Belief Distortions and Unemployment Fluctuations*

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Abstract

This paper studies the joint dynamics of asset prices and unemployment when expectations deviate from a rational benchmark. I decompose firms' hiring decisions into expected future cash flows and discount rates, both in the aggregate market (U.S. job filling rate) and the cross section (firm hiring rate). Under subjective beliefs implied by survey forecasts, hiring is driven by predictable errors in expected cash flows, while discount rates play a limited role. In contrast, rational expectations assign a dominant role to discount rates, suggesting that subjective beliefs overestimate the importance of cash flows. A search and matching model in which agents learn with fading memory about stock prices and cash flows can reproduce these patterns and generate realistic unemployment volatility.

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1 Introduction

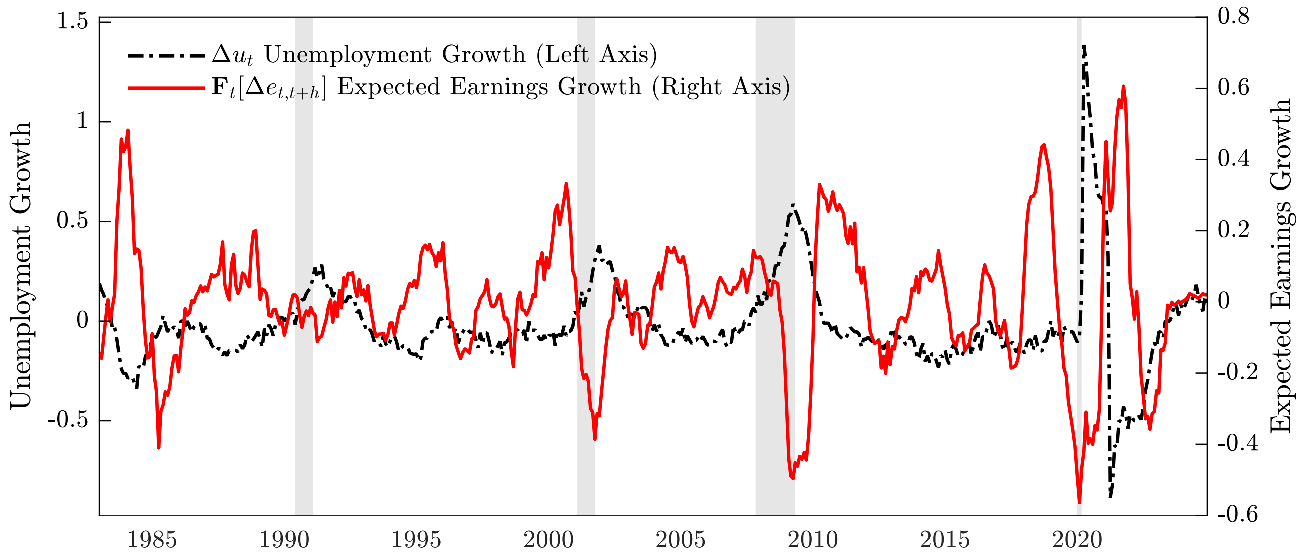
Aggregate unemployment is an important indicator of the business cycle, and the fact that unemployment rises so sharply during recessions is one of the main reasons why business cycle fluctuations are viewed as undesirable. Despite its importance, the standard model of unemployment, the search-and-matching model, struggles to account for the observed volatility in unemployment fluctuations, a disconnect known as the "unemployment volatility puzzle" (Shimer, 2005). Even when calibrated to plausible parameters, the search model fails to explain the magnitude of unemployment fluctuations and the procyclicality of the job-finding rate observed in the data.

Recent work has addressed this puzzle by emphasizing the role of time-varying discount rates under rational expectations. The discount rate is the rate of return that a firm requires when evaluating the expected cash flows from new investments or hiring decisions, capturing both a risk-free rate and a risk premium. Under rational expectations, discount rates rise during recessions as risk premia increase, which reduces the present discounted value of the cash flows expected to be generated by the newly hired worker. This mechanism amplifies unemployment volatility by making the value of job creation sensitive to fluctuations in risk premia (Hall, 2017; Borovickova and Borovička, 2017; Kehoe et al., 2022). Therefore, these models predict that, under rational beliefs, news about future discount rates should be the primary driver of fluctuations in unemployment, not news about productivity or cash flows.

In this paper, I offer an alternative behavioral explanation to the puzzle, that distortions in subjective beliefs can explain the volatility of unemployment fluctuations. I interpret the data through the lens of the Diamond-Mortensen-Pissarides search and matching model of the labor market, while allowing for beliefs to deviate from full information rational expectations. I use the firm's optimal hiring condition in the search model to derive an explicit link that relates the equilibrium job filling rate with subjective expectations of the firm's future cash flows and discount rates. Firms in the model make hiring decisions based on their subjective expectations about the discounted value of a newly hired worker. If deviations from rational expectations lead firms to over-react to news about cash flows, the distortion could provide an additional source of fluctuation to the expected value of job creation.

In particular, fluctuations in subjective cash flow expectations, marked by periods of excessive optimism followed by sharp reversals, can drive boom-bust cycles in labor market activity, even if subjective discount rates remain unchanged. During expansions, firms may become overly optimistic about its future cash flows, leading them to post vacancies and hire more aggressively than justified by fundamentals. But when these beliefs eventually disappoint, the economy can enter a downturn in which pessimism overshoots to the downside, prompting firms to cut back sharply on hiring. Figure 1 illustrates this pattern by showing that surges in unemployment growth during recessions coincide not just with declining cash flow expectations, but with reversals of earlier run-ups in optimism. The subjective expectations series, constructed from IBES analyst survey forecasts of S&P 500 earnings growth, closely tracks these boom-bust cycles.

Figure 1: Unemployment and Subjective Cash Flow Expectations



Notes: Figure plots U.S. unemployment growth Δu_t (left axis) alongside the $h = 5$ year survey forecast of S&P 500 earnings growth $\mathbb{F}_t[\Delta e_{t,t+h}]$ (right axis). Unemployment growth is measured as the 12-month log difference in the U.S. unemployment rate (UNRATE). Survey expectations are annualized S&P 500 earnings growth forecasts $\mathbb{F}_t[\Delta e_{t+h}]$ constructed from IBES median analyst forecasts for the next four fiscal years and long-term growth (LTG). The sample is quarterly from 1983Q1 to 2023Q4. Gray shaded areas indicate NBER recessions.

I quantify the importance of these subjective beliefs along two dimensions. First, I estimate a time-series decomposition of the aggregate job filling rate, which is a key determinant of fluctuations in aggregate unemployment (Shimer, 2012). The decomposition attributes fluctuations in the job filling rate to either subjective expectations of future cash flows or discount rates. Second, I extend this framework to the cross-section, decomposing variation in the hiring rate across book-to-market sorted portfolios using subjective expectations of firms' future discount rates and cash flows. The decompositions I derive from the search model are analogous to the Campbell and Shiller (1988) approximate present value identity for stock market valuation ratios, which attributes fluctuations in price-dividend and price-earnings ratios to changes in expected future cash flows and discount rates.

To isolate the role of belief distortions embedded in subjective beliefs, I compare survey-based subjective expectations against an undistorted benchmark based on machine learning forecasts. The gap between the two measures captures the extent of distortion in subjective beliefs. To measure subjective expectations of cash flows, I use survey forecasts of S&P 500 earnings growth from financial analysts in the IBES database. For subjective discount rates, I use survey forecasts of S&P 500 stock returns from Chief Financial Officers (CFOs). These survey responses proxy for subjective beliefs that inform the hiring and investment decisions of firm managers. As a proxy for rational expectations, I use out-of-sample forecasts from a Long Short-Term Memory (LSTM) neural network. Rational expectations require agents to form beliefs by efficiently processing all available information. A high-dimensional neural network trained on a rich set of macroeconomic and financial variables can approximate this benchmark by learning complex nonlinear relationships without imposing strong parametric assumptions about the underlying data-generating process. To avoid look-ahead bias, the machine forecasts are constructed in real time using only information available at each date.

The results reveal a stark contrast between subjective and rational expectations. In the time series, subjective cash flow expectations at the 5-year horizon account up to 96.7% of variation in the job filling rate, while subjective discount rates play only a limited role at -1.0%. In the cross-section, subjective cash flow expectations at the 5-year horizon explain up to 83.3% of cross-sectional dispersion in hiring, with limited role for subjective discount rates. Under rational expectations, the pattern reverses: discount rates dominate both time-series and cross-sectional variation, with rational discount rates explaining up to 69.1% and 71.6% of variation in time-series and cross-sectional variation in hiring, respectively. This reversal is consistent with asset pricing studies that find subjective cash flow growth expectations, rather than subjective discount rates, to explain a large share of valuation cycles in the stock market (Nagel and Xu, 2021; Bordalo et al., 2024a; De La O et al., 2024). My paper extends these insights to real decisions, showing that the same belief distortions shape both stock market valuations and labor market outcomes.

These belief distortions can have important implications for aggregate unemployment fluctuations. Incorporating subjective cash flow expectations into predictive models of the unemployment rate significantly improves their explanatory power compared to models based solely on rational discount rates. Subjective beliefs can better explain the sharp and persistent spikes in unemployment during downturns. Cross-sectional evidence further shows that firms with more pessimistic subjective cash flow expectations hire less than firms with optimistic expectations. These findings suggest that misperceptions about future cash flows can substantially influence hiring behavior across firms and drive fluctuations in unemployment over the business cycle.

To interpret these findings, I develop a search-and-matching model in which firms engage in constant-gain learning about the long-run mean of cash flow growth and stock price growth. Firms update their subjective expectations at a slow constant learning rate and choose hiring based on their beliefs. Simulations from the model generate patterns consistent with the empirical decomposition. Under subjective beliefs, firms overstate job filling rate fluctuations to persistent shifts in expected cash flows, both in the time-series and the cross-section. Importantly, the model with belief distortions can generate about 74.6% of observed unemployment volatility, a substantial improvement over standard models that underpredict it by an order of magnitude.

Taken together, the results show that labor market fluctuations reflect not only rational changes in discount rates, but also systematic distortions in belief formation. Rational models that emphasize time-varying discount rates may understate the role of subjective cash flow expectations in driving labor market volatility. These findings highlight the need for macroeconomic models to incorporate subjective beliefs more explicitly, and for policy tools to address the behavioral biases they generate.

Related Literature This paper contributes to several strands of literature on unemployment fluctuations, labor markets, asset prices, and expectation formation.

First, it relates to the literature on the unemployment volatility puzzle in search-and-matching models. A central challenge in macroeconomics is to explain why unemployment is highly volatile relative to productivity (Shimer, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Pissarides, 2009; Elsby and Michaels, 2013; Kudlyak, 2014; Chodorow-Reich and Karabarbounis, 2016; Ljungqvist and Sargent, 2017). Traditional search and matching models struggle to generate sufficient volatility in unemployment unless firms' responses to shocks are amplified through mechanisms such as rational expectations of time-varying discount rates (Merz and Yashiv, 2007; Donangelo, 2014; Belo et al., 2014; Favilukis and Lin, 2015; Hall, 2017; Borovickova and Borovička, 2017; Kuehn et al., 2017; Kilic and Wachter, 2018; Mitra and Xu, 2019; Donangelo et al., 2019; Kehoe et al., 2019; Liu, 2021; Kehoe et al., 2022; Belo et al., 2023; Meeuwis et al., 2023). These models assume that firms rationally process information about cash flows and discount rates. My approach of introducing subjective expectations complements these rational models. Belief distortions, particularly about cash flows, can better explain variation in hiring and unemployment, offering an alternative resolution to the unemployment volatility puzzle.

A growing literature embeds non-rational expectations in macro models with labor market frictions (Venkateswaran, 2014; Acharya and Wee, 2020; Mueller et al., 2021; Menzio, 2023; Faberman et al., 2022; Bhandari et al., 2024; Wang et al., 2025). Notably, Bhandari et al. (2024) show that systematic pessimism in households and firms can explain the volatility of unemployment fluctuations. My paper complements their findings by providing direct survey evidence on the content and cyclicality of firm expectations, showing that over-reaction to cash flow news is the main driver of excess unemployment volatility. The cross-sectional analysis in my paper also adds another dimension to belief-driven labor market volatility by showing that firms with more distorted beliefs experience larger swings in hiring.

The empirical analysis of this paper builds on existing survey-based evidence on the empirical properties of firm expectations. Ben-David et al. (2013) document persistent over-optimism in CFO forecasts. Gennaioli et al. (2016) document that extrapolative CFO expectations of earnings growth predict corporate investment. Ma et al. (2020) link systematic biases in managerial forecasts to distortions in firm investment. Coibion et al. (2018) and Candia et al. (2020) find that firm managers' inflation expectations adjust slowly and display substantial dispersion. My paper builds on this work by showing how distortions in survey expectations shape labor markets.

The variance decomposition and learning model in this paper builds on recent work using survey-based expectations to reassess the drivers of asset prices (Timmermann, 1993; Barberis et al., 1998; Chen et al., 2013; Greenwood and Shleifer, 2014; Collin-Dufresne et al., 2016; Adam et al., 2016; Giglio et al., 2021; De La O and Myers, 2021; Nagel and Xu, 2022; Jin and Sui, 2022; De La O et al., 2024; Adam and Nagel, 2023; Bordalo et al., 2024a; Décaire and Graham, 2024). The variance decomposition method adapts the Campbell-Shiller framework (Campbell and Shiller, 1988; Cochrane, 2007), which attributes price-dividend and price-earnings ratio variation to expected cash flows and discount rates. Recent applications of this framework using survey-based expectations have challenged traditional views about the sources of asset price volatility. De La O and Myers (2021) show that subjective expectations of cash flow growth, rather than discount rates, explain most of the variation in price-dividend and price-earnings ratios, challenging standard decompositions that assume rational expectations. Bordalo et al. (2024a) find that over-reaction in long-term earnings growth expectations accounts for a substantial share of aggregate and cross-sectional return predictability. My contribution is to show that a similar decomposition can be adapted to study real decisions by linking asset valuation to hiring through the firm's optimality condition, revealing that the same belief distortions operate in both financial markets and labor markets.

Informed by this literature, I adopt a machine learning approach to measure rational expectations using a dynamic real-time forecasting framework developed in Bianchi et al. (2022) and Bianchi et al. (2024b). It is based on the principle that rational expectations require agents to efficiently use the full set of real-time information available to them. The algorithm uses high-dimensional prediction models estimated on rolling samples of real-time data to produce a benchmark that is free from human cognitive biases and look-ahead bias, while also addressing overfitting and structural change. The method uses tools from machine learning by training LSTM networks with recursive re-estimation and hyperparameter tuning (Gu et al., 2020, Cong et al., 2020, Bybee et al., 2024). The resulting forecasts are fully ex-ante and provide high-dimensional empirical counterparts to rational expectations for evaluating belief distortions.

The rest of the paper proceeds as follows. Section 2 presents a search and matching model with belief distortions and derives a decomposition of the job filling rate. Section 3 describes the data used in the empirical analysis. Section 4 compares the predictive performance of machine and survey forecasts. Section 5 presents the estimated variance decomposition of the aggregate job filling rate. Section 6 presents cross-sectional evidence motivated by a firm-level extension of the baseline model. Section 7 presents evidence that subjective cash flow expectations predict aggregate unemployment and cross-sectional hiring rates. Section 8 introduces a model of constant-gain learning about future earnings that could match the decompositions estimated from the data. Section 9 discusses model extensions and robustness checks. Finally, section 10 concludes.

2 Theoretical Framework

This section develops a search and matching model of the labor market in which firms' expectations about future cash flows and discount rates may be distorted, leading to fluctuations in job filling rates and unemployment. The model builds on the Diamond (1982), Mortensen (1982), and Pissarides (2009) framework but departs from the standard rational expectations assumption, allowing firms' hiring decisions to be influenced by biased subjective beliefs.

Environment Consider a discrete time economy populated by a representative household and a representative firm that hires workers in a frictional labor market. The firm uses labor as a single input to production. The household's population is normalized to one and has a continuum of members, where a fraction L_t are employed and the rest are unemployed $U_t = 1 - L_t$. The household's intertemporal consumption decision gives rise to a stochastic discount factor M_{t+1}.

Each period, the firm posts job vacancies at a cost $\kappa > 0$ to maximize its cum-dividend value of equity. Employment L_t reflects the number of workers at the beginning of period t before any separations or new hires.[1] During the period, a fraction δ_t of employed workers separate, while the firm posts vacancies V_t to search for unemployed workers U_t. Matches are formed at the end of period t according to a matching function $m(U_t, V_t)$, with job filling rate $q_t \equiv m(U_t, V_t)/V_t$ and job finding rate $f_t \equiv m(U_t, V_t)/U_t$. These new hires enter employment at the start of period $t + 1$, so employment L_t and unemployment $U_t = 1 - L_t$ evolve according to the law of motion:

$$L_{t+1} = (1 - \delta_t)L_t + q_t V_t \tag{1}$$

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{2}$$

where $\theta_t = V_t/U_t$ denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

Firm's Technology and Cash Flow The firm produces output using a Cobb-Douglas production function with labor L_t as it input. Period cash flows (earnings) E_t are given by:

$$E_t = A_t L_t^{\alpha} - W_t L_t - \kappa V_t \tag{3}$$

where $A_t L_t^{\alpha}$ is gross output with total factor productivity A_t and returns to scale parameter α. $W_t L_t$ total wage payments, and κV_t vacancy posting costs. I assume that the household owns the equity of the firm and the firm pays out all of its earnings E_t as dividends (Petrosky-Nadeau et al., 2018), and that the firm's manager has access to complete markets so that the return to hiring equals the stock market return in equilibrium (Cochrane, 1991).

[1] I adopt an end-of-period matching convention following Petrosky-Nadeau et al. (2018). See Hansen et al. (2005) and Kogan and Papanikolaou (2012) for similar conventions applied for the q theory of investment.

Firm's Problem The firm chooses vacancy postings V_t to maximize the present discounted value of future cash flows. The firm's value function $\mathcal{V}$ satisfies the Bellman equation:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \{E_t + \mathbb{F}_t[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})]\} \tag{4}$$

subject to the employment accumulation equation (1). $\mathbb{F}_t[\cdot]$ is the firm's subjective expectations conditional on information available at the beginning of period t.[2] These beliefs may depart from rational expectations $\mathbb{E}_t[\cdot]$, with the nature and magnitude of the deviation disciplined using survey data. M_{t+1} is the stochastic discount factor that prices the firm's cash flows. The firm does not observe this discount factor directly and needs to form expectations about it by forecasting the household's marginal utility of consumption (Venkateswaran, 2014).

Hiring Condition Under search frictions, hiring is forward-looking investment. The firm's optimal hiring decision equates the marginal cost of posting a vacancy with the expected discounted marginal value of employment:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \underbrace{\mathbb{F}_t\left[M_{t+1}\frac{\partial \mathcal{V}(A_{t+1}, L_{t+1})}{\partial L_{t+1}}\right]}_{\text{Expected discounted value of hiring}} \tag{5}$$

The left side represents the expected cost of hiring one additional worker, accounting for the probability q_t that a posted vacancy will be filled. The right side captures the expected discounted value of the marginal worker, incorporating both the firm's subjective beliefs about future conditions and the appropriate discount rate for valuing risky cash flows.[3] Subjective distortions in beliefs can thus shift the perceived value of hiring through $\mathbb{F}_t[\cdot]$ and affect equilibrium job filling rates, which in turn affects unemployment through its law of motion in equation (2). Assuming constant returns to scale $\alpha = 1$, the marginal value of hiring coincides with its average value:

$$\frac{\partial \mathcal{V}(A_{t+1}, L_{t+1})}{\partial L_{t+1}} = \frac{\mathcal{V}(A_{t+1}, L_{t+1})}{L_{t+1}} \tag{6}$$

Define the firm's ex-dividend market value as $P_t \equiv \mathbb{F}_t[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})]$ to derive a direct link between the job filling rate and the firm's market value per worker:

$$\frac{\kappa}{q_t} = \frac{P_t}{L_{t+1}} \tag{7}$$

[2] I use the term "firm's beliefs" as shorthand to refer to the expectations held by decision makers within firms (Coibion et al., 2018; Candia et al., 2020).

[3] The hiring equation is the labor market analogue of the optimality condition for physical capital in the q theory of investment (Hayashi, 1982), where the upfront cost of hiring κ/q_t is analogous to Tobin's marginal q and the separation rate δ_{t+1} is analogous to the depreciation rate (Borovickova and Borovička, 2017). See Lettau and Ludvigson (2002) and Kogan and Papanikolaou (2012) for a similar log-linearization applied for the q theory of physical capital investment.

where employment L_{t+1} is determined at the end of date t under our timing convention from equation (1). Take logarithms, rearrange terms, and expand the price-employment ratio P_t/L_{t+1}:

$$\log q_t = \log \kappa - \log\left(\frac{P_t}{E_t}\right) - \log\left(\frac{E_t}{L_{t+1}}\right) \tag{8}$$

Defining log price-earnings $pe_t = \log(P_t/E_t)$ and earnings-employment $el_t = \log(E_t/L_{t+1})$:

$$\log q_t = \log \kappa - pe_t - el_t \tag{9}$$

Log-linear Approximation of Price-Earnings Ratio To decompose the job filling rate into economically meaningful components, I apply the Campbell and Shiller (1988) present value identity to the price-earnings ratio. Log-linearize the price-earnings ratio $pe_t \equiv \ln(P_t/E_t)$ around its long-run mean $\overline{pe}$ to obtain the approximate relationship:

$$pe_t = c_{pe} - r_{t+1} + \Delta e_{t+1} + \rho pe_{t+1} \tag{10}$$

where c_{pe} is a linearization constant, $\rho = \exp(\overline{pe})/(1+\exp(\overline{pe})) \approx 0.98$ is the time discount factor from the log-linearization, $r_{t+1} = \log((P_{t+1}+E_{t+1})/P_t)$ represents the stock return assuming that the firm pays out its earnings as dividends, and Δe_{t+1} denotes earnings growth. This identity holds approximately even when earnings can differ from dividends because the payout ratio term that will be introduced to the identity is quantitatively small and can be approximated as a constant (De La O et al., 2024).[4] Substituting recursively for the next h periods yields the Campbell and Shiller (1988) present value identity:

$$pe_t = \sum_{j=1}^{h} \rho^{j-1} c_{pe} - \sum_{j=1}^{h} \rho^{j-1} r_{t+j} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j} + \rho^h pe_{t+h} \tag{11}$$

Decomposition of Job Filling Rate Substitute log-linearized price-earnings from (11) into the hiring equation from (9) to obtain a decomposition of the job filling rate q_t:

$$\underbrace{\log q_t}_{\text{Job Filling Rate}} = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{\substack{\text{Discount Rate} \\ \equiv\ r_{t,t+h}}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}\right]}_{\substack{\text{Cash Flow} \\ \equiv\ e_{t,t+h}}} - \underbrace{\rho^h pe_{t+h}}_{\substack{\text{Future Price-Earnings} \\ \equiv\ pe_{t,t+h}}} \tag{12}$$

where $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The job filling rate has been decomposed into three forward-looking components: the present value of future discount rates $r_{t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$, cash flows $e_{t,t+h} \equiv el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$, and price-earnings ratio $pe_{t,t+h} \equiv \rho^h pe_{t+h}$. The cash flow component consists of the current earnings-employment ratio el_t, which captures short-term

[4] See Appendix Section B for a derivation.

fluctuations in cash flows, and $j = 1, \ldots, h$ period ahead earnings growth Δe_{t+j}, which captures news about future cash flows.

Since equation (12) holds both ex-ante and ex-post, it can be evaluated under either subjective or rational expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their ex-ante subjective expectation $\mathbb{F}_t[\cdot]$:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] \tag{13}$$

The equation implies that the job filling rate is high when firms subjectively expect future returns to be high, expected cash flows to be low, or both. Alternatively, the *rational decomposition* replaces ex-post realizations of future outcomes with their ex-ante rational expectation $\mathbb{E}_t[\cdot]$:

$$\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}] \tag{14}$$

Comparing these decompositions can quantify how belief distortions affect the job filling rate.

Estimation The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current job filling rate. For the subjective decomposition, demean each variable in equation (13), multiply both sides by the current log job filling rate $\log q_t$, and take the sample average:

$$Var\left[\log q_t\right] = Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right] \tag{15}$$

where $Var[\cdot]$ and $Cov[\cdot]$ are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by $Var\left[\log q_t\right]$ to decompose its variance:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{16}$$

The left-hand side represents the full variability in job filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or future price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, and $\mathbb{F}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively.

Finally, the decomposition under rational expectations can be estimated similarly based on equation (14) by replacing the subjective expectation $\mathbb{F}_t[\cdot]$ with its rational counterpart $\mathbb{E}_t[\cdot]$. This comparison allows us to assess the role of belief distortions in explaining labor market dynamics and determine whether firms systematically mis-perceive economic conditions when making hiring decisions. Although the variance decomposition does not necessarily capture causal relationships, it has the advantage of not requiring the researcher to take a stand on the deep determinants of job filling rates because the evolution of discount rates and cash flows summarize the combined effects of these deep determinants.

3 Data

This section describes the data used to estimate the time-series and cross-sectional variance decompositions. For each outcome variable, I use survey forecasts to measure subjective expectations $\mathbb{F}_t[\cdot]$ and machine learning forecasts to measure rational expectations $\mathbb{F}_t[\cdot]$. The final estimation sample is quarterly and spans 2005Q1 to 2023Q4. See Appendix C for more details.

Job Filling Rate Vacancies V_t is measured using JOLTS job openings starting 2000:12 and help-wanted index for earlier periods (Barnichon, 2010). Unemployment U_t is measured from the BLS unemployment series (UNEMPLOY). The job filling rate q_t is defined as the share of filled vacancies out of unemployment:

$$q_t = \frac{f_t V_t}{U_t}$$

The job finding rate f_t is the share of unemployed workers that find jobs within the period:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}}$$

where U_t^s is short-term unemployment less than 5 weeks (UEMPLT5). I construct the variables at a monthly frequency, time-aggregate to quarterly averages, and detrend using an HP filter with a smoothing parameter of 10^5 to ensure stationarity (Shimer, 2005). Labor market tightness is defined as the vacancy-to-unemployment ratio, $\theta_t \equiv V_t/U_t$. The job separation rate δ_t uses the corresponding series from JOLTS.

Employment Employment $L_{i,t}$ is measured using annual total employee counts (EMP) for S&P 500 firms from the CRSP/Compustat Merged Annual Industrial Files. I aggregate the firm-level employment data to construct a total employment series L_t for the S&P 500. I interpolate the annual series to a quarterly frequency by using quarterly averages of the fitted values from regressing annual S&P 500 employment on the monthly BLS nonfarm payrolls.

Earnings Quarterly earnings for the S&P 500 are sourced from IBES street earnings per share (EPS) data that starts in 1983Q4 (Hillenbrand and McCarthy, 2024). Street earnings, which serve as the forecast target for IBES analysts, differ from standard GAAP earnings by excluding discontinued operations, extraordinary charges, and other non-operating items. This adjustment makes street earnings a cleaner measure of recurring performance and a more relevant proxy for expected cash flows. As shown by Hillenbrand and McCarthy, 2024, street earnings exhibit less transitory volatility and are more informative about firm fundamentals and valuation than standard earnings measures.

To construct subjective expectations of future cash flows, I use survey forecasts of S&P 500 earnings from the IBES database (De La O and Myers, 2021; Bordalo et al., 2019). IBES provides firm-level forecasts from financial analysts, which I aggregate to form market-wide earnings expectations for the S&P 500. These forecasts reflect the views of professionals who actively track firms for investment research and have strong reputational incentives to report them accurately, as they are not anonymous (Cooper et al., 2001; De La O et al., 2024). Prior research shows that these forecasts are widely followed by market participants and are priced into asset values, supporting their use as proxies for subjective expectations (Kothari et al., 2016).[5]

IBES provides monthly median analyst forecasts for earnings per share (EPS) at one through four year horizons, as well as long-term growth (LTG) forecasts.[6] One through four year ahead forecasts of annual log earnings growth $\mathbb{F}_t[\Delta e_{t+h}]$ for $h = 1, 2, 3, 4$ are constructed as log differences between level forecasts from adjacent horizons. For the five year horizon $\mathbb{F}_t[\Delta e_{t+5}]$, I interpret the LTG forecast as the expected log growth in earnings from year four to five (Bianchi et al., 2024b). The sample spans 1982 to 2021 at a monthly frequency, which are time-aggregated to quarterly averages. The forecasts cover approximately 80% of total market capitalization, providing broad coverage of U.S. public firms.

Stock Returns Stock returns are measured using monthly Center for Research in Security Prices (CRSP) value-weighted returns with dividends (VWRETD). Annualized cumulative h-year log stock returns are compounded from monthly returns.

For survey expectations of aggregate stock returns, I use the quarterly CFO survey from 2001Q4 to 2023Q4. The CFO survey is a quarterly survey that asks respondents about their expectations for the S&P 500 return over the next 12 months and 10 years ahead. For intermediate horizons between 1 and 10 years, I interpolate linearly between the 1 and 10 year ahead forecasts. The CFO survey panel includes firms that range from small operations to Fortune 500 companies across all major industries. Respondents include chief financial officers, owner-operators, vice presidents and directors of finance, and others with financial decision-making roles.

For survey expectations of firm-level stock returns, I use survey data on stock price targets (De La O et al., 2024). Specifically, I proxy for stock return expectations by constructing expected price growth from IBES 12-month median price targets and Value Line 3–5 year median price targets. I interpret the Value Line price targets as a 5 year ahead forecast and interpolate linearly to impute expectations for intermediate horizons between 1 and 5 years.

Price-Earnings Ratio The current price-earnings ratio is defined as $PE_t \equiv P_t/E_t$, where P_t is the end-of-quarter S&P 500 stock price index and E_t denotes quarterly total earnings for

[5] As a robustness check, Appendix Table A.7 considers using earnings forecasts from the CFO survey, which reflect managerial expectations. The two series have a relatively strong correlation of 0.60 at the 1-year horizon, suggesting that analyst and managerial beliefs are broadly aligned.

[6] Long-Term Growth (LTG) is defined in IBES as the "expected annual increase in operating earnings over the company's next full business cycle. These forecasts refer to a period of between three to five years."

the S&P 500. I construct subjective expectations of log price-earnings $\mathbb{F}_t[pe_{t+h}]$ by applying the Campbell and Shiller (1988) approximate present value identity (De La O and Myers, 2021):

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \mathbb{F}_t[\Delta e_{t+j}] - \mathbb{F}_t[r_{t+j}])$$

where expected returns and earnings growth come from the CFO survey and IBES, respectively.

Earnings-Employment Ratio The current earnings-employment ratio is defined as $EL_t \equiv E_t/L_{t+1}$, where E_t denotes quarterly total earnings for the S&P 500 and L_{t+1} denotes total employment for the S&P 500 at the beginning of period $t+1$. I measure L_{t+1} using employment levels as of the end of each quarter, which aligns with the timing conventions of the search model in Section 2 while ensuring that the measure is known to firms by the end of period t.

Machine Learning Forecasts For each survey forecast, I construct the corresponding machine learning forecast using a Long Short-Term Memory (LSTM) neural network:

$$\mathbb{E}_t[y_{t,t+h}] = G(\boldsymbol{\mathcal{X}}_t, \boldsymbol{\beta}_{h,t})$$

where $y_{t,t+h}$ denotes the outcome variable (stock returns or earnings growth) to be predicted h periods ahead of time t. $\boldsymbol{\mathcal{X}}_t$ is a large input dataset of macroeconomic, financial, and textual predictors (Appendix D.2). The input dataset also includes the survey forecast of y, allowing the machine to extract intangible information and correct for potential biases embedded in the survey responses. $G(\boldsymbol{\mathcal{X}}_t, \boldsymbol{\beta}_{h,t})$ denotes predicted values from Long Short-Term Memory (LSTM) neural networks that can be represented by a potentially high dimensional set of parameters $\boldsymbol{\beta}_{h,t}$. The parameters are estimated using a dynamic algorithm from Bianchi et al. (2022, 2024b) that takes into account the data-rich environment in which firms operate in (Appendix D.1).

To obtain more granular measures of undistorted expectations with a cross-sectional dimension across firms, I construct analogous machine learning forecasts at the portfolio level:

$$\mathbb{E}_t[y_{i,t,t+h}] = G(\boldsymbol{\mathcal{X}}_{i,t}, \boldsymbol{\beta}_{i,h,t})$$

where $y_{i,t,t+h}$ is the outcome to be predicted for portfolio i. The predictor set $\boldsymbol{\mathcal{X}}_{i,t} = \boldsymbol{\mathcal{X}}_t \otimes \boldsymbol{\mathcal{C}}_{i,t}$ augments the aggregate predictors $\boldsymbol{\mathcal{X}}_t$ with firm-level characteristics $\boldsymbol{\mathcal{C}}_{i,t}$ (Gu et al., 2020, Appendix D.3). Firms are sorted into five value-weighted book-to-market portfolios, with predictor variables aggregated to the portfolio level using market cap weights (De La O et al., 2024).

4 Forecasting Performance

Accuracy of Machine Learning vs. Survey Forecasts To assess whether survey respondents systematically misweight relevant information, Figure 2 evaluates the out-of-sample accuracy of machine learning forecasts relative to survey forecasts for discount rates $r_{t,t+h}$ and cash flows $e_{t,t+h}$, as defined in equation (12). These variables are factors that can influence the job filling rate through the firm's optimal hiring decision in the search model. I measure the relative predictive performance using the ratio of mean-square-forecast-error of the machine ($MSE_{\mathbb{E}}$) over that of the survey ($MSE_{\mathbb{F}}$). The out-of-sample testing period spans 2005Q1 to 2023Q4.

Figure 2 shows that machine learning forecasts consistently outperform survey forecasts across all variables and horizons, with MSE ratios well below one. The performance gap widens with forecast horizon, indicating larger belief distortions in long-term expectations. The machine outperforms the survey for both aggregate S&P 500 level and portfolio level forecasts across book-to-market sorted groups, suggesting that belief distortions affect not only aggregate expectations but also the dispersion of beliefs across firms.

These findings suggest that survey expectations about factors that influence hiring decisions systematically deviate from an unbiased benchmark, both in the time-series and the cross-section. If survey respondents were rational in forming their beliefs, their forecasts would have performed at least on par with the machine.[7] The superior performance of the machine also highlights its ability to process a large amount of real-time data efficiently and objectively, supporting its use as a reliable benchmark of undistorted beliefs.

Predictability of Survey Forecast Errors To assess whether survey expectations systematically deviate from rational expectations, panel (1) of Table 1 estimates Coibion and Gorodnichenko (2015) regressions of survey forecast errors on survey forecast revisions:

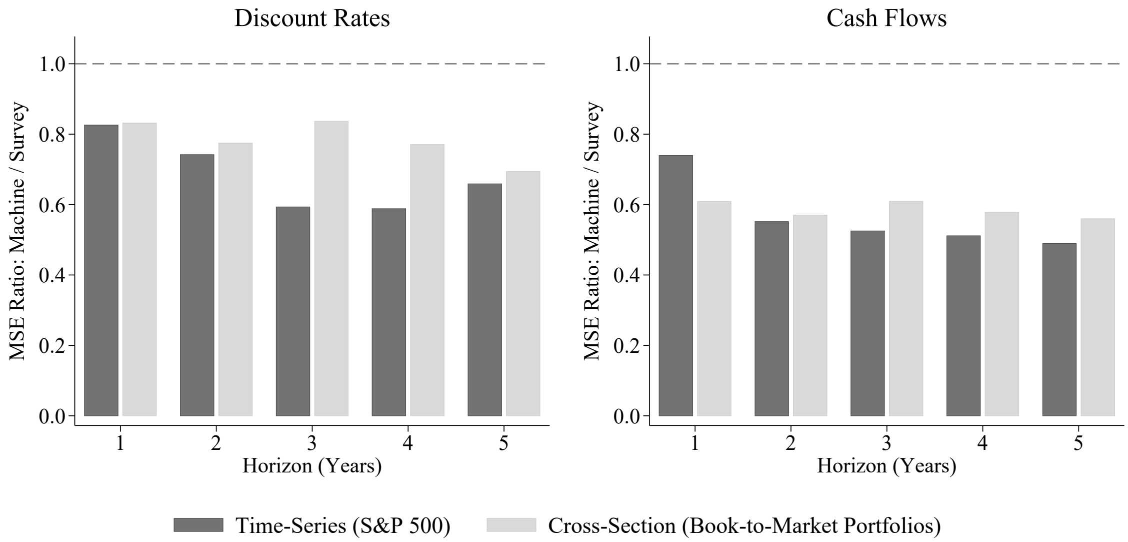
$$y_{t,t+h} - \mathbb{F}_t[y_{t,t+h}] = \beta_0 + \beta_1[\mathbb{F}_t[y_{t,t+h}] - \mathbb{F}_{t-1}[y_{t,t+h}]] + \beta_2\mathbb{F}_{t-1}[y_{t,t+h}] + \varepsilon_t \tag{17}$$

The results reveal predictable forecast errors in subjective beliefs. For cash flow expectations, the coefficients on forecast revisions are negative, ranging from -0.263 at the one-year horizon to -0.968 at five years. These results indicate over-reaction. Upward revisions in survey forecasts are followed by negative forecast errors, suggesting that survey respondents respond too strongly to negative earnings news and generate overly pessimistic forecasts. For discount rate expectations, the coefficients are negative and significant at longer horizons, with -0.998 at five years, indicating that respondents over-react to discount rate news.[8]

[7]One natural question is whether survey respondents report rational, risk-neutral expectations rather than truly subjective beliefs. However, estimates of risk premia typically range from 5-10% annually (Adam et al., 2021), which is insufficient to explain the 15-30% deterioration in MSE ratios observed in Figure 2. The magnitude and persistence of forecast errors across horizons instead point to behavioral biases (e.g., extrapolation) rather than rational risk compensation.

[8]For future price-earnings expectations, the coefficient is also negative at -0.919, showing that price-earnings forecasts over-react to news. Through the Campbell-Shiller present-value identity, this suggests cash flow over-reaction dominates discount rate over-reaction in driving price-earnings forecasts. While both discount rates and cash flow expectations over-react, the cash flow component (which enters positively in the identity) has a stronger influence on price-earnings ratio variation than the discount rate component (which enters negatively), leading to net over-reaction in price-earnings forecasts.

Figure 2: Accuracy of Machine Learning vs. Survey Forecasts



Notes: Figure plots $MSE_{\mathbb{E}}/MSE_{\mathbb{F}}$, the ratio of mean squared forecast errors between machine learning and survey forecasts. Lower values indicate greater accuracy of the machine learning forecast. $MSE_{\mathbb{E}}$ and $MSE_{\mathbb{F}}$ refer to out-of-sample forecast errors from machine and survey forecasts, respectively. The out-of-sample testing period is 2005Q1–2023Q4. Dark bars correspond to aggregate time-series forecasts for the S&P 500; light bars correspond to cross-sectional forecasts across five book-to-market sorted portfolios. The forecast target $y_{t,t+h}$ is the present discounted value of discount rates $r_{t,t+h}$ and cash flows $e_{t,t+h}$, as defined in equation (12). Time-series survey forecasts $\mathbb{F}_t$ come from the CFO survey (discount rates) and IBES (cash flows). Cross-sectional survey forecasts $\mathbb{F}_t$ come from IBES (discount rates and cash flows). Time-series and cross-sectional machine learning expectations $\mathbb{E}_t$ are generated using a Long Short-Term Memory (LSTM) model trained in real time on macroeconomic, financial, and textual data.

Panel (2) repeats the analysis using machine learning forecasts in place of survey expectations. In contrast to the strong predictability in survey forecast errors, the coefficients on machine forecast revisions are small and statistically insignificant at all horizons, with values near zero (0.096 for discount rates, -0.070 for cash flows, -0.150 for future price-earnings at 5-year horizon). This lack of systematic forecast error is consistent with the behavior of rational expectations, under which forecast errors should be unpredictable.

Panels (3) and (4) of Table 1 presents a complementary analysis using cross-sectional regressions of forecast errors for book-to-market sorted portfolios. These regressions include portfolio and time fixed effects, implying that the identifying variation comes from revisions in expectations that are idiosyncratic to each portfolio. At the five year horizon, the coefficients on survey forecast revisions are large and negative for both discount rates (-0.730) and cash flows (-0.715), indicating that survey respondents over-react to portfolio-specific news. Upward revisions in a given portfolio's forecasts beyond the average are associated with subsequent forecast errors in the opposite direction.

In contrast, the corresponding coefficients for machine forecasts are small and close to zero (-0.051 for discount rates and 0.033 for cash flows), suggesting that machine expectations do not systematically over-react to idiosyncratic information. These results reinforce the conclusion that survey expectations exhibit predictable bias even at the cross-sectional level, while machine expectations remain consistent with a rational benchmark.

Table 1: Predictability of Survey Forecast Errors

(a) Aggregate Forecasts

Horizon h (Years)	1	2	3	4	5
(1) Survey: $y_{t,t+h} - \mathbb{F}_t[y_{t,t+h}] = \beta_0 + \beta_1[\mathbb{F}_t[y_{t,t+h}] - \mathbb{F}_{t-1}[y_{t,t+h}]] + \beta_2\mathbb{F}_{t-1}[y_{t,t+h}] + \varepsilon_t$					
Discount Rates	-0.581^{***}	-0.646^{**}	-0.658^{***}	-0.681^{***}	-0.998^{***}
t-stat	(-2.916)	(-2.360)	(-2.821)	(-3.047)	(-2.758)
Cash Flows	-0.263	-0.463^{***}	-0.801^{***}	-0.833^{***}	-0.968^{***}
t-stat	(-1.413)	(-3.793)	(-5.682)	(-5.898)	(-8.242)
N	76	76	76	76	76
(2) Machine: $y_{t,t+h} - \mathbb{E}_t[y_{t,t+h}] = \beta_0 + \beta_1[\mathbb{E}_t[y_{t,t+h}] - \mathbb{E}_{t-1}[y_{t,t+h}]] + \beta_2\mathbb{E}_{t-1}[y_{t,t+h}] + \varepsilon_t$					
Discount Rates	0.057	-0.005	0.109	0.010	0.096
t-stat	(0.249)	(-0.036)	(0.710)	(0.114)	(0.973)
Cash Flows	-0.064	-0.114	-0.056	-0.133	-0.070
t-stat	(-0.507)	(-1.387)	(-1.255)	(-1.563)	(-1.594)
N	76	76	76	76	76

(b) Cross-Sectional Forecasts

Horizon h (Years)	1	2	3	4	5
(3) Survey: $y_{i,t,t+h} - \mathbb{F}_t[y_{i,t,t+h}] = \beta_1[\mathbb{F}_t[y_{i,t,t+h}] - \mathbb{F}_{t-1}[y_{i,t,t+h}]] + \beta_2\mathbb{F}_{t-1}[y_{i,t,t+h}] + \alpha_i + \alpha_t + \varepsilon_{i,t}$					
Discount Rates	-0.837^{***}	-0.853^{***}	-0.696^{**}	-0.845^{***}	-0.730^{***}
t-stat	(-5.883)	(-6.133)	(-2.354)	(-5.376)	(-4.124)
Cash Flows	-0.665^{**}	-0.581^{***}	-0.588^{***}	-1.092^{***}	-0.715^{***}
t-stat	(-2.297)	(-2.899)	(-3.690)	(-7.989)	(-5.605)
N	380	380	380	380	380
(4) Machine: $y_{i,t,t+h} - \mathbb{E}_t[y_{i,t,t+h}] = \beta_1[\mathbb{E}_t[y_{i,t,t+h}] - \mathbb{E}_{t-1}[y_{i,t,t+h}]] + \beta_2\mathbb{E}_{t-1}[y_{i,t,t+h}] + \alpha_i + \alpha_t + \varepsilon_{i,t}$					
Discount Rates	0.038	-0.041	0.025	-0.151	-0.051
t-stat	(0.838)	(-0.744)	(0.806)	(-1.301)	(-0.707)
Cash Flows	-0.018	-0.004	0.027	0.025	0.033
t-stat	(-1.055)	(-0.143)	(1.028)	(0.760)	(1.022)
N	380	380	380	380	380

Notes: Table reports regression coefficients from forecast error regressions of the form: forecast error regressed on the revision in the forecast and the lagged forecast level. The forecast target is either the present discounted value of discount rates $r_{t,t+h}$ or cash flows $e_{t,t+h}$ over horizon h, as defined in equation (12). Panel (a) presents results using aggregate time-series forecasts for the S&P 500: (1) survey forecast errors on survey forecast revisions, and (2) machine forecast errors on machine forecast revisions. Panel (b) presents results from cross-sectional regressions of forecast errors on forecast revisions and lagged forecast levels for five portfolios sorted by book-to-market ratios, including portfolio and time fixed effects. Specifications (3) and (4) report cross-sectional results based on survey-based and machine-based forecasts, respectively. Time-series survey forecasts $\mathbb{F}_t$ come from the CFO survey (discount rates) and IBES (cash flows). Cross-sectional survey forecasts $\mathbb{F}_t$ come from IBES (discount rates and cash flows). Time-series and cross-sectional machine learning expectations $\mathbb{E}_t$ are generated using a Long Short-Term Memory (LSTM) model trained in real time on macroeconomic, financial, and textual data. The sample covers quarterly data from 2005Q1 to 2023Q4. All t-statistics are Newey-West corrected with 4 lags in Panel (a), two-way clustering by portfolio and quarter in Panel (b). Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5 Time-Series Decomposition of the Job Filling Rate

The superior forecasting performance of machine learning over survey forecasts suggests the presence of systematic distortions in subjective expectations. This section quantifies how those distortions affect hiring behavior by estimating the contribution of discount rate and cash flow expectations to fluctuations in the aggregate job filling rate. Based on the search model in Section 2, the job filling rate q_t can be decomposed as:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{18}$$

where $\mathbb{F}_t[\cdot]$ represents subjective expectations measured with survey forecasts. $r_{t,t+h}$, $e_{t,t+h}$, and $pe_{t,t+h}$ are h-year present discounted values of future discount rates, cash flows, and price-earnings ratios (terminal value), respectively, as defined in equation (12). To quantify the contribution of each term, I estimate variance decompositions by regressing each forecasted component on $\log q_t$.

A similar decomposition under rational expectations can be estimated by replacing the survey forecast $\mathbb{F}_t[\cdot]$ with its machine learning counterpart as a proxy for rational beliefs $\mathbb{E}_t[\cdot]$. Comparing the decompositions implied by subjective and rational expectations can highlight the role of belief distortions, which I define as the gap between the survey and machine forecasts: $\mathbb{F}_t - \mathbb{E}_t$.

Rational Expectations Figure 3 presents the variance decomposition of the job filling rate, with more detailed statistics reported in Table A.3. The figure shows that, under rational beliefs, discount rate news is the dominant driver of variation in job filling rates. At the five-year horizon, rational discount rates explain 69.1% of the variation in job filling rates, while rational cash flow news accounts for 6.6%.[9] Consistent with the predictions of the search and matching model, higher job filling rates are associated with higher discount rates and lower expected cash flows. The contribution from terminal price-earnings ratios still remains sizable at the five-year horizon, accounting for 20.1%.[10] The combined contribution from the three components sum to 95.8% at the five-year horizon, a value reasonably close to 100.0% suggesting that the decomposition is empirically accurate despite being estimated freely without imposing this constraint.

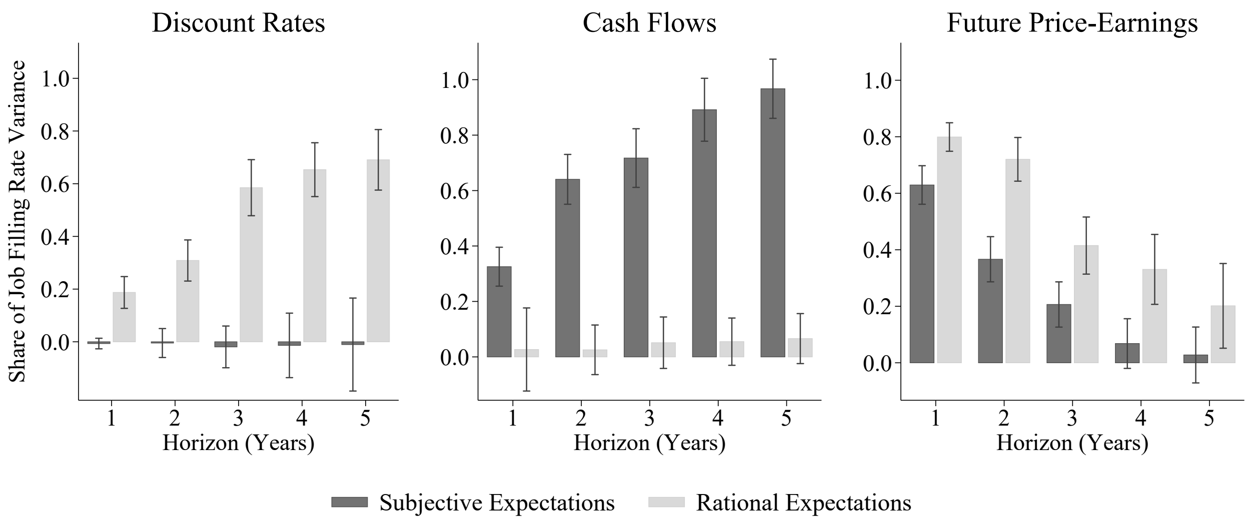
These findings are consistent with with predictions from rational search-and-matching models that emphasize time-varying risk premia. The large contribution from discount rate news is consistent with rational models that introduce time-varying discount rates to explain unemployment fluctuations (Hall, 2017). The increasing importance of discount rate news at longer

[9] First-differenced estimates in Figure A.3 show similar patterns, with rational discount rates explaining 58.7% and cash flows explaining only 10.0% of job filling rate variation. Figure A.2 uses a VAR to extend the decomposition to the infinite horizon, showing that rational discount rates explain 78.1% of job filling rate variation.

[10] Hyatt and Spletzer (2016) document that about half of U.S. workers have job tenures exceeding five years, reflecting the prevalence of long-term employment relationships. Despite relatively long job tenures, time discounting and mean-reversion in cash flows could limit the variance contribution of long-horizon cash flows.

horizons is consistent with rational models that match observed fluctuations in unemployment by modeling hiring as a risky investment with long-duration returns (Kehoe et al., 2022). On the relative importance of risk-free rates and risk premia, Figure A.7 shows that rational risk-free rate expectations explain less than 5% of the variation in the job filling rate. This implies that the explanatory power of discount rate news is driven primarily by the risk premium component, consistent with rational models of labor markets with time-varying risk premia (Borovickova and Borovička, 2017). Finally, the small rational cash flow component aligns with the unemployment volatility puzzle, as Shimer (2005) showed that standard search models without time-varying discount rates cannot generate enough unemployment volatility from productivity shocks, which would mainly be reflected in the cash flow component.

