{E}_t^Q[r_{t,t+h}] = \sum_{j=1}^{h} \rho^{j-1} (f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500}) \tag{A.12}$$

where $f_{t,t+j}^{sp500}$ denotes the log futures price of the S&P 500 at time t for delivery at $t+j$, and $f_{t,t}^{sp500} \equiv p_t$ is the log spot price. Similarly, I measure expected dividend growth using dividend futures:

$$\mathbb{E}_t^Q[d_{t,t+h}] = dl_t + \sum_{j=1}^{h} \rho^{j-1} (f_{t,t+j}^{div} - f_{t,t+j-1}^{div}) \tag{A.13}$$

where $f_{t,t+j}^{div}$ is the log price of the dividend future for maturity $t+j$, and $f_{t,t}^{div} \equiv d_t$ is the log of current dividends. To compute the terminal price-dividend ratio $\mathbb{E}_t^Q[pd_{t+h}]$, I apply a forward iteration of the log-linear price-dividend identity:

$$\mathbb{E}_t^Q[pd_{t+h}] = \frac{1}{\rho^h} pd_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pd} + \mathbb{E}_t^Q[\Delta d_{t+j}] - \mathbb{E}_t^Q[r_{t+j}]) \tag{A.14}$$

where c_{pd} is a constant from the log-linearization. Since market data on futures prices is typically limited to near-term maturities (e.g., 1-year ahead), I extrapolate longer-horizon expectations using fitted values from autoregressive models. Specifically, I estimate AR(1) processes for the 1-year futures returns and dividend growth:

$$f_{t,t+1}^{sp500} - p_t = \mu_{sp500} + \rho_{sp500}(p_t - p_{t-1}) + \varepsilon_t \tag{A.15}$$

$$f_{t,t+1}^{div} - d_t = \mu_{div} + \rho_{div}(d_t - d_{t-1}) + \varepsilon_t \tag{A.16}$$

and then forecast growth at horizons $j > 1$ recursively:

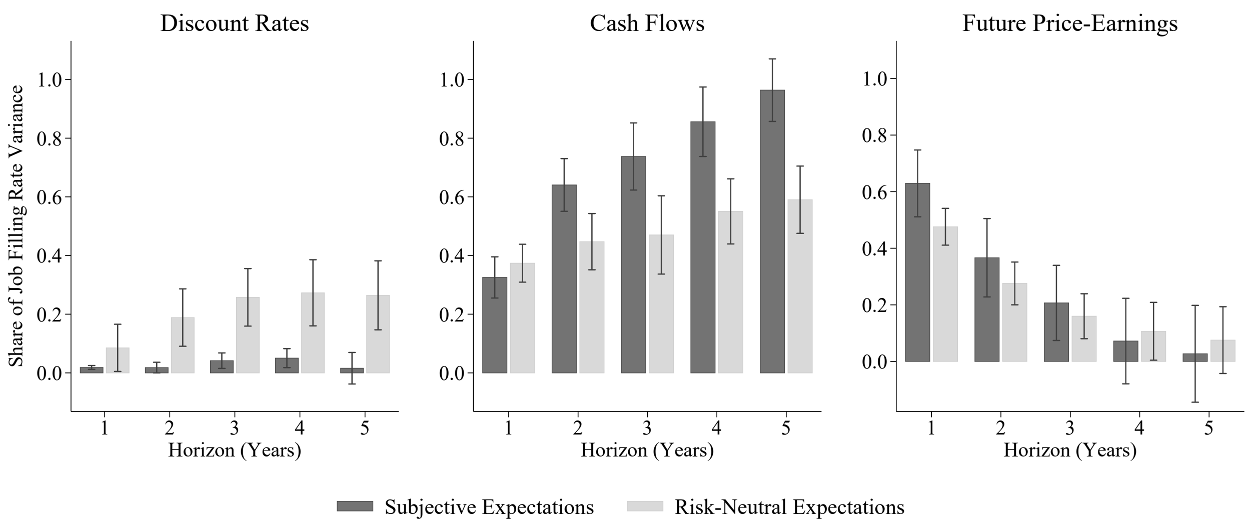
$$f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500} = \frac{\mu_{sp500}(1 - \rho_{sp500}^{j})}{1 - \rho_{sp500}} + \rho_{sp500}^{j-1}(f_{t,t+1}^{sp500} - p_t) \tag{A.17}$$

$$f_{t,t+j}^{div} - f_{t,t+j-1}^{div} = \frac{\mu_{div}(1 - \rho_{div}^{j})}{1 - \rho_{div}} + \rho_{div}^{j-1}(f_{t,t+1}^{div} - d_t) \tag{A.18}$$

Using these forward-imputed values, I compute the full set of risk-neutral expectations required for the decomposition.

The results of this exercise are shown in Figure A.1. Compared to subjective expectations, risk-neutral expectations attribute a smaller role to future cash flows and a greater role to discount rates in explaining the variation in the job filling rate. This contrast suggests that belief distortions in survey forecasts may overweight the informational content of short-term earnings outlooks and underweight changes in risk premia, leading to distorted hiring incentives.

Figure A.1: Variance Decomposition of Job Filling Rate: Risk-Neutral Expectations

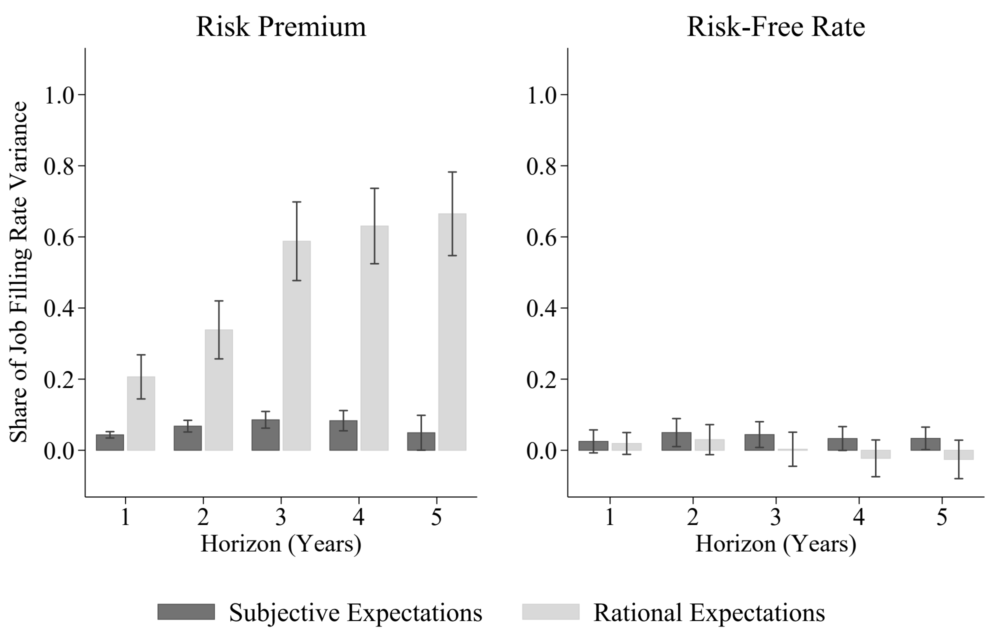


Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate. Light bars show the contribution under risk-neutral expectations implied by S&P 500 and dividend futures. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.4.9 Risk Premia vs. Risk-Free Rate

Risk-free rates play only a small role in explaining fluctuations in job filling rates. Figure A.2 plots estimates from regressing subjective expectations implied by forecasts from the Survey of Professional Forecasters (SPF), and machine expectations of h year ahead annualized log 3-month Treasury bill rates on the the job filling rate. Under all measures of beliefs and all horizons considered, the contribution from risk-free rates explain less than 5% of the variation in job filling rates. The result suggests that the significant contribution of rational discount rates in Table 2 is driven by fluctuations in risk premia instead of risk-free rates.

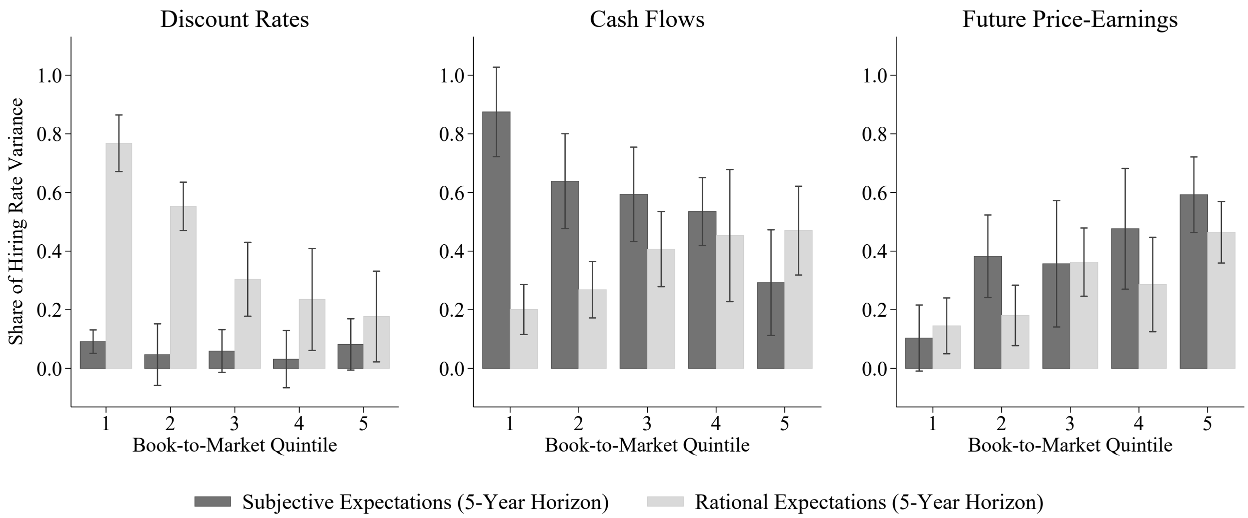
Figure A.2: Variance Decomposition of Job Filling Rate: Risk Premia vs. Risk-Free Rate



Notes: This figure plots estimates from regressing h year present discounted value of annualized log 3-month Treasury bill rates $\sum_{j=1}^{h} \rho^{j-1} r^{f}_{t+j}$ on the the job filling rate under alternative assumptions about the firm's beliefs. Subjective expectations $\mathbb{F}_t$ of risk-free rates are based on survey forecasts from the Survey of Professional Forecasters. Subjective expectations of the equity risk premium is defined as the difference between CFO survey S&P 500 stock return forecast and the SPF risk-free rate forecast. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The sample is quarterly from 2005Q1 to 2021Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.4.10 Cross-Sectional Decomposition of Hiring Rate: By B/M Portfolio

Figure A.3: Cross-Sectional Decomposition of Hiring Rate: By B/M Portfolio



Notes: Figure estimates time-series decomposition of hiring rate separately for each of the five book-to-market portfolios. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.4.11 Job Filling Rate and Valuation Ratios

The job filling rate in the search model is closely related to price-employment ratio. Under constant returns to scale, the hiring equation directly relates the job filling rate with the price-employment ratio:

$$\frac{Cov\left[pl_t, \log q_t\right]}{Var\left[\log q_t\right]} \approx -1 \tag{A.19}$$

where pl_t is the log stock price to employment ratio. Table A.10 tests this prediction by regressing log price-employment on log job filling rate:

$$pl_t = \alpha + \beta \log q_t + \varepsilon_t \tag{A.20}$$

The estimates show that β not significantly different from -1, and the R^2 of the regression is relatively high, consistent with predictions of the model.

Table A.10: Role of Price-Employment Ratios

Dep. Var.	Log Price-Employment pl_t	
Specification	Levels	First Differences
Share of job filling rate variation (β)	-0.988	-0.977
t-stat	(0.204)	(0.203)
Adj. R^2	0.895	0.780
N	76	76

Notes: Table reports slope coefficient from regressing the aggregate log price-employment ratio on the aggregate log job filling rate. The sample is quarterly from 2002Q1 to 2020Q4. Newey-West corrected t-statistics for a two-sided t-test of $H_0 : \beta = -1$ with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.5 Alternative Survey Expectations

A.5.1 Alternative Measures of Subjective Cash Flow Expectations

The large role played by subjective cash flow expectations in explaining the job filling rate holds more generally across alternative survey forecasts of earnings growth. Table A.11 re-estimates the subjective variance decomposition while replacing IBES survey forecasts of earnings growth with the corresponding forecast from the Bloomberg (BBG) survey. Table A.11 re-estimates the subjective variance decomposition while replacing 1 year ahead IBES survey forecasts of earnings growth with the corresponding forecast from the CFO survey.[1]

To summarize the alternative survey measures into a single series, I extract the common component of subjective discount rates using a Kalman filter. The state variable is a latent h-month ahead expected stock return capturing investors' subjective beliefs $S_t \equiv \mathbb{F}_t[r_{t+h}]$, which evolves according to an AR(1) state equation $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$, where C, T, R are matrices of the model's primitive parameters $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$. ε_t is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster. α is the intercept, ρ is the persistence, and σ_ε is the standard deviation of the latent innovation error. The Observation equation takes the form $X_t = D + ZS_t + Uv_t$, where h is a fixed forecast horizon. The observation vector X_t contains measures of survey expected cash flows from IBES, BBG, and CFO surveys over the next h periods. v_t is a vector of observation errors with standard deviations in the diagonal matrix U. Z and D are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters $\alpha, \rho, \sigma_\varepsilon, U$. Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table A.11: Variance Decomposition of Job Filling Rate: Alternative Measures of Subjective Cash Flow Expectations

Horizon h (Years)	1	2	3	4	5
			Subjective Expectations $\mathbb{F}_t[\cdot]$ $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$		
			(a) Kalman Filter (KF) Composite		
(-) Cash Flow	0.578***	0.625***	0.682***	0.877***	0.933***
t-stat	(3.046)	(4.275)	(4.722)	(5.891)	(7.640)
			(b) Bloomberg (BBG) Survey		
(-) Cash Flow	0.586***	0.830***	0.851***	0.896***	0.949***
t-stat	(8.476)	(8.317)	(7.213)	(5.288)	(4.541)
			(c) CFO Survey		
(-) Cash Flow	0.637*				
t-stat	(1.934)				

Notes: Table reports variance decompositions of the job filling rate while replacing IBES earnings growth forecast with alternative surveys as measures of subjective cash flows. KF summarizes the alternative survey measures into a single series using a Kalman filter. The sample for BBG and KF is quarterly from 2006Q1 to 2022Q4. The sample for CFO is quarterly from 2005Q1 to 2019Q3. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

[1] The forecast horizon for the CFO survey has been limited to $h = 1$ year ahead and the sample covers a shorter period over 2005Q1 to 2019Q3 due to missing earnings growth forecasts in the CFO survey.

A.5.2 Alternative Measures of Subjective Discount Rates

The small role played by subjective discount rate expectations in explaining the job filling rate holds more generally across alternative survey forecasts of stock returns. Table A.12 reports estimates from regressing 1 year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log job filling rate q_t under alternative survey forecasts of stock returns. In all survey measures, the estimates suggest a weak relationship between subjective stock return expectations $\mathbb{F}_t^s[r_{t,t+h}]$ and the job filling rate q_t.

$r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2023). The forecast horizon has been limited to 1 year ahead due to limited data availability in the alternative surveys. The sample is quarterly over 2005Q1 to 2021Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2007Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2021Q4 from Q2 and Q4 of each calendar year for Livingston.

To summarize the alternative survey measures into a single series, I extract the common component of subjective discount rates using a Kalman filter. The state variable is a latent h-month ahead expected stock return capturing investors' subjective beliefs $S_t \equiv \mathbb{F}_t[r_{t+h}]$, which evolves according to an AR(1) state equation $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$, where C, T, R are matrices of the model's primitive parameters $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$. ε_t is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster. α is the intercept, ρ is the persistence, and σ_ε is the standard deviation of the latent innovation error. The Observation equation takes the form $X_t = D + ZS_t + Uv_t$, where $h = 12$ months is a fixed forecast horizon. The observation vector X_t contains measures of survey expected returns listed above over the next h periods. v_t is a vector of observation errors with standard deviations in the diagonal matrix U. Z and D are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters $\alpha, \rho, \sigma_\varepsilon, U$. Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table A.12: Variance Decomposition of Job Filling Rate: Alternative Discount Rates

Regression: $\mathbb{F}_t^s[r_{t,t+h}] = \beta_{0,\mathbb{F}} + \beta_{1,\mathbb{F}} \log q_t + \varepsilon_{t,\mathbb{F}}$								
Dep Var	Discount Rate	Discount Rate	Discount Rate	Discount Rate	Discount Rate	Discount Rate	Discount Rate	Discount Rate
Survey s	KF	NX	CB	SOC	Gallup	Liv	CoC	Hurdle
Horizon h (Years)	1	1	1	1	1	1	1	1
β_1	0.013	−0.011	0.026	0.002	−0.065	0.067	0.024	0.013
t-stat	(0.614)	(−0.249)	(0.504)	(0.103)	(−0.922)	(0.181)	(0.734)	(0.522)
Adj. R^2	0.070	0.012	0.069	0.009	0.216	0.045	0.232	0.154
N	68	68	68	68	16	34	68	68

Notes: Table reports slope (β_1) estimates from regressing $h = 1$ year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log job filling rate q_t. $r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2023). KF summarizes the alternative survey measures into a single series using a Kalman filter. The sample is quarterly over 2005Q1 to 2021Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2007Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2021Q4 from Q2 and Q4 of each calendar year for Livingston. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.6 Variance Decomposition of Aggregate Labor Market Tightness

Table A.13 estimates a variance decomposition of aggregate labor market tightness (θ_t), defined as the ratio of vacancies to unemployment, under subjective expectations and rational expectations. Under rational expectations, discount rates and cash flows both contribute to fluctuations in labor market tightness. Under subjective expectations, cash flow news dominates, explaining nearly 80% of the variation in labor market tightness. Fluctuations in subjective discount rates play no significant role. These results suggest that belief distortions in cash flow expectations are a primary driver of aggregate labor market dynamics, leading to excessive volatility in unemployment and hiring decisions.

Table A.13: Variance Decomposition of Labor Market Tightness

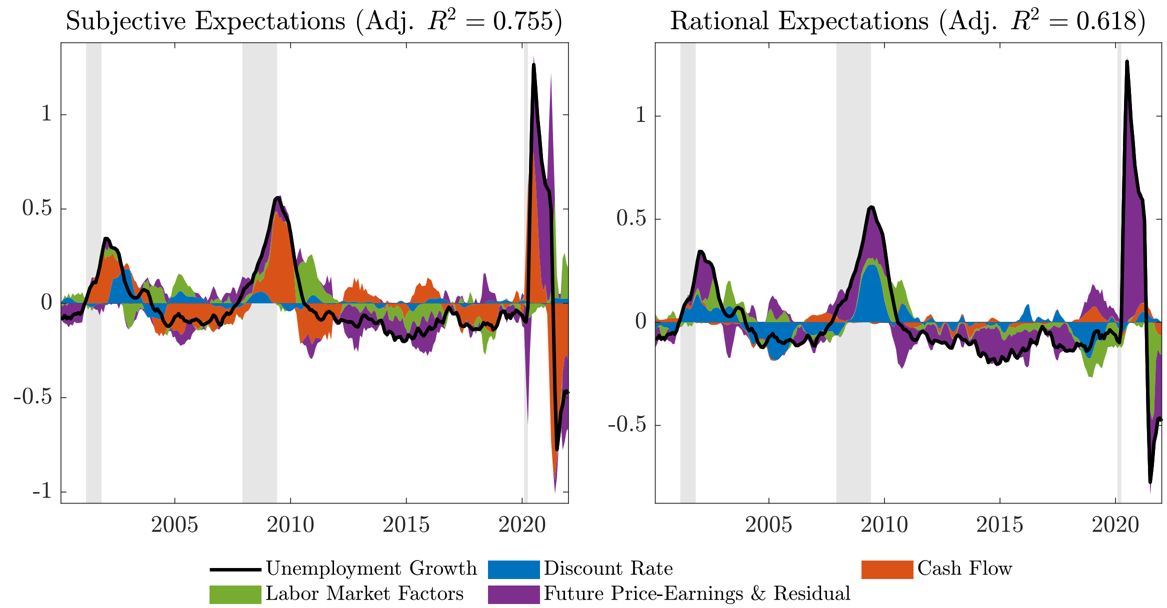
Horizon h (Years)	1	2	3	4	5
			Rational Expectations $\theta_t = c_\theta - \mathbb{E}_t[r_{t,t+h}] + \mathbb{E}_t[e_{t,t+h}] + \mathbb{E}_t[pe_{t,t+h}]$		
(-) Discount Rate	0.131***	0.275***	0.435***	0.475***	0.515***
t-stat	(3.478)	(3.577)	(3.793)	(3.045)	(2.678)
Cash Flow	0.060	0.133	0.149	0.159	0.171*
t-stat	(1.331)	(1.560)	(1.376)	(0.836)	(1.722)
Price-Earnings	0.795***	0.617***	0.450	0.371	0.288
t-stat	(4.452)	(2.751)	(1.168)	(0.935)	(0.555)
Residual	0.013	−0.025	−0.033	−0.006	0.026
t-stat	(0.069)	(−0.099)	(−0.080)	(−0.013)	(0.045)
N	68	68	68	68	68
			Subjective Expectations $\theta_t = c_\theta - \mathbb{F}_t[r_{t,t+h}] + \mathbb{F}_t[e_{t,t+h}] + \mathbb{F}_t[pe_{t,t+h}]$		
(-) Discount Rate	0.015	0.018	0.033	−0.006	−0.094
t-stat	(1.128)	(0.771)	(0.940)	(−0.090)	(−1.060)
Cash Flow	0.688***	0.698***	0.666***	0.737***	0.772***
t-stat	(5.599)	(5.772)	(4.586)	(4.182)	(4.623)
Price-Earnings	0.331**	0.253	0.287	0.237	0.322
t-stat	(2.044)	(1.420)	(0.882)	(0.726)	(1.124)
Residual	−0.034	0.032	0.014	0.032	0.001
t-stat	(−0.165)	(0.146)	(0.038)	(0.086)	(0.002)
N	68	68	68	68	68

Notes: Table reports variance decompositions of aggregate labor market tightness, defined as the ratio of vacancies to unemployment, under subjective expectations or rational expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.7 Time-Series Decomposition of the Unemployment Rate

Figure A.4 plots the actual annual change in unemployment against its model-implied decomposition using both subjective and rational expectations based on equation (24). Under subjective beliefs, fluctuations in unemployment closely track the component attributed to expected cash flow news. The model captures the sharp rise and fall in unemployment during the COVID-19 crisis with considerable precision. The fit of the model in terms of R^2 is notably better under subjective expectations than under rational expectations, indicating that incorporating belief distortions improves the model's explanatory power.

Figure A.4: Time-Series Decomposition of the Unemployment Rate



Notes: Figure plots decompositions of log annual growth in the unemployment rate from equation (A.98), under subjective (left panel) or rational (right panel) expectations. Labor market factors include the log annual growth of lagged unemployment Δu_t, labor market tightness $\Delta\theta_t$ and job separations $\Delta\delta_t$. Residual term represents the variation in job filling rates that are not captured by the other components. NBER recessions are shown with gray shaded bars.

A.8 Regional Model and Shift-Share Instrument

The aggregate analysis in Section 5 shows that belief distortions in subjective expectations play an important role in explaining hiring fluctuations. This section extends that analysis by exploiting cross-sectional variation in state-level data to strengthen identification and test whether the theoretical mechanism generalizes beyond aggregate dynamics.

Overview While the aggregate-level variance decompositions are informative, they cannot establish causality. The limited number of business cycles in the time series also restricts inference. This section addresses these challenges by extending the aggregate model to a regional framework. In estimating the regional model, I introduce a Bartik shift-share instrument for survey expectations to address endogeneity challenges in identifying the relative importance of subjective discount rate and cash flow expectations. Specifically, I investigate whether regional labor markets characterized by more distorted subjective cash flow expectations experience larger swings in job filling rates. This analysis is motivated by empirical evidence of substantial geographic variation in unemployment dynamics, especially during crises (Beraja et al., 2019, Kehoe et al., 2019; Chodorow-Reich and Wieland, 2020). While existing work studies these regional differences under a rational expectations framework, differences in subjective beliefs may also be an important explanatory factor.

Regional Model To guide the empirical strategy, I extend the baseline search model to a multi-region, multi-sector environment, building from the models in Kehoe et al. (2019) and Chodorow-Reich and Wieland (2020). The economy consists of a continuum of islands indexed by s. Each island produces a differentiated variety of tradable goods that is consumed everywhere and a nontradable good. Both of these goods are produced using intermediate goods. Each consumer is endowed with one of two types of skills which are used in different intensities in the nontradable and tradable goods sectors. Labor is immobile across islands but can switch sectors.[2] Consumers receive utility from a composite consumption good that is either purchased in the market or produced at home. Consumers and firms are ex-ante homogeneous and share the same subjective expectation $\mathbb{F}_t[\cdot]$. The islands only differ in the shocks that hit them.

Predictability of Regional Unemployment Rates In this environment, the log unemployment rate $u_{s,t+1}$ in region s approximately satisfies the following predictive relationship:[3]

$$u_{s,t+1} = \beta_r \mathbb{F}_t[r_{s,t,t+h}] + \beta_e \mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.21}$$

where $X_{s,t} \equiv [u_{s,t}, \log \theta_{s,t}, \log \delta_{s,t}]'$ collects standard labor market controls: the lagged unemployment rate $u_{s,t}$, the log vacancy-to-unemployment ratio $\log \theta_{s,t}$, and the log separation rate $\log \delta_{s,t}$. The cross-sectional unit s corresponds to U.S. states, and time t is measured at the monthly frequency. Following Korniotis (2008), each firm is assigned to the state in which it is headquartered. The regression includes state fixed effects α_s to absorb time-invariant regional heterogeneity and time fixed effects α_t to capture national shocks. The coefficients of interest, β_r and β_e, quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

[2]This assumption aligns with empirical evidence indicating that labor markets are predominantly local in nature (Manning and Petrongolo, 2017).

[3]See Appendix Section B.3 for a derivation.

This regional equation extends the aggregate specification in equation (24), and is designed to test whether perceived shocks to discount rates or earnings forecasts help explain variation in unemployment across local labor markets. If firms form biased beliefs about future returns or earnings, those belief distortions should manifest in regional hiring behavior and thus influence unemployment at the state level. A counterpart regression can be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with machine learning-based forecasts $\mathbb{E}_t[\cdot]$.

Empirical Specification: OLS As a baseline, I estimate the regression above using multivariate OLS applied to a panel of state-level data. This allows for a direct assessment of whether variation in firm-level beliefs, aggregated to the state level, predicts changes in unemployment.[4] State-level forecasts of discount rates $\mathbb{F}_t[r_{s,t,t+h}]$ are constructed from IBES price target forecasts. These targets are used to infer expected returns by back-solving from analysts' price projections. Forecasts are assigned to states based on firm headquarters and then aggregated using value-weighted averages. Expected cash flows $\mathbb{F}_t[e_{s,t,t+h}]$ are constructed analogously from IBES analyst forecasts of earnings per share.

Regional labor market variables are constructed from publicly available BLS datasets. Unemployment rates $u_{s,t}$ are sourced from the Local Area Unemployment Statistics (LAUS). The vacancy-to-unemployment ratio $\theta_{s,t}$ is computed using job openings from the state-level Job Openings and Labor Turnover Survey (JOLTS) combined with unemployment counts from LAUS. Separation rates $\delta_{s,t}$ are also taken from JOLTS. Monthly series are time-aggregated to the quarterly frequency by averaging values within each quarter.

Empirical Specification: Bartik Shift-Share Instrument A key challenge in estimating the regional decomposition is that regional labor market conditions and subjective expectations may be jointly determined, potentially leading to biased estimates. For example, firms might revise their beliefs in response to local shocks in unemployment or hiring, making it difficult to separate cause from effect. Additionally, state-level aggregates of firm-level forecasts may suffer from measurement error if the geographic scope of a firm's operations does not align with the location of its headquarters.

To address these concerns, I construct a Bartik-style shift-share instrument $\widehat{\mathbb{F}}_t[y_{s,t,t+h}]$ to isolate exogenous variation in subjective expectations at the regional level:

$$\widehat{\mathbb{F}}_t[y_{s,t,t+h}] = \sum_{i \in I} s_{s,i,t-1}\mathbb{F}_t[y_{i,t,t+h}], \qquad s_{s,i,t} = \frac{L_{s,i,t}}{\sum_{i' \in I} L_{s,i',t}}, \qquad x = r, e \tag{A.22}$$

where $s_{s,i,t}$ is the share of state s's employment in industry i, sourced from the Quarterly Census of Employment and Wages (QCEW). $\mathbb{F}_t[x_{i,t,t+h}]$ is the national forecast of variable $y \in \{r, e\}$ for industry i based on IBES survey data. Industries are defined at the 2-digit NAICS level. The instrument thus captures the component of state-level expectations that is driven by exogenous national industry trends, weighted by state-level industrial composition.

Using the Bartik instrument, I estimate the following predictive regression:

$$u_{s,t+1} = \beta_r\widehat{\mathbb{F}}_t[r_{s,t,t+h}] + \beta_e\widehat{\mathbb{F}}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.23}$$

The coefficients β_r and β_e now reflect the causal impact of plausibly exogenous variation in discount rate and earnings expectations on future unemployment at the state level.

[4] The future price-earnings ratio term $\mathbb{F}_t[pe_{s,t,t+h}]$ is omitted from the regression due to its near collinearity with forecasted discount rates and cash flows via the present-value identity of Campbell and Shiller (1988).

Identification Assumptions Compared to the OLS specification, the Bartik approach offers stronger identification by addressing both measurement error and endogeneity concerns. First, it reduces measurement error by replacing noisy state-level aggregates of firm-level forecasts with industry-level forecasts weighted by predetermined employment shares. Second, it mitigates endogeneity by exploiting the fact that national industry trends in expectations are unlikely to respond to contemporaneous state-level labor market shocks.

The identifying assumption is that, conditional on fixed effects and controls, there are no omitted factors that simultaneously affect both national industry-level expectations and local hiring behavior in states more exposed to those industries. This assumption would be violated, for example, if pre-existing trends in local demand systematically coincided with national shocks. To mitigate this concern, I include a rich set of controls and fixed effects. Specifically, state fixed effects α_s absorb time-invariant differences in labor market characteristics across states. Time fixed effects α_t account for common national shocks such as business cycles or federal policy changes. By leveraging only the cross-sectional variation in state exposure to national shocks, the Bartik specification helps isolate the exogenous component of belief-driven hiring fluctuations.

Cross-Sectional Decomposition of the Regional Job Filling Rate Table A.14 reports regression estimates that evaluate the predictive power of state-level expectations for future unemployment. Each column adds different combinations of rational or subjective forecasts for discount rates and cash flows, with all specifications controlling for standard labor market factors and including both state and time fixed effects.