Figure 3: Time-Series Decomposition of the Job Filling Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components from the time-series decomposition of the U.S. aggregate job filling rate. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. Subjective expectations $\mathbb{F}_t$ are constructed from CFO survey forecasts (discount rates) and IBES analyst forecasts (cash flows). Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. x-axis denotes the forecast horizon h. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Subjective Expectations On the other hand, Figure 3 reveals a striking reversal under subjective expectations. At the five-year horizon, subjective cash flow news explains 96.7% of the variation in job filling rates, while subjective discount rate news accounts for only -1.0%.[11] These results suggest that firms systematically over-estimate the importance of cash flows and under-estimate the importance of discount rates when hiring workers. The direction of the cash flow effect aligns with rational expectations, but the discount rate effect does not. The negative sign

[11] First-differenced estimates in Figure A.3 show similar results, with subjective cash flows explaining 90.6% and discount rates explaining only -1.3% of the job filling rate. Figure A.2 uses a VAR to extend the decomposition to the infinite horizon, showing that subjective cash flow expectations explain 95.4% of job filling rate variation.

on the discount rate component indicates that survey respondents predict lower future returns during recessions, contrary to a rational forecast. Since the subjective discount rate component is small and insignificant, belief distortions appear to mainly reflect errors in magnitudes.

The contribution from the terminal price-earnings ratio falls with horizon and is negligible by year five (2.8%), in contrast to the higher terminal value assigned under rational expectations (20.1%). This implies that subjective beliefs place excessive weight on near-term cash flows relative to long-run fundamentals. Finally, the three components sum to 98.5% at the five-year horizon, showing that survey expectations are internally consistent and the model's approximation is reasonablly accurate. The smaller approximation residual under subjective expectation ($98.5 - 100.0 = 1.5\%$ vs. $100.0 - 95.8 = 4.2\%$) suggests that allowing for subjective beliefs can improve the model's ability to explain job filling rate fluctuations more accurately, with any remaining gap likely due to measurement error in survey data (e.g., Ma et al., 2020).

Compared to the rational benchmark, the implied over-reaction to cash flow news is substantial. Low job filling rates during expansions are associated with a significant disappointment in future cash flows. Defining the belief distortion as the difference between subjective and rational expectations $\mathbb{F}_t - \mathbb{E}_t$, the estimates imply that, at the five-year horizon, $96.7\% - 6.6\% = 90.1\%$ of variation in job filling rates can be attributed to the fact that the job filling rate predicts distortions in cash flows expectations with a significant positive relationship (Table A.4). These distortions capture systematic inefficiencies or behavioral biases in survey respondents' subjective beliefs that the machine learning model could have identified ex-ante.

Discussion Although the decomposition does not necessarily estimate causal relationships, it can account for possible sources of variation in the job filling rate. A large estimate for subjective cash flow news means that, whatever shocks drive the job filling rate, they must have a larger impact on subjective cash flow expectations than subjective discount rates. Under rational expectations, by contrast, firms correctly interpret those same shocks as signals about future risk compensation embedded in discount rates. This divergence points to belief distortions as a key source of job filling rate fluctuations. By over-reacting to perceived changes in future cash flows, firms may cut hiring and vacancies too sharply during downturns, amplifying unemployment volatility beyond what rational models predict.

Several robustness checks confirm this interpretation. The patterns persist when comparing survey-based subjective expectations against risk-neutral expectations extracted from futures prices, confirming that the observed distortions reflect genuine departures from rational belief formation rather than respondents merely reporting forecasts under a rational risk-neutral measure (Figure A.8). At the five-year horizon, cash flow expectations explain 96.7% of the variation in job filling rates under subjective beliefs, compared to just 59.6% under risk-neutral expectations, with the gap between the two capturing the extent of over-reaction in subjective beliefs.

Additionally, extending the baseline model to introduce financial constraints does not overturn the over-reaction in subjective cash flow expectations, suggesting that belief distortions rather than financial frictions drive these hiring patterns (Figure A.10).

While Table 1 has shown subjective discount rate forecasts to exhibit over-reaction, their contribution to the variance decomposition of job filling rates remains small in Figure 3. This can be reconciled by the fact that subjective discount rate expectations display relatively little time-series variation, so even biased revisions have limited impact on hiring decisions. In contrast, subjective cash flow expectations vary much more over time and across firms, making them the primary driver of belief-driven fluctuations in hiring. Figure A.1 illustrates this point visually by showing that subjective expectations exhibit excessive cyclicality in cash flow forecasts and muted responses in discount rate forecasts compared to their machine-based counterparts.

The large contribution of subjective cash flows in shaping hiring decisions is consistent with models that introduce non-rational expectations about earnings growth to account for fluctuations in asset prices (Bordalo et al., 2024a; Bianchi et al., 2024b) and the business cycle (Bordalo et al., 2024b). This parallel implies that the belief distortions known to influence asset valuations can also extend to real economic behavior through the labor market. The increasing contribution at longer horizons is consistent with behavioral models in which the over-reaction grows with the forecast horizon (Bordalo et al., 2020; Bianchi et al., 2024a; Augenblick et al., 2024).

On the other hand, the small and negative contribution of subjective discount rates is consistent with existing survey evidence showing that subjective return expectations are acyclical (Nagel and Xu, 2022) or even procyclical (Greenwood and Shleifer, 2014; Adam et al., 2016), contrary to the countercyclical discount rate variation implied by rational models (Cochrane, 2017). In standard asset pricing models, discount rates reflect the firm's market-based cost of capital, such as the weighted average cost of debt and cost of equity (WACC). In contrast, survey evidence suggests that CFOs likely rely on internal discount rates that are persistent and often unresponsive to market conditions, even when firms are not financially constrained (Gormsen and Huber, 2025). My findings extend this evidence to labor markets, where hiring decisions appear similarly detached from subjective beliefs about risk premia or financial constraints.

6 Cross-Sectional Decomposition of the Hiring Rate

To analyze the sources of dispersion in hiring across firms, I implement a cross-sectional decomposition of the log hiring rate based on the same theoretical framework developed for the time-series decomposition. The log hiring rate for each firm can be constructed using the employment accumulation equation:

$$hl_{i,t} = \log\left(\frac{q_t V_{i,t}}{L_{i,t}}\right) = \log\left(\frac{L_{i,t+1}}{L_{i,t}} - (1 - \delta^l_{i,t})\right) \tag{19}$$

where $L_{i,t}$ uses data from Compustat number of employees (EMP) and $\delta^l_{i,t}$ uses JOLTS industry-level job separation rate. The hiring rate captures the fraction of workers hired per existing employee, conditional on vacancies filled at rate q_t. This demeaned hiring rate is then decomposed into three components:

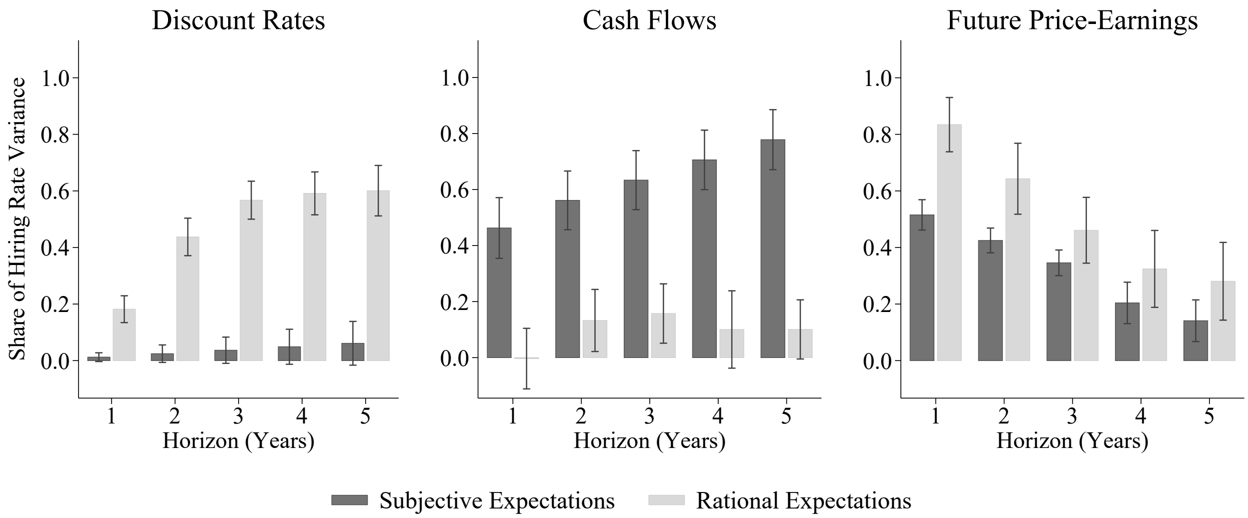
$$\widetilde{hl}_{i,t} = \underbrace{-\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\substack{\text{Discount Rate} \\ \equiv \mathbb{F}_t[\widetilde{r}_{i,t,t+h}]}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}]\right]}_{\substack{\text{Cash Flow} \\ \equiv \mathbb{F}_t[\widetilde{e}_{i,t,t+h}]}} + \underbrace{\rho^h \mathbb{F}_t\left[\widetilde{pe}_{i,t+h}\right]}_{\substack{\text{Future Price-Earnings} \\ \equiv \mathbb{F}_t[\widetilde{pe}_{i,t,t+h}]}} \tag{20}$$

The first term represents cross-sectional dispersion in expected returns, which affect the discount rate at which future expected cash flows are converted to present value. The second term captures dispersion in the current earnings per worker, $\widetilde{el}_{i,t}$, and the sum of expected earnings growth over the forecast horizon h. The third term is the dispersion in expected future price-earnings ratios, which is a terminal value that captures longer-run influences not already captured in discount rates and expected cash flows by horizon h. All expectations are formed using either survey (subjective expectation) or machine learning forecasts (rational expectation benchmark). To isolate cross-sectional variation, I demean each variable across firms indexed by I, defining $\widetilde{hl}_{i,t} = hl_{i,t} - \frac{1}{I}\sum_{j \in I} hl_{j,t}$, so that the decomposition isolates the extent to which deviations from the average hiring rate can be traced to each component.

Under stationarity, the econometrician can estimate these shares using OLS coefficients from regressing $\mathbb{F}_t[\widetilde{r}_{i,t,t+h}]$, $\mathbb{F}_t[\widetilde{e}_{i,t,t+h}]$, and $\mathbb{F}_t[\widetilde{pe}_{i,t,t+h}]$ on the current log hiring rate $\widetilde{hl}_{i,t}$, respectively. I estimate the decomposition using a panel of five value-weighted portfolios sorted by book-to-market ratio, which serve as representative groups for capturing cross-sectional dispersion in subjective beliefs across firms. Aggregating firms to portfolios smooth out firm-level measurement errors and occasional negative values for earnings (De La O et al., 2024). At each point in time, firms are assigned to one of five bins based on their book-to-market ranking, and portfolio-level variables are computed using value weights. The sample covers all common stocks (share code 10 and 11) listed on NYSE, AMEX, and NASDAQ, restricted to firms that have sufficient data to construct total employee counts (EMP) from Compustat and the median analyst stock return and earnings growth forecasts at the 5-year horizon from IBES, as described in Section 3. For each portfolio, I construct the hiring rate by fixing portfolio membership at time t based on lagged book-to-market sorting, then measuring the change in total employment from t to $t+1$ for firms in each portfolio.

Figure 4 shows that under subjective expectations, cross-sectional dispersion in the hiring rate is dominated by differences in expected cash flows. At the five-year horizon, 83.3% of the cross-sectional variance is explained by the expected cash flows. In contrast, only 4.4% of the variation is explained by discount rates, and 14.1% is attributed to differences in the terminal future price-earnings expectation. These results indicate that firms sorted into different book-to-market portfolios have sharply different expectations about future cash flows when expectations are subjective, and these differences in beliefs translate into differences in perceived hiring incentives. The combined contribution of the three components sum to 101.8% at the five-year horizon, a value close to 100.0% suggesting that the approximations used in the decomposition is reasonably accurate despite being freely estimated without imposing this constraint.

Figure 4: Cross-Sectional Decomposition of the Hiring Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components from the cross-sectional decomposition of the hiring rate. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. Subjective expectations $\mathbb{F}_t$ are constructed from IBES analyst forecasts (discount rates and cash flows). Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. x-axis denotes the forecast horizon h. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Under rational expectations, the decomposition reverses. At horizon five, 71.6% of cross-sectional variation in hiring is explained by differences in expected discount rates, while only 22.6% is explained by expected cash flows. This pattern is consistent with existing estimates showing that, under rational expectations, much of the variation across firms in asset valuations comes from dispersion in risk premia rather than expected cash flows (De La O et al., 2024). Finally, the contribution of the terminal price-earnings component is 28.0%, which is limited by still remains sizeable and is larger than the same component estimated under subjective beliefs. Under both set of beliefs, the direction of the estimated relationship is consistent with the predictions of the search model, suggesting that the errors in subjective beliefs are about magnitudes, not about directions. The hiring rate is high either because future discount rates are low, expected cash flows are high, or both.[12]

[12]Figure A.9 examines the nature of this cross-sectional dispersion more closely by estimating separate time-series decompositions of the hiring rate for each of the five book-to-market portfolios. Belief distortions are most pronounced among growth firms with low book-to-market ratios, suggesting greater sensitivity to subjective beliefs about long-term fundamentals. In contrast, hiring in high book-to-market (value) firms is less affected,

Taken together, the results reveal that under subjective beliefs, cross-sectional variation in hiring is driven primarily by firms overreacting to news about their future cash flows. This provides a micro-foundation for the aggregate results by showing that the same type of belief distortion that drives fluctuations in aggregate unemployment also operates at the firm level, where hiring decisions are actually made.[13] The result can be consistent with a model of extrapolative beliefs. If expectations of future cash flows are updated sluggishly, then firms that recently experienced positive shocks to their cash flows continue to expect strong growth, leading to persistent differences in hiring rates across firms. Under rational expectations, such distortions are absent, and the primary driver of hiring differences is variation in required returns.

7 Predictability of Unemployment and Hiring Rates

Time-Series Predictability of Aggregate Unemployment Rate To complement the decomposition of the job filling rate, this section analyzes the unemployment rate directly. While the job filling rate captures the main driver of unemployment dynamics in search models, the unemployment rate is the key macroeconomic outcome of interest and the direct target of policy. Start from the unemployment accumulation equation of the search model in Section 2:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{21}$$

which states that the number of unemployed workers at the beginning of next period U_{t+1} equals the number of unemployed worker who fail to find a job in the current period $(1 - q_t\theta_t)U_t$ plus the number of employed workers who lose their jobs due to separations $\delta_t(1 - U_t)$. Log-linearize around the steady state and substitute in equation (12), which is a decomposition of the job filling rate q_t into discount rate, cash flow, and future price-earnings components. As shown in Section B.2, the log unemployment rate u_{t+1} satisfies the following predictive relationship:

$$u_{t+1} = \alpha + \beta_r\mathbb{F}_t[r_{t,t+h}] + \beta_e\mathbb{F}_t[e_{t,t+h}] + \gamma' X_t + \varepsilon_{s,t+1} \tag{22}$$

where $X_t \equiv [u_t, \log\theta_t, \log\delta_t]'$ collects labor market factors including the lagged log unemployment rate u_t, vacancy-to-unemployment ratio $\log\theta_t$, and job separation rate $\log\delta_t$. The coefficients of interest, β_r and β_e, quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

To isolate the contribution of belief distortions, I further decompose each subjective expec-

with cash flow expectations contributing similarly under rational and subjective benchmarks.

[13] The cross-sectional results suggest that much of the dispersion in hiring rates reflects belief-driven forecast errors rather than differences in fundamentals. Such distorted expectations can act as a wedge that misallocates labor by inducing over-hiring at optimistic firms and under-hiring at pessimistic firms (Ma et al., 2020; David et al., 2022; Ropele et al., 2024).

tation $\mathbb{F}_t$ into its rational expectation $\mathbb{E}_t$ and its distortion $\mathbb{F}_t - \mathbb{E}_t$:

$$\begin{aligned} u_{t+1} = \alpha &+ \beta_{r,\mathbb{E}}\mathbb{E}_t[r_{t,t+h}] + \beta_{r,\mathbb{F}}(\mathbb{F}_t[r_{t,t+h}] - \mathbb{E}_t[r_{t,t+h}]) \\ &+ \beta_{e,\mathbb{E}}\mathbb{E}_t[e_{t,t+h}] + \beta_{e,\mathbb{F}}(\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]) + \gamma' X_t + \varepsilon_{s,t+1} \end{aligned} \tag{23}$$

I estimate equation (23) using multivariate OLS regressions, allowing the data to inform the relative importance of each component.[14] To ensure stationarity and remove seasonal effects, I estimate the regression in log growth rates relative to the same quarter of the previous year. The regression is designed to test whether perceived shocks to discount rates or earnings forecasts help predict fluctuations in unemployment rates. If firms form distorted beliefs about future returns or earnings, they should manifest in hiring behavior and thus influence unemployment.

Table 2 reports the results. Column (1) predicts the unemployment rate based on a benchmark model using only machine-based forecasts of discount rates and cash flows. Rational discount rates $\mathbb{E}_t[r_{t,t+h}]$ significantly predict unemployment (coefficient 0.551), consistent with rational models that introduce time-varying discount rates to generate realistic fluctuations in unemployment. The rational cash flow expectation $\mathbb{E}_t[e_{t,t+h}]$ is not a significant predictor (-0.041), consistent with the unemployment volatility puzzle where productivity shocks on its own struggle to generate sufficient unemployment fluctuations. Overall, the sign of the estimated coefficients are consistent with the implications of the search model, since higher discount rates or low expected cash flows depress the expected discounted value of job creation, leading to reduced hiring and higher future unemployment.

Column (2) extends the baseline model by incorporating belief distortions in subjective discount rate and cash flow expectations. The distortion in subjective cash flow expectation $\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]$ emerges as the strongest predictor of future unemployment, with a large statistically significant coefficient of -0.701.The inclusion of belief distortions improves model performance substantially. The adjusted R^2 increases from 0.528 to 0.745 in-sample and the out-of-sample R^2 increases from 0.149 to 0.254, where the out-of-sample R^2 implies an improvement in the MSE ratio relative to the Survey of Professional Forecasters (SPF) by $0.254 - 0.149 = 0.105$.[15] These results suggest that the distortions embedded in survey expectations contain valuable information not captured by other rational forecasts and pre-existing labor market factors.

Strikingly, distortions in subjective cash flow expectations drive out the predictive power of the machine-based discount rate forecast, whose coefficient has been reduced to 0.236 and is no longer statistically significant. This result suggests that behavioral factors can crowd out

[14] The future price-earnings ratio term $\mathbb{F}_t[pe_{t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\mathbb{F}_t[r_{t,t+h}]$ and cash flows $\mathbb{F}_t[e_{t,t+h}]$ as long as the Campbell and Shiller (1988) present value identity holds in equation (11).

[15] Traditional labor market factors including lagged unemployment, labor market tightness, and separations explain only a modest portion of unemployment fluctuations, with an in-sample adjusted R^2 of 0.260. In terms of out-of-sample performance, a model that excludes expectations entirely performs worse than the Survey of Professional Forecasters (SPF) benchmark with a negative OOS R^2 of -0.094.

Table 2: Time-Series and Cross-Sectional Predictability

Forecast Target: Unemployment Growth Δu_{t+1}	(1)	(2)
$\mathbb{E}_t[r_{t,t+h}]$	0.551***	0.236
t-stat	(5.046)	(0.893)
$\mathbb{E}_t[e_{t,t+h}]$	−0.041	−0.018
t-stat	(−0.108)	(−0.050)
$\mathbb{F}_t[r_{t,t+h}] - \mathbb{E}_t[r_{t,t+h}]$		−0.006
t-stat		(−0.033)
$\mathbb{F}_t[e_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}]$		−0.701***
t-stat		(−5.584)
Labor Market Factors	Yes	Yes
N	76	76
Adj. R^2	0.528	0.745
OOS R^2	0.149	0.254

Forecast Target: Employment Growth $\Delta\widetilde{l}_{i,t+1}$	(3)	(4)
$\mathbb{E}_t[\widetilde{r}_{i,t,t+h}]$	−0.498***	−0.119
t-stat	(−3.058)	(−0.734)
$\mathbb{E}_t[\widetilde{e}_{i,t,t+h}]$	0.154	0.053
t-stat	(1.304)	(0.754)
$\mathbb{F}_t[\widetilde{r}_{i,t,t+h}] - \mathbb{E}_t[\widetilde{r}_{i,t,t+h}]$		−0.043
t-stat		(−0.410)
$\mathbb{F}_t[\widetilde{e}_{i,t,t+h}] - \mathbb{E}_t[\widetilde{e}_{i,t,t+h}]$		0.759***
t-stat		(6.412)
Labor Market Factors	Yes	Yes
N	380	380
Adj. R^2	0.135	0.253
OOS R^2	0.207	0.447

Notes: This table reports decompositions of log annual growth in the unemployment rate from equation (23), under subjective or rational expectations. Labor market factors X_t include the log annual growth of lagged log unemployment rate u_t, log labor market tightness $\log\theta_t$ and log job separation rate $\log\delta_t$. The sample is quarterly from 2005Q1 to 2023Q4. OOS R^2 is defined as $1 - MSE_{\text{Model}}/MSE_{\text{Benchmark}}$. Out-of-sample forecasts are constructed as 1-year-ahead predictions using model parameters estimated over a rolling 10-year window. $MSE_{\text{Model}}/MSE_{\text{Benchmark}}$ denotes the ratio of each model's out-of-sample mean squared forecast error to that of a benchmark, which is the Survey of Professional Forecasters (SPF) consensus for time-series predictions and an AR(1) model for cross-sectional predictions. Newey-West corrected (time-series) and two-way clustering by portfolio and quarter (cross-sectional) t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

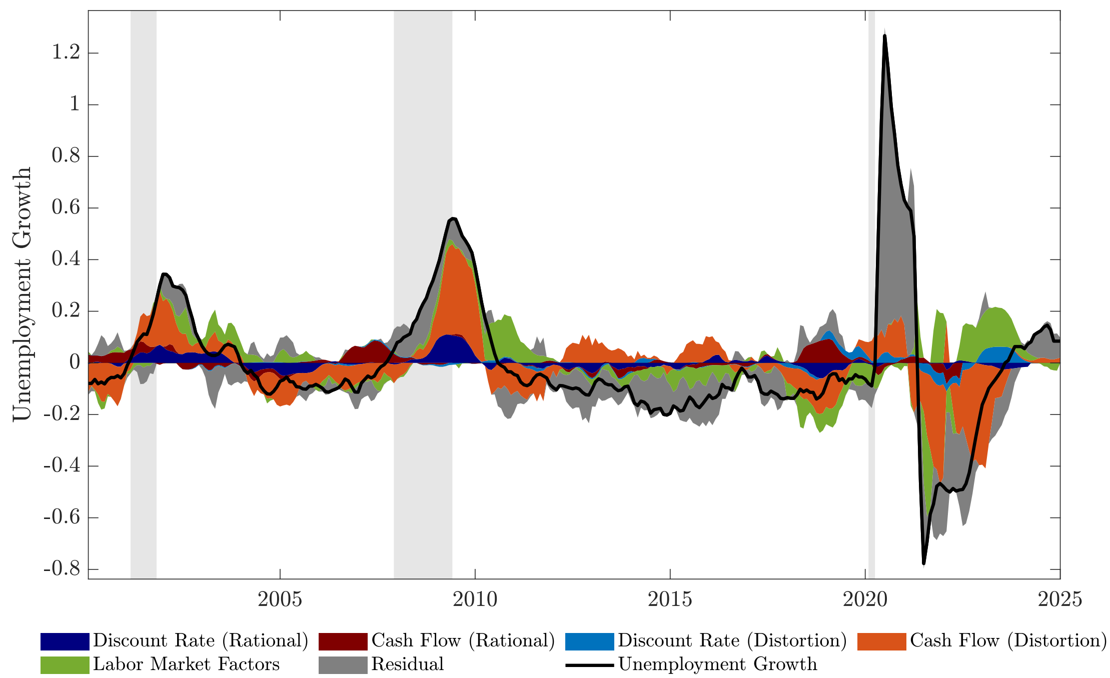
rational forces in driving labor market fluctuations, consistent with models of behavioral overreaction where salient signals can dominate decision making (Bordalo et al., 2020). Since machine forecasts already incorporate a high-dimensional set of real-time predictors, this displacement likely reflect systematic misperceptions of underlying economic shocks rather than statistical bias due to omitted variables.

Figure 5 illustrates the result by plotting the actual annual change in unemployment against its model-implied decomposition using both rational expectations and belief distortions based on equation (23). Fluctuations in unemployment closely track the component attributed to the distortion in expected cash flows. In particular, the cash flow distortion component captures the sharp rise and fall in unemployment during the global financial crisis and COVID-19 recessions with considerable precision.

Cross-Sectional Predictability of Employment Growth To complement the aggregate analysis, I examine whether belief distortions also explain cross-sectional differences in hiring behavior across firms. Start from the employment accumulation equation:

$$L_{i,t+1} = (1 - \delta_{i,t})L_{i,t} + H_{i,t} \tag{24}$$

Figure 5: Time-Series Decomposition of the U.S. Unemployment Rate



Notes: Figure plots decompositions of log annual growth in the unemployment rate from equation (A.48), using rational expectations $\mathbb{E}_t$ and belief distortions $\mathbb{F}_t - \mathbb{E}_t$ of expected cash flows and discount rates. Labor market factors include the log annual growth of lagged unemployment Δu_t, labor market tightness $\Delta\theta_t$ and job separations $\Delta\delta_t$. Residual (dark gray) represents the variation in job filling rates that are not captured by the other components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. NBER recessions are shown with light gray shaded bars.

for firm i, where $\delta_{i,t}$ is the job separation rate and $H_{i,t}$ denotes hires. Then we can approximate employment growth $\Delta l_{i,t+1} \equiv \Delta \log L_{i,t+1}$ as:

$$\Delta l_{i,t+1} \approx hl_{i,t} - \delta_{i,t} \tag{25}$$

where $h_{i,t} = H_{i,t}/L_{i,t}$ is the hiring rate. As shown in Section 2, the hiring rate reflects the firm's valuation of a job match and embeds forward-looking expectations of return, cash flow, and terminal value:

$$hl_{i,t} = -\mathbb{F}_t[r_{i,t,t+j}] + \mathbb{F}_t[e_{i,t,t+j}] + \mathbb{F}_t[pe_{i,t,t+j}], \tag{26}$$

where expectations are formed under the firm's subjective belief measure $\mathbb{F}_t$. Substituting into the employment growth approximation yields a predictive regression:[16]

$$\Delta\widetilde{l}_{i,t+1} = \alpha_i - \beta_1\mathbb{F}_t[\widetilde{r}_{i,t,t+j}] + \beta_2\mathbb{F}_t[\widetilde{e}_{i,t,t+j}] + \beta_3\widetilde{\delta}_{i,t} + \varepsilon_{i,t+1}, \tag{27}$$

where α_i denotes a firm fixed effect, and $\delta_{i,t}$ is included directly as a control for firm-level separations. The sample consists of the five book-to-market sorted portfolios, which serve as

[16]The terminal price-earnings term $\mathbb{F}_t[\widetilde{pe}_{i,t,t+j}]$ has been dropped due to its near collinearity with expected returns and expected earnings growth under the Campbell and Shiller (1988) present value identity.

representative groups for capturing belief heterogeneity across firms (Section 6). To isolate cross-sectional variation, I demean each variable across the five portfolios, defining $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{5}\sum_{j=1}^{5} x_{j,t}$ for variable x. This specification can be estimated using panel methods with firm and time fixed effects. To isolate the contribution of belief distortions, I further decompose each subjective expectation $\mathbb{F}_t$ into its rational expectation $\mathbb{E}_t$ and its distortion $\mathbb{F}_t - \mathbb{E}_t$:

$$\begin{aligned}\Delta\widetilde{l}_{i,t+1} = \alpha_i &- \beta_{1,\mathbb{E}}\mathbb{E}_t[\widetilde{r}_{i,t,t+j}] - \beta_{1,\mathbb{F}}\left(\mathbb{F}_t[\widetilde{r}_{i,t,t+j}] - \mathbb{E}_t[\widetilde{r}_{i,t,t+j}]\right) \\ &+ \beta_{2,\mathbb{E}}\mathbb{E}_t[\widetilde{e}_{i,t,t+j}] + \beta_{2,\mathbb{F}}\left(\mathbb{F}_t[\widetilde{e}_{i,t,t+j}] - \mathbb{E}_t[\widetilde{e}_{i,t,t+j}]\right) + \beta_3\widetilde{\delta}_{i,t} + \varepsilon_{i,t+1}.\end{aligned} \tag{28}$$

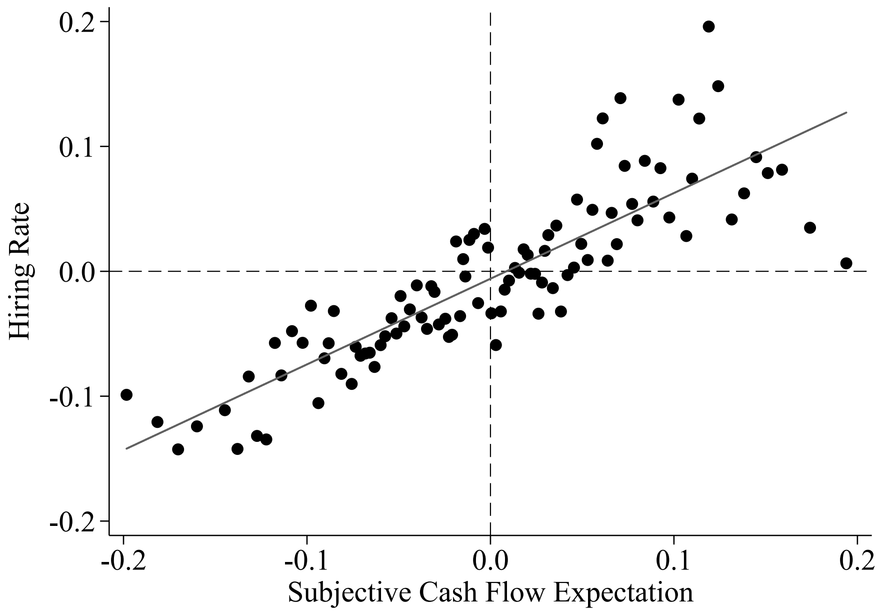
If firms over-react to news about cash flows, we expect significant positive coefficients on $\beta_{2,\mathbb{F}}$, reflecting inflated expectations of future cash flows that induce excessive hiring. Similarly, if firms over-react to news about discount rates, we may observe large distortions in $\beta_{1,\mathbb{F}}$.

Table 2 column (3) predicts portfolio-level employment growth using only machine forecasts of future returns and earnings growth. Rational return expectations $\mathbb{E}_t[\widetilde{r}_{i,t,t+j}]$ significantly predict future employment growth (coefficient -0.498), consistent with the search model's implication that firms hire more when the expected value of a match rises due to lower discounting. In contrast, the rational cash flow expectation $\mathbb{E}_t[\widetilde{e}_{i,t,t+j}]$ is not a significant predictor, although the size of the estimate remains nontrivial (coefficient 0.154).

Column (4) extends the baseline model by incorporating belief distortions in subjective return and cash flow expectations. Strikingly, distortions in subjective cash flow expectations $\mathbb{F}_t[\widetilde{e}_{i,t,t+j}] - \mathbb{E}_t[\widetilde{e}_{i,t,t+j}]$ emerge as the dominant predictor of future employment growth, with a large and statistically significant coefficient of 0.759. At the same time, the coefficient on the machine return forecast falls to -0.119 and becomes statistically insignificant. The inclusion of belief distortions substantially improves the model's predictive accuracy. The adjusted R^2 rises from 0.135 to 0.253 in-sample, and the out-of-sample R^2 rises from 0.207 to 0.443, indicating that distorted expectations provide explanatory power beyond what is captured by rational benchmarks. These cross-sectional findings reinforce the aggregate evidence that survey expectations embed economically meaningful belief distortions driven by boom-bust cycles that help explain differences in hiring across firms.

Figure 6 illustrates this key relationship by plotting the cross-sectional correlation between actual hiring rates and cash flow belief distortions across the five book-to-market portfolios. The scatter plot reveals a clear positive relationship, where portfolios with more optimistic cash flow expectations relative to the machine learning benchmark exhibit systematically higher hiring rates. Each point in the binned scatter represents a percentile of the joint distribution, and the strong positive slope confirms that belief distortions in expected cash flows translate directly into observable differences in labor demand across firms. This pattern is consistent with the large coefficient on cash flow distortions (0.759) found in the predictive regression, demonstrating that the statistical relationship captures an economically meaningful channel through which subjective beliefs influence real hiring decisions.

Figure 6: Cross-Sectional Hiring Rates and Belief Distortions



Notes: Figure plots the relationship between hiring rates and belief distortions in subjective cash flow expectations across five book-to-market portfolios. x-axis reports the cross-sectionally demeaned distortion in subjective expectations of future earnings, defined as $\mathbb{F}_t[\widetilde{e}_{i,t,t+j}] - \mathbb{E}_t[\widetilde{e}_{i,t,t+j}]$, where $\mathbb{F}_t$ is based on IBES forecasts and $\mathbb{E}_t$ is based on machine learning (LSTM) forecasts. y-axis reports the corresponding cross-sectionally demeaned log hiring rate, $\widetilde{hl}_{i,t}$. Each dot is a bin scatter representing one percentile of the pooled distribution across all observations in the sample. A positive slope implies that portfolios with upward-biased cash flow expectations tend to hire more, consistent with the model's prediction that belief distortions influence firm-level employment decisions. The sample is quarterly from 2005Q1 to 2023Q4.

Discussion The results can be informative about whether the survey-based subjective expectation is observationally equivalent to rational expectations. If subjective beliefs differ from rational beliefs only through a change of measure based on a Radon-Nikodym derivative that preserves its pricing implications, then subjective and rational forecasts should have equal predictive power for unemployment and hiring. In that case, the difference between the two expectations should be pure noise and should not improve predictions. However, the predictive regressions show that the belief distortion component $\mathbb{F}_t - \mathbb{E}_t$ has a highly significant explanatory power for both aggregate unemployment and cross-sectional employment growth. These results reject the null of observational equivalence and suggest that the implied stochastic discount factor under subjective beliefs is distinct from the one used under rational expectations. This difference implies that deviations from rational expectations can meaningfully influence real decisions.

In particular, the cross-sectional predictability results point to a meaningful departure from standard search models that assume a common, rational stochastic discount factor across firms. Rather than rational variation in discount rates, the evidence indicates that distorted beliefs about future cash flows are the main driver of both aggregate unemployment fluctuations and cross-sectional differences in hiring. If subjective and rational beliefs differed only by a change of measure, they would have similar predictive power. The result that belief distortions in cash flows predict cross-sectional differences in hiring better than rational discount rate forecasts suggests that the distortion term varies substantially across firms. Firm-specific differences in the distortion term implies that subjective beliefs influence the perceived value of job creation in firm-specific ways, possibly reflecting differences in perceived patience or risk even when fundamentals are held constant. These findings suggest the need for models that allow for heterogeneous and biased beliefs, rather than relying on a uniform stochastic discount factor with no distortions.

8 Model of Constant-Gain Learning

In this section, I introduce a model of hiring in which firms form subjective beliefs about cash flows and prices using a constant-gain learning rule. The evolving expectations shape firms' vacancy posting decisions and drive variation in hiring and job filling rates. The model embeds belief distortions in a search-and-matching framework and generates decompositions that can match those estimated from the data in Sections 5 and 6.

Environment and Firm Problem The model features a frictional labor market in which unemployed workers are matched with job vacancies using a Cobb-Douglas matching function:

$$\mathcal{M}(U_t, V_t) = BU_t^{\eta} V_t^{1-\eta} \tag{29}$$

where $\mathcal{M}(U_t, V_t)$ denotes the total number of matches in period t and is a function of aggregate unemployment U_t and job vacancies V_t. B is the matching efficiency parameter, and $\eta \in (0, 1)$ governs the elasticity of matches with respect to unemployment. The probability that a firm fills a posted vacancy, the job filling rate, is then given by:

$$q_t = \frac{\mathcal{M}(U_t, V_t)}{V_t} = B\left(\frac{U_t}{V_t}\right)^{\eta} = B\theta_t^{-\eta} \tag{30}$$

where $\theta_t \equiv V_t/U_t$ denotes labor market tightness. A firm that posts a vacancy incurs a cost $\kappa > 0$ per period. Matches dissolve at an exogenous separation rate δ, and each firm hires new workers by posting vacancies in anticipation of future returns. Each firm i uses labor to produce output via a constant returns to scale (CRS) production function:

$$Y_{i,t} = A_{i,t} L_{i,t} \tag{31}$$

where $A_{i,t}$ is firm-level productivity and $L_{i,t}$ is the level of employment. The firm pays wages $W_{i,t}$, incurs hiring costs $\kappa V_{i,t}$, and generates earnings:

$$E_{i,t} = Y_{i,t} - W_t L_{i,t} - \kappa V_{i,t} \tag{32}$$

Earnings represent the net flow profits from operating the firm: output net of the wage bill and the costs associated with posting vacancies. Firms maximize the expected present discounted value of earnings. Let $\mathcal{V}(A_{i,t}, L_{i,t})$ denote the value of the firm as a function of current productivity and employment. The Bellman equation for the firm's dynamic problem is:

$$\mathcal{V}(A_{i,t}, L_{i,t}) = \max_{V_{i,t}, L_{i,t+1}} \{E_{i,t} + \mathbb{F}_t [M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})]\} \tag{33}$$

The firm chooses the number of vacancies $V_{i,t}$ to post and the resulting employment $L_{i,t+1}$ to maximize the sum of current earnings and the discounted continuation value, formed under subjective expectations $\mathbb{F}_t[\cdot]$ and a stochastic discount factor M_{t+1}. Employment evolves according to the accumulation equation:

$$L_{i,t+1} = (1-\delta)L_{i,t} + q_t V_{i,t} \tag{34}$$

which states that next period's employment depends on retained workers $(1-\delta)L_{i,t}$ and new hires $q_t V_{i,t}$ from current vacancies. Under constant returns to scale, the firm's marginal value of labor equals average value, and the first-order condition with respect to $V_{i,t}$ simplifies to:

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \frac{\partial \mathcal{V}(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}} \right] = \frac{\mathbb{F}_t [M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})]}{L_{i,t+1}} \equiv \frac{P_{i,t}}{L_{i,t+1}} \tag{35}$$

This condition equates the marginal cost of hiring a worker today, κ/q_t, to the expected marginal benefit of that hire, defined as the expected continuation value per worker. The term $P_{i,t} \equiv \mathbb{F}_t [M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})]$ denotes the firm's ex-dividend market value. Rewriting in logs:

$$\log q_t = \log \kappa - \log \left(\frac{P_{i,t}}{L_{i,t+1}} \right) = \log \kappa - \underbrace{\ln \left(\frac{P_{i,t}}{E_{i,t}} \right)}_{\equiv pe_{i,t}} - \underbrace{\ln \left(\frac{E_{i,t}}{L_{i,t+1}} \right)}_{\equiv el_{i,t}} \tag{36}$$

where $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$ is the log price-earnings ratio and $el_{i,t} \equiv \log(E_{i,t}/L_{i,t+1})$ is the log earnings per worker.

Cash Flow Process Assume that the firm's cash flow process consists of aggregate and idiosyncratic components. Firm i's earnings at time t are given by:

$$E_{i,t} = \exp(e_{i,t}) = E_t \cdot \widetilde{E}_{i,t} \tag{37}$$

where E_t represents the aggregate component and $\widetilde{E}_{i,t}$ captures firm-specific variation. The log aggregate earnings follow a random walk with drift:

$$\Delta e_t = \log a + \log \varepsilon_t, \quad \log \varepsilon_t \sim \mathcal{N}(-\tfrac{s^2}{2}, s^2) \tag{38}$$

while the log idiosyncratic component evolves as:

$$\Delta \widetilde{e}_{i,t} = \log \widetilde{a}_i + \log \widetilde{\varepsilon}_{i,t}, \quad \log \widetilde{\varepsilon}_{i,t} \sim \mathcal{N}(-\tfrac{\widetilde{s}_i^2}{2}, \widetilde{s}_i^2) \tag{39}$$

For simplicity, I assume that the aggregate ε_t and idiosyncratic $\widetilde{\varepsilon}_{i,t}$ shocks are independently distributed, and that subjective beliefs preserve this independence.

Full Information Rational Expectations Under full information rational expectations, agents know the true drift and volatility parameters and form expectations using the true data generating process. Let $g_{i,t}^{RE}$ and $m_{i,t}^{RE}$ denote expected earnings and price growth for firm i under rational beliefs, decomposed into aggregate (g_t^{RE}, m_t^{RE}) and idiosyncratic ($\widetilde{g}_{i,t}^{RE}$, $\widetilde{m}_{i,t}^{RE}$) components. Under rational expectations, all growth expectations equal the corresponding true drift parameters: $g_t^{RE} = m_t^{RE} = a$ and $\widetilde{g}_{i,t}^{RE} = \widetilde{m}_{i,t}^{RE} = \widetilde{a}_i$. By independence of shocks, firm-level expectations are:

$$g_{i,t}^{RE} = g_t^{RE} \cdot \widetilde{g}_{i,t}^{RE} = a \cdot \widetilde{a}_i, \tag{40}$$

$$m_{i,t}^{RE} = m_t^{RE} \cdot \widetilde{m}_{i,t}^{RE} = a \cdot \widetilde{a}_i \tag{41}$$

Since the true drift parameters are constant, this also makes the price-earnings ratio $P_{i,t}^{RE}/E_{i,t} = \beta g_{i,t}^{RE}/(1 - \beta m_{i,t}^{RE})$ constant under rational expectations.

Subjective Expectations Under Constant-Gain Learning Suppose that agents do not observe the true drift terms a and $\widetilde{a}_i$ in the cash flow process, and the firms do not know how their stock price $P_{i,t}$ is determined. Instead, they form beliefs and update these beliefs recursively as new information arrives. Firms form subjective expectations about the aggregate and idiosyncratic components of both cash flow growth and stock price growth:

$$\mathbb{F}_t[E_{t+1}] = g_t E_t, \quad \mathbb{F}_t[P_{t+1}] = m_t P_t, \tag{42}$$

$$\mathbb{F}_t[\widetilde{E}_{i,t+1}] = \widetilde{g}_{i,t} \widetilde{E}_{i,t}, \quad \mathbb{F}_t[\widetilde{P}_{i,t+1}] = \widetilde{m}_{i,t} \widetilde{P}_{i,t} \tag{43}$$

where g_t, m_t denote expectations about growth in the aggregate component and $\widetilde{g}_{i,t}$ and $\widetilde{m}_{i,t}$ denote expectations about growth in the idiosyncratic component. Under the independence of aggregate and idiosyncratic shocks, beliefs about total firm-level growth can be written as:

$$\mathbb{F}_t[E_{i,t+1}] = g_{i,t} E_{i,t} = g_t \widetilde{g}_{i,t} \cdot E_t \widetilde{E}_{i,t} \tag{44}$$

$$\mathbb{F}_t[P_{i,t+1}] = m_{i,t} P_{i,t} = m_t \widetilde{m}_{i,t} \cdot P_t \widetilde{P}_{i,t} \tag{45}$$

where $g_{i,t} = g_t \widetilde{g}_{i,t}$ and $m_{i,t} = m_t \widetilde{m}_{i,t}$. I assume that firms employ constant-gain learning to update their expectations using the rule:

$$g_t = g_{t-1} + \nu \left(\frac{E_{t-1}}{E_{t-2}} - g_{t-1} \right), \quad m_t = m_{t-1} + \nu \left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1} \right) \tag{46}$$

for the aggregate components, and

$$\widetilde{g}_{i,t} = \widetilde{g}_{i,t-1} + \nu \left(\frac{\widetilde{E}_{i,t-1}}{\widetilde{E}_{i,t-2}} - \widetilde{g}_{i,t-1} \right), \quad \widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} + \nu \left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1} \right) \tag{47}$$

for the idiosyncratic components, where ν is the constant gain parameter that governs the speed of learning. Suppose the initial beliefs are set equal to the growth rates under full information rational expectations:

$$g_0 = m_0 = a, \quad \widetilde{g}_{i,0} = \widetilde{m}_{i,0} = \widetilde{a}_i \tag{48}$$

Note that the current price and current cash flows do not enter the learning rule for $g_{i,t}$ and $m_{i,t}$. The belief updates incorporate information with a lag by using information only up to period $t-1$, which eliminates the simultaneity between prices and price growth expectations. The lag in the updating equation can be motivated by an information structure in which agents observe only part of the lagged transitory shocks to stock price growth (Adam et al., 2016).

Under constant-gain learning, agents update their beliefs using a fixed gain, which causes past observations to receive exponentially decreasing weights. As a result, memory fades over time and beliefs never fully converge to rational expectations, even in a stationary environment (Nagel and Xu, 2021). This learning scheme has the advantage of allowing beliefs to remain responsive to structural changes in the data-generating process. Compared to ordinary least squares (OLS) learning, where the gain vanishes over time, constant-gain learning avoids the counterfactual implication of declining volatility in predicted variables, and is often more realistic in environments with potential regime shifts.[17]

For parsimony and interpretability, the updating rules use the same constant gain parameter ν across all components. This reflects a shared degree of rate at which firms update beliefs about different components of prices and cash flows. Existing estimates of the constant gain parameter ν are deliberately small, meaning that learning is slow and allows subjective beliefs to remain persistently distorted even after observing large forecast errors (Malmendier and Nagel, 2015; Adam et al., 2016).[18] This persistence plays an important role for generating the sustained belief distortions needed to explain fluctuations in hiring and unemployment.

Subjective Firm Valuation To highlight how learning can improve the model's performance, I consider the simplest asset pricing model by assuming risk-neutral agents and time separable preferences (Adam et al., 2016). The firm's equilibrium stock price under subjective beliefs is:

$$P_{i,t} = \beta \mathbb{F}_t[P_{i,t+1} + E_{i,t+1}] = \beta(m_{i,t}P_{i,t} + g_{i,t}E_{i,t}) \tag{49}$$

[17] Constant-gain learning can be micro-founded in two complementary ways. First, when agents are internally rational but lack external knowledge of market dynamics, they optimally forecast next-period prices using past data (Adam et al., 2016). Alternatively, when agents learn from recent experience, as older generations pass and newer ones rely more on recent data, the aggregation of their belief updates approximates a constant-gain rule (Nagel and Xu, 2021).

[18] The constant-gain learning specification for cash flow growth is supported by empirical evidence showing that survey respondents update their long-run earnings expectations only gradually following short-term earnings surprises (Nagel and Xu, 2021; De La O et al., 2024). The learning specification for stock price growth is motivated by empirical evidence showing that the implied return expectation can reproduce the dynamics of various survey based measures of subjective return expectations (Adam et al., 2016).

which implies $P_{i,t}(1-\beta m_{i,t}) = \beta g_{i,t} E_{i,t}$ and thus we have

$$P_{i,t} = \frac{\beta g_{i,t}}{1-\beta m_{i,t}} \cdot E_{i,t} \tag{50}$$

where β is the time discount factor. The equation shows that the firm's value rises with expected cash flow growth $g_{i,t}$ and falls with expected price growth $m_{i,t}$. The belief distortions captured in these expectation terms will affect the firm's hiring decisions through its valuation.

Projection Facility To prevent agents from having an infinite demand for stocks based on the valuations in (50), I assume that the subjective beliefs about price growth are bounded by:

$$0 < m_t < \beta^{-1}, \quad 0 < m_{i,t} = m_t \widetilde{m}_{i,t} < \beta^{-1} \tag{51}$$

which rules out the case $m_{i,t} \geq \beta^{-1}$ where the expected stock returns are greater than the inverse of the time discount factor. To prevent perceived stock price growth from violating the bounds in (51), I apply a projection facility which makes a smooth modification to the belief-updating equation (Timmermann, 1993; Cogley and Sargent, 2005; Adam et al., 2016). If the updated belief from (46) exceeds a constant $m^U \leq \beta^{-1}$, then the update is ignored:

$$m_t = m_{t-1} \quad \text{if } m_{t-1} + \nu\left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1}\right) \geq m^U. \tag{52}$$

For the idiosyncratic component, the bound applies to the firm-level expectation $m_{i,t} = m_t \widetilde{m}_{i,t}$. Given beliefs about the aggregate component m_t, the projection rule therefore becomes:

$$\widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} \quad \text{if } \; m_t\left[\widetilde{m}_{i,t-1} + \nu\left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1}\right)\right] \geq m^U \tag{53}$$

This procedure can be interpreted as an approximate Bayesian updating scheme where agents have a truncated prior that assigns probability zero to $m_t \geq m^U$ and $m_{i,t} \geq m^U$ (Adam et al., 2016). It can be viewed as agents ignoring observations that would lead to beliefs implying infinite demand for stocks, which would represent economically implausible behavior.[19]

Hiring Condition I close the model by connecting asset valuations to firm hiring behavior. The connection to labor markets operates through the hiring condition. Firms post vacancies until the marginal cost of hiring equals its marginal value:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of Hiring}} = \underbrace{\frac{P_{i,t}}{L_{i,t+1}}}_{\text{Value of Hiring}} \tag{54}$$

[19] Applying the projection facility is equivalent to imposing that firm-level price-earnings ratios remain below an upper bound $U^{PE} \equiv \beta a/(1-\beta m^U)$. One interpretation is that, if the price-earnings ratio exceeds this upper bound, either market participants begin to fear a sharp downturn or some regulatory authority intervenes to bring prices down. In the simulations below, this upper bound U^{PE} has been set to 500. The results are not sensitive to the exact value of U^{PE} provided it is sufficiently high, since the bounding facility binds only rarely.

where κ is the cost per vacancy, q_t is the job filling rate, and $L_{i,t+1}$ represents future employment. When firms are overly pessimistic about their expected cash flows (low $g_{i,t}$), this leads to lower firm value $P_{i,t}$, which reduces the value of hiring and leads to fewer job postings. The resulting decrease in vacancy creation drives up unemployment and reduces the job filling rate q_t.

Let $el_{i,t} \equiv e_{i,t} - l_{i,t+1} = \log E_{i,t} - \log L_{i,t+1}$ denote log earnings per worker. Given values for $\kappa, \delta, B, \eta, P_{i,t}$ and initial values for employment $L_{i,0}$, one can construct the sequence of vacancies $V_{i,t}$, employment $L_{i,t+1}$, labor market tightness θ_t, job filling rates q_t, and unemployment rate U_t by solving for the employment accumulation (34), firm valuation (50), and optimal hiring (54) equations under a Cobb-Douglas matching function (30). See Appendix B.3 for details.

Long-Horizon Cash Flow Growth and Stock Returns In this learning environment, the realized $j \geq 1$ period ahead log cash flow growth $\Delta e_{i,t+j} \equiv \log(E_{i,t+j}/E_{i,t+j-1})$ follows:

$$\Delta e_{i,t+j} = \log a_i + \log \varepsilon_{i,t+j} \tag{55}$$

and stock returns $r_{i,t+j} \equiv \log((P_{i,t+j} + E_{i,t+j})/P_{i,t+j-1})$ follow:

$$\begin{aligned} r_{i,t+j} &= \log\left(\frac{E_{i,t+j}}{E_{i,t+j-1}}\frac{E_{i,t+j-1}}{P_{i,t+j-1}}\left(\frac{P_{i,t+j}}{E_{i,t+j}} + 1\right)\right) && (56) \\ &= \Delta e_{i,t+j} + \log\left(\frac{1-\beta m_{i,t+j-1}}{\beta g_{i,t+j-1}}\right) + \log\left(\frac{1-\beta m_{i,t+j} + \beta g_{i,t+j}}{1-\beta m_{i,t+j}}\right) && (57) \end{aligned}$$

where $a_i \equiv a \cdot \widetilde{a}_i$ and $\varepsilon_{i,t} \equiv \varepsilon_t \cdot \widetilde{\varepsilon}_{i,t}$. The price-earnings ratios are based on the firm valuations implied by equation (50).

Subjective expectations of these variables reflect beliefs about future cash flow growth $g_{i,t+j}$ and capital gains $m_{i,t+j}$. In models with constant-gain learning, beliefs evolve with fading memory, breaking the law of iterated expectations and making resale and buy-and-hold valuation methods non-equivalent. The buy-and-hold approach evaluates long-run payoffs under today's beliefs, while the resale method prices assets through a sequence of one-period-ahead valuations, each using updated beliefs. Following Nagel and Xu (2021), I adopt the resale valuation approach because it ensures time consistency under belief updating and reflects the idea that assets are effectively resold across agents with evolving expectations. I assume that the manager and the representative investor share the same beliefs and both apply the resale method, ensuring consistency between decision-making and valuation.

Let x_t and $\widetilde{x}_{i,t}$ denote the aggregate and idiosyncratic level of a variable $x \in \{E, P\}$ at time t, which are either aggregate cash flows or prices. Define:

$$R^x_t \equiv \frac{x_t}{x_{t-1}}, \quad Z^x_t \equiv (1-\nu)Z^x_{t-1} + \nu R^x_{t-1} \tag{58}$$

$$\widetilde{R}^x_{i,t} \equiv \frac{\widetilde{x}_{i,t}}{\widetilde{x}_{i,t-1}}, \quad \widetilde{Z}^x_{i,t} \equiv (1-\nu)\widetilde{Z}^x_{i,t-1} + \nu \widetilde{R}^x_{i,t-1} \tag{59}$$

That is, $Z_t^x, \widetilde{Z}_{i,t}^x$ denotes the subjective expectation of the growth rate of variable x, formed using constant-gain learning based on past realized growth $R_{t-1}^x, \widetilde{R}_{i,t-1}^x$, respectively. It can be shown by induction that the j-step-ahead expectation at time t is given by:[20]

$$\mathbb{F}_t[Z_{t+j}^x] = a_j^x Z_t^x + b_j^x R_t^x, \tag{60}$$
$$\mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] = a_j^x \widetilde{Z}_{i,t}^x + b_j^x \widetilde{R}_{i,t}^x, \tag{61}$$
$$\mathbb{F}_t[Z_{i,t+j}^x] = \mathbb{F}_t[Z_{t+j}^x] \cdot \mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] \tag{62}$$

with recursively defined coefficients:

$$a_0 = 1, \quad b_0 = 0, \quad a_1 = 1 - \nu, \quad b_1 = \nu, \tag{63}$$
$$a_j = (1-\nu)a_{j-1} + \nu a_{j-2}, \quad b_j = (1-\nu)b_{j-1} + \nu b_{j-2}, \quad j \geq 2 \tag{64}$$

After making a first-order approximation $\mathbb{F}_t[\log(X)] \approx \log(\mathbb{F}_t[X])$, subjective expectations of log cash flow growth can be written as:

$$\mathbb{F}_t[\Delta e_{i,t+j}] \approx \log\left(\mathbb{F}_t\left[g_{i,t+j-1}\right]\right) \tag{65}$$

Similarly, subjective expectations of log stock returns can be written as:

$$\begin{aligned}\mathbb{F}_t[r_{i,t+j}] \approx (1-\beta)\log\left(\mathbb{F}_t[g_{i,t+j-1}]\right) + \log\left(\frac{1-\beta\mathbb{F}_t[m_{i,t+j-1}]}{1-\beta\mathbb{F}_t[m_{i,t+j}]}\right) \\ + \log\left(1+\beta\mathbb{F}_t[g_{i,t+j} - m_{i,t+j}]\right)\end{aligned} \tag{66}$$

where $\mathbb{F}_t[g_{i,t+j}]$ and $\mathbb{F}_t[m_{i,t+j}]$ are determined by the recursion in equations (62) through (64). Under constant-gain learning, expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$ from equation (65) can fluctuate substantially due to large and persistent distortions in subjective beliefs embedded in $g_{i,t+j}$. Similarly, distortions in subjective beliefs about capital gains $m_{i,t+j}$ can drive fluctuations in realized stock returns $r_{i,t+j}$ through equation (57). In contrast, expected stock returns $\mathbb{F}_t[r_{i,t+j}]$ from equation (66) likely shows only small fluctuations because its variation depends mainly on the gap between expected cash flow growth and price growth $g_{i,t+j} - m_{i,t+j}$.[21] Since both $g_{i,t+j}$ and $m_{i,t+j}$ terms adjust slowly and move together, their difference remains relatively stable. This generates the empirically observed pattern of high volatility in realized returns but low volatility in subjective expected returns, consistent with survey evidence on return expectations.

Model-Implied Decompositions I use data simulated from the learning model to decompose the job filling rate at the aggregate level and hiring rates at the firm level. The time-series

[20] See Appendix B.3 for a proof.

[21] Since the time discount rate β is typically close to one, the first term in equation (66) will be quantitatively small since it is scaled by $1-\beta \approx 0$. Since the learning rate ν is small, the one-period belief revisions captured in $\mathbb{F}_t[m_{i,t+j}]/\mathbb{F}_t[m_{i,t+j-1}]$ will also be quantitatively small in the second term of equation (66).

decomposition of the aggregate job filling rate q_t is given by:

$$\log q_t = \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\text{Discount Rate}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\text{Cash Flow}} - \underbrace{\rho^h \mathbb{F}_t \left[pe_{t+h} \right]}_{\text{Future Price-Earnings}} \tag{67}$$

where $x_t = \sum_{i \in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. To analyze differences across firms, I estimate a cross-sectional decomposition of hiring rates using simulated firm-level data:

$$\widetilde{hl}_{i,t} = \underbrace{- \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}] \right]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t \left[\widetilde{pe}_{i,t+h} \right]}_{\text{Future Price-Earnings}} \tag{68}$$

where $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{I} \sum_i x_{i,t}$ denotes a cross-sectional deviation from the mean at time t.

Firms' hiring decisions reflect their evolving beliefs about cash flow growth $g_{i,t+j}$ and stock price growth $m_{i,t+j}$, which are updated according to the constant-gain learning rules. The slow learning rate can generate persistent distortions in $g_{i,t+j}$ which drives fluctuations in expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$, and distortions in $m_{i,t+j}$ can drive fluctuations in realized stock returns $r_{i,t+j}$. In contrast, the model likely produces a low volatility in expected returns $\mathbb{F}_t[r_{i,t+j}]$ because their variation depends mostly on the gap between cash flow growth and price growth $g_{i,t+j} - m_{i,t+j}$, which is relatively stable over time. Therefore under ex-ante subjective beliefs, the cash flow component in the decomposition will be highly volatile while the discount rate component remains muted. Consequently, subjective expectations will systematically over-weight the role of cash flows relative to discount rates, generating the empirical pattern observed in the data. This contrasts sharply with the decomposition estimated under rational beliefs, where the cash flow component will only have a minor contribution because the rational expectation of future cash flow growth equals the constant drift term.

Simulation Details I simulate a panel of 300 firms over 500 periods, where the first 150 periods are discarded as a burn-in to eliminate the influence of initial conditions. Under constant-gain learning, each firm updates its beliefs using the updating rules in equations (46) and (47). All expectations, returns, and decompositions are computed at a monthly frequency using the model equations derived above. At each horizon h, I compute the model-implied time-series decomposition of the aggregate job filling rate based on equation (67) and the cross-sectional decomposition of the firm-level hiring rates (68). I then compare these model-implied decompositions to those estimated from the observed data from Figures 3 and 4.

Model Estimation Table 3 reports the parameters used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The drift $a = 1.0035$ and volatility $s = 0.0298$ of aggregate cash flow growth is set to match the long-run mean and standard deviation of aggregate U.S. dividend growth (Adam et al., 2016). The drift $\widetilde{a} = 1.00$ and volatility $\widetilde{s}_i = 0.0345$ of idiosyncratic earnings growth is set to match the long-run mean and standard deviation of dividend growth across five value-weighted book-to-market sorted portfolios, after cross-sectionally demeaning the variable. The time discount rate $\rho = \exp(\overline{pe})/(1 + \exp(\overline{pe})) = 0.98$ is chosen to be consistent with a steady-state price-earnings ratio from the Campbell and Shiller (1988) present value identity, where $\overline{pe}$ is the long-run average of the log price-earnings ratio over 1983–2023.

The speed at which agents discount past observations of realized cash flow growth depends on the constant gain parameter ν in the learning rule. This parameter shapes the persistence and volatility of the price-earnings ratio and the extent of return predictability. I take the value directly from survey-based estimates in Malmendier and Nagel (2015), setting it to $\nu = 0.018$ at the quarterly frequency. This implies that in forming expectations, agents assign a weight of 0.018 to the most recent growth surprise and $1 - \nu = 0.982$ to their previous estimate, making the perceived growth rate evolve slowly over time.

Labor market parameters are mainly adopted from Kehoe et al. (2022). Following Shimer (2005), I normalize the value of labor market tightness θ to one in the deterministic steady state, which implies an an efficiency of the matching function $B = 0.562$ by noting from the matching function that $q = B\theta^{-\eta}$. I set the elasticity of the matching function to $\eta = 0.5$ following Ljungqvist and Sargent (2017). I use an annual job separation rate of $\delta = 0.286$, which is the annualized value of the Abowd-Zellner corrected estimate by Krusell et al. (2017) based on data from the Current Population Survey (CPS). Following Elsby and Michaels (2013), per-worker vacancy posting cost 0.314 is targeted to match a per-worker hiring cost κ/q equal to 14 percent of the quarterly worker compensation. In the context of the annual calibration of this model, this implies a value approximately equal $\kappa = 4 \times 0.14 \times q \times el = 0.314$, where 4×0.14 is the annualized percent of worker compensation, while $q = 0.562$ and $el = 3.750$ are long-run averages of the log job filling rate and earnings per employee in the historical sample from 1983 to 2023.

Model vs. Data: Variance Decompositions The model successfully replicates the empirical variance decompositions from the data. Figure 7 shows that the model can reproduce the finding that subjective beliefs over-estimate the role of expected cash flows and underestimate the role of discount rates in explaining labor market fluctuations.

Panel (a) presents the time-series decomposition of the job filling rate, comparing contributions under subjective and rational expectations. The model captures the empirical pattern where subjective expectations (dark bars) assign a larger role to cash flows compared to rational expectations (light bars). The model-implied values (circles and triangles) align closely with the empirical estimates, demonstrating the model's ability to match the data.

Table 3: Model Parameters

Parameter	Value	Moments
ν	0.0180	Constant-gain learning (Malmendier and Nagel (2015))
a	1.0035	Mean of U.S. aggregate dividend growth (Adam et al. (2016))
s	0.0298	S.D. of U.S. aggregate dividend growth (Adam et al. (2016))
$\tilde{a}_i$	1.0000	Mean of U.S. idiosyncratic dividend growth
$\tilde{s}_i$	0.0345	S.D. of U.S. idiosyncratic dividend growth
ρ	0.980	Average price-earnings ratio
B	0.562	Matching function efficiency (Kehoe et al. (2022))
η	0.500	Matching function elasticity (Kehoe et al. (2022))
δ	0.286	Separation rate (Kehoe et al. (2022))
κ	0.314	Per worker hiring cost (Elsby and Michaels (2013))

Notes: Table reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The model is calibrated at a monthly frequency.

Panel (b) shows the cross-sectional decomposition of hiring rates across firms. Again, the model captures the empirical pattern that subjective beliefs over-estimate the contribution of earnings expectations and under-estimate the variation in firm-level discount rates. This cross-sectional fit is particularly important as it shows that the model can explain not just aggregate patterns but also the heterogeneity in hiring behavior across different firms.

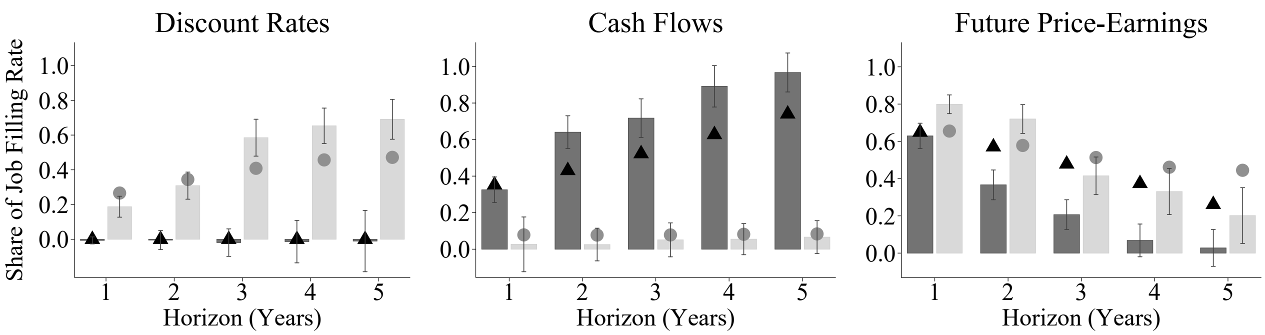
The large estimated discount rate component under rational beliefs is consistent with existing search models formulated under rational expectations, which have emphasized time-varying discount rates to match the volatility of unemployment fluctuations. Table A.6 compares the empirical variance decomposition from Figure 3 with those implied by a selection of search-and-matching models.[22] The Diamond-Mortensen-Pisarides and Hall (2017) models predict that discount rate fluctuations should explain more than 78.2% of the variance of job filling rates. The Kehoe et al. (2022) (KLMP) model predicts a more balanced decomposition, attributing 54.3% to discount rates and 31.9% to cash flows, consistent with the model's amplification mechanism based on human capital accumulation.

Model vs. Data: Moments Table 4 demonstrates that the constant-gain learning model successfully matches both asset market and labor market moments. The table compares moments generated by the learning model against those generated from a rational model under no learning, where all agents have full information rational expectations. To generate simulations under the

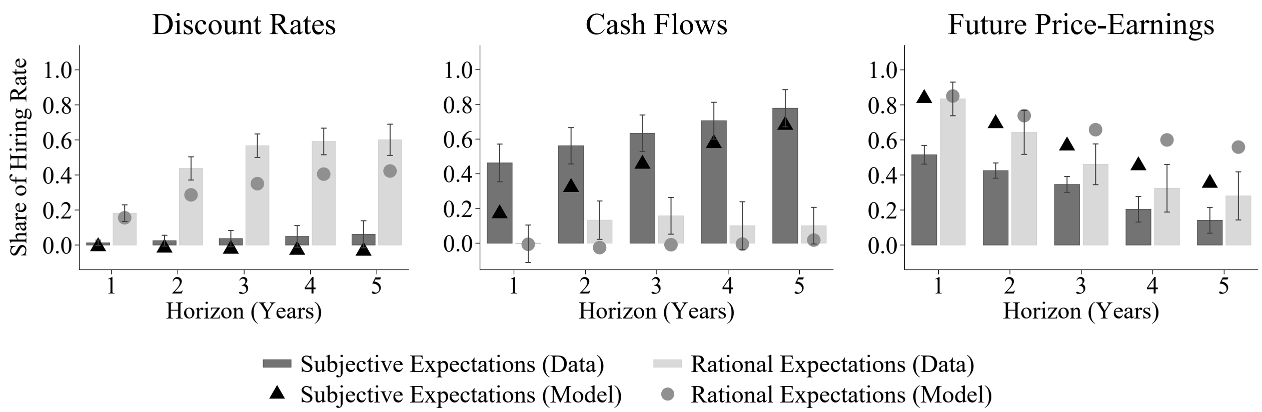
[22]For each model, I set a sample length of 500 periods, produce 300 simulations, and discard the first 150 periods as a burn-in sample. I use the simulated data from each model to estimate a variance decomposition of the job filling rate according to equation (14), and report the average across the simulated runs. All parameter values in the calibration use estimates from the original papers.

Figure 7: Model vs. Data: Variance Decompositions

(a) Time-Series Decomposition of the Job Filling Rate



(b) Cross-Sectional Decomposition of the Hiring Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate (panel (a)) and cross-sectional decomposition of the hiring rate (panel (b)). Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4. Circle and triangle dots show the values of rational and subjective expectations implied by the model, respectively.

rational model, I employ the same sequence of shocks as in the baseline learning specification but set the learning rate parameter to zero. This eliminates belief updating and, conditional on the true initial values, reduces the model exactly to its rational expectations counterpart in equations (40) and (41).

Panel (a) reports time-series and cross-sectional moments for asset prices. The learning model broadly matches the mean and volatility of price-earnings ratios, the persistence in valuations, and the volatility of returns and expected returns. In contrast, the rational expectations model severely understates price-earnings volatility and generates virtually no variation in expected returns, confirming that belief distortions are essential for matching observed financial market behavior (Adam et al., 2016). For the cross-sectional moments, the learning model captures the dispersion in price-earnings ratios, expected earnings growth, returns, and expected returns. These moments confirm that the firm-specific beliefs $\widetilde{\gamma}_{i,t}$ and $\widetilde{\beta}_{i,t}$ generate realistic heterogeneity

Table 4: Model vs. Data: Asset Market and Labor Market Moments

Moment	Data	Learning Model	Rational Model
(a) Asset Market			
$Mean(pe_t)$	2.98	2.26	3.15
$\mathrm{SD}(pe_t) \times 100$	47.4	40.2	0.0
$\mathrm{AC}(pe_t)$	0.75	0.70	1.00
$\mathrm{SD}(r_t) \times 100$	16.0	14.7	2.0
$\mathrm{SD}(\mathbb{F}_t[r_{t+1}]) \times 100$	1.1	0.0	0.0
$\mathrm{SD}(\mathbb{F}_t[\Delta e_{t+1}]) \times 100$	26.8	20.8	0.0
$\mathrm{SD}_i(pe_{i,t}) \times 100$	22.6	15.9	0.0
$\mathrm{SD}_i(r_{i,t}) \times 100$	5.7	4.8	2.4
$\mathrm{SD}_i(\mathbb{F}_t[r_{i,t+1}]) \times 100$	2.6	0.0	0.0
$\mathrm{SD}_i(\mathbb{F}_t[\Delta e_{i,t+1}]) \times 100$	14.0	10.0	0.0
(b) Labor Market			
$\mathrm{SD}(u_t) \times 100$	2.09	1.56	0.07
$\mathrm{AC}(u_t)$	0.91	0.87	0.98
$\mathrm{SD}(q_t) \times 100$	8.71	6.50	0.21
$\mathrm{AC}(q_t)$	0.94	0.97	0.98
$\mathrm{Corr}(u_t, q_t)$	0.82	0.91	1.00
$\mathrm{SD}_i(hl_{i,t}) \times 100$	15.70	12.13	1.13

Notes: This table compares empirical moments with model-generated moments with and without constant-gain learning. $\mathrm{SD}(\cdot)$ denotes the time-series standard deviation of aggregate variables. $\mathrm{SD}_i(\cdot)$ denotes the cross-sectional standard deviation across firms at each point in time, averaged over time. $\mathrm{AC}(\cdot)$ denotes the first-order autocorrelation coefficient. $\mathrm{Corr}(\cdot)$ denotes the correlation between two time series. pe_t is the log price-earnings ratio, r_t is the log stock return, Δe_t is log earnings growth, q_t is the job-filling rate, u_t is the unemployment rate, and $hl_{i,t}$ is the firm-level hiring rate. $\mathbb{F}_t[\cdot]$ denotes subjective expectations formed at time t. Data column reports empirical moments estimated from historical data. Learning model reports moments from simulations of the constant-gain learning model. Rational model reports moments from the rational expectations benchmark where agents have perfect knowledge of the earnings process.

in firm valuations and expectations. The rational expectations model, by construction, produces minimal cross-sectional variation in expectations, highlighting how constant-gain learning creates the belief heterogeneity observed in the data.

Panel (b) reports moments related to the labor market. The learning model broadly matches key labor market statistics including the volatility and persistence of the job filling rate q_t and unemployment rate u_t. The constant-gain learning model only slightly undershoots the volatility of the unemployment rate, which is a substantial improvement over the rational expectations model where unemployment volatility is typically an order of magnitude too small. The learning model's ability to match these moments demonstrates that the constant-gain learning mechanism provides a coherent explanation for both asset market and labor market fluctuations.

9 Robustness Checks and Extensions

This section presents additional results that reinforce the main finding. Across multiple robustness checks, the evidence consistently shows that firms overweight expected cash flows and underweight discount rates under subjective expectations.

Subjective vs. Risk-Neutral Expectations A natural question is whether subjective beliefs implied by survey expectations reflect a risk-neutral measure rather than genuine belief distortions (Cochrane, 2017). While one might argue that these survey forecasts reflect a risk premium, this interpretation is inconsistent with several lines of evidence. First, the magnitude of the forecast errors far exceeds what standard risk premia can explain. Typical annual equity risk premia are on the order of 5% to 10%, yet the survey forecasts exhibit mean squared errors that are 15% to 30% larger than machine forecasts (Figure 2). Second, decompositions using risk-neutral expectations implied by futures prices show that subjective expectations overweight long-horizon cash flows even relative to risk-neutral counterparts (Figure A.8). Third, survey forecasts of stock returns consistently exceed risk-free rates, and the expected excess returns they imply vary predictably over time (Adam et al., 2021). Moreover, rather than being systematically pessimistic, these forecasts are often predictably optimistic, contradicting the idea that they reflect ambiguity aversion or robustness-driven pessimism. These findings are inconsistent with rational or risk-neutral pricing and suggests that subjective beliefs reflect genuine behavioral distortions rather than a rational risk-neutral measure.

Capital Investment Appendix Section A.6 extends the baseline framework to include firm investment decisions, distinguishing between tangible and intangible capital. Firms choose investment and hiring jointly to maximize value, facing convex adjustment costs and forming expectations over future productivity, returns, and earnings. A decomposition of investment rates in Figures A.12 (time-series) and A.13 (cross-section) reveals that distortions in subjective beliefs play a central role in driving capital allocation, mirroring results for hiring. Using IBES and Compustat data, the decomposition shows that subjective expectations substantially overstate the role of expected earnings and understate the importance of discount rates.

Regional Model using Shift-Share Instrument Appendix Section A.5 strengthens the causal interpretation of belief distortions by using a Bartik shift-share instrument to isolate exogenous variation in regional hiring conditions. By leveraging national industry-level hiring shocks weighted by historical state industry shares, the instrument generates plausibly exogenous shifts in local job filling rates. State-level regressions reveal that subjective forecasts of earnings growth respond strongly to these local shocks, even after controlling for state and time fixed effects, while discount rates respond less (Table A.9). The results confirm that belief distortions, especially over-reaction to perceived cash flow opportunities, are not merely correlated with labor demand, but causally influence hiring decisions across regions.

Additional Results Table A.1 reports summary statistics for each variable used in the empirical analysis. Table A.2 shows that survey respondents only partially incorporate short-term earnings surprises into their long-run expectations, demonstrating gradual belief adjustment consistent with constant-gain learning rather than rational expectations. The baseline variance decomposition of the job filling rate (Table A.3) is robust to alternative specifications: Using Vector Autoregressions (Figure A.2), first-differences (Figure A.3), including all publicly listed firms (Figure A.4), extended historical sample from 1983Q4 to 2023Q4 (Figure A.5), replacing machine learning forecasts with their ex-post realized values (Figure A.6), conditioning on lagged job filling rates and survey forecasts (Table A.5), alternative survey sources (Tables A.7 and A.8), decreasing returns to scale in the firm's production function (Figure A.14), on-the-job search (Figure A.15), financial constraints (Figure A.10), and time-varying estimates (Figure A.11). Table A.7 substitutes CFO survey forecasts for IBES analyst forecasts to directly capture managerial expectations and, despite limited data availability, finds comparable results. Table A.6 compares the variance decomposition against theoretical predictions from a selection of existing search-and-matching models. Predictable deviations from rational expectations can explain a substantial share of job filling rate variation since the job filling rate strongly predicts survey biases (Table A.4). Figure A.9 shows that subjective beliefs strongly overstate cash flow effects for low book-to-market (growth) firms and small firms. The large contribution of rational discount rate is driven by fluctuations in risk premia instead of risk-free rates (Figure A.7). Figure A.16 and Table A.10 show that survey-based wage expectations are far less cyclical than realized wages, leading firms to perceive the user cost of labor as relatively rigid over the business cycle. As a result, firms may fail to anticipate declines in wages during downturns, which keeps the perceived user cost of labor high and discourages job creation.

10 Conclusion

This paper examines how belief distortions can resolve the unemployment volatility puzzle by comparing survey-based subjective expectations with machine learning forecasts that proxy for rational expectations. Motivated by the unemployment volatility puzzle, I reinterpret hiring behavior through the lens of a Diamond-Mortensen-Pissarides search-and-matching model, allowing firms to form beliefs that deviate from full-information rational expectations.

Using a decomposition of the job filling rate grounded in the search model, I uncover a stark contrast between how subjective and rational beliefs drive unemployment fluctuations. Under subjective expectations, firms' hiring decisions are driven almost entirely by predictable errors in expected future cash flows, which account for up to 96.7% of variation in the aggregate job filling rate and 83.3% of cross-sectional hiring dispersion across firms at the 5-year horizon. Subjective discount rates play only a limited role. This pattern reverses completely under rational expectations, where discount rates dominate both time-series and cross-sectional variation, explaining up to 69.1% and 71.6% of hiring fluctuations, respectively.

To interpret these findings, I develop a model in which firms engage in constant-gain learning about the long-run growth of their cash flows and stock prices. Firms slowly update their beliefs in response to forecast errors and base hiring decisions on these evolving expectations. The model reproduces the empirical patterns observed in the data: subjective expectations over-attribute fluctuations in the value of job creation to cash flows, both in the aggregate and across firms. The learning model can generate realistic fluctuations in aggregate unemployment, substantially outperforming standard rational models that fall short by an order of magnitude.

Together, the results suggest that labor market fluctuations are shaped not only by rational responses to discount rate news, but also by systematic distortions in belief formation. Accounting for these distortions helps reconcile the sharp and persistent spikes in unemployment during downturns that standard models struggle to explain. More broadly, the findings highlight a behavioral channel through which expectations formed under limited information and learning can amplify unemployment volatility. Incorporating subjective beliefs into macroeconomic models can thus offer a richer and more realistic account of labor market dynamics.

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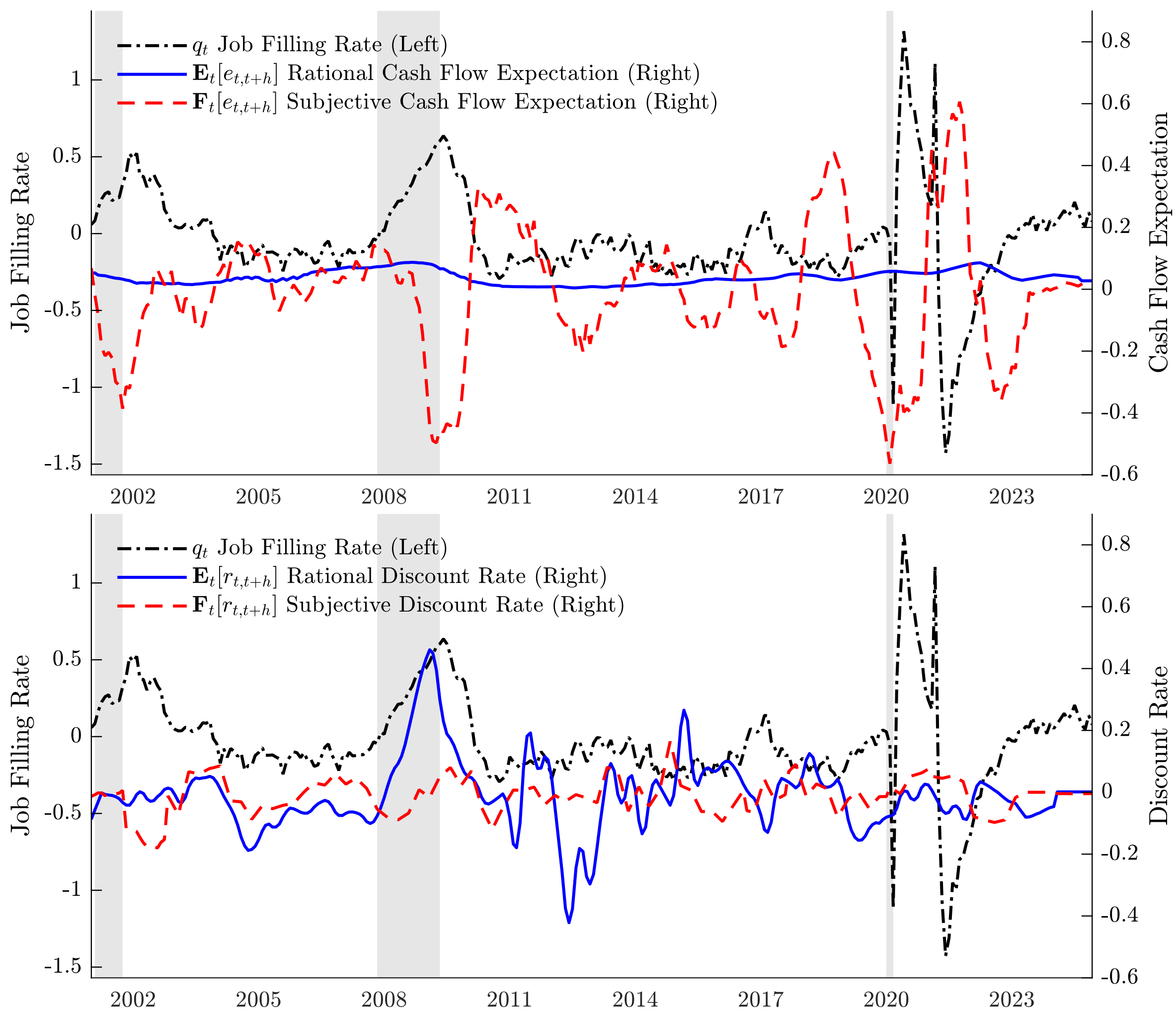
A Appendix: Additional Results

A.1 Stylized Facts

Figure A.1 compares subjective and machine expectations for discount rates and cash flows, plotted against the job filling rate. These series represent the specific theoretical components that drive hiring decisions in the DMP framework: discount rates capture the firm's intertemporal trade-offs when evaluating the present value of a new hire, while cash flows reflect expected future productivity gains from employment. Through the lens of the decomposition in equation (12), these components should move systematically with job filling rates if firms correctly interpret economic conditions.

Figure A.1: Job Filling Rates, Discount Rates, and Expected Cash Flows



Notes: Figure plots $h = 5$ year ahead survey forecasts $\mathbb{F}_t[\cdot]$ and machine learning forecasts $\mathbb{E}_t[\cdot]$ of discount rates $r_{t,t+h}$ and cash flows $e_{t,t+h}$ (left axis) against the current job filling rate q_t (right axis). x axis denotes the date on which each forecast has been made and the job filling rate was realized. Each series is expressed in annual log growth rates. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The out-of-sample forecast testing period is quarterly and spans 2005Q1 to 2023Q4. NBER recessions are shown with gray shaded bars.

Machine expectations of discount rates exhibit a strong positive relationship with job filling rates, particularly around the Global Financial Crisis. This pattern aligns with the theoretical prediction that higher discount rates (reflecting greater compensation for risk) should coincide with lower hiring as firms perceive a lower present discounted value of employment. Survey expectations of discount rates, by contrast, are relatively flat and display little sensitivity to the business cycle, consistent with studies that find acyclical subjective risk premia (Nagel and Xu, 2022). This disconnect suggests that firms fail to internalize how macroeconomic conditions affect the risk-adjusted value of hiring decisions.

For cash flows, the pattern reverses. Survey expectations show exaggerated cyclical variation, becoming sharply pessimistic during downturns, such as the Global Financial Crisis, when job filling rates are high. Machine forecasts also vary cyclically but to a much lesser extent, indicating that survey respondents tend to over-react to macroeconomic conditions when forming cash flow expectations. This over-reaction manifests in the decomposition as an outsized role for subjective cash flow news in explaining job filling rate variation, even when a model under rational beliefs suggests that discount rate changes should be the primary driver of hiring fluctuations.

Appendix Table A.1 summarizes the distributions of survey-based and machine learning forecasts for the key components of the variance decomposition. The most notable pattern is the contrast in time-series volatility and cross-sectional dispersion between the two sources of expectations. In the time series, 5-year survey-based discount rate expectations $\mathbb{F}_t[r_{t,t+5}]$ are substantially less volatile than machine forecasts, with standard deviations of 0.037 and 0.118, respectively. In contrast, 5-year survey-based cash flow expectations $\mathbb{F}_t[e_{t,t+5}]$ exhibit much higher volatility than machine forecasts, with standard deviations of 0.299 and 0.058, respectively. In the cross section across book-to-market portfolios, survey-based expectations display greater dispersion for both components: the standard deviation of $\mathbb{F}_t[r_{i,t,t+5}]$ is 0.510 versus 0.103 for machine forecasts, and for $\mathbb{F}_t[e_{i,t,t+5}]$, the standard deviation is 0.079 compared to 0.038.

Table A.1: Summary statistics

Panel (a): Aggregate U.S.	Obs	Mean	St. Dev.	Min	p25	Median	p75	Max
$r_{t,t+5}$	72	0.284	0.283	-0.279	0.131	0.330	0.464	0.789
$\mathbb{F}_t[r_{t,t+5}]$	72	0.226	0.037	0.147	0.195	0.229	0.251	0.327
$\mathbb{E}_t[r_{t,t+5}]$	72	0.287	0.118	0.036	0.209	0.284	0.362	0.572
$e_{t,t+5}$	72	3.739	0.300	2.353	3.741	3.777	3.905	4.288
$\mathbb{F}_t[e_{t,t+5}]$	72	3.908	0.299	3.264	3.768	3.892	4.101	4.423
$\mathbb{E}_t[e_{t,t+5}]$	72	3.801	0.058	3.704	3.763	3.793	3.823	3.936
pe_{t+5}	72	3.553	0.294	3.084	3.332	3.527	3.642	4.594
$\mathbb{F}_t[pe_{t,t+5}]$	72	3.654	0.146	3.321	3.537	3.686	3.761	3.925
$\mathbb{E}_t[pe_{t,t+5}]$	72	3.603	0.284	2.864	3.408	3.590	3.803	4.208
q_t	72	0.596	0.236	0.211	0.408	0.587	0.731	1.202
U_t	72	0.061	0.021	0.036	0.046	0.054	0.078	0.130
θ_t	72	0.598	0.315	0.160	0.339	0.558	0.747	1.438
δ_t	72	0.350	0.058	0.265	0.316	0.354	0.370	0.689

Panel (b): Book-to-Market Portfolios	Obs	Mean	St. Dev.	Min	p25	Median	p75	Max
$r_{i,t,t+5}$	360	0.168	0.185	-0.349	0.048	0.166	0.271	0.752
$\mathbb{F}_t[r_{i,t,t+5}]$	360	0.136	0.051	0.024	0.097	0.137	0.178	0.243
$\mathbb{E}_t[r_{i,t,t+5}]$	360	0.190	0.103	-0.101	0.124	0.189	0.262	0.428
$e_{i,t,t+5}$	360	3.912	0.042	3.796	3.885	3.911	3.941	4.016
$\mathbb{F}_t[e_{i,t,t+5}]$	360	3.821	0.079	3.610	3.766	3.824	3.874	4.012
$\mathbb{E}_t[e_{i,t,t+5}]$	360	3.903	0.038	3.791	3.879	3.904	3.929	4.002
$pe_{i,t,t+5}$	360	3.599	0.284	2.891	3.385	3.585	3.791	4.302
$\mathbb{F}_t[pe_{i,t,t+5}]$	360	3.642	0.219	3.032	3.463	3.652	3.806	4.138
$\mathbb{E}_t[pe_{i,t,t+5}]$	360	3.628	0.167	3.190	3.509	3.640	3.752	4.047
$hl_{i,t}$	360	0.039	0.093	-0.220	-0.010	0.031	0.085	0.336

Notes: This table reports summary statistics for ex-post realized outcomes (Actual), subjective expectations (Survey), and machine expectations (Machine) of key variables used in the variance decomposition. Panel (a) reports aggregate U.S. statistics, and Panel (b) reports statistics from a sample of five value-weighted book-to-market sorted portfolios. The forecasted variables are $h = 5$ year present discounted values of discount rates $r_{t,t+h}$, cash flows $e_{t,t+h}$, and price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Aggregate labor market variables include the job filling rate q_t, unemployment rate U_t, vacancy-to-unemployment ratio θ_t, and job separation rate δ_t. Portfolio-level variables are constructed by aggregating employment and forecast data across firms within each book-to-market group, holding portfolio assignment fixed at the time of portfolio formation. Subjective expectations at the aggregate level $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns and from IBES for earnings growth. Subjective expectations at the portfolio level $\mathbb{F}_t$ are based on survey forecasts from the IBES survey for both stock returns and earnings growth. Machine expectations $\mathbb{E}_t$ are based on forecasts from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, where parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large-scale dataset of macroeconomic, financial, and textual data. The sample is quarterly and spans 2005Q1 to 2023Q4.

A.2 Gradual Adjustment of Expectations

To provide evidence on the dynamics of belief formation, this section examines how survey respondents revise their expectations about future earnings following an earnings surprise. The following regression estimates the responsiveness of long-horizon forecasts to short-term earnings news:

$$\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j},$$

where $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}]$ denotes the expectation formed at time $t+j$ for earnings-related variable $\widetilde{x}$ at horizon h, and $\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]$ captures the earnings surprise. The coefficient $\gamma_{h,j}$ measures how much of the surprise is incorporated into expectations for long-run outcomes.

Table A.2 reports estimates for two forward-looking variables: (a) long-run earnings growth, and (b) the long-run ratio of earnings to employment. The target horizon is fixed at $h = 5$ years, while the revision horizon j ranges from 1 to 4 years. The estimated $\gamma_{h,j}$ coefficients are uniformly small and often statistically insignificant, indicating that respondents only partially incorporate short-term earnings news into their long-run expectations. This pattern is consistent with models of belief formation under constant-gain learning, in which agents update expectations gradually and exhibit fading memory. In such models, a fixed updating gain leads to persistent deviations from rational expectations and a breakdown of the law of iterated expectations.

Table A.2: Gradual adjustment of expectations

Target Horizon h (Years)	5	5	5	5
Revision Horizon j (Years)	1	2	3	4
Survey Forecast Revisions: $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j}$				
(a) Earnings Growth	0.0929	0.0934	0.1121	0.1245
	(0.0734)	(0.0455)	(0.0776)	(0.0743)
(b) Earnings to Employment	0.0600	0.0508	0.0697	0.0745
	(0.1281)	(0.0725)	(0.0321)	(0.0419)

Notes: Table shows the gradual adjustment of expectations about future earnings $\widetilde{x}_{i,t+h}$ after an earnings surprise at $t+1$. Sample: 2005Q1 to 2023Q4. Newey-West t-statistics with lags = 4 reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3 Variance Decomposition of Job Filling Rate

A.3.1 Baseline Specification

Table A.3 reports a variance decomposition of the aggregate job filling rate based on equation (18). Under rational expectations, discount rate fluctuations explain the largest share of variation, accounting for 69.1% at the 5-year horizon. Under subjective expectations, cash flow beliefs dominate at all horizons, accounting for 96.7% in the 5-year horizon.

Table A.3: Time-Series Decomposition of the Job Filling Rate

Horizon h (Years)	1	2	3	4	5
(a) Rational Expectations: $\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}]$					
Discount Rate	0.187***	0.309***	0.585***	0.653***	0.691***
t-stat	(3.310)	(4.708)	(5.977)	(6.974)	(6.659)
Cash Flow	0.027	0.026	0.051	0.055	0.066
t-stat	(0.090)	(0.181)	(0.364)	(0.459)	(0.472)
Price-Earnings	0.799***	0.720***	0.415***	0.331***	0.201**
t-stat	(5.620)	(4.322)	(3.332)	(2.845)	(1.716)
Residual	−0.013	−0.054	−0.051	−0.039	0.042
t-stat	(−0.030)	(−0.141)	(−0.076)	(−0.046)	(0.049)
N	76	76	76	76	76
(b) Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$					
Discount Rate	−0.007	−0.005	−0.019	−0.014	−0.010
t-stat	(−0.457)	(−0.130)	(−0.400)	(−0.157)	(−0.091)
Cash Flow	0.325***	0.641***	0.717***	0.892***	0.967***
t-stat	(3.939)	(4.500)	(4.661)	(5.572)	(7.097)
Price-Earnings	0.629***	0.366***	0.206***	0.068	0.028
t-stat	(8.383)	(4.231)	(2.896)	(0.701)	(0.313)
Residual	0.052	−0.002	0.096	0.054	0.015
t-stat	(0.186)	(−0.008)	(0.292)	(0.126)	(0.039)
N	76	76	76	76	76

Notes: This table reports variance decompositions of the aggregate job filling rate under rational expectations (panel (a)) or subjective expectations (panel (b)). Each row reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Residual term represents the variation in job filling rates that are not captured by the other components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3.2 Biases in Subjective Beliefs and Job Filling Rate

To directly quantify the importance of biases in subjective beliefs, I consider predictive regressions of biases in subjective expectations of discount rates, cash flows, and price-earnings ratios on the job filling rate. I define the bias as the difference between subjective and machine expectations. Table A.4 reports estimates $\beta_{1,B}$ from regressing biases in subjective discount rate, cash flow, and log price-earnings expectations on the job filling rate:

$$Bias_t[y_{t,t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \qquad y = r, e, pe$$

where the $Bias_t[y_{t,t+h}] \equiv \mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}]$ is defined as the difference between subjective and machine expectations of the same variable.

The results indicate that biases in survey forecasts are important contributors to fluctuations in job filling rates, especially at longer horizons. At the 5-year horizon, biases in cash flow expectations lead survey respondents to over-weight 90.1% of the variation in job filling rates to the cash flow component. This mis-perception is counteracted by biases in subjective discount rate expectations, which leads survey respondents to under-weight 70.1% of the variation in the job filling rate. These findings emphasize the importance of belief distortions in driving labor market fluctuations. The profile of the response across forecast horizons is broadly consistent with the profile of the MSE ratios across horizons in Figure 2. For discount rate and cash flow expectations, the machine outperformed the survey by a wider margin over longer horizons, suggesting that the bias in survey responses likely play a bigger role over these longer horizons.

Table A.4: Biases in Subjective Beliefs and the Job Filling Rate

Horizon h (Years)	1	2	3	4	5
Biases: $\mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \quad y = r, e, pe$					
Discount Rate	-0.194	-0.313^{**}	-0.604^{***}	-0.667^{***}	-0.701^{***}
t-stat	(-1.574)	(-2.167)	(-2.896)	(-2.918)	(-2.740)
(-) Cash Flow	0.299	0.615^{***}	0.666^{***}	0.837^{***}	0.901^{***}
t-stat	(1.421)	(5.476)	(5.703)	(7.365)	(6.665)
(-) Price-Earnings	-0.170	-0.354^{**}	-0.209	-0.262	-0.174
t-stat	(-0.464)	(-2.373)	(-0.503)	(-0.479)	(-0.292)
Residual	-0.065	-0.052	-0.147	-0.093	0.026
t-stat	(-0.148)	(-0.219)	(-0.306)	(-0.154)	(0.040)
N	76	76	76	76	76

Notes: This table reports estimates $\beta_{1,B}$ from regressing the survey bias $\mathbb{F}_t[y_{t,t+h}] - \mathbb{E}_t[y_{t,t+h}]$ on the job filling rate q_t. $y_{t,t+h}$ denotes the dependent variable of type j to be predicted h years ahead of time t. The components of the decomposition are h-year present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$. The residual term captures variation in the bias that cannot be explained by the three components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The bias is defined as the difference between subjective and machine expectations: $Bias_t = \mathbb{F}_t - \mathbb{E}_t$. The sample is quarterly from 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3.3 Additional Controls

The large contribution from subjective long-term cash flow expectations in explaining the job filling rate is robust to conditioning on additional variables that could distort the relationship. Table A.5 re-estimates the subjective variance decomposition at the 5 year horizon with additional control variables on the right-hand side of the regression: 1 year lag of the log job filling rate and the dependent variable, and the 1 year ahead survey forecast of the same variable. Controlling for the short-term expectation $\mathbb{F}_t[y_{t+1}]$ accounts for the possibility that survey respondents' long-term forecasts could be influenced by the short-term component of cash flows (Nagel and Xu, 2021).

Table A.5: Variance Decomposition of Job Filling Rate: Additional Controls

Dep. Var.	Discount Rate	(-) Cash Flow	(-) Price-Earnings
Horizon h (Years)	5	5	5
Subjective Expectations: $\mathbb{F}_t[y_{t,t+h}] = \beta_{0,\mathbb{F}} + \beta_{1,\mathbb{F}} \log q_t + \beta_{2,\mathbb{F}} \log q_{t-1} + \beta_{3,\mathbb{F}} \mathbb{F}_{t-1}[y_{t+h-1}] + \beta_{4,\mathbb{F}} \mathbb{F}_t[y_{t+1}] + \varepsilon_{t,\mathbb{F}}$			
Share of job filling rate variation	-0.007	0.855^{***}	0.049
t-stat	(-0.108)	(4.865)	(0.455)
Adj. R^2	0.456	0.514	0.533
N	76	76	76
Controls	Yes	Yes	Yes

Notes: Table reports variance decompositions of the job filling rate under subjective expectations $\mathbb{F}_t$ implied by survey forecasts. $y_{t,t+h}$ denotes the dependent variable of type j to be predicted $h = 5$ years ahead of time t: h year present discounted values of discount rates ($r_{t,t+h} = \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$), cash flows ($e_{t,t+h} = el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$), and log price-earnings ratios ($pe_{t,t+h} = \rho^h pe_{t+h}$). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. The sample is quarterly over 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3.4 Model vs. Data

Table A.6 compares the variance decomposition obtained using survey-based subjective expectations and machine learning-based rational expectations against the theoretical predictions from prominent search-and-matching models in the literature. Hall (2017) and DMP (Diamond-Mortensen-Pisarides) models predict that discount rate fluctuations should explain 78.2% of the variance of job filling rates. Kehoe et al. (2022) (KLMP model) predict a more balanced role for both discount rates and cash flows, attributing 54.3% to discount rates and 31.9% to cash flows. Empirical results using data on subjective expectations differ from these models, showing that firms place almost no weight on discount rates (-1.0%) and instead attribute 96.7% of the variance to cash flows. These differences suggest that belief distortions play a substantial role in shaping labor market fluctuations and challenge the standard rational expectations assumption in existing search models.

Table A.6: Variance Decomposition of Job Filling Rate: Model vs. Data

Dep. Var.	Discount Rate	(-) Cash Flow	(-) Price-Earnings	Residual
Horizon h (Years)	5	5	5	5
(a) Rational Expectations: $\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}]$				
Data (Machine)	0.691***	0.066	0.201	0.042
t-stat	(3.329)	(0.472)	(0.245)	
Model (DMP)	0.782***	0.017**	0.201***	0.000
t-stat	(12.334)	(1.992)	(47.883)	
Model (Hall)	0.838***	0.073	0.088	0.000
t-stat	(12.000)	(1.387)	(1.074)	
Model (KLMP)	0.543***	0.319	0.138***	0.000
t-stat	(4.484)	(0.937)	(16.392)	
Model (Learning)	0.472**	0.084	0.445	0.000
t-stat	(2.541)	(0.922)	(1.023)	
(b) Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$				
Data (Survey)	-0.010	0.967***	0.028	0.015
t-stat	(-0.091)	(7.097)	(0.078)	
Model (Learning)	-0.001	0.740***	0.261	0.000
t-stat	(-0.011)	(6.689)	(1.706)	

Notes: Table compares the variance decomposition estimated from the data (Table A.3) against the implied decomposition from simulations of alternative search-and-matching models. The models are simulated annually over 500 periods and 300 firms, discarding the first 150 periods as a burn-in, All parameter values in the calibration use estimates from the original papers. Learning: Constant-gain learning model from Section 8; DMP: Diamond-Mortensen-Pissarides Model; Hall: Hall (2017); KLMP: Kehoe et al. (2022). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.3.5 VAR Estimates

To validate the robustness of the variance decompositions, I estimate a Vector Autoregression (VAR) for the log job filling rate $\log q_t$ and its forward-looking components under subjective or rational expectations. For the case of subjective beliefs, the VAR is estimated using survey expectations for future returns, earnings growth, and price-earnings ratios:

$$X_{t+1} = AX_t + \varepsilon_{t+1}, \quad X_t = [\mathbb{F}_t[r_{t,t+1}] \quad \mathbb{F}_t[e_{t,t+1}] \quad \mathbb{F}_t[pe_{t,t+1}] \quad \log q_t]'.$$

From the theoretical framework in Section 2, the log job filling rate can be decomposed as:

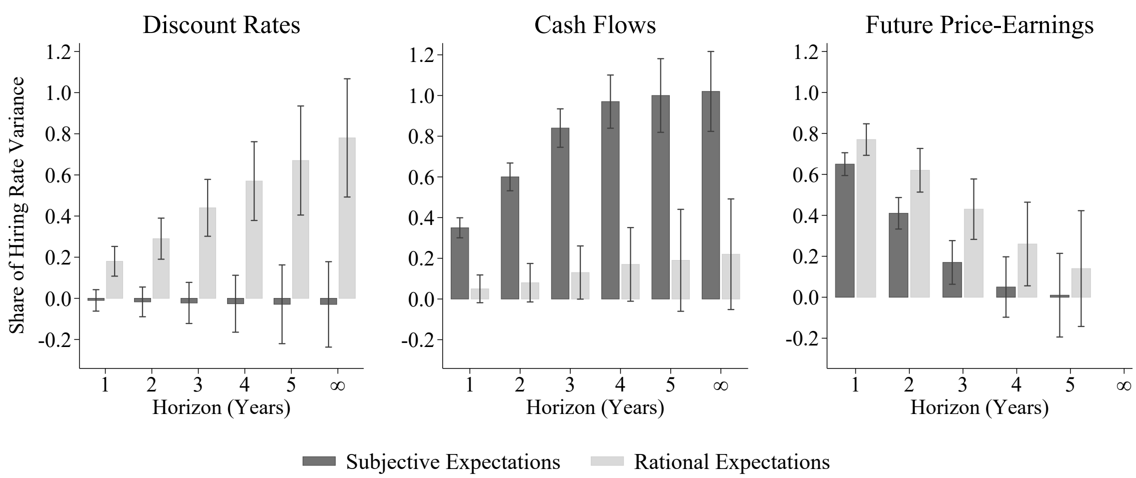
$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \rho^h \mathbb{F}_t[pe_{t+h}]$$

where the expected present values $\mathbb{F}_t[r_{t,t+h}]$ and $\mathbb{F}_t[e_{t,t+h}]$ are constructed recursively using the VAR forecast. As $h \to \infty$, the terminal value $\rho^h \mathbb{F}_t[pe_{t+h}]$ converges to zero under a transversality condition, yielding the long-run decomposition:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+\infty}] - \mathbb{F}_t[e_{t,t+\infty}].$$

The same procedure is repeated using machine learning forecasts $\mathbb{E}_t[\cdot]$ to obtain the decomposition under rational expectations. Figure A.2 reports variance shares across horizons $h = 1$ to $h = 5$, as well as the full-horizon case $h = \infty$. Under rational expectations, discount rate fluctuations explain an increasing share of variation, rising to 78.1% at long horizons. Under subjective expectations, cash flow beliefs dominate at all horizons, accounting for 102.0% in the long run.