The estimates demonstrate that subjective earnings expectations are not only informative about regional unemployment but crowd out the predictive power of rational components. Column (1) shows that rational discount rate expectations $\mathbb{E}_t[r_{s,t,t+5}]$ significantly predict unemployment, with a coefficient of 0.725 and R^2 of 0.414. This implies that a one standard deviation increase in rational discount rate expectations predicts a 0.240 percentage point increase in the unemployment rate. Column (2) shows that among subjective forecasts, only expected earnings $\mathbb{F}_t[e_{s,t,t+5}]$ matter, with a large negative coefficient (-0.817) and higher explanatory power ($R^2 = 0.558$). A one standard deviation increase in expected earnings predicts a 0.129 percentage point decrease in the unemployment rate. Column (3) includes both sets of expectations. Subjective earnings dominate: their coefficient remains significant (-0.791), while rational expectations become insignificant.

Column (4) repeats the rational-only regression using Bartik instruments; the discount rate remains significant (0.572), implying a 0.181 percentage point increase in unemployment per standard deviation increase in instrumented discount rate expectations. In Column (5), only instrumented subjective earnings are significant (-0.690), with a standard deviation of 0.168 implying a 0.116 percentage point decrease in unemployment. Column (6) confirms that instrumented subjective earnings expectations (-0.708) continue to drive out all other predictors, implying a 0.119 percentage point decline in unemployment for a one standard deviation increase.

The shift-share estimates are generally smaller in magnitude than their OLS counterparts, as expected, since the shift-share instrument isolates only variation that is plausibly exogenous to regional labor market conditions. The attenuation suggests that some of the OLS signal reflects endogenous responses to regional shocks—such as changes in local labor supply—that amplify belief-driven dynamics. Nevertheless, the fact that the earnings coefficient remains large and significant under instrumentation supports a causal interpretation: belief distortions about cash flows play a central role in driving unemployment fluctuations across regions.

Taken together, the results provide robust evidence that distorted beliefs about future earnings are a key driver of regional labor market volatility. The strong and consistent link between subjective earnings expectations and unemployment—even when instrumented—suggests that firms' hiring decisions are shaped not only by fundamentals but also by biased beliefs. Regions where firms overreact to cash flow news experience deeper hiring cuts during downturns and more aggressive expansions during booms, thereby driving business cycle volatility. These findings indicate that persistent regional differences in unemployment may arise not only from structural characteristics such as industry mix or demographics, but also from variation in how firms perceive and respond to economic signals.

Table A.14: Predictability of the State-Level Unemployment Rate

	Dependent Variable: Log Unemployment Rate u_{t+1}					
	OLS			Shift-Share Instrument		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[r_{s,t,t+h}]$	0.725***		0.470	0.572***		0.207
	(0.235)		(0.780)	(0.222)		(0.240)
$\mathbb{E}_t[e_{s,t,t+h}]$	-0.247		-0.065	-0.064		0.005
	(0.499)		(0.182)	(0.075)		(0.168)
$\mathbb{F}_t[r_{s,t,t+h}]$		0.248	0.233		0.052	0.052
		(0.297)	(0.300)		(0.228)	(0.228)
$\mathbb{F}_t[e_{s,t,t+h}]$		-0.817***	-0.791***		-0.690***	-0.708***
		(0.236)	(0.242)		(0.160)	(0.200)
R^2	0.414	0.558	0.558	0.414	0.549	0.549
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Labor Market Factors	Yes	Yes	Yes	Yes	Yes	Yes
N	4,358	4,358	4,358	4,358	4,358	4,358

Notes: Labor market factors include the log annual growth of lagged log unemployment rate $u_{s,t}$, log labor market tightness $\log\theta_{s,t}$ and log job separation rate $\log\delta_{s,t}$. The sample is quarterly from 2005Q1 to 2021Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.9 Capital Investment

This section extends the baseline model by incorporating firm investment decisions and distinguishing between tangible and intangible capital. I show how belief distortions about future returns and earnings influence not only hiring decisions, but also capital investment behavior. I then decompose the investment rate into components associated with discount rates and cash flows.

Model Setup I assume firms produce output using a Cobb-Douglas production function that depends on both capital and labor inputs:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{A.24}$$

where $A_{i,t}$ denotes total factor productivity, $K_{i,t} = K_{i,t}^{\text{phy}} + K_{i,t}^{\text{int}}$ is total capital input composed of tangible and intangible capital, and $L_{i,t}$ is labor input. Following Hall (2001) and Hansen et al. (2005), I treat tangible and intangible capital as perfect substitutes. Earnings are defined as:

$$E_{i,t} = Y_{i,t} - W_{i,t} L_{i,t} - \kappa V_{i,t} - I_{i,t} - \phi\left(\frac{I_{i,t}}{K_{i,t}}\right) K_{i,t} \tag{A.25}$$

where $W_{i,t}$ is the wage rate, $\kappa V_{i,t}$ is the vacancy posting cost, $I_{i,t} = I_{i,t}^{\text{phy}} + I_{i,t}^{\text{int}}$ is total investment, and $\phi(\cdot)$ denotes convex adjustment costs. I adopt a piecewise-quadratic specification for $\phi(\cdot)$ with different coefficients for expansion and contraction:

$$\phi\left(\frac{I_{i,t}}{K_{i,t}}\right) = \begin{cases} \frac{c_k^+}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} \geq 0 \\ \frac{c_k^-}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} < 0 \end{cases} \tag{A.26}$$

Firms choose investment $I_{i,t}$ and vacancies $V_{i,t}$ to maximize firm value:

$$V(A_{i,t}, K_{i,t}, L_{i,t}) = \max_{I_{i,t}, V_{i,t}} \left\{E_{i,t} + \mathbb{F}_t\left[M_{t+1} V(A_{i,t+1}, K_{i,t+1}, L_{i,t+1})\right]\right\} \tag{A.27}$$

subject to both capital and employment accumulation equations:

$$K_{i,t+1} = (1-\delta_{i,t}^k) K_{i,t} + I_{i,t} \tag{A.28}$$

$$L_{i,t+1} = (1-\delta_{i,t}^l) L_{i,t} + q_t V_{i,t} \tag{A.29}$$

The first order condition with respect to investment implies:

$$1 + \phi'\left(\frac{K_{i,t+1} - (1-\delta_{i,t}^k) K_{i,t}}{K_{i,t}}\right) = \frac{P_{i,t}}{K_{i,t+1}} \tag{A.30}$$

where $P_{i,t} = \mathbb{F}_t[M_{t+1} V(A_{i,t+1}, K_{i,t+1})]$ is the ex-dividend firm value.

Recovering Intangible Capital To estimate intangible capital, I construct a panel of five value-weighted portfolios sorted by book-to-market ratio. For each portfolio, I measure realized data on physical capital $K_{i,t}^{\text{phy}}$, tangible investment $I_{i,t}^{\text{phy}}$, depreciation rates $\delta_{i,t}^k$, and market value $P_{i,t}$. The physical capital stock $K_{i,t}^{\text{phy}}$ is measured using Compustat's PPEGT item, and tangible investment $I_{i,t}^{\text{phy}}$ is measured using capital expenditures (CAPX). The depreciation rate $\delta_{i,t}^k$ is calculated as depreciations

(DP) as a share of physical capital stock (PPEGT), and applied to both tangible and intangible capital (Hall, 2001). I construct the firm's total market value $P_{i,t}$ as the sum of the market value of equity, the book value of debt, minus current assets. Starting from an initial value $K_{i,1970Q1} = P_{i,1970Q1}$, I recursively solve the first order condition for $K_{i,t+1}$, using observed investment, depreciation, and market value. Intangible capital is then recovered as the residual:

$$K_{i,t}^{\text{int}} = K_{i,t} - K_{i,t}^{\text{phy}} \tag{A.31}$$

Decomposition of Investment Rates Taking logs and linearizing the first order condition:

$$\log\left(1 + c_k \frac{I_{i,t}}{K_{i,t}}\right) \approx \log c_k + \log\left(\frac{I_{i,t}}{K_{i,t}}\right) = \log\left(\frac{P_{i,t}}{K_{i,t+1}}\right) \tag{A.32}$$

I decompose the right-hand side into price-to-earnings and earnings-to-capital terms:

$$\underbrace{\log\left(\frac{I_{i,t}}{K_{i,t}}\right)}_{ik_{i,t}} = -\log c_k + \underbrace{\log\left(\frac{P_{i,t}}{E_{i,t}}\right)}_{pe_{i,t}} + \underbrace{\log\left(\frac{E_{i,t}}{K_{i,t+1}}\right)}_{ek_{i,t}} \tag{A.33}$$

Using a Campbell-Shiller log-linear approximation for the price-earnings ratio:

$$pe_{i,t} = \sum_{j=1}^{h} \rho^{j-1}(c_{pe} + \Delta e_{i,t+j} - r_{i,t+j}) + \rho^h pe_{i,t+h} \tag{A.34}$$

Substituting yields the final decomposition:

$$ik_{i,t} = c_{ik} - \sum_{j=1}^{h} \rho^{j-1} r_{i,t+j} + \left(ek_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{i,t+j}\right) + \rho^h pe_{i,t+h} \tag{A.35}$$

where $c_{ik} \equiv \frac{c_{pe}(1-\rho^h)}{1-\rho} - \log c_k$. To separately analyze tangible and intangible investment, I define $ik_{i,t}^m \equiv \log(\frac{I_{i,t}^m}{K_{i,t}})$ and $s_{i,t}^m \equiv \log(\frac{I_{i,t}^m}{I_{i,t}})$ so that:

$$ik_{i,t}^m = s_{i,t}^m + ik_{i,t}, \qquad m = phy, int \tag{A.36}$$

implying the decomposition structure remains unchanged up to an additive shift $s_{i,t}^m$. I estimate the decomposition separately for tangible and intangible investment. The time-series decomposition of the aggregate investment rate is:

$$ik_t^m \approx -\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}] + \left(ek_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}]\right) + \rho^h \mathbb{F}_t[pe_{t+h}] \tag{A.37}$$

where $x_t = \sum_{i\in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. For the cross-section, demeaned variables yield:

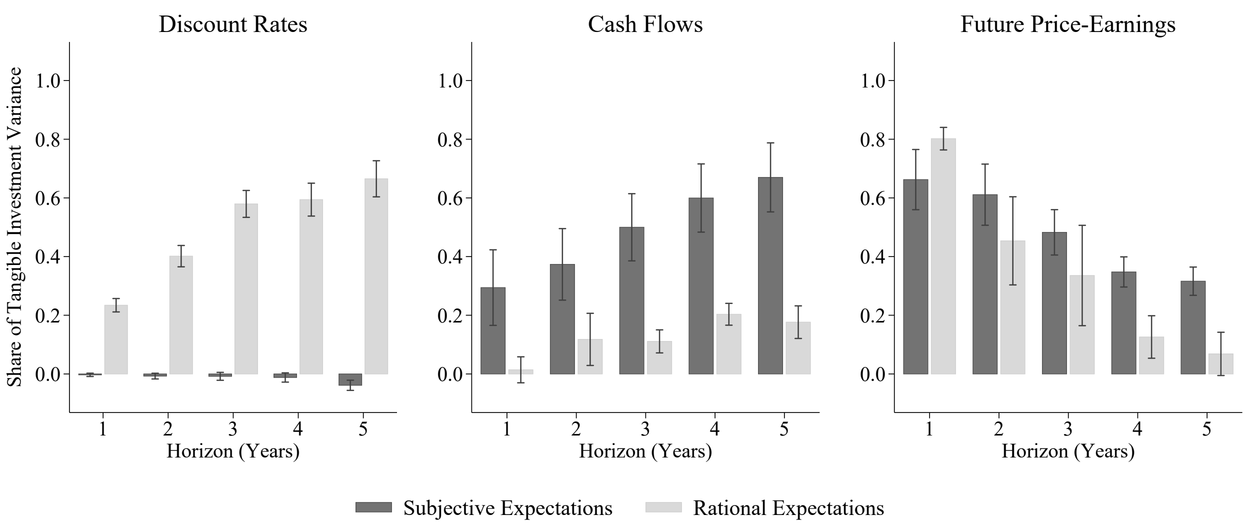
$$\tilde{ik}_{i,t}^m \approx -\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\tilde{r}_{i,t+j}] + \left(\tilde{ek}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \tilde{e}_{i,t+j}]\right) + \rho^h \mathbb{F}_t[\tilde{pe}_{i,t+h}] \tag{A.38}$$

where $\widetilde{x}_{i,t} = x_{i,t} - \sum_{i\in I} x_{i,t}$ cross-sectionally demeans variable $x_{i,t}$.

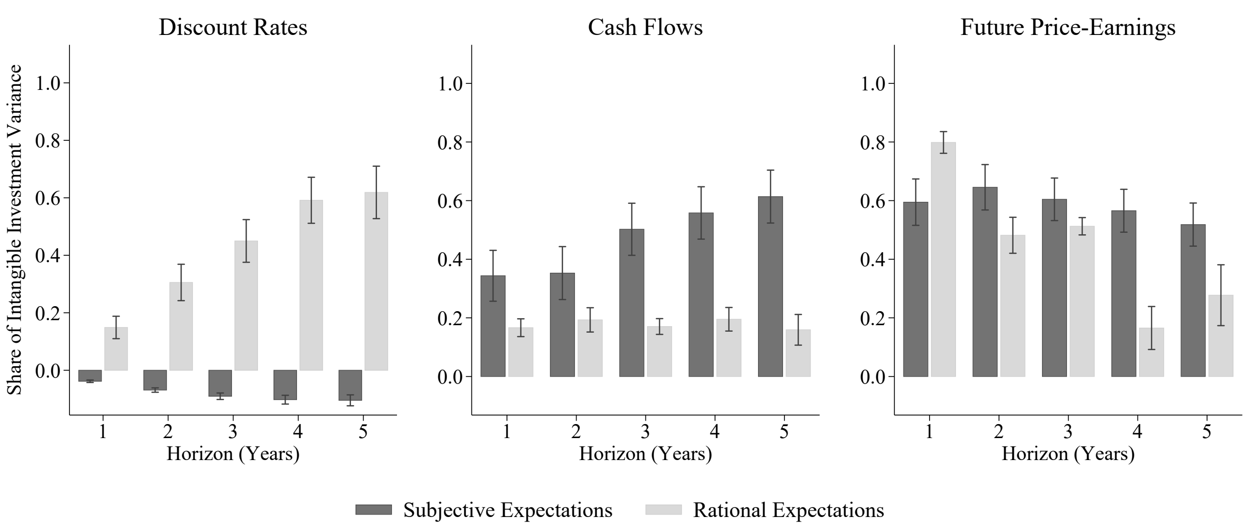
Results The empirical results mirror those for hiring rates, both in the time series (Figure A.5) and the cross-section (Figure A.6). Subjective expectations substantially overstate the contribution of cash flows and understate that of discount rates, both for tangible and intangible investment. Notably, the distortions are stronger for intangible investment, consistent with greater uncertainty and measurement error in expectations about intangible value creation. These findings highlight how belief distortions affect not only labor demand but also capital allocation decisions across asset types.

Figure A.5: Time-Series Decomposition of Capital Investment

(1) Tangible Capital Investment Rate



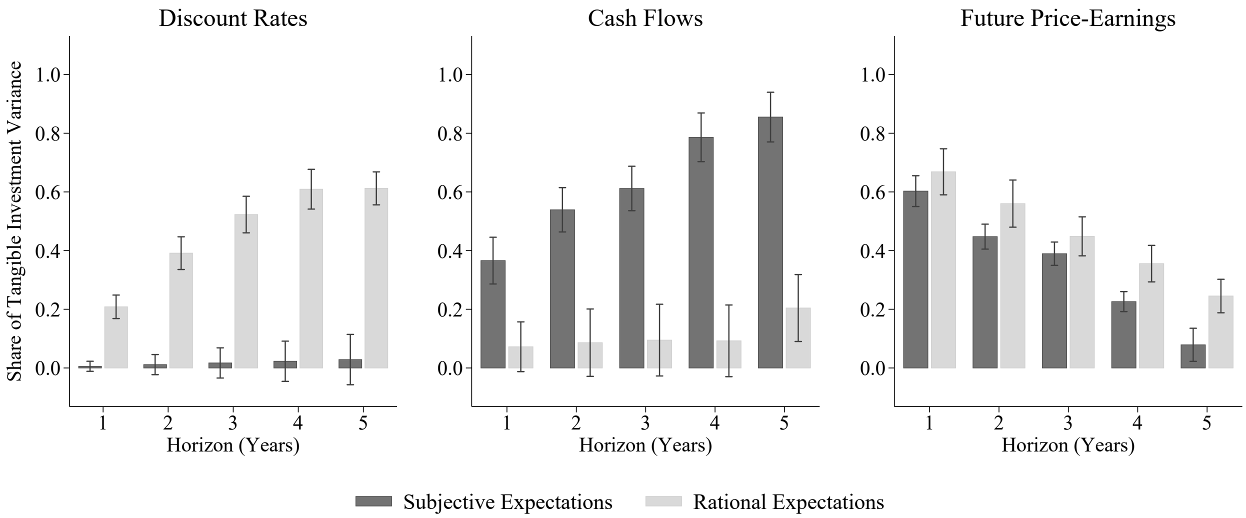
(2) Intangible Capital Investment



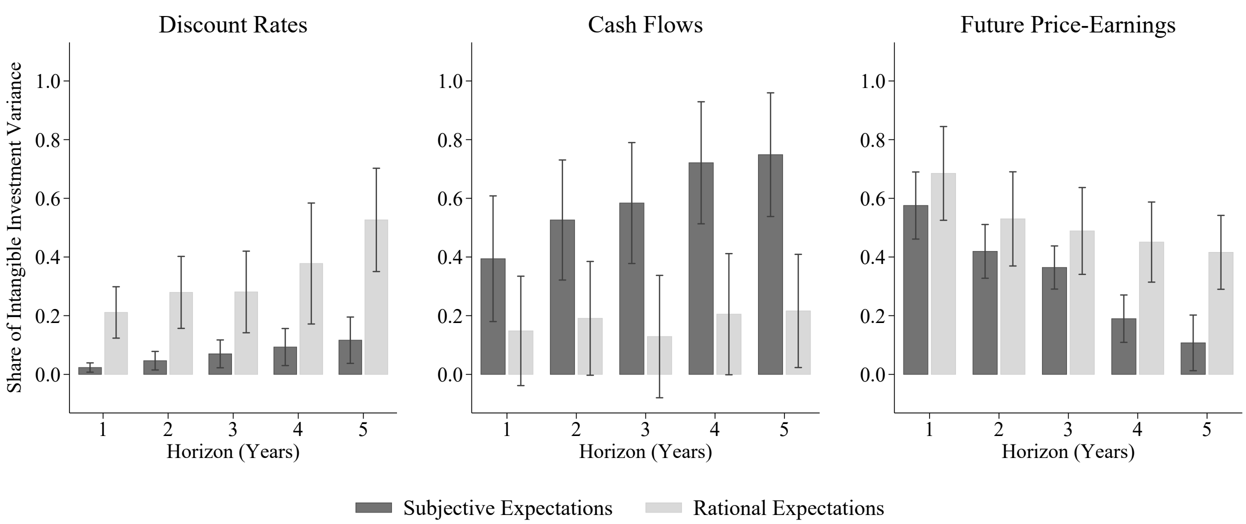
Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate tangible and intangible capital investment rate. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Figure A.6: Cross-Sectional Decomposition of Capital Investment

(1) Tangible Capital Investment Rate



(2) Intangible Capital Investment



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the cross-sectional decomposition to the dispersion of the current tangible and intangible capital investment rate. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.10 Decreasing Returns to Scale and Composition Effects

Stock market valuations reflect average profits, while hiring decisions depend on marginal profits (Borovickova and Borovička, 2017). Decreasing returns to scale can amplify unemployment fluctuations even under a rational framework by making the marginal value of hiring more sensitive to productivity shocks, prompting firms to adjust vacancies more aggressively in response (Elsby and Michaels, 2013; Kaas and Kircher, 2015). Allowing for decreasing returns to scale introduces the notion of firm size. Changes in the equilibrium firm size distribution can thus introduce a composition effect that also contributes to fluctuations in the job filling rate (Solon et al. (1994)).

This section relaxes the constant returns to scale (CRS) assumption by allowing for decreasing returns to scale (DRS) in the production function. Assume that firm i's output is $Y_{i,t} = F(L_{i,t}) = A_{i,t}L^{\alpha}_{i,t}$, where $A_{i,t}$ is an exogenous productivity process and $0 < \alpha < 1$. This introduces a "DRS wedge" between marginal and average profits:

$$\pi_{i,t}L_{i,t} - \kappa V_{i,t} = \alpha A_{i,t}L^{\alpha}_{i,t} - W_{i,t}L_{i,t} - \kappa V_{i,t} = E_{i,t} - (1-\alpha)Y_{i,t} \tag{A.39}$$

where $E_{i,t} \equiv \Pi_{i,t} - \kappa V_{i,t}$ is the firm's earnings, $\Pi_{i,t} \equiv Y_{i,t} - W_{i,t}L_{i,t} = A_{i,t}L^{\alpha}_{i,t} - W_{i,t}L_{i,t}$ is the total profit before wages $W_{i,t}L_{i,t}$ and vacancy posting costs $\kappa V_{i,t}$, and $\pi_{i,t} = \frac{\partial \Pi_{i,t}}{\partial L_{i,t}}$ is the marginal profit from hiring. The second term $(1-\alpha)Y_{i,t}$ is a "DRS wedge" that captures the gap between the average profit and marginal profit. Under DRS, the firm's hiring condition becomes:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[\sum_{j=1}^{\infty}\frac{1}{R_{i,t,t+j}}\left(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha)\frac{Y_{i,t+j}}{L_{i,t+1}}\right)\right] \tag{A.40}$$

where firm i takes the aggregate job filling rate q_t as given. Express aggregate earning-employment and output-employment ratios as the employment-weighted average of firm-level ratios:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\Big[\sum_i\sum_{j=1}^{\infty}\frac{1}{R_{i,t,t+j}}\Big(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha)\frac{Y_{i,t+j}}{L_{i,t+1}}\Big)\frac{L_{i,t+1}}{L_{t+1}}\Big] \tag{A.41}$$

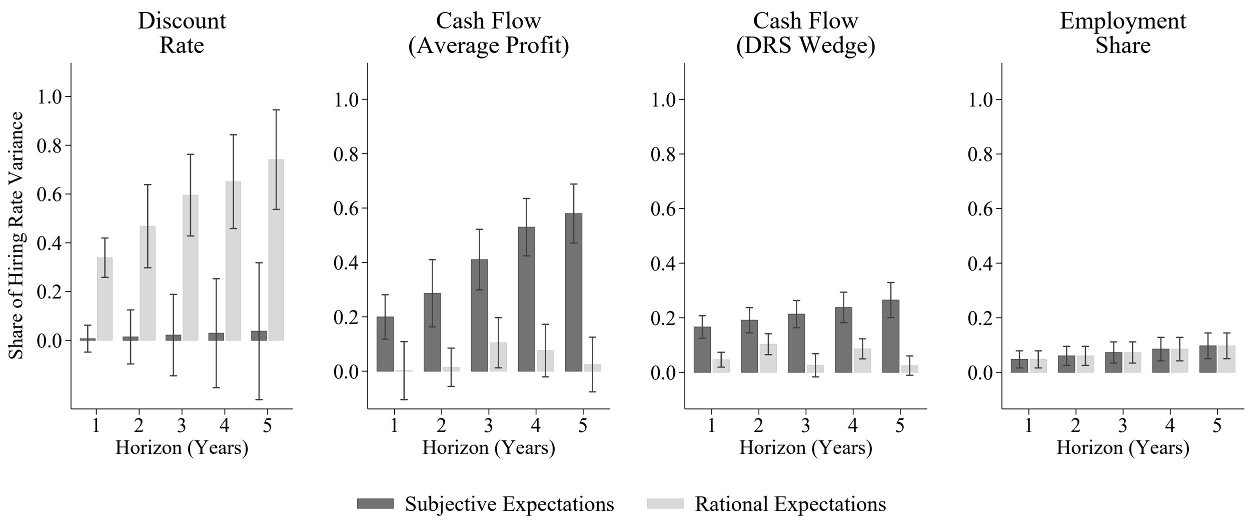
Define $S_{i,t+1} \equiv \frac{L_{i,t+1}}{L_{t+1}}$ as the employment share, $EL_{i,t+j} \equiv E_{i,t+j}/L_{i,t+1}$ the earnings-employment ratio, and $YL_{i,t+j} \equiv Y_{i,t+j}/L_{i,t+1}$ the output-employment ratio of firm i. Log linearize the expression around the steady state:

$$\log q_t = \sum_{j=1}^{\infty}\sum_i\left[\underbrace{\mathbb{F}_t\left[w_{r,i,j}r_{i,t,t+j}\right]}_{\text{Discount Rate}} - \underbrace{\mathbb{F}_t\left[w_{el,i,j}el_{i,t+j}\right]}_{\substack{\text{Cash Flow}\\ \text{(Earnings-Employment)}}} + \underbrace{\mathbb{F}_t\left[w_{yl,i,j}yl_{i,t+j}\right]}_{\substack{\text{Cash Flow}\\ \text{(Output-Employment)}}} - \underbrace{\mathbb{F}_t\left[w_{s,i,j}s_{i,t+1}\right]}_{\text{Employment Share}}\right] \tag{A.42}$$

where $r_{i,t,t+j}$, $el_{i,t+j}$, $yl_{i,t+j}$, and $s_{i,t+1}$ denote log deviations of $R_{i,t,t+j}$, $EL_{i,t+j}$, $YL_{i,t+j}$, and $S_{i,t+1}$ from the steady state state, respectively. The coefficients $w_{r,i,j} = w_{s,i,j} \equiv \frac{\overline{q}}{\kappa}\frac{(\overline{EL}_i + (1-\alpha)\overline{YL}_i)\cdot\overline{S}_i}{\overline{R}_i^j}$, $w_{el,i,j} \equiv \frac{\overline{q}}{\kappa}\frac{\overline{EL}_i\cdot\overline{S}_i}{\overline{R}_i^j}$, and $w_{yl,i,j} \equiv (1-\alpha)\frac{\overline{q}}{\kappa}\frac{\overline{YL}_i\cdot\overline{S}_i}{\overline{R}_i^j}$ are functions of steady-state values and linearization constants. $\alpha = 0.72$ comes from the labor share, $\kappa = 0.133$ comes from the flow vacancy cost (Elsby and Michaels, 2013). $\overline{q} = 0.631$, $\overline{R}_i = 1.04$, $\overline{EL} = 0.014$, $\overline{YL} = 0.074$ are long-run sample averages. Finally, approximate the infinite sum by truncating up to h periods.

The expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ captures the DRS wedge, and the employment share $s_{i,t+1}$ captures composition effects of changes in the firm size distribution. I measure the expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ by using IBES sales forecasts. Figure A.7 shows that under subjective expectations, the output-employment term accounts for roughly 30% of the variation in the job filling rate, while the earnings-employment term explains slightly less than 60%. The compositional term is small. These results confirm that even under DRS, subjective cash flow expectations—whether expressed in average or marginal terms—remain the dominant driver of hiring fluctuations.

Figure A.7: Time-Series Decomposition of the Job Filling Rate: Decreasing Returns to Scale



Notes: This figure illustrates the components of the time-series decomposition of aggregate job filling rate under decreasing returns to scale, based on equation (A.42). The components of the decomposition are expected present discounted values of discount rate, earnings-employment ratio, output-employment ratio, and the employment share. The light bars show the contributions to the job filling rate obtained under rational expectations. The dark bars show the contributions to the time-series variation in the job filling rate obtained in subjective expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2021Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.11 On-the-Job Search

The baseline model assumes that all hires come from the pool of unemployed workers. However, measured earnings and hiring flows reflect contributions from both unemployed-to-employed (UE) and job-to-job (J2J) transitions. To better capture the sources of observed hiring, this section extends the baseline model to allow for on-the-job search. This modification draws on recent work modeling labor market flows with job-to-job transitions (Kuhn et al., 2021; Faberman et al., 2022).

Let a fraction ϕ of employed workers search for jobs each period, in addition to the unemployed. The total number of searchers is:

$$S_t = U_t + \phi L_t = U_t + \phi(1 - U_t), \tag{A.43}$$

where U_t is the unemployment rate and $L_t = 1 - U_t$ is the employment rate. Vacant firms post V_t vacancies, and matches form via a constant returns to scale matching function $\mathcal{M}(S_t, V_t)$.

Not all on-the-job searchers who receive an offer accept it. Let $\chi \in (0, 1)$ denote the fraction of employed searchers who accept a job offer. The effective hiring efficiency from the firm's perspective is:

$$\varphi_t = \frac{U_t + \chi\phi(1 - U_t)}{U_t + \phi(1 - U_t)}. \tag{A.44}$$

The law of motion for employment becomes:

$$L_{t+1} = (1 - \delta_t)L_t + q_t\varphi_t V_t, \tag{A.45}$$

where δ_t is the separation rate and $q_t = \frac{\mathcal{M}(S_t,V_t)}{V_t}$ is the job filling rate. The Bellman equation for the firm's value is updated to reflect turnover due to J2J transitions:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \left\{E_t + (1 - \phi\chi f_t)\mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})\right]\right\}, \tag{A.46}$$

subject to the employment accumulation equation above. The term $1 - \phi\chi f_t$ reflects the retention rate, accounting for voluntary separations from employed workers who successfully switch jobs. Under constant returns to scale, the firm's optimal vacancy posting condition implies:

$$\frac{\kappa}{q_t\varphi_t} = (1 - \phi\chi f_t) \cdot \frac{P_t}{L_{t+1}}, \tag{A.47}$$

where $P_t = \mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})\right]$ is the ex-dividend firm value and κ is the flow cost of posting a vacancy. Taking logs and rearranging, the log job filling rate can be written as:

$$\log q_t = c_q - \log(1 - \phi\chi f_t) + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}], \tag{A.48}$$

where $c_q = \log\kappa - \log\varphi_t - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant, $r_{t,t+h}$ is the present value of expected discount rates, $e_{t,t+h}$ is expected cumulative earnings growth, and $pe_{t,t+h}$ is the expected terminal price-earnings ratio. This decomposition extends the Campbell-Shiller present value identity to account for hiring frictions due to job-to-job transitions. The job filling rate q_t is computed as the ratio of total hires to vacancies:

$$q_t = \frac{H_t}{V_t}, \tag{A.49}$$

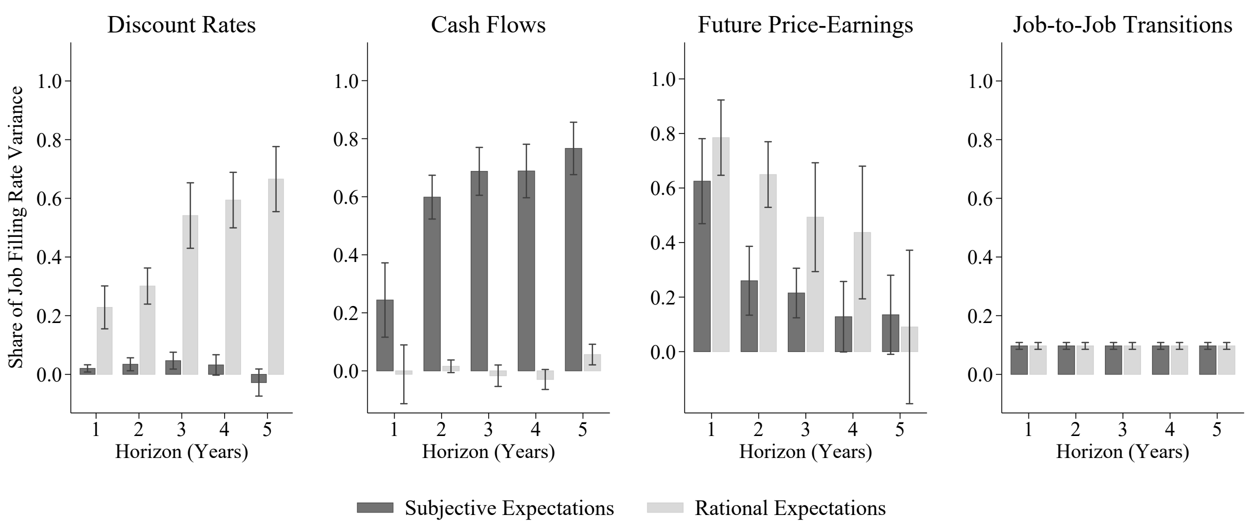
using JOLTS data for hires and job openings. The total search pool S_t includes both unemployed and a fraction $\phi = 0.12$ of employed workers, based on estimates from Kuhn et al. (2021) and Faberman et al. (2022). The job finding rate is then inferred from the matching function as $f_t = q_t \cdot \theta_t$, where labor market tightness is defined as:

$$\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_t + \phi(1 - U_t)}. \tag{A.50}$$

I assume that $\chi = 0.75$ of employed job seekers accept offers. These parameter values imply an endogenous efficiency term φ_t and a retention rate $1 - \phi\chi f_t$, which are used to adjust the firm's hiring incentives and derive the decomposition. Subjective expectations of earnings growth are from IBES, which aggregates analyst forecasts of total firm earnings and therefore reflect both UE and J2J hires.