Figure A.2: Variance Decomposition of Job Filling Rate: VAR Estimates



Notes: Figure reports variance decompositions of the aggregate job filling rate based on a Vector Autoregression (VAR). Each panel reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows bootstrapped 95% confidence intervals.

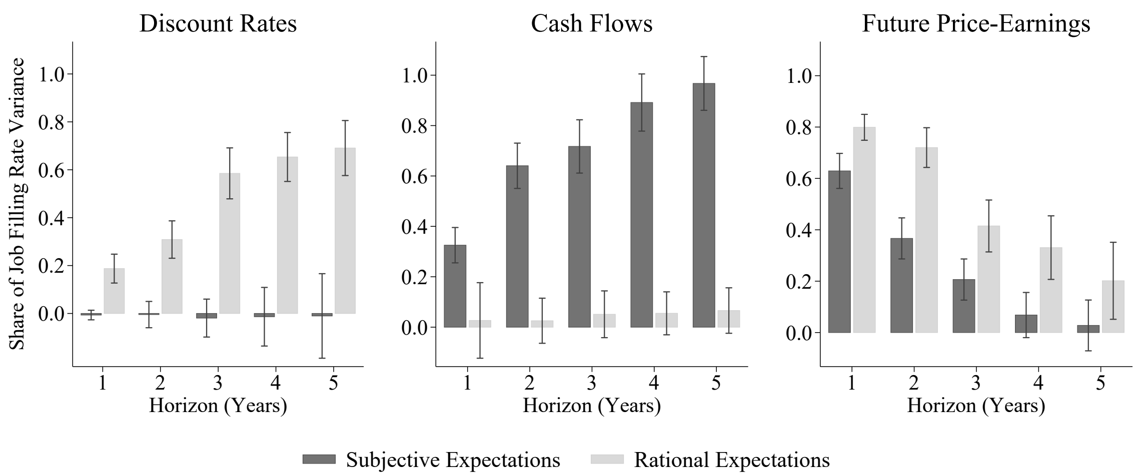
A.3.6 First Differences

The decomposition in equation (18) may be more accurate in first differences than in levels, as low-frequency variation in the job filling rate or subjective expectations can introduce measurement error. This concern is similar to the argument in Cochrane (1991), who points to low-frequency changes in fundamentals as a potential source of measurement error in the context of the q-theory of investment. Figure A.3 estimates the variance decomposition of the job filling rate from equation (18) in first differences:

$$\Delta \log q_t = \Delta \mathbb{E}_t[r_{t,t+h}] - \Delta \mathbb{E}_t[e_{t,t+h}] - \Delta \mathbb{E}_t[pe_{t,t+h}]$$
$$\Delta \log q_t = \Delta \mathbb{F}_t[r_{t,t+h}] - \Delta \mathbb{F}_t[e_{t,t+h}] - \Delta \mathbb{F}_t[pe_{t,t+h}]$$

Under rational expectations, discount rate fluctuations explain the largest share of variation, accounting for 58.7% at the 5-year horizon. Under subjective expectations, cash flow beliefs dominate, accounting for 90.6% at the 5-year horizon.

Figure A.3: Variance Decomposition of Job Filling Rate: First Differences



Notes: Figure reports variance decompositions of the aggregate job filling rate in first differences. Each panel reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

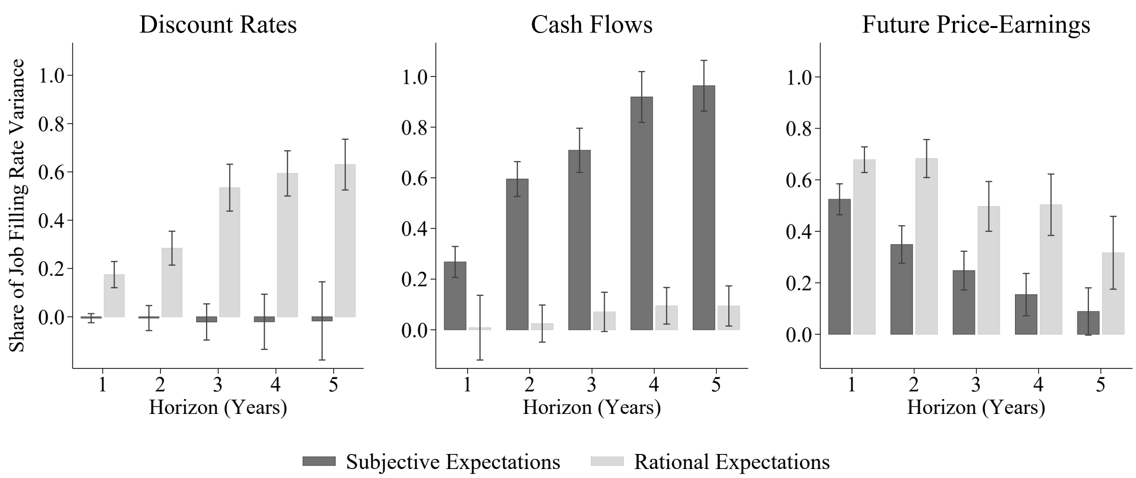
A.3.7 All Listed Firms

Figure A.4 reports the variance decomposition of the job filling rate from equation (18) using an expanded definition of subjective expectations that includes all publicly listed firms with IBES analyst coverage, rather than restricting to the S&P 500. Subjective cash flow expectations are computed as value-weighted aggregates of IBES median forecasts of long-horizon earnings growth across all covered firms. Subjective discount rate expectations are constructed analogously, using the same survey-based measures as in the baseline but applying the expanded firm universe for consistency in coverage.

The results are similar to the baseline. Under rational expectations, discount rate fluctuations explain 63.0% of the variation in job filling rates at the 5-year horizon, while under subjective expectations, distorted cash flow beliefs remain dominant, accounting for 96.3%. The similarity in results suggests that the dominance of cash flow distortions under subjective beliefs is not specific to large-cap firms in the S&P 500 but holds more broadly across publicly listed firms with analyst coverage.

While this paper focuses on publicly listed firms due to data limitations, preliminary evidence suggest that similar patterns likely emerge among smaller private businesses. A 2010 report from the National Federation of Independent Business (NFIB) on small business credit during the recession shows that hiring decisions were primarily driven by pessimism about future sales rather than financing constraints. At the time, 51% of small employers cited weak sales expectations as their top concern, compared to just 8% who cited access to credit (Dennis, 2010). To the extent that access to credit capture financial frictions that would show up in discount rates, this survey suggests that subjective beliefs about future cash flows also shape employment decisions in the small business sector.

Figure A.4: Variance Decomposition of Job Filling Rate: All Listed Firms

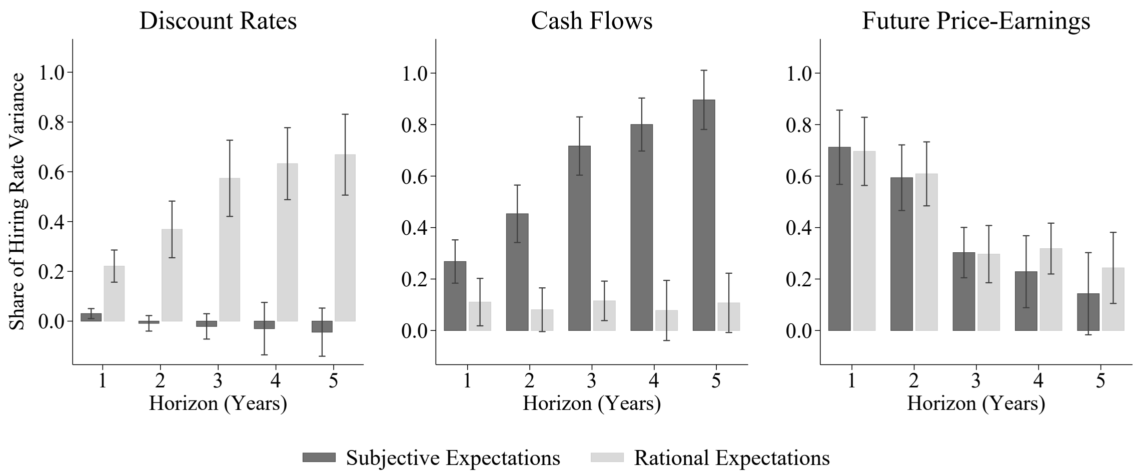


Notes: Figure reports variance decompositions of the aggregate job filling rate using forecasts aggregated over all publicly listed firms with IBES analyst coverage. Each panel reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on IBES survey forecasts of financial analysts aggregated over all covered firms. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

A.3.8 Extended Historical Sample

Figure A.5 reports the variance decomposition of the job filling rate from equation (18) using an extended quarterly sample from 1983Q4 to 2023Q4. Subjective cash flow expectations are measured using IBES survey forecasts of earnings growth, available from 1983Q4. Subjective discount rate expectations are extended by extracting a common latent component from multiple historical survey sources using a state-space model estimated via the Kalman filter, where the latent state $S_t \equiv \mathbb{F}_t[r_{t+h}]$ captures subjective beliefs about h-month ahead stock returns. The observation vector includes return expectations from the Gallup/UBS, CFO, SOC, and Livingston surveys, with missing data handled through the Kalman filter. The extended sample results are consistent with the baseline. Under rational expectations, discount rate fluctuations explain 66.9% of job filling rate variation at the 5-year horizon. Under subjective expectations, distorted cash flow beliefs dominate, accounting for 89.6%.

Figure A.5: Variance Decomposition of Job Filling Rate: Extended Sample 1983Q4–2023Q4



Notes: Figure reports variance decompositions of the aggregate job filling rate using an extended sample from 1983Q4 to 2023Q4. Each panel reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

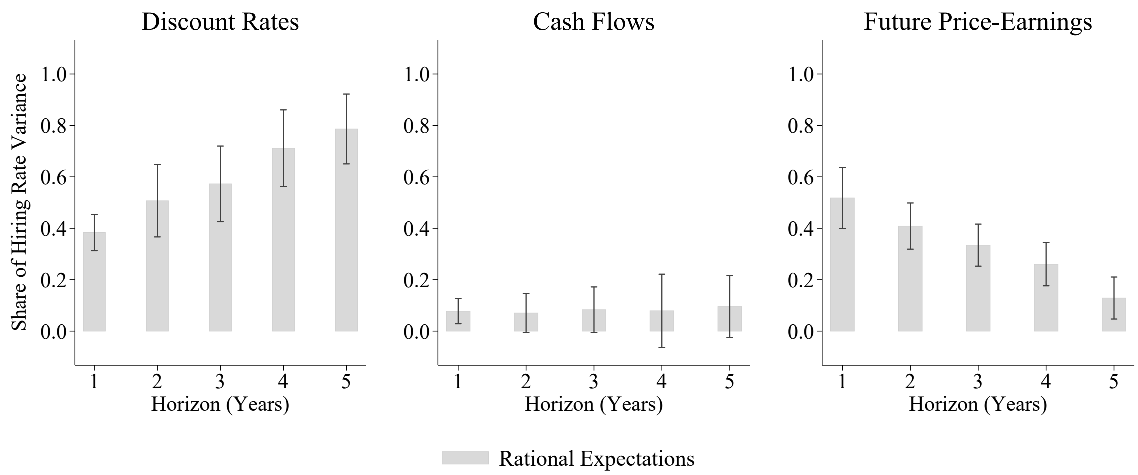
A.3.9 Ex-post Decomposition

Since the log-linear decomposition of the job filling rate holds both ex-ante and ex-post, a variance decomposition of the job filling rate can also be estimated using ex-post realized data, under the assumption of the firm's perfect foresight:

$$1 \approx \underbrace{\frac{Cov\left[r_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate news}} - \underbrace{\frac{Cov\left[e_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[pe_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}}$$

Table A.6 reports the estimates. For the main sample covering 2005Q1 to 2023Q4, at the 5 year horizon, 79.4% of the variation in the job filling rate is driven by discount rate news. In contrast, cash flow news has a smaller effect, contributing only 10.3% over the same period. For the full sample covering 1965Q1 to 2023Q4, at the 5 year horizon, 78.6% of the variation in the job filling rate is driven by discount rate news. In contrast, cash flow news has a smaller effect, contributing only 9.5% over the same period.

Figure A.6: Variance Decomposition of Job Filling Rate: Ex-Post Measure 1965Q1–2023Q4

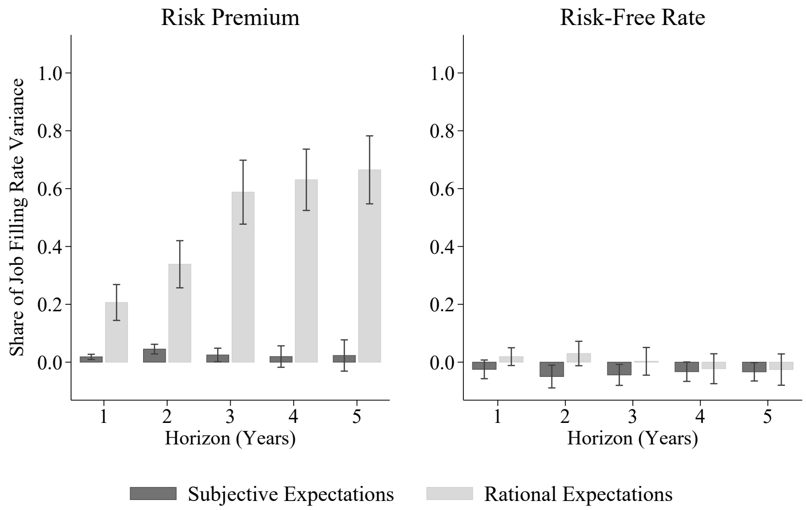


Notes: Figure reports variance decompositions of the job filling rate from equation using ex-post realized outcomes. Each panel reports the share of the variation in job filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (12). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 1965Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

A.3.10 Risk Premia vs. Risk-Free Rate

Risk-free rates play only a small role in explaining fluctuations in job filling rates. Figure A.7 plots estimates from regressing subjective expectations implied by forecasts from the Survey of Professional Forecasters (SPF), and machine expectations of h year ahead annualized log 3-month Treasury bill rates on the the job filling rate. Under all measures of beliefs and all horizons considered, the contribution from risk-free rates explain less than 5% of the variation in job filling rates. The result suggests that the significant contribution of rational discount rates in Table A.3 is driven by fluctuations in risk premia instead of risk-free rates.

Figure A.7: Variance Decomposition of Job Filling Rate: Risk Premia vs. Risk-Free Rate



Notes: Figure plots estimates from regressing h year present discounted value of annualized log 3-month Treasury bill rates $\sum_{j=1}^{h} \rho^{j-1} r_{t+j}^{f}$ on the the job filling rate under alternative assumptions about the firm's beliefs. Subjective expectations $\mathbb{F}_t$ of risk-free rates are based on survey forecasts from the Survey of Professional Forecasters. Subjective expectations of the equity risk premium is defined as the difference between CFO survey S&P 500 stock return forecast and the SPF risk-free rate forecast. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.3.11 Risk-Neutral Measure Implied by Futures Prices

To address whether systematic forecast errors simply reflect risk compensation rather than belief distortions, I re-evaluate the decomposition using risk-neutral expectations extracted from futures prices. Under risk-neutral pricing, forecast errors should equal risk premia plus noise, with no systematic patterns beyond those explained by time-varying risk compensation. In contrast to subjective survey forecasts, which may reflect belief distortions, risk-neutral expectations are extracted directly from financial market prices and reflect the valuations of marginal investors in the economy. The decomposition parallels the earlier analysis based on subjective beliefs but replaces the expectations operator $F_t[\cdot]$ with the risk-neutral operator $\mathbb{E}_t^Q[\cdot]$, where Q denotes the risk-neutral probability measure. I begin with the ex-post decomposition of the job filling rate $\log q_t$, which can be expressed as:

$$\log q_t = c_q + \sum_{j=1}^{h} \rho^{j-1} r_{t+j} - \left(dl_t + \sum_{j=1}^{h} \rho^{j-1} \Delta d_{t+j} \right) - \rho^h pd_{t+h}$$

where r_{t+j} denotes the return on the S&P 500 index, Δd_{t+j} denotes the change in log dividends, and pd_{t+h} is the terminal log price-dividend ratio. To evaluate this decomposition under the risk-neutral measure, I replace each future variable with its risk-neutral expectation. Using the standard no-arbitrage pricing result that the futures price equals the risk-neutral expectation of the future spot price (Ait-Sahalia et al., 2001), I compute the expected return over horizon h using log differences of S&P 500 futures prices:

$$\mathbb{E}_t^Q[r_{t,t+h}] = \sum_{j=1}^{h} \rho^{j-1} (f_{t,t+j}^{sp500} - f_{t,t+j-1}^{sp500})$$

where $f^{sp500}_{t,t+j}$ denotes the log futures price of the S&P 500 at time t for delivery at $t+j$, and $f^{sp500}_{t,t} \equiv p_t$ is the log spot price. Similarly, I measure expected dividend growth using dividend futures:

$$\mathbb{E}^Q_t[d_{t,t+h}] = dl_t + \sum_{j=1}^{h} \rho^{j-1}(f^{div}_{t,t+j} - f^{div}_{t,t+j-1})$$

where $f^{div}_{t,t+j}$ is the log price of the dividend future for maturity $t+j$, and $f^{div}_{t,t} \equiv d_t$ is the log of current dividends. To compute the terminal price-dividend ratio $\mathbb{E}^Q_t[pd_{t+h}]$, I apply a forward iteration of the log-linear price-dividend identity:

$$\mathbb{E}^Q_t[pd_{t+h}] = \frac{1}{\rho^h} pd_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1}(c_{pd} + \mathbb{E}^Q_t[\Delta d_{t+j}] - \mathbb{E}^Q_t[r_{t+j}])$$

where c_{pd} is a constant from the log-linearization. Since market data on futures prices is typically limited to near-term maturities (e.g., 1-year ahead), I extrapolate longer-horizon expectations using fitted values from autoregressive models. Specifically, I estimate AR(1) processes for the 1-year futures returns and dividend growth:

$$\begin{aligned} f^{sp500}_{t,t+1} - p_t &= \mu_{sp500} + \rho_{sp500}(p_t - p_{t-1}) + \varepsilon_t \\ f^{div}_{t,t+1} - d_t &= \mu_{div} + \rho_{div}(d_t - d_{t-1}) + \varepsilon_t \end{aligned}$$

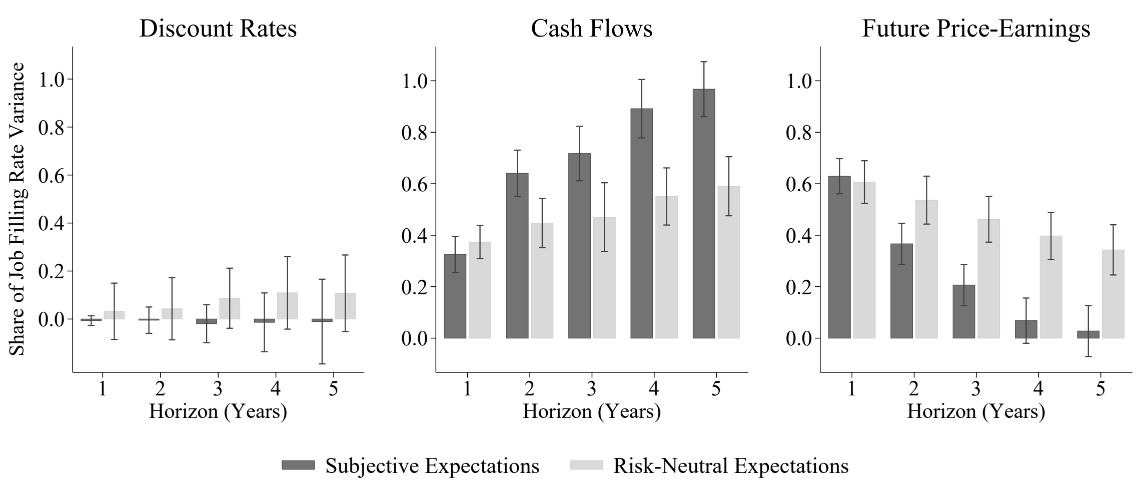
and then forecast growth at horizons $j > 1$ recursively:

$$\begin{aligned} f^{sp500}_{t,t+j} - f^{sp500}_{t,t+j-1} &= \frac{\mu_{sp500}(1-\rho^j_{sp500})}{1-\rho_{sp500}} + \rho^{j-1}_{sp500}(f^{sp500}_{t,t+1} - p_t) \\ f^{div}_{t,t+j} - f^{div}_{t,t+j-1} &= \frac{\mu_{div}(1-\rho^j_{div})}{1-\rho_{div}} + \rho^{j-1}_{div}(f^{div}_{t,t+1} - d_t) \end{aligned}$$

Using these forward-imputed values, I compute the full set of risk-neutral expectations required for the decomposition.

The results of this exercise are shown in Figure A.8. Compared to subjective expectations, risk-neutral expectations attribute a smaller role to future cash flows and a greater role to discount rates in explaining the variation in the job filling rate. This contrast suggests that belief distortions in survey forecasts may overweight the informational content of short-term earnings outlooks and underweight changes in risk premia, leading to distorted hiring incentives.

Figure A.8: Variance Decomposition of Job Filling Rate: Risk-Neutral Expectations



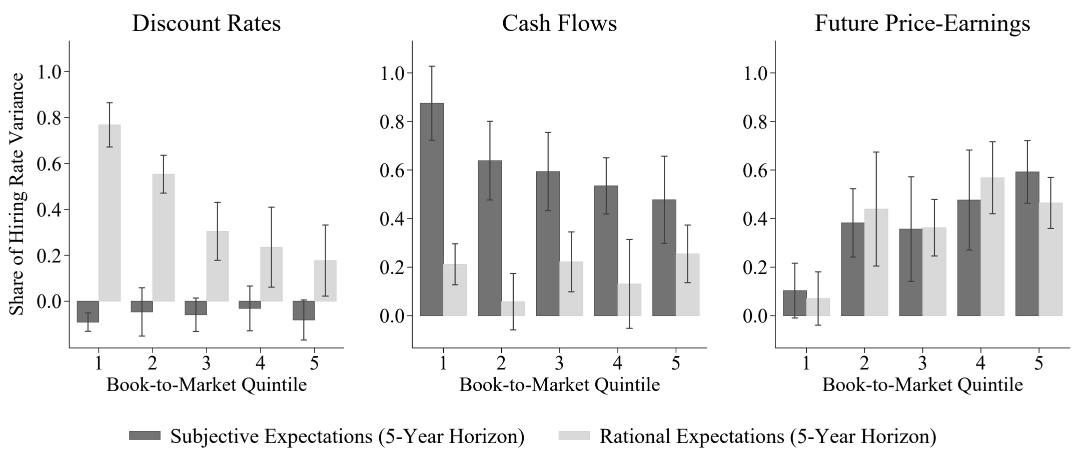
Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate. Light bars show the contribution under risk-neutral expectations implied by S&P 500 and dividend futures. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.3.12 Cross-Sectional Decomposition of Hiring Rate: By B/M and Size Portfolios

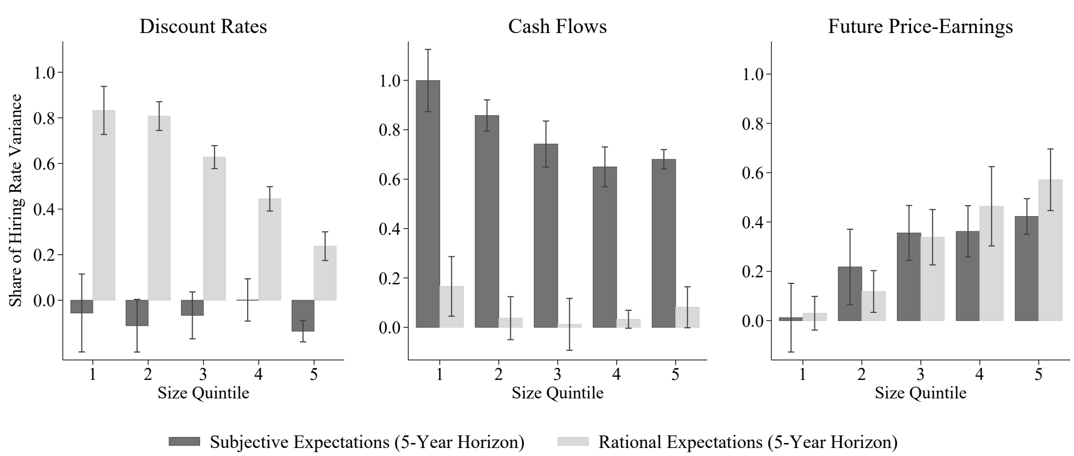
Figure A.9 shows that belief distortions play a significant role in explaining the cross-sectional variation in hiring across book-to-market portfolios (panel (a)) and size portfolios (panel (b)). The decomposition reveals that under subjective expectations, distorted beliefs about future cash flows account for a larger share of hiring rate variation, particularly among low book-to-market (growth) firms and small firms. This pattern is consistent with the idea that growth firms and small firms are more sensitive to subjective beliefs about long-term fundamentals, amplifying the role of distorted expectations in their hiring decisions. In contrast, for high book-to-market (value) firms and large firms, the contribution of cash flow expectations remains relatively stable across subjective and rational benchmarks, suggesting their hiring is less exposed to belief distortions.

Figure A.9: Cross-Sectional Decomposition of Hiring Rate: By Portfolio

(a) By Book-to-Market Portfolio



(b) By Size Portfolio



Notes: Figure estimates time-series decomposition of hiring rate separately for each of the five book-to-market (panel (a)) and size (panel (b)) portfolios. Firms have been sorted into five value-weighted portfolios by book-to-market ratio or size (market capitalization). Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

A.3.13 Financial Constraints

A natural concern is that variation in hiring may reflect differences in financial constraints rather than distortions in beliefs. In a rational expectations model, financial constraints appear as a Lagrange multiplier that tightens the firm's stochastic discount factor (SDF), raising internal hurdle rates and suppressing hiring (Kehoe et al., 2019). In this setting, constraint-induced fluctuations in hiring would be rationally attributed to higher discount rates. By contrast, under subjective expectations, survey respondents may misattribute the effect of constraints to lower future cash flows, especially if internal hurdle rates are persistent, upward-biased, and unresponsive to market conditions (Gormsen and Huber, 2025). Financial constraints could also allow the effects of belief distortions to persist by limiting arbitrage that would otherwise correct them (De La O et al., 2024).

Measures of Financial Constraints To test these hypotheses, I incorporate firm-level financial constraint measures into the decomposition framework:

- Firm Size (Total Assets): Firms in the bottom tertile of the asset size distribution are classified as financially constrained, while those in the top tertile are unconstrained (Erickson and Whited, 2000).
- Payout Ratio: Defined as dividends plus stock repurchases scaled by total assets. Firms with the lowest (highest) payout ratios are classified as constrained (unconstrained), consistent with the idea that constrained firms conserve internal funds (Fazzari et al., 1988).
- SA Index: The size-age index developed by Hadlock and Pierce (2010), constructed as $\text{SA} = -0.737 \cdot \text{Size} + 0.043 \cdot \text{Size}^2 - 0.040 \cdot \text{Age}$, where Size is log real assets and Age is years since listing. Higher SA values indicate tighter constraints.
- Expected Free Cash Flow: Based on Lewellen and Lewellen (2016), firms are sorted into constraint groups using predicted free cash flow, estimated from cross-sectional regressions on lagged characteristics. Low expected FCF implies tighter constraints.
- WW Index: The Whited-Wu index (Whited and Wu, 2006), a linear combination of cash flow, dividend status, leverage, size, and sales growth, where higher index values imply greater constraints.

Each measure is updated annually and firms are classified based on terciles or continuous index values. Each measure is aggregated to the portfolio level and standardized before entering the regression as controls.

Decomposition with Financial Constraints I modify the baseline decomposition regression as follows:

$$\mathbb{F}_t[\Delta e_{i,t,t+h}] = \beta \cdot \log q_{i,t} + \Gamma \cdot \text{FC}_{i,t} + \alpha_i + \alpha_t + \varepsilon_{i,t}$$

where $\text{FC}_{i,t}$ is a vector of standardized financial constraint measures for portfolio i at time t, aggregated from firm-level values to five value-weighted book-to-market portfolios. As before, the parameter of interest is β, which captures the share of variation in the hiring rate $\log q_{i,t}$ explained by subjective expectations, but this time after controlling for financial constraints. I run analogous regressions to estimate the contributions of discount rate expectations and future price-earnings ratios. I also replace survey forecasts with machine learning forecasts to estimate the decomposition under rational expectations, again controlling for financial constraints using the same specification.

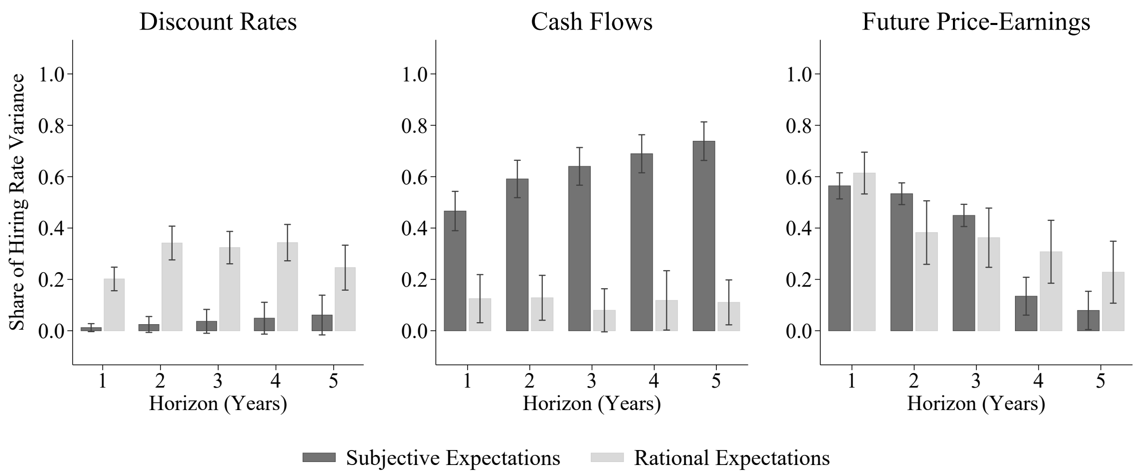
Results Figure A.10 presents the decomposition estimates with and without financial constraint controls, under both subjective and rational expectations. Under subjective expectations, the contribution of expected earnings to hiring variation remains large and significant, with only a modest reduction in explanatory power after controlling for financial constraints. This suggests that distorted beliefs about cash flows persist even after adjusting for observable constraint-related fundamentals. These findings are consistent with the view that constrained firms over-react to cash flow news or internalize persistent pessimism about earnings. Under rational expectations, however, the contribution of discount rate expectations drops substantially once constraint controls are included. This is consistent with a rational model in which financial constraints tighten the SDF and raise internal hurdle rates. When this variation is accounted for, the rational model assigns less importance to discount rate news in explaining hiring variation. The results supports the interpretation that financial constraints can explain a nontrivial share, but do not fully explain, variation in hiring. While rational forecasts attribute constraint effects to discount rates, subjective expectations appear to reflect persistent pessimism about cash flows.

A.3.14 Time-Varying Parameters

Figure A.11 estimates time-series variance decompositions of the job filling rate over rolling samples of trailing 15-year windows, along with 95% confidence intervals based on Newey-West standard errors. The estimated rational discount rate component is large and the rational cash flow component is small throughout the rolling samples. In contrast, the subjective discount rate component is small and the subjective cash flow component is large throughout the rolling samples. The persistent dominance of subjective cash flow expectations across all time periods confirms that belief distortions are not episodic phenomena but represent systematic and enduring features of expectation formation.

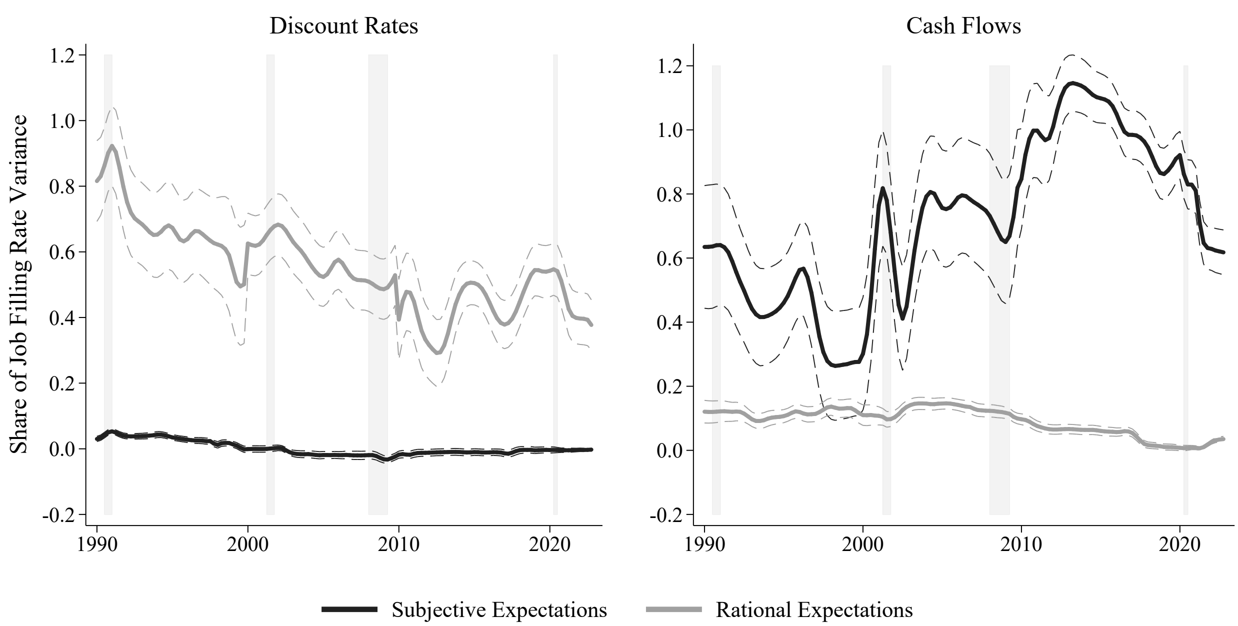
Nevertheless, there is notable variation in the estimated components over time, as the subjective cash flow component shows large increases during recessions. The sharp increases in the subjective cash flow component during recession periods indicate that firms respond to economic downturns by becoming excessively pessimistic about future cash flows. The rational discount rate component exhibits a gradual decline over the sample period, potentially reflecting structural changes in risk premia or monetary policy regimes.

Figure A.10: Cross-Sectional Decomposition of Hiring Rate: Control for Financial Constraints



Notes: Figure estimates time-series decomposition of hiring rate separately for each of the five book-to-market portfolios, controlling for measures of financial constraints. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. Financial constraint controls include firm size, payout ratio, SA index, expected free cash flow, and the Whited-Wu index. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

Figure A.11: Time-Series Decomposition of Job Filling Rate: Time-Varying Parameters



Notes: Figure estimates time-series decomposition of the job filling rate over rolling samples of trailing 15-year windows. Grey line show the contribution under rational expectations. Dark line show the contribution under subjective expectations. Each dashed line shows Newey-West 95% confidence intervals with lags = 4. NBER recessions are shown with light gray shaded bars.

A.4 Alternative Measures of Survey Expectations

A.4.1 Subjective Cash Flow Expectations

The large role played by subjective cash flow expectations in explaining the job filling rate holds more generally across alternative survey forecasts of earnings growth. Table A.7 re-estimates the subjective variance decomposition while replacing IBES survey forecasts of earnings growth with the corresponding forecast from the Bloomberg (BBG) survey. Table A.7 re-estimates the subjective variance decomposition while replacing 1 year ahead IBES survey forecasts of earnings growth with the corresponding forecast from the CFO survey. The forecast horizon for the CFO survey has been limited to $h = 1$ year ahead and the sample covers a shorter period over 2002Q1 to 2019Q3 due to missing earnings growth forecasts in the CFO survey.

To summarize the alternative survey measures into a single series, Filtered Investor (FI) expectations extract the common component of subjective discount rates using a Kalman filter. The state variable is a latent h-month ahead expected stock return capturing investors' subjective beliefs $S_t \equiv \mathbb{F}_t[r_{t+h}]$, which evolves according to an AR(1) state equation $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$, where C, T, R are matrices of the model's primitive parameters $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$. ε_t is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster. α is the intercept, ρ is the persistence, and σ_ε is the standard deviation of the latent innovation error. The Observation equation takes the form $X_t = D + ZS_t + Uv_t$, where h is a fixed forecast horizon. The observation vector X_t contains measures of survey expected cash flows from IBES, BBG, and CFO surveys over the next h periods. v_t is a vector of observation errors with standard deviations in the diagonal matrix U. Z and D are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters $\alpha, \rho, \sigma_\varepsilon, U$. Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table A.7: Variance Decomposition of Job Filling Rate: Alternative Subjective Cash Flow Expectations

Horizon h (Years)	1	2	3	4	5
Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$					
(a) Filtered Investor (FI) Expectations					
(-) Cash Flow	0.578***	0.625***	0.684***	0.887***	0.933***
t-stat	(3.046)	(4.275)	(4.894)	(6.019)	(7.612)
N	76	76	76	76	76
(b) Bloomberg (BBG) Survey					
(-) Cash Flow	0.586***	0.830***	0.851***	0.896***	0.949***
t-stat	(8.476)	(8.317)	(7.213)	(5.288)	(4.541)
N	76	76	76	76	76
(c) CFO Survey					
(-) Cash Flow	0.637*				
t-stat	(1.934)				
N	71				

Notes: Table reports variance decompositions of the job filling rate while replacing IBES earnings growth forecast with alternative surveys as measures of subjective cash flows. FI summarizes the alternative survey measures into a single series using a Kalman filter. The sample for BBG and FI is quarterly from 2005Q1 to 2023Q4. The sample for CFO is quarterly from 2002Q1 to 2019Q3. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.4.2 Subjective Discount Rates

The small role played by subjective discount rate expectations in explaining the job filling rate holds more generally across alternative survey forecasts of stock returns. Table A.8 reports estimates from regressing 1 year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log job filling rate q_t under alternative survey forecasts of stock returns. In all survey measures, the estimates suggest a weak relationship between subjective stock return expectations $\mathbb{F}_t^s[r_{t,t+h}]$ and the job filling rate q_t.

$r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2023). The forecast horizon has been limited to 1 year ahead due to limited data availability in the alternative surveys. The sample is quarterly over 2005Q1 to 2023Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2008Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2023Q4 from Q2 and Q4 of each calendar year for Livingston.

To summarize the alternative survey measures into a single series, I extract the common component of subjective discount rates using a Kalman filter. The state variable is a latent h-month ahead expected stock return capturing investors' subjective beliefs $S_t \equiv \mathbb{F}_t[r_{t+h}]$, which evolves according to an AR(1) state equation $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$, where C, T, R are matrices of the model's primitive parameters $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$. ε_t is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster. α is the intercept, ρ is the persistence, and σ_ε is the standard deviation of the latent innovation error. The Observation equation takes the form $X_t = D + ZS_t + Uv_t$, where $h = 12$ months is a fixed forecast horizon. The observation vector X_t contains measures of survey expected returns listed above over the next h periods. v_t is a vector of observation errors with standard deviations in the diagonal matrix U. Z and D are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters $\alpha, \rho, \sigma_\varepsilon, U$. Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table A.8: Variance Decomposition of Job Filling Rate: Alternative Discount Rates

Horizon h (Years)	1	1	1	1	1	1	1	1
	Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t^s[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$							
Survey s	FI	NX	CB	SOC	Gallup	Liv	CoC	Hurdle
Discount Rate	0.013	−0.011	0.026	0.002	−0.065	0.067	0.024	0.013
t-stat	(0.614)	(−0.249)	(0.504)	(0.103)	(−0.922)	(0.181)	(0.734)	(0.522)
Adj. R^2	0.070	0.012	0.069	0.009	0.216	0.045	0.232	0.154
N	76	76	76	76	16	40	76	76

Notes: Table reports slope (β_1) estimates from regressing $h = 1$ year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log job filling rate q_t. $r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2023). Filtered Investor (FI) expectations summarize the alternative survey measures into a single series using a Kalman filter. The sample is quarterly over 2005Q1 to 2023Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2008Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2023Q4 from Q2 and Q4 of each calendar year for Livingston. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.5 Regional Model and Shift-Share Instrument

The aggregate analysis in Section 5 shows that belief distortions in subjective expectations play an important role in explaining hiring fluctuations. This section extends that analysis by exploiting cross-sectional variation in state-level data to strengthen identification and test whether the theoretical mechanism generalizes beyond aggregate dynamics.

Overview While the aggregate-level variance decompositions are informative, they cannot establish causality. The limited number of business cycles in the time series also restricts inference. This section addresses these challenges by extending the aggregate model to a regional framework. In estimating the regional model, I introduce a Bartik shift-share instrument for survey expectations to address endogeneity challenges in identifying the relative importance of subjective discount rate and cash flow expectations. Specifically, I investigate whether regional labor markets characterized by more distorted subjective cash flow expectations experience larger swings in job filling rates. This analysis is motivated by empirical evidence of substantial geographic variation in unemployment dynamics, especially during crises (Beraja et al., 2019, Kehoe et al., 2019; Chodorow-Reich and Wieland, 2020). While existing work studies these regional differences under a rational expectations framework, differences in subjective beliefs may also be an important explanatory factor.

Regional Model To guide the empirical strategy, I extend the baseline search model to a multi-region, multi-sector environment, building from the models in Kehoe et al. (2019) and Chodorow-Reich and Wieland (2020). The economy consists of a continuum of islands indexed by s. Each island produces a differentiated variety of tradable goods that is consumed everywhere and a nontradable good. Both of these goods are produced using intermediate goods. Each consumer is endowed with one of two types of skills which are used in different intensities in the nontradable and tradable goods sectors. Labor is immobile across islands but can switch sectors. This assumption aligns with empirical evidence indicating that labor markets are predominantly local in nature (Manning and Petrongolo, 2017). Consumers receive utility from a composite consumption good that is either purchased in the market or produced at home. Consumers and firms are ex-ante homogeneous and share the same subjective expectation $\mathbb{F}_t[\cdot]$. The islands only differ in the shocks that hit them.

Predictability of Regional Unemployment Rates In this environment, the log unemployment rate $u_{s,t+1}$ in region s approximately satisfies the following predictive relationship (Section B.4):

$$u_{s,t+1} = \beta_r \mathbb{F}_t[r_{s,t,t+h}] + \beta_e \mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.1}$$

where $X_{s,t} \equiv [u_{s,t}, \log \theta_{s,t}, \log \delta_{s,t}]'$ collects standard labor market controls: the lagged unemployment rate $u_{s,t}$, the log vacancy-to-unemployment ratio $\log \theta_{s,t}$, and the log separation rate $\log \delta_{s,t}$. The cross-sectional unit s corresponds to U.S. states, and time t is measured at the monthly frequency. Following Korniotis (2008), each firm is assigned to the state in which it is headquartered. The regression includes state fixed effects α_s to absorb time-invariant regional heterogeneity and time fixed effects α_t to capture national shocks. The coefficients of interest, β_r and β_e, quantify the effect of subjective expectations about discount rates and cash flows, respectively, on future unemployment.

This regional equation extends the aggregate specification in equation (23), and is designed to test whether perceived shocks to discount rates or earnings forecasts help explain variation in unemployment across local labor markets. If firms form biased beliefs about future returns or earnings, those belief distortions should manifest in regional hiring behavior and thus influence unemployment at the state level. A counterpart regression can be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with machine learning-based forecasts $\mathbb{E}_t[\cdot]$.

Empirical Specification: OLS As a baseline, I estimate the regression above using multivariate OLS applied to a panel of state-level data. This allows for a direct assessment of whether variation in firm-level beliefs, aggregated to the state level, predicts changes in unemployment. The future price-earnings ratio term $\mathbb{F}_t[pe_{s,t,t+h}]$ is omitted from the regression due to its near collinearity with forecasted discount rates and cash flows via the present-value identity of Campbell and Shiller (1988). State-level forecasts of discount rates $\mathbb{F}_t[r_{s,t,t+h}]$ are constructed from IBES price target forecasts. These targets are used to infer expected returns by back-solving from analysts' price projections. Forecasts are assigned to states based on firm headquarters and then aggregated using value-weighted averages. Expected cash flows $\mathbb{F}_t[e_{s,t,t+h}]$ are constructed analogously from IBES analyst forecasts of earnings per share.

Regional labor market variables are constructed from publicly available BLS datasets. Unemployment rates $u_{s,t}$ are sourced from the Local Area Unemployment Statistics (LAUS). The vacancy-to-unemployment ratio $\theta_{s,t}$ is computed using job openings from the state-level Job Openings and Labor Turnover Survey (JOLTS) combined with unemployment counts from LAUS. Separation rates $\delta_{s,t}$ are also taken from JOLTS. Monthly series are time-aggregated to the quarterly frequency by averaging values within each quarter.

Empirical Specification: Bartik Shift-Share Instrument A key challenge in estimating the regional decomposition is that regional labor market conditions and subjective expectations may be jointly determined, potentially leading to biased estimates. For example, firms might revise their beliefs in response to local shocks in unemployment or hiring, making it difficult to separate cause from effect. Additionally, state-level aggregates of firm-level forecasts may suffer from measurement error if the geographic scope of a firm's operations does not align with the location of its headquarters.

To address these concerns, I construct a leave-one-out Bartik-style shift-share instrument $\widehat{\mathbb{F}}_t[y_{s,t,t+h}]$ that isolates plausibly exogenous variation in subjective expectations at the regional level, while avoiding mechanical feedback between local shocks and the national forecast component:

$$\widehat{\mathbb{F}}_t[y_{s,t,t+h}] = \sum_{i \in I} \phi_{s,i,t-1} \cdot \mathbb{F}_t^{-s}[y_{i,t,t+h}], \qquad \phi_{s,i,t} = \frac{L_{s,i,t}}{\sum_{i' \in I} L_{s,i',t}}, \qquad y \in \{r, e\} \tag{A.2}$$

Here, $\phi_{s,i,t}$ denotes the lagged employment share of industry i in state s, sourced from the Quarterly Census of Employment and Wages (QCEW). $\mathbb{F}_t^{-s}[y_{i,t,t+h}]$ is the national IBES forecast for industry i constructed by excluding all firms headquartered in state s. The leave-one-out structure ensures that local shocks in state s do not mechanically influence the national industry-level forecasts used to construct the instrument, strengthening the validity of the exogeneity assumption. Using the leave-one-out Bartik instrument, I estimate the following predictive regression:

$$u_{s,t+1} = \beta_r \widehat{\mathbb{F}}_t[r_{s,t,t+h}] + \beta_e \widehat{\mathbb{F}}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.3}$$

The coefficients β_r and β_e now reflect the causal effect of variation in subjective discount rate and earnings expectations that is exogenous to state-specific labor market conditions.

Identification Assumptions Compared to the OLS specification, the Bartik approach offers stronger identification by addressing both measurement error and endogeneity concerns. First, it reduces measurement error by replacing noisy state-level aggregates of firm-level forecasts with industry-level forecasts weighted by predetermined employment shares. Second, it mitigates endogeneity by exploiting the fact that national industry trends in expectations are unlikely to respond to contemporaneous state-level labor market shocks.

For example, consider a scenario where national energy sector earnings expectations surge due to geopolitical developments. The shift-share instrument would assign Texas (with high energy employment shares) a much larger increase in instrumented expectations than Vermont (with minimal energy exposure). Crucially, this variation stems from predetermined industrial composition interacted with national sectoral trends, rather than from endogenous responses to Texas-specific labor market conditions or measurement error in aggregating individual firm forecasts within Texas.

The identifying assumption is that, conditional on fixed effects and controls, there are no omitted factors that simultaneously affect both national industry-level expectations and local hiring behavior in states more exposed to those industries. While many shift-share designs rely on the exogenous shocks assumption, in our setting the exogenous shares assumption is likely more appropriate. In sectors where specific regions have large exposures to (e.g., Texas in oil energy), national energy industry-level expectations $\mathbb{F}_t[e_{i,t,t+h}]$ may be influenced by news from firms headquartered in those regions. For example, a slowdown in hiring or disappointing earnings guidance from large Texas energy firms could cause IBES analysts to revise downward their national energy sector earnings forecasts. If so, the national shock would be endogenous to Texas-specific developments, violating the exogenous shock assumption. In contrast, the state-level industry shares $s_{s,i,t-1}$, measured using lagged QCEW employment data, reflect slow-moving industrial structure and are plausibly predetermined. We therefore treat industry shares as conditionally exogenous and interpret our identification through the lens of the exogenous shares assumption following Borusyak et al. (2025).

This assumption would be violated, for example, if pre-existing trends in local demand systematically coincided with national shocks. To mitigate this concern, I include a rich set of controls and fixed effects. Specifically, state fixed effects α_s absorb time-invariant differences in labor market characteristics across states. Time fixed effects α_t account for common national shocks such as business cycles or federal policy changes. By leveraging only the cross-sectional variation in state exposure to national shocks, the Bartik specification helps isolate the exogenous component of belief-driven hiring fluctuations.

Cross-Sectional Decomposition of the Regional Job Filling Rate Table A.9 reports regression estimates that evaluate the predictive power of state-level expectations for future unemployment. Each column adds different combinations of rational or subjective forecasts for discount rates and cash flows, with all specifications controlling for standard labor market factors and including both state and time fixed effects.

The estimates demonstrate that subjective earnings expectations are not only informative about regional unemployment but crowd out the predictive power of rational components. Column (1) shows that rational discount rate expectations $\mathbb{E}_t[r_{s,t,t+5}]$ significantly predict unemployment, with a coefficient of 0.725 and R^2 of 0.414. This implies that a one standard deviation increase in rational discount rate expectations predicts a 0.240 percentage point increase in the unemployment rate. Column (2) shows that among subjective forecasts, only expected earnings $\mathbb{F}_t[e_{s,t,t+5}]$ matter, with a large negative coefficient (-0.817) and higher explanatory power ($R^2 = 0.558$). A one standard deviation increase in expected earnings predicts a 0.129 percentage point decrease in the unemployment rate. Column (3) includes both sets of expectations. Subjective earnings dominate: their coefficient remains significant (-0.791), while rational expectations become insignificant.

Column (4) repeats the rational-only regression using Bartik instruments; the discount rate remains significant (0.572), implying a 0.181 percentage point increase in unemployment per standard deviation increase in instrumented discount rate expectations. In Column (5), only instrumented subjective earnings are significant (-0.690), with a standard deviation of 0.168 implying a 0.116 percentage point decrease in unemployment. Column (6) confirms that instrumented subjective earnings expectations (-0.708) continue to drive out all other predictors, implying a 0.119 percentage point decline in unemployment for a one standard deviation increase.

The shift-share estimates are generally smaller in magnitude than their OLS counterparts, as expected, since the shift-share instrument isolates only variation that is plausibly exogenous to regional labor market conditions. The attenuation suggests that some of the OLS signal reflects endogenous responses to regional shocks, such as changes in local labor supply, that amplify belief-driven dynamics. Nevertheless, the fact that the earnings coefficient remains large and significant under instrumentation supports a causal interpretation: belief distortions about cash flows play a central role in driving unemployment fluctuations across regions.

Taken together, the results provide robust evidence that distorted beliefs about future earnings are a key driver of regional labor market volatility. The strong and consistent link between subjective earnings expectations and unemployment, even when instrumented, suggests that firms' hiring decisions are shaped not only by fundamentals but also by biased beliefs. Regions where firms over-react to cash flow news experience deeper hiring cuts during downturns and more aggressive expansions during booms, thereby driving business cycle volatility. These findings indicate that persistent regional differences in unemployment may arise not only from structural characteristics such as industry mix or demographics, but also from variation in how firms perceive and respond to economic signals.

Table A.9: Predictability of the State-Level Unemployment Rate

	Dependent Variable: Log Unemployment Rate u_{t+1}					
	OLS			Shift-Share Instrument		
	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbb{E}_t[r_{s,t,t+h}]$	0.725***		0.470	0.572***		0.207
	(0.235)		(0.780)	(0.222)		(0.240)
$\mathbb{E}_t[e_{s,t,t+h}]$	-0.247		-0.065	-0.064		0.005
	(0.499)		(0.182)	(0.075)		(0.168)
$\mathbb{F}_t[r_{s,t,t+h}]$		0.248	0.233		0.052	0.052
		(0.297)	(0.300)		(0.228)	(0.228)
$\mathbb{F}_t[e_{s,t,t+h}]$		-0.817***	-0.791***		-0.690***	-0.708***
		(0.236)	(0.242)		(0.160)	(0.200)
R^2	0.414	0.558	0.558	0.414	0.549	0.549
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Labor Market Factors	Yes	Yes	Yes	Yes	Yes	Yes
N	4,358	4,358	4,358	4,358	4,358	4,358

Notes: Labor market factors include the log annual growth of lagged log unemployment rate $u_{s,t}$, log labor market tightness $\log \theta_{s,t}$ and log job separation rate $\log \delta_{s,t}$. The sample is quarterly from 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

A.6 Capital Investment

This section extends the baseline model by incorporating firm investment decisions and distinguishing between tangible and intangible capital. I show how belief distortions about future returns and earnings influence not only hiring decisions, but also capital investment behavior. I then decompose the investment rate into components associated with discount rates and cash flows.

Model Setup I assume firms produce output using a Cobb-Douglas production function that depends on both capital and labor inputs:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}$$

where $A_{i,t}$ denotes total factor productivity, $K_{i,t} = K_{i,t}^{\text{phy}} + K_{i,t}^{\text{int}}$ is total capital input composed of tangible and intangible capital, and $L_{i,t}$ is labor input. Following Hall (2001) and Hansen et al. (2005), I treat tangible and intangible capital as perfect substitutes. Earnings are defined as:

$$E_{i,t} = Y_{i,t} - W_{i,t}L_{i,t} - \kappa V_{i,t} - I_{i,t} - \phi\left(\frac{I_{i,t}}{K_{i,t}}\right)K_{i,t}$$

where $W_{i,t}$ is the wage rate, $\kappa V_{i,t}$ is the vacancy posting cost, $I_{i,t} = I_{i,t}^{\text{phy}} + I_{i,t}^{\text{int}}$ is total investment, and $\phi(\cdot)$ denotes convex adjustment costs. I adopt a piecewise-quadratic specification for $\phi(\cdot)$ with different coefficients for expansion and contraction:

$$\phi\left(\frac{I_{i,t}}{K_{i,t}}\right) = \begin{cases} \frac{c_k^+}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} \geq 0 \\ \frac{c_k^-}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 & \text{if } I_{i,t} < 0 \end{cases}$$

Firms choose investment $I_{i,t}$ and vacancies $V_{i,t}$ to maximize firm value:

$$V(A_{i,t}, K_{i,t}, L_{i,t}) = \max_{I_{i,t}, V_{i,t}} \{E_{i,t} + \mathbb{F}_t\left[M_{t+1}V(A_{i,t+1}, K_{i,t+1}, L_{i,t+1})\right]\}$$

subject to both capital and employment accumulation equations:

$$K_{i,t+1} = (1 - \delta_{i,t}^k)K_{i,t} + I_{i,t}$$
$$L_{i,t+1} = (1 - \delta_{i,t}^l)L_{i,t} + q_t V_{i,t}$$

The first order condition with respect to investment implies:

$$1 + \phi'\left(\frac{K_{i,t+1} - (1 - \delta_{i,t}^k)K_{i,t}}{K_{i,t}}\right) = \frac{P_{i,t}}{K_{i,t+1}}$$

where $P_{i,t} = \mathbb{F}_t[M_{t+1}V(A_{i,t+1}, K_{i,t+1})]$ is the ex-dividend firm value.

Recovering Intangible Capital To estimate intangible capital, I construct a panel of five value-weighted portfolios sorted by book-to-market ratio. For each portfolio, I measure realized data on physical capital $K_{i,t}^{\text{phy}}$, tangible investment $I_{i,t}^{\text{phy}}$, depreciation rates $\delta_{i,t}^k$, and market value $P_{i,t}$. The physical capital stock $K_{i,t}^{\text{phy}}$ is measured using Compustat's PPEGT item, and tangible investment $I_{i,t}^{\text{phy}}$ is measured using capital expenditures (CAPX). The depreciation rate $\delta_{i,t}^k$ is calculated as depreciations (DP) as a share of physical capital stock (PPEGT), and applied to both tangible and intangible capital (Hall, 2001). I construct the firm's total market value $P_{i,t}$ as the sum of the market value of equity, the book value of debt, minus current assets. Starting from an initial value $K_{i,1970Q1} = P_{i,1970Q1}$, I recursively solve the first order condition for $K_{i,t+1}$, using observed investment, depreciation, and market value. Intangible capital is then recovered as the residual:

$$K_{i,t}^{\text{int}} = K_{i,t} - K_{i,t}^{\text{phy}}$$

Decomposition of Investment Rates Taking logs and linearizing the first order condition:

$$\log\left(1 + c_k\frac{I_{i,t}}{K_{i,t}}\right) \approx \log c_k + \log\left(\frac{I_{i,t}}{K_{i,t}}\right) = \log\left(\frac{P_{i,t}}{K_{i,t+1}}\right)$$

I decompose the right-hand side into price-to-earnings and earnings-to-capital terms:

$$\underbrace{\log\left(\frac{I_{i,t}}{K_{i,t}}\right)}_{ik_{i,t}} = -\log c_k + \underbrace{\log\left(\frac{P_{i,t}}{E_{i,t}}\right)}_{pe_{i,t}} + \underbrace{\log\left(\frac{E_{i,t}}{K_{i,t+1}}\right)}_{ek_{i,t}}$$

Using a Campbell and Shiller (1988) log-linear approximation for the price-earnings ratio:

$$pe_{i,t} = \sum_{j=1}^{h}\rho^{j-1}(c_{pe} + \Delta e_{i,t+j} - r_{i,t+j}) + \rho^h pe_{i,t+h}$$

Substituting yields the final decomposition:

$$ik_{i,t} = c_{ik} - \sum_{j=1}^{h} \rho^{j-1} r_{i,t+j} + \left(ek_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{i,t+j} \right) + \rho^h pe_{i,t+h}$$

where $c_{ik} \equiv \frac{c_{pe}(1-\rho^h)}{1-\rho} - \log c_k$. To separately analyze tangible and intangible investment, I define $ik^m_{i,t} \equiv \log(\frac{I^m_{i,t}}{K_{i,t}})$ and $s^m_{i,t} \equiv \log(\frac{I^m_{i,t}}{I_{i,t}})$ so that:

$$ik^m_{i,t} = s^m_{i,t} + ik_{i,t}, \qquad m = phy, int$$

implying the decomposition structure remains unchanged up to an additive shift $s^m_{i,t}$. I estimate the decomposition separately for tangible and intangible investment. The time-series decomposition of the aggregate investment rate is:

$$ik^m_t \approx -\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}] + \left(ek_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right) + \rho^h \mathbb{F}_t[pe_{t+h}]$$

where $x_t = \sum_{i \in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. For the cross-section, demeaned variables yield:

$$\tilde{ik}^m_{i,t} \approx -\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\tilde{r}_{i,t+j}] + \left(\tilde{ek}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \tilde{e}_{i,t+j}] \right) + \rho^h \mathbb{F}_t[\tilde{pe}_{i,t+h}]$$

where $\widetilde{x}_{i,t} = x_{i,t} - \sum_{i \in I} x_{i,t}$ cross-sectionally demeans variable $x_{i,t}$.

Results The empirical results mirror those for hiring rates, both in the time series (Figure A.12) and the cross-section (Figure A.13). Subjective expectations substantially overstate the contribution of cash flows and understate that of discount rates, both for tangible and intangible investment. Notably, the distortions are stronger for intangible investment, consistent with greater uncertainty and measurement error in expectations about intangible value creation. These findings highlight how belief distortions affect not only labor demand but also capital allocation decisions across asset types.

A.7 Decreasing Returns to Scale and Composition Effects

Stock market valuations reflect average profits, while hiring decisions depend on marginal profits (Borovickova and Borovička, 2017). Decreasing returns to scale can amplify unemployment fluctuations even under a rational framework by making the marginal value of hiring more sensitive to productivity shocks, prompting firms to adjust vacancies more aggressively in response (Elsby and Michaels, 2013; Kaas and Kircher, 2015). Allowing for decreasing returns to scale introduces the notion of firm size. Changes in the equilibrium firm size distribution can thus introduce a composition effect that also contributes to fluctuations in the job filling rate (Solon et al. (1994)).

This section relaxes the constant returns to scale (CRS) assumption by allowing for decreasing returns to scale (DRS) in the production function. Assume that firm i's output is $Y_{i,t} = F(L_{i,t}) = A_{i,t} L^{\alpha}_{i,t}$, where $A_{i,t}$ is an exogenous productivity process and $0 < \alpha < 1$. This introduces a "DRS wedge" between marginal and average profits:

$$\pi_{i,t} L_{i,t} - \kappa V_{i,t} = \alpha A_{i,t} L^{\alpha}_{i,t} - W_{i,t} L_{i,t} - \kappa V_{i,t} = E_{i,t} - (1-\alpha) Y_{i,t}$$

where $E_{i,t} \equiv \Pi_{i,t} - \kappa V_{i,t}$ is the firm's earnings, $\Pi_{i,t} \equiv Y_{i,t} - W_{i,t} L_{i,t} = A_{i,t} L^{\alpha}_{i,t} - W_{i,t} L_{i,t}$ is the total profit before wages $W_{i,t} L_{i,t}$ and vacancy posting costs $\kappa V_{i,t}$, and $\pi_{i,t} = \frac{\partial \Pi_{i,t}}{\partial L_{i,t}}$ is the marginal profit from hiring. The second term $(1-\alpha) Y_{i,t}$ is a "DRS wedge" that captures the gap between the average profit and marginal profit. Under DRS, the firm's hiring condition becomes:

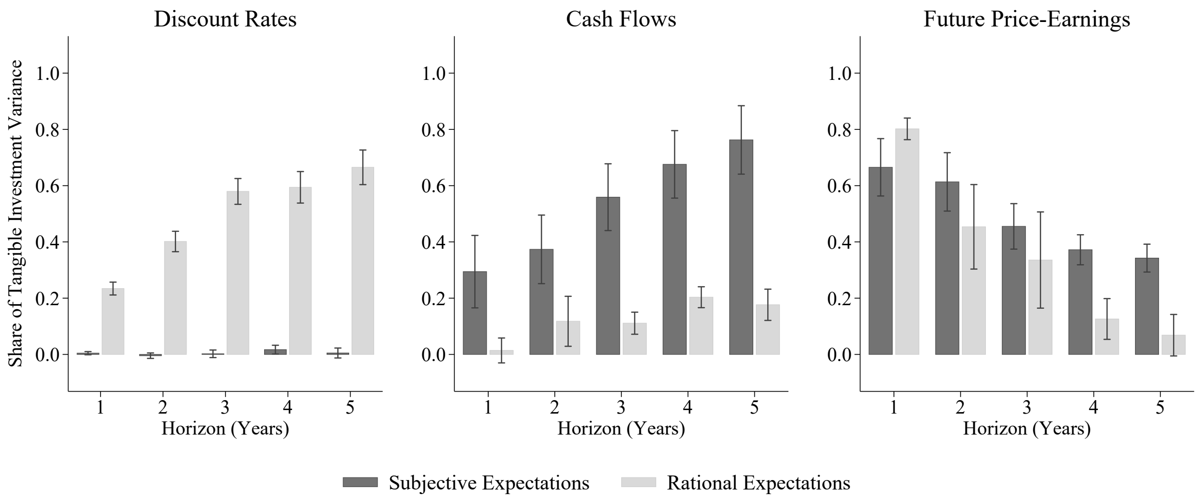
$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[\sum_{j=1}^{\infty} \frac{1}{R_{i,t,t+j}} \left(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha) \frac{Y_{i,t+j}}{L_{i,t+1}} \right) \right]$$

where firm i takes the aggregate job filling rate q_t as given. Express aggregate earning-employment and output-employment ratios as the employment-weighted average of firm-level ratios:

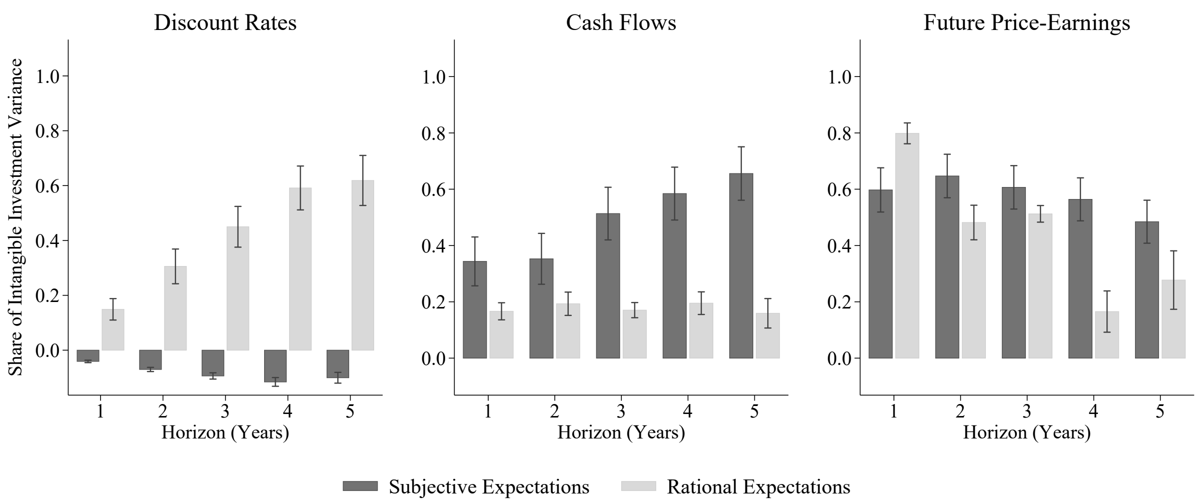
$$\frac{\kappa}{q_t} = \mathbb{F}_t \Big[\sum_i \sum_{j=1}^{\infty} \frac{1}{R_{i,t,t+j}} \Big(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha) \frac{Y_{i,t+j}}{L_{i,t+1}} \Big) \frac{L_{i,t+1}}{L_{t+1}} \Big]$$

Figure A.12: Time-Series Decomposition of Capital Investment

(a) Tangible Capital Investment Rate



(b) Intangible Capital Investment



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate tangible and intangible capital investment rate. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

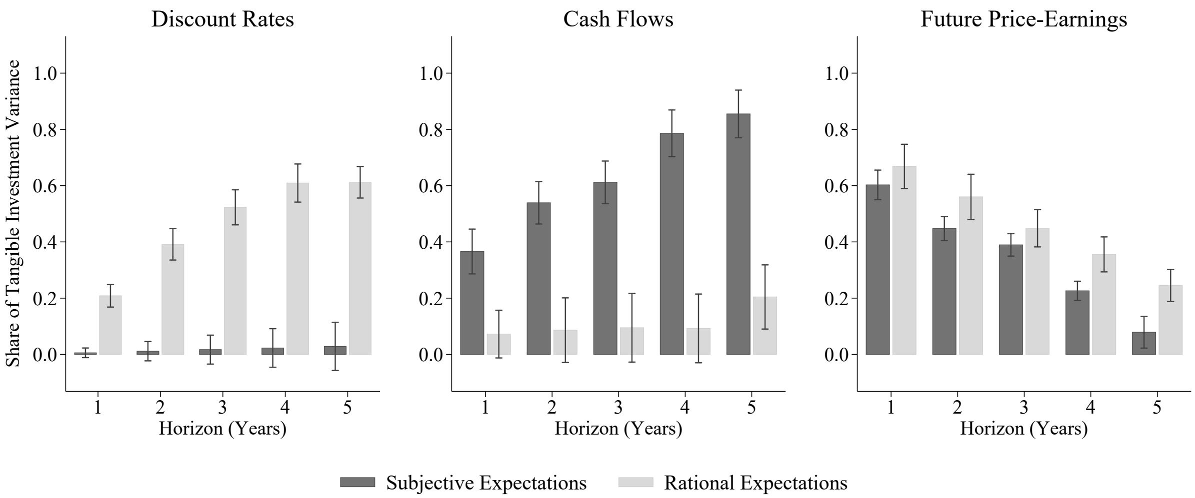
Define $S_{i,t+1} \equiv \frac{L_{i,t+1}}{L_{t+1}}$ as the employment share, $EL_{i,t+j} \equiv E_{i,t+j}/L_{i,t+1}$ the earnings-employment ratio, and $YL_{i,t+j} \equiv Y_{i,t+j}/L_{i,t+1}$ the output-employment ratio of firm i. Log linearize the expression around the steady state:

$$\log q_t = \sum_{j=1}^{\infty} \sum_i \Bigg[\underbrace{\mathbb{F}_t \left[w_{r,i,j} r_{i,t,t+j} \right]}_{\text{Discount Rate}} - \underbrace{\mathbb{F}_t \left[w_{el,i,j} el_{i,t+j} \right]}_{\substack{\text{Cash Flow} \\ \text{(Earnings-Employment)}}} + \underbrace{\mathbb{F}_t \left[w_{yl,i,j} yl_{i,t+j} \right]}_{\substack{\text{Cash Flow} \\ \text{(Output-Employment)}}} - \underbrace{\mathbb{F}_t \left[w_{s,i,j} s_{i,t+1} \right]}_{\text{Employment Share}} \Bigg] \tag{A.4}$$

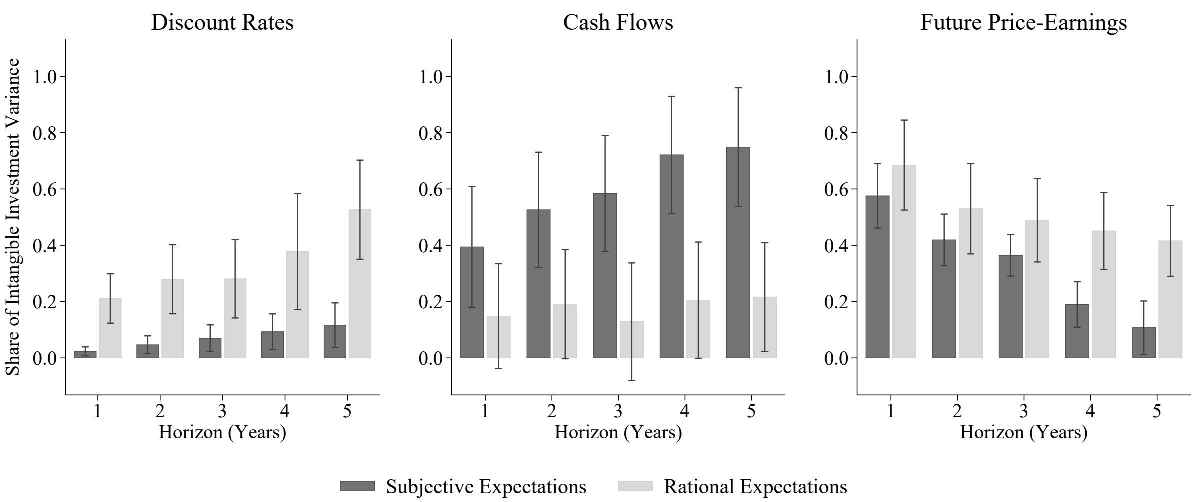
where $r_{i,t,t+j}$, $el_{i,t+j}$, $yl_{i,t+j}$, and $s_{i,t+1}$ denote log deviations of $R_{i,t,t+j}$, $EL_{i,t+j}$, $YL_{i,t+j}$, and $S_{i,t+1}$ from the steady state state, respectively. The coefficients $w_{r,i,j} = w_{s,i,j} \equiv \frac{\overline{q}}{\kappa} \frac{(\overline{EL}_i + (1-\alpha)\overline{YL}_i) \cdot \overline{S}_i}{\overline{R}_i^j}$, $w_{el,i,j} \equiv \frac{\overline{q}}{\kappa} \frac{\overline{EL}_i \cdot \overline{S}_i}{\overline{R}_i^j}$, and $w_{yl,i,j} \equiv (1-\alpha) \frac{\overline{q}}{\kappa} \frac{\overline{YL}_i \cdot \overline{S}_i}{\overline{R}_i^j}$ are functions of steady-state values and linearization constants. $\alpha = 0.72$ comes from the labor share, $\kappa = 0.133$ comes from the flow vacancy cost (Elsby and Michaels, 2013). $\overline{q} = 0.631$, $\overline{R}_i = 1.04$, $\overline{EL} = 0.014$, $\overline{YL} = 0.074$ are long-run sample averages. Finally, approximate the infinite sum by truncating up to h periods.

Figure A.13: Cross-Sectional Decomposition of Capital Investment

(a) Tangible Capital Investment Rate



(b) Intangible Capital Investment



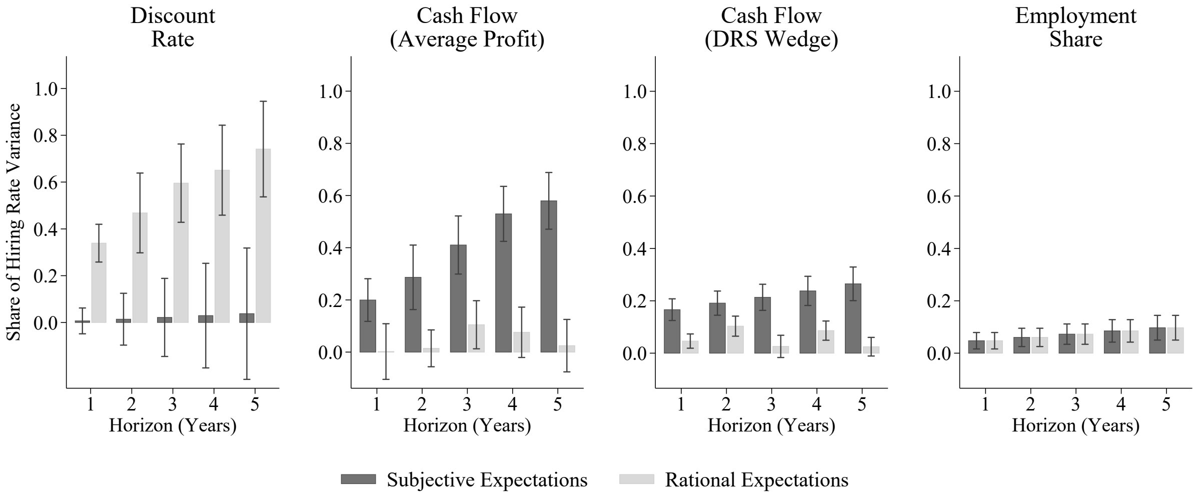
Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the cross-sectional decomposition to the dispersion of the current tangible and intangible capital investment rate. Firms have been sorted into five value-weighted portfolios by book-to-market ratio. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

The expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ captures the DRS wedge, and the employment share $s_{i,t+1}$ captures composition effects of changes in the firm size distribution. I measure the expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ by using IBES sales forecasts. Figure A.14 shows that under subjective expectations, the output-employment term accounts for roughly 30% of the variation in the job filling rate, while the earnings-employment term explains slightly less than 60%. The compositional term is small. These results confirm that even under DRS, subjective cash flow expectations, whether expressed in average or marginal terms, remain the dominant driver of hiring fluctuations.

A.8 On-the-Job Search

The baseline model assumes that all hires come from the pool of unemployed workers. However, measured earnings and hiring flows reflect contributions from both unemployed-to-employed (UE) and job-to-job (J2J) transitions. To better capture the sources of observed hiring, this section extends the baseline model to allow for on-the-job search. This

Figure A.14: Time-Series Decomposition of the Job Filling Rate: Decreasing Returns to Scale



Notes: This figure illustrates the components of the time-series decomposition of aggregate job filling rate under decreasing returns to scale, based on equation (A.4). The components of the decomposition are expected present discounted values of discount rate, earnings-employment ratio, output-employment ratio, and the employment share. The light bars show the contributions to the job filling rate obtained under rational expectations. The dark bars show the contributions to the time-series variation in the job filling rate obtained in subjective expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

modification draws on recent work modeling labor market flows with job-to-job transitions (Kuhn et al., 2021; Faberman et al., 2022). Let a fraction ϕ of employed workers search for jobs each period, in addition to the unemployed. The total number of searchers is:

$$S_t = U_t + \phi L_t = U_t + \phi(1 - U_t), \tag{A.5}$$

where U_t is the unemployment rate and $L_t = 1 - U_t$ is the employment rate. Vacant firms post V_t vacancies, and matches form via a constant returns to scale matching function $\mathcal{M}(S_t, V_t)$. Not all on-the-job searchers who receive an offer accept it. Let $\chi \in (0, 1)$ denote the fraction of employed searchers who accept a job offer. The effective hiring efficiency from the firm's perspective is:

$$\varphi_t = \frac{U_t + \chi\phi(1 - U_t)}{U_t + \phi(1 - U_t)}. \tag{A.6}$$

The law of motion for employment becomes:

$$L_{t+1} = (1 - \delta_t)L_t + q_t\varphi_t V_t, \tag{A.7}$$

where δ_t is the separation rate and $q_t = \frac{\mathcal{M}(S_t,V_t)}{V_t}$ is the job filling rate. The Bellman equation for the firm's value is updated to reflect turnover due to J2J transitions:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \left\{E_t + (1 - \phi\chi f_t)\mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})\right]\right\}, \tag{A.8}$$

subject to the employment accumulation equation above. The term $1 - \phi\chi f_t$ reflects the retention rate, accounting for voluntary separations from employed workers who successfully switch jobs. Under constant returns to scale, the firm's optimal vacancy posting condition implies:

$$\frac{\kappa}{q_t\varphi_t} = (1 - \phi\chi f_t) \cdot \frac{P_t}{L_{t+1}}, \tag{A.9}$$

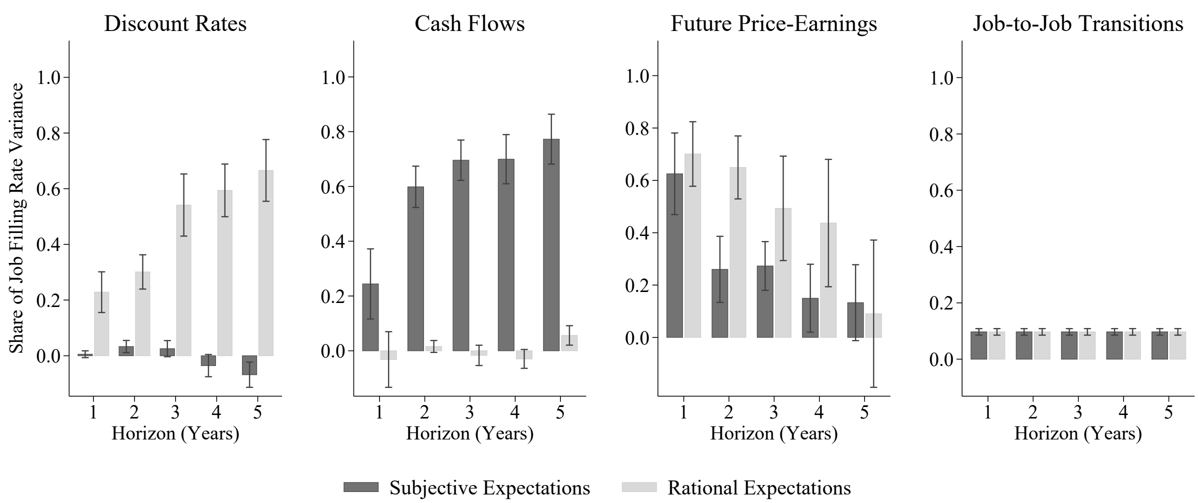
where $P_t = \mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1})\right]$ is the ex-dividend firm value and κ is the flow cost of posting a vacancy. Taking logs and rearranging, the log job filling rate can be written as:

$$\log q_t = c_q - \log(1 - \phi\chi f_t) + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}], \tag{A.10}$$

where $c_q = \log \kappa - \log \varphi_t - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant, $r_{t,t+h}$ is the present value of expected discount rates, $e_{t,t+h}$ is expected cumulative earnings growth, and $pe_{t,t+h}$ is the expected terminal price-earnings ratio. This decomposition extends the Campbell and Shiller (1988) present value identity to account for hiring frictions due to job-to-job transitions. The job filling rate q_t is computed as the ratio of total hires to vacancies $q_t = \frac{H_t}{V_t}$ using JOLTS data for hires and job openings. The total search pool S_t includes both unemployed and a fraction $\phi = 0.12$ of employed workers, based on estimates from Kuhn et al. (2021) and Faberman et al. (2022). The job finding rate is then inferred from the matching function as $f_t = q_t \cdot \theta_t$, where labor market tightness is defined as $\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_t + \phi(1-U_t)}$. I assume that $\chi = 0.75$ of employed job seekers accept offers. These parameter values imply an endogenous efficiency term φ_t and a retention rate $1 - \phi\chi f_t$, which are used to adjust the firm's hiring incentives and derive the decomposition. Subjective expectations of earnings growth are from IBES, which aggregates analyst forecasts of total firm earnings and therefore reflect both UE and J2J hires.

Figure A.15 presents the decomposition of the job filling rate under this extended model with on-the-job search. Consistent with the baseline analysis, the cash flow component remains the dominant driver of variation in the job filling rate under subjective expectations. However, accounting for job-to-job transitions modestly shifts the decomposition: the log retention rate term $\log(1 - \phi\chi f_t)$ explains 8.9% of the variation in $\log q_t$. This adjustment reflects the influence of selective separations on firms' incentives to post vacancies. Overall, the results reinforce the finding that distorted cash flow expectations are the primary driver of hiring fluctuations. The extension confirms that even when allowing for endogenous separations due to on-the-job search, subjective belief distortions about firm-level earnings continue to dominate the variation in hiring behavior.

Figure A.15: Time-Series Decomposition of the Job Filling Rate: On-the-Job Search



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate job filling rate. Light bars show the contribution under rational expectations. Dark bars show the contribution under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

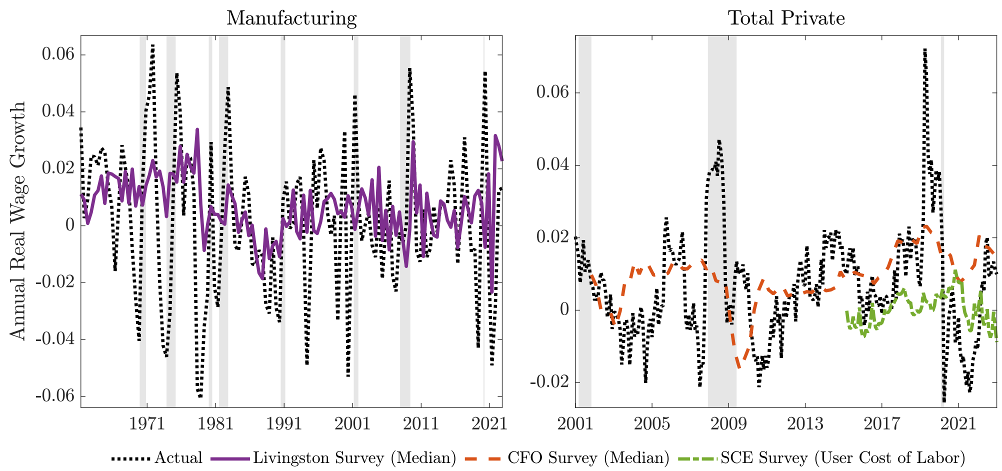
A.9 Subjective User Cost of Labor

Overview The previous sections show that firms' hiring decisions are heavily influenced by subjective cash flow expectations. This section examines whether expectations about the user cost of labor also contribute to hiring behavior, since it is a key component of the firm's cash flows. Using survey data, I show that subjective wage expectations are significantly less cyclical than realized wages, implying that firms perceive labor costs as more rigid than they actually are. To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor under subjective expectations. See Section C for more details about its measurement.

In the search and matching model, the user cost of labor is the difference in the expected present value of wages between two firm-worker matches that are formed in two consecutive periods. Existing work assumes full information rational expectations and show that this user cost is more cyclical than flow wages, as workers hired in recessions earn lower wages both when hired and over time (Kudlyak, 2014; Bils et al., 2023). This section relaxes that assumption by using survey-based measures of subjective wage expectations. If firms and workers perceive the future path of wages as rigid, the subjective user cost of labor may remain high even during recessions, dampening hiring and amplifying unemployment fluctuations.

Time-series evidence Figure A.16 compares realized real wage growth with 1-year-ahead subjective wage growth forecasts from three sources: the Livingston Survey, the CFO Survey, and the Survey of Consumer Expectations (SCE). Actual wage growth is clearly cyclical, with declines during downturns and strong rebounds during recoveries. In contrast, subjective wage forecasts are far more stable over time. Even during major shocks, such as the 2008 financial crisis and the COVID-19 recession, survey respondents anticipated only modest wage adjustments. Forecast errors are persistent and systematically biased: wage growth forecasts overestimate during downturns and underestimate during expansions.

Figure A.16: Real Wage Growth: Actual vs. Subjective Expectations



Notes: This figure plots ex-post realized outcomes (Actual) and 1-year ahead subjective expectations (Survey) of real wage growth. x axis denotes the date on which actual values were realized and the period on which the survey forecast is made, making the vertical distance between the actual and survey lines the forecast error. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Left panel compares actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Right panel compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Actual values are deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2023Q4, SCE is monthly spanning 2015M5 to 2022M12. NBER recessions are shown with gray shaded bars.

To formally assess the cyclicality of real wage growth, Table A.10 panel (a) compares the relationship between changes in the unemployment rate and real wage growth across rational and subjective expectations. As a rational expectations benchmark, I use historical data on actual real wage growth to estimate the following regression, replicating existing estimates in the literature (e.g., Bils, 1985; Solon et al., 1994; Gertler et al., 2020):

$$\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t$$

where $\Delta \log w_t$ represents the actual annual log growth rate of real wages, Δu_t is the annual change in the unemployment rate, and ε_t is the error term. β_1 is the coefficient of interest and captures the cyclicality of real wage growth.

Under subjective expectations, I use survey data on expected real wage growth to estimate:

$$F_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 F_{t-1}[\Delta u_t] + \varepsilon_t$$

where $F_{t-1}[\Delta \log w_t]$ is the median survey forecast for the annual log growth rate of real wages, where the surveys are either from Livingston, CFO, or SCE. $F_{t-1}[\Delta u_t]$ is the median survey forecast of the annual change in the unemployment rate from the Survey of Professional Forecasters (SPF). The coefficient of interest β_1 measures the cyclicality of expected real wage growth as perceived by survey respondents.

Table A.10 panel (a) reports the estimates. Under rational expectations, actual real wage growth is clearly cyclical since it is significantly negatively related to changes in unemployment rates. The magnitude of the estimate is also consistent with prior estimates in the literature, with elasticities ranging from -3.05 to -3.46 depending on the sample period (Solon et al., 1994). In contrast, subjective wage growth expectations are acyclical, with small and statistically insignificant coefficients across all survey sources and sample periods. Notably, the magnitude of the estimated elasticity is an order of magnitude smaller, ranging from -0.20 to -0.97 depending on the survey measure and sample period.

Table A.10: Cyclicality of Real Wage Growth: Actual vs. Subjective Expectations

(a) Aggregate Time-Series
Actual: $\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t$
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \varepsilon_t$

	1961S1-2022S2		2001Q4-2023Q4		2015M5-2022M12	
	Actual	Survey Median (Liv)	Actual	Survey Median (CFO)	Actual	Survey User Cost (SCE)
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment Rate	-0.0340^{***}	-0.0020	-0.0305^{***}	0.0006	-0.0346^{***}	-0.0086
t-stat	(-3.8684)	(-0.1568)	(-4.2477)	(0.0800)	(-6.6994)	(-1.6332)
Adj. R^2	0.1021	0.0003	0.2557	0.0001	0.4719	0.0498
N	124	124	85	85	92	92
Frequency	SA	SA	Q	Q	M	M
Sector	Mfg	Mfg	Pvt	Pvt	Pvt	Pvt

(b) Worker-Level New Hire Effect
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t}$

	2015M5-2022M12	
	Survey (SCE)	Survey (SCE)
	(1)	(2)
	First Difference	Fixed Effects
Unemployment Rate	-0.0048	-0.0028
	(0.0029)	(0.0026)
New Hire	0.0036^{***}	0.0003
	(0.0009)	(0.0013)
Unemployment Rate × New Hire	-0.0026	-0.0059
	(0.0020)	(0.0035)
Adj. R^2	0.0011	0.0036
N	39,832	39,832
Frequency	M	M
Sector	Pvt	Pvt

Notes: Table reports estimates from time-series and worker-level regressions of annual log real wage growth on unemployment growth. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Panel (a) reports estimates from time-series regressions using the aggregate series. Panel (a) Columns (1)-(2) compare actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Panel (a) Columns (3)-(6) compare compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Panel (b) reports worker-level estimates from regressions of SCE survey expectations of wage growth on survey expectations of unemployment growth, an indicator of whether the worker is a new hire, and the interaction between the two. Actual wage growth is deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. Subjective expectations of unemployment rates are from 1-year ahead consensus median forecasts from the SPF. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2023Q4, SCE is monthly spanning 2015M5 to 2022M12. Panel (a): Newey-West corrected t-statistics with lags 2 (semi-annual), 4 (quarterly), 12 (monthly) are reported in parentheses; Panel (b): Standard errors clustered by worker are reported in parentheses. *sig. at 10%. **sig. at 5%. ***sig. at 1%.

Cross-Sectional evidence To explore these patterns at the individual level, I use microdata from the SCE to estimate subjective wage cyclicality separately for new hires and incumbents. The regression specification relaxes the rational expectations assumption from Gertler et al. (2020) and includes an interaction between expected unemployment growth and the probability of being a new hire:

$$\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t}$$

where $\mathbb{F}_{t-1}[\Delta \log w_{i,t}]$ represents the time $t-1$ subjective expectation of wage growth for worker i at time t. $\mathbb{F}_{t-1}[\Delta u_t]$ is the survey expectation of aggregate unemployment growth. The indicator variable $\mathbb{I}\{N_{i,t} = 1\}$ equals one if the worker is newly hired and zero otherwise. Its expectation $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}]$ is thus the subjective probability that the worker will be newly hired next period. The interaction term $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot \mathbb{F}_{t-1}[\Delta u_t]$ captures the differential sensitivity of expected wage growth to unemployment changes for new hires relative to incumbents. The error term $\varepsilon_{i,t}$ accounts for individual-level deviations in expectations. The coefficient β_1 captures the overall cyclicality of subjective wage expectations, reflecting how much workers expect wages to change in response to shifts in aggregate unemployment. The coefficient β_2 measures the baseline difference in expected wage growth between new hires and existing workers. The interaction term β_3 determines whether new hires expect wages to be more sensitive to unemployment fluctuations than incumbents do.

The results in Table A.10 panel (b) column (1) show that, even after controlling for differences between job stayers and new hires, subjective wage expectations are highly rigid and exhibit weak cyclicality. The coefficient β_1 is negative but small, confirming the aggregate result in panel (a) that workers that are not new hires expect only mild wage adjustments in response to unemployment fluctuations. The estimate for β_2 is positive, suggesting that, on average, new hires expect higher wage growth than job stayers. The interaction term β_3 is negative but small in magnitude, implying that new hires do not expect substantially greater cyclicality in wages compared to incumbents. Column (2) extends column (1) by including worker fixed effects to find similar results. These findings extend the results from aggregate regressions by showing that subjective wage expectations are highly rigid even at the individual level, regardless of job transitions. Both new hires and incumbents perceive only weak cyclical variation in wages.

Implications for macroeconomic models These findings could have important implications for macroeconomic models of unemployment fluctuations. If firms do not expect wages to fall during downturns, then the subjective user cost of labor remains high even as demand declines, suppressing job creation. This mechanism is consistent with models that rely on wage rigidity to explain labor market volatility (Shimer, 2005; Hall, 2005; Christiano et al., 2016). These results suggest that it could be reasonable for macroeconomists to introduce rigid wages under subjective expectations to explain the volatility of business cycle fluctuations. Moreover, the persistence of subjective wage expectations may reflect underlying frictions in information processing. Survey data on wage expectations can help distinguish between alternative theories of wage formation. Unlike rational models where the timing of wage payments is irrelevant (Barro, 1977), models with sticky or inattentive expectations, such as those in Mankiw and Reis (2002) or Coibion and Gorodnichenko (2015), can be better suited to capture the persistent behavior of expected wages. Finally, the finding that subjective cost of labor is rigid suggests that volatile subjective cash flow expectations are unlikely to be driven by fluctuations in the user cost of labor. Instead, firms may be over-reacting to other components of profitability, such as revenue expectations or perceived demand conditions, rather than expected changes in labor costs.

B Model Details

B.1 Representative Agent Model

In this section, I present a search and matching model based on Diamond (1982), Mortensen (1982), and Pissarides (2009). The model introduces subjective beliefs that may depart from rational expectations, thereby capturing the impact of belief distortions on labor market dynamics. See Petrosky-Nadeau et al. (2018) for a standard search and matching model formulated under rational expectations. Consider a discrete time economy populated by a representative household and a representative firm that uses labor as a single input to production.

Representative Household The household has a continuum of mass 1 members who are either employed L_t or unemployed U_t at any point in time. The population is normalized to 1, i.e., $L_t + U_t = 1$, meaning that L_t and U_t are also the rates of employment and unemployment, respectively. The household's consumption decision implies a stochastic discount factor M_{t+1}. The household pools the income of all members before making its consumption decision. Assume that the household has perfect consumption insurance and its members have access to complete contingent claims against aggregate risk. Risk sharing implies each member consumes the same amount regardless of idiosyncratic shocks.

Search and Matching At the start of period t, the employment stock L_t reflects the total number of workers carried over from the previous period before any separations or new hires in period t. A fraction δ_t of these workers separate during the period, so the number of continuing employees becomes $(1 - \delta_t)L_t$. The representative firm posts job vacancies

V_t and engage in search over the course of the period to attract unemployed workers U_t. Matches are formed at the end of period t according to a matching function $m(U_t, V_t)$, where $q_t \equiv m(U_t, V_t)/V_t$ is the *job filling rate*, and $f_t \equiv m(U_t, V_t)/U_t$ is the *job finding rate*. These new matches become part of the workforce starting in period $t+1$, so employment evolves according to the employment accumulation equation:

$$L_{t+1} = (1 - \delta_t)L_t + q_t V_t \tag{A.11}$$

The job filling rate q_t maps vacancy posting decisions made during period t into employment outcomes observed at the beginning of period $t+1$. The variance decomposition does not require us to fully specify the the matching function m. Posting a vacancy costs the firm $\kappa > 0$ per period, reflecting fixed hiring costs such as training and administrative setup. Jobs are destroyed at a time-varying job separation rate δ_t. Unemployment $U_t = 1 - L_t$ evolves according to:

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{A.12}$$

where $\theta_t = V_t/U_t$ denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

Representative Firm The firm has access to a production function F which uses labor L_t as an input to produce output $Y_t = F(L_t)$. Dividends to the firm's shareholders E_t are defined as $E_t \equiv \Pi_t - \kappa V_t$, where $\Pi_t \equiv Y_t - W_t L_t$ is the total profit before vacancy posting costs κV_t and W_t is the wage rate. As in Petrosky-Nadeau et al. (2018), I assume that the representative household owns the equity of the firm, and that the firm pays out all of its earnings as dividends. I also assume that firms have the same unconstrained access to financing as investors in the financial market. The firm posts the optimal number of vacancies to maximize the cum-dividend market value of equity S_t:

$$S_t = \max_{\{V_{t+j}, L_{t+j}\}_{j=0}^{\infty}} \mathbb{F}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} E_{t+j} \right] \tag{A.13}$$

subject to the employment accumulation equation (A.11). The firm takes the wage rate W_t, household's stochastic discount factor $M_{t,t+j} = \prod_{s=1}^{j} M_{t+s}$, and job filling rate q_t as given. $\mathbb{F}_t[\cdot]$ denotes expectations conditional on information available at period t, computed based on the firm's possibly distorted beliefs. These beliefs may depart from rational expectations $\mathbb{E}_t[\cdot]$, with the nature and magnitude of the deviation disciplined using survey data.

Hiring Equation The firm's optimal hiring decision equates the expected discounted value of hiring a marginal worker with its marginal cost. Rewrite the firm's problem in equation (A.13) from infinite-horizon to recursive form:

$$S_t = \max_{V_t, L_{t+1}} \Pi_t - \kappa V_t + \mathbb{F}_t \left[M_{t+1} S_{t+1} \right] \tag{A.14}$$

$$\text{s.t. } L_{t+1} = (1 - \delta_t)L_t + q_t V_t \tag{A.15}$$

The first-order condition with respect to V_t is:

$$\frac{\partial S_t}{\partial V_t} = -\kappa + \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \frac{\partial L_{t+1}}{\partial V_t} \right] = 0 \tag{A.16}$$

Substitute $\frac{\partial L_{t+1}}{\partial V_t} = q_t$ and $\frac{\partial L_{t+1}}{\partial L_t} = (1 - \delta_t)$ from the employment accumulation equation (A.15), and rearrange (A.16) in terms of the marginal cost of hiring κ/q_t:

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \right] \tag{A.17}$$

Next, differentiate S_t with respect to L_t:

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + \mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \frac{\partial L_{t+1}}{\partial L_t} \right] \tag{A.18}$$

Substitute $\frac{\partial L_{t+1}}{\partial L_t} = (1 - \delta_t)$ from the employment accumulation equation (A.15):

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + (1 - \delta_t)\mathbb{F}_t \left[M_{t+1} \frac{\partial S_{t+1}}{\partial L_{t+1}} \right] \tag{A.19}$$

Substitute equation (A.19) for period $t+1$ into equation (A.17):

$$\frac{\kappa}{q_t} = \mathbb{F}_t \left[M_{t+1} \left(\frac{\partial \Pi_{t+1}}{\partial L_{t+1}} + (1-\delta_{t+1}) \mathbb{F}_{t+1} \left[M_{t+2} \frac{\partial S_{t+2}}{\partial L_{t+2}} \right] \right) \right] \tag{A.20}$$

Finally, substitute in (A.17) for period $t+1$ to arrive at the *hiring equation*:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \underbrace{\mathbb{F}_t \left[M_{t+1} \left(\pi_{t+1} + (1-\delta_{t+1}) \frac{\kappa}{q_{t+1}} \right) \right]}_{\text{Expected discounted value of hiring}} \tag{A.21}$$

where $\pi_t \equiv \frac{\partial \Pi_t}{\partial L_t}$ is the profit flow from the marginal hired worker. The hiring equation relates the marginal cost of hiring $\frac{\kappa}{q_t}$ with the expected marginal value of hiring to the firm, which equals the future expected marginal benefits of hiring discounted to present value with the stochastic discount factor M_{t+1}. The future marginal benefits of hiring include π_{t+1}, the future marginal product of labor net of the wage rate, plus the future marginal value of hiring, which equals the future marginal cost of hiring $\frac{\kappa}{q_{t+1}}$ net of separation $(1-\delta_{t+1})$. During recessions, job filling rates q_t are high, which makes the cost of hiring κ/q_t low. The low cost of hiring must be rationalized by either low expected discounted profit flows $\mathbb{F}_t[M_{t+1}\pi_{t+1}]$ or low future value of hiring $(1-\delta_{t+1})\frac{\kappa}{q_{t+1}}$. The hiring equation is the labor market analogue of the optimality condition for physical capital in the q theory of investment (Hayashi, 1982), where κ/q_t is the upfront cost of investment analogous to Tobin's marginal q and δ_{t+1} is the depreciation rate.