Figure A.8 presents the decomposition of the job filling rate under this extended model with on-the-job search. Consistent with the baseline analysis, the cash flow component remains the dominant driver of variation in the job filling rate under subjective expectations. However, accounting for job-to-job transitions modestly shifts the decomposition: the log retention rate term $\log(1 - \phi\chi f_t)$ explains 8.9% of the variation in $\log q_t$. This adjustment reflects the influence of selective separations on firms' incentives to post vacancies. Overall, the results reinforce the finding that distorted cash flow expectations are the primary driver of hiring fluctuations. The extension confirms that even when allowing for endogenous separations due to on-the-job search, subjective belief distortions about firm-level earnings continue to dominate the variation in hiring behavior.

Figure A.8: Time-Series Decomposition of the Job Filling Rate: On-the-Job Search



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2020Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.12 Subjective User Cost of Labor

Overview The previous sections show that firms' hiring decisions are heavily influenced by subjective cash flow expectations. This section examines whether expectations about the user cost of labor also contribute to hiring behavior, since it is a key component of the firm's cash flows. Using survey data, I show that subjective wage expectations are significantly less cyclical than realized wages, implying that firms perceive labor costs as more rigid than they actually are. To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor under subjective expectations.[5]

In the search and matching model, the user cost of labor is the difference in the expected present value of wages between two firm-worker matches that are formed in two consecutive periods. Existing work assumes full information rational expectations and show that this user cost is more cyclical than flow wages, as workers hired in recessions earn lower wages both when hired and over time (Kudlyak, 2014; Bils et al., 2023). This section relaxes that assumption by using survey-based measures of subjective wage expectations. If firms and workers perceive the future path of wages as rigid, the subjective user cost of labor may remain high even during recessions, dampening hiring and amplifying unemployment fluctuations.

Time-series evidence Figure A.9 compares realized real wage growth with 1-year-ahead subjective wage growth forecasts from three sources: the Livingston Survey, the CFO Survey, and the Survey of Consumer Expectations (SCE). Actual wage growth is clearly cyclical, with declines during downturns and strong rebounds during recoveries. In contrast, subjective wage forecasts are far more stable over time. Even during major shocks, such as the 2008 financial crisis and the COVID-19 recession, survey respondents anticipated only modest wage adjustments. Forecast errors are persistent and systematically biased: wage growth forecasts overestimate during downturns and underestimate during expansions.

To formally assess the cyclicality of real wage growth, Table A.15 panel (a) compares the relationship between changes in the unemployment rate and real wage growth across rational and subjective expectations. As a rational expectations benchmark, I use historical data on actual real wage growth to estimate the following regression, replicating existing estimates in the literature (e.g., Bils, 1985; Solon et al., 1994; Gertler et al., 2020):

$$\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t \tag{A.51}$$

where $\Delta \log w_t$ represents the actual annual log growth rate of real wages, Δu_t is the annual change in the unemployment rate, and ε_t is the error term. β_1 is the coefficient of interest and captures the cyclicality of real wage growth.

Under subjective expectations, I use survey data on expected real wage growth to estimate:

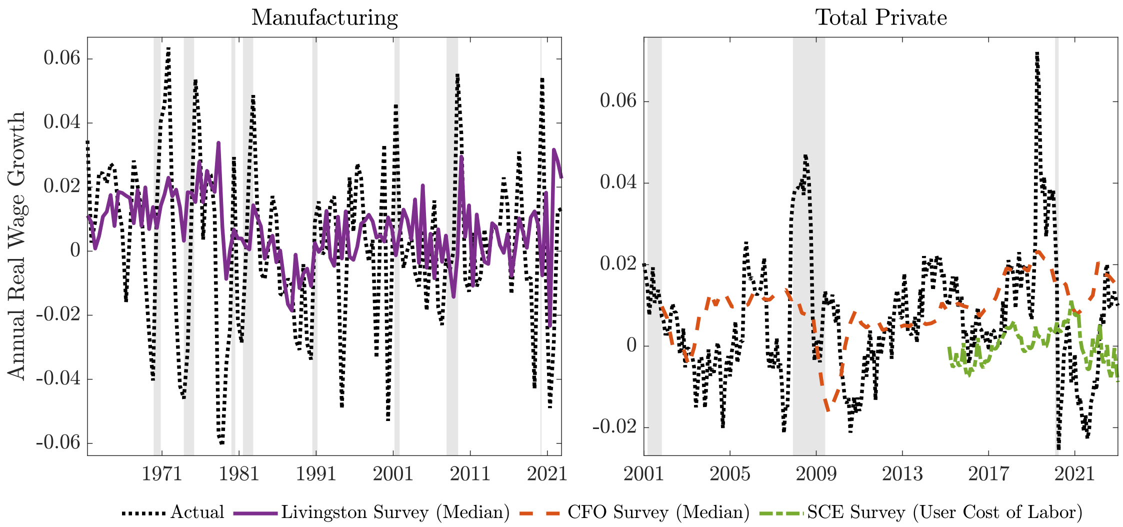
$$F_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 F_{t-1}[\Delta u_t] + \varepsilon_t \tag{A.52}$$

where $F_{t-1}[\Delta \log w_t]$ is the median survey forecast for the annual log growth rate of real wages, where the surveys are either from Livingston, CFO, or SCE. $F_{t-1}[\Delta u_t]$ is the median survey forecast of the annual change in the unemployment rate from the Survey of Professional Forecasters (SPF). The coefficient of interest β_1 measures the cyclicality of expected real wage growth as perceived by survey respondents.

Table A.15 panel (a) reports the estimates. Under rational expectations, actual real wage growth is clearly cyclical since it is significantly negatively related to changes in unemployment rates. The

[5] See Section C for more details about its measurement.

Figure A.9: Real Wage Growth: Actual vs. Subjective Expectations



Notes: This figure plots ex-post realized outcomes (Actual) and 1-year ahead subjective expectations (Survey) of real wage growth. x axis denotes the date on which actual values were realized and the period on which the survey forecast is made, making the vertical distance between the actual and survey lines the forecast error. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Left panel compares actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Right panel compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Actual values are deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2022Q4, SCE is monthly spanning 2015M5 to 2022M12. NBER recessions are shown with gray shaded bars.

magnitude of the estimate is also consistent with prior estimates in the literature, with elasticities ranging from -3.05 to -3.46 depending on the sample period (Solon et al., 1994). In contrast, subjective wage growth expectations are acyclical, with small and statistically insignificant coefficients across all survey sources and sample periods. Notably, the magnitude of the estimated elasticity is an order of magnitude smaller, ranging from -0.20 to -0.97 depending on the survey measure and sample period.

Cross-Sectional evidence To explore these patterns at the individual level, I use microdata from the SCE to estimate subjective wage cyclicality separately for new hires and incumbents. The regression specification relaxes the rational expectations assumption from Gertler et al. (2020) and includes an interaction between expected unemployment growth and the probability of being a new hire:

$$\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t} \tag{A.53}$$

where $\mathbb{F}_{t-1}[\Delta \log w_{i,t}]$ represents the time $t-1$ subjective expectation of wage growth for worker i at time t. $\mathbb{F}_{t-1}[\Delta u_t]$ is the survey-based expectation of aggregate unemployment growth. The indicator variable $\mathbb{I}\{N_{i,t} = 1\}$ equals one if the worker is newly hired and zero otherwise. Its expectation $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}]$ is thus the subjective perceived probability that the worker will be newly hired next period. The interaction term $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot \mathbb{F}_{t-1}[\Delta u_t]$ captures the differential sensitivity of expected wage growth to unemployment changes for new hires relative to incumbents. The error term $\varepsilon_{i,t}$ accounts for individual-level deviations in expectations.

The coefficient β_1 captures the overall cyclicality of subjective wage expectations, reflecting how much workers expect wages to change in response to shifts in aggregate unemployment. The coef-

Table A.15: Cyclicality of Real Wage Growth: Actual vs. Subjective Expectations

(a) Aggregate Time-Series
Actual: $\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t$
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \varepsilon_t$

	1961S1-2022S2		2001Q4-2022Q4		2015M5-2022M12	
	Actual	Survey Median (Liv)	Actual	Survey Median (CFO)	Actual	Survey User Cost (SCE)
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment Rate	-0.0340^{***}	-0.0020	-0.0305^{***}	0.0006	-0.0346^{***}	-0.0086
t-stat	(-3.8684)	(-0.1568)	(-4.2477)	(0.0800)	(-6.6994)	(-1.6332)
Adj. R^2	0.1021	0.0003	0.2557	0.0001	0.4719	0.0498
N	124	124	85	85	92	92
Frequency	SA	SA	Q	Q	M	M
Sector	Mfg	Mfg	Pvt	Pvt	Pvt	Pvt

(b) Worker-Level New Hire Effect
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t}$

	2015M5-2022M12	
	Survey (SCE)	Survey (SCE)
	(1)	(2)
	First Difference	Fixed Effects
Unemployment Rate	-0.0048	-0.0028
	(0.0029)	(0.0026)
New Hire	0.0036^{***}	0.0003
	(0.0009)	(0.0013)
Unemployment Rate × New Hire	-0.0026	-0.0059
	(0.0020)	(0.0035)
Adj. R^2	0.0011	0.0036
N	39,832	39,832
Frequency	M	M
Sector	Pvt	Pvt

Notes: Table reports estimates from time-series and worker-level regressions of annual log real wage growth on unemployment growth. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Panel (a) reports estimates from time-series regressions using the aggregate series. Panel (a) Columns (1)-(2) compare actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Panel (a) Columns (3)-(6) compare compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Panel (b) reports worker-level estimates from regressions of SCE survey expectations of wage growth on survey expectations of unemployment growth, an indicator of whether the worker is a new hire, and the interaction between the two. Actual wage growth is deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. Subjective expectations of unemployment rates are from 1-year ahead consensus median forecasts from the SPF. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2022Q4, SCE is monthly spanning 2015M5 to 2022M12. Panel (a): Newey-West corrected t-statistics with lags 2 (semi-annual), 4 (quarterly), 12 (monthly) are reported in parentheses; Panel (b): Standard errors clustered by worker are reported in parentheses. *sig. at 10%. **sig. at 5%. ***sig. at 1%.

ficient β_2 measures the baseline difference in expected wage growth between new hires and existing workers. The interaction term β_3 determines whether new hires expect wages to be more sensitive to unemployment fluctuations than incumbents do.

The results in Table A.15 panel (b) column (1) show that, even after controlling for differences between job stayers and new hires, subjective wage expectations are highly rigid and exhibit weak cyclicality. The coefficient β_1 is negative but small, confirming the aggregate result in panel (a) that workers that are not new hires expect only mild wage adjustments in response to unemployment fluctuations. The estimate for β_2 is positive, suggesting that, on average, new hires expect higher wage growth than job stayers. The interaction term β_3 is negative but small in magnitude, implying that new hires do not expect substantially greater cyclicality in wages compared to incumbents. Column (2) extends column (1) by including worker fixed effects to find similar results. These findings extend the results from aggregate regressions by showing that subjective wage expectations are highly rigid even at the individual level, regardless of job transitions. Both new hires and incumbents perceive only weak cyclical variation in wages.

Implications for macroeconomic models These findings could have important implications for macroeconomic models of unemployment fluctuations. If firms do not expect wages to fall during downturns, then the subjective user cost of labor remains high even as demand declines, suppressing job creation. This mechanism is consistent with models that rely on wage rigidity to explain labor market volatility (Shimer, 2005; Hall, 2005; Christiano et al., 2016). These results suggest that it could be reasonable for macroeconomists to introduce rigid wages under subjective expectations to explain the volatility of business cycle fluctuations.

Moreover, the persistence of subjective wage expectations may reflect underlying frictions in information processing. Survey data on wage expectations can help distinguish between alternative theories of wage formation. Unlike rational models where the timing of wage payments is irrelevant (Barro, 1977), models with sticky or inattentive expectations—such as those in Mankiw and Reis (2002) or Coibion and Gorodnichenko (2015)—can be better suited to capture the persistent behavior of expected wages.

Finally, the finding that subjective cost of labor is rigid suggests that volatile subjective cash flow expectations are unlikely to be driven by fluctuations in the user cost of labor. Instead, firms may be over-reacting to other components of profitability, such as revenue expectations or perceived demand conditions, rather than expected changes in labor costs.

B Model Details

B.1 Representative Agent Model

In this section, I present a search and matching model based on Diamond (1982), Mortensen (1982), and Pissarides (2009). The model introduces subjective beliefs that may depart from rational expectations, thereby capturing the impact of belief distortions on labor market dynamics. See Petrosky-Nadeau et al. (2018) for a standard search and matching model formulated under rational expectations.

Environment Consider a discrete time economy populated by a representative household and a representative firm that uses labor as a single input to production.

Representative Household The household has a continuum of mass 1 members who are either employed L_t or unemployed U_t at any point in time. The population is normalized to 1, i.e., $L_t + U_t = 1$, meaning that L_t and U_t are also the rates of employment and unemployment, respectively. The household's consumption decision implies a stochastic discount factor M_{t+1}. The household pools the income of all members before making its consumption decision. Assume that the household's members have access to complete contingent claims against aggregate risk. Assume that the household has perfect consumption insurance. Risk sharing implies each member consumes the same amount regardless of idiosyncratic shocks.

Search and Matching We adopt a standard end-of-period matching convention (Petrosky-Nadeau et al., 2018). At the start of period t, the employment stock L_t reflects the total number of workers carried over from the previous period before any separations or new hires in period t. A fraction δ_t of these workers separate during the period, so the number of continuing employees becomes $(1 - \delta_t)L_t$.

The representative firm posts job vacancies V_t and engage in search over the course of the period to attract unemployed workers U_t. Matches are formed at the end of period t according to a matching function $m(U_t, V_t)$, where $q_t \equiv m(U_t, V_t)/V_t$ is the *job filling rate*, and $f_t \equiv m(U_t, V_t)/U_t$ is the *job finding rate*. These new matches become part of the workforce starting in period $t + 1$, so employment evolves according to the employment accumulation equation:

$$L_{t+1} = (1 - \delta_t)L_t + q_t V_t \tag{A.54}$$

The job filling rate q_t maps vacancy posting decisions made during period t into employment outcomes observed at the beginning of period $t + 1$. The variance decomposition does not require specifying the functional form of the matching function m. As explained in Section C, I measure the number of matches directly from the data using changes in unemployment and the number of short-term unemployed (Shimer, 2012). Posting a vacancy costs the firm $\kappa > 0$ per period, reflecting fixed hiring costs such as training and administrative setup. Jobs are destroyed at a time-varying job separation rate δ_t. Unemployment $U_t = 1 - L_t$ evolves according to:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{A.55}$$

where $\theta_t = V_t/U_t$ denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

Representative Firm The firm has access to a production function F which uses labor L_t as an input to produce output $Y_t = F(L_t)$. Dividends to the firm's shareholders E_t are defined as $E_t \equiv \Pi_t - \kappa V_t$, where $\Pi_t \equiv Y_t - W_t L_t$ is the total profit before vacancy posting costs κV_t and W_t is the wage rate. As in Petrosky-Nadeau et al. (2018), I assume that the representative household owns the equity of the firm, and that the firm pays out all of its earnings as dividends. I also assume that firms have the same unconstrained access to financing as investors in the financial market. The firm posts the optimal number of vacancies to maximize the cum-dividend market value of equity S_t:

$$S_t = \max_{\{V_{t+j}, L_{t+j}\}_{j=0}^{\infty}} \mathbb{F}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} E_{t+j} \right] \tag{A.56}$$

subject to the employment accumulation equation (A.54). The firm takes the wage rate W_t, household's stochastic discount factor $M_{t,t+j} = \prod_{s=1}^{j} M_{t+s}$, and job filling rate q_t as given.

Expectations Let $\mathbb{F}_t[\cdot]$ denote expectations conditional on information available at the beginning of period t, computed based on the firm's possibly distorted beliefs. These beliefs may depart from rational expectations $\mathbb{E}_t[\cdot]$, with the nature and magnitude of the deviation disciplined using survey data (e.g., Bhandari et al. (2024)).

Hiring Equation The firm's optimal hiring decision implies the *hiring equation*, which equates the expected discounted value of hiring a marginal worker with its marginal cost. Rewrite the infinite-horizon value-maximization problem of the firm into recursive form:

$$S_t = \max_{V_t, L_{t+1}} \Pi_t - \kappa V_t + \mathbb{F}_t \left[M_{t+1} S_{t+1} \right] \tag{A.57}$$

$$\text{s.t. } L_{t+1} = (1 - \delta_t) L_t + q_t V_t \tag{A.58}$$

The first-order condition with respect to V_t is:

$$\frac{\partial S_t}{\partial V_t} = -\kappa + \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \frac{\partial L_{t+1}}{\partial V_t} \right] = 0 \tag{A.59}$$

Substitute $\frac{\partial L_{t+1}}{\partial V_t} = q_t$ and $\frac{\partial L_{t+1}}{\partial L_t} = (1 - \delta_t)$ from the employment accumulation equation (A.58), and rearrange (A.59) in terms of the marginal cost of hiring κ / q_t:

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \right] \tag{A.60}$$

Next, differentiate S_t with respect to L_t:

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \frac{\partial L_{t+1}}{\partial L_t} \right] \tag{A.61}$$

Substitute $\frac{\partial L_{t+1}}{\partial L_t} = (1 - \delta_t)$ from the employment accumulation equation (A.58):

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + (1 - \delta_t) \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \right] \tag{A.62}$$

Substitute equation (A.62) for period $t+1$ into equation (A.60):

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial L_{t+1}} + (1-\delta_{t+1})\mathbb{F}_{t+1}\left[M_{t+2}\frac{\partial S_{t+2}}{\partial L_{t+2}}\right]\right)\right] \tag{A.63}$$

Finally, substitute in (A.60) for period $t+1$ to arrive at the *hiring equation*:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \underbrace{\mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1} + (1-\delta_{t+1})\frac{\kappa}{q_{t+1}}\right)\right]}_{\text{Expected discounted value of hiring}} \tag{A.64}$$

where $\pi_t \equiv \frac{\partial \Pi_t}{\partial L_t}$ is the profit flow from the marginal hired worker.

The hiring equation relates the marginal cost of hiring $\frac{\kappa}{q_t}$ with the expected marginal value of hiring to the firm, which equals the future expected marginal benefits of hiring discounted to present value with the stochastic discount factor M_{t+1}. The future marginal benefits of hiring include π_{t+1}, the future marginal product of labor net of the wage rate, plus the future marginal value of hiring, which equals the future marginal cost of hiring $\frac{\kappa}{q_{t+1}}$ net of separation $(1-\delta_{t+1})$. During recessions, job filling rates q_t are high, which makes the cost of hiring κ/q_t low. The low cost of hiring must be rationalized by either low expected discounted profit flows $\mathbb{F}_t[M_{t+1}\pi_{t+1}]$ or low future value of hiring $(1-\delta_{t+1})\frac{\kappa}{q_{t+1}}$.

The hiring equation is the labor market analogue of the optimality condition for physical capital in the q theory of investment (Hayashi, 1982), where κ/q_t is the upfront cost of investment analogous to Tobin's marginal q and δ_{t+1} is the depreciation rate.

Firm Value and Hiring Next, I derive the firm's stock price implied by the optimal hiring decision (Liu et al. (2009), Belo et al. (2023)). Assume a constant returns to scale (CRS) production function so that marginal profits equal average profits:

$$\pi_{t+1}L_{t+1} = \frac{\partial \Pi_{t+1}}{\partial L_{t+1}}L_{t+1} = \Pi_{t+1} \tag{A.65}$$

Multiply both sides of the hiring equation by the number of employees L_{t+1}:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} + (1-\delta_{t+1})\frac{\kappa}{q_{t+1}}L_{t+1}\right)\right] \tag{A.66}$$

Substitute in the employment accumulation equation (A.58) and rearrange terms:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} + \frac{\kappa}{q_{t+1}}(L_{t+2} - q_{t+1}V_{t+1})\right)\right] \tag{A.67}$$

$$= \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} - \kappa V_{t+1} + \frac{\kappa}{q_{t+1}}L_{t+2}\right)\right] \tag{A.68}$$

Use the constant returns to scale assumption to simplify $\pi_{t+1}L_{t+1} - \kappa V_{t+1} = \Pi_{t+1} - \kappa V_{t+1} = E_{t+1}$:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(E_{t+1} + \frac{\kappa}{q_{t+1}}L_{t+2}\right)\right] \tag{A.69}$$

Substitute the equation recursively:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[\sum_{j=1}^{\infty} M_{t,t+j}E_{t+j}\right] + \lim_{T\to\infty}\mathbb{F}_t\left[M_{t,t+T}\frac{\kappa}{q_{t+T}}L_{t+T+1}\right] \tag{A.70}$$

The first term on the right-hand side is the firm's stock price $P_t \equiv S_t - E_t$, which is the firm's ex-dividend equity value. Take the second term to zero by applying a transversality condition to arrive at an equation that relates the total cost of hiring with the firm's stock price:

$$\frac{\kappa}{q_t} L_{t+1} = P_t \tag{A.71}$$

where employment L_{t+1} is determined at the end of date t under our timing convention from equation (A.54).

Log-linear Approximation Take logarithms of both sides of the firm's stock price equation (A.71) and rearrange terms:

$$\log \kappa - \log q_t = \log \frac{P_t}{L_{t+1}} \tag{A.72}$$

$$= \log \frac{P_t}{E_t} - \log \frac{E_t}{L_{t+1}} \tag{A.73}$$

$$\equiv pe_t - el_t \tag{A.74}$$

where I define $pe_t \equiv \log \frac{P_t}{E_t}$ and $el_t \equiv \log \frac{E_t}{L_{t+1}}$ for notational convenience. To express the price-earnings ratio pe_t in terms of forward-looking variables, start by log-linearizing the price-dividend ratio $pd_t = \log(P_t/D_t)$ around its long-term average $\overline{pd}$ (Campbell and Shiller, 1988):

$$pd_t = c_{pd} + \Delta d_{t+1} - r_{t+1} + \rho pd_{t+1} \tag{A.75}$$

where c_{pd} is a linearization constant, $r_{t+1} \equiv \log(\frac{P_{t+1}+D_{t+1}}{P_t})$ is the log stock return (with dividends), and $\rho \equiv \frac{\exp(\overline{pd})}{1+\exp(\overline{pd})} = 0.98$ is a persistence parameter that arises from the log linearization. Rewrite the equation in terms of log price-earnings instead of log price-dividends by using the identity $pe_t = pd_t + de_t$, where de_t log payout ratio:

$$pe_t = c_{pd} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} + (1-\rho) de_{t+1} \tag{A.76}$$

Since $1 - \rho \approx 0$ and the payout ratio de_t is bounded, $(1-\rho)de_{t+1}$ can be approximated as a constant, i.e., $c_{pe} \approx c_{pd} + (1-\rho)de_{t+1}$ (De La O et al., 2024):

$$pe_t \approx c_{pe} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} \tag{A.77}$$

Recursively substitute for the next h periods

$$pe_t = \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h} \tag{A.78}$$

Decomposition of Job Filling Rate Substitute the log-linearized price-earnings ratio in equation (A.78) into the log stock price in equation (A.74):

$$\log q_t = \log \kappa - pe_t - el_t \tag{A.79}$$

$$= \log \kappa - \left[\sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h} \right] - el_t \tag{A.80}$$

Rearrange and collect terms to obtain an ex-post decomposition of the job filling rate:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{r_{t,t+h}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j} \right]}_{e_{t,t+h}} - \underbrace{\rho^h pe_{t+h}}_{pe_{t,t+h}} \tag{A.81}$$

where $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The equation decomposes the job filling rate into future discount rates $r_{t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$, cash flows $e_{t,t+h} \equiv el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$, and price-earnings $pe_{t,t+h} \equiv \rho^h pe_{t+h}$. The cash flow component consists of one period ahead log earnings-employment el_t, which captures news about current cash flow fluctuations, and $j = 1, \ldots, h$ period ahead log earnings growth Δe_{t+j}, which captures news about future cash flows.[6] $pe_{t,t+h}$ is a terminal value that captures other long-term influences beyond h periods into the future not already captured in discount rates and cash flows.

Since equation (A.81) holds both ex-ante and ex-post, it can be evaluated under either subjective or rational expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their subjective expectations $\mathbb{F}_t[\cdot]$:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\mathbb{F}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\mathbb{F}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{F}_t[pe_{t+h}]}_{\mathbb{F}_t[pe_{t,t+h}]} \tag{A.82}$$

Alternatively, the *rational decomposition* replaces ex-post realizations of future outcomes with their rational expectations $\mathbb{E}_t[\cdot]$:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{E}_t[r_{t+j}]}_{\mathbb{E}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{E}_t[\Delta e_{t+j}] \right]}_{\mathbb{E}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{E}_t[pe_{t+h}]}_{\mathbb{E}_t[pe_{t,t+h}]} \tag{A.83}$$

Comparing these decompositions can quantify how belief distortions affect the job filling rate.

Estimation The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current job filling rate. For the subjective decomposition, demean each variable in equation (A.82), multiply both sides by the current log job filling rate $\log q_t$, and take the sample average:

$$Var\left[\log q_t\right] = Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right] \tag{A.84}$$

where $Var[\cdot]$ and $Cov[\cdot]$ are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by $Var\left[\log q_t\right]$ to decompose its variance:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{A.85}$$

[6]The earnings-employment ratio can be interpreted as a measure of the marginal product of labor under constant returns to scale (David et al., 2022).

The left-hand side represents the full variability in job filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, and $\mathbb{F}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively.

Finally, the decomposition under rational expectations can be estimated similarly based on equation (A.83) by replacing the subjective expectation $\mathbb{F}_t[\cdot]$ with its rational counterpart $\mathbb{E}_t[\cdot]$:

$$1 = \underbrace{\frac{Cov\left[\mathbb{E}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{A.86}$$

Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{E}_t[r_{t,t+h}]$, $\mathbb{E}_t[e_{t,t+h}]$, and $\mathbb{E}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively.

B.2 Decomposition of Unemployment Rates

I derive a decomposition of the unemployment rate by log linearizing the unemployment accumulation equation from equation (2):

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{A.87}$$

Denote the steady state values without time subscripts: U, δ, q, and θ. In steady state:

$$U = \delta(1 - U) + (1 - q\theta)U \tag{A.88}$$

Define log deviations from steady state as $\hat{x}_t = \log(X_t) - \log(X)$ for some variable X. Log-linearizing the accumulation equation around the steady state involves taking a first-order Taylor approximation:

$$Ue^{\hat{u}_{t+1}} \approx \delta e^{\hat{\delta}_t}(1 - Ue^{\hat{u}_t}) + (1 - q\theta e^{\hat{q}_t+\hat{\theta}_t})Ue^{\hat{u}_t} \tag{A.89}$$

Use the approximation $Xe^{x_t} \approx X(1 + x_t)$, expand, and simplify:

$$U(1 + \hat{u}_{t+1}) \approx \delta(1 + \hat{\delta}_t)(1 - U(1 + \hat{u}_t)) + (1 - q\theta(1 + \hat{q}_t + \hat{\theta}_t))U(1 + \hat{u}_t) \tag{A.90}$$

or equivalently

$$U + U\hat{u}_{t+1} \approx \delta(1 + \hat{\delta}_t)(1 - U - U\hat{u}_t) + (1 - q\theta - q\theta\hat{q}_t - q\theta\hat{\theta}_t)U(1 + \hat{u}_t) \tag{A.91}$$

Use the steady state equation and collect terms with log deviations:

$$U\hat{u}_{t+1} \approx \delta(1 - U)\hat{\delta}_t - \delta U\hat{u}_t - q\theta U\hat{q}_t - q\theta U\hat{\theta}_t + U(1 - q\theta)\hat{u}_t \tag{A.92}$$

Divide both sides by U:

$$\hat{u}_{t+1} \approx \frac{\delta(1 - U)}{U}\hat{\delta}_t - \delta\hat{u}_t - q\theta\hat{q}_t - q\theta\hat{\theta}_t + (1 - q\theta)\hat{u}_t \tag{A.93}$$

or equivalently:

$$\hat{u}_{t+1} \approx \frac{\delta(1 - U)}{U}\hat{\delta}_t + (1 - \delta - q\theta)\hat{u}_t - q\theta\hat{q}_t - q\theta\hat{\theta}_t \tag{A.94}$$

The steady state relationship $\delta(1-U) = q\theta U$ implies:

$$\frac{\delta(1-U)}{U} = q\theta \tag{A.95}$$

Substitute this back into our equation:

$$\hat{u}_{t+1} \approx q\theta\hat{\delta}_t + (1-\delta-q\theta)\hat{u}_t - q\theta\hat{q}_t - q\theta\hat{\theta}_t \tag{A.96}$$

The result is a linear relationship between a decomposition of the log job filling rate and the log-linearized unemployment accumulation equation.