Constant Returns to Scale (CRS) Next, I derive the firm's stock price implied by the optimal hiring decision. Assume a constant returns to scale (CRS) production function so that marginal profits equal average profits:

$$\pi_{t+1} L_{t+1} = \frac{\partial \Pi_{t+1}}{\partial L_{t+1}} L_{t+1} = \Pi_{t+1} \tag{A.22}$$

Multiply both sides of the hiring equation by the number of employees L_{t+1}:

$$\frac{\kappa}{q_t} L_{t+1} = \mathbb{F}_t \left[M_{t+1} \left(\pi_{t+1} L_{t+1} + (1-\delta_{t+1}) \frac{\kappa}{q_{t+1}} L_{t+1} \right) \right] \tag{A.23}$$

Substitute in the employment accumulation equation (A.15) and rearrange terms:

$$\frac{\kappa}{q_t} L_{t+1} = \mathbb{F}_t \left[M_{t+1} \left(\pi_{t+1} L_{t+1} + \frac{\kappa}{q_{t+1}} (L_{t+2} - q_{t+1} V_{t+1}) \right) \right] \tag{A.24}$$

$$= \mathbb{F}_t \left[M_{t+1} \left(\pi_{t+1} L_{t+1} - \kappa V_{t+1} + \frac{\kappa}{q_{t+1}} L_{t+2} \right) \right] \tag{A.25}$$

Use the constant returns to scale assumption to simplify $\pi_{t+1} L_{t+1} - \kappa V_{t+1} = \Pi_{t+1} - \kappa V_{t+1} = E_{t+1}$:

$$\frac{\kappa}{q_t} L_{t+1} = \mathbb{F}_t \left[M_{t+1} \left(E_{t+1} + \frac{\kappa}{q_{t+1}} L_{t+2} \right) \right] \tag{A.26}$$

Substitute the equation recursively:

$$\frac{\kappa}{q_t} L_{t+1} = \mathbb{F}_t \left[\sum_{j=1}^{\infty} M_{t,t+j} E_{t+j} \right] + \lim_{T \to \infty} \mathbb{F}_t \left[M_{t,t+T} \frac{\kappa}{q_{t+T}} L_{t+T+1} \right] \tag{A.27}$$

The first term on the right-hand side is the firm's stock price $P_t \equiv S_t - E_t$, which is the firm's ex-dividend equity value. Take the second term to zero by applying a transversality condition to arrive at an equation that relates the total cost of hiring with the firm's stock price:

$$\frac{\kappa}{q_t} L_{t+1} = P_t \tag{A.28}$$

where employment L_{t+1} is determined at the end of date t under our timing convention from equation (A.11). Take logarithms of both sides of the firm's stock price equation (A.28) and rearrange terms:

$$\log \kappa - \log q_t = \log \frac{P_t}{L_{t+1}} = \log \frac{P_t}{E_t} - \log \frac{E_t}{L_{t+1}} \equiv pe_t - el_t \tag{A.29}$$

where I define $pe_t \equiv \log \frac{P_t}{E_t}$ and $el_t \equiv \log \frac{E_t}{L_{t+1}}$ for notational convenience.

Log-linear Approximation of Price-Earnings Ratio To express the price-earnings ratio pe_t in terms of forward-looking variables, start by log-linearizing the price-dividend ratio $pd_t = \log(P_t/D_t)$ around its long-term average $\overline{pd}$ (Campbell and Shiller, 1988):

$$pd_t = c_{pd} + \Delta d_{t+1} - r_{t+1} + \rho pd_{t+1} \tag{A.30}$$

where c_{pd} is a linearization constant, $r_{t+1} \equiv \log(\frac{P_{t+1}+D_{t+1}}{P_t})$ is the log stock return (with dividends), and $\rho \equiv \exp(\overline{pd})/(1+\exp(\overline{pd})) = 0.98$ is a persistence parameter that arises from the log linearization. Rewrite the equation in terms of log price-earnings instead of log price-dividends by using the identity $pe_t = pd_t + de_t$, where de_t log payout ratio:

$$pe_t = c_{pd} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} + (1-\rho)de_{t+1} \tag{A.31}$$

Since $1 - \rho \approx 0$ and the payout ratio de_t is bounded, $(1-\rho)de_{t+1}$ can be approximated as a constant, i.e., $c_{pe} \approx c_{pd} + (1-\rho)de_{t+1}$ (De La O et al., 2024):

$$pe_t \approx c_{pe} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} \tag{A.32}$$

Recursively substitute for the next h periods

$$pe_t = \sum_{j=1}^{h} \rho^{j-1}(c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h} \tag{A.33}$$

Decomposition of Job Filling Rate Substitute the log-linearized price-earnings ratio in equation (A.33) into the hiring equation in equation (A.29):

$$\log q_t = \log\kappa - pe_t - el_t = \log\kappa - \left[\sum_{j=1}^{h} \rho^{j-1}(c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h}\right] - el_t \tag{A.34}$$

Rearrange and collect terms to obtain an ex-post decomposition of the job filling rate:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{t+j}}_{r_{t,t+h}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1}\Delta e_{t+j}\right]}_{e_{t,t+h}} - \underbrace{\rho^h pe_{t+h}}_{pe_{t,t+h}} \tag{A.35}$$

where $c_q \equiv \log\kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The equation decomposes the job filling rate into future discount rates $r_{t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$, cash flows $e_{t,t+h} \equiv el_t + \sum_{j=1}^{h} \rho^{j-1}\Delta e_{t+j}$, and price-earnings $pe_{t,t+h} \equiv \rho^h pe_{t+h}$. The cash flow component consists of one period ahead log earnings-employment el_t, which captures news about current cash flow fluctuations, and $j = 1, \ldots, h$ period ahead log earnings growth Δe_{t+j}, which captures news about future cash flows. The earnings-employment ratio can be interpreted as a measure of the marginal product of labor under constant returns to scale (David et al., 2022). $pe_{t,t+h}$ is a terminal value that captures other long-term influences beyond h periods into the future not already captured in discount rates and cash flows. Since equation (A.35) holds both ex-ante and ex-post, it can be evaluated under either subjective or rational expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their subjective expectations:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\mathbb{F}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1}\mathbb{F}_t[\Delta e_{t+j}]\right]}_{\mathbb{F}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{F}_t[pe_{t+h}]}_{\mathbb{F}_t[pe_{t,t+h}]} \tag{A.36}$$

Alternatively, the *rational decomposition* replaces ex-post realizations of future outcomes with their rational expectations:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{E}_t[r_{t+j}]}_{\mathbb{E}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1}\mathbb{E}_t[\Delta e_{t+j}]\right]}_{\mathbb{E}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{E}_t[pe_{t+h}]}_{\mathbb{E}_t[pe_{t,t+h}]} \tag{A.37}$$

Comparing these decompositions can quantify how belief distortions affect the job filling rate.

Estimation The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current job filling rate. For the subjective decomposition, demean each variable in equation (A.36), multiply both sides by the current log job filling rate $\log q_t$, and take the sample average:

$$Var\left[\log q_t\right] = Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right] \tag{A.38}$$

where $Var[\cdot]$ and $Cov[\cdot]$ are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by $Var\left[\log q_t\right]$ to decompose its variance:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{A.39}$$

The left-hand side represents the full variability in job filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, and $\mathbb{F}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively. Finally, the decomposition under rational expectations can be estimated similarly based on equation (A.37) by replacing the subjective expectation $\mathbb{F}_t[\cdot]$ with its rational counterpart $\mathbb{E}_t[\cdot]$:

$$1 = \underbrace{\frac{Cov\left[\mathbb{E}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{A.40}$$

Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{E}_t[r_{t,t+h}]$, $\mathbb{E}_t[e_{t,t+h}]$, and $\mathbb{E}_t[pe_{t,t+h}]$ on the current log job filling rate $\log q_t$, respectively.

B.2 Decomposition of Unemployment Rates

The unemployment rate can be decomposed into components similar to the decomposition for job filling rates from equation (A.36). Log linearize the unemployment accumulation equation from equation (2):

$$U_{t+1} = \delta_t(1 - U_t) + (1 - q_t\theta_t)U_t \tag{A.41}$$

Denote the steady state values without time subscripts: U, δ, q, and θ. Define log deviations from steady state as $\hat{x}_t = \log(X_t) - \log(X)$ for some variable X. Log-linearizing the accumulation equation around the steady state involves taking a first-order Taylor approximation:

$$Ue^{\hat{u}_{t+1}} \approx \delta e^{\hat{\delta}_t}(1 - Ue^{\hat{u}_t}) + (1 - q\theta e^{\hat{q}_t + \hat{\theta}_t})Ue^{\hat{u}_t} \tag{A.42}$$

Use the approximation $Xe^{x_t} \approx X(1 + x_t)$, expand, and simplify:

$$U(1 + \hat{u}_{t+1}) \approx \delta(1 + \hat{\delta}_t)(1 - U(1 + \hat{u}_t)) + (1 - q\theta(1 + \hat{q}_t + \hat{\theta}_t))U(1 + \hat{u}_t) \tag{A.43}$$

Use the steady state equation and collect terms with log deviations:

$$U\hat{u}_{t+1} \approx \delta(1 - U)\hat{\delta}_t - \delta U\hat{u}_t - q\theta U\hat{q}_t - q\theta U\hat{\theta}_t + U(1 - q\theta)\hat{u}_t \tag{A.44}$$

Divide both sides by U:

$$\hat{u}_{t+1} \approx \frac{\delta(1 - U)}{U}\hat{\delta}_t - \delta\hat{u}_t - q\theta\hat{q}_t - q\theta\hat{\theta}_t + (1 - q\theta)\hat{u}_t \tag{A.45}$$

The steady state relationship $\delta(1 - U) = q\theta U$ implies: $\frac{\delta(1-U)}{U} = q\theta$. Substitute this back into our equation:

$$\hat{u}_{t+1} \approx -q\theta\hat{q}_t + (1 - \delta - q\theta)\hat{u}_t - q\theta\hat{\theta}_t + q\theta\hat{\delta}_t \tag{A.46}$$

Finally, substitute in equation (A.36), which is a decomposition of the job filling rate $\hat{q}_t$ into discount rate, cash flow, and future price-earnings under subjective expectations:

$$\hat{u}_{t+1} \approx -\underbrace{q\theta \cdot \mathbb{F}_t[\hat{r}_{t,t+h}]}_{\text{Discount Rate}} + \underbrace{q\theta \cdot \mathbb{F}_t[\hat{e}_{t,t+h}]}_{\text{Cash Flow}} + \underbrace{q\theta \cdot \mathbb{F}_t[\hat{pe}_{t,t+h}]}_{\text{Future Price-Earning}} + \underbrace{(1 - \delta - q\theta) \cdot \hat{u}_t - q\theta \cdot \hat{\theta}_t + q\theta \cdot \hat{\delta}_t}_{\text{Lag Unemployment, Tightness, Separations}} \tag{A.47}$$

The equation holds both ex-ante and ex-post. Therefore, I compare results from evaluating the equation under subjective $\mathbb{F}_t[\cdot]$ or rational $\mathbb{E}_t[\cdot]$ expectations. The decomposition can be estimated using regressions of the log unemployment rate on each of the components shown in the equation:

$$u_{t+1} = \beta_0 + \beta_1 u_t + \beta_2 \log \theta_t + \beta_3 \log \delta_t + \beta_4 \mathbb{F}_t[r_{t,t+h}] + \beta_5 \mathbb{F}_t[e_{t,t+h}] + \varepsilon_{t+1} \tag{A.48}$$

where lowercase variables denote log deviations from steady state. I estimate the decomposition using multivariate OLS regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations. To ensure stationarity and remove seasonal effects, I estimate the regression in log annual growth rates relative to the same quarter of the previous year. The future price-earnings ratio term $\Delta\mathbb{F}_t[pe_{t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\Delta\mathbb{F}_t[r_{t,t+h}]$ and cash flows $\Delta\mathbb{F}_t[e_{t,t+h}]$ through the Campbell and Shiller (1988) present value identity in equation (11). Similarly, the equation can also be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with its rational expectations counterpart $\mathbb{E}_t[\cdot]$ based on machine learning forecasts.

B.3 Constant-Gain Learning Model

In this section, I introduce a model of hiring in which firms form subjective beliefs about cash flows and prices using a constant-gain learning rule. The evolving expectations shape firms' vacancy posting decisions and drive variation in hiring and job filling rates. The model embeds belief distortions in a search-and-matching framework and generates decompositions that can match those estimated from the data in Sections 5 and 6.

Environment and Firm Problem The model features a frictional labor market in which unemployed workers are matched with job vacancies using a Cobb-Douglas matching function:

$$\mathcal{M}(U_t, V_t) = B U_t^{\eta} V_t^{1-\eta} \tag{A.49}$$

where $\mathcal{M}(U_t, V_t)$ denotes the total number of matches in period t and is a function of aggregate unemployment U_t and job vacancies V_t. B is the matching efficiency parameter, and $\eta \in (0,1)$ governs the elasticity of matches with respect to unemployment. The probability that a firm fills a posted vacancy, the job filling rate, is then given by:

$$q_t = \frac{\mathcal{M}(U_t, V_t)}{V_t} = B\left(\frac{U_t}{V_t}\right)^{\eta} = B\theta_t^{-\eta} \tag{A.50}$$

where $\theta_t \equiv V_t/U_t$ denotes labor market tightness. A firm that posts a vacancy incurs a cost $\kappa > 0$ per period. Matches dissolve at an exogenous separation rate δ, and each firm hires new workers by posting vacancies in anticipation of future returns. Each firm i uses labor to produce output via a constant returns to scale (CRS) production function:

$$Y_{i,t} = A_{i,t} L_{i,t} \tag{A.51}$$

where $A_{i,t}$ is firm-level productivity and $L_{i,t}$ is the level of employment. The firm pays wages $W_{i,t}$, incurs hiring costs $\kappa V_{i,t}$, and generates earnings:

$$E_{i,t} = Y_{i,t} - W_t L_{i,t} - \kappa V_{i,t} \tag{A.52}$$

Earnings represent the net flow profits from operating the firm: output net of the wage bill and the costs associated with posting vacancies. Firms maximize the expected present discounted value of earnings. Let $\mathcal{V}(A_{i,t}, L_{i,t})$ denote the value of the firm as a function of current productivity and employment. The Bellman equation for the firm's dynamic problem is:

$$\mathcal{V}(A_{i,t}, L_{i,t}) = \max_{V_{i,t}, L_{i,t+1}} \{E_{i,t} + \mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]\} \tag{A.53}$$

The firm chooses the number of vacancies $V_{i,t}$ to post and the resulting employment $L_{i,t+1}$ to maximize the sum of current earnings and the discounted continuation value, formed under subjective expectations $\mathbb{F}_t[\cdot]$ and a stochastic discount factor M_{t+1}. Employment evolves according to the accumulation equation:

$$L_{i,t+1} = (1-\delta)L_{i,t} + q_t V_{i,t} \tag{A.54}$$

which states that next period's employment depends on retained workers $(1-\delta)L_{i,t}$ and new hires $q_t V_{i,t}$ from current vacancies. Under constant returns to scale, the firm's marginal value of labor equals average value, and the first-order condition with respect to $V_{i,t}$ simplifies to:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[M_{t+1}\frac{\partial \mathcal{V}(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}}\right] = \frac{\mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]}{L_{i,t+1}} \equiv \frac{P_{i,t}}{L_{i,t+1}} \tag{A.55}$$

This condition equates the marginal cost of hiring a worker today, κ/q_t, to the expected marginal benefit of that hire, defined as the expected continuation value per worker. The term $P_{i,t} \equiv \mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{i,t+1}, L_{i,t+1})\right]$ denotes the firm's ex-dividend market value. Rewriting in logs:

$$\log q_t = \log \kappa - \log\left(\frac{P_{i,t}}{L_{i,t+1}}\right) = \log \kappa - \underbrace{\ln\left(\frac{P_{i,t}}{E_{i,t}}\right)}_{\equiv pe_{i,t}} - \underbrace{\ln\left(\frac{E_{i,t}}{L_{i,t+1}}\right)}_{\equiv el_{i,t}} \tag{A.56}$$

where $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$ is the log price-earnings ratio and $el_{i,t} \equiv \log(E_{i,t}/L_{i,t+1})$ is the log earnings per worker.

Cash Flow Process Assume that the firm's cash flow process consists of aggregate and idiosyncratic components. Firm i's earnings at time t are given by:

$$E_{i,t} = \exp(e_{i,t}) = E_t \cdot \widetilde{E}_{i,t} \tag{A.57}$$

where E_t represents the aggregate component and $\widetilde{E}_{i,t}$ captures firm-specific variation. The log aggregate earnings follow a random walk with drift:

$$\Delta e_t = \log a + \log \varepsilon_t, \quad \log \varepsilon_t \sim \mathcal{N}(-\tfrac{s^2}{2}, s^2) \tag{A.58}$$

while the log idiosyncratic component evolves as:

$$\Delta \widetilde{e}_{i,t} = \log \widetilde{a}_i + \log \widetilde{\varepsilon}_{i,t}, \quad \log \widetilde{\varepsilon}_{i,t} \sim \mathcal{N}(-\tfrac{\widetilde{s}_i^2}{2}, \widetilde{s}_i^2) \tag{A.59}$$

For simplicity, I assume that the aggregate ε_t and idiosyncratic $\widetilde{\varepsilon}_{i,t}$ are independently distributed, and that subjective beliefs preserve this independence.

Full Information Rational Expectations Under full information rational expectations, agents know the true drift and volatility parameters and form expectations using the true data generating process. Let $g_{i,t}^{RE}$ and $m_{i,t}^{RE}$ denote expected earnings and price growth for firm i under rational beliefs, decomposed into aggregate (g_t^{RE}, m_t^{RE}) and idiosyncratic ($\widetilde{g}_{i,t}^{RE}$, $\widetilde{m}_{i,t}^{RE}$) components. Under rational expectations, all growth expectations equal the corresponding true drift parameters: $g_t^{RE} = m_t^{RE} = a$ and $\widetilde{g}_{i,t}^{RE} = \widetilde{m}_{i,t}^{RE} = \widetilde{a}_i$. This makes the price-earnings ratio $P_{i,t}/E_{i,t} = \beta g_{i,t}^{RE}/(1-\beta m_{i,t}^{RE})$ constant. By independence of shocks, firm-level expectations are:

$$g_{i,t}^{RE} = g_t^{RE} \cdot \widetilde{g}_{i,t}^{RE} = a \cdot \widetilde{a}_i, \tag{A.60}$$
$$m_{i,t}^{RE} = m_t^{RE} \cdot \widetilde{m}_{i,t}^{RE} = a \cdot \widetilde{a}_i \tag{A.61}$$

Subjective Expectations Under Constant-Gain Learning Suppose that agents do not observe the true drift terms a and $\widetilde{a}_i$ in the cash flow process, and the firms do not know how their stock price $P_{i,t}$ is determined. Instead, they form beliefs and update these beliefs recursively as new information arrives. Firms form subjective expectations about the aggregate and idiosyncratic components of both cash flow growth and stock price growth:

$$\mathbb{F}_t[E_{t+1}] = g_t E_t, \quad \mathbb{F}_t[P_{t+1}] = m_t P_t, \tag{A.62}$$
$$\mathbb{F}_t[\widetilde{E}_{i,t+1}] = \widetilde{g}_{i,t}\widetilde{E}_{i,t}, \quad \mathbb{F}_t[\widetilde{P}_{i,t+1}] = \widetilde{m}_{i,t}\widetilde{P}_{i,t} \tag{A.63}$$

where g_t, m_t denote expectations about growth in the aggregate component and $\widetilde{g}_{i,t}$ and $\widetilde{m}_{i,t}$ denote expectations about growth in the idiosyncratic component. Under the independence of aggregate and idiosyncratic shocks, beliefs about total firm-level growth can be written as:

$$\mathbb{F}_t[E_{i,t+1}] = g_{i,t}E_{i,t} = g_t\widetilde{g}_{i,t} \cdot E_t\widetilde{E}_{i,t} \tag{A.64}$$
$$\mathbb{F}_t[P_{i,t+1}] = m_{i,t}P_{i,t} = m_t\widetilde{m}_{i,t} \cdot P_t\widetilde{P}_{i,t} \tag{A.65}$$

where $g_{i,t} = g_t\widetilde{g}_{i,t}$ and $m_{i,t} = m_t\widetilde{m}_{i,t}$. I assume that firms employ constant-gain learning to update their expectations using the rule:

$$g_t = g_{t-1} + \nu\left(\frac{E_{t-1}}{E_{t-2}} - g_{t-1}\right), \quad m_t = m_{t-1} + \nu\left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1}\right) \tag{A.66}$$

for the aggregate components, and

$$\widetilde{g}_{i,t} = \widetilde{g}_{i,t-1} + \nu \left(\frac{\widetilde{E}_{i,t-1}}{\widetilde{E}_{i,t-2}} - \widetilde{g}_{i,t-1} \right), \quad \widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} + \nu \left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1} \right) \tag{A.67}$$

for the idiosyncratic components, where ν is the constant gain parameter that governs the speed of learning. Suppose the initial beliefs are set equal to the growth rates under full information rational expectations:

$$g_0 = m_0 = a, \quad \widetilde{g}_{i,0} = \widetilde{m}_{i,0} = \widetilde{a}_i \tag{A.68}$$

Note that the current price and current cash flows do not enter the learning rule for $g_{i,t}$ and $m_{i,t}$. The belief updates incorporate information with a lag by using information only up to period $t-1$, which eliminates the simultaneity between prices and price growth expectations. The lag in the updating equation can be motivated by an information structure in which agents observe part of the lagged transitory shocks to stock price growth (Adam et al., 2016).

Under constant-gain learning, agents update their beliefs using a fixed gain, which causes past observations to receive exponentially decreasing weights. As a result, memory fades over time and beliefs never fully converge to rational expectations, even in a stable environment (Nagel and Xu, 2021). This learning scheme has the advantage of allowing beliefs to remain responsive to structural changes in the data-generating process, such as shifts in the trend growth rate of fundamentals. Compared to ordinary least squares (OLS) learning, where the gain vanishes over time, constant-gain learning avoids the counterfactual implication of declining volatility in predicted variables, and is often more realistic in environments with potential regime shifts or time-varying dynamics.

Constant-gain learning can be micro-founded in two complementary ways. First, when agents are internally rational but lack external knowledge of market dynamics, they optimally forecast next-period prices using past data (Adam et al., 2016). Alternatively, when agents learn from experience with fading memory, as older generations pass and newer ones rely more on recent data, the aggregation of their belief updates approximates a constant-gain rule (Nagel and Xu, 2021). The constant-gain learning specification for cash flow growth is supported by empirical evidence showing that survey respondents update their long-run earnings expectations only gradually following short-term earnings surprises (Nagel and Xu, 2021; De La O et al., 2024). The learning specification for stock price growth is motivated by empirical evidence showing that the implied return expectation can reproduce the dynamics of various survey based measures of subjective return expectations (Adam et al., 2016).

For parsimony and interpretability, the updating rules use the same constant gain parameter ν across all components. This reflects a shared degree of sluggishness in how firms update beliefs about different components of prices and cash flows. Existing estimates of the constant gain parameter ν are deliberately small, meaning that learning is slow and allows subjective beliefs to remain persistently distorted even after observing large forecast errors (Malmendier and Nagel, 2015; Adam et al., 2016). This persistence plays an important role for generating the sustained belief distortions needed to explain fluctuations in hiring and unemployment.

Subjective Firm Valuation To highlight how learning can improve the model's performance, I consider the simplest asset pricing model by assuming risk-neutral agents and time separable preferences (Adam et al., 2016). In this case, the aggregate stock price under subjective beliefs satisfies:

$$P_t = \beta \mathbb{F}_t[P_{t+1} + E_{t+1}] = \beta(m_t P_t + g_t E_t) \tag{A.69}$$

which implies $P_t(1 - \beta m_t) = \beta g_t E_t$ and thus we have

$$P_t = \frac{\beta g_t}{1 - \beta m_t} \cdot E_t \tag{A.70}$$

The firm's equilibrium stock price under subjective beliefs is:

$$P_{i,t} = \beta \mathbb{F}_t[P_{i,t+1} + E_{i,t+1}] = \beta(m_{i,t} P_{i,t} + g_{i,t} E_{i,t}) \tag{A.71}$$

which implies $P_{i,t}(1 - \beta m_{i,t}) = \beta g_{i,t} E_{i,t}$ and thus we have

$$P_{i,t} = \frac{\beta g_{i,t}}{1 - \beta m_{i,t}} \cdot E_{i,t} \tag{A.72}$$

where β is the time discount factor. The equation shows that the firm's value rises with expected cash flow growth $g_{i,t}$ and falls with expected price growth $m_{i,t}$. The belief distortions captured in these expectation terms will affect the firm's hiring decisions through its valuation.

Projection Facility To prevent agents from having an infinite demand for stocks based on the valuations in (A.70) and (A.72), I assume that the subjective beliefs about price growth are bounded such that

$$0 < m_t < \beta^{-1}, \quad 0 < m_{i,t} = m_t \widetilde{m}_{i,t} < \beta^{-1} \tag{A.73}$$

which rules out the case $m_{i,t} \geq \beta^{-1}$ where the expected stock returns are greater than the inverse of the time discount factor. To prevent perceived stock price growth from violating the bounds in (A.73), I apply a projection facility which makes a smooth modification to the belief-updating equation (Timmermann, 1993; Cogley and Sargent, 2005; Adam et al., 2016). If the updated belief from (A.66) exceeds a constant $m^U \leq \beta^{-1}$, then the update is ignored:

$$m_t = m_{t-1} \quad \text{if } m_{t-1} + \nu \left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1} \right) \geq m^U. \tag{A.74}$$

For the idiosyncratic component, the bound applies to the firm-level expectation $m_{i,t} = m_t \widetilde{m}_{i,t}$. Given beliefs about the aggregate component m_t, the projection rule therefore becomes:

$$\widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} \quad \text{if } \; m_t \left[\widetilde{m}_{i,t-1} + \nu \left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1} \right) \right] \geq m^U \tag{A.75}$$

This procedure can be interpreted as an approximate Bayesian updating scheme where agents have a truncated prior that assigns probability zero to $m_t \geq m^U$ and $m_{i,t} \geq m^U$ (Adam et al., 2016). It can be viewed as agents ignoring observations that would lead to beliefs implying infinite demand for stocks, which would represent economically implausible behavior.

Applying the projection facility is equivalent to imposing that firm-level price-earnings ratios remain below an upper bound $U^{PE} \equiv \beta a/(1 - \beta m^U)$. One interpretation is that, if the price-earnings ratio exceeds this upper bound, either market participants begin to fear a sharp downturn or some regulatory authority intervenes to bring prices down. In the simulations below, the results are not sensitive to the exact value of U^{PE} provided it is sufficiently high, since the bounding facility binds only rarely.

Hiring Condition I close the model by connecting asset valuations to firm hiring behavior. The connection to labor markets operates through the hiring condition. Firms post vacancies until the marginal cost of hiring equals its marginal value:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of Hiring}} = \underbrace{\frac{P_{i,t}}{L_{i,t+1}}}_{\text{Value of Hiring}} \tag{A.76}$$

where κ is the cost per vacancy, q_t is the job filling rate, and $L_{i,t+1}$ represents future employment. When firms are overly pessimistic about their expected cash flows (low $g_{i,t}$), this leads to lower firm value $P_{i,t}$, which reduces the value of hiring and leads to fewer job postings. The resulting decrease in vacancy creation drives up unemployment and reduces the job filling rate q_t.

Let $el_{i,t} \equiv e_{i,t} - l_{i,t+1} = \log E_{i,t} - \log L_{i,t+1}$ denote log earnings per worker. Given values for $\kappa, \delta, B, \eta, P_{i,t}$ and initial values for employment $L_{i,0}$, one can construct the sequence of vacancies $V_{i,t}$, employment $L_{i,t+1}$, labor market tightness θ_t, job filling rates q_t, and unemployment rate U_t by solving for the firm valuation, optimal hiring, and employment accumulation equations:

1. Initialize labor market tightness: $\theta_t^{(0)} = 1$

2. At iteration s, construct job filling rate under Cobb-Douglas matching in equation (A.50):

$$q_t^{(s)} = B(\theta_t^{(s)})^{-\eta} \tag{A.77}$$

3. Update each firm's employment policy using the hiring equation (A.76):

$$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa} \tag{A.78}$$

where $P_{i,t}$ is determined by the firm valuation equation (A.72) under the constant-gain learning rules in (A.66) and (A.67).

4. Update each firm's vacancy posting using the employment accumulation equation (A.54):

$$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}}(L_{i,t+1}^{(s)} - (1-\delta)L_{i,t}) \tag{A.79}$$

5. Aggregate firm-level variables over the set of firms I:

$$P_t = \sum_{i\in I} P_{i,t}, \quad V_t^{(s)} = \sum_{i\in I} V_{i,t}^{(s)}, \quad L_{t+1}^{(s)} = \sum_{i\in I} L_{i,t+1}^{(s)}, \quad U_t^{(s)} = 1 - \sum_{i\in I} L_{i,t} \tag{A.80}$$

6. Update labor market tightness: $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$. Check convergence: $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$ for some small tolerance $\varepsilon > 0$. If not, return to step 2 with the updated values.

Long-Horizon Cash Flow Growth and Stock Returns In this learning environment, the realized $j \geq 1$ period ahead log cash flow growth $\Delta e_{i,t+j} \equiv \log(E_{i,t+j}/E_{i,t+j-1})$ follows:

$$\Delta e_{i,t+j} = \log a_i + \log \varepsilon_{i,t+j} \tag{A.81}$$

and stock returns $r_{i,t+j} \equiv \log((P_{i,t+j} + E_{i,t+j})/P_{i,t+j-1})$ follow:

$$r_{i,t+j} = \log\left(\frac{E_{i,t+j}}{E_{i,t+j-1}}\frac{E_{i,t+j-1}}{P_{i,t+j-1}}\left(\frac{P_{i,t+j}}{E_{i,t+j}} + 1\right)\right) \tag{A.82}$$

$$= \Delta e_{i,t+j} + \log\left(\frac{1-\beta m_{i,t+j-1}}{\beta g_{i,t+j-1}}\right) + \log\left(\frac{1-\beta m_{i,t+j} + \beta g_{i,t+j}}{1-\beta m_{i,t+j}}\right) \tag{A.83}$$

where $a_i \equiv a \cdot \widetilde{a}_i$ and $\varepsilon_{i,t} \equiv \varepsilon_t \cdot \widetilde{\varepsilon}_{i,t}$. The price-earnings ratios are based on the firm valuations implied by equation (A.72).

Subjective expectations of these variables reflect beliefs about future earnings and capital gains. In models with constant-gain learning, beliefs evolve with fading memory, breaking the law of iterated expectations and making resale and buy-and-hold valuation methods non-equivalent. The buy-and-hold approach evaluates long-run payoffs under today's beliefs, while the resale method prices assets through a sequence of one-period-ahead valuations, each using updated beliefs. Following Nagel and Xu (2021), I adopt the resale valuation approach because it ensures time consistency under belief updating and reflects the idea that assets are effectively resold across agents with evolving expectations. I assume that the manager and the representative investor share the same beliefs and both apply the resale method, ensuring consistency between decision-making and valuation.

Let x_t and $\widetilde{x}_{i,t}$ denote the aggregate and idiosyncratic level of a variable $x \in \{E, P\}$ at time t, which are either aggregate cash flows or prices. Define:

$$R_t^x \equiv \frac{x_t}{x_{t-1}}, \quad Z_t^x \equiv (1-\nu)Z_{t-1}^x + \nu R_{t-1}^x \tag{A.84}$$

$$\widetilde{R}_{i,t}^x \equiv \frac{\widetilde{x}_{i,t}}{\widetilde{x}_{i,t-1}}, \quad \widetilde{Z}_{i,t}^x \equiv (1-\nu)\widetilde{Z}_{i,t-1}^x + \nu \widetilde{R}_{i,t-1}^x \tag{A.85}$$

That is, $Z_t^x, \widetilde{Z}_{i,t}^x$ denotes the subjective expectation of the growth rate of variable x, formed using constant-gain learning based on past realized growth $R_{t-1}^x, \widetilde{R}_{i,t-1}^x$, respectively. It can be shown by induction that the j-step-ahead expectation at time t is given by:

$$\mathbb{F}_t[Z_{t+j}^x] = a_j^x Z_t^x + b_j^x R_t^x, \tag{A.86}$$

$$\mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] = a_j^x \widetilde{Z}_{i,t}^x + b_j^x \widetilde{R}_{i,t}^x, \tag{A.87}$$

$$\mathbb{F}_t[Z_{i,t+j}^x] = \mathbb{F}_t[Z_{t+j}^x] \cdot \mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] \tag{A.88}$$

with recursively defined coefficients:

$$a_0 = 1, \quad b_0 = 0, \quad a_1 = 1-\nu, \quad b_1 = \nu, \tag{A.89}$$

$$a_j = (1-\nu)a_{j-1} + \nu a_{j-2}, \quad b_j = (1-\nu)b_{j-1} + \nu b_{j-2}, \quad j \geq 2 \tag{A.90}$$

Base case ($j = 0$). At time t, the value Z_t^x is known:

$$\mathbb{F}_t[Z_t^x] = Z_t^x = a_0 Z_t^x + b_0 R_t^x. \tag{A.91}$$

Base case ($j = 1$). From the learning rule:

$$Z^x_{t+1} = (1-\nu)Z^x_t + \nu R^x_t, \tag{A.92}$$

Taking expectations at time t:

$$\mathbb{F}_t[Z^x_{t+1}] = (1-\nu)Z^x_t + \nu R^x_t = a_1 Z^x_t + b_1 R^x_t. \tag{A.93}$$

Inductive step. Assume for $j-1$ and $j-2$ that:

$$\mathbb{F}_t[Z^x_{t+j-1}] = a_{j-1}Z^x_t + b_{j-1}R^x_t, \tag{A.94}$$

$$\mathbb{F}_t[Z^x_{t+j-2}] = a_{j-2}Z^x_t + b_{j-2}R^x_t. \tag{A.95}$$

Then, by the learning rule:

$$Z^x_{t+j} = (1-\nu)Z^x_{t+j-1} + \nu R^x_{t+j-1}. \tag{A.96}$$

Taking expectations at time t, and using $\mathbb{F}_t[R^x_{t+j-1}] = \mathbb{F}_t[Z^x_{t+j-2}]$ from the definition of Z^x_{t+j-2}:

$$\begin{aligned}
\mathbb{F}_t[Z^x_{t+j}] &= (1-\nu)\mathbb{F}_t[Z^x_{t+j-1}] + \nu\mathbb{F}_t[Z^x_{t+j-2}] && \text{(A.97)}\\
&= (1-\nu)(a_{j-1}Z^x_t + b_{j-1}R^x_t) + \nu(a_{j-2}Z^x_t + b_{j-2}R^x_t) && \text{(A.98)}\\
&= \left[(1-\nu)a_{j-1} + \nu a_{j-2}\right] Z^x_t + \left[(1-\nu)b_{j-1} + \nu b_{j-2}\right] R^x_t. && \text{(A.99)}
\end{aligned}$$

Thus, the recursion holds for j, completing the induction.

After making a first-order approximation $\mathbb{F}_t[\log(X)] \approx \log(\mathbb{F}_t[X])$, subjective expectations of log cash flow growth can be written as:

$$\mathbb{F}_t[\Delta e_{t+j}] = \mathbb{F}_t\left[\log\left(\frac{E_{t+j}}{E_{t+j-1}}\right)\right] \approx \log\left(\mathbb{F}_t\left[\frac{E_{t+j}}{E_{t+j-1}}\right]\right) = \log\left(\mathbb{F}_t\left[g_{t+j-1}\right]\right) \tag{A.100}$$

$$\mathbb{F}_t[\Delta e_{i,t+j}] = \mathbb{F}_t\left[\log\left(\frac{E_{i,t+j}}{E_{i,t+j-1}}\right)\right] \approx \log\left(\mathbb{F}_t\left[\frac{E_{i,t+j}}{E_{i,t+j-1}}\right]\right) = \log\left(\mathbb{F}_t\left[g_{i,t+j-1}\right]\right) \tag{A.101}$$

Similarly, subjective expectations of log stock returns can be written as:

$$\begin{aligned}
\mathbb{F}_t[r_{t+j}] &= \mathbb{F}_t\left[\Delta e_{t+j} + \log\left(\frac{1-\beta m_{t+j-1}}{\beta g_{t+j-1}}\right) + \log\left(\frac{1-\beta m_{t+j} + \beta g_{t+j}}{1-\beta m_{t+j}}\right)\right]\\
&\approx \log\left(\mathbb{F}_t[g_{t+j-1}]\right) + \log\left(\frac{1-\beta\mathbb{F}_t[m_{t+j-1}]}{\beta\mathbb{F}_t[g_{t+j-1}]}\right)\\
&\quad + \log\left(\frac{1-\beta\mathbb{F}_t[m_{t+j}] + \beta\mathbb{F}_t[g_{t+j}]}{1-\beta\mathbb{F}_t[m_{t+j}]}\right)\\
&\approx (1-\beta)\log\left(\mathbb{F}_t[g_{t+j-1}]\right) + \log\left(\frac{1-\beta\mathbb{F}_t[m_{t+j-1}]}{1-\beta\mathbb{F}_t[m_{t+j}]}\right)\\
&\quad + \log\left(1-\beta\mathbb{F}_t[m_{t+j}] + \beta\mathbb{F}_t[g_{t+j}]\right)
\end{aligned} \tag{A.102}$$

$$\begin{aligned}
\mathbb{F}_t[r_{i,t+j}] &= \mathbb{F}_t\left[\Delta e_{i,t+j} + \log\left(\frac{1-\beta m_{i,t+j-1}}{\beta g_{i,t+j-1}}\right) + \log\left(\frac{1-\beta m_{i,t+j} + \beta g_{i,t+j}}{1-\beta m_{i,t+j}}\right)\right]\\
&\approx \log\left(\mathbb{F}_t[g_{i,t+j-1}]\right) + \log\left(\frac{1-\beta\mathbb{F}_t[m_{i,t+j-1}]}{\beta\mathbb{F}_t[g_{i,t+j-1}]}\right)\\
&\quad + \log\left(\frac{1-\beta\mathbb{F}_t[m_{i,t+j}] + \beta\mathbb{F}_t[g_{i,t+j}]}{1-\beta\mathbb{F}_t[m_{i,t+j}]}\right)\\
&\approx (1-\beta)\log\left(\mathbb{F}_t[g_{i,t+j-1}]\right) + \log\left(\frac{1-\beta\mathbb{F}_t[m_{i,t+j-1}]}{1-\beta\mathbb{F}_t[m_{i,t+j}]}\right)\\
&\quad + \log\left(1-\beta\mathbb{F}_t[m_{i,t+j}] + \beta\mathbb{F}_t[g_{i,t+j}]\right)
\end{aligned} \tag{A.103}$$

where $\mathbb{F}_t[g_{i,t+j}]$ and $\mathbb{F}_t[m_{i,t+j}]$ are determined by the recursion in equations (A.88) through (A.90). Under constant-gain learning, realized stock returns $r_{i,t+j}$ and expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$ can fluctuate substantially due to large and persistent distortions in subjective beliefs embedded in $g_{i,t}$. In contrast, expected stock returns $\mathbb{F}_t[r_{i,t+j}]$ from equation (A.103) can show only small fluctuations because its variation depends mainly on the gap between expected cash flow growth and price growth $g_{i,t} - m_{i,t}$.[1] Since both $g_{i,t}$ and $m_{i,t}$ terms adjust slowly and often move together, their difference remains relatively stable. This generates the empirically observed pattern of high volatility in realized returns but low volatility in expected returns, consistent with survey evidence on return expectations.

[1]Since β is a number close to one, the first term in equation (A.103) involving $1-\beta$ will be quantitatively small. Since the learning rate ν is small, the one-period belief revisions in $\mathbb{F}_t[m_{i,t+j}]$ will also be quantitatively small in equation (A.103).

Model-Implied Decompositions I use data simulated from the learning model to decompose the job filling rate at the aggregate level and hiring rates at the firm level. The time-series decomposition of the aggregate job filling rate q_t is given by:

$$\log q_t = \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\text{Discount Rate}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\text{Cash Flow}} - \underbrace{\rho^h \mathbb{F}_t\left[pe_{t+h}\right]}_{\text{Future Price-Earnings}} \tag{A.104}$$

where $x_t = \sum_{i \in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. To analyze differences across firms, I estimate a cross-sectional decomposition of hiring rates using simulated firm-level data:

$$\widetilde{hl}_{i,t} = \underbrace{-\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}] \right]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t\left[\widetilde{pe}_{i,t+h}\right]}_{\text{Future Price-Earnings}} \tag{A.105}$$

where $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{I} \sum_i x_{i,t}$ denotes a cross-sectional deviation from the mean at time t.

Firms' hiring decisions reflect their evolving beliefs about cash flow growth $g_{i,t}$ and stock price growth $m_{i,t}$, which are updated according to the constant-gain learning rules. The slow learning rate in the model can generate large and persistent fluctuations in $g_{i,t}$ which drives fluctuations in expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$ and realized stock returns $r_{i,t+j}$. In contrast, the model can produce a low volatility in expected returns $\mathbb{F}_t[r_{i,t+j}]$ because their variation depends only on the gap between cash flow growth and price growth $g_{i,t} - m_{i,t}$, which is relatively stable over time. Therefore under subjective beliefs, the cash flow component in the decompositions will be highly volatile while the discount rate component remains relatively muted. Consequently, subjective expectations will systematically over-weight the role of cash flows relative to discount rates, generating the empirical pattern observed in the data. This contrasts sharply with rational expectations where the cash flow component contributes zero to the variance because expected future cash flow growth equals the constant drift term.

B.4 Regional Model

Model Setup This section presents a multi-area, multi-sector search-and-matching model with imperfect mobility across sectors, building from the models in Kehoe et al. (2019) and Chodorow-Reich and Wieland (2020). The economy consists of a continuum of islands indexed by s. Each island produces a differentiated variety of tradable goods that is consumed everywhere and a nontradable good. Both of these goods are produced using intermediate goods. Each consumer is endowed with one of two types of skills which are used in different intensities in the nontradable and tradable goods sectors. Labor is immobile across islands but can switch sectors. This assumption aligns with empirical evidence indicating that labor markets are predominantly regional in nature (Manning and Petrongolo, 2017). Consumers receive utility from a composite consumption good that is either purchased in the market or produced at home. Consumers and firms are ex-ante homogeneous and share the same subjective expectation $\mathbb{F}_t[\cdot]$. The islands only differ in the shocks that hit them.

Preferences and demand The composite consumption good on island s is produced from nontradable goods $X_{s,N,t}$ and tradable goods $X_{s,T,t}$

$$X_{s,t} = \left[\tau^{\frac{1}{\mu}} (X_{s,N,t})^{1-\frac{1}{\mu}} + (1-\tau)^{\frac{1}{\mu}} (X_{s,T,t})^{1-\frac{1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \tag{A.106}$$

where μ is the elasticity of substitution between tradable and nontradable goods. Demand for nontradable and tradable goods on island s is

$$X_{s,N,t} = \tau \left(\frac{P_{s,N,t}}{P_{s,t}} \right)^{-\mu} X_{s,t}, \qquad X_{s,T,t} = (1-\tau) \left(\frac{P_{s,T,t}}{P_{s,t}} \right)^{-\mu} X_{s,t} \tag{A.107}$$

where $P_{s,N,t}$ is the price of the nontradable good and $P_{s,T,t}$ is the world price of the composite tradable good. The price of the composite consumption good on island s is

$$P_{s,t} = \left[\tau P_{s,N,t}^{1-\mu} + (1-\tau) P_{s,T,t}^{1-\mu} \right]^{\frac{1}{1-\mu}} \tag{A.108}$$

The tradable good is a composite of varieties of differentiated tradable goods produced in all islands s'

$$X_{s,T,t} = \left[\int X_{s,T,t,s'}{}^{\frac{\mu_T - 1}{\mu_T}} ds' \right]^{\frac{\mu_T}{\mu_T - 1}} \tag{A.109}$$

where $X_{s,T,t,s'}$ is the amount of the variety of tradable good produced on island s' and consumed on island s. μ_T is the elasticity of substitution between varieties produced on different islands. Let $P_{s',T,t}$ be the price of tradable variety produced on island s'. Assume that there are no costs of shipping goods from one island to another, so that the law of one price holds and all islands purchase the variety s at the common price $P_{s,T,t}$. The price of the composite tradable good is common to all islands

$$P_{T,t} = \left[\int P_{s,T,t}^{1-\mu_T} ds\right]^{\frac{1}{1-\mu_T}} \tag{A.110}$$

The demand on island s' for a tradable variety produced on island s is therefore

$$X_{s',T,t,s} = \left[\frac{P_{s,T,t}}{P_{T,t}}\right]^{-\mu_T} X_{s',T,t} \tag{A.111}$$

so that the world demand for tradable goods produced by island s is

$$Y_{s,T,t} = \int X_{s',T,t,s} ds' = \left[\frac{P_{s,T,t}}{P_{T,t}}\right]^{-\mu_T} Y_{T,t} \tag{A.112}$$

where $Y_{T,t} = \int X_{s',T,t} ds'$ is the world demand for the composite tradable good. Since any individual island is of measure zero, shocks to an individual island do not affect either the world aggregate price of tradables $P_{T,t}$ or the world demand for tradables $Y_{T,t}$. Normalize the constant world price of the composite tradable good $P_{T,t}$ to one so that the composite tradable good is the numeraire.

Family's problem Each family of workers on island s chooses sequences for consumption $\{C_{s,t}\}$ and assets $\{A_{s,t+1}\}$ to maximize the present discounted value of consumption

$$\max_{C_{s,t},A_{s,t+1}} \sum_{t=0}^{\infty} \beta^t u(C_{s,t}) \tag{A.113}$$

where the family's consumption $C_{s,t} = X_{s,t} + b_{s,t}$ is the sum of goods purchased in the market $X_{s,t}$, and produced at home $b_{s,t}$ which can be consumed only by that family. The budget constraint is

$$P_{s,t}X_{s,t} + q^A A_{s,t+1} = Y_{s,t} + E_{s,t} + A_{s,t} \tag{A.114}$$

where $P_{s,t}$ is the price of the composite consumption good on the island, $A_{s,t}$ are the family's assets, and the family saves or borrows at a constant world bond price $q^A > \beta$. $Y_{s,t}$ is the income of the family's workers in the form of wages

$$Y_{s,t} \equiv \sum_{i\in\{\mathcal{N},\mathcal{T}\}} w_{s,i,t}L_{s,i,t} \tag{A.115}$$

and $E_{s,t}$ are profits from the firms the family owns on island s

$$E_{s,t} \equiv \sum_{i\in\{\mathcal{N},\mathcal{T}\}} E_{s,i,t} = \sum_{i\in\{\mathcal{N},\mathcal{T}\}} [(z_{s,i,t} - w_{s,i,t})L_{s,i,t} - \kappa V_{s,i,t}] \tag{A.116}$$

where $z_{s,i,t}$ is a sectoral labor productivity shock, $w_{s,i,t}$ is the wage of an employed worker, $L_{s,i,t}$ is the measure of employed workers, and $V_{s,i,t}$ is the measure of vacancies for producing intermediate goods of type i on island s. From the first-order condition for the family's problem, we can derive the shadow price of the composite consumption good at date t in units of the composite consumption good at date 0 on island s as

$$M_{s,t} = \beta^t \frac{u'(C_{s,t})/P_{s,t}}{u'(C_{s,0})/P_{s,0}} \tag{A.117}$$

Technology Nontradable and tradable goods are produced with locally produced intermediate goods. These intermediate goods are used by the nontradable and tradable sectors in different proportions. This setup effectively introduces costs of sectoral reallocations of workers because it implies a curved production possibility frontier between nontradable and tradable goods. The economy has two types of intermediate goods: Type $\mathcal{N}$ and type $\mathcal{T}$ goods. The technology for producing nontradable goods disproportionately uses type $\mathcal{N}$ goods, whereas the technology for producing tradable goods disproportionately uses type $\mathcal{T}$ goods according to the production technologies

$$Y_{s,N,t} = A(Y^{\mathcal{N}}_{s,N,t})^{\nu}(Y^{\mathcal{T}}_{s,N,t})^{1-\nu}, \qquad Y_{s,T,t} = A(Y^{\mathcal{N}}_{s,T,t})^{1-\nu}(Y^{\mathcal{T}}_{s,T,t})^{\nu} \tag{A.118}$$

with $\nu \geq 1/2$. $Y^{\mathcal{N}}_{s,N,t}$ and $Y^{\mathcal{N}}_{s,T,t}$ denote the use of intermediate inputs of type $\mathcal{N}$ in the production of nontradable and tradable goods, whereas $Y^{\mathcal{T}}_{s,N,t}$ and $Y^{\mathcal{T}}_{s,T,t}$ denote the use of intermediate inputs of type $\mathcal{T}$ in the production of nontradable and tradable goods. Both nontradable goods producers and tradable goods producers are competitive and take as given the price of their goods, $P_{s,N,t}$ and $P_{s,T,t}$. The demands for intermediate inputs in the nontradable goods sector are

$$Y^{\mathcal{N}}_{s,N,t} = \nu\left(\frac{P^{\mathcal{T}}_{s,t}}{P^{\mathcal{N}}_{s,t}}\right)^{1-\nu} Y_{s,N,t}, \qquad Y^{\mathcal{T}}_{s,N,t} = (1-\nu)\left(\frac{P^{\mathcal{N}}_{s,t}}{P^{\mathcal{T}}_{s,t}}\right)^{\nu} Y_{s,N,t} \tag{A.119}$$

where $P^{\mathcal{N}}_{s,t}$ and $P^{\mathcal{T}}_{s,t}$ are prices of the intermediate goods $\mathcal{N}$ and $\mathcal{T}$. The equation was derived under the normalization $A = \nu^{-\nu}(1-\nu)^{1-\nu}$. Likewise, the demands for intermediate inputs in the tradable goods sector are

$$Y^{\mathcal{N}}_{s,T,t} = (1-\nu)\left(\frac{P^{\mathcal{N}}_{s,t}}{P^{\mathcal{T}}_{s,t}}\right)^{\nu} Y_{s,T,t}, \qquad Y^{\mathcal{T}}_{s,T,t} = \nu\left(\frac{P^{\mathcal{T}}_{s,t}}{P^{\mathcal{N}}_{s,t}}\right)^{1-\nu} Y_{s,T,t} \tag{A.120}$$

Adding up the demands for each type of intermediate good by the two sectors gives the total demand on island s for intermediate goods of type i

$$Y^{i}_{s,t} = Y^{i}_{s,N,t} + Y^{i}_{s,T,t} \tag{A.121}$$

Production of these intermediate goods is given by

$$Y^{i}_{s,t} = z_{s,i,t} \cdot L_{s,i,t} \tag{A.122}$$

where $L_{s,i,t}$ is the measure of employed workers producing intermediate goods of type i on island s. $z_{s,i,t}$ represents exogenous labor productivity for producing intermediate goods of type i on island s. Zero profit conditions in nontradable and tradable goods sectors imply

$$P_{s,N,t} = (P^{\mathcal{N}}_{s,t})^{\nu}(P^{\mathcal{T}}_{s,t})^{1-\nu}, \qquad P_{s,T,t} = (P^{\mathcal{N}}_{s,t})^{1-\nu}(P^{\mathcal{T}}_{s,t})^{\nu} \tag{A.123}$$

Assume that there are measures of consumers $\pi^{\mathcal{N}}$ and $\pi^{\mathcal{T}} = 1 - \pi^{\mathcal{N}}$ in occuprations $\mathcal{N}$ and $\mathcal{T}$ who supply labor to produce the two types of intermediate goods $\mathcal{N}$ and $\mathcal{T}$, respectively. Consumers in occupation $\mathcal{N}$ can produce good $\mathcal{N}$, and consumers in occupation $\mathcal{T}$ can produce good $\mathcal{T}$. Consumers are hired by intermediate goods firms that produce intermediate goods of either type $\mathcal{N}$ or $\mathcal{T}$. These goods are then sold at competitive prices $P^{\mathcal{N}}_{s,t}$ and $P^{\mathcal{T}}_{s,t}$ to firms in the nontradable and tradable goods sectors. It is equivalent to think that the consumers in each occupation work in the sector that purchases the goods they produce. Under this interpretation, we can think of consumers in occupation $\mathcal{T}$ as being employed in sectors N and T and consumers in occupation $\mathcal{N}$ as also being employed in sectors N and T in different proportions. Sector N employs consumers in occupation $\mathcal{N}$ relatively more intensively, whereas sector T employs consumers in occupation $\mathcal{T}$ relatively more intensely.