Finally, substitute in equation (12), which is a decomposition of the job filling rate $\hat{q}_t$ into discount rate, cash flow, and future price-earnings components:

$$\hat{u}_{t+1} \approx -\underbrace{q\theta\cdot\hat{r}_{t,t+h}}_{\text{Discount Rate}} + \underbrace{q\theta\cdot\hat{e}_{t,t+h}}_{\text{Cash Flow}} + \underbrace{q\theta\cdot\hat{pe}_{t,t+h}}_{\text{Future Price-Earning}} + \underbrace{(1-\delta-q\theta)\cdot\hat{u}_t - q\theta\cdot\hat{\theta}_t + q\theta\cdot\hat{\delta}_t}_{\text{Lag Unemployment, Tightness, Separations}} \tag{A.97}$$

The equation holds both ex-ante and ex-post. Therefore, I compare results from evaluating the equation under subjective $\mathbb{F}_t[\cdot]$ or rational $\mathbb{E}_t[\cdot]$ expectations. The decomposition can be estimated using regressions of the log unemployment rate on each of the components shown in the equation:

$$u_{t+1} = \beta_0 + \beta_1 u_t + \beta_2 \log\theta_t + \beta_3 \log\delta_t + \beta_4\mathbb{F}_t[r_{t,t+h}] + \beta_5\mathbb{F}_t[e_{t,t+h}] + \varepsilon_{t+1} \tag{A.98}$$

where lowercase variables denote log deviations from steady state. I estimate the decomposition using multivariate OLS regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations. To ensure stationarity and remove seasonal effects, I estimate the regression in log annual growth rates relative to the same quarter of the previous year. The future price-earnings ratio term $\Delta\mathbb{F}_t[pe_{t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\Delta\mathbb{F}_t[r_{t,t+h}]$ and cash flows $\Delta\mathbb{F}_t[e_{t,t+h}]$ through the Campbell and Shiller (1988) present value identity in equation (11). Similarly, the equation can also be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with its rational expectations counterpart $\mathbb{E}_t[\cdot]$ based on machine learning forecasts.

B.3 Regional Model

Model Setup This section presents a multi-area, multi-sector search-and-matching model with imperfect mobility across sectors, building from the models in Kehoe et al. (2019) and Chodorow-Reich and Wieland (2020). The economy consists of a continuum of islands indexed by s. Each island produces a differentiated variety of tradable goods that is consumed everywhere and a nontradable good. Both of these goods are produced using intermediate goods. Each consumer is endowed with one of two types of skills which are used in different intensities in the nontradable and tradable goods sectors. Labor is immobile across islands but can switch sectors.[7] Consumers receive utility from a composite consumption good that is either purchased in the market or produced at home. Consumers and firms are ex-ante homogeneous and share the same subjective expectation $\mathbb{F}_t[\cdot]$. The islands only differ in the shocks that hit them.

[7] This assumption aligns with empirical evidence indicating that labor markets are predominantly regional in nature (Manning and Petrongolo, 2017).

Preferences and demand The composite consumption good on island s is produced from non-tradable goods $X_{s,N,t}$ and tradable goods $X_{s,T,t}$

$$X_{s,t} = \left[\tau^{\frac{1}{\mu}} (X_{s,N,t})^{1-\frac{1}{\mu}} + (1-\tau)^{\frac{1}{\mu}} (X_{s,T,t})^{1-\frac{1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{A.99}$$

where μ is the elasticity of substitution between tradable and nontradable goods. The demand for nontradable and tradable goods on island s is

$$X_{s,N,t} = \tau \left(\frac{P_{s,N,t}}{P_{s,t}}\right)^{-\mu} X_{s,t}, \qquad X_{s,T,t} = (1-\tau)\left(\frac{P_{s,T,t}}{P_{s,t}}\right)^{-\mu} X_{s,t} \tag{A.100}$$

where $P_{s,N,t}$ is the price of the nontradable good and $P_{s,T,t}$ is the world price of the composite tradable good. The price of the composite consumption good on island s is

$$P_{s,t} = \left[\tau P_{s,N,t}^{1-\mu} + (1-\tau) P_{s,T,t}^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{A.101}$$

The tradable good is a composite of varieties of differentiated tradable goods produced in all islands s'

$$X_{s,T,t} = \left[\int X_{s,T,t,s'}{}^{\frac{\mu_T-1}{\mu_T}} ds'\right]^{\frac{\mu_T}{\mu_T-1}} \tag{A.102}$$

where $X_{s,T,t,s'}$ is the amount of the variety of tradable good produced on island s' and consumed on island s. μ_T is the elasticity of substitution between varieties produced on different islands. Let $P_{s',T,t}$ be the price of tradable variety produced on island s'. Assume that there are no costs of shipping goods from one island to another, so that the law of one price holds and all islands purchase the variety s at the common price $P_{s,T,t}$. The price of the composite tradable good is common to all islands

$$P_{T,t} = \left[\int P_{s,T,t}^{1-\mu_T} ds\right]^{\frac{1}{1-\mu_T}} \tag{A.103}$$

The demand on island s' for a tradable variety produced on island s is therefore

$$X_{s',T,t,s} = \left[\frac{P_{s,T,t}}{P_{T,t}}\right]^{-\mu_T} X_{s',T,t} \tag{A.104}$$

so that the world demand for tradable goods produced by island s is

$$Y_{s,T,t} = \int X_{s',T,t,s} ds' = \left[\frac{P_{s,T,t}}{P_{T,t}}\right]^{-\mu_T} Y_{T,t} \tag{A.105}$$

where $Y_{T,t} = \int X_{s',T,t} ds'$ is the world demand for the composite tradable good. Since any individual island is of measure zero, shocks to an individual island do not affect either the world aggregate price of tradables $P_{T,t}$ or the world demand for tradables $Y_{T,t}$. Normalize the constant world price of the composite tradable good $P_{T,t}$ to one so that the composite tradable good is the numeraire.

Family's problem Each family of workers on island s chooses sequences for consumption $\{C_{s,t}\}$ and assets $\{A_{s,t+1}\}$ to maximize the present discounted value of consumption

$$\max_{C_{s,t}, A_{s,t+1}} \sum_{t=0}^{\infty} \beta^t u(C_{s,t}) \tag{A.106}$$

where the family's consumption $C_{s,t} = X_{s,t} + b_{s,t}$ is the sum of goods purchased in the market $X_{s,t}$, and produced at home $b_{s,t}$ which can be consumed only by that family. The budget constraint is

$$P_{s,t}X_{s,t} + q^A A_{s,t+1} = Y_{s,t} + E_{s,t} + A_{s,t} \tag{A.107}$$

where $P_{s,t}$ is the price of the composite consumption good on the island, $A_{s,t}$ are the family's assets, and the family saves or borrows at a constant world bond price $q^A > \beta$. $Y_{s,t}$ is the income of the family's workers in the form of wages

$$Y_{s,t} \equiv \sum_{i \in \{\mathcal{N},\mathcal{T}\}} w_{s,i,t} L_{s,i,t} \tag{A.108}$$

and $E_{s,t}$ are profits from the firms the family owns on island s

$$E_{s,t} \equiv \sum_{i \in \{\mathcal{N},\mathcal{T}\}} E_{s,i,t} = \sum_{i \in \{\mathcal{N},\mathcal{T}\}} [(z_{s,i,t} - w_{s,i,t})L_{s,i,t} - \kappa V_{s,i,t}] \tag{A.109}$$

where $z_{s,i,t}$ is a sectoral labor productivity shock, $w_{s,i,t}$ is the wage of an employed worker, $L_{s,i,t}$ is the measure of employed workers, and $V_{s,i,t}$ is the measure of vacancies for producing intermediate goods of type i on island s.

From the first-order condition for the family's problem, we can derive the shadow price of the composite consumption good at date t in units of the composite consumption good at date 0 on island s as

$$M_{s,t} = \beta^t \frac{u'(C_{s,t})/P_{s,t}}{u'(C_{s,0})/P_{s,0}} \tag{A.110}$$

Technology Nontradable and tradable goods are produced with locally produced intermediate goods. These intermediate goods are used by the nontradable and tradable sectors in different proportions. This setup effectively introduces costs of sectoral reallocations of workers because it implies a curved production possibility frontier between nontradable and tradable goods.

The economy has two types of intermediate goods: Type $\mathcal{N}$ and type $\mathcal{T}$ goods. The technology for producing nontradable goods disproportionately uses type $\mathcal{N}$ goods, whereas the technology for producing tradable goods disproportionately uses type $\mathcal{T}$ goods according to the production technologies

$$Y_{s,N,t} = A(Y^{\mathcal{N}}_{s,N,t})^{\nu}(Y^{\mathcal{T}}_{s,N,t})^{1-\nu}, \qquad Y_{s,T,t} = A(Y^{\mathcal{N}}_{s,T,t})^{1-\nu}(Y^{\mathcal{T}}_{s,T,t})^{\nu} \tag{A.111}$$

with $\nu \geq 1/2$. $Y^{\mathcal{N}}_{s,N,t}$ and $Y^{\mathcal{N}}_{s,T,t}$ denote the use of intermediate inputs of type $\mathcal{N}$ in the production of nontradable and tradable goods, whereas $Y^{\mathcal{T}}_{s,N,t}$ and $Y^{\mathcal{T}}_{s,T,t}$ denote the use of intermediate inputs of type $\mathcal{T}$ in the production of nontradable and tradable goods. Both nontradable goods producers and tradable goods producers are competitive and take as given the price of their goods, $P_{s,N,t}$ and $P_{s,T,t}$. The demands for intermediate inputs in the nontradable goods sector are

$$Y^{\mathcal{N}}_{s,N,t} = \nu \left(\frac{P^{\mathcal{T}}_{s,t}}{P^{\mathcal{N}}_{s,t}}\right)^{1-\nu} Y_{s,N,t}, \qquad Y^{\mathcal{T}}_{s,N,t} = (1-\nu) \left(\frac{P^{\mathcal{N}}_{s,t}}{P^{\mathcal{T}}_{s,t}}\right)^{\nu} Y_{s,N,t} \tag{A.112}$$

where $P^{\mathcal{N}}_{s,t}$ and $P^{\mathcal{T}}_{s,t}$ are prices of the intermediate goods $\mathcal{N}$ and $\mathcal{T}$. The equation was derived under the normalization $A = \nu^{-\nu}(1-\nu)^{1-\nu}$. Likewise, the demands for intermediate inputs in the tradable goods sector are

$$Y^{\mathcal{N}}_{s,T,t} = (1-\nu) \left(\frac{P^{\mathcal{N}}_{s,t}}{P^{\mathcal{T}}_{s,t}}\right)^{\nu} Y_{s,T,t}, \qquad Y^{\mathcal{T}}_{s,T,t} = \nu \left(\frac{P^{\mathcal{T}}_{s,t}}{P^{\mathcal{N}}_{s,t}}\right)^{1-\nu} Y_{s,T,t} \tag{A.113}$$

Adding up the demands for each type of intermediate good by the two sectors gives the total demand on island s for intermediate goods of type i

$$Y_{s,t}^{i} = Y_{s,N,t}^{i} + Y_{s,T,t}^{i} \tag{A.114}$$

Production of these intermediate goods is given by

$$Y_{s,t}^{i} = z_{s,i,t} \cdot L_{s,i,t} \tag{A.115}$$

where $L_{s,i,t}$ is the measure of employed workers producing intermediate goods of type i on island s. $z_{s,i,t}$ represents exogenous labor productivity for producing intermediate goods of type i on island s. Zero profit conditions in nontradable and tradable goods sectors imply

$$P_{s,N,t} = (P_{s,t}^{\mathcal{N}})^{\nu}(P_{s,t}^{\mathcal{T}})^{1-\nu}, \qquad P_{s,T,t} = (P_{s,t}^{\mathcal{N}})^{1-\nu}(P_{s,t}^{\mathcal{T}})^{\nu} \tag{A.116}$$

Assume that there are measures of consumers $\pi^{\mathcal{N}}$ and $\pi^{\mathcal{T}} = 1 - \pi^{\mathcal{N}}$ in occuprations $\mathcal{N}$ and $\mathcal{T}$ who supply labor to produce the two types of intermediate goods $\mathcal{N}$ and $\mathcal{T}$, respectively. Consumers in occupation $\mathcal{N}$ can produce good $\mathcal{N}$, and consumers in occupation $\mathcal{T}$ can produce good $\mathcal{T}$. Consumers are hired by intermediate goods firms that produce intermediate goods of either type $\mathcal{N}$ or $\mathcal{T}$. These goods are then sold at competitive prices $P_{s,t}^{\mathcal{N}}$ and $P_{s,t}^{\mathcal{T}}$ to firms in the nontradable and tradable goods sectors.[8]

This setup captures in a simple way the idea that switching sectors is relatively easy, whereas switching occupations is difficult. Note that any individual consumer faces no cost of switching sectors. But if a positive measure of consumer moves from one sector to the other, then the marginal revenue product of the consumers in the new sector falls and so do wages. This reduction in marginal revenue products acts like a switching cost in the aggregate.

Labor market Firms that produce intermediate good $i \in \{\mathcal{N}, \mathcal{T}\}$ post vacancies for consumers in occupation i, who produce intermediate good i when matched. Assume that consumers cannot switch occupations, so the measure of consumers in each occupation is fixed. The values of consumers in occupation i of island s are

$$\widetilde{W}_{s,i,t}(z_{s,i,t}) = w_{s,i,t}(z_{s,i,t}) \tag{A.117}$$
$$+ (1-\delta)\mathbb{F}_t[M_{s,t,t+1}\widetilde{W}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.118}$$
$$+ \delta\mathbb{F}_t[M_{s,t,t+1}\widetilde{U}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.119}$$

for employed consumers, and

$$\widetilde{U}_{s,i,t}(z_{s,i,t}) = P_t b_{s,t}(z_{s,i,t}) \tag{A.120}$$
$$+ \mathbb{F}_t[M_{s,t,t+1} f_{s,i,t}(z_{s,i,t})\widetilde{W}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.121}$$
$$+ \mathbb{F}_t[M_{s,t,t+1}(1 - f_{s,i,t}(z_{s,i,t}))\widetilde{U}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.122}$$

[8] It is equivalent to think that the consumers in each occupation work in the sector that purchases the goods they produce. Under this interpretation, we can think of consumers in occupation $\mathcal{T}$ as being employed in sectors N and T and consumers in occupation $\mathcal{N}$ as also being employed in sectors N and T in different proportions. Sector N employs consumers in occupation $\mathcal{N}$ relatively more intensively, whereas sector T employs consumers in occupation $\mathcal{T}$ relatively more intensely.

for nonemployed consumers. $w_{s,i,t}(z_{s,i,t})$ is the wage received by a consumer in occupation i. $f_{s,i,t}(z_{s,i,t})$ is the job-finding probability of a consumer in occupation i. $b_{s,t}(z_{s,i,t})$ is the output of a consumer when not employed. δ is an exogenous separation probability. Subjective expectations $\mathbb{F}_t[\cdot]$ are with respect to next period's productivity $z_{s,i,t+1}$.

The value of a firm producing intermediate good i matched with a consumer in occupation i with productivity $z_{s,i,t}$ is

$$\widetilde{J}_{s,i,t}(z_{s,i,t}) = P_{s,i,t}z_{s,i,t} - w_{s,i,t}(z_{s,i,t}) + (1-\delta)\mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.123}$$

At date t a consumer in occupation i matched with a firm in intermediate good sector i produces $z_{s,i,t}$ units of good i, which sells for $P_{s,i,t}z_{s,i,t}$, and the firm pays the consumer $w_{s,i,t}(z_{s,i,t})$. The cost of posting a vacancy is κ units of the composite tradable good. Free entry for intermediate goods producers in the labor market for workers in occupation i implies

$$\kappa = q_{s,i,t}(z_{s,i,t}) \cdot \mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.124}$$

The matches of firms that produce intermediate good i with consumers are

$$L_{s,i,t}(z_{s,i,t}) = \frac{U_{s,i,t}(z_{s,i,t})V_{s,i,t}(z_{s,i,t})}{[U_{s,i,t}(z_{s,i,t})^\eta + V_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.125}$$

where $U_{s,i,t}(z_{s,i,t})$ is the measure of nonemployed consumers and $V_{s,i,t}(z_{s,i,t})$ is the measure of posted vacancies to attract such consumers. The parameter η governs the sensitivity of $f_{s,i,t}(z_{s,i,t})$ to $\theta_{s,i,t}$. The worker job-finding rate $f_{s,i,t}(z_{s,i,t})$ and firm job-filling rate $q_{s,i,t}(z_{s,i,t})$ are

$$f_{s,i,t}(z_{s,i,t}) = \frac{L_{s,i,t}(z_{s,i,t})}{U_{s,i,t}(z_{s,i,t})} = \frac{\theta_{s,i,t}(z_{s,i,t})}{[1+\theta_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.126}$$

$$q_{s,i,t}(z_{s,i,t}) = \frac{L_{s,i,t}(z_{s,i,t})}{V_{s,i,t}(z_{s,i,t})} = \frac{1}{[1+\theta_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.127}$$

where $\theta_{s,i,t}(z_{s,i,t}) = V_{s,i,t}(z_{s,i,t})/U_{s,i,t}(z_{s,i,t})$ is the vacancy to nonemployment ratio.

The Nash bargaining problem determines the wage $w_{s,i,t}(z_{s,i,t})$ in any given match

$$\max_w [\widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t})]^\gamma \widetilde{J}_{s,i,t}(z_{s,i,t})^{1-\gamma} \tag{A.128}$$

where γ is a consumer's bargaining weight. Defining the surplus of a match between a firm and a consumer as $\widetilde{S}_{s,i,t}(z_{s,i,t}) = \widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t}) + \widetilde{J}_{s,i,t}(z_{s,i,t})$, Nash bargaining implies that firms and consumers split this surplus according to

$$\widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t}) = \gamma\widetilde{S}_{s,i,t}(z_{s,i,t}), \qquad \widetilde{J}_{s,i,t}(z_{s,i,t}) = (1-\gamma)\widetilde{S}_{s,i,t}(z_{s,i,t}) \tag{A.129}$$

Equilibrium Consider now the market-clearing conditions. Market clearing for the two types of intermediate goods requires that

$$Y_{s,i,t} = z_{s,i,t} \cdot L_{s,i,t} = Y^i_{s,N,t} + Y^i_{s,T,t}, \qquad i \in \{\mathcal{N}, \mathcal{T}\} \tag{A.130}$$

the left side of this equation is the total amount of intermediate goods of type i produced by employed workers in occupation i on island s, $L_{s,i,t}$. The right side is the total amount of these intermediate goods used by firms in the nontradable and tradable goods sectors on that island. Employment in the nontradable goods sector on island s is

$$\frac{Y^{\mathcal{N}}_{s,N,t}}{Y^{\mathcal{N}}_{s,t}} L_{s,\mathcal{N},t} + \frac{Y^{\mathcal{T}}_{s,N,t}}{Y^{\mathcal{T}}_{s,t}} L_{s,\mathcal{T},t} \tag{A.131}$$

Employment in the tradable goods sector on island s is

$$\frac{Y^{\mathcal{N}}_{s,T,t}}{Y^{\mathcal{N}}_{s,t}} L_{s,\mathcal{N},t} + \frac{Y^{\mathcal{T}}_{s,T,t}}{Y^{\mathcal{T}}_{s,t}} L_{s,\mathcal{T},t} \tag{A.132}$$

The relative demand effect on employment in the two sectors captures the idea that, since $Y^i_{s,N,t}/Y_{s,i,t} + Y^i_{s,T,t}/Y_{s,i,t} = 1$ for $i \in \{\mathcal{N}, \mathcal{T}\}$, any shift in demand from the nontradable goods sector on an island, holding fixed total employment on the island, decreases employment in the nontradable goods sector and increases it in the tradable goods sector on the island.

Market clearing for nontradable goods requires that the demand for nontradable goods on island s equal the amount on nontradable goods produced on island s

$$X_{s,N,t} = A(Y^{\mathcal{N}}_{s,N,t})^{\nu}(Y^{\mathcal{T}}_{s,N,t})^{1-\nu} \tag{A.133}$$

Similarly, market clearing for tradable goods requires that the world demand for tradable goods produced from island s equal the amount of tradable goods produced on island s

$$Y_{s,T,t} = A(Y^{\mathcal{N}}_{s,T,t})^{1-\nu}(Y^{\mathcal{T}}_{s,T,t})^{\nu} \tag{A.134}$$

Decomposition of Regional Job Filling Rates Combine the value of the worker to the firm with the zero-profit condition for entering firms, substitute recursively:

$$\frac{\kappa}{q_{s,i,t}} = \mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}] \tag{A.135}$$

$$= \mathbb{F}_t[M_{s,t,t+1}(P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1} + (1-\delta)\mathbb{F}_{t+1}[M_{s,t+1,t+2}\widetilde{J}_{s,i,t+2}])] \tag{A.136}$$

$$= \mathbb{F}_t\left[M_{s,t,t+1}\left(P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1} + (1-\delta)\frac{\kappa}{q_{s,i,t+1}}\right)\right] \tag{A.137}$$

Multiply both sides by the level of employment in sector i island s

$$\frac{\kappa}{q_{s,i,t}} L_{s,i,t+1} = \mathbb{F}_t\left[M_{s,t,t+1}\left((P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1})L_{s,i,t+1} + (1-\delta)\frac{\kappa}{q_{s,i,t+1}} L_{s,i,t+1}\right)\right] \tag{A.138}$$

Substitute in the law of motion for employment $L_{s,i,t+1} = (1-\delta)L_{s,i,t} + q_{s,i,t}V_{s,i,t}$

$$\frac{\kappa}{q_{s,i,t}} L_{s,i,t+1} = \mathbb{F}_t\left[M_{s,t,t+1}\left((P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1})L_{s,i,t+1} - \kappa V_{s,i,t+1} + \frac{\kappa}{q_{s,i,t+1}} L_{s,i,t+2}\right)\right] \tag{A.139}$$

Define the firm's total earnings as profits after vacancy posting costs $E_{s,i,t} \equiv (P_{s,i,t}z_{s,i,t} - w_{s,i,t})L_{s,i,t} - \kappa V_{s,i,t}$. Substitute recursively:

$$\frac{\kappa}{q_{s,i,t}} L_{s,i,t+1} = \mathbb{F}_t\left[\sum_{j=1}^{T-1}\left(\prod_{k=1}^{j} M_{s,t+k-1,t+k}\right) E_{s,i,t+j}\right] + \mathbb{F}_t\left[M_{s,t+T-1,t+T}\frac{\kappa}{q_{s,i,t+T}} L_{s,i,t+T+1}\right] \tag{A.140}$$

Take limits as $T \to \infty$ while applying a transversality condition to rule out bubbles

$$\frac{\kappa}{q_{s,i,t}} L_{s,i,t+1} = \mathbb{F}_t \left[\sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} M_{s,t+k-1,t+k} \right) E_{s,i,t+j} \right] \equiv P^E_{s,i,t} \tag{A.141}$$

where $P^E_{s,i,t}$ is the firm's ex-dividend equity value. Aggregate the hiring condition to the regional level as employment-weighted averages:

$$\frac{\kappa}{q_{s,t}} = \frac{P^E_{s,t}}{L_{s,t+1}} \tag{A.142}$$

where $S_{s,i,t+1} \equiv \frac{L_{s,i,t+1}}{L_{s,t+1}} = \frac{L_{s,i,t+1}}{\sum_{i' \in I} L_{s,i',t+1}}$ is industry i's employment share in region s. Define $q_{s,t} \equiv (\sum_{i \in I} \frac{1}{q_{s,i,t}} S_{s,i,t+1})^{-1}$, $P^E_{s,t} \equiv \sum_{i \in I} P^E_{s,i,t}$, and $E_{s,t} \equiv \sum_{i \in I} E_{s,i,t}$ as the employment-weighted job filling rate, total equity value, and total earnings of firms in region s. Take logarithms on both sides and expand the price-employment ratio $P^E_{s,t}/L_{s,t+1}$

$$\log q_{s,t} = \log \kappa - \log \left(\frac{P^E_{s,t}}{E_{s,t}} \right) - \log \left(\frac{E_{s,t}}{L_{s,t+1}} \right) \equiv \log \kappa - pe_{s,t} - el_{s,t} \tag{A.143}$$

where $pe_{s,t} \equiv \log(P^E_{s,t}/E_{s,t})$ is the log price-earnings ratio and $el_{s,t} \equiv \log(E_{s,t}/L_{s,t+1})$ is the log earnings-employment ratio. Next, log-linearize the price-earnings ratio by first log-linearizing the price-dividend ratio $pd_{s,t}$ around its long-term average $\overline{pd}$ (Campbell and Shiller, 1988):

$$pd_{s,t} = c_{pd} + \Delta d_{s,t+1} - r_{s,t+1} + \rho pd_{s,t+1} \tag{A.144}$$

where c_{pd} is a linearization constant, $r_{s,t+1} \equiv \log(\frac{P_{s,t+1}+D_{s,t+1}}{P_{s,t}})$ is the log stock return (with dividends), and $\rho \equiv \frac{\exp(\overline{pd})}{1+\exp(\overline{pd})} = 0.98$ is a persistence parameter that arises from the log linearization. Rewrite the equation in terms of log price-earnings instead of log price-dividends by using the identity $pe_{s,t} = pd_{s,t} + de_{s,t}$, where $de_{s,t}$ log payout ratio:

$$pe_{s,t} = c_{pd} + \Delta e_{s,t+1} - r_{s,t+1} + \rho pe_{s,t+1} + (1-\rho) de_{s,t+1} \tag{A.145}$$

Since $1-\rho \approx 0$ and the payout ratio $de_{s,t}$ is bounded, $(1-\rho)de_{s,t+1}$ can be approximated as a constant, i.e., $c_{pe} \approx c_{pd} + (1-\rho)de_{s,t+1}$ (De La O et al., 2024):

$$pe_{s,t} \approx c_{pe} + \Delta e_{s,t+1} - r_{s,t+1} + \rho pe_{s,t+1} \tag{A.146}$$

Recursively substitute for the next h periods

$$pe_{s,t} = \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{s,t+j} - r_{s,t+j}) + \rho^h pe_{s,t+h} \tag{A.147}$$

Substitute log-linearized price-earnings into the hiring equation:

$$\log q_{s,t} = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{s,t+j}}_{r_{s,t,t+h}} - \underbrace{\left[el_{s,t} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{s,t+j} \right]}_{e_{s,t,t+h}} - \underbrace{\rho^h pe_{s,t+h}}_{pe_{s,t,t+h}} \tag{A.148}$$

where $c_q \equiv \log\kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. $r_{s,t,t+h}$ captures news about discount rates. $e_{s,t,t+h}$ captures news about cash flows, where the one year ahead log earnings-employment ratio $el_{s,t}$ captures news about current cash flow fluctuations and $j = 1, \ldots, h$ year ahead log earnings growth $\Delta e_{s,t+j}$ capture news about future cash flows. $pe_{s,t,t+h}$ is a terminal value that captures other long-term influences beyond h years into the future that is not already captured in discount rates and cash flows. The decomposition above holds both ex-ante and ex-post. I consider ex-ante decompositions under subjective expectations $\mathbb{F}_t[\cdot]$:

$$\log q_{s,t} = c_q + \mathbb{F}_t[r_{s,t,t+h}] - \mathbb{F}_t[e_{s,t,t+h}] - \mathbb{F}_t[pe_{s,t,t+h}] \tag{A.149}$$

where $\mathbb{F}_t[x_{s,t,t+h}]$ denotes the h period ahead subjective expectations of variable x for the firm in region s under the time t information set.

Predictability of Regional Unemployment Rates To derive implications for unemployment at the regional level, start with the disaggregated accumulation equation:

$$U_{s,i,t+1} = \delta_{s,i,t}(1 - U_{s,i,t}) + (1 - f_{s,i,t})U_{s,i,t} \tag{A.150}$$

where $U_{s,i,t}$ denotes the measure of unemployed workers in occupation i region s. Aggregate across the occupations to get total regional unemployment $U_{i,t} \equiv \sum_{i\in I} U_{s,i,t}$:

$$U_{s,t+1} = \delta_{s,t}(1 - U_{s,t}) + (1 - f_{s,t})U_{s,t} \tag{A.151}$$

where it is assumed that $\delta_{s,i,t} = \delta_{s,t}$ and $f_{s,i,t} = f_{s,t}$ for all i. A log-linearization around the steady state similar to Section B.2 gives us:

$$u_{s,t+1} \approx q\theta \log\delta_{s,t} + (1 - \delta - q\theta)u_{s,t} - q\theta\log q_{s,t} - q\theta\log\theta_{s,t} \tag{A.152}$$

where lower-cased variables denote log deviations from steady state. Finally, substitute in equation (A.149), which is a decomposition of the job filling rate $\log q_{s,t}$ into discount rate, cash flow, and future price-earnings components:

$$u_{s,t+1} \approx - \underbrace{q\theta c_q}_{\text{Constant}} - \underbrace{q\theta \cdot \mathbb{F}_t[r_{s,t,t+h}]}_{\text{Discount Rate}} + \underbrace{q\theta \cdot \mathbb{F}_t[e_{s,t,t+h}]}_{\text{Cash Flow}} + \underbrace{q\theta \cdot \mathbb{F}_t[pe_{s,t,t+h}]}_{\text{Future Price-Earning}} \tag{A.153}$$

$$+ \underbrace{(1 - \delta - q\theta)\cdot u_{s,t} - q\theta \cdot \log\theta_{s,t} + q\theta\cdot\log\delta_{s,t}}_{\text{Lag Unemployment, Tightness, Separations}} \tag{A.154}$$

The equation holds both ex-ante and ex-post. Therefore, I compare results from evaluating the equation under subjective $\mathbb{F}_t[\cdot]$ or rational $\mathbb{E}_t[\cdot]$ expectations. The decomposition can be estimated using regressions of the log unemployment rate on each of the components shown in the equation:

$$u_{s,t+1} = \beta_r\mathbb{F}_t[r_{s,t,t+h}] + \beta_e\mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.155}$$

where $X_{s,t} \equiv [u_{s,t}, \log\theta_{s,t}, \log\delta_{s,t}]'$ collects the labor market factors. I estimate the decomposition using multivariate regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations. To ensure stationarity and remove seasonal effects, I estimate the regression in log annual growth rates relative to the same quarter of the previous year. The future price-earnings ratio term $\mathbb{F}_t[pe_{s,t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\mathbb{F}_t[r_{s,t,t+h}]$ and cash flows $\mathbb{F}_t[e_{s,t,t+h}]$ through the Campbell and Shiller (1988) present value identity in equation (11). Similarly, the equation can also be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with its rational expectations counterpart $\mathbb{E}_t[\cdot]$ based on machine learning forecasts.

C Data Details

This section describes the data sources used in the estimation. I use quarterly data on the macroeconomic and financial series represented in the decomposition from equations (A.82) and (A.83): employment L_t, unemployment U_t, job filling rates q_t, stock returns $r_{t,t+h}$, earnings growth $\Delta e_{t,t+h}$, price-earnings ratio pe_{t+h}, and earnings-employment ratio el_{t+h}. For each dependent variable of the decomposition, I also construct their corresponding survey expectations $\mathbb{F}_t$ and machine expectations $\mathbb{E}_t$.