This setup captures in a simple way the idea that switching sectors is relatively easy, whereas switching occupations is difficult. Note that any individual consumer faces no cost of switching sectors. But if a positive measure of consumer moves from one sector to the other, then the marginal revenue product of the consumers in the new sector falls and so do wages. This reduction in marginal revenue products acts like a switching cost in the aggregate.

Labor market Firms that produce intermediate good $i \in \{\mathcal{N}, \mathcal{T}\}$ post vacancies for consumers in occupation i, who produce intermediate good i when matched. Assume that consumers cannot switch occupations, so the measure of consumers in each occupation is fixed. The values of consumers in occupation i of island s are

$$\widetilde{W}_{s,i,t}(z_{s,i,t}) = w_{s,i,t}(z_{s,i,t}) \tag{A.124}$$

$$+ (1-\delta)\mathbb{F}_t[M_{s,t,t+1}\widetilde{W}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.125}$$

$$+ \delta\mathbb{F}_t[M_{s,t,t+1}\widetilde{U}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.126}$$

for employed consumers, and

$$\widetilde{U}_{s,i,t}(z_{s,i,t}) = P_t b_{s,t}(z_{s,i,t}) \tag{A.127}$$

$$+ \mathbb{F}_t[M_{s,t,t+1} f_{s,i,t}(z_{s,i,t}) \widetilde{W}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.128}$$

$$+ \mathbb{F}_t[M_{s,t,t+1}(1 - f_{s,i,t}(z_{s,i,t}))\widetilde{U}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.129}$$

for nonemployed consumers. $w_{s,i,t}(z_{s,i,t})$ is the wage received by a consumer in occupation i. $f_{s,i,t}(z_{s,i,t})$ is the job-finding probability of a consumer in occupation i. $b_{s,t}(z_{s,i,t})$ is the output of a consumer when not employed. δ is an exogenous separation probability. Subjective expectations $\mathbb{F}_t[\cdot]$ are with respect to next period's productivity $z_{s,i,t+1}$.

The value of a firm producing intermediate good i matched with a consumer in occupation i with productivity $z_{s,i,t}$ is

$$\widetilde{J}_{s,i,t}(z_{s,i,t}) = P_{s,i,t} z_{s,i,t} - w_{s,i,t}(z_{s,i,t}) + (1-\delta)\mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.130}$$

A consumer in occupation i matched with a firm in intermediate good sector i produces $z_{s,i,t}$ units of good i, which sells for $P_{s,i,t}z_{s,i,t}$, and the firm pays the consumer $w_{s,i,t}(z_{s,i,t})$. The cost of posting a vacancy is κ units of the composite tradable good. Free entry for intermediate goods producers in the labor market for workers in occupation i implies

$$\kappa = q_{s,i,t}(z_{s,i,t}) \cdot \mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}(z_{s,i,t+1})] \tag{A.131}$$

The matches of firms that produce intermediate good i with consumers are

$$L_{s,i,t}(z_{s,i,t}) = \frac{U_{s,i,t}(z_{s,i,t})V_{s,i,t}(z_{s,i,t})}{[U_{s,i,t}(z_{s,i,t})^\eta + V_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.132}$$

where $U_{s,i,t}(z_{s,i,t})$ is the measure of nonemployed consumers and $V_{s,i,t}(z_{s,i,t})$ is the measure of posted vacancies to attract such consumers. The parameter η governs the sensitivity of $f_{s,i,t}(z_{s,i,t})$ to $\theta_{s,i,t}$. The worker job-finding rate $f_{s,i,t}(z_{s,i,t})$ and firm job-filling rate $q_{s,i,t}(z_{s,i,t})$ are

$$f_{s,i,t}(z_{s,i,t}) = \frac{L_{s,i,t}(z_{s,i,t})}{U_{s,i,t}(z_{s,i,t})} = \frac{\theta_{s,i,t}(z_{s,i,t})}{[1+\theta_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.133}$$

$$q_{s,i,t}(z_{s,i,t}) = \frac{L_{s,i,t}(z_{s,i,t})}{V_{s,i,t}(z_{s,i,t})} = \frac{1}{[1+\theta_{s,i,t}(z_{s,i,t})^\eta]^{1/\eta}} \tag{A.134}$$

where $\theta_{s,i,t}(z_{s,i,t}) = V_{s,i,t}(z_{s,i,t})/U_{s,i,t}(z_{s,i,t})$ is the vacancy to nonemployment ratio.

The Nash bargaining problem determines the wage $w_{s,i,t}(z_{s,i,t})$ in any given match

$$\max_w [\widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t})]^\gamma \widetilde{J}_{s,i,t}(z_{s,i,t})^{1-\gamma} \tag{A.135}$$

where γ is a consumer's bargaining weight. Defining the surplus of a match between a firm and a consumer as $\widetilde{S}_{s,i,t}(z_{s,i,t}) = \widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t}) + \widetilde{J}_{s,i,t}(z_{s,i,t})$, Nash bargaining implies that firms and consumers split this surplus according to

$$\widetilde{W}_{s,i,t}(z_{s,i,t}) - \widetilde{U}_{s,i,t}(z_{s,i,t}) = \gamma\widetilde{S}_{s,i,t}(z_{s,i,t}), \qquad \widetilde{J}_{s,i,t}(z_{s,i,t}) = (1-\gamma)\widetilde{S}_{s,i,t}(z_{s,i,t}) \tag{A.136}$$

Equilibrium Market clearing for the two types of intermediate goods requires that

$$Y_{s,i,t} = z_{s,i,t} \cdot L_{s,i,t} = Y^i_{s,N,t} + Y^i_{s,T,t}, \quad i \in \{\mathcal{N}, \mathcal{T}\} \tag{A.137}$$

the left side of this equation is the total amount of intermediate goods of type i produced by employed workers in occupation i on island s, $L_{s,i,t}$. The right side is the total amount of these intermediate goods used by firms in the nontradable and tradable goods sectors on that island. Employment in the nontradable goods sector on island s is

$$\frac{Y^{\mathcal{N}}_{s,N,t}}{Y^{\mathcal{N}}_{s,t}} L_{s,\mathcal{N},t} + \frac{Y^{\mathcal{T}}_{s,N,t}}{Y^{\mathcal{T}}_{s,t}} L_{s,\mathcal{T},t} \tag{A.138}$$

Employment in the tradable goods sector on island s is

$$\frac{Y^{\mathcal{N}}_{s,T,t}}{Y^{\mathcal{N}}_{s,t}} L_{s,\mathcal{N},t} + \frac{Y^{\mathcal{T}}_{s,T,t}}{Y^{\mathcal{T}}_{s,t}} L_{s,\mathcal{T},t} \tag{A.139}$$

The relative demand effect on employment in the two sectors captures the idea that, since $Y^i_{s,N,t}/Y_{s,i,t} + Y^i_{s,T,t}/Y_{s,i,t} = 1$ for $i \in \{\mathcal{N}, \mathcal{T}\}$, any shift in demand from the nontradable goods sector on an island, holding fixed total employment on the island, decreases employment in the nontradable goods sector and increases it in the tradable goods sector on the island.

Market clearning for nontradable goods requires that the demand for nontradable goods on island s equal the amount on nontradable goods produced on island s

$$X_{s,N,t} = A(Y^{\mathcal{N}}_{s,N,t})^{\nu}(Y^{\mathcal{T}}_{s,N,t})^{1-\nu} \tag{A.140}$$

Similarly, market clearing for tradable goods requires that the world demand for tradable goods produced from island s equal the amount of tradable goods produced on island s

$$Y_{s,T,t} = A(Y^{\mathcal{N}}_{s,T,t})^{1-\nu}(Y^{\mathcal{T}}_{s,T,t})^{\nu} \tag{A.141}$$

Decomposition of Regional Job Filling Rates Combine the value of the worker to the firm with the zero-profit condition for entering firms, and substitute recursively:

$$\frac{\kappa}{q_{s,i,t}} = \mathbb{F}_t[M_{s,t,t+1}\widetilde{J}_{s,i,t+1}] = \mathbb{F}_t\left[M_{s,t,t+1}\left(P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1} + (1-\delta)\frac{\kappa}{q_{s,i,t+1}}\right)\right] \tag{A.142}$$

Multiply both sides by employment $L_{s,i,t+1}$, and substitute in the law of motion $L_{s,i,t+1} = (1-\delta)L_{s,i,t} + q_{s,i,t}V_{s,i,t}$

$$\frac{\kappa}{q_{s,i,t}}L_{s,i,t+1} = \mathbb{F}_t\left[M_{s,t,t+1}\left((P_{s,i,t+1}z_{s,i,t+1} - w_{s,i,t+1})L_{s,i,t+1} - \kappa V_{s,i,t+1} + \frac{\kappa}{q_{s,i,t+1}}L_{s,i,t+2}\right)\right] \tag{A.143}$$

Define earnings as profits after vacancy posting costs $E_{s,i,t} \equiv (P_{s,i,t}z_{s,i,t} - w_{s,i,t})L_{s,i,t} - \kappa V_{s,i,t}$, and substitute recursively:

$$\frac{\kappa}{q_{s,i,t}}L_{s,i,t+1} = \mathbb{F}_t\left[\sum_{j=1}^{T-1}\left(\prod_{k=1}^{j} M_{s,t+k-1,t+k}\right)E_{s,i,t+j}\right] + \mathbb{F}_t\left[M_{s,t+T-1,t+T}\frac{\kappa}{q_{s,i,t+T}}L_{s,i,t+T+1}\right] \tag{A.144}$$

Take limits as $T \to \infty$ while applying a transversality condition to rule out bubbles

$$\frac{\kappa}{q_{s,i,t}}L_{s,i,t+1} = \mathbb{F}_t\left[\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} M_{s,t+k-1,t+k}\right)E_{s,i,t+j}\right] \equiv P^E_{s,i,t} \tag{A.145}$$

where $P^E_{s,i,t}$ is the firm's ex-dividend value. Aggregate hiring condition to regional level as employment-weighted averages:

$$\frac{\kappa}{q_{s,t}} = \frac{P^E_{s,t}}{L_{s,t+1}} \tag{A.146}$$

where $S_{s,i,t+1} \equiv \frac{L_{s,i,t+1}}{L_{s,t+1}} = \frac{L_{s,i,t+1}}{\sum_{i' \in I} L_{s,i',t+1}}$ is industry i's employment share in region s. Define $q_{s,t} \equiv (\sum_{i\in I}\frac{1}{q_{s,i,t}}S_{s,i,t+1})^{-1}$, $P^E_{s,t} \equiv \sum_{i\in I} P^E_{s,i,t}$, and $E_{s,t} \equiv \sum_{i\in I} E_{s,i,t}$ as the employment-weighted job filling rate, total equity value, and total earnings of firms in region s. Take logarithms on both sides and expand the price-employment ratio $P^E_{s,t}/L_{s,t+1}$

$$\log q_{s,t} = \log\kappa - \log\left(\frac{P^E_{s,t}}{E_{s,t}}\right) - \log\left(\frac{E_{s,t}}{L_{s,t+1}}\right) \equiv \log\kappa - pe_{s,t} - el_{s,t} \tag{A.147}$$

where $pe_{s,t} \equiv \log(P^E_{s,t}/E_{s,t})$ is the log price-earnings ratio and $el_{s,t} \equiv \log(E_{s,t}/L_{s,t+1})$ is the log earnings-employment ratio. Next, log-linearize the price-earnings ratio by first log-linearizing the price-dividend ratio $pd_{s,t}$ around its long-term average $\overline{pd}$ (Campbell and Shiller, 1988):

$$pd_{s,t} = c_{pd} + \Delta d_{s,t+1} - r_{s,t+1} + \rho pd_{s,t+1} \tag{A.148}$$

where c_{pd} is a linearization constant, $r_{s,t+1} \equiv \log(\frac{P_{s,t+1}+D_{s,t+1}}{P_{s,t}})$ is the log stock return (with dividends), and $\rho \equiv \exp(\overline{pd})/(1+\exp(\overline{pd})) = 0.98$ is a persistence parameter that arises from the log linearization. Rewrite the equation in terms of log price-earnings by using the identity $pe_{s,t} = pd_{s,t} + de_{s,t}$, where $de_{s,t}$ is the log payout ratio:

$$pe_{s,t} = c_{pd} + \Delta e_{s,t+1} - r_{s,t+1} + \rho pe_{s,t+1} + (1-\rho)de_{s,t+1} \tag{A.149}$$

Since $1-\rho \approx 0$ and the payout ratio $de_{s,t}$ is bounded, $(1-\rho)de_{s,t+1}$ can be approximated as a constant, i.e., $c_{pe} \approx c_{pd} + (1-\rho)de_{s,t+1}$ (De La O et al., 2024):

$$pe_{s,t} \approx c_{pe} + \Delta e_{s,t+1} - r_{s,t+1} + \rho pe_{s,t+1} \tag{A.150}$$

Recursively substitute for the next h periods

$$pe_{s,t} = \sum_{j=1}^{h} \rho^{j-1}(c_{pe} + \Delta e_{s,t+j} - r_{s,t+j}) + \rho^h pe_{s,t+h} \tag{A.151}$$

Substitute log-linearized price-earnings into the hiring equation:

$$\log q_{s,t} = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} r_{s,t+j}}_{r_{s,t,t+h}} - \underbrace{\left[el_{s,t} + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{s,t+j}\right]}_{e_{s,t,t+h}} - \underbrace{\rho^h pe_{s,t+h}}_{pe_{s,t,t+h}} \tag{A.152}$$

where $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. $r_{s,t,t+h}$ captures news about discount rates. $e_{s,t,t+h}$ captures news about cash flows. $pe_{s,t,t+h}$ is a terminal value that captures other long-term influences beyond h years into the future that is not already captured in discount rates and cash flows. The decomposition above holds both ex-ante and ex-post. I consider ex-ante decompositions under subjective expectations $\mathbb{F}_t[\cdot]$:

$$\log q_{s,t} = c_q + \mathbb{F}_t[r_{s,t,t+h}] - \mathbb{F}_t[e_{s,t,t+h}] - \mathbb{F}_t[pe_{s,t,t+h}] \tag{A.153}$$

where $\mathbb{F}_t[x_{s,t,t+h}]$ denotes the h period ahead subjective conditional expectation of variable x for the firm in region s.

Predictability of Regional Unemployment Rates To derive implications for unemployment at the regional level, start with the disaggregated accumulation equation:

$$U_{s,i,t+1} = \delta_{s,i,t}(1 - U_{s,i,t}) + (1 - f_{s,i,t})U_{s,i,t} \tag{A.154}$$

where $U_{s,i,t}$ denotes the measure of unemployed workers in occupation i region s. Aggregate across the occupations to get total regional unemployment $U_{i,t} \equiv \sum_{i \in I} U_{s,i,t}$:

$$U_{s,t+1} = \delta_{s,t}(1 - U_{s,t}) + (1 - f_{s,t})U_{s,t} \tag{A.155}$$

where it is assumed that $\delta_{s,i,t} = \delta_{s,t}$, $f_{s,i,t} = f_{s,t}$. A log-linearization around steady state similar to Section B.2 gives us:

$$u_{s,t+1} \approx q\theta \log \delta_{s,t} + (1 - \delta - q\theta)u_{s,t} - q\theta \log q_{s,t} - q\theta \log \theta_{s,t} \tag{A.156}$$

where lower-cased variables denote log deviations from steady state. Finally, substitute in equation (A.153), which is a decomposition of the job filling rate $\log q_{s,t}$ into discount rate, cash flow, and future price-earnings components:

$$u_{s,t+1} \approx - \underbrace{q\theta c_q}_{\text{Constant}} - \underbrace{q\theta \cdot \mathbb{F}_t[r_{s,t,t+h}]}_{\text{Discount Rate}} + \underbrace{q\theta \cdot \mathbb{F}_t[e_{s,t,t+h}]}_{\text{Cash Flow}} + \underbrace{q\theta \cdot \mathbb{F}_t[pe_{s,t,t+h}]}_{\text{Future Price-Earning}} \tag{A.157}$$

$$+ \underbrace{(1 - \delta - q\theta) \cdot u_{s,t} - q\theta \cdot \log \theta_{s,t} + q\theta \cdot \log \delta_{s,t}}_{\text{Lag Unemployment, Tightness, Separations}} \tag{A.158}$$

The equation holds both ex-ante and ex-post. Therefore, I compare results from evaluating the equation under subjective $\mathbb{F}_t[\cdot]$ or rational $\mathbb{E}_t[\cdot]$ expectations. The decomposition can be estimated using regressions of the log unemployment rate on each of the components shown in the equation:

$$u_{s,t+1} = \beta_r \mathbb{F}_t[r_{s,t,t+h}] + \beta_e \mathbb{F}_t[e_{s,t,t+h}] + \gamma' X_{s,t} + \alpha_s + \alpha_t + \varepsilon_{s,t+1} \tag{A.159}$$

where $X_{s,t} \equiv [u_{s,t}, \log \theta_{s,t}, \log \delta_{s,t}]'$ collects the labor market factors. I estimate the decomposition using multivariate regressions to jointly identify the relative contributions of each component to observed unemployment fluctuations. To ensure stationarity and remove seasonal effects, I estimate the regression in log annual growth rates relative to the same quarter of the previous year. The future price-earnings ratio term $\mathbb{F}_t[pe_{s,t,t+h}]$ has been omitted in the multivariate regression because it is nearly collinear with future discount rates $\mathbb{F}_t[r_{s,t,t+h}]$ and cash flows $\mathbb{F}_t[e_{s,t,t+h}]$ through the Campbell and Shiller (1988) present value identity in equation (11). Similarly, the equation can also be estimated under rational expectations by replacing $\mathbb{F}_t[\cdot]$ with its rational expectations counterpart $\mathbb{E}_t[\cdot]$ based on machine learning forecasts.

C Data Details

This section describes the time-series and cross-sectional data sources used in the estimation. I use quarterly data on the variables represented in the decomposition from equations (A.36) and (A.37): employment L_t, unemployment U_t, job filling rates q_t, stock returns $r_{t,t+h}$, earnings growth $\Delta e_{t,t+h}$, price-earnings ratio pe_{t+h}, and earnings-employment ratio el_{t+h}. For each dependent variable of the decomposition, I also construct their corresponding survey expectations $\mathbb{F}_t$ and machine expectations $\mathbb{E}_t$.

C.1 Employment

For realized values of employment, I first construct an annual series for the aggregate number of employees (EMP) of the S&P 500 constituents by using accounting information from the CRSP and Compustat Merged Annual Industrial Files. The data spans 1970 to 2022 and was downloaded from WRDS on July 26, 2023. I aggregate the firm-level employment data to construct a total employment series for the S&P 500. I interpolate this series to a monthly frequency by using the fitted values from real-time regressions of log annual Compustat employment series on the log monthly BLS series for total nonfarm payrolls (PAYEMS). The regressions are estimated over recursively expanding samples from an initial monthly sample that begins on 1970:01 and ends on the month of the data release for each month's total nonfarm payrolls. To ensure that the fitted values do not use future information not available on each data release, I align each monthly BLS nonfarm payroll release with the annual Compustat S&P 500 employment series from the previous calendar year. To obtain a measure of employment L_{t+1} at the beginning of period $t+1$, I convert the monthly interpolated values to a quarterly frequency by taking the value of the series as of the last month of each calendar quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section B while still remaining known to firms by the end of period t. Data on nonfarm payrolls was downloaded through FRED on May 15, 2024.

C.2 Job Filling Rate

I construct a monthly series for the number of vacancies V_t following Barnichon (2010), by using JOLTS job openings starting 2000:12 (JTS00000000JOL) and extending the series back in time using the help-wanted index before 2000:12. The vacancies data has been downloaded from available on the author's website on May 19, 2024. For realized values of unemployment U_t, I use the BLS monthly series for the unemployment level (UNEMPLOY), downloaded through FRED on May 15, 2024. Labor market tightness $\theta_t = V_t/U_t$ is the ratio between vacancies and unemployment. The job separation rate δ_t uses the corresponding series from JOLTS.

I follow Shimer (2012) in constructing the job separation rate δ_t, job finding rate f_t, and job filling rate q_t. Job separation rate is the share of short-term unemployed out of total employment $\delta_t = U_t^s/L_t$, where U_t^s is the BLS series for the number of unemployed less than 5 weeks (UEMPLT5) that was downloaded through FRED on May 15, 2024. The job finding rate is:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}}$$

The expression for the job finding rate follows from the unemployment accumulation equation:

$$U_t = (1 - f_t)U_{t-1} + U_t^s$$

which states that unemployment U_t consists of either the previously unemployed U_{t-1} who did not find a job $(1 - f_t)$, or the short-term unemployed U_t^s that lost a job during the current period. The job filling rate is defined as the share of filled vacancies f_tV_t out of unemployment U_t:

$$q_t = \frac{f_t}{\theta_t} = \frac{f_tV_t}{U_t}$$

I first construct the job filling rate q_t at the monthly frequency. To remove high-frequency fluctuations that likely reflect measurement errors, I time-aggregate the monthly series to a quarterly frequency by taking a 3-month trailing average that ends on the first month of each calendar quarter. This timing assumption ensures that the survey and machine expectations in the variance decomposition do not use advance information about job filling rates that were not published at the time of each forecast. To ensure that all variables used in the variance decomposition are stationary, I follow Shimer (2012) by detrending the quarterly job filling rate q_t using an HP filter with a smoothing parameter of 10^5.

C.3 Wages

To assess the cyclicality of subjective wage expectations, I use publicly available survey and macroeconomic data to construct measures of actual real wage growth, subjective wage expectations, and unemployment rate changes. The Livingston Survey (semi-annual, 1961S1–2022S2), the CFO Survey (quarterly, 2001Q4–2023Q4), and the Survey of Consumer Expectations (SCE) (monthly, 2015M5–2022M12) provide the necessary data. I derive subjective wage growth expectations from median consensus forecasts of nominal wage growth in these surveys. The Livingston Survey forecasts are deflated using its own median CPI inflation forecast, while the CFO and SCE survey forecasts are deflated using CPI inflation expectations from the Survey of Professional Forecasters (SPF).

To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor UC_t^W under subjective expectations. In the search and matching model, the user cost of labor is the difference in the present value of wages between two firm-worker matches that are formed in two consecutive periods. Existing work measures the user cost of labor under full information rational expectations and finds that the user cost is more cyclical than the flow wage, suggesting that workers hired in recessions earn lower wages not only when hired but also in subsequent periods (Kudlyak, 2014; Bils et al., 2023). The survey-based measure this this paper relaxes the rational expectations assumption maintained in existing work. Consider the free-entry condition in the search and matching model:

$$\frac{\kappa}{q_t} = J_{t,t}$$

where a firm must pay a per vacancy cost of κ and vacancies are filled with probability q_t. $J_{t,\tau}$ is the value of a firm with a worker at time τ such that the productive match started at time t:

$$J_{t,\tau} \equiv z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}]$$

where $\mathbb{F}_t[\cdot]$ denotes subjective expectations based on survey data. $\beta = 0.9569$ is a discount factor and $\delta = 0.295$ is the probability that a employment relationship is terminated, both from Kudlyak (2014). Each period τ, a firm-worker match produces a per period output of z_τ and an employed worker received wage $w_{t,\tau}$ where t is the period when the worker is hired. $w_{t,t}$ is the new-hire wage. Note that the free entry condition is only required to hold for newly created matches for $\tau = t$. The expected difference between the firm's value of a newly created match in time t and the discounted value of a newly created match in period $t+1$ is

$$\begin{aligned} J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] = z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}] \\ - \beta(1-\delta)\mathbb{F}_t \left[z_{t+1} - w_{t+1,t+1} + \sum_{\tau=t+2}^{\infty} (\beta(1-\delta))^{\tau-(t+1)} \mathbb{F}_{t+1}[z_\tau - w_{t+1,\tau}] \right] \end{aligned}$$

Apply the Law of Iterated Expectations and collect terms

$$J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] = z_t - w_{t,t} - \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}]$$

Substitute the free-entry condition to the left-hand side

$$\underbrace{\frac{\kappa}{q_t} - \beta(1-\delta)\mathbb{F}_t\left[\frac{\kappa}{q_{t+1}}\right]}_{\text{Non-wage component of user cost } UC_t^V} = \underbrace{z_t}_{\text{Benefit}} - \underbrace{\left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}] \right]}_{\text{Wage component of user cost } UC_t^W}$$

The equation shows that the firm faces two sources of costs from a match: wage payments to a worker UC_t^W and vacancy opening costs UC_t^V. The firm creates jobs as long as the marginal benefit from adding a worker exceeds the user cost of labor. Note that the wage component of the user cost of labor UC_t^W, not the wage $w_{t,t}$, is the allocative price of labor.

I use worker-level data from the Survey of Consumer Expectations (SCE) to construct the user cost of labor UC_t^W under the survey respondents' subjective expectations. The SCE asks respondents about: the *month and year on which their current employment relationship started* (i.e., t in $w_{t,\tau}$); *"annual earnings, before taxes and other deductions, on your [current/main] job"* ($w_{t,\tau}$); short-term expectations on what their *"annual earnings will be in 4 months"* ($\mathbb{F}_t[w_{t,t+\frac{4}{12}}]$) and long-term expectations on *"annual earnings to be at your current job in 10 years"* ($\mathbb{F}_t[w_{t,t+10}]$). I obtain survey expectations about medium-term earnings between 4 months to 10 years by linearly interpolating between the two horizons:

$$\mathbb{F}_t[w_{t,t+h}] = \frac{10-h}{10-\frac{4}{12}} \mathbb{F}_t[w_{t,t+\frac{4}{12}}] + \frac{h-\frac{4}{12}}{10-\frac{4}{12}} \mathbb{F}_t[w_{t,t+10}], \quad h = 1, 2, \ldots, 10$$

The user cost of labor formulation assumes infinitely lived firms and workers, while empirical data are inherently finite. I truncate the horizon at 10 years given the availability of the survey data. Longer horizons reduce the weight of future terms due to discounting and job separations. In addition, if unemployment follows a mean-reverting process, wages in long-term employment relationships will eventually converge to the long-term mean, which after discounting would limit the size of very long-term influences (Kudlyak, 2014).

I measure actual real wage growth using two BLS wage series. The Livingston Survey forecasts target annual log real wage growth based on average weekly earnings of production and nonsupervisory employees in manufacturing (CES3000000030). The CFO and SCE surveys target annual log real wage growth based on average hourly earnings of private-sector employees (CEU0500000008). I deflate nominal wages using the Consumer Price Index (CPIAUCSL) to adjust for purchasing power.

For unemployment rates used to assess the cyclicality of wages, I use both actual data and subjective forecasts. Actual seasonally adjusted U.S. unemployment rate (UNRATE) comes from the BLS Current Population Survey (CPS). Subjective unemployment expectations are derived from median consensus SPF forecasts of future unemployment rates.

C.4 Stock Returns

C.4.1 Realized Stock Returns

Stock market returns use monthly data on CRSP value-weighted returns including dividends (VWRETD) from the Center for Research in Security Prices (CRSP). I compute annualized log stock returns by compounding the monthly returns using $r_{t+h} \equiv \frac{1}{h}\sum_{j=1}^{12h}\log(1+VWRETD_{t+j/12})$. The data was downloaded from WRDS on February 12, 2023. When evaluating the MSE ratios of the machine relative to that of a benchmark survey, I compute machine forecasts for either annual CRSP returns or S&P 500 price growth depending on which value most closely aligns with the concept that survey respondents are asked to predict. To measure one-year stock market price growth, I use the one-year log cumulative growth rate of the S&P 500 index, $\Delta p_{t+1} \equiv \log(P_{t+1}/P_t)$. The monthly S&P index series spans the period 1957:03 to 2022:12 and was downloaded from WRDS on January 24, 2024 from the Annual Update data of the Index File on the S&P 500.

C.4.2 Survey Expectations of Stock Returns

CFO Survey I use survey forecasts of S&P 500 stock returns from the CFO survey to measure subjective return expectations. The CFO survey is a quarterly survey that asks respondents about their expectations for the S&P 500 return over the next 12 months and 10 years ahead, obtained from `https://www.richmondfed.org/-/media/RichmondFedOrg/research/national_economy/cfo_survey/current_historical_cfo_data.xlsx`. I use the mean point forecast for the value of the "most likely" future stock return in the estimation. More specifically, the survey asks the respondent "*over the next 12 months, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*". The survey question for stock return expectations 10 years ahead is "*over the next 10 years, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*". The CFO survey panel includes firms that range from small operations to Fortune 500 companies across all major industries. Respondents include chief financial officers, owner-operators, vice presidents and directors of finance, and others with financial decision-making roles. The CFO panel has 1,600 members as of December 2022.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Because the CFO survey releases quarterly forecasts at the end of each quarter, I conservatively set the response deadline for the machine forecast to be the first day of the last month of each quarter (e.g., March 1st). The data spans the periods 2001Q4 to 2021Q1 and were downloaded on August 8th, 2022. Mean point forecasts before 2020Q3 are available in column `sp_1_exp` of sheet `through_Q1_2020`; mean point forecasts from 2020Q3 and onwards are available in column `sp_12moexp_2` of sheet `CFO_SP500`. The forecast is not available in 2019Q1, 2019Q4, 2020Q1, and 2020Q2. I impute the missing forecast for 2019Q1 by linearly interpolating between the available forecasts from 2018Q4 and 2019Q2. I impute the missing forecasts for 2019Q4, 2020Q1, and 2020Q2 by interpolating with the nearest available forecast between 2019Q3 and 2020Q3. Following Nagel and Xu (2022), I assume that the forecasted S&P 500 return includes dividends and capture expectations about annualized cumulative simple net returns compounded from time t to $t+h$, i.e., $\mathbb{F}_t[R_{t,t+h}]$. To obtain survey expectations of log returns $\mathbb{F}_t[\log(1+r_{t,t+h})]$ from a survey expectation of net simple returns $\mathbb{F}_t[R_{t,t+h}]$, I use the approximation $\mathbb{F}_t[\log(1+r_{t,t+h})] \approx \log(1+\mathbb{F}_t[R_{t,t+h}])$.

To obtain long-horizon survey expectations of annualized cumulative log S&P 500 returns over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year cumulative log return expectations:

$$\mathbb{F}_t[r_{t,t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[r_{t,t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[r_{t,t+10}], \quad h = 1, 2, \ldots, 10$$

Finally, I use the difference between cumulative long-horizon log return expectations between adjacent years (i.e., $\mathbb{F}_t[r_{t,t+h-1}]$ and $\mathbb{F}_t[r_{t,t+h}]$) to obtain $\mathbb{F}_t[r_{t+h}]$, the survey expectation of forward one-year log stock returns h years ahead:

$$\mathbb{F}_t[r_{t+h}] = h \times \mathbb{F}_t[r_{t,t+h}] - (h-1) \times \mathbb{F}_t[r_{t,t+h-1}], \quad h = 1, 2, \ldots, 10$$

IBES and Value Line I proxy expected firm-level stock returns using price growth expectations following De La O et al. (2024). Specifically, I construct expected price growth from IBES 12-month median price targets and Value Line 3–5 year median price targets, interpolating linearly for intermediate horizons.

To construct expected price growth, I combine short- and long-term price targets from two sources. For the short horizon, I use the 12-month median price targets from the Institutional Brokers Estimate System (IBES) database. For longer horizons, I use the median price targets from Value Line, which provide the expected stock price level approximately 3–5 years into the future for each firm. These targets reflect analysts' consensus expectations for each firm's stock price. I interpret the Value Line price target as the expected price level five years ahead and interpolate linearly between the IBES 12-month price target and the Value Line five-year price target to construct expected price growth for intermediate horizons between one and five years. For each firm i, expected annualized price growth over horizon h is given by:

$$\mathbb{F}_t[r_{i,t+h}] \approx \frac{1}{h} \log\left(\frac{\mathbb{F}_t[P_{i,t+h}]}{P_{i,t}}\right)$$

where $\mathbb{F}_t[P_{i,t+h}]$ is the forecasted price at horizon h, constructed through linear interpolation of IBES and Value Line targets, and $P_{i,t}$ is the observed stock price at time t. As shown in De La O et al. (2024), using price growth expectations to approximate expected firm-level stock returns is reasonably accurate, as dividends represent a relatively small component of total returns for most firms.

Gallup/UBS Survey The UBS/Gallup is a monthly survey of one-year-ahead stock market return expectations. I use the mean point forecast in our estimation and compare these to machine forecasts of the annual CRSP return. Gallup conducted 1,000 interviews of investors during the first two weeks of every month and results were reported on the last Monday of the month. The first survey was conducted on 1998:05. Until 1992:02, the survey was conducted quarterly on 1998:05, 1998:09, and 1998:11. The data on 1998:06, 1998:07, 1998:08, 1998:10, 1998:12, 1999:01, and 2006:01 are missing because the survey was not conducted on these months. I follow Adam et al. (2021) in starting the sample after 1999:02 due to missing values at the beginning of the sample.

For each month when the survey was conducted, respondents are asked about the return they expect on their own portfolio. The survey question is *"What overall rate of return do you expect to get on your portfolio in the next twelve months?"* Before 2003:05, respondents are also asked about the return they expect from an investment in the stock market during the next 12 months. The survey question is *"Thinking about the stock market more generally, what overall rate of return do you think the stock market will provide investors during the coming twelve months?"* For each month, I calculate the average expectations of returns on their own portfolio and returns on the market index. When calculating the average, survey respondents are weighted by the weight factor provided in the survey. I exclude extreme observations where a respondent reported expected returns higher than 95% or lower than -95%.

In order to construct a consistent measure of stock market return expectations over the entire sample period, I impute missing market return expectations using the fitted values from two regressions. First, I impute missing values during 1999:02-2005:12 and 2006:02-2007:10 with the fitted value from regressing expected market returns on own portfolio expectations contemporaneously, where the regression is estimated using the part of the sample where both are available. Second, I impute the one missing observation in both market and own portfolio return expectations for 2006:01 with the fitted value from regressing the market return expectations on the lagged own portfolio return expectations, where the coefficients are estimated using part of the sample where both are available, and the fitted value combines the estimated coefficients with lagged own portfolio expectations data from 2005:12. Following Nagel and Xu (2022), I assume that the forecasted stock market return includes dividends and capture expectations about annual simple net stock returns $\mathbb{F}_t[R_{t+1}]$. To obtain survey expectations of annual log returns $\mathbb{F}_t[\log(1 + r_{t+1})]$ from a survey expectation of annual net simple returns $\mathbb{F}_t[R_{t+1}]$, I use the approximation $\mathbb{F}_t[\log(1 + r_{t+1})] \approx \log(1 + \mathbb{F}_t[R_{t+1}])$. After applying all the procedures, the Gallup market return expectations series spans the periods 1999:02 to 2007:10. The data were downloaded on August 1st, 2024 from Roper iPoll: `http://ropercenter.cornell.edu/ubs-index-investor-optimism/`.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are in the first two weeks of a month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all UBS/Gallup respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "one-year-ahead" I interpret the question to be asking about the forecast period spanning from the current survey month to the same month one year ahead.

Michigan Survey of Consumers (SOC) The SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. Table 20 of the SOC reports the probability of an increase in stock market in next year. The survey question was *"The next question is about investing in the stock market. Please think about the type of mutual fund known as a diversified stock fund. This type of mutual fund holds stock in many different companies engaged in a wide variety of business activities. Suppose that tomorrow someone were to invest one thousand dollars in such a mutual fund. Please think about how much money this investment would be worth one year from now. What do you think the percent chance that this one thousand dollar investment will increase in* *value in the year ahead, so that it is worth more than one thousand dollars one year from now?"* When using this survey forecast to compare to machine forecasts, I impute a point forecast for stock market returns using the method described in Section C.4.2 below. I compare the imputed point forecast to machine forecasts of CRSP returns.

For the SOC, interviews are conducted monthly typically over the course of an entire month. (In rare cases, interviews may commence at the end of the previous month, as in February 2018 when interviews began on January 31st 2018.) I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are almost always conducted over the course of an entire month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "year ahead" I interpret the question to be asking about the forecast period spanning the period running from the current survey month to the same month one year ahead. The data spans 2002:06 to 2021:12. The SOC responses were obtained from `https://data.sca.isr.umich.edu/data-archive/mine.php` and downloaded on August 13th, 2022.

Livingston Survey Stock Price Forecast I obtain the Livingston Survey S&P 500 index forecast (SPIF) from the Federal Reserve Bank of Philadelphia, and use the mean values in our structural and forecasting models. I compare the one-year growth in these forecasts to machine forecasts of S&P 500 price growth. Our sample spans 1947:06 to 2021:06. The forecast series were downloaded on September 20, 2021.

The survey provides semi-annual forecasts on the level of the S&P 500 index. Participants are asked to provide forecasts for the level of the S&P 500 index for the end of the current survey month, 6 months ahead, and 12 months ahead. I use the mean of the respondents' forecasts each period, where the sample is based on about 50 observations. Most of the survey participants are professional forecasters with "formal and advanced training in economic theory and forecasting and use econometric models to generate their forecasts." Participants receive questionnaires for the survey in May and November, after the Consumer Price Index (CPI) data release for the previous month. All forecasts are typically submitted by the end of the respective month of May and November. The results of the survey are released near the end of the following month, on June and December of each calendar year. The exact release dates are available on the Philadelphia Fed website, at the header of each news release. I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since all forecasts are typically submitted by the end of May and November of each calendar year, I set the response deadline for the machine forecast to be the first day of the last month of June and December, implying that I allow the machine to use information only up through the end of the May and November.

I follow Nagel and Xu (2021) in constructing one-year stock price growth expectations from the level forecasts. Starting from June 1992, I use the ratio between the 12-month level forecast ($\texttt{SPIF\_12M}_t$) and 0-month level nowcasts ($\texttt{SPIF\_ZM}_t$) of the S&P 500 index. Before June 1992, the 0-month nowcast is not available. Therefore I use the annualized ratio between the 12-month ($\texttt{spi12}_t$) and 6-month ($\texttt{spi6}_t$) level forecast of the S&P 500 index

$$\mathbb{F}_t^{(Liv)}\left[\frac{P_{t+1}}{P_t}\right] \approx \begin{cases} \frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_t]} = \frac{\texttt{SPIF\_12M}_t}{\texttt{SPIF\_ZM}_t} & \text{if } t \geq 1992M6 \\ \left(\frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_{t+6}]}\right)^2 = \left(\frac{\texttt{spi12}_t}{\texttt{spi6}_t}\right)^2 & \text{if } t < 1992M6 \end{cases}$$

where P_t is the S&P 500 index and t indexes the survey's response deadline. To obtain a survey expectation of the log change in price growth I use the approximation $\mathbb{F}_t(\Delta p_{t+1}) \approx \log(\mathbb{F}_t[P_{t+1}]) - \log(P_t)$.

Conference Board (CB) Survey Respondents provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. Since the survey asks respondents about stock prices in the "year ahead," I interpret the question to be asking about the forecast period from the end of the current survey month to the end of the same month one year ahead. When we use this qualitative survey forecast to compare to machine forecasts, we impute a point forecast for stock market returns using the method described in Section C.4.2 below. I compare the imputed point forecast to machine forecasts of CRSP returns.

The survey is conducted monthly and I use the survey responses over 1987:04 to 2022:08. The data was downloaded on September 26, 2022. The survey uses an address-based mail sample design. Questionnaires are mailed to households on or about the first of each month. Survey responses flow in throughout the collection period, with the sample close-out for preliminary estimates occurring around the 18th of the month. Any responses received after then are used to produce final estimates for the month, which are published with the following month's data. Conversations with those knowledgeable about the survey suggested that most panelists respond early. Any responses received after around the 20th of the month–regardless of when they are filled out–are included in the final (but not preliminary) numbers.

I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since questionnaires reach households on or about the first of each month (e.g., February 1st) and most respondents respond early, I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., January 31st).

Converting Qualitative Forecasts to Point Forecasts (SOC and CB) I use the SOC probability to impute a quantitative point forecast of stock returns using a linear regression of CFO point forecasts for returns onto the SOC probablity of a price increase. The SOC asks respondents about the percent chance that an investment will "increase in value in the year ahead." I interpret this as asking about the ex dividend value, i.e., about price price growth. The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The SOC survey is conducted monthly, where survey months span 2002:06 to 2021:12. Since the CFO is a quarterly survey, the regression is estimated in real-time over a quarterly overlapping sample. Since the CFO survey is conducted during the last month of the quarter while the SOC is conducted monthly, I align the survey months between CFO and SOC by regressing the quarterly CFO survey point forecast with the qualitative SOC survey response during the last month of the quarter.

Since the SOC survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed SOC point forecast of stock returns including dividends. Specifically, at time t, I assume that the CFO forecast of stock returns, $\mathbb{F}_t^{\text{CFO}}[r_{t,t+1}]$, minus the current dividend yield, D_t/P_t, is related to the contemporaneous SOC probability of an increase in the stock market next year, $P_{t,t+1}^{\text{SOC}}$, by:

$$\mathbb{F}_t^{\text{CFO}}[r_{t,t+1}] - D_t/P_t = \beta_0 + \beta_1 P_{t,t+1}^{\text{SOC}} + \epsilon_t.$$

The final imputed SOC point forecast is constructed as $\mathbb{F}_t^{\text{SOC}}[r_{t,t+1}] = \hat{\beta}_0 + \hat{\beta}_1 P_{t,t+1}^{\text{SOC}} + D_t/P_t$. I first estimate the coefficients of the above regression over an initial overlapping sample of 2002Q2 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the SOC survey responses from the last month of each calendar quarter. Using the estimated coefficients and the SOC probability from 2005:03 gives us the point forecast of the one-year stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one quarterly observation to the end of the sample at a time, and storing the fitted values. This results in a time series of SOC point forecasts $\mathbb{F}_t^{\text{SOC}}[r_{t,t+1}]$ spanning 2005Q1 to 2021Q1.

The same procedure is done for the Conference Board Survey, except I replace $P_{t,t+1}^{\text{SOC}}$ by $P_{t,t+1}^{\text{CB}}$, a ratio of the proportion of those who respond with "increase" to the sum of "decrease" and "same." The CB survey asks respondents to provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. I interpret this as asking about price price growth. Since the CB survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed CB point forecast of stock returns including dividends.

The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The CB survey is conducted monthly, where survey months span 1987:04 to 2022:08. The regression is first estimated over an initial overlapping sample of 2001Q4 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the CB survey responses from the last month of each calendar quarter. Using the estimated coefficients and the CB survey response $P_{t,t+1}^{\text{CB}}$ from 2005:03 gives us the point forecast of the stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one observation to the end of the sample at a time, and storing the fitted values. This results in a time series of CB point forecasts $\mathbb{F}_t^{\text{CB}}[r_{t,t+1}]$ over 2005Q1 to 2021Q1.

Nagel and Xu Individual Investor Expectations Nagel and Xu (2021)'s individual investor expectations series for returns covers 1972-1977 (Annual) and 1987Q2-2023Q4 (Quarterly) and combine data from the following surveys:

1. UBS/Gallup: 1998:06-2007:10; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

2. Michigan Survey of Consumers (SOC): 2002:04-2022:12; Respondents provide the probability of a rise in the stock market over a 1-year horizon.

3. Conference Board (CB): 1987:04-2022:08; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

4. Vanguard Research Initiative (VRI): 2014:08; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

5. Roper: 1974-1977, annual, observed June of each calendar year; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

6. Lease, Lewellen, and Schlarbaum (1974, 1977): 1972-1973, annual, observed July of each calendar year; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

Among these sources, UBS/Gallup and VRI provide direct, point forecasts of expected stock returns, while SOC, CB, and Roper offer qualitative or probabilistic information that requires conversion to consistent return expectations. Nagel and Xu (2021) construct their final series using the following procedure:

1. Start with UBS/Gallup for 1998:06-2007:10 and VRI for 2014:08 since they capture the respondents' expected stock returns relatively closely (other surveys only provide qualitative measures).

2. Regress SOC on UBS/Gallup and VRI using periods of overlapping coverage (2002:04-2007:10). Use the fitted values from this regression to impute missing data for 2007:11-2022:12 (excluding 2014:08).

3. Regress CB on UBS/Gallup and VRI using periods of overlapping coverage (1998:06-2007:10). Use the fitted values from this regression to impute missing data for 1987:04-1998:05 (using CB) and 1974-1977 (using Roper).

4. Use the coefficients from regressing CB on UBS/Gallup and VRI (from step 3) to compute fitted values that convert the probabilistic forecast from Roper into point forecasts of stock returns.

5. Convert expected returns to expected excess returns by subtracting the average 1-year Treasury yield measured at the beginning of the survey month.

6. Aggregate monthly series to a quarterly frequency by taking the average expectation within calendar quarters.

C.5 Risk-Free Rates

Realized Risk-Free Rates As a measure of realized risk-free rates r_t^f, I obtain daily series for the annualized three-month Treasury bill rate (DTB3), downloaded from FRED on May 15, 2024. To match the definition used as the target variable in the Survey of Professional Forecasters (SPF), I time-aggregate the daily realized risk-free rate series to a quarterly frequency by taking the quarterly average, as discussed below.

Survey Expectations of Risk-Free Rates I obtain subjective expectations about risk-free rates from median forecasts for the annualized three-month Treasury bill rate from the Survey of Professional Forecasters (SPF). The SPF provides forecasts at the one and ten year horizons. For one year ahead forecasts (TBILL), respondents are asked to provide quarterly forecasts of the quarterly average three-month Treasury bill rate, in percentage points, where the forecasts are for the quarterly average of the underlying daily levels. I interpret the survey to be asking about annual net simple rates $\mathbb{F}_t[R^f_{t,t+1}]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r^f_{t,t+1}] \approx \log(1+\mathbb{F}_t[R^f_{t,t+1}])$. For ten year ahead forecasts (BILL10), respondents are asked to provide forecasts for the annual-average rate of return to three-month Treasury bills over the next 10 years, in percentage points. The ten year ahead forecasts are available only for surveys conducted in the first quarter of each calendar year. I interpret the survey to be asking about annualized cumulative net simple rates compounded from the survey quarter to the same quarter that is ten years after the survey year $\mathbb{F}_t[R^f_{t,t+10}]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r^f_{t,t+10}] \approx \log(1+\mathbb{F}_t[R^f_{t,t+10}])$. To obtain long-horizon survey expectations of annualized log three-month Treasury bill rates over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year return expectations:

$$\mathbb{F}_t[r^f_{t,t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[r^f_{t,t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[r^f_{t,t+10}], \quad h = 1,2,\ldots,10$$

Finally, I use the difference between the cumulative annualized long-horizon log three-month Treasury bill rate expectations between adjacent years (i.e., $\mathbb{F}_t[r^f_{t,t+h-1}]$ and $\mathbb{F}_t[r^f_{t,t+h}]$) to obtain $\mathbb{F}_t[r^f_{t+h}]$, the time t survey expectation of annualized forward log three-month Treasury bill rate h years ahead:

$$\mathbb{F}_t[r^f_{t+h}] = h \times \mathbb{F}_t[r^f_{t,t+h}] - (h-1) \times \mathbb{F}_t[r^f_{t,t+h-1}], \quad h = 1,2,\ldots,10$$

The surveys are sent out at the end of the first month of each quarter, and collected in the second or third week of the middle month of each quarter. When constructing machine learning forecasts for the risk-free rate, I assume that forecasters could have used all data released before the survey deadlines for the SPF, which are posted online at the Federal Reserve Bank of Philadelphia website. Since surveys are typically sent out at the end of the first month of each quarter, I make the conservative assumption that respondents only had data released by the first day of the second month of each quarter.

C.6 Earnings

C.6.1 Realized Earnings

To measure corporate earnings, I use quarterly S&P 500 IBES street earnings per share (EPS) data that starts in 1983Q4 from Hillenbrand and McCarthy (2024). Street earnings differ from GAAP earnings by excluding discontinued operations, extraordinary charges, and other non-operating items. According to the IBES user guide, analysts submit forecasts after backing out these transitory components, and IBES constructs the realized series to align with those forecasts. While analysts have some discretion over which items to exclude, Hillenbrand and McCarthy (2024) demonstrate that the target of these forecasts corresponds closely to earnings before special items in Compustat, suggesting that street earnings accurately reflect the measure analysts are targeting.

To convert EPS to total earnings, I multiply the resulting quarterly EPS series by the quarterly S&P 500 divisor, available at: `https://ycharts.com/indicators/sp_500_divisor`. Finally, to obtain a monthly S&P 500 earnings series, we linearly interpolate the resulting quarterly total earnings series. The final monthly total earnings series spans the period 1983:12 to 2021:12. I obtained quarterly IBES street earnings data from the authors of Hillenbrand and McCarthy (2024) on June 3, 2025. The divisor data were downloaded on March 13, 2022. To extend the sample back to 1965Q1, I use quarterly Compustat data on earnings before special items. As noted in Hillenbrand and McCarthy (2024), this measure closely tracks IBES street earnings, indicating it accurately reflects analysts' forecast targets.

C.6.2 Survey Expectations of Earnings

I obtain monthly survey data for the median analyst earnings per share forecast and actual earnings per share from the Institutional Brokers Estimate System (IBES) via Wharton Research Data Services (WRDS). The data spans the period 1976:01 to 2021:12 and was downloaded on October 2022.

Short-Term Growth (STG) Expectations I build measures of aggregate S&P 500 earnings expectations growth using the constituents of the S&P 500 at each point in time following De La O and Myers (2021). I first construct expected earnings expectations for aggregate earnings h-months-ahead as:

$$\mathbb{F}_t[E_{t+h}] = \varOmega_t \left[\sum_{i \in x_{t+h}} \mathbb{F}_t \left[EPS_{i,t+h} \right] S_{i,t} \right] / Divisor_t,$$

where $\mathbb{F}$ is the median analyst survey forecast, E is aggregate S&P 500 earnings, EPS_i is earning per share of firm i among all S&P 500 firms x_{t+h} for which I have forecasts in IBES for $t+h$, S_i is shares outstanding of firm i, and $Divisor_t$ is calculated as the S&P 500 market capitalization divided by the S&P 500 index. I obtain the number of outstanding shares for all companies in the S&P500 from Compustat. All data from Compustat were downloaded on November 17th, 2022. IBES estimates are available for most but not all S&P 500 companies. Following De La O and Myers (2021), I multiply this aggregate by $\varOmega_{t+h}$, a ratio of total S&P 500 market value to the market value of the forecasted companies at $t+h$ to account for the fact that IBES does not provide earnings forecasts for all firms in the S&P 500 in every period.