C.1 Employment

C.1.1 Realized Employment

For realized values of employment, I first construct an annual series for the aggregate number of employees (EMP) of the S&P 500 constituents by using accounting information from the CRSP and Compustat Merged Annual Industrial Files. The data spans 1970 to 2022 and was downloaded from WRDS on July 26, 2023. I interpolate the annual series to a monthly frequency by using the fitted values from real-time regressions of log annual Compustat employment series on the log monthly BLS series for total nonfarm payrolls (PAYEMS). The regressions are estimated over recursively expanding samples from an initial monthly sample that begins on 1970:01 and ends on the month of the data release for each month's total nonfarm payrolls. To ensure that the fitted values do not use future information not available on each data release, I align each monthly BLS nonfarm payroll release with the annual Compustat S&P 500 employment series from the previous calendar year. To obtain a measure of employment L_{t+1} at the beginning of period $t+1$, I convert the monthly interpolated values to a quarterly frequency by taking the value of the series as of the last month of each calendar quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section B while still remaining known to firms by the end of period t. Data on nonfarm payrolls was downloaded through FRED on May 15, 2024.

C.1.2 Survey Expectations of Employment

For subjective expectations about employment growth, I use mean point forecasts from the CFO survey, obtained from `https://www.richmondfed.org/-/media/RichmondFedOrg/research/national_economy/cfo_survey/current_historical_cfo_data.xlsx`. Prior to 2020Q1, the survey asks respondents about their expectations for their company's annual employment growth during the next 12 months: *"Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, 0%, −2%, etc.) [Leave blank if not applicable]."* I interpret the survey to be asking about $\mathbb{F}_t[100 \times (L_{t+1}/L_t - 1)]$, the annual net simple employment growth between the survey month at period t and the same month one year later at period $t+1$. To obtain survey expectations of annual log employment growth $\mathbb{F}_t[\Delta l_{t+1}]$ from a survey expectation of annual net simple employment growth, I use the approximation

$$\mathbb{F}_t[\Delta l_{t+1}] \approx \log(1 + \mathbb{F}_t[100 \times (L_{t+1}/L_t - 1)]/100) \quad \text{if } t < 2020Q1 \tag{A.156}$$

After 2020Q1, the survey asks respondents about their expectations for their company's employment level for the current and next calendar years: *"What number of employees do you expect to have at the end of calendar years 2021 and 2022? We will assume a value of 0 if left blank."* Since the reference periods for these forecasts are at the end of the calendar year, I follow De La O and Myers (2021) by interpolating across the end of current and next year expectations to obtain rolling 12-month ahead expectations. For example, if the fiscal year of firm XYZ ends nine months after the survey date, the end of current year expectation has a forecast horizon of 9 months, and the end of next year expectation has a forecast horizon of 21 months. I then obtain the 12-month ahead forecast by interpolating these two forecasts as follows:

$$\mathbb{F}_t[L_{t+1}] = \frac{9}{12}\mathbb{F}_t[L_{t+9/12}] + \frac{3}{12}\mathbb{F}_t[L_{t+21/12}] \quad \text{if } t \geq 2020Q1 \tag{A.157}$$

The survey also asks respondents about the current number of employees at the time of the survey: *"What is your current number of employees? Precise values are preferred. We will assume a value of 0 if left blank."* I convert the level forecasts to one-year growth forecasts by constructing the log difference of the level forecasts one-year ahead relative to the current number of employees:

$$\mathbb{F}_t[\Delta l_{t+1}] \equiv \mathbb{F}_t[\Delta \log L_{t+1}] \approx \log(\mathbb{F}_t[L_{t+1}]) - \log(L_t) \quad \text{if } t \geq 2020Q1 \tag{A.158}$$

To obtain long-horizon survey expectations of annualized long-horizon log employment growth for $h > 1$ years ahead, I assume that survey respondents expect employment growth to revert back to its historical mean over the next 10 years. Specifically, I interpolate the h year ahead forecast between the 1 year ahead forecast and the historical mean annual log employment growth, with a relative weight on the 1 year ahead forecast that decays linearly over the next 10 years:

$$\mathbb{F}_t[\Delta l_{t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[\Delta l_{t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[\Delta l_{t+10}], \qquad h = 1, 2, \ldots, 10 \tag{A.159}$$

where $\mathbb{F}_t[\Delta l_{t+10}]$ is the historical mean of the realized log annual employment growth up to the time of the forecast.

The CFO survey panel includes firms that range from small operations to Fortune 500 companies across all major industries. Respondents include chief financial officers, owner-operators, vice presidents and directors of finance, and others with financial decision-making roles. The CFO panel has 1,600 members as of December 2022. I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Because the CFO survey releases quarterly forecasts at the end of each quarter, I conservatively set the response deadline for the machine forecast to be the first day of the last month of each quarter (e.g., March 1st).

The data spans the periods 2001Q4 to 2021Q1 and were downloaded on August 8th, 2022. The forecast is not available in 2019Q1, 2019Q4, 2020Q1, and 2020Q2. I impute the missing forecast for 2019Q1 by linearly interpolating between the available forecasts from 2018Q4 and 2019Q2. I impute the missing forecasts for 2019Q4, 2020Q1, and 2020Q2 by interpolating with the nearest available forecast between 2019Q3 and 2020Q3.

C.2 Job Filling Rate

I construct a monthly series for the number of vacancies V_t following Barnichon (2010), by using JOLTS job openings starting 2000:12 (JTS00000000JOL) and extending the series back in time using the help-wanted index before 2000:12. The vacancies data has been downloaded from available on the author's website on May 19, 2024. For realized values of unemployment U_t, I use the BLS monthly series for the unemployment level (UNEMPLOY), downloaded through FRED on May 15, 2024. Labor market tightness $\theta_t = V_t/U_t$ is the ratio between vacancies and unemployment.

I follow Shimer (2012) in constructing the job separation rate δ_t, job finding rate f_t, and job filling rate q_t. Job separation rate is the share of short-term unemployed out of total employment $\delta_t = U_t^s/L_t$, where U_t^s is the BLS series for the number of unemployed less than 5 weeks (UEMPLT5) that was downloaded through FRED on May 15, 2024. The job finding rate is:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}} \tag{A.160}$$

The expression for the job finding rate follows from the unemployment accumulation equation:

$$U_t = (1 - f_t)U_{t-1} + U_t^s \tag{A.161}$$

which states that unemployment U_t consists of either the previously unemployed U_{t-1} who did not find a job $(1 - f_t)$, or the short-term unemployed U_t^s that lost a job during the current period. The job filling rate is defined as the share of filled vacancies $f_t V_t$ out of unemployment U_t:

$$q_t = \frac{f_t}{\theta_t} = \frac{f_t V_t}{U_t} \tag{A.162}$$

I first construct the job filling rate q_t at the monthly frequency. To remove high-frequency fluctuations that likely reflect measurement errors, I time-aggregate the monthly series to a quarterly frequency by taking a 3-month trailing average that ends on the first month of each calendar quarter. This timing assumption ensures that the survey and machine expectations in the variance decomposition do not use advance information about job filling rates that were not published at the time of each forecast. To ensure that all variables used in the variance decomposition are stationary, I follow Shimer (2012) by detrending the quarterly job filling rate q_t using an HP filter with a smoothing parameter of 10^5.

C.3 Wages

To assess the cyclicality of subjective wage expectations, I use publicly available survey and macroeconomic data to construct measures of actual real wage growth, subjective wage expectations, and unemployment rate changes. The Livingston Survey (semi-annual, 1961S1–2022S2), the CFO Survey (quarterly, 2001Q4–2022Q4), and the Survey of Consumer Expectations (SCE) (monthly, 2015M5–2022M12) provide the necessary data.

I derive subjective wage growth expectations from median consensus forecasts of nominal wage growth in these surveys. The Livingston Survey forecasts are deflated using its own median CPI inflation forecast, while the CFO and SCE survey forecasts are deflated using CPI inflation expectations from the Survey of Professional Forecasters (SPF).

To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor UC_t^W under subjective expectations. In the search and matching model, the user cost of labor is the difference in the present value of wages between two firm-worker matches that are formed in two consecutive periods. Existing work measures the user cost of labor under full information rational expectations and finds that the user cost is more cyclical than the flow wage, suggesting that workers hired in recessions earn lower wages not only when hired but also in subsequent periods (Kudlyak, 2014; Bils et al., 2023). The survey-based measure this this paper relaxes the rational expectations assumption maintained in existing work.

Consider the free-entry condition in the search and matching model:

$$\frac{\kappa}{q_t} = J_{t,t} \tag{A.163}$$

where a firm must pay a per vacancy cost of κ and vacancies are filled with probability q_t. $J_{t,\tau}$ is the value of a firm with a worker at time τ such that the productive match started at time t:

$$J_{t,\tau} \equiv z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}] \tag{A.164}$$

where $\mathbb{F}_t[\cdot]$ denotes subjective expectations based on survey data. $\beta = 0.9569$ is a discount factor and $\delta = 0.295$ is the probability that a employment relationship is terminated, both from Kudlyak (2014). Each period τ, a firm-worker match produces a per period output of z_τ and an employed worker received wage $w_{t,\tau}$ where t is the period when the worker is hired. $w_{t,t}$ is the new-hire wage. Note that the free entry condition is only required to hold for newly created matches for $\tau = t$. The expected difference between the firm's value of a newly created match in time t and the discounted value of a newly created match in period $t+1$ is

$$J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] = z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}] \tag{A.165}$$

$$- \beta(1-\delta)\mathbb{F}_t \left[z_{t+1} - w_{t+1,t+1} + \sum_{\tau=t+2}^{\infty} (\beta(1-\delta))^{\tau-(t+1)} \mathbb{F}_{t+1}[z_\tau - w_{t+1,\tau}] \right] \tag{A.166}$$

Apply the Law of Iterated Expectations and collect terms

$$J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] = z_t - w_{t,t} - \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}] \tag{A.167}$$

Substitute the free-entry condition to the left-hand side

$$\underbrace{\frac{\kappa}{q_t} - \beta(1-\delta)\mathbb{F}_t \left[\frac{\kappa}{q_{t+1}} \right]}_{\text{Non-wage component of user cost } UC_t^V} = \underbrace{z_t}_{\text{Benefit}} - \underbrace{\left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}] \right]}_{\text{Wage component of user cost } UC_t^W} \tag{A.168}$$

The equation shows that the firm faces two sources of costs from a match: wage payments to a worker UC_t^W and vacancy opening costs UC_t^V. The firm creates jobs as long as the marginal benefit from adding a worker exceeds the user cost of labor. It is worth noting that the wage component of the user cost of labor UC_t^W, not the wage $w_{t,t}$, is the allocative price of labor.

I use worker-level data from the Survey of Consumer Expectations (SCE) to construct the user cost of labor UC_t^W under the survey respondents' subjective expectations. The SCE asks respondents about: the *month and year on which their current employment relationship started* (i.e., t in $w_{t,\tau}$); *"annual earnings, before taxes and other deductions, on your [current/main] job"* ($w_{t,\tau}$); short-term expectations on what their *"annual earnings will be in 4 months"* ($\mathbb{F}_t[w_{t,t+\frac{4}{12}}]$) and long-term expectations on *"annual earnings to be at your current job in 10 years"* ($\mathbb{F}_t[w_{t,t+10}]$). I obtain survey expectations about medium-term earnings between 4 months to 10 years by linearly interpolating between the two horizons:

$$\mathbb{F}_t[w_{t,t+h}] = \frac{10-h}{10-\frac{4}{12}} \mathbb{F}_t[w_{t,t+\frac{4}{12}}] + \frac{h-\frac{4}{12}}{10-\frac{4}{12}} \mathbb{F}_t[w_{t,t+10}], \qquad h = 1, 2, \ldots, 10 \tag{A.169}$$

The user cost of labor formulation assumes infinitely lived firms and workers, while empirical data are inherently finite. I truncate the horizon at 10 years given the availability of the survey data. Longer horizons reduce the weight of future terms due to discounting and job separations. In addition, if unemployment follows a mean-reverting process, wages in long-term employment relationships will eventually converge to the long-term mean, which after discounting would limit the size of very long-term influences (Kudlyak, 2014).

I measure actual real wage growth using two BLS wage series. The Livingston Survey forecasts target annual log real wage growth based on average weekly earnings of production and nonsupervisory employees in manufacturing (CES3000000030). The CFO and SCE surveys target annual log real wage growth based on average hourly earnings of private-sector employees (CEU0500000008). I deflate nominal wages using the Consumer Price Index (CPIAUCSL) to adjust for purchasing power.

For unemployment rates used to assess the cyclicality of wages, I use both actual data and subjective forecasts. The actual seasonally adjusted U.S. unemployment rate (UNRATE) comes from the BLS Current Population Survey (CPS), while subjective unemployment expectations are derived from median consensus SPF forecasts of future unemployment rates.

C.4 Stock Returns

C.4.1 Realized Stock Returns

To measure stock market returns, I use monthly data on CRSP value-weighted returns including dividends (VWRETD) from the Center for Research in Security Prices (CRSP). I compute annualized log stock returns by compounding the monthly returns using $r_{t+h} \equiv (1/h)\sum_{j=1}^{12h} \log(1 + VWRETD_{t+j/12})$. The data was downloaded from WRDS on February 12, 2023. When evaluating the MSE ratios of the machine relative to that of a benchmark survey, I compute machine forecasts for either annual CRSP returns or S&P 500 price growth depending on which value most closely aligns with the concept that survey respondents are asked to predict. To measure one-year stock market price growth, I use the one-year log cumulative growth rate of the S&P 500 index, $\Delta p_{t+1} \equiv \log(P_{t+1}/P_t)$. The monthly S&P index series spans the period 1957:03 to 2022:12 and was downloaded from WRDS on January 24, 2024 from the Annual Update data of the Index File on the S&P 500.

C.4.2 Survey Expectations of Stock Returns

CFO Survey I use survey forecasts of S&P 500 stock returns from the CFO survey to measure subjective return expectations. The CFO survey is a quarterly survey that asks respondents about their expectations for the S&P 500 return over the next 12 months and 10 years ahead, obtained from `https://www.richmondfed.org/-/media/RichmondFedOrg/research/national_economy/cfo_survey/current_historical_cfo_data.xlsx`. I use the mean point forecast for the value of the "most likely" future stock return in the estimation. More specifically, the survey asks the respondent "*over the next 12 months, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*". The survey question for stock return expectations 10 years ahead is "*over the next 10 years, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*".

The CFO survey panel includes firms that range from small operations to Fortune 500 companies across all major industries. Respondents include chief financial officers, owner-operators, vice presidents and directors of finance, and others with financial decision-making roles. The CFO panel has 1,600 members as of December 2022.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Because the CFO survey releases quarterly forecasts at the end of each quarter, I conservatively set the response deadline for the machine forecast to be the first day of the last month of each quarter (e.g., March 1st).

The data spans the periods 2001Q4 to 2021Q1 and were downloaded on August 8th, 2022. Mean point forecasts before 2020Q3 are available in column `sp_1_exp` of sheet `through_Q1_2020`; mean point forecasts from 2020Q3 and onwards are available in column `sp_12moexp_2` of sheet `CFO_SP500`. The forecast is not available in 2019Q1, 2019Q4, 2020Q1, and 2020Q2. I impute the missing forecast for 2019Q1 by linearly interpolating between the available forecasts from 2018Q4 and 2019Q2. I impute the missing forecasts for 2019Q4, 2020Q1, and 2020Q2 by interpolating with the nearest available forecast between 2019Q3 and 2020Q3.

Following Nagel and Xu (2022), I assume that the forecasted S&P 500 return includes dividends and capture expectations about annualized cumulative simple net returns compounded from time t to $t+h$, i.e., $\mathbb{F}_t[R_{t,t+h}]$. To obtain survey expectations of log returns $\mathbb{F}_t[\log(1+r_{t,t+h})]$ from a survey expectation of net simple returns $\mathbb{F}_t[R_{t,t+h}]$, I use the approximation $\mathbb{F}_t[\log(1+r_{t,t+h})] \approx \log(1+\mathbb{F}_t[R_{t,t+h}])$.

To obtain long-horizon survey expectations of annualized cumulative log S&P 500 returns over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year cumulative log return expectations:

$$\mathbb{F}_t[r_{t,t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[r_{t,t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[r_{t,t+10}], \quad h = 1, 2, \ldots, 10 \tag{A.170}$$

Finally, I use the difference between the cumulative annualized long-horizon log return expectations between adjacent years (i.e., $\mathbb{F}_t[r_{t,t+h-1}]$ and $\mathbb{F}_t[r_{t,t+h}]$) to obtain $\mathbb{F}_t[r_{t+h}]$, the time t survey expectation of forward one-year log stock returns h years ahead:

$$\mathbb{F}_t[r_{t+h}] = h \times \mathbb{F}_t[r_{t,t+h}] - (h-1) \times \mathbb{F}_t[r_{t,t+h-1}], \quad h = 1, 2, \ldots, 10 \tag{A.171}$$

Gallup/UBS Survey The UBS/Gallup is a monthly survey of one-year-ahead stock market return expectations. I use the mean point forecast in our estimation and compare these to machine forecasts of the annual CRSP return. Gallup conducted 1,000 interviews of investors during the first two weeks of every month and results were reported on the last Monday of the month. The first survey was conducted on 1998:05. Until 1992:02, the survey was conducted quarterly on 1998:05, 1998:09, and 1998:11. The data on 1998:06, 1998:07, 1998:08, 1998:10, 1998:12, 1999:01, and 2006:01 are missing because the survey was not conducted on these months. I follow Adam et al. (2021) in starting the sample after 1999:02 due to missing values at the beginning of the sample.

For each month when the survey was conducted, respondents are asked about the return they expect on their own portfolio. The survey question is *"What overall rate of return do you expect to get on your portfolio in the next twelve months?"* Before 2003:05, respondents are also asked about the return they expect from an investment in the stock market during the next 12 months. The survey question is *"Thinking about the stock market more generally, what overall rate of return do you think the stock market will provide investors during the coming twelve months?"* For each month, I calculate the average expectations of returns on their own portfolio and returns on the market index. When calculating the average, survey respondents are weighted by the weight factor provided in the survey. I exclude extreme observations where a respondent reported expected returns higher than 95% or lower than -95% on either their own portfolio or the market index.

In order to construct a consistent measure of stock market return expectations over the entire sample period, I impute missing market return expectations using the fitted values from two regressions. First, I impute missing values during 1999:02-2005:12 and 2006:02-2007:10 with the fitted value from regressing expected market returns on own portfolio expectations contemporaneously, where the regression is estimated using the part of the sample where both are available. Second, I impute the one missing observation in both market and own portfolio return expectations for 2006:01 with the fitted value from regressing the market return expectations on the lagged own portfolio return expectations, where the coefficients are estimated using part of the sample where both are available, and the fitted value combines the estimated coefficients with lagged own portfolio expectations data from 2005:12.

Following Nagel and Xu (2022), I assume that the forecasted stock market return includes dividends and capture expectations about annual simple net stock returns $\mathbb{F}_t[R_{t+1}]$. To obtain survey expectations of annual log returns $\mathbb{F}_t[\log(1 + r_{t+1})]$ from a survey expectation of annual net simple returns $\mathbb{F}_t[R_{t+1}]$, I use the approximation $\mathbb{F}_t[\log(1 + r_{t+1})] \approx \log(1 + \mathbb{F}_t[R_{t+1}])$. After applying all the procedures, the Gallup market return expectations series spans the periods 1999:02 to 2007:10. The data were downloaded on August 1st, 2024 from Roper iPoll: `http://ropercenter.cornell.edu/ubs-index-investor-optimism/`.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are in the first two weeks of a month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all UBS/Gallup respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "one-year-ahead" I interpret the question to be asking about the forecast period spanning from the current survey month to the same month one year ahead.

Michigan Survey of Consumers (SOC) The SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. Table 20 of the SOC reports the probability of an increase in stock market in next year. The survey question was *"The next question is about investing in the stock market. Please think about the type of mutual fund known as a diversified stock fund. This type of mutual fund holds stock in many different companies engaged in a wide variety of business activities. Suppose that tomorrow someone were to invest one thousand dollars in such a mutual fund. Please think about how much money this investment would be worth one year from now. What do you think the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?"*

When I use this survey forecast to compare to machine forecasts, I impute a point forecast for stock market returns using the method described in Section C.4.2 below. I compare the imputed point forecast to machine forecasts of CRSP returns.

For the SOC, interviews are conducted monthly typically over the course of an entire month. (In rare cases, interviews may commence at the end of the previous month, as in February 2018 when interviews began on January 31st 2018.) I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are almost always conducted over the course of an entire month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "year ahead" I interpret the question to be asking about the forecast period spanning the period running from the current survey month to the same month one year ahead. The data spans 2002:06 to 2021:12. The SOC responses were obtained from `https://data.sca.isr.umich.edu/data-archive/mine.php` and downloaded on August 13th, 2022.

Livingston Survey Stock Price Forecast I obtain the Livingston Survey S&P 500 index forecast (SPIF) from the Federal Reserve Bank of Philadelphia, URL: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-historical-data`, and use the mean values in our structural and forecasting models. I compare the one-year growth in these forecasts to machine forecasts of S&P 500 price growth. Our sample spans 1947:06 to 2021:06. The forecast series were downloaded on September 20, 2021.

The survey provides semi-annual forecasts on the level of the S&P 500 index. Participants are asked to provide forecasts for the level of the S&P 500 index for the end of the current survey month, 6 months ahead, and 12 months ahead. I use the mean of the respondents' forecasts each period, where the sample is based on about 50 observations. Most of the survey participants are professional forecasters with "formal and advanced training in economic theory and forecasting and use econometric models to generate their forecasts."

Participants receive questionnaires for the survey in May and November, after the Consumer Price Index (CPI) data release for the previous month. All forecasts are typically submitted by the end of the respective month of May and November. The results of the survey are released near the end of the following month, on June and December of each calendar year. The exact release dates are available on the Philadelphia Fed website, at the header of each news release. I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since all forecasts are typically submitted by the end of May and November of each calendar year, I set the response deadline for the machine forecast to be the first day of the last month of June and December, implying that I allow the machine to use information only up through the end of the May and November.

I follow Nagel and Xu (2021) in constructing one-year stock price growth expectations from the level forecasts. Starting from June 1992, I use the ratio between the 12-month level forecast ($\texttt{SPIF\_12M}_t$) and 0-month level nowcasts ($\texttt{SPIF\_ZM}_t$) of the S&P 500 index. Before June 1992, the 0-month nowcast is not available. Therefore I use the annualized ratio between the 12-month ($\texttt{spi12}_t$) and 6-month ($\texttt{spi6}_t$) level forecast of the S&P 500 index

$$\mathbb{F}_t^{(Liv)}\left[\frac{P_{t+1}}{P_t}\right] \approx \begin{cases} \frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_t]} = \frac{\texttt{SPIF\_12M}_t}{\texttt{SPIF\_ZM}_t} & \text{if } t \geq 1992M6 \\ \left(\frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_{t+6}]}\right)^2 = \left(\frac{\texttt{spi12}_t}{\texttt{spi6}_t}\right)^2 & \text{if } t < 1992M6 \end{cases} \tag{A.172}$$

where P_t is the S&P 500 index and t indexes the survey's response deadline. To obtain a survey expectation of the log change in price growth I use the approximation:

$$\mathbb{F}_t(\Delta p_{t+1}) \approx \log(\mathbb{F}_t[P_{t+1}]) - \log(P_t)$$

Conference Board (CB) Survey Respondents provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. Since the survey asks respondents about stock prices in the "year ahead," I interpret the question to be asking about the forecast period from the end of the current survey month to the end of the same month one year ahead. When we use this qualitative survey forecast to compare to machine forecasts, we impute a point forecast for stock market returns using the method described in Section C.4.2 below. I compare the imputed point forecast to machine forecasts of CRSP returns.

The survey is conducted monthly and I use the survey responses over 1987:04 to 2022:08. The data was downloaded on September 26, 2022. The survey uses an address-based mail sample design. Questionnaires are mailed to households on or about the first of each month. Survey responses flow in throughout the collection period, with the sample close-out for preliminary estimates occurring around the 18th of the month. Any responses received after then are used to produce final estimates for the month, which are published with the following month's data. Conversations with those knowledgeable about the survey suggested that most panelists respond early. Any responses received after around the 20th of the month–regardless of when they are filled out–are included in the final (but not preliminary) numbers.

I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since questionnaires reach households on or about the first of each month (e.g., February 1st) and most respondents respond early, I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., January 31st).

Converting Qualitative Forecasts to Point Forecasts (SOC and CB) I use the SOC probability to impute a quantitative point forecast of stock returns using a linear regression of CFO point forecasts for returns onto the SOC probablity of a price increase. The SOC asks respondents about the percent chance that an investment will "increase in value in the year ahead." I interpret this as asking about the ex dividend value, i.e., about price price growth. The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The SOC survey is conducted monthly, where survey months span 2002:06 to 2021:12. Since the CFO is a quarterly survey, the regression is estimated in real-time over a quarterly overlapping sample. Since the CFO survey is conducted during the last month of the quarter while the SOC is conducted monthly, I align the survey months between CFO and SOC by regressing the quarterly CFO survey point forecast with the qualitative SOC survey response during the last month of the quarter.

Since the SOC survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed SOC point forecast of stock returns including dividends.

Specifically, at time t, I assume that the CFO forecast of stock returns, $\mathbb{F}_t^{\text{CFO}}[r_{t,t+1}]$, minus the current dividend yield, D_t/P_t, is related to the contemporaneous SOC probability of an increase in the stock market next year, $P_{t,t+1}^{\text{SOC}}$, by:

$$\mathbb{F}_t^{\text{CFO}}[r_{t,t+1}] - D_t/P_t = \beta_0 + \beta_1 P_{t,t+1}^{\text{SOC}} + \epsilon_t.$$

The final imputed SOC point forecast is constructed as $\mathbb{F}_t^{\text{SOC}}[r_{t,t+1}] = \hat{\beta}_0 + \hat{\beta}_1 P_{t,t+1}^{\text{SOC}} + D_t/P_t$. I first estimate the coefficients of the above regression over an initial overlapping sample of 2002Q2 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the SOC survey responses from the last month of each calendar quarter. Using the estimated coefficients and the SOC probability from 2005:03 gives us the point forecast of the one-year stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one quarterly observation to the end of the sample at a time, and storing the fitted values. This results in a time series of SOC point forecasts $\mathbb{F}_t^{\text{SOC}}[r_{t,t+1}]$ spanning 2005Q1 to 2021Q1.

The same procedure is done for the Conference Board Survey, except I replace $P_{t,t+1}^{\text{SOC}}$ by $P_{t,t+1}^{\text{CB}}$, a ratio of the proportion of those who respond with "increase" to the sum of "decrease" and "same." The CB survey asks respondents to provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. I interpret this as asking about price price growth. Since the CB survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed CB point forecast of stock returns including dividends.

The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The CB survey is conducted monthly, where survey months span 1987:04 to 2022:08. The regression is first estimated over an initial overlapping sample of 2001Q4 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the CB survey responses from the last month of each calendar quarter. Using the estimated coefficients and the CB survey response $P_{t,t+1}^{\text{CB}}$ from 2005:03 gives us the point forecast of the stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one observation to the end of the sample at a time, and storing the fitted values. This results in a time series of CB point forecasts $\mathbb{F}_t^{\text{CB}}[r_{t,t+1}]$ over 2005Q1 to 2021Q1.

Nagel and Xu Individual Investor Expectations Nagel and Xu (2021)'s individual investor expectations series for returns covers 1972-1977 (Annual) and 1987Q2-2021Q4 (Quarterly) and combine data from the following surveys:

1. UBS/Gallup: 1998:06-2007:10; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

2. Michigan Survey of Consumers (SOC): 2002:04-2022:12; Respondents provide the probability of a rise in the stock market over a 1-year horizon.

3. Conference Board (CB): 1987:04-2022:08; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

4. Vanguard Research Initiative (VRI): 2014:08; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

5. Roper: 1974-1977, annual, observed June of each calendar year; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

6. Lease, Lewellen, and Schlarbaum (1974, 1977): 1972-1973, annual, observed July of each calendar year; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

NX arrive at their final series through the following imputation steps. UBS/Gallup measures the investors' expected stock market returns most closely, but it covers a relatively short period. SOC and CB cover a relatively longer period, but they are qualitative forecasts that need to be converted to point forecasts.

1. Start with UBS/Gallup for 1998:06-2007:10 and VRI for 2014:08 since they capture the respondents' expected stock returns relatively closely (other surveys only provide qualitative measures).

2. Regress SOC on UBS/Gallup and VRI using periods of overlapping coverage (2002:04-2007:10). Use the fitted values from this regression to impute missing data for 2007:11-2022:12 (excluding 2014:08).

3. Regress CB on UBS/Gallup and VRI using periods of overlapping coverage (1998:06-2007:10). Use the fitted values from this regression to impute missing data for 1987:04-1998:05 (using CB) and 1974-1977 (using Roper).

4. Use the coefficients from regressing CB on UBS/Gallup and VRI (from step 3) to compute fitted values that convert the probabilistic forecast from Roper into point forecasts of stock returns.

5. Convert expected returns to expected excess returns by subtracting the average 1-year Treasury yield measured at the beginning of the survey month.

6. Aggregate monthly series to a quarterly frequency by taking the average expectation within calendar quarters.

C.5 Risk-Free Rates

Realized Risk-Free Rates As a measure of realized risk-free rates r_t^f, I obtain daily series for the annualized three-month Treasury bill rate (DTB3), downloaded from FRED on May 15, 2024. To match the definition used as the target variable in the Survey of Professional Forecasters (SPF), I time-aggregate the daily realized risk-free rate series to a quarterly frequency by taking the quarterly average, as discussed below.

Survey Expectations of Risk-Free Rates I obtain subjective expectations about risk-free rates from median forecasts for the annualized three-month Treasury bill rate from the Survey of Professional Forecasters (SPF). The SPF provides forecasts at the one and ten year horizons.

For one year ahead forecasts (TBILL), respondents are asked to provide quarterly forecasts of the quarterly average three-month Treasury bill rate, in percentage points, where the forecasts are for the quarterly average of the underlying daily levels. I interpret the survey to be asking about annual net simple rates $\mathbb{F}_t[R^f_{t,t+1}]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r^f_{t,t+1}] \approx \log(1+\mathbb{F}_t[R^f_{t,t+1}])$.

For ten year ahead forecasts (BILL10), respondents are asked to provide forecasts for the annual-average rate of return to three-month Treasury bills over the next 10 years, in percentage points. The ten year ahead forecasts are available only for surveys conducted in the first quarter of each calendar year. I interpret the survey to be asking about annualized cumulative net simple rates compounded from the survey quarter to the same quarter that is ten years after the survey year $\mathbb{F}_t[R^f_{t,t+10}]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r^f_{t,t+10}] \approx \log(1 + \mathbb{F}_t[R^f_{t,t+10}])$.