IBES database contains earning forecasts up to five annual fiscal periods (FY1 to FY5) and as a result, I interpolate across the different horizons to obtain the expectation over the next 12 months. This procedure has been used in the literature, including De La O and Myers (2021). Specifically, if the fiscal year of firm XYZ ends nine months after the survey date, I have a 9-month earning forecast $\mathbb{F}_t[E_{t+9}]$ from FY1 and a 21-month forecast $\mathbb{F}_t[E_{t+21}]$ from FY2. I then obtain the 12-month ahead forecast by interpolating these two forecasts as follows,

$$\mathbb{F}_t[E_{t+12}] = \frac{9}{12}\mathbb{F}_t[E_{t+9}] + \frac{3}{12}\mathbb{F}_t[E_{t+21}].$$

To convert the monthly forecast to quarterly frequency, I use the forecast made in the middle month of each quarter, and construct one-year earnings expectations from 1976Q1 to 2023Q4 and the earning expectation growth is calculated as an approximation following following De La O and Myers (2021):

$$\mathbb{F}_t \left(\Delta e_{t+12} \right) \approx \ln \left(\mathbb{F}_t[E_{t+12}] \right) - e_t$$

where e_t is log earnings for S&P 500 at time t calculated as $e_t = \log \left(EPS_t \cdot Divisor_t \right)$, where EPS_t is the earnings per share for the S&P 500 obtained from Shiller's data depository and S&P Global, as described above.

Long-Term Growth (LTG) Expectations I construct long term expected earnings growth (LTG) for the S&P 500 following Bordalo et al. (2019). Specifically, I obtain the median firm-level LTG forecast from IBES, and aggregate the value-weighted firm-level forecasts,

$$LTG_t = \sum_{i=1}^{S} LTG_{i,t} \frac{P_{i,t} Q_{i,t}}{\sum_{i=1}^{S} P_{i,t} Q_{i,t}}$$

where S is the number of firms in the S&P 500 index, and where $P_{i,t}$ and $Q_{i,t}$ are the stock price and the number of shares outstanding of firm i at time t, respectively. $LTG_{i,t}$ is the median forecast of firm i's long term expected earnings growth. The data spans the periods from 1981:12 to 2021:12. All data were downloaded in February 2023.

Finally, I use the difference between survey expectations of log earnings between adjacent years (i.e., $\mathbb{F}_t[e_{t+h-1}]$ and $\mathbb{F}_t[e_{t+h}]$) to obtain $\mathbb{F}_t[\Delta e_{t+h}] = \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}]$, the time t survey expectation of forward one-year log earnings growth $h = 1, 2, 3, 4$ years ahead. For the $h = 5$ year horizon, I interpret the IBES's Long-Term Growth (LTG) forecast as the 5-year forward annual log earnings growth from 4 to 5 years ahead:

$$\mathbb{F}_t[\Delta e_{t+h}] = \begin{cases} \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}] & \text{if } h = 1, 2, 3, 4 \text{ years} \\ LTG_t & \text{if } h = 5 \text{ years} \end{cases}$$

To estimate any biases in IBES analyst forecasts, the dynamic machine algorithm takes as an input a likely date corresponding to information analysts could have known at the time of their forecast. IBES does not provide an explicit deadline for their forecasts to be returned. Therefore I instead use the "statistical period" day (the day when the set of summary statistics was calculated) as a proxy for the deadline. I set the machine deadline to be the day before this date. The statistical period date is typically between day 14 and day 20 of a given month, implying that the machine deadline varies from month to month. As the machine learning algorithm uses mixed-frequency techniques adapted to quarterly sampling intervals, while the IBES forecasts are monthly, I compare machine and IBES analyst forecasts as of the middle month of each quarter, considering 12-month ahead forecast from the beginning of the month following the survey month.

C.7 Price-Earnings Ratio

I construct a quarterly series for the price-earnings ratio $PE_t \equiv P_t/E_t$ using the end-of-quarter S&P 500 stock price index P_t and the S&P 500 quarterly total earnings E_t. I infer subjective expectations of the log price-earnings ratio $\mathbb{F}_t[pe_{t+h}]$ by combining the current log price-earnings ratio pe_t with h year ahead subjective expectations of annual log stock returns $\mathbb{F}_t[r_{t+h}]$ and annual log earnings growth $\mathbb{F}_t[\Delta e_{t+h}]$, following the approach used in De La O and Myers (2021). Rearrange the Campbell and Shiller (1988) present value identity for the price-earnings ratio in equation (A.33) to express the future log price-earnings ratio as a function of current log price-earnings, log earnings growth, and log stock returns:

$$pe_{t+h} = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{t+j} - r_{t+j})$$

where the equation holds both ex-ante and ex-post. Apply subjective expectations $\mathbb{F}_t$ on both sides of the equation:

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \underbrace{\mathbb{F}_t[\Delta e_{t+j}]}_{\text{Survey (IBES)}} - \underbrace{\mathbb{F}_t[r_{t+j}]}_{\text{Survey (CFO)}}) \tag{A.160}$$

where subjective expectations about j years ahead forward annual log stock returns $\mathbb{F}_t[r_{t+j}]$ and forward annual log earnings growth $\mathbb{F}_t[\Delta e_{t+j}]$ use survey forecasts from the CFO survey and IBES, respectively. I construct firm-level price-earnings expectations by applying the same log-linear approximation to firm-level expectations of stock returns (from IBES and Value Line) and earnings growth (from IBES).

C.8 Earnings-Employment Ratio

The current earnings-employment ratio is defined as $EL_t \equiv E_t/L_{t+1}$, where E_t denotes quarterly total earnings for the S&P 500 and L_{t+1} is the employment stock at the beginning of period $t+1$. I measure L_{t+1} using end-of-period employment levels within each quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section B while still remaining known to firms by the end of period t.

C.9 Machine Learning Forecasts

For each survey forecast, I also construct their corresponding machine learning forecast by estimating a Long Short-Term Memory (LSTM) neural network:

$$\mathbb{E}_t[y_{t,t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$$

where $y_{t,t+h}$ denotes the variable y to be predicted h years ahead of time t, and $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept. $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$ denotes predicted values from a LSTM neural network that can be represented by a (potentially) high dimensional set of finite-valued parameters $\boldsymbol{\beta}_{h,t}$. The machine learning model is estimated using an algorithm that takes into account the data-rich environment in which firms operate in (Bianchi et al., 2022 and Bianchi et al., 2024b). When constructing machine learning forecasts of each variable, I allow the machine to use only information that would have been available to all survey respondents at the time of each forecast. See Section D for details about the machine learning algorithm and predictor variables. Machine expectations about the price-earnings ratio $\mathbb{E}_t[pe_{t+h}]$ is constructed similarly to the survey counterpart, by replacing the survey forecasts of stock returns and earnings growth on the right-hand side of equation (A.160) with the corresponding machine learning forecasts.

For the cross-sectional decomposition, I construct analogous machine learning forecasts of returns, earnings growth, and price-earnings ratios at the portfolio level using the same LSTM framework, applied to portfolio-specific predictors and outcomes. Firms are first sorted into five value-weighted portfolios based on book-to-market ratios, and all firm-level variables are aggregated to the portfolio level using market capitalization weights prior to estimation.

D Machine Learning

D.1 Machine Algorithm Details

The basic dynamic algorithm follows the six step approach of Bianchi et al. (2022) of 1. Sample partitioning, 2. In-sample estimation, 3. Training and cross-validation, 4. Grid reoptimization, 5. Out-of-sample prediction, and 6. Roll forward and repeat. We refer the interested reader to that paper for details and discuss details of the implementation here only insofar as they differ. At time t, a prior sample of size $\dot{T}$ is partitioned into two subsample windows: a *training sample* consisting of the first T_E observations, and a hold-out *validation sample* of T_V subsequent observations so that $\dot{T} = T_E + T_V$. The training sample is used to estimate the model subject to a specific set of tuning parameter values, and the validation sample is used for tuning the hyperparameters. The model to be estimated over the training sample is

$$y_{t,t+h} = G^e\left(\mathcal{X}_t, \boldsymbol{\beta}_{h,t}\right) + \epsilon_{t+h}.$$

where $y_{t,t+h}$ is a time series indexed by j whose value in period $h \geq 1$ the machine is asked to predict at time t, $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept, and $G^e(\cdot)$ is a machine learning estimator that can be represented by a (potentially) high dimensional set of finite-valued parameters $\boldsymbol{\beta}^e_{h,t}$. We consider two estimators for $G^e(\cdot)$: Elastic Net $G^{\text{EN}}(\mathcal{X}_t, \boldsymbol{\beta}^{\text{EN}}_{j,h})$, and Long Short-Term Memory (LSTM) network $G^{\text{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}^{\text{LSTM}}_{j,h})$. The $e \in \{\text{EN}, \text{LSTM}\}$ superscripts on $\boldsymbol{\beta}$ indicate that the parameters depend on the estimator being used (See the next section for a description of EN and LSTM). $\mathcal{X}_t$ always denotes the most recent data that would have been in real time prior to the date on which the forecast was submitted. To ensure that the effect of each variable in the input vector is regularized fairly during the estimation, we standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. It should be noted that the most recent observation on the left-hand-side is generally available in real time only with a one-period lag, thus the forecasting estimations can only be run with data over a sample that stops one period later than today in real time. The parameters $\boldsymbol{\beta}^e_{h,t}$ are estimated by minimizing the mean-square loss function over the training sample with L_1 and L_2 penalties

$$L(\boldsymbol{\beta}^e_{h,t}, \mathbf{X}_{T_E}, \boldsymbol{\lambda}^e_t) \equiv \underbrace{\frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - G^e\left(\mathcal{X}_\tau, \boldsymbol{\beta}^e_{h,t}\right)\right)^2}_{\text{Mean Square Error}} + \underbrace{\lambda^e_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^e_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^e_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^e_{j,h,t,k})^2}_{L_2 \text{ Penalty}}$$

where $\mathbf{X}_{T_E} = (y_{t-T_E}, \ldots, y_t, \mathcal{X}'_{t-T_E}, \ldots, \mathcal{X}'_t)'$ is the vector containing all observations in the training sample of size T_E. The estimated $\boldsymbol{\beta}^e_{h,t}$ is a function of the data $\boldsymbol{X}_{T_E}$ and a non-negative regularization parameter vector $\boldsymbol{\lambda}^e_t = \left(\lambda^e_{1,t}, \lambda^e_{2,t}, \boldsymbol{\lambda}^{LSTM}_{0,t}\right)'$ where $\boldsymbol{\lambda}^{LSTM}_{0,t}$ is a set of hyperparameters only relevant when using the LSTM estimator for $G^e(\cdot)$ (see below). For the EN case there are only two hyperparameters, which determine the optimal shrinkage and sparsity of the time t machine specification. The regularization parameters $\boldsymbol{\lambda}^e_t$ are estimated by minimizing the mean-square loss over pseudo-out-of-sample forecast errors generated from rolling regressions through the validation sample:

$$\widehat{\boldsymbol{\lambda}}^e_t, \widehat{T}_E, \widehat{T}_V = \underset{\boldsymbol{\lambda}^e_t, T_E, T_V}{argmin}\left\{\frac{1}{T_V - h}\sum_{\tau=T_E}^{T_E+T_V-h}\left(y_{\tau+h} - G^e(\mathcal{X}_\tau, \widehat{\boldsymbol{\beta}}^e_{j,h,\tau}(\mathbf{X}_{T_E}, \boldsymbol{\lambda}^e_t))\right)^2 + \underbrace{\lambda^e_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^e_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^e_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^e_{j,h,t,k})^2}_{L_2 \text{ Penalty}}\right\}$$

where $\widehat{\boldsymbol{\beta}}^e_{j,h,\tau}(\cdot)$ for $e \in \{\text{EN}, \text{LSTM}\}$ is the time τ estimate of $\boldsymbol{\beta}^e_{j,h}$ given $\boldsymbol{\lambda}^e_t$ and data through time τ in a training sample of size T_E. Denote the combined final estimator $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$, where the regularization parameter $\widehat{\boldsymbol{\lambda}}^e_t$ is estimated using cross-validation dynamically over time. Note that the algorithm also asks the machine to dynamically choose both the optimal training window $\widehat{T}_E$ and the optimal validation window $\widehat{T}_V$ by minimizing the pseudo-out-of-sample MSE.

The estimation of $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$ is repeated sequentially in rolling subsamples, with parameters estimated from information known at time t. Note that the time t subscripts of $\widehat{\boldsymbol{\beta}}^e_{h,t}$ and $\widehat{\boldsymbol{\lambda}}^e_t$ denote one in a sequence of time-invariant parameter estimates obtained from rolling subsamples, rather than estimates that vary over time within a sample. Likewise, we denote the time t machine belief about $y_{t,t+h}$ as $\mathbb{E}_t^e[y_{t,t+h}]$, defined by

$$\mathbb{E}_t^e[y_{t,t+h}] \equiv G^e\left(\mathcal{X}_t, \widehat{\boldsymbol{\beta}}_{h,t}^e(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}_t^e)\right)$$

Finally, the machine MSE is computed by averaging across the sequence of squared forecast errors in the true out-of-sample forecasts for periods $t = (\dot{T} + h), \ldots, T$ where T is the last period of our sample. The true out-of-sample forecasts used for neither estimation nor tuning is the *testing subsample* used to evaluate the model's predictive performance.

On rare occasions, one or more of the explanatory variables used in the machine forecast specification assumes a value that is order of magnitudes different from its historical value. This is usually indicative of a measurement problem in the raw data. We therefore program the machine to detect in real-time whether its forecast is an extreme outlier, and in that case to discard the forecast replacing it with the historical mean. Specifically, at each t, the machine forecast $\mathbb{E}_t^e[y_{t,t+h}]$ is set to be the historical mean calculated up to time t whenever the former is five or more standard deviations above its own rolling mean over the most recent 20 quarters.

We include the contemporaneous survey forecasts $\mathbb{F}_t[y_{t,t+h}]$ for the median respondent only for inflation and GDP forecasts, following Bianchi et al. (2022). This procedure allows the machine to capture intangible information due to judgement or private signals. Specifically, for these forecasts of inflation and GDP growth, we consider the following machine learning empirical specification for forecasting $y_{t,t+h}$ given information at time t, to be benchmarked against the time t survey forecast of respondent-type X, where this type is the median here:

$$y_{t,t+h} = G_{jh}^e(\mathbf{Z}_t) + \gamma_{jh\mathbb{M}}\mathbb{F}_t[y_{t,t+h}] + \epsilon_{t+h}, \qquad h \geq 1$$

where $\gamma_{jh\mathbb{M}}$ is a parameter to be estimated, and where $G_{jh\mathbb{M}}(\mathbf{Z}_t)$ represents a ML estimator as function of big data. Note that the intercept α_{jh} from Bianchi et al. (2022) gets absorbed into the $G_{jh}^e(\mathbf{Z}_t)$ in LSTM via the outermost bias term.

D.1.1 Elastic Net (EN)

We use the Elastic Net (EN) estimator, which combines Least Absolute Shrinkage and Selection Operator (LASSO) and ridge type penalties. The model can be written as:

$$y_{t,t+h} = \mathcal{X}'_{tj}\boldsymbol{\beta}_{j,h}^{\text{EN}} + \epsilon_{t+h}$$

where $\mathcal{X}_t = (1, \mathcal{X}_{1t,...,}\mathcal{X}_{Kt})'$ include the independent variable observations $(\mathbb{F}_t[y_{t,t+h}], \mathcal{Z}_{j,t})$ into a vector with "1" and $\boldsymbol{\beta}_{j,h}^{\text{EN}} = (\alpha_{j,h}, \beta_{j,h\mathbb{F}}, vec(\mathbf{B}_{j,h\mathcal{Z}}))' \equiv (\beta_0, \beta_1, ...\beta_K)'$ collects all the coefficients.

It is customary to standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. The coefficient estimates are then put back in their original scale by multiplying the slope coefficients by their respective standard deviations, and adding back the mean (scaled by slope coefficient over standard deviation.) The EN estimator incorporates both an L_1 and L_2 penalty:

$$\widehat{\boldsymbol{\beta}}_{j,h}^{\text{EN}} = \underset{\beta_0,\beta_1,\ldots,\beta_K}{argmin} \frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - \mathcal{X}_\tau^{'}\boldsymbol{\beta}_{j,h}\right)^2 + \underbrace{\lambda_1\sum_{k=1}^{K}\left|\boldsymbol{\beta}_{j,h,k}\right|}_{\text{LASSO}} + \underbrace{\lambda_2\sum_{k=1}^{K}(\boldsymbol{\beta}_{j,h,k})^2}_{\text{ridge}}$$

By minimizing the MSE over the training samples, we choose the optimal λ_1 and λ_2 values simultaneously.

In the implementation, the EN estimator is sometimes used as an input into the algorithm using the LSTM estimator. Specifically, we ensure that the machine forecast can only differ from the relevant benchmark if it demonstrably improves the pseudo out-of-sample prediction in the training samples *prior* to making a true out-of-sample forecast. Otherwise, the machine is replaced by the benchmark calculated up to time t. In some cases the benchmark is a survey forecast, in others it could be a historical mean value for the variable. However, for the implementation using LSTM, we also use the EN forecast as a benchmark.

D.1.2 Long Short-Term Memory (LSTM) Network

An LSTM network is a type of Recurrent Neural Network (RNN), which are neural networks used to learn about sequential data such as time series or natural language. In particular, LSTM networks can learn long-term dependencies between across time periods by introducing hidden layers and memory cells to control the flow of information over longer time periods. The general case of the LSTM network with up to N hidden layers is defined as

$$\underbrace{G^{\text{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}_{j,h}^{\text{LSTM}})}_{1\times 1} = \underbrace{W^{(yh^N)}}_{1\times D_{h^N}} \underbrace{h_t^N}_{D_{h^N}\times 1} + \underbrace{b_y}_{1\times 1} \quad \text{(Output layer)}$$

$$\underbrace{h_t^n}_{D_{h^n}\times 1} = \underbrace{o_t^n}_{D_{h^n}\times 1} \odot \tanh(\underbrace{c_t^n}_{D_{h^n}\times 1}) \quad \text{(Hidden layer)}$$

$$\underbrace{c_t^n}_{D_{h^n}\times 1} = \underbrace{f_t^n}_{D_{h^n}\times 1} \odot \underbrace{c_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{i_t^n}_{D_{h^n}\times 1} \odot \underbrace{\widetilde{c}_t^n}_{D_{h^n}\times 1} \quad \text{(Final memory)}$$

$$\underbrace{\widetilde{c}_t^n}_{D_{h^n}\times 1} = \tanh(\underbrace{W^{(c^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(c^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{c^n}}_{D_{h^n}\times 1}) \quad \text{(New memory)}$$

$$\underbrace{f_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(f^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(f^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{f^n}}_{D_{h^n}\times 1}) \quad \text{(Forget gate)}$$

$$\underbrace{i_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(i^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(i^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{i^n}}_{D_{h^n}\times 1}) \quad \text{(Input gate)}$$

$$\underbrace{o_t^n}_{D_{h^n}\times 1} = \sigma(\underbrace{W^{(o^n h^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}} \underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times 1} + \underbrace{W^{(o^n h^n)}}_{D_{h^n}\times D_{h^n}} \underbrace{h_{t-1}^n}_{D_{h^n}\times 1} + \underbrace{b_{o^n}}_{D_{h^n}\times 1}) \quad \text{(Output gate)}$$

where $n = 1, \dots, N$ indexes each hidden layer. $h_t^n \in \mathbb{R}^{D_{h^n}}$ is the n-th *hidden layer*, where D_{h^n} is the number of *neurons* or *nodes* in the hidden layer. The 0-th layer is defined as the input data: $h_t^0 \equiv \mathcal{X}_t$. The memory cell c_t^n allows the LSTM network to retain information over longer time periods. The output gate o_t^n controls the extent to which the memory cell c_t^n maps to the hidden layer h_t^n. The forget gate f_t^n controls the flow of information carried over from the final memory in the previous timestep c_{t-1}^n. The input gate i_t^n controls the flow of information from the new memory cell $\widetilde{c}_t^n$. The initial states for the hidden layers $(h_0^n)_{n=1}^N$ and memory cells $(c_0^n)_{n=1}^N$ are set to zeros. $\sigma(\cdot)$ and $\tanh(\cdot)$ are *activation functions* that introduce non-linearities in the LSTM network, applied elementwise. $\sigma : \mathbb{R} \to \mathbb{R}$ is the sigmoid function: $\sigma(x) = (1 + e^{-x})^{-1}$. $\tanh : \mathbb{R} \to \mathbb{R}$ is the hyperbolic tangent function: $\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$. The $\odot$ operator refers to elementwise multiplication. $\boldsymbol{\beta}_{j,h}^{\text{LSTM}} \equiv (((vec(W^{(g^n h^{n-1})})', vec(W^{(g^n h^n)})', b_{g^n}')_{g\in\{c,f,i,o\}})_{n=1}^N, vec(W^{(yh^N)})', b_y)'$ are parameters to be estimated. We will refer to parameters indexed with W as *weights*; parameters indexed with b are *biases*. We estimate the parameters $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$ for the LSTM network using Stochastic Gradient Decent (SGD), which is an iterative algorithm for minimizing the loss function and proceeds as follows:

1. *Initialization.* Fix a random seed R and draw a starting value of the parameters $\boldsymbol{\beta}_{j,h}^{(0)}$ randomly, where the superscript (0) in parentheses indexes the iteration for an estimate of $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$.

   (a) Initialize input weights $W^{(g^n h^{n-1})} \in \mathbb{R}^{D_{h^n}\times D_{h^{n-1}}}$ for $g \in \{c, f, i, o\}$ using the *Glorot* initializer. Draw from a uniform distribution with zero mean and a variance that depends on the dimensions of the matrix:

   $$W_{ij}^{(g^n h^{n-1})} \overset{iid}{\sim} U\left[-\sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}, \sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}\right]$$

   for each $i = 1, \dots, D_{h^n}$ and $j = 1, \dots, D_{h^{n-1}}$.

   (b) Initialize the recurrent weights $W^{(g^n h^n)} \in \mathbb{R}^{D_{h^n}\times D_{h^n}}$ for $g \in \{c, f, i, o\}$ using the *Orthogonal* initializer. Use the orthogonal matrix obtained from the QR decomposition of a $D_{h^n} \times D_{h^n}$ matrix of random numbers drawn from a standard normal distribution.

   (c) Initialize biases $(b_{g^n})_{g\in\{c,f,i,o\}}$, hidden layers h_0^n, and memory cells c_0^n with zeros.

2. *Mini-batches.* Prepare the input data by dividing the training sample into a collection of *mini-batches*.

   (a) Suppose that we have a multi-variate time-series training sample with dimensions (T_E, K) whose time steps t are indexed by $t = 1, \dots, T_E$ and K is the number of predictors. We transform this training sample into a 3-D tensor with dimensions (N_S, M, K) where

   - N_S = Total number of sequences in training sample
   - M = Sequence length, i.e., number of time steps in each sequence
   - K = Input size, i.e., number of predictors in each time step

This can be done by creating overlapping sequences from the time series:

- Sequence 1 contains time steps $1, \ldots, M$
- Sequence 2 contains time steps $2, \ldots, M+1$
- Sequence 3 contains time steps $3, \ldots, M+2$
- $\ldots$
- Sequence $T_E - M$ contains time steps $T_E - M, \ldots, T_E - 1$
- Sequence $N_S = T_E - M + 1$ contains time steps $T_E - M + 1, \ldots, T_E$

(b) Randomly shuffle the N_S sequences by randomly sampling a permutation without replacement.

(c) Partition the N_S shuffled sequences into $\lceil N_S/N_B \rceil$ mini-batches. We partition the N_S sequences in the training sample ((N_S, M, K) tensor) into a list of $\lceil N_S/N_B \rceil$ mini-batches. A mini-batch is a (N_B, M, K)-dimensional tensor containing N_B out of N_S randomly shuffled sequences. When N_S/N_B is not a whole number, $\lfloor N_S/N_B \rfloor$ of the mini-batches will be 3-D tensors with dimensions (N_B, M, K). One batch will contain leftover sequences and will have dimensions $(N_S\%N_B, M, K)$ where % is the modulus operator. Let $B^{(1)}, \ldots, B^{\lceil N_S/N_B \rceil}$ denote the list of mini-batches.

- $N_S =$ Total number of sequences in training sample
- $N_B =$ Mini-batch size, i.e., number of sequences in each partition.
- $M =$ Sequence length, i.e., number of time steps in each sequence
- $K =$ Input size, i.e., number of predictors in each time step

3. Repeat until the stopping condition is satisfied ($k = 1, 2, 3, \ldots$):

(a) *Dropout.* Apply dropout to the mini-batch. To obtain the n-th hidden layer under dropout, multiply the current value of the $n-1$-th hidden layer h_t^{n-1} and the lagged value of the n-th hidden layer h_{t-1}^n with binary masks $r_{t,h_t^{n-1}}^{(k)} \in \mathbb{R}^{D_{h^{n-1}}}$ and $r_{t,h_{t-1}^n}^{(k)} \in \mathbb{R}^{D_{h^n}}$, respectively:

$$\underbrace{\overline{h}_t^{n-1}}_{D_{h^{n-1}} \times 1} = \underbrace{r_{t,h_t^{n-1}}^{(k)}}_{D_{h^{n-1}} \times 1} \odot \underbrace{h_t^{n-1}}_{D_{h^{n-1}} \times 1}, \quad r_{t,h_t^{n-1},i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_t^{n-1}}), \quad i = 1, \ldots, D_{h^{n-1}}$$

$$\underbrace{\overline{h}_{t-1}^{n}}_{D_{h^n} \times 1} = \underbrace{r_{t,h_{t-1}^n}^{(k)}}_{D_{h^n} \times 1} \odot \underbrace{h_{t-1}^{n}}_{D_{h^n} \times 1}, \quad r_{t,h_{t-1}^n,i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_{t-1}^n}), \quad i = 1, \ldots, D_{h^n}$$

where $t \in B^{(k)}$ and $n = 1, \ldots, N$ indexes the hidden layer and it is understood that the 0-th layer is the input vector $h_t^0 \equiv \mathcal{X}_t$. $p_{h_t^{n-1}}, p_{h_{t-1}^n} \in [0,1]$ is the probability that time t nodes in the $n-1$-th hidden layer and time $t-1$ nodes in the n-th hidden layer are retained, respectively.

(b) *Stochastic Gradient.* Average the gradient over observations in the mini-batch

$$\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}) = \frac{1}{M} \sum_{t \in B^{(k)}} \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$$

where $\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$ is the gradient of the loss function with respect to the parameters $\boldsymbol{\beta}_{j,h}^{(k-1)}$, evaluated at the time t observation $\mathbf{X}_t = (y_{t,t+h}, \widehat{\mathcal{X}}_t')'$ after applying dropout.

(c) *Learning rate shrinkage.* Update the parameters to $\boldsymbol{\beta}_{j,h}^{(k)}$ using the Adaptive Moment Estimation (Adam) algorithm. The method uses the first and second moments of the gradients to shrink the overall learning rate to zero as the gradient approaches zero.

$$\boldsymbol{\beta}_{j,h}^{(k)} = \boldsymbol{\beta}_{j,h}^{(k-1)} - \gamma \frac{m^{(k)}}{\sqrt{v^{(k)}} + \varepsilon}$$

where $m^{(k)}$ and $v^{(k)}$ are weighted averages of first two moments of past gradients:

$$m^{(k)} = \frac{1}{1-\pi_1^k}(\pi_1 m^{(k-1)} + (1-\pi_1)\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}))$$

$$v^{(k)} = \frac{1}{1-\pi_2^k}(\pi_2 v^{(k-1)} + (1-\pi_2)\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}})^2)$$

π^k denotes the k-the power of $\pi \in (0,1)$, and $/$, $\sqrt{\cdot}$, and $(\cdot)^2$ are applied elementwise. The default values of the hyperparameters are $m^{(0)} = v^{(0)} = 0$ (initial moment vectors), $\gamma = 0.001$ (initial learning rate), $(\pi_1, \pi_2) = (0.9, 0.999)$ (decay rates), and $\varepsilon = 10^{-7}$ (prevent zero denominators).

(d) *Stopping Critera.* Stop iterating and return $\boldsymbol{\beta}_{j,h}^{(k)}$ if one of the following holds:

- *Early stopping.* At each iteration, use the updated $\boldsymbol{\beta}_{j,h}^{(k)}$ to calculate the loss from the validation sample. Stop when the validation loss has not improved for S steps, where S is a "patience" hyperparameter. By updating the parameters for fewer iterations, early stopping shrinks the final parameters $\boldsymbol{\beta}_{j,h}$ towards the initial guess $\boldsymbol{\beta}_{j,h}^{(0)}$, and at a lower computational cost than ℓ_2 regularization.
- *Maximum number of epochs.* Stop if the number of iterations reaches the maximum number of epochs E. An epoch happens when the full set of the training sample has been used to update the parameters. If the training sample has T_E observations and each mini-batch has M observations, then each epoch would contain $\lceil T_E/M \rceil$ iterations (after rounding up as needed). So the maximum number of iterations is bounded by $E \times \lceil T_E/M \rceil$.

4. *Ensemble forecasts.* Repeat steps 1. and 2. over different random seeds R and save each of the estimated parameters $\widehat{\boldsymbol{\beta}}_{j,h,T_E}^{LSTM}(\boldsymbol{X}_{T_E}, \boldsymbol{\lambda}^{\text{LSTM}}, R)$. Then construct out-of-sample forecasts using the top 10 out of 20 starting values with the best performance in the validation sample. Ensemble can be considered as a regularization method because it aims to guard against overfitting by shrinking the forecasts toward the average across different random seeds. The random seed affects the random draws of the parameter's initial starting value $\boldsymbol{\beta}_{j,h}^{(0)}$, the sequences selected in each mini-batch $B^{(k)}$, and the dropout mask $r_t^{(k)}$.

Hyperparameters Let $\boldsymbol{\lambda}^{\text{LSTM}} \equiv [\lambda_1, \lambda_2, \gamma, \pi_1, \pi_2, p, N, (D_{h^n})_{n=1}^N, M, E, S]'$ collect all the hyper-parameters that control the LSTM network's complexity and prevent the model from overfitting the data. The number of hidden layers N and the number of neurons $D_{h^1}, \ldots, D_{h^N}$ in each hidden layer are hyper-parameters that characterize the network's architecture. To choose the number of neurons in each layer, we apply a geometric pyramid rule where the dimension of each additional hidden layer is half that of the previous hidden layer. We select the best LSTM architecture iteratively by minimizing the pseudo out-of-sample mean-squared error from rolling forecasts over the validation sample. Table A.11 reports the hyper-parameters for the LSTM network and its estimation. Hyper-parameters reported as a range or a set of values are cross-validated. The hyper-parameters are estimated by minimizing the mean-square loss over pseudo out-of-sample forecast errors generated from rolling regressions through the validation sample. The pseudo out-of-sample forecasts are ensemble averages implied by parameters based on different random seeds R.

Adaptive Architecture Selection We allow the LSTM architecture to evolve over time using a simple, adaptive updating procedure. At each period in the testing sample, the machine selects the architecture (number of hidden layers and neurons per layer) that minimized out-of-sample forecast errors in the preceding period. The candidate architectures considered span various combinations of hidden layers and neurons per layer, as listed in Table A.11. The architecture is updated quarterly by using the forecast performance from the most recent quarter. This systematic approach allows the machine to adjust its specification over time based on evolving patterns in the data, while avoiding look-ahead bias or overfitting to future outcomes.

D.2 Data Inputs for Machine Learning Algorithm

D.2.1 Macro Data Surprises

These data are used as inputs into the machine learning forecasts. I obtain median forecasts for GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change) from the Money Market Service Survey. The median market survey forecasts are compiled and published by the Money Market Services (MMS) the Friday before each release. I apply the approach used in Bauer and Swanson (2023) and define macroeconomic data surprise as the actual value of the data release minus the median expectation from MMS on the Friday immediately prior to that data release. The GDP growth forecasts are available quarterly from 1990Q1 to 2022Q1. The core CPI forecast is available monthly from July 1989 to April 2022. The median forecasts for the unemployment rate and nonfarm payrolls are available monthly from Jan 1980 to May 2022, and Jan. 1985 to May 2022, respectively. All survey forecasts were downloaded from Haver Analytics on December 17, 2022. To pin down the timing of when the news was actually released I follow the published tables of releases from the Bureau of Labor Statistics (BLS), discussed below.

The macro news events are indexed by their date and time of the data release, while the machine learning algorithm is adapted to quarterly sampling frequencies. When including the macro data surprises as additional predictors for the machine forecast, I time-aggregate the macro data surprises to a quarterly frequency by taking the sum of the surprises across data releases that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from data releases up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

Table A.11: Candidate hyper-parameters for the machine learning forecast

Variable	Earnings Growth	Earnings Growth	Earnings Growth	Stock Returns	Price Growth	CPI Inflation
Horizon (Years)	1,2,3,4	4-5 LTG	1-10 LTG	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5
(a) Elastic Net						
L_1 penalty λ_1	$[10^{-2}, 10^{1}]$	$[10^{-2}, 10^{1}]$	$[10^{-2}, 10^{1}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$	$[10^{-4}, 10^{1}]$
L_2 penalty λ_2	$[10^{-2}, 10^{1}]$	$[10^{-2}, 10^{1}]$	$[10^{-2}, 10^{1}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$	$[10^{-4}, 10^{1}]$
Training window T_E	$4, 6, 8, 10$	$4, 6, 8, 10, 12$	$4, 6, 8, 10, 12$	$5, 7$	$5, 7$	$3, 4, 5, 6, 7$
Validation window T_V	$4, 6, 8, 10$	$4, 6, 8, 10, 12$	$4, 6, 8, 10, 12$	$5, 7, 20$	$5, 7, 20$	$6, 7, \ldots, 14, 15$
(b) Long Short-Term Memory Network						
L_1 penalty λ_1	$[10^{-6}, 10^{-2}]$	$[10^{-5}, 10^{-1}]$	$[10^{-5}, 10^{-1}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$
L_2 penalty λ_2	$[10^{-6}, 10^{-2}]$	$[10^{-5}, 10^{-1}]$	$[10^{-5}, 10^{-1}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$
Learning rate γ	0.001	0.001	0.001	0.001	0.001	0.001
Gradient decay π_1, π_2	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$	$0.9, 0.999$
Dropout input p_x	0.8	0.8	0.8	0.8	0.8	0.8
Dropout recurrent p_h	0.8	0.8	0.8	0.5	0.5	0.5
Hidden layers N	$1, 3, 5$	$1, 3, 5$	$1, 3, 5$	$1, 3, 5$	$1, 3, 5$	1
Neurons per layer	$16, 32, 64$	$16, 32, 64$	$16, 32, 64$	$4, 8, 16$	$4, 8, 16$	4
Mini-batch size M	4	4	4	4	4	4
Max epochs E	$10,000$	$10,000$	$10,000$	$10,000$	$10,000$	$10,000$
Early stopping S	20	20	20	80	20	20
Random seeds R	$1, \ldots, 20$	$1, \ldots, 20$	$1, \ldots, 20$	$1, \ldots, 20$	$1, \ldots, 20$	$1, \ldots, 20$
Training window T_E	$4, 8, 12$	$3, 7, 12$	$3, 7, 12$	$5, 7$	$5, 7$	$5, 7$
Validation window T_V	$4, 8, 12$	$3, 7, 12, 20$	$3, 7, 12, 20$	$5, 7, 20$	$5, 7, 20$	$6, 9, 12, 15$

Notes: This table reports the hyperparameters considered in the machine learning algorithm for each estimator.

D.2.2 FOMC Surprises

FOMC surprises are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contract rate, from 10 minutes before to 20 minutes after each U.S. Federal Reserve Federal Open Market Committee (FOMC) announcement. The data on FFF and ED were downloaded on June 3rd 2022. When benchmarking against a survey, I use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, I compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, I use the last FOMC meeting before the end of the first month in each quarter to compute surprises.

When including the FOMC surprises as additional predictors for the machine forecast, I time-aggregate the FOMC surprises to a quarterly frequency by taking the sum of the surprises across FOMC announcements that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from FOMC announcements up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

D.2.3 S&P 500 Jumps

As a measure of the market's reaction to news shocks, I use the jump in the S&P 500 pre- and post- a 30-minute window around major news events. The events in our analysis include (i) 1,482 macroeconomic data releases for U.S. GDP, Consumer Price Index (CPI), unemployment, and payroll data spanning 1980:01-2021:12, (ii) 16 corporate earnings announcement days spanning 1999:03-2020:05, and (iii) 219 Federal Open Market Committee (FOMC) press releases from the Fed spanning 1994:02-2021:12. The corporate earnings news events are from Baker et al. (2019) who conduct textual analyses of *Wall Street Journal* articles to identify days in which there were large jumps in the aggregate stock market that could be attributed to corporate earnings news with high confidence. The jump in the S&P 500 for a given event is defined as $j_\tau = p_{\tau+\delta_{post}} - p_{\tau-\delta_{pre}}$, where τ indexes the time of an event and $p_\tau = \log(P_\tau)$ is the log S&P 500 index. δ_{pre} and δ_{post} denote the pre and post event windows, which is 10 minutes before and 20 minutes after the event, respectively. I obtain data on P_τ using tick-by-tick data on the S&P 500 index from tickdata.com. The series was purchased and downloaded on 7/2/2022 from `https://www.tickdata.com/`. I create the minutely data using the close price within each minute. I supplement the S&P 500 index using S&P500 E-mini futures for events that occur in off-market hours. I use the current-quarter contract futures. I purchased the S&P 500 E-mini futures from CME group on 7/2/2022 at `https://datamine.cmegroup.com/`. Our sample spans 1/2/1986 to 6/30/2022.

For each event, I separate out the events for which the S&P 500 increased over the window ($j_\tau^{(+)} \geq 0$) and those for which the market decreased ($j_\tau^{(-)} \leq 0$). I aggregate the event-level jumps to monthly time series by summing over all the relevant events within the month, where the events are partitioned into two groups based on the sign of the jump: $J_t^{(+)} = \sum_{\tau \in x(t)} j_\tau^{(+)}, J_t^{(-)} = \sum_{\tau \in x(t)} j_\tau^{(-)}$, where t indexes the month and $x(t)$ is the set of all events that occurred within month t. The procedure results in two monthly variables, $J_t^{(+)}$ and $J_t^{(-)}$, which capture total market reaction to news events in either direction during the quarter. The series spans the period 1994:02 to 2022:03. Separating out the events based on the sign of the jump allows us to capture any differential effects on return predictability based on whether the market perceived the news as good or bad. The partition also allows us to accurately capture the total extent of over- or under-reaction. Otherwise, mixing all the events would only capture the net effect of the jumps and bias the market reaction towards zero.

When used as additional predictors in the for the machine forecast, the jumps need to be converted to quarterly time series because the machine learning algorithm is adapted to a quarterly sampling frequency. The set of events in $x(t)$ is chosen so that the machine only sees the news events that would have been available to the real-time firm. When combining the events within a quarter, I impose the response deadline used to produce the machine forecast. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I use the jumps from the events up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

D.2.4 Real-Time Macro Data

This section gives details on the real time macro data inputs used in the machine learning forecasts. A subset of these series are used in the structural estimation. At each forecast date in the sample, I construct a dataset of macro variables that could have been observed on or before the day of the survey deadline. I use the Philadelphia Fed's Real-Time Data Set to obtain vintages of macro variables. The real-time data sets are available at https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files. These vintages capture changes to historical data due to periodic revisions made by government statistical agencies. The vintages for a particular series can be available at the monthly and/or quarterly frequencies, and the series have monthly and/or quarterly observations. In cases where a variable has both frequencies available for its vintages and/or its observations, I choose one format of the variable. For instance, nominal personal consumption expenditures on goods is quarterly data with both monthly and quarterly vintages available; in this case, I use the version with monthly vintages.

Table A.12 gives the complete list of real-time macro variables. Included in the table is the first available vintages for each variable that has multiple vintages. I do not include the last vintage because most variables have vintages through the present. For variables BASEBASAQVMD, NBRBASAQVMD, NBRECBASAQVMD, and TRBASAQVMD, the last available vintage is 2013Q2. Table A.12 also lists the transformation applied to each variable to make them stationary before generating factors. Let $X_{i,t}$ denote variable i at time t after the transformation, and let $X^A_{i,t}$ be the untransformed series. Let $\Delta = (1 - L)$ with $LX_{i,t} = X_{it-1}$. There are seven possible transformations with the following codes:

1 Code lv: $X_{i,t} = X^A_{i,t}$

2 Code Δlv: $X_{i,t} = X^A_{i,t} - X^A_{it-1}$

3 Code $\Delta^2 lv$: $X_{i,t} = \Delta^2 X^A_{i,t}$

4 Code ln: $X_{i,t} = \ln(X^A_{i,t})$

5 Code Δln: $X_{i,t} = \ln(X^A_{i,t}) - \ln(X^A_{it-1})$

6 Code $\Delta^2 ln$: $X_{i,t} = \Delta^2 \ln(X^A_{i,t})$

7 Code $\Delta lv/lv$: $X_{i,t} = (X^A_{i,t} - X^A_{it-1})/X^A_{it-1}$

Table A.12: List of Macro Dataset Variables

No.	Short Name	Source	Tran	Description	First Vintage
			Group 1: Output and Income		
1	IPMMVMD	Philly Fed	Δln	Ind. production index - Manufacturing	1962M11
2	IPTMVMD	Philly Fed	Δln	Ind. production index - Total	1962M11
3	CUMMVMD	Philly Fed	lv	Capacity utilization - Manufacturing	1979M8
4	CUTMVMD	Philly Fed	lv	Capacity utilization - Total	1983M7
5	NCPROFATMVQD	Philly Fed	Δln	Nom. corp. profits after tax without IVA/CCAdj	1965Q4
6	NCPROFATWMVQD	Philly Fed	Δln	Nom. corp. profits after tax with IVA/CCAdj	1981Q1
7	OPHMVQD	Philly Fed	Δln	Output per hour - Business sector	1998Q4
8	NDPIQVQD	Philly Fed	Δln	Nom. disposable personal income	1965Q4
9	NOUTPUTQVQD	Philly Fed	Δln	Nom. GNP/GDP	1965Q4
10	NPIQVQD	Philly Fed	Δln	Nom. personal income	1965Q4
11	NPSAVQVQD	Philly Fed	Δlv	Nom. personal saving	1965Q4
12	OLIQVQD	Philly Fed	Δln	Other labor income	1965Q4
13	PINTIQVQD	Philly Fed	Δln	Personal interest income	1965Q4
14	PINTPAIDQVQD	Philly Fed	Δln	Interest paid by consumers	1965Q4
15	PROPIQVQD	Philly Fed	Δln	Proprietors' income	1965Q4
16	PTAXQVQD	Philly Fed	Δln	Personal tax and nontax payments	1965Q4
17	RATESAVQVQD	Philly Fed	Δlv	Personal saving rate	1965Q4
18	RENTIQVQD	Philly Fed	Δlv	Rental income of persons	1965Q4
19	ROUTPUTQVQD	Philly Fed	Δln	Real GNP/GDP	1965Q4
20	SSCONTRIBQVQD	Philly Fed	Δln	Personal contributions for social insurance	1965Q4

No.	Short Name	Source	Tran	Description	First Vintage
21	TRANPFQVQD	Philly Fed	Δln	Personal transfer payments to foreigners	1965Q4
22	TRANRQVQD	Philly Fed	Δln	Transfer payments	1965Q4
23	CUUR0000SA0E	BLS	$\Delta^2 ln$	Energy in U.S. city avg., all urban consumers, not seasonally adj	
			Group 2: Employment		
24	EMPLOYMVMD	Philly Fed	Δln	Nonfarm payroll	1946M12
25	HMVMD	Philly Fed	lv	Aggregate weekly hours - Total	1971M9
26	HGMVMD	Philly Fed	lv	Agg. weekly hours - Goods-producing	1971M9
27	HSMVMD	Philly Fed	lv	Agg. weekly hours - Service-producing	1971M9
28	LFCMVMD	Philly Fed	Δln	Civilian labor force	1998M11
29	LFPARTMVMD	Philly Fed	lv	Civilian participation rate	1998M11
30	POPMVMD	Philly Fed	Δln	Civilian noninstitutional population	1998M11
31	ULCMVQD	Philly Fed	Δln	Unit labor costs - Business sector	1998Q4
32	RUCQVMD	Philly Fed	Δlv	Unemployment rate	1965Q4
33	WSDQVQD	Philly Fed	Δln	Wage and salary disbursements	1965Q4
			Group 3: Orders, Investment, Housing		
34	HSTARTSMVMD	Philly Fed	Δln	Housing starts	1968M2
35	RINVBFMVQD	Philly Fed	Δln	Real gross private domestic inv. - Nonresidential	1965Q4
36	RINVCHIMVQD	Philly Fed	Δlv	Real gross private domestic inv. - Change in private inventories	1965Q4
37	RINVRESIDMVQD	Philly Fed	Δln	Real gross private domestic inv. - Residential	1965Q4
38	CASESHILLER	S&P	Δln	Case-Shiller US National Home Price index/CPI	1987M1
			Group 4: Consumption		
39	NCONGMMVMD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009M8
40	NCONHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp.	2009M8
41	NCONSHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009M8
42	NCONSNPMMVMD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009M8
43	RCONDMMVMD	Philly Fed	Δln	Real personal cons. exp. - Durables	1998M11
44	RCONGMMVMD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009M8
45	RCONHHMMVMD	Philly Fed	Δln	Real hh. cons. exp.	2009M8
46	RCONMMVMD	Philly Fed	Δln	Real personal cons. exp. - Total	1998M11
47	RCONNDMVMD	Philly Fed	Δln	Real personal cons. exp. - Nondurables	1998M11
48	RCONSHHMMVMD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009M8
49	RCONSMMVMD	Philly Fed	Δln	Real personal cons. exp. - Services	1998M11
50	RCONSNPMMVMD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009M8
51	NCONGMVQD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009Q3
52	NCONHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp.	0209Q3
53	NCONSHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009Q3
54	NCONSNPMVQD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009Q3
55	RCONDMVQD	Philly Fed	Δln	Real personal cons. exp. - Durable goods	1965Q4
56	RCONGMVQD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009Q3
57	RCONHHMVQD	Philly Fed	Δln	Real hh. cons. exp.	2009Q3
58	RCONMVQD	Philly Fed	Δln	Real personal cons. exp. - Total	1965Q4
59	RCONNDMVQD	Philly Fed	Δln	Real pesonal cons. exp. - Nondurable goods	1965Q4
60	RCONSHHMVQD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009Q3
61	RCONSMVQD	Philly Fed	Δln	Real personal cons. exp. - Services	1965Q4
62	RCONSNPMVQD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009Q3
63	NCONQVQD	Philly Fed	Δln	Nom. personal cons. exp.	1965Q4
			Group 5: Prices		
64	PCONGMMVMD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp. - Goods	2009M8
65	PCONHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009M8
66	PCONSHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009M8
67	PCONSNPMMVMD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009M8
68	PCPIMVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1998M11
69	PCPIXMVMD	Philly Fed	$\Delta^2 ln$	Core consumer price index	1998M11
70	PPPIMVMD	Philly Fed	$\Delta^2 ln$	Producer price index	1998M11
71	PPPIXMVMD	Philly Fed	$\Delta^2 ln$	Core producer price index	1998M11
72	PCONGMVQD	Philly Fed	$\Delta^2 ln$	Price index for personal. cons. exp. - Goods	2009Q3
73	PCONHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009Q3
74	PCONSHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009Q3
75	PCONSNPMVQD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009Q3
76	PCONXMVQD	Philly Fed	$\Delta^2 ln$	Core price index for personal cons. exp.	1996Q1
77	CPIQVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1994Q3
78	PQVQD	Philly Fed	$\Delta^2 ln$	Price index for GNP/GDP	1965Q4
79	PCONQVQD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp.	1965Q4
80	PIMPQVQD	Philly Fed	$\Delta^2 ln$	Price index for imports of goods and services	1965Q4
			Group 6: Trade and Government		
81	REXMVQD	Philly Fed	Δln	Real exports of goods and services	1965Q4
82	RGMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Total	1965Q4
83	RGFMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Federal	1965Q4

No.	Short Name	Source	Tran	Description	First Vintage
84	RGSLMVQD	Philly Fed	Δln	Real government cons. and gross. inv. - State and local	1965Q4
85	RIMPMVQD	Philly Fed	Δln	Real imports of goods and services	1965Q4
86	RNXMVQD	Philly Fed	Δlv	Real net exports of goods and services	1965Q4
			Group 7: Money and Credit		
87	BASEBASAQVMD	Philly Fed	$\Delta^2 ln$	Monetary base	1980Q2
88	M1QVMD	Philly Fed	$\Delta^2 ln$	M1 money stock	1965Q4
89	M2QVMD	Philly Fed	$\Delta^2 ln$	M2 money stock	1971Q2
90	NBRBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves	1967Q3
91	NBRECBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves plus extended credit	1984Q2
92	TRBASAQVMD	Philly Fed	$\Delta^2 ln$	Total reserves	1967Q3
93	DIVQVQD	Philly Fed	Δln	Dividends	1965Q4

D.2.5 Monthly Financial Data

The 147 financial series in this data set are versions of the financial dataset used in Jurado et al. (2015) and Ludvigson et al. (2021). It consists of a number of indicators measuring the behavior of a broad cross-section of asset returns, as well as some aggregate financial indicators not included in the macro dataset. These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry equity returns. Following Fama and French (1992), returns on 100 portfolios of equities sorted into 10 size and 10 book-to-market categories. The dataset X^f also includes a group of variables we call "risk-factors," since they have been used in cross-sectional or time-series studies to uncover variation in the market risk-premium. These risk-factors include the three Fama and French (1993) risk factors, namely the excess return on the market MKT_t, the "small-minus-big" (SMB_t) and "high-minus-low" (HML_t) portfolio returns, the momentum factor UMD_t, and the small stock value spread $R15 - R11$.

The raw data used to form factors are always transformed to achieve stationarity. In addition, when forming forecasting factors from the large macro and financial datasets, the raw data (which are in different units) are standardized before performing PCA. When forming common uncertainty from estimates of individual uncertainty, the raw data (which are in this case in the same units) are demeaned, but we do not divide by the observation's standard deviation before performing PCA. Throughout, the factors are estimated by the method of static principal components (PCA). Specifically, the $T \times r_F$ matrix $\widehat{F}_t$ is $\sqrt{T}$ times the r_F eigenvectors corresponding to the r_F largest eigenvalues of the $T \times T$ matrix $xx'/(TN)$ in decreasing order. In large samples (when $\sqrt{T}/N \to \infty$), Bai and Ng (