To obtain long-horizon survey expectations of annualized log three-month Treasury bill rates over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year return expectations:

$$\mathbb{F}_t[r^f_{t,t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[r^f_{t,t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[r^f_{t,t+10}], \quad h = 1, 2, \ldots, 10 \tag{A.173}$$

Finally, I use the difference between the cumulative annualized long-horizon log three-month Treasury bill rate expectations between adjacent years (i.e., $\mathbb{F}_t[r^f_{t,t+h-1}]$ and $\mathbb{F}_t[r^f_{t,t+h}]$) to obtain $\mathbb{F}_t[r^f_{t+h}]$, the time t survey expectation of annualized forward log three-month Treasury bill rate h years ahead:

$$\mathbb{F}_t[r^f_{t+h}] = h \times \mathbb{F}_t[r^f_{t,t+h}] - (h-1) \times \mathbb{F}_t[r^f_{t,t+h-1}], \quad h = 1, 2, \ldots, 10 \tag{A.174}$$

The surveys are sent out at the end of the first month of each quarter, and they are collected in the second or third week of the middle month of each quarter. When constructing machine learning forecasts for the risk-free rate, I assume that forecasters could have used all data released before the survey deadlines for the SPF, which are posted online at the Federal Reserve Bank of Philadelphia website. Since surveys are typically sent out at the end of the first month of each quarter, I make the conservative assumption that respondents only had data released by the first day of the second month of each quarter.

C.6 Earnings

C.6.1 Realized Earnings

To measure corporate earnings, I use S&P 500 earnings divided by GDP as a noisy signal of the payout share K_t in the structural estimation. To map into a monthly estimation, we ideally would use monthly earnings data. Instead, we have quarterly S&P 500 IBES street earnings per share (EPS) data that starts in 1983:Q4 from Hillenbrand and McCarthy (2024). Street earnings differ from GAAP earnings by excluding discontinued operations, extraordinary charges, and other non-operating items. According to the IBES user guide, analysts submit forecasts after backing out these transitory components, and IBES constructs the realized series to align with those forecasts. While analysts have some discretion over which items to exclude, Hillenbrand and McCarthy (2024) demonstrate that the target of these forecasts corresponds closely to earnings before special items in Compustat, suggesting that street earnings accurately reflect the measure analysts are targeting.

To convert EPS to total earnings, I multiply the resulting quarterly EPS series by the quarterly S&P 500 divisor, available at: `https://ycharts.com/indicators/sp_500_divisor`. Finally, to obtain a monthly S&P 500 earnings series, we linearly interpolate the resulting quarterly total earnings series. The final monthly total earnings series spans the period 1983:12 to 2021:12. I obtained quarterly IBES street earnings data from the authors of Hillenbrand and McCarthy (2024) on June 3, 2025. The divisor data were downloaded on March 13, 2022.

C.6.2 Survey Expectations of Earnings

I obtain monthly survey data for the median analyst earnings per share forecast and actual earnings per share from the Institutional Brokers Estimate System (IBES) via Wharton Research Data Services (WRDS). The data spans the period 1976:01 to 2021:12 and was downloaded on October 2022.

Short-Term Growth (STG) Expectations I build measures of aggregate S&P 500 earnings expectations growth using the constituents of the S&P 500 at each point in time following De La O and Myers (2021). I first construct expected earnings expectations for aggregate earnings h-months-ahead as:

$$\mathbb{F}_t[E_{t+h}] = \Omega_t \left[\sum_{i \in x_{t+h}} \mathbb{F}_t \left[EPS_{i,t+h}\right] S_{i,t} \right] / Divisor_t, \tag{A.175}$$

where $\mathbb{F}$ is the median analyst survey forecast, E is aggregate S&P 500 earnings, EPS_i is earning per share of firm i among all S&P 500 firms x_{t+h} for which I have forecasts in IBES for $t+h$, S_i is shares outstanding of firm i, and $Divisor_t$ is calculated as the S&P 500 market capitalization divided by the S&P 500 index. I obtain the number of outstanding shares for all companies in the S&P500 from Compustat. All data from Compustat were downloaded on November 17th, 2022. IBES estimates are available for most but not all S&P 500 companies. Following De La O and Myers (2021), I multiply this aggregate by Ω_{t+h}, a ratio of total S&P 500 market value to the market value of the forecasted companies at $t+h$ to account for the fact that IBES does not provide earnings forecasts for all firms in the S&P 500 in every period.

IBES database contains earning forecasts up to five annual fiscal periods (FY1 to FY5) and as a result, I interpolate across the different horizons to obtain the expectation over the next 12 months. This procedure has been used in the literature, including De La O and Myers (2021). Specifically, if the fiscal year of firm XYZ ends nine months after the survey date, I have a 9-month earning forecast $\mathbb{F}_t[E_{t+9}]$ from FY1 and a 21-month forecast $\mathbb{F}_t[E_{t+21}]$ from FY2. I then obtain the 12-month ahead forecast by interpolating these two forecasts as follows,

$$\mathbb{F}_t[E_{t+12}] = \frac{9}{12}\mathbb{F}_t[E_{t+9}] + \frac{3}{12}\mathbb{F}_t[E_{t+21}]. \tag{A.176}$$

For the forecasting performance estimates, I use quarterly data. To convert the monthly forecast to quarterly frequency, I use the forecast made in the middle month of each quarter, and construct one-year earnings expectations from 1976Q1 to 2021Q4 and the earning expectation growth is calculated as an approximation following following De La O and Myers (2021):

$$\mathbb{F}_t\left(\Delta e_{t+12}\right) \approx \ln\left(\mathbb{F}_t[E_{t+12}]\right) - e_t \tag{A.177}$$

where e_t is log earnings for S&P 500 at time t calculated as $e_t = \log\left(EPS_t \cdot Divisor_t\right)$, where EPS_t is the earnings per share for the S&P 500 obtained from Shiller's data depository and S&P Global, as described above.

Long-Term Growth (LTG) Expectations I construct long term expected earnings growth (LTG) for the S&P 500 following Bordalo et al. (2019). Specifically, I obtain the median firm-level LTG forecast from IBES, and aggregate the value-weighted firm-level forecasts,

$$LTG_t = \sum_{i=1}^{S} LTG_{i,t} \frac{P_{i,t}Q_{i,t}}{\sum_{i=1}^{S} P_{i,t}Q_{i,t}} \tag{A.178}$$

where S is the number of firms in the S&P 500 index, and where $P_{i,t}$ and $Q_{i,t}$ are the stock price and the number of shares outstanding of firm i at time t, respectively. $LTG_{i,t}$ is the median forecast of firm i's long term expected earnings growth. The data spans the periods from 1981:12 to 2021:12. All data were downloaded in February 2023.

Finally, I use the difference between survey expectations of log earnings between adjacent years (i.e., $\mathbb{F}_t[e_{t+h-1}]$ and $\mathbb{F}_t[e_{t+h}]$) to obtain $\mathbb{F}_t[\Delta e_{t+h}] = \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}]$, the time t survey expectation of forward one-year log earnings growth $h = 1, 2, 3, 4$ years ahead. For the $h = 5$ year horizon, I interpret the IBES's Long-Term Growth (LTG) forecast as the 5-year forward annual log earnings growth from 4 to 5 years ahead:

$$\mathbb{F}_t[\Delta e_{t+h}] = \begin{cases} \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}] & \text{if } h = 1, 2, 3, 4 \text{ years} \\ LTG_t & \text{if } h = 5 \text{ years} \end{cases} \tag{A.179}$$

To estimate any biases in IBES analyst forecasts, the dynamic machine algorithm takes as an input a likely date corresponding to information analysts could have known at the time of their forecast. IBES does not provide an explicit deadline for their forecasts to be returned. Therefore I instead use the "statistical period" day (the day when the set of summary statistics was calculated) as a proxy for the deadline. I set the machine deadline to be the day before this date. The statistical period date is typically between day 14 and day 20 of a given month, implying that the machine deadline varies from month to month. As the machine learning algorithm uses mixed-frequency techniques adapted to quarterly sampling intervals, while the IBES forecasts are monthly, I compare machine and IBES analyst forecasts as of the middle month of each quarter, considering 12-month ahead forecast from the beginning of the month following the survey month.

C.7 Price-Earnings Ratio

I construct a quarterly series for the price-earnings ratio $PE_t \equiv P_t/E_t$ as the ratio between the end-of-quarter S&P 500 stock price index P_t and the S&P 500 quarterly total earnings E_t. I infer subjective expectations of the log price-earnings ratio $\mathbb{F}_t[pe_{t+h}]$ by combining the current log price-earnings ratio pe_t with h year ahead subjective expectations of annual log stock returns $\mathbb{F}_t[r_{t+h}]$ and annual log earnings growth $\mathbb{F}_t[\Delta e_{t+h}]$, following the approach used in De La O and Myers (2021). Rearrange the Campbell and Shiller (1988) present value identity for the price-earnings ratio in equation (A.78) to express the future log price-earnings ratio as a function of current log price-earnings, log earnings growth, and log stock returns:

$$pe_{t+h} = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{t+j} - r_{t+j}) \tag{A.180}$$

where the equation holds both ex-ante and ex-post. Apply subjective expectations $\mathbb{F}_t$ on both sides of the equation:

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \underbrace{\mathbb{F}_t[\Delta e_{t+j}]}_{\text{Survey (IBES)}} - \underbrace{\mathbb{F}_t[r_{t+j}]}_{\text{Survey (CFO)}}) \tag{A.181}$$

where subjective expectations about j years ahead forward annual log stock returns $\mathbb{F}_t[r_{t+j}]$ and forward annual log earnings growth $\mathbb{F}_t[\Delta e_{t+j}]$ use survey forecasts from the CFO survey and IBES, respectively.

C.8 Earnings-Employment Ratio

The current earnings-employment ratio is defined as $EL_t \equiv E_t/L_{t+1}$, where E_t denotes quarterly total earnings for the S&P 500 and L_{t+1} is the employment stock at the beginning of period $t+1$. I measure L_{t+1} using end-of-period employment levels within each quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section B while still remaining known to firms by the end of period t. Let $el_t = \log(EL_t)$ denote the log earnings-employment ratio. Subjective expectations of future values $\mathbb{F}_t[el_{t+1}] \equiv \mathbb{F}_t[\log E_{t+1} - \log L_{t+1}]$ are constructed as:

$$\mathbb{F}_t[el_{t+1}] \equiv \mathbb{F}_t[\log E_{t+1} - \log L_{t+1}] \tag{A.182}$$

$$= \mathbb{F}_t[\log E_{t+1} - \log E_t + \log E_t - \log L_{t+1}] \tag{A.183}$$

$$= el_t + \underbrace{\mathbb{F}_t[\Delta e_{t,t+1}]}_{\text{Survey (IBES)}} \tag{A.184}$$

where survey expectations of earnings growth $\mathbb{F}_t[\Delta e_{t,t+h}]$ come from IBES.

C.9 Machine Learning Forecasts

For each survey forecast, I also construct their corresponding machine learning forecast by estimating a Long Short-Term Memory (LSTM) neural network:

$$\mathbb{E}_t[y_{t,t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t}) \tag{A.185}$$

where $y_{t,t+h}$ denotes the variable y to be predicted h years ahead of time t, and $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept. $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$ denotes predicted values from a LSTM neural network that can be represented by a (potentially) high dimensional set of finite-valued parameters $\boldsymbol{\beta}_{h,t}$. The machine learning model is estimated using an algorithm that takes into account the data-rich environment in which firms operate in (Bianchi et al., 2022 and Bianchi et al., 2024b). When constructing machine learning forecasts of each variable, I allow the machine to use only information that would have been available to all survey respondents at the time of each forecast. See Section D for details about the machine learning algorithm and predictor variables. Machine expectations about the price-earnings ratio $\mathbb{E}_t[pe_{t+1}]$ and earnings-employment ratio $\mathbb{E}_t[el_{t+1}]$ are constructed similarly to the survey counterpart, by replacing the survey forecasts of stock returns, earnings growth, and employment growth on the right-hand side of equation (A.181) and (A.184) with the corresponding machine learning forecasts.

D Machine Learning

D.1 Data Inputs for Machine Learning Algorithm

D.1.1 Macro Data Surprises

These data are used as inputs into the machine learning forecasts. I obtain median forecasts for GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change) from the Money Market Service Survey. The median market survey forecasts are compiled and published by the Money Market Services (MMS) the Friday before each release. I apply the approach used in Bauer and Swanson (2023) and define macroeconomic data surprise as the actual value of the data release minus the median expectation from MMS on the Friday immediately prior to that data release. The GDP growth forecasts are available quarterly from 1990Q1 to 2022Q1. The core CPI forecast is available monthly from July 1989 to April 2022. The median forecasts for the unemployment rate and nonfarm payrolls are available monthly from Jan 1980 to May 2022, and Jan. 1985 to May 2022, respectively. All survey forecasts were downloaded from Haver Analytics on December 17, 2022. To pin down the timing of when the news was actually released I follow the published tables of releases from the Bureau of Labor Statistics (BLS), discussed below.

The macro news events are indexed by their date and time of the data release, while the machine learning algorithm is adapted to quarterly sampling frequencies. When including the macro data surprises as additional predictors for the machine forecast, I time-aggregate the macro data surprises to a quarterly frequency by taking the sum of the surprises across data releases that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from data releases up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

D.1.2 FOMC Surprises

FOMC surprises are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contract rate, from 10 minutes before to 20 minutes after each U.S. Federal Reserve Federal Open Market Committee (FOMC) announcement. The data on FFF and ED were downloaded on June 3rd 2022. When benchmarking against a survey, I use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, I compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, I use the last FOMC meeting before the end of the first month in each quarter to compute surprises.

When including the FOMC surprises as additional predictors for the machine forecast, I time-aggregate the FOMC surprises to a quarterly frequency by taking the sum of the surprises across FOMC announcements that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from FOMC announcements up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

D.1.3 S&P 500 Jumps

As a measure of the market's reaction to news shocks, I use the jump in the S&P 500 pre- and post- a 30-minute window around major news events. The events in our analysis include (i) 1,482 macroeconomic data releases for U.S. GDP, Consumer Price Index (CPI), unemployment, and payroll data spanning 1980:01-2021:12, (ii) 16 corporate earnings announcement days spanning 1999:03-2020:05, and (iii) 219 Federal Open Market Committee (FOMC) press releases from the Fed spanning 1994:02-2021:12. The corporate earnings news events are from Baker et al. (2019) who conduct textual analyses of *Wall Street Journal* articles to identify days in which there were large jumps in the aggregate stock market that could be attributed to corporate earnings news with high confidence.

The jump in the S&P 500 for a given event is defined as

$$j_\tau = p_{\tau+\delta_{post}} - p_{\tau-\delta_{pre}} \tag{A.186}$$

where τ indexes the time of an event and $p_\tau = \log(P_\tau)$ is the log S&P 500 index. δ_{pre} and δ_{post} denote the pre and post event windows, which is 10 minutes before and 20 minutes after the event, respectively.

I obtain data on P_τ using tick-by-tick data on the S&P 500 index from tickdata.com. The series was purchased and downloaded on 7/2/2022 from `https://www.tickdata.com/`. I create the minutely data using the close price within each minute. I supplement the S&P 500 index using S&P500 E-mini futures for events that occur in off-market hours. I use the current-quarter contract futures. I purchased the S&P 500 E-mini futures from CME group on 7/2/2022 at `https://datamine.cmegroup.com/`. Our sample spans 1/2/1986 to 6/30/2022.

For each event, I separate out the events for which the S&P 500 increased over the window ($j_\tau^{(+)} \geq 0$) and those for which the market decreased ($j_\tau^{(-)} \leq 0$):

$$j_\tau^{(+)} = \max\{0, j_\tau\} \tag{A.187}$$

$$j_\tau^{(-)} = \min\{0, j_\tau\} \tag{A.188}$$

I aggregate the event-level jumps to monthly time series by summing over all the relevant events within the month, where the events are partitioned into two groups based on the sign of the jump:

$$J_t^{(+)} = \sum_{\tau \in x(t)} j_\tau^{(+)} \tag{A.189}$$

$$J_t^{(-)} = \sum_{\tau \in x(t)} j_\tau^{(-)} \tag{A.190}$$

where t indexes the month and $x(t)$ is the set of all events that occurred within month t. The procedure results in two monthly variables, $J_t^{(+)}$ and $J_t^{(-)}$, which capture total market reaction to news events in either direction during the quarter. The series spans the period 1994:02 to 2022:03. Separating out the events based on the sign of the jump allows us to capture any differential effects on return predictability based on whether the market perceived the news as good or bad. The partition also allows us to accurately capture the total extent of over- or under-reaction. Otherwise, mixing all the events would only capture the net effect of the jumps and bias the market reaction towards zero.

When used as additional predictors in the for the machine forecast, the jumps need to be converted to quarterly time series because the machine learning algorithm is adapted to a quarterly sampling frequency. The set of events in $x(t)$ is chosen so that the machine only sees the news events that would have been available to the real-time firm. When combining the events within a quarter, I impose the response deadline used to produce the machine forecast. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I use the jumps from the events up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

D.1.4 Real-Time Macro Data

This section gives details on the real time macro data inputs used in the machine learning forecasts. A subset of these series are used in the structural estimation. At each forecast date in the sample, I construct a dataset of macro variables that could have been observed on or before the day of the survey deadline. I use the Philadelphia Fed's Real-Time Data Set to obtain vintages of macro variables.[9]

[9] The real-time data sets are available at https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files.

These vintages capture changes to historical data due to periodic revisions made by government statistical agencies. The vintages for a particular series can be available at the monthly and/or quarterly frequencies, and the series have monthly and/or quarterly observations. In cases where a variable has both frequencies available for its vintages and/or its observations, I choose one format of the variable. For instance, nominal personal consumption expenditures on goods is quarterly data with both monthly and quarterly vintages available; in this case, I use the version with monthly vintages.

Table A.16 gives the complete list of real-time macro variables. Included in the table is the first available vintages for each variable that has multiple vintages. I do not include the last vintage because most variables have vintages through the present.[10] Table A.16 also lists the transformation applied to each variable to make them stationary before generating factors. Let $X_{i,t}$ denote variable i at time t after the transformation, and let $X^A_{i,t}$ be the untransformed series. Let $\Delta = (1 - L)$ with $LX_{i,t} = X_{it-1}$. There are seven possible transformations with the following codes:

1 Code lv: $X_{i,t} = X^A_{i,t}$

2 Code Δlv: $X_{i,t} = X^A_{i,t} - X^A_{it-1}$

3 Code $\Delta^2 lv$: $X_{i,t} = \Delta^2 X^A_{i,t}$

4 Code ln: $X_{i,t} = \ln(X^A_{i,t})$

5 Code Δln: $X_{i,t} = \ln(X^A_{i,t}) - \ln(X^A_{it-1})$

6 Code $\Delta^2 ln$: $X_{i,t} = \Delta^2 \ln(X^A_{i,t})$

7 Code $\Delta lv/lv$: $X_{i,t} = (X^A_{i,t} - X^A_{it-1})/X^A_{it-1}$

Table A.16: List of Macro Dataset Variables

No.	Short Name	Source	Tran	Description	First Vintage
		Group 1: Output and Income			
1	IPMMVMD	Philly Fed	Δln	Ind. production index - Manufacturing	1962M11
2	IPTMVMD	Philly Fed	Δln	Ind. production index - Total	1962M11
3	CUMMVMD	Philly Fed	lv	Capacity utilization - Manufacturing	1979M8
4	CUTMVMD	Philly Fed	lv	Capacity utilization - Total	1983M7
5	NCPROFATMVQD	Philly Fed	Δln	Nom. corp. profits after tax without IVA/CCAdj	1965Q4
6	NCPROFATWMVQD	Philly Fed	Δln	Nom. corp. profits after tax with IVA/CCAdj	1981Q1
7	OPHMVQD	Philly Fed	Δln	Output per hour - Business sector	1998Q4
8	NDPIQVQD	Philly Fed	Δln	Nom. disposable personal income	1965Q4
9	NOUTPUTQVQD	Philly Fed	Δln	Nom. GNP/GDP	1965Q4
10	NPIQVQD	Philly Fed	Δln	Nom. personal income	1965Q4
11	NPSAVQVQD	Philly Fed	Δlv	Nom. personal saving	1965Q4
12	OLIQVQD	Philly Fed	Δln	Other labor income	1965Q4
13	PINTIQVQD	Philly Fed	Δln	Personal interest income	1965Q4
14	PINTPAIDQVQD	Philly Fed	Δln	Interest paid by consumers	1965Q4
15	PROPIQVQD	Philly Fed	Δln	Proprietors' income	1965Q4
16	PTAXQVQD	Philly Fed	Δln	Personal tax and nontax payments	1965Q4
17	RATESAVQVQD	Philly Fed	Δlv	Personal saving rate	1965Q4
18	RENTIQVQD	Philly Fed	Δlv	Rental income of persons	1965Q4
19	ROUTPUTQVQD	Philly Fed	Δln	Real GNP/GDP	1965Q4
20	SSCONTRIBQVQD	Philly Fed	Δln	Personal contributions for social insurance	1965Q4
21	TRANPFQVQD	Philly Fed	Δln	Personal transfer payments to foreigners	1965Q4
22	TRANRQVQD	Philly Fed	Δln	Transfer payments	1965Q4

[10] For variables BASEBASAQVMD, NBRBASAQVMD, NBRECBASAQVMD, and TRBASAQVMD, the last available vintage is 2013Q2.

No.	Short Name	Source	Tran	Description	First Vintage
23	CUUR0000SA0E	BLS	$\Delta^2 ln$	Energy in U.S. city avg., all urban consumers, not seasonally adj	
			Group 2: Employment		
24	EMPLOYMVMD	Philly Fed	Δln	Nonfarm payroll	1946M12
25	HMVMD	Philly Fed	lv	Aggregate weekly hours - Total	1971M9
26	HGMVMD	Philly Fed	lv	Agg. weekly hours - Goods-producing	1971M9
27	HSMVMD	Philly Fed	lv	Agg. weekly hours - Service-producing	1971M9
28	LFCMVMD	Philly Fed	Δln	Civilian labor force	1998M11
29	LFPARTMVMD	Philly Fed	lv	Civilian participation rate	1998M11
30	POPMVMD	Philly Fed	Δln	Civilian noninstitutional population	1998M11
31	ULCMVQD	Philly Fed	Δln	Unit labor costs - Business sector	1998Q4
32	RUCQVMD	Philly Fed	Δlv	Unemployment rate	1965Q4
33	WSDQVQD	Philly Fed	Δln	Wage and salary disbursements	1965Q4
			Group 3: Orders, Investment, Housing		
34	HSTARTSMVMD	Philly Fed	Δln	Housing starts	1968M2
35	RINVBFMVQD	Philly Fed	Δln	Real gross private domestic inv. - Nonresidential	1965Q4
36	RINVCHIMVQD	Philly Fed	Δlv	Real gross private domestic inv. - Change in private inventories	1965Q4
37	RINVRESIDMVQD	Philly Fed	Δln	Real gross private domestic inv. - Residential	1965Q4
38	CASESHILLER	S&P	Δln	Case-Shiller US National Home Price index/CPI	1987M1
			Group 4: Consumption		
39	NCONGMMVMD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009M8
40	NCONHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp.	2009M8
41	NCONSHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009M8
42	NCONSNPMMVMD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009M8
43	RCONDMMVMD	Philly Fed	Δln	Real personal cons. exp. - Durables	1998M11
44	RCONGMMVMD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009M8
45	RCONHHMMVMD	Philly Fed	Δln	Real hh. cons. exp.	2009M8
46	RCONMMVMD	Philly Fed	Δln	Real personal cons. exp. - Total	1998M11
47	RCONNDMVMD	Philly Fed	Δln	Real personal cons. exp. - Nondurables	1998M11
48	RCONSHHMMVMD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009M8
49	RCONSMMVMD	Philly Fed	Δln	Real personal cons. exp. - Services	1998M11
50	RCONSNPMMVMD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009M8
51	NCONGMVQD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009Q3
52	NCONHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp.	0209Q3
53	NCONSHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009Q3
54	NCONSNPMVQD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009Q3
55	RCONDMVQD	Philly Fed	Δln	Real personal cons. exp. - Durable goods	1965Q4
56	RCONGMVQD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009Q3
57	RCONHHMVQD	Philly Fed	Δln	Real hh. cons. exp.	2009Q3
58	RCONMVQD	Philly Fed	Δln	Real personal cons. exp. - Total	1965Q4
59	RCONNDMVQD	Philly Fed	Δln	Real pesonal cons. exp. - Nondurable goods	1965Q4
60	RCONSHHMVQD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009Q3
61	RCONSMVQD	Philly Fed	Δln	Real personal cons. exp. - Services	1965Q4
62	RCONSNPMVQD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009Q3
63	NCONQVQD	Philly Fed	Δln	Nom. personal cons. exp.	1965Q4
			Group 5: Prices		
64	PCONGMMVMD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp. - Goods	2009M8
65	PCONHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009M8
66	PCONSHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009M8
67	PCONSNPMMVMD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009M8
68	PCPIMVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1998M11
69	PCPIXMVMD	Philly Fed	$\Delta^2 ln$	Core consumer price index	1998M11
70	PPPIMVMD	Philly Fed	$\Delta^2 ln$	Producer price index	1998M11
71	PPPIXMVMD	Philly Fed	$\Delta^2 ln$	Core producer price index	1998M11
72	PCONGMVQD	Philly Fed	$\Delta^2 ln$	Price index for personal. cons. exp. - Goods	2009Q3
73	PCONHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009Q3
74	PCONSHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009Q3

No.	Short Name	Source	Tran	Description	First Vintage
75	PCONSNPMVQD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009Q3
76	PCONXMVQD	Philly Fed	$\Delta^2 ln$	Core price index for personal cons. exp.	1996Q1
77	CPIQVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1994Q3
78	PQVQD	Philly Fed	$\Delta^2 ln$	Price index for GNP/GDP	1965Q4
79	PCONQVQD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp.	1965Q4
80	PIMPQVQD	Philly Fed	$\Delta^2 ln$	Price index for imports of goods and services	1965Q4
			Group 6: Trade and Government		
81	REXMVQD	Philly Fed	Δln	Real exports of goods and services	1965Q4
82	RGMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Total	1965Q4
83	RGFMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Federal	1965Q4
84	RGSLMVQD	Philly Fed	Δln	Real government cons. and gross. inv. - State and local	1965Q4
85	RIMPMVQD	Philly Fed	Δln	Real imports of goods and services	1965Q4
86	RNXMVQD	Philly Fed	Δlv	Real net exports of goods and services	1965Q4
			Group 7: Money and Credit		
87	BASEBASAQVMD	Philly Fed	$\Delta^2 ln$	Monetary base	1980Q2
88	M1QVMD	Philly Fed	$\Delta^2 ln$	M1 money stock	1965Q4
89	M2QVMD	Philly Fed	$\Delta^2 ln$	M2 money stock	1971Q2
90	NBRBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves	1967Q3
91	NBRECBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves plus extended credit	1984Q2
92	TRBASAQVMD	Philly Fed	$\Delta^2 ln$	Total reserves	1967Q3
93	DIVQVQD	Philly Fed	Δln	Dividends	1965Q4

D.1.5 Daily Financial Data

Daily Data and construction of daily factors These data are used in the machine learning forecasts. The daily financial series in this data set are from the daily financial dataset used in Andreou et al. (2013). I create a smaller daily database which is a subset of the large cross-section of 991 daily series in their dataset. Our dataset covers five classes of financial assets: (i) the Commodities class; (ii) the Corporate Risk category; (iii) the Equities class; (iv) the Foreign Exchange Rates class and (v) the Government Securities.

The dataset includes up to 87 daily predictors in a daily frequency from 23-Oct-1959 to 24-Oct-2021 (14852 trading days) from the above five categories of financial assets. I remove series with fewer than ten years of data and time periods with no variables observed, which occurs for some series in the early part of the sample. For those years, I have less than 87 series. There are 39 commodity variables which include commodity indices, prices and futures, 16 corporate risk series, 9 equity series which include major US stock market indices and the 500 Implied Volatility, 16 government securities which include the federal funds rate, government treasury bills of securities from three months to ten years, and 7 foreign exchange variables which include the individual foreign exchange rates of major five US trading partners and two effective exchange rate. I choose these daily predictors because they are proposed in the literature as good predictors of economic growth.

I construct daily financial factors in a quarterly frequency in two steps. First, I use these daily financial time series to form factors at a daily frequency. The raw data used to form factors are always transformed to achieve stationarity and standardized before performing factor estimation (see generic description below). I re-estimate factors at each date in the sample recursively over time using the entire history of data available in real time prior to each out-of-sample forecast.

In the second step, I convert these daily financial indicators to quarterly weighted variables to form quarterly factors by selecting an optimal weighting scheme according to the method described below (see the weighting scheme section).

The data series used in this dataset are listed below in Table A.17 by data source. The tables also list the transformation applied to each variable to make them stationary before generating factors. The transformations used to stationarize a time series are the same as those explained in the section "Monthly financial factor data".

Table A.17: List of Daily Financial Dataset Variables

No.	Short Name	Source	Tran	Description
		Group 1: Commodities		
1	GSIZSPT	Data Stream	Δln	S&P GSCI Zinc Spot - PRICE INDEX
2	GSSBSPT	Data Stream	Δln	S&P GSCI Sugar Spot - PRICE INDEX
3	GSSOSPT	Data Stream	Δln	S&P GSCI Soybeans Spot - PRICE INDEX
4	GSSISPT	Data Stream	Δln	S&P GSCI Silver Spot - PRICE INDEX
5	GSIKSPT	Data Stream	Δln	S&P GSCI Nickel Spot - PRICE INDEX
6	GSLCSPT	Data Stream	Δln	S&P GSCI Live Cattle Spot - PRICE INDEX
7	GSLHSPT	Data Stream	Δln	S&P GSCI Lean Hogs Index Spot - PRICE INDEX
8	GSILSPT	Data Stream	Δln	S&P GSCI Lead Spot - PRICE INDEX
9	GSGCSPT	Data Stream	Δln	S&P GSCI Gold Spot - PRICE INDEX
10	GSCTSPT	Data Stream	Δln	S&P GSCI Cotton Spot - PRICE INDEX
11	GSKCSPT	Data Stream	Δln	S&P GSCI Coffee Spot - PRICE INDEX
12	GSCCSPT	Data Stream	Δln	S&P GSCI Cocoa Index Spot - PRICE INDEX
13	GSIASPT	Data Stream	Δln	S&P GSCI Aluminum Spot - PRICE INDEX
14	SGWTSPT	Data Stream	Δln	S&P GSCI All Wheat Spot - PRICE INDEX
15	EIAEBRT	Data Stream	Δln	Europe Brent Spot FOB U$/BBL Daily
16	CRUDOIL	Data Stream	Δln	Crude Oil-WTI Spot Cushing U$/BBL - MID PRICE
17	LTICASH	Data Stream	Δln	LME-Tin 99.85% Cash U$/MT
18	CWFCS00	Data Stream	Δln	CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE
19	CCFCS00	Data Stream	Δln	CBT-CORN COMP. CONTINUOUS - SETT. PRICE
20	CSYCS00	Data Stream	Δln	CBT-SOYBEANS COMP. CONT. - SETT. PRICE
21	NCTCS20	Data Stream	Δln	CSCE-COTTON #2 CONT.2ND FUT - SETT. PRICE
22	NSBCS00	Data Stream	Δln	CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE
23	NKCCS00	Data Stream	Δln	CSCE-COFFEE C CONTINUOUS - SETT. PRICE
24	NCCCS00	Data Stream	Δln	CSCE-COCOA CONTINUOUS - SETT. PRICE
25	CZLCS00	Data Stream	Δln	ECBOT-SOYBEAN OIL CONTINUOUS - SETT. PRICE
26	COFC01	Data Stream	Δln	CBT-OATS COMP. TRc1 - SETT. PRICE
27	CLDCS00	Data Stream	Δln	CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE
28	CLGC01	Data Stream	Δln	CME-LEAN HOGS COMP. TRc1 - SETT. PRICE
29	NGCCS00	Data Stream	Δln	CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE
30	LAH3MTH	Data Stream	Δln	LME-Aluminium 99.7% 3 Months U$/MT
31	LED3MTH	Data Stream	Δln	LME-Lead 3 Months U$/MT
32	LNI3MTH	Data Stream	Δln	LME-Nickel 3 Months U$/MT
33	LTI3MTH	Data Stream	Δln	LME-Tin 99.85% 3 Months U$/MT
34	PLNYD	www.macrotrends.net	Δln	Platinum Cash Price (U$ per troy ounce)
35	XPDD	www.macrotrends.net	Δln	Palladium (U$ per troy ounce)
36	CUS2D	www.macrotrends.net	Δln	Corn Spot Price (U$/Bushel)
37	SoybOil	www.macrotrends.net	Δln	Soybean Oil Price (U$/Pound)
38	OATSD	www.macrotrends.net	Δln	Oat Spot Price (US$/Bushel)
39	WTIOilFut	US EIA	Δln	Light Sweet Crude Oil Futures Price: 1St Expiring Contract Settlement ($/Bbl)
		Group 2: Equities		
40	S&PCOMP	Data Stream	Δln	S&P 500 COMPOSITE - PRICE INDEX
41	ISPCS00	Data Stream	Δln	CME-S&P 500 INDEX CONTINUOUS - SETT. PRICE
42	SP5EIND	Data Stream	Δln	S&P500 ES INDUSTRIALS - PRICE INDEX
43	DJINDUS	Data Stream	Δln	DOW JONES INDUSTRIALS - PRICE INDEX
44	CYMCS00	Data Stream	Δln	CBT-MINI DOW JONES CONTINUOUS - SETT. PRICE

No.	Short Name	Source	Tran	Description
45	NASCOMP	Data Stream	Δln	NASDAQ COMPOSITE - PRICE INDEX
46	NASA100	Data Stream	Δln	NASDAQ 100 - PRICE INDEX
47	CBOEVIX	Data Stream	lv	CBOE SPX VOLATILITY VIX (NEW) - PRICE INDEX
48	S&P500toVIX	Data Stream	Δln	S&P500/VIX
		Group 3: Corporate Risk		
49	LIBOR	FRED	Δlv	Overnight London Interbank Offered Rate (%)
50	1MLIBOR	FRED	Δlv	1-Month London Interbank Offered Rate (%)
51	3MLIBOR	FRED	Δlv	3-Month London Interbank Offered Rate (%)
52	6MLIBOR	FRED	Δlv	6-Month London Interbank Offered Rate (%)
53	1YLIBOR	FRED	Δlv	One-Year London Interbank Offered Rate (%)
54	1MEuro-FF	FRED	lv	1-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
55	3MEuro-FF	FRED	lv	3-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
56	6MEuro-FF	FRED	lv	6-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
57	APFNF-AANF	Data Stream	lv	1-Month A2/P2/F2 Nonfinancial Commercial Paper (NCP) (% P. A.) minus 1-Month Aa NCP (% P.A.)
58	APFNF-AAF	Data Stream	lv	1-Month A2/P2/F2 NCP (% P.A.) minus 1-Month Aa Financial Commercial Paper (% P.A.)
59	TED	Data Stream, FRED	lv	3Month Tbill minus 3-Month London Interbank Offered Rate (%)
60	MAaa-10YTB	Data Stream	lv	Moody Seasoned Aaa Corporate Bond Yield (% P.A.) minus Y10-Tbond
61	MBaa-10YTB	Data Stream	lv	Moody Seasoned Baa Corporate Bond Yield (% P.A.) minus Y10-Tbond
62	MLA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: A Rated: Effective Yield (%) minus Y10-Tbond
63	MLAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aa Rated: Effective Yield (%) minus Y10-Tbond
64	MLAAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aaa Rated: Effective Yield (%) minus Y10-Tbond
		Group 4: Treasuries		
65	FRFEDFD	Data Stream	Δlv	US FED FUNDS EFF RATE (D) - MIDDLE RATE
66	FRTBS3M	Data Stream	Δlv	US T-BILL SEC MARKET 3 MONTH (D) - MIDDLE RATE
67	FRTBS6M	Data Stream	Δlv	US T-BILL SEC MARKET 6 MONTH (D) - MIDDLE RATE
68	FRTCM1Y	Data Stream	Δlv	US TREASURY CONST MAT 1 YEAR (D) - MIDDLE RATE
69	FRTCM10	Data Stream	Δlv	US TREASURY CONST MAT 10 YEAR (D) - MIDDLE RATE
70	6MTB-FF	Data Stream	lv	6-month treasury bill market bid yield at constant maturity (%) minus Fed Funds
71	1YTB-FF	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus Fed Funds
72	10YTB-FF	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus Fed Funds
73	6MTB-3MTB	Data Stream	lv	6-month treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
74	1YTB-3MTB	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
75	10YTB-3MTB	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus 3M-Tbills
76	BKEVEN05	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 5-year (%)
77	BKEVEN10	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 10-year (%)
78	BKEVEN1F4	FRB	lv	BKEVEN1F4
79	BKEVEN1F9	FRB	lv	BKEVEN1F9

No.	Short Name	Source	Tran	Description
80	BKEVEN5F5	FRB	lv	US Inflation compensation: coupon equivalent forward rate: 5-10 years (%)
		Group 5: Foreign Exchange (FX)		
81	US_CWBN	Data Stream	Δln	US NOMINAL DOLLAR BROAD INDEX - EXCHANGE INDEX
82	US_CWMN	Data Stream	Δln	US NOMINAL DOLLAR MAJOR CURR INDEX - EXCHANGE INDEX
83	US_CSFR2	Data Stream	Δln	CANADIAN $ TO US $ NOON NY - EXCHANGE RATE
84	EU_USFR2	Data Stream	Δln	EURO TO US$ NOON NY - EXCHANGE RATE
85	US_YFR2	Data Stream	Δln	JAPANESE YEN TO US $ NOON NY - EXCHANGE RATE
86	US_SFFR2	Data Stream	Δln	SWISS FRANC TO US $ NOON NY - EXCHANGE RATE
87	US_UKFR2	Data Stream	Δln	UK POUND TO US $ NOON NY - EXCHANGE RATE

D.1.6 LDA Data

The LDA data are used as inputs into the machine learning forecasts. The database for our Latent Dirichlet Allocation (LDA) analysis contains around one million articles published in *Wall Street Journal* between January 1984 to June 2022. The current vintage of the results reported here is based a randomly selected sub-sample of 200,000 articles over the same period, one-fifth size of the entire database. The sample selection procedures follows Bybee et al. (2021). First, I remove all articles prior to January 1984 and after June 2022 and exclude articles published in weekends. Second, I exclude articles with subject tags associated with obviously non-economic content such as sports. Third, I exclude articles with the certain headline patterns, such as those associated with data tables or those corresponding to regular sports, leisure, or books columns. I filter the articles using the same list of exclusions provided by Bybee et al. (2021). Last, I exclude articles with less than 100 words.

Processing of texts The processing of the texts can be summarized into five steps:

1. Tokenization: parse each article's text into a white-space-separated word list retaining the article's word ordering.

2. I drop all non-alphabetical characters and set the remaining characters to lower-case, remove words with less than 3 letters, and remove common stop words and URL-based terms. I use a standard list of stop words from the Python library *gensim.parsing.preprocessing*.

3. Lemmatization and Stemming: lemmatization returns the original form of a word using external dictionary *Textblob.Word* in Python and based on the context of the word. For instance, as a verb, "went" is converted to"go". Stemming usually refers to a heuristic process that remove the trailing letters at the end of the words, such as from "assesses" to "assess', and "really" to "real". I use the Python library *Textblob.Word* to implement the lemmatization and *SnowballStemmer* for the stemming. The results are not very sensitive to the particular Python packages being used.

4. From the first three steps, I obtain a list of uni-grams which are a list of singular words. For example, "united" and "states" are uni-grams from "united states". From the list of uni-grams, I generate a set of bi-grams as all pairs of (ordered) adjacent uni-grams. For example, "united states" together is one bi-gram. I then exclude uni-grams and bi-grams appearing in less than 0.1% of articles.

5. Last, I convert an article's word list into a vector of counts for each uni-gram and bi-gram. For example, the vector of counts $[5, 7, 2]$ corresponds to the number of times the words $[''federal'', ''reserve'', ''bank'']$ appear in the article.

The LDA Model The LDA model Blei et al. (2003) essentially achieves substantial dimension reduction of the word distribution of each article using the following assumptions. I assume a factor structure on the vectors of word counts. Each factor is a topic and each article is a parametric distribution of topics, specified as follows,

$$\underbrace{\overbrace{w_i}^{V\times 1}}_{\text{word dist of article } i} \sim \text{Mult}\left(\underbrace{\overbrace{\Phi'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\theta_i}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_i}_{\text{\# of words}} \right) \tag{A.191}$$

where Mult is the multinomial distribution. In the above equation, w_i is a vector of word counts of each unique term (uni-gram or bi-gram) in article i, whose size is equal to the number of unique terms V. K is the number of factors in article i. In the estimation, I assume $K = 180$ following Bybee et al. (2021). Φ is a matrix sized $K \times V$, whose kth row and vth column is equal to the probability of the unique term v showing up in topic k. θ_i stores the weights of all k topics contained in article i, which sum up to one. Dimension reduction is achieved as long as $K << V$ (the number of topics are significantly smaller than the number of unique terms). More specifically, it reduces the dimension from $T \times V$ to $T \times K$ (the size of θ) + $K \times V$ (the size of Φ).

Real-time news factors. I also generate real-time news factors for each month t starting from January 1991. In theory, I could train the LDA model using each real-time monthly vintage but it is computationally challenging. Instead, I simplify the procedure by training the LDA model using quarterly vintages t, $t + 3$, $t + 6$, etc, and use the LDA model parameters estimated at t to filter news paper articles within the quarter and generate news factors for those months. More specifically, given every article's word distribution $w_{i,t+s}$,for $s = 0, 1, 2$, and the estimated real-time topic-word distribution parameters $\hat{\Phi}_t$ using articles till date t, one can obtain the filtered topic distribution of each article $\hat{\theta}_{i,t+s}$, as follows,

$$\underbrace{\overbrace{w_{i,t+s}}^{V\times 1}}_{\text{word dist of article } i \text{ at time } t+s} \sim \text{Mult}\left(\underbrace{\overbrace{\hat{\Phi}'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\hat{\theta}_{i,t+s}}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_{i,t+s}}_{\text{\# of words}} \right). \tag{A.192}$$

LDA Estimation I use the built-in LDA model estimation toolbox in the Python library `https://pypi.org/project/gensim/`Gensim to implement the model estimation. The model requires following initial inputs and parameters and it is estimated using Bayesian methods.[11]

1. I create a document-term matrix $\mathbf{W}$ as a collection of w_i for all articles i in the sample. The number of rows in $\mathbf{W}$ is equal to the number of articles in our sample and the number of columns in $\mathbf{W}$ is equal to the number of unique uni-gram and bi-grams (after being filtered) across all articles. The matrix $\mathbf{W}$ is used as an input for the LDA model estimation. I then follow Bybee et al. (2021) and set the number of topics K to be 180.[12]

[11]In theory, maximum-likelihood estimation is possible but it is computationally challenging.
[12]The authors used Bayesian criteria to find 180 to be an optimal number of topics.

2. In the Python library Gensim, the key parameters of the LDA estim are α and β.With a higher value of α, the documents are composed of more topics. With a higher values of β, each topic contains more terms (uni- or bi-grams). In the implementations, I do not impose any explicit restrictions on initial values of those parameters and set them to be "auto". These two parameters, alongside Φ' and $\{\theta_i\}_i$, are estimated by the toolbox from Python library `https://pypi.org/project/gensim/`Gensim.

Real-time LDA Factors With the estimated topic weights $\theta_{i,t}$ of each article i from the LDA model, I fruther construct time series of the overall news attention to each topic, or a news factor. The value of the topic k at time t is the average weights of topic k of all articles published at t, specified as follows,

$$F_{k,t} = \frac{\sum_i \hat{\theta}_{i,k,t}}{\text{\# of articles at } t} \tag{A.193}$$

for all topics k.

D.2 Machine Algorithm Details

The basic dynamic algorithm follows the six step approach of Bianchi et al. (2022) of 1. Sample partitioning, 2. In-sample estimation, 3. Training and cross-validation, 4. Grid reoptimization, 5. Out-of-sample prediction, and 6. Roll forward and repeat. We refer the interested reader to that paper for details and discuss details of the implementation here only insofar as they differ.

At time t, a prior sample of size $\dot{T}$ is partitioned into two subsample windows: a *training sample* consisting of the first T_E observations, and a hold-out *validation sample* of T_V subsequent observations so that $\dot{T} = T_E + T_V$. The training sample is used to estimate the model subject to a specific set of tuning parameter values, and the validation sample is used for tuning the hyperparameters. The model to be estimated over the training sample is

$$y_{t,t+h} = G^e\left(\mathcal{X}_t, \boldsymbol{\beta}_{h,t}\right) + \epsilon_{t+h}.$$

where $y_{t,t+h}$ is a time series indexed by j whose value in period $h \geq 1$ the machine is asked to predict at time t, $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept, and $G^e(\cdot)$ is a machine learning estimator that can be represented by a (potentially) high dimensional set of finite-valued parameters $\boldsymbol{\beta}^e_{h,t}$. We consider two estimators for $G^e(\cdot)$: Elastic Net $G^{\text{EN}}(\mathcal{X}_t, \boldsymbol{\beta}^{\text{EN}}_{j,h})$, and Long Short-Term Memory (LSTM) network $G^{\text{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}^{\text{LSTM}}_{j,h})$. The $e \in \{\text{EN}, \text{LSTM}\}$ superscripts on $\boldsymbol{\beta}$ indicate that the parameters depend on the estimator being used (See the next section for a description of EN and LSTM). $\mathcal{X}_t$ always denotes the most recent data that would have been in real time prior to the date on which the forecast was submitted. To ensure that the effect of each variable in the input vector is regularized fairly during the estimation, we standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. It should be noted that the most recent observation on the left-hand-side is generally available in real time only with a one-period lag, thus the forecasting estimations can only be run with data over a sample that stops one period later than today in real time.

The parameters $\boldsymbol{\beta}^e_{h,t}$ are estimated by minimizing the mean-square loss function over the training sample with L_1 and L_2 penalties

$$L(\boldsymbol{\beta}^e_{h,t}, \mathbf{X}_{T_E}, \boldsymbol{\lambda}^e_t) \equiv \underbrace{\frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - G^e\left(\mathcal{X}_\tau, \boldsymbol{\beta}^e_{h,t}\right)\right)^2}_{\text{Mean Square Error}} + \underbrace{\lambda^e_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^e_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^e_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^e_{j,h,t,k})^2}_{L_2 \text{ Penalty}}$$

where $\mathbf{X}_{T_E} = (y_{t-T_E}, \ldots, y_t, \mathcal{X}'_{t-T_E}, \ldots, \mathcal{X}'_t)'$ is the vector containing all observations in the training sample of size T_E. The estimated $\boldsymbol{\beta}^e_{h,t}$ is a function of the data $\boldsymbol{X}_{T_E}$ and a non-negative regularization parameter vector $\boldsymbol{\lambda}^e_t = \left(\lambda^e_{1,t}, \lambda^e_{2,t}, \boldsymbol{\lambda}^{LSTM}_{0,t}\right)'$ where $\boldsymbol{\lambda}^{LSTM}_{0,t}$ is a set of hyperparameters only relevant when using the LSTM estimator for $G^e(\cdot)$ (see below). For the EN case there are only two hyperparameters, which determine the optimal shrinkage and sparsity of the time t machine specification. The regularization parameters $\boldsymbol{\lambda}^e_t$ are estimated by minimizing the mean-square loss over pseudo-out-of-sample forecast errors generated from rolling regressions through the validation sample:

$$\begin{aligned}\widehat{\boldsymbol{\lambda}}^{EN}_t, \widehat{T}_E, \widehat{T}_V = \underset{\boldsymbol{\lambda}^{EN}_t, T_E, T_V}{argmin}\Bigg\{&\frac{1}{T_V - h}\sum_{\tau=T_E}^{T_E+T_V-h}\left(y_{\tau+h} - G^{EN}(\mathcal{X}_\tau, \widehat{\boldsymbol{\beta}}^{EN}_{j,h,\tau}(\mathbf{X}_{T_E}))\right)^2 \\ &+ \underbrace{\lambda^{EN}_{1,t}\sum_{k=1}^{K}\boldsymbol{\beta}^{EN}_{j,h,t,k}}_{L_1 \text{ Penalty}} + \underbrace{\lambda^{EN}_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^{EN}_{j,h,t,k})^2}_{L_2 \text{ Penalty}}\Bigg\}\end{aligned}$$

$$\begin{aligned}\widehat{\boldsymbol{\lambda}}^{LSTM}_t, \widehat{T}_E, \widehat{T}_V = \underset{\boldsymbol{\lambda}^{LSTM}_t, T_E, T_V}{argmin}\Bigg\{&\frac{1}{T_V - h}\sum_{\tau=T_E}^{T_E+T_V-h}\left(y_{\tau+h} - G^{LSTM}(\mathcal{X}_\tau, \widehat{\boldsymbol{\beta}}^{LSTM}_{j,h,\tau}(\mathbf{X}_{T_E}, \boldsymbol{\lambda}^{LSTM}_t))\right)^2 \\ &+ \underbrace{\lambda^{LSTM}_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^{LSTM}_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^{LSTM}_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^{LSTM}_{j,h,t,k})^2}_{L_2 \text{ Penalty}}\Bigg\}\end{aligned}$$

where $\widehat{\boldsymbol{\beta}}^e_{j,h,\tau}(\cdot)$ for $e \in \{\text{EN}, \text{LSTM}\}$ is the time τ estimate of $\boldsymbol{\beta}^e_{j,h}$ given $\boldsymbol{\lambda}^e_t$ and data through time τ in a training sample of size T_E. Denote the combined final estimator $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$, where the regularization parameter $\widehat{\boldsymbol{\lambda}}^e_t$ is estimated using cross-validation dynamically over time. Note that the algorithm also asks the machine to dynamically choose both the optimal training window $\widehat{T}_E$ and the optimal validation window $\widehat{T}_V$ by minimizing the pseudo-out-of-sample MSE.

The estimation of $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$ is repeated sequentially in rolling subsamples, with parameters estimated from information known at time t. Note that the time t subscripts of $\widehat{\boldsymbol{\beta}}^e_{h,t}$ and $\widehat{\boldsymbol{\lambda}}^e_t$ denote one in a sequence of time-invariant parameter estimates obtained from rolling subsamples, rather than estimates that vary over time within a sample. Likewise, we denote the time t machine belief about $y_{t,t+h}$ as $\mathbb{E}^e_t[y_{t,t+h}]$, defined by

$$\mathbb{E}^e_t[y_{t,t+h}] \equiv G^e\left(\mathcal{X}_t, \widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)\right)$$

Finally, the machine MSE is computed by averaging across the sequence of squared forecast errors in the true out-of-sample forecasts for periods $t = (\dot{T} + h), \ldots, T$ where T is the last period of our sample. The true out-of-sample forecasts used for neither estimation nor tuning is the *testing subsample* used to evaluate the model's predictive performance.

On rare occasions, one or more of the explanatory variables used in the machine forecast specification assumes a value that is order of magnitudes different from its historical value. This is usually indicative of a measurement problem in the raw data. We therefore program the machine to detect in real-time whether its forecast is an extreme outlier, and in that case to discard the forecast replacing it with the historical mean. Specifically, at each t, the machine forecast $\mathbb{E}_t^e[y_{t,t+h}]$ is set to be the historical mean calculated up to time t whenever the former is five or more standard deviations above its own rolling mean over the most recent 20 quarters.

We include the contemporaneous survey forecasts $\mathbb{F}_t[y_{t,t+h}]$ for the median respondent only for inflation and GDP forecasts, following Bianchi et al. (2022). This procedure allows the machine to capture intangible information due to judgement or private signals. Specifically, for these forecasts of inflation and GDP growth, we consider the following machine learning empirical specification for forecasting $y_{t,t+h}$ given information at time t, to be benchmarked against the time t survey forecast of respondent-type X, where this type is the median here:

$$y_{t,t+h} = G_{jh}^e(\mathbf{Z}_t) + \gamma_{jh\mathbb{M}}\mathbb{F}_t[y_{t,t+h}] + \epsilon_{t+h}, \qquad h \geq 1 \tag{A.194}$$

where $\gamma_{jh\mathbb{M}}$ is a parameter to be estimated, and where $G_{jh\mathbb{M}}(\mathbf{Z}_t)$ represents a ML estimator as function of big data. Note that the intercept α_{jh} from Bianchi et al. (2022) gets absorbed into the $G_{jh}^e(\mathbf{Z}_t)$ in LSTM via the outermost bias term.

D.2.1 Elastic Net (EN)

We use the Elastic Net (EN) estimator, which combines Least Absolute Shrinkage and Selection Operator (LASSO) and ridge type penalties. The model can be written as:

$$y_{t,t+h} = \mathcal{X}_{tj}'\boldsymbol{\beta}_{j,h}^{\mathrm{EN}} + \epsilon_{t+h}$$

where $\mathcal{X}_t = (1, \mathcal{X}_{1t,\dots,}\mathcal{X}_{Kt})'$ include the independent variable observations $(\mathbb{F}_t[y_{t,t+h}], \mathcal{Z}_{j,t})$ into a vector with "1" and $\boldsymbol{\beta}_{j,h}^{\mathrm{EN}} = (\alpha_{j,h}, \beta_{j,h\mathbb{F}}, vec(\mathbf{B}_{j,h\mathcal{Z}}))' \equiv (\beta_0, \beta_1, ...\beta_K)'$ collects all the coefficients.

It is customary to standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. The coefficient estimates are then put back in their original scale by multiplying the slope coefficients by their respective standard deviations, and adding back the mean (scaled by slope coefficient over standard deviation.) The EN estimator incorporates both an L_1 and L_2 penalty:

$$\widehat{\boldsymbol{\beta}}_{j,h}^{\mathrm{EN}} = \underset{\beta_0,\beta_1,\dots,\beta_K}{argmin}\left\{\frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - \mathcal{X}_\tau'\boldsymbol{\beta}_{j,h}\right)^2 + \underbrace{\lambda_1\sum_{k=1}^{K}|\boldsymbol{\beta}_{j,h,k}|}_{\text{LASSO}} + \underbrace{\lambda_2\sum_{k=1}^{K}(\boldsymbol{\beta}_{j,h,k})^2}_{\text{ridge}}\right\}$$

By minimizing the MSE over the training samples, we choose the optimal λ_1 and λ_2 values simultaneously.

In the implementation, the EN estimator is sometimes used as an input into the algorithm using the LSTM estimator. Specifically, we ensure that the machine forecast can only differ from the relevant benchmark if it demonstrably improves the pseudo out-of-sample prediction in the training samples *prior* to making a true out-of-sample forecast. Otherwise, the machine is replaced by the benchmark calculated up to time t. In some cases the benchmark is a survey forecast, in others it could be a historical mean value for the variable. However, for the implementation using LSTM, we also use the EN forecast as a benchmark.

D.2.2 Long Short-Term Memory (LSTM) Network

An LSTM network is a type of Recurrent Neural Network (RNN), which are neural networks used to learn about sequential data such as time series or natural language. In particular, LSTM networks can learn long-term dependencies between across time periods by introducing hidden layers and memory cells to control the flow of information over longer time periods. The general case of the LSTM network with up to N hidden layers is defined as

$$\underbrace{G^{\text{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}_{j,h}^{\text{LSTM}})}_{1\times 1} = \underbrace{W^{(yh^N)}}_{1\times D_{h^N}} \underbrace{h_t^N}_{D_{h^N}\times 1} + \underbrace{b_y}_{1\times 1} \qquad \text{(Output layer)}$$

$$\underbrace{h_t^n}_{D_{h^n}\times 1} = \underbrace{o_t^n}_{D_{h^n}\times 1} \odot \tanh(\underbrace{c_t^n}_{D_{h^n}\times 1}) \qquad \text{(Hidden layer)}$$

$$\underbrace{c_t^n}_{D_{h^n}\times 1} = \underbrace{f_t^n}_{D_{h^n}\times 1} \odot \underbrace{c_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{i_t^n}_{D_{h^n}\times 1} \odot \underbrace{\tilde{c}_t^n}_{D_{h^n}\times 1} \qquad \text{(Final memory)}$$

$$\underbrace{\tilde{c}_t^n}_{D_{h^n}\times 1} = \tanh(\underbrace{W^{(c^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(c^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{c^n}}_{D_{h^n}\times 1}) \qquad \text{(New memory)}$$

$$\underbrace{f_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(f^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(f^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{f^n}}_{D_{h^n}\times 1}) \qquad \text{(Forget gate)}$$

$$\underbrace{i_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(i^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(i^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{i^n}}_{D_{h^n}\times 1}) \qquad \text{(Input gate)}$$

$$\underbrace{o_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(o^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(o^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{o^n}}_{D_{h^n}\times 1}) \qquad \text{(Output gate)}$$

where $n = 1, \ldots, N$ indexes each hidden layer. $h_t^n \in \mathbb{R}^{D_{h^n}}$ is the n-th *hidden layer*, where D_{h^n} is the number of *neurons* or *nodes* in the hidden layer. The 0-th layer is defined as the input data: $h_t^0 \equiv \mathcal{X}_t$. The memory cell c_t^n allows the LSTM network to retain information over longer time periods. The output gate o_t^n controls the extent to which the memory cell c_t^n maps to the hidden layer h_t^n. The forget gate f_t^n controls the flow of information carried over from the final memory in the previous timestep c_{t-1}^n. The input gate i_t^n controls the flow of information from the new memory cell $\tilde{c}_t^n$. The initial states for the hidden layers $(h_0^n)_{n=1}^N$ and memory cells $(c_0^n)_{n=1}^N$ are set to zeros.

$\sigma(\cdot)$ and $\tanh(\cdot)$ are *activation functions* that introduce non-linearities in the LSTM network, applied elementwise. $\sigma : \mathbb{R} \to \mathbb{R}$ is the sigmoid function: $\sigma(x) = (1 + e^{-x})^{-1}$. $\tanh : \mathbb{R} \to \mathbb{R}$ is the hyperbolic tangent function: $\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$. The $\odot$ operator refers to elementwise multiplication.

$\boldsymbol{\beta}_{j,h}^{\text{LSTM}} \equiv (((vec(W^{(g^n h^{n-1})})', vec(W^{(g^n h^n)})', b_{g^n}')_{g\in\{c,f,i,o\}})_{n=1}^N, vec(W^{(yh^N)})', b_y)'$ are parameters to be estimated. We will refer to parameters indexed with W as *weights*; parameters indexed with b are *biases*. We estimate the parameters $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$ for the LSTM network using Stochastic Gradient Decent (SGD), which is an iterative algorithm for minimizing the loss function and proceeds as follows:

1. *Initialization.* Fix a random seed R and draw a starting value of the parameters $\boldsymbol{\beta}_{j,h}^{(0)}$ randomly, where the superscript (0) in parentheses indexes the iteration for an estimate of $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$.

   (a) Initialize the input weights $W^{(g^n h^{n-1})} \in \mathbb{R}^{D_{h^n}\times D_{h^{n-1}}}$ for $g \in \{c, f, i, o\}$ using the *Glorot* initializer. Draw randomly from a uniform distribution with zero mean and a variance that depends on the dimensions of the matrix:

$$W_{ij}^{(g^n h^{n-1})} \overset{iid}{\sim} U\left[-\sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}, \sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}\right]$$

for each $i = 1, \dots, D_{h^n}$ and $j = 1, \dots, D_{h^{n-1}}$.

(b) Initialize the recurrent weights $W^{(g^n h^n)} \in \mathbb{R}^{D_{h^n} \times D_{h^n}}$ for $g \in \{c, f, i, o\}$ using the *Orthogonal* initializer. Use the orthogonal matrix obtained from the QR decomposition of a $D_{h^n} \times D_{h^n}$ matrix of random numbers drawn from a standard normal distribution.

(c) Initialize biases $(b_{g^n})_{g \in \{c,f,i,o\}}$, hidden layers h_0^n, and memory cells c_0^n with zeros.

2. *Mini-batches.* Prepare the input data by dividing the training sample into a collection of *mini-batches.*

(a) Suppose that we have a multi-variate time-series training sample with dimensions (T_E, K) whose time steps t are indexed by $t = 1, \dots, T_E$ and K is the number of predictors. We transform this training sample into a 3-D tensor with dimensions (N_S, M, K) where

- $N_S =$ Total number of sequences in training sample
- $M =$ Sequence length, i.e., number of time steps in each sequence
- $K =$ Input size, i.e., number of predictors in each time step

This can be done by creating overlapping sequences from the time series:

- Sequence 1 contains time steps $1, \dots, M$
- Sequence 2 contains time steps $2, \dots, M + 1$
- Sequence 3 contains time steps $3, \dots, M + 2$
- ...
- Sequence $T_E - M$ contains time steps $T_E - M, \dots, T_E - 1$
- Sequence $N_S = T_E - M + 1$ contains time steps $T_E - M + 1, \dots, T_E$

(b) Randomly shuffle the N_S sequences by randomly sampling a permutation of the sequences without replacement.

(c) Partition the N_S shuffled sequences into $\lceil N_S/N_B \rceil$ mini-batches. We partition the N_S sequences in the training sample ((N_S, M, K) tensor) into a list of $\lceil N_S/N_B \rceil$ mini-batches. A mini-batch is a (N_B, M, K)-dimensional tensor containing N_B out of N_S randomly shuffled sequences.[13] Let $B^{(1)}, \dots, B^{\lceil N_S/N_B \rceil}$ denote the list of mini-batches.

- $N_S =$ Total number of sequences in training sample
- $N_B =$ Mini-batch size, i.e., number of sequences in each partition.
- $M =$ Sequence length, i.e., number of time steps in each sequence
- $K =$ Input size, i.e., number of predictors in each time step

3. Repeat until the stopping condition is satisfied ($k = 1, 2, 3, \dots$):

(a) *Dropout.* Apply dropout to the mini-batch. To obtain the n-th hidden layer under dropout, multiply the current value of the $n-1$-th hidden layer h_t^{n-1} and the lagged value of the n-th

[13] When N_S/N_B is not a whole number, $\lfloor N_S/N_B \rfloor$ of the mini-batches will be 3-D tensors with dimensions (N_B, M, K). One batch will contain leftover sequences and will have dimensions $(N_S \% N_B, M, K)$ where % is the modulus operator.

hidden layer h_{t-1}^n with binary masks $r_{t,h_t^{n-1}}^{(k)} \in \mathbb{R}^{D_{h^{n-1}}}$ and $r_{t,h_{t-1}^n}^{(k)} \in \mathbb{R}^{D_{h^n}}$, respectively:

$$\underbrace{\overline{h}_t^{n-1}}_{D_{h^{n-1}}\times 1} = \underbrace{r_{t,h_t^{n-1}}^{(k)}}_{D_{h^{n-1}}\times 1} \odot \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1}, \quad r_{t,h_t^{n-1},i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_t^{n-1}}), \quad i = 1, \ldots, D_{h^{n-1}}$$

$$\underbrace{\overline{h}_{t-1}^{n}}_{D_{h^{n}}\times 1} = \underbrace{r_{t,h_{t-1}^{n}}^{(k)}}_{D_{h^{n}}\times 1} \odot \underbrace{h_{t-1}^{n}}_{D_{h^{n}}\times 1}, \quad r_{t,h_{t-1}^{n},i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_{t-1}^{n}}), \quad i = 1, \ldots, D_{h^{n}}$$

where $t \in B^{(k)}$ and $n = 1, \ldots, N$ indexes the hidden layer and it is understood that the 0-th layer is the input vector $h_t^0 \equiv \mathcal{X}_t$. $p_{h_t^{n-1}}, p_{h_{t-1}^n} \in [0, 1]$ is the probability that time t nodes in the $n-1$-th hidden layer and time $t-1$ nodes in the n-th hidden layer are retained, respectively.

(b) *Stochastic Gradient.* Average the gradient over observations in the mini-batch

$$\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}) = \frac{1}{M} \sum_{t \in B^{(k)}} \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$$

where $\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$ is the gradient of the loss function with respect to the parameters $\boldsymbol{\beta}_{j,h}^{(k-1)}$, evaluated at the time t observation $\mathbf{X}_t = (y_{t,t+h}, \widehat{\mathcal{X}}_t')'$ after applying dropout.

(c) *Learning rate shrinkage.* Update the parameters to $\boldsymbol{\beta}_{j,h}^{(k)}$ using the Adaptive Moment Estimation (Adam) algorithm. The method uses the first and second moments of the gradients to shrink the overall learning rate to zero as the gradient approaches zero.

$$\boldsymbol{\beta}_{j,h}^{(k)} = \boldsymbol{\beta}_{j,h}^{(k-1)} - \gamma \frac{m^{(k)}}{\sqrt{v^{(k)}} + \varepsilon}$$

where $m^{(k)}$ and $v^{(k)}$ are weighted averages of first two moments of past gradients:

$$m^{(k)} = \frac{1}{1 - \pi_1^k} (\pi_1 m^{(k-1)} + (1 - \pi_1) \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}))$$

$$v^{(k)} = \frac{1}{1 - \pi_2^k} (\pi_2 v^{(k-1)} + (1 - \pi_2) \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}})^2)$$

π^k denotes the k-the power of $\pi \in (0, 1)$, and $/$, $\sqrt{\cdot}$, and $(\cdot)^2$ are applied elementwise. The default values of the hyperparameters are $m^{(0)} = v^{(0)} = 0$ (initial moment vectors), $\gamma = 0.001$ (initial learning rate), $(\pi_1, \pi_2) = (0.9, 0.999)$ (decay rates), and $\varepsilon = 10^{-7}$ (prevent zero denominators).

(d) *Stopping Critera.* Stop iterating and return $\boldsymbol{\beta}_{j,h}^{(k)}$ if one of the following holds:

- *Early stopping.* At each iteration, use the updated $\boldsymbol{\beta}_{j,h}^{(k)}$ to calculate the loss from the validation sample. Stop when the validation loss has not improved for S steps, where S is a "patience" hyperparameter. By updating the parameters for fewer iterations, early stopping shrinks the final parameters $\boldsymbol{\beta}_{j,h}$ towards the initial guess $\boldsymbol{\beta}_{j,h}^{(0)}$, and at a lower computational cost than ℓ_2 regularization.

- *Maximum number of epochs.* Stop if the number of iterations reaches the maximum number of epochs E. An epoch happens when the full set of the training sample has been used to update the parameters. If the training sample has T_E observations and each mini-batch has M observations, then each epoch would contain $\lceil T_E/M \rceil$ iterations (after rounding up as needed). So the maximum number of iterations is bounded by $E \times \lceil T_E/M \rceil$.

4. *Ensemble forecasts.* Repeat steps 1. and 2. over different random seeds R and save each of the estimated parameters $\widehat{\boldsymbol{\beta}}^{LSTM}_{j,h,T_E}(\boldsymbol{X}_{T_E}, \boldsymbol{\lambda}^{\text{LSTM}}, R)$. Then construct out-of-sample forecasts using the top 10 out of 20 starting values with the best performance in the validation sample. Ensemble can be considered as a regularization method because it aims to guard against overfitting by shrinking the forecasts toward the average across different random seeds. The random seed affects the random draws of the parameter's initial starting value $\boldsymbol{\beta}^{(0)}_{j,h}$, the sequences selected in each mini-batch $B^{(k)}$, and the dropout mask $r^{(k)}_t$.

D.2.3 Hyperparameters

Let $\boldsymbol{\lambda}^{\text{LSTM}} \equiv [\lambda_1, \lambda_2, \gamma, \pi_1, \pi_2, p, N, (D_{h^n})_{n=1}^N, M, E, S]'$ collect all the hyperparameters that control the LSTM network's complexity and prevent the model from overfitting the data. The number of hidden layers N and the number of neurons $D_{h^1}, \ldots, D_{h^N}$ in each hidden layer are hyperparameters that characterize the network's *architecture*. The hyperparameters are estimated by minimizing the mean-square loss over pseudo out-of-sample forecast errors generated from rolling regressions through the validation sample.

Table A.18 reports the hyperparameters for the LSTM network and its estimation. Hyperparameters reported as a range or a set of values are cross-validated. We ask the machine to cross validate the L_1 and L_2 penalties by adjusting the relative weight on the L_1 penalty, α, and the overall multiplier, λ, within the range of values listed in the table. α and λ are converted back to the L_1 and L_2 penalties using the mapping $\lambda_1 = \alpha\lambda$ and $\lambda_2 = \frac{1-\alpha}{2}\lambda$. The window lengths for the training sample T_E and validation sample T_V denote the number of quarterly observations. $T_V = \textit{Expand}$ denotes the case of recursively expanding windows, where pseudo out-of-sample forecasts are generated from rolling estimates based on a training sample that expands recursively over time starting from 1984Q1 for returns and price growth, 1995Q1 for earnings growth.

Table A.18: Hyperparameters for the Machine Algorithm

(1) Machine Forecast: Stock Returns and Price Growth (1 Year Ahead)

Description / Horizon	Returns (1 Year)	Price Growth (1 Year)
(a) Elastic Net ($\boldsymbol{\lambda}^{\text{EN}}$)		
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 penalties (λ)	$[10^{-2}, 10]$	$[10^{-2}, 10]$
Size of training sample (T_E)	$4, 5, 6, 7$	$4, 5, 6, 7$
Size of validation sample (T_V)	$4, 5, 6, 7, \textit{Expand}$	$4, 5, 6, 7, \textit{Expand}$
(b) Long Short-Term Memory Network ($\boldsymbol{\lambda}^{\text{LSTM}}$)		
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 penalties (λ)	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$
Initial learning rate (γ)	0.001	0.001
Gradient decay rates (π_1, π_2)	$0.9, 0.999$	$0.9, 0.999$
Probability of dropout ($p_{\mathcal{X}}, p_{h^n}$)	$0.8, 0.5$	$0.8, 0.5$
Number of hidden layers (N)	1	1
Number of neurons $(D_{h^n})_{n=1}^{N}$	4	4
Mini-batch size (N_B)	4	4
Sequence length (M)	4	4
Patience for early stopping (S)	10	20
Maximum number of epochs (E)	$10,000$	$10,000$
Random seeds (R)	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$
Size of training sample (T_E)	$5, 7$	$5, 7$
Size of validation sample (T_V)	$5, 7, 20$	$5, 7, 20$

(2) Machine Forecast: Stock Returns (2, 3, 4, and 5 Years Ahead)

Description / Horizon	2 Years	3 Years	4 Years	5 Years
(a) Elastic Net ($\boldsymbol{\lambda}^{\text{EN}}$)				
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.1, 0.9]$	$[0.1, 0.9]$	$[0.1, 0.9]$
Multiplier on L_1 and L_2 (λ)	$[10^{-2}, 10]$	$[10^{-1}, 10]$	$[10^{-1}, 10]$	$[10^{-1}, 10]$
Size of training sample (T_E)	$4, 5, 6, 7$	$4, 5, 6, 7$	$4, 5, 6, 7$	$4, 5, 6, 7$
Size of validation sample (T_V)	$4, 5, 6, 7$ *Expand*	$4, 5, 6, 7$ *Expand*	$4, 5, 6, 7$ *Expand*	$4, 5, 6, 7$ *Expand*
(b) Long Short-Term Memory Network ($\boldsymbol{\lambda}^{\text{LSTM}}$)				
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 (λ)	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$
Initial learning rate (γ)	0.001	0.001	0.001	0.001
Gradient decay rates (π_1, π_2)	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$
Probability of dropout ($1 - p$)	0.5	0.5	0.5	0.05
Number of hidden layers (N)	1	1	1	1
Number of neurons $(D_{h^n})_{n=1}^{N}$	4	4	4	4
Mini-batch size (M)	2	2	2	2
Patience for early stopping (S)	3	3	3	20
Maximum number of epochs (E)	$1,000$	$1,000$	$1,000$	$10,000$
Random seeds (R)	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$
Size of training sample (T_E)	$3, 5, 7$	$3, 5, 7$	$3, 5, 7$	$3, 5, 7$
Size of validation sample (T_V)	$3, 5, 7$	$3, 5, 7$	$3, 5, 7$	$3, 5, 7$

Notes: This table reports the hyperparameters considered in the machine learning algorithm for each estimator.

(3) Machine Forecast: Earnings Growth (1, 2, 3, and 4 Years Ahead)

Description / Horizon	1 Year	2 Years	3 Years	4 Years
(a) Elastic Net ($\boldsymbol{\lambda}^{\text{EN}}$)				
Weight on L_1 penalty (α)	$[0.1, 0.9]$	$[0.1, 0.9]$	$[0.1, 0.9]$	$[0.1, 0.9]$
Multiplier on L_1 and L_2 (λ)	$[1, 10]$	$[1, 10]$	$[1, 10]$	$[1, 10]$
Size of training sample (T_E)	$4, 6, 8, 10, 12$	$4, 6, 8, 10, 12$	$4, 6, 8, 10, 12$	$4, 6, 8, 10, 12$
Size of validation sample (T_V)	$4, 6, 8, 10, 12$ *Expand*	$4, 6, 8, 10, 12$ *Expand*	$4, 6, 8, 10, 12$ *Expand*	$4, 6, 8, 10, 12$ *Expand*
(b) Long Short-Term Memory Network ($\boldsymbol{\lambda}^{\text{LSTM}}$)				
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 (λ)	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$
Initial learning rate (γ)	0.001	0.001	0.001	0.001
Gradient decay rates (π_1, π_2)	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$
Probability of dropout ($p_{\mathcal{X}}, p_{h^n}$)	$0.05, 0.05$	$0.05, 0.05$	$0.05, 0.05$	$0.05, 0.05$
Number of hidden layers (N)	1	1	1	1
Number of neurons $(D_{h^n})_{n=1}^N$	4	4	4	4
Mini-batch size (N_B)	4	4	4	4
Sequence length (M)	4	4	4	4
Patience for early stopping (S)	20	20	20	20
Maximum number of epochs (E)	$10,000$	$10,000$	$10,000$	$10,000$
Random seeds (R)	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$
Size of training sample (T_E)	$4, 8, 12$	$4, 8, 12$	$4, 8, 12$	$4, 8, 12$
Size of validation sample (T_V)	$4, 8, 12, 20$	$4, 8, 12, 20$	$4, 8, 12, 20$	$4, 8, 12, 20$

(4) Machine Forecast: Long-Term Growth (LTG)

Description / Horizon	4-to-5-years	0-to-5-years	1-to-10-years
(a) Elastic Net ($\boldsymbol{\lambda}^{\text{EN}}$)			
Weight on L_1 penalty (α)	$[0.1, 0.9]$	$[0.1, 0.9]$	$[0.1, 0.9]$
Multiplier on L_1 and L_2 (λ)	$[1, 10]$	$[1, 10]$	$[1, 10]$
Size of training sample (T_E)	$6, 8, 10, 12$	$6, 8, 10, 12$	$6, 8, 10, 12$
Size of validation sample (T_V)	$6, 8, 10, 12$ *Expand*	$6, 8, 10, 12$ *Expand*	$6, 8, 10, 12$ *Expand*
(b) Long Short-Term Memory Network ($\boldsymbol{\lambda}^{\text{LSTM}}$)			
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 (λ)	$[10^{-5}, 10^{-1}]$	$[10^{-4}, 10^{-2}]$	$[10^{-5}, 10^{-1}]$
Initial learning rate (γ)	0.001	0.001	0.001
Gradient decay rates (π_1, π_2)	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$
Probability of dropout ($p_{\mathcal{X}}, p_{h^n}$)	$0.05, 0.05$	$0.05, 0.05$	$0.05, 0.05$
Number of hidden layers (N)	1	1	1
Number of neurons $(D_{h^n})_{n=1}^N$	4	4	4
Mini-batch size (N_B)	4	4	4
Sequence length (M)	4	4	4
Patience for early stopping (S)	80	20	20
Maximum number of epochs (E)	$10,000$	$10,000$	$10,000$
Random seeds (R)	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$
Size of training sample (T_E)	$3, 7, 12$; $T_E = 3$ if $T_V = 20$	$4, 8, 12$	$3, 7, 12$; $T_E = 3$ if $T_V = 20$
Size of validation sample (T_V)	$3, 7, 12, 20$	$4, 8, 12, 20$	$3, 7, 12, 20$

Notes: This table reports the hyperparameters considered in the machine learning algorithm for each estimator. The LSTM forecast is replaced with the corresponding EN forecast if the LSTM cannot improve on the EN over the pseudo out-of-sample predictions. The LSTM forecast for 0-to-5 year ahead LTG is implemented as an estimation using forecast errors as the dependent variable.

(5) Machine Forecast: Inflation and Employment Growth (1 Year Ahead)

Description / Horizon	Inflation (1 Year)	Employment Growth (1 Year)
(a) Elastic Net ($\boldsymbol{\lambda}^{\text{EN}}$)		
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 penalties (λ)	$[10^{-2}, 3]$	$[10^{-1}, 10]$
Size of training sample (T_E)	$3, 4, 5, 6, 7$	$3, 4, 5, 6, 7$
Size of validation sample (T_V)	$6, 7, \ldots, 14, 15$	$6, 7, \ldots, 14, 15$
(b) Long Short-Term Memory Network ($\boldsymbol{\lambda}^{\text{LSTM}}$)		
Weight on L_1 penalty (α)	$[0.01, 0.99]$	$[0.01, 0.99]$
Multiplier on L_1 and L_2 penalties (λ)	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-2}]$
Initial learning rate (γ)	0.001	0.001
Gradient decay rates (π_1, π_2)	$0.9, 0.999$	$0.9, 0.999$
Probability of dropout ($p_{\mathcal{X}}, p_{h^n}$)	$0.8, 0.5$	$0.005, 0.005$
Number of hidden layers (N)	1	1
Number of neurons $(D_{h^n})_{n=1}^{N}$	4	4
Mini-batch size (N_B)	4	4
Sequence length (M)	4	4
Patience for early stopping (S)	20	5
Maximum number of epochs (E)	$10,000$	$10,000$
Random seeds (R)	$1, 2, \ldots, 20$	$1, 2, \ldots, 20$
Size of training sample (T_E)	$5, 7$	$3, 5$
Size of validation sample (T_V)	$6, 9, 12, 15$	$6, 9, 12$

Notes: This table reports the hyperparameters considered in the machine learning algorithm for each estimator.

D.2.4 Machine Variables to Be forecast

Returns and price growth When evaluating the MSE ratio of the machine relative to that of a benchmark survey, we use the machine forecast for the return or price growth measure that most closely corresponds to the concept that survey respondents are asked to predict:

1. CFO survey asks respondents about their expectations for the S&P 500 return over the next 12 months. Following Nagel and Xu (2021), we interpret the survey to be asking about the one-year CRSP value-weighted return (including dividends) from the current survey month to the same month one year ahead.

2. Gallup/UBS survey respondents report the return (including dividends) they expect on their own portfolio one year ahead. We interpret the survey to be asking about the one-year CRSP value-weighted return(including dividends) from the current survey month to the same month one year ahead.

3. Livingston survey respondents provide 12-month ahead forecasts of the S&P 500 index. We convert the level forecast to price growth forecast by taking the log difference between the 12-month ahead level forecast and the nowcast of the S&P 500 index for the current survey month. Therefore, we interpret the survey to be asking about the one-year price growth in the S&P 500 index.

4. Bloomberg Consensus Forecasts asks survey respondents about the end-of-year closing value of the S&P 500 index. We interpret the survey to be asking about the h-month price growth in the S&P 500 index. The horizon of the forecast changes depending on when in the year the panelists are answering the survey.

5. Michigan Survey of Consumers (SOC) asks respondents about their perceived probability that an investment in a diversified stock fund would increase in value in the year ahead. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

6. Conference Board (CB) survey asks respondents about their categorical belief on whether they expect stock prices to increase, decrease, or stay the same over the next year. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

7. When the machine forecast is compared with the historical mean as the benchmark, we compare the forecasts for annualized CRSP return. For forecast horizons longer than one year, we compare the machine forecast of annualized cumulative log CRSP return from the time of the forecast to the end of the forecast horizon less the current short rate against the historical mean of the same variable.

Earning Growth For earning growth forecasts, we use a quarterly S&P 500 total earnings series that combines data from S&P Global, Shiller, and the S&P 500 divisor, as described above. Since the machine learning algorithm has been adapted to a quarterly forecasting frequency, we use the quarterly series before the monthly interpolation. The quarterly series spans the period 1959:Q1 to 2021:Q4.

For Long-Term Growth (LTG) forecasts, IBES defines LTG as the "expected annual increase in operating earnings over the company's next full business cycle. These forecasts refer to a period of between three to five years." We compare survey responses of LTG against machine forecasts under alternative interpretations of LTG. First, we consider machine forecasts of annual 5-year forward growth, i.e., annual earnings growth from four to five years ahead. Second, we consider machine forecasts of annualized 5-year growth, i.e., annual earnings growth from current quarter to five years ahead, following the interpretation in Bordalo et al. (2019). Third, we consider machine forecasts of annualized earnings growth from one to 10 years ahead, following the interpretation in Nagel and Xu (2021)

Inflation We construct forecasts of annual inflation defined as

$$\pi_{t+4,t} = \log\left(\frac{PGDP_{t+4}}{PGDP_t}\right)$$

where $PGDP_t$ is the quarterly level of the chain-weighted GDP price index. Following Coibion and Gorodnichenko (2015), we use the vintage of data that are available four quarters after the period being forecast.

Employment growth We construct forecasts of annual employment growth defined as

$$y_{t+4,t} = \log\left(\frac{EMP_{t+4}}{EMP_t}\right)$$

Following Coibion and Gorodnichenko (2015), we use the vintage of data that are available four quarters after the period being forecast.

D.2.5 Machine Input Data: Predictor Variables

The vector $\mathbf{Z}_{jt} \equiv \left(y_t, \hat{\mathbf{G}}_t', \mathbf{W}_{jt}'\right)'$ is an $r = 1 + r_G + r_W$ vector which collects the data at time t with $\mathcal{Z}_{jt} \equiv \left(y_t, ..., y_{t-p_y}, \hat{\mathbf{G}}_t', ..., \hat{\mathbf{G}}_{t-p_G}', \mathbf{W}_{jt}', ..., \mathbf{W}_{jt-p_W}'\right)'$ a vector of contemporaneous and lagged values of $\mathbf{Z}_{jt}$, where p_y, p_G, p_W denote the total number of lags of y_t, $\hat{\mathbf{G}}_t'$, $\mathbf{W}_{jt}'$, respectively. The predictors below are listed as elements of y_t, $\hat{\mathbf{G}}_{jt}'$, or $\mathbf{W}_{jt}'$ for variables.

Stock return and price growth predictor variables and specifications For y_j equal to CRSP value-weighted returns or S&P 500 price index growth, we first predict the one-year log stock return or price growth that is expected to occur h quarters into the future from time $t+h-4$ to $t+h$, i.e., $\mathbb{E}_t[r_{t+h-4,t+h}]$. For horizons longer than one year, since the h-quarter long horizon return is the sum of one-year returns between time t to $t+h$, we first forecast the forward one-year returns separately and then add the components together to get machine forecasts of h-quarter long horizon returns. The forecasting model considers the following variables:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}_{D,t-k}^Q$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the

daily factors are aggregated up to quarterly observations $\mathbf{G}_{D,t}^{Q}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{t,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

4. *LDA topics* $F_{k,t}$, for topic $k = 1, 2, ...50$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

5. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

6. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 3, 4, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 3, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

7. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

8. y_{t-k} for $k = 1, 2$ are lags of the dependent variable. For price growth forecasts, include lags of CRSP returns. We include this term for forecast horizons of 1 and 2 years.

9. $\bar{\mu}_{t-k}$ for $k = 1, 2$ is the historical mean of the dependent variable calculated up to the middle month of the quarter at time t. The initial period is 1959Q1. For price growth forecasts, include the historical mean of CRSP returns.

10. *Long-term growth of earnings*: Annualized 5-year log growth rate of quarterly S&P 500 total earnings at the end of the previous quarter. We include this term for forecast horizons of 1 and 2 years.

11. *Short rates.* When forecasting returns or price growth, the machine controls for the current nominal short rate, $\log(1+3MTB_t/100)$, imposing a unit coefficient. This is equivalent to forecasting the future return minus the current short rate.

12. For LSTM forecasts only: EN forecast of the variable to be predicted. Missing EN forecasts during earlier periods are imputed with the historical mean of the dependent variable. For price growth forecasts, include the EN forecast of CRSP returns.

The 92 macro series in $\mathcal{D}^M$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^f$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns.[14] The 87 daily financial indicators in $\mathcal{D}^D$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

After constructing machine forecasts of forward one-year stock returns $\mathbb{E}_t[r_{t+h-4,t+h}]$, we construct machine forecasts of cumulative h-quarter long horizon returns between time t to $t+h$, i.e., $\mathbb{E}_t[r_{t,t+h}]$, as the sum of the forward one-year return expectations up to h quarters ahead:

$$\mathbb{E}_t[r_{t,t+h}] = \mathbb{E}_t[r_{t,t+4}] + \mathbb{E}_t[r_{t+4,t+8}] + \cdots + \mathbb{E}_t[r_{t+h-4,t+h}]$$

Earning growth predictor variables and specifications For earning growth forecasts, we first detrend the (log) earnings level in real time by, starting with an initial sample, recursively running the following regression at each point in time t

$$\log(earnings_t) = \alpha_t + \beta_t t + y_t$$

For y_t equal to the detrended (log) earning level, we construct a forecasted value for y_t, denoted $\hat{y}_{t|t-h}$, based on information known up to time t using the following variables:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of y_t on $\mathbf{G}_{D,t}(\mathbf{w})$.

4. *LDA factors* $F_{k,t-j}$, for topic $k = 1, 2, ...50$ and $j = 0, 1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

5. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

6. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 3, 4, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 3, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC

[14] A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf

announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

7. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

8. y_{t-k} for $k = 1, 2$ are lags of the dependent variable.

9. $\bar{\mu}_{t-k}$ for $k = 1, 2$ is the historical mean of the dependent variable calculated up to time t. The initial period is 1959Q1.

10. $\mathbb{F}_{t-k}[y_{t+h-k}]$ for $k = 1, 2$ are lags of the survey forecasts of the dependent variable.

11. For LSTM forecasts only: EN forecast of the variable to be predicted. Missing EN forecasts during earlier periods are imputed with the historical mean of the dependent variable.

The 92 macro series in $\mathcal{D}^M$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^f$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns.[15] The 87 daily financial indicators in $\mathcal{D}^D$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

After we obtain the machine forecast for the detrended level of earnings, y, we obtain the h-horizon machine earnings growth forecast (from $t-h$ to t denoted $\mathbb{E}_{t-h}\left[\Delta \log\left(earnings_t^M\right)\right]$) by constructing

$$\mathbb{E}_{t-h}\left[\Delta \log\left(earnings_t^M\right)\right] = \hat{\alpha}_{t-h} + \hat{\beta}_{t-h}t + \hat{y}^M_{t|t-h} - \log\left(earnings_{t-h}\right)$$

where $\log\left(earnings_{t-h}\right)$ is the realized log earning level at time $t-h$, and $\hat{y}^M_{t|t-h}$ is the machine forecast of the detrended log earnings based on information up to time $t-h$. Following De La O and Myers (2021) and Bordalo et al. (2019), we assume that the survey respondents and the machine could observe $\log\left(earnings_{t-h}\right)$, the log earnings level for the quarter on which the forecast is made.

To use this approach to forecast the 20-quarter ahead annual forward earnings i.e., (from $t-4$ to t on basis of information at $t-20$), we would construct

$$\mathbb{E}_{t-20}\left[\log\left(earnings_t^M\right)\right] = \hat{\alpha}_{t-20} + \hat{\beta}_{t-20}t + \hat{y}^M_{t|t-20}.$$

[15] A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf

To construct 20-quarter ahead annual earnings growth forecast we compute

$$\mathbb{E}_{t-20}\left[\log\left(earnings_{t-4}^{M}\right)\right] = \hat{\alpha}_{t-20} + \hat{\beta}_{t-20}(t-4) + \hat{y}_{t-4|t-20}^{M}$$

to get the machine forecast of 20-quarter forward annual earnings log growth as

$$\mathbb{E}_{t-20}\left[\log\left(earnings_{t}^{M}\right) - \log\left(earnings_{t-4}^{M}\right)\right] = \hat{\beta}_{t-20}4 + \hat{y}_{t|t-20}^{M} - \hat{y}_{t-4|t-20}^{M}.$$

An alternative is to use the machine inputs to directly forecast 20-quarter forward annual earnings log growth $\mathbb{E}_{t-20}\left[\log\left(earnings_{t}^{M}\right) - \log\left(earnings_{t-4}^{M}\right)\right]$.

Inflation predictor variables For y_j equal to inflation, the forecasting model considers the following variables:

1. $\mathbb{F}_{jt-k}^{(i)}[y_{jt+h-k}]$, lagged values of the ith type's forecast, where $k = 1, 2$
2. $\mathbb{F}_{jt-1}^{(s \neq i)}[y_{jt+h-1}]$, lagged values of other type's forecasts, $s \neq i$
3. $var_N\left(\mathbb{F}_{t-1}^{(\cdot)}[y_{jt+h-1}]\right)$, where $var_N(\cdot)$ denotes the cross-sectional variance of lagged survey forecasts
4. $skew_N\left(\mathbb{F}_{t-1}^{(\cdot)}[y_{jt+h-1}]\right)$, where $skew_N(\cdot)$ denotes the cross-sectional skewness of lagged survey forecasts
5. Trend inflation measured as $\overline{\pi}_{t-1} = \begin{cases} \rho\overline{\pi}_{t-2} + (1-\rho)\pi_{t-1}, \rho = 0.95 & \text{if } t < \text{1991:Q4} \\ \text{CPI10}_{t-1} & \text{if } t \geq \text{1991:Q4} \end{cases}$, where CPI10 is the median SPF forecast of annualized average inflation over the current and next nine years. Trend inflation is intended to capture long-run trends. When long-run forecasts of inflation are not available, as is the case pre-1991:Q4, we us a moving average of past inflation.
6. $G\dot{D}P_{t-1}$ = detrended gross domestic product, defined as the residual from a regression of GDP_{t-1} on a constant and the four most recent values of GDP as of date $t-8$. See Hamilton (2018).
7. $E\dot{M}P_{t-1}$ = detrended employment, defined as the residual from a regression of EMP_{t-1} on a constant and the four most recent values of EMP as of date $t-8$. See Hamilton (2018).
8. $\mathbb{N}_t^{(i)}[\pi_{t,t-h}]$ = Nowcast as of time t of the ith percentile of inflation over the period $t-h$ to t.

Lags of the dependent variable:

1. $y_{t-1,t-h-1}$ one quarter lagged inflation.

The factors in $\hat{\mathbf{G}}_{jt}'$ include factors formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.
2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^{Q}_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^{D}$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^{Q}_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{t,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

4. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

5. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 3, 4, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 3, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

6. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

The 92 macro series in $\mathcal{D}^{M}$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^{f}$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns.[16] The 87 daily financial indicators in $\mathcal{D}^{D}$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

Employment growth predictor variables and specifications For earning growth forecasts, I first detrend the (log) employment level in real time by, starting with an initial sample, recursively running the following regression at each point in time t

$$\log(employment_t) = \alpha_t + \beta_t t + y_t \tag{A.195}$$

For y_t equal to the detrended (log) employment level, I construct a forecasted value for y_t, denoted $\hat{y}_{t|t-h}$, based on information known up to time t using the following variables:

[16] A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of y_t on $\mathbf{G}_{D,t}(\mathbf{w})$.

4. *LDA factors* $F_{k,t-j}$, for topic $k = 1, 2, ...50$ and $j = 0, 1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

5. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). I include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

6. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 3, 4, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 3, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, I use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, I compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, I use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

7. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

8. y_{t-k} for $k = 1, 2$ are lags of the dependent variable.

9. $\bar{\mu}_{t-k}$ for $k = 1, 2$ is the historical mean of the dependent variable calculated up to time t. The initial period is 1959Q1.

10. $\mathbb{F}_{t-k}[y_{t+h-k}]$ for $k = 1, 2$ are lags of the survey forecasts of the dependent variable.

11. For LSTM forecasts only: EN forecast of the variable to be predicted. Missing EN forecasts during earlier periods are imputed with the historical mean of the dependent variable.

The 92 macro series in $\mathcal{D}^M$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^f$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng

(2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns.[17] The 87 daily financial indicators in $\mathcal{D}^D$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

After I obtain the machine forecast for the detrended level of employment, y, I obtain the h-horizon machine employment growth forecast (from $t-h$ to t denoted $\mathbb{E}_{t-h}\left[\Delta \log\left(employment_t^M\right)\right]$) by constructing

$$\mathbb{E}_{t-h}\left[\Delta \log\left(employment_t^M\right)\right] = \hat{\alpha}_{t-h} + \hat{\beta}_{t-h} t + \hat{y}_{t|t-h}^M - \log\left(employment_{t-h}\right) \tag{A.196}$$

where $\log\left(employment_{t-h}\right)$ is the realized log employment level at time $t-h$, and $\hat{y}_{t|t-h}^M$ is the machine forecast of the detrended log employment based on information up to time $t-h$. Following De La O and Myers (2021) and Bordalo et al. (2019), I assume that the survey respondents and the machine could observe $\log\left(employment_{t-h}\right)$, the log employment level for the quarter on which the forecast is made.

[17] A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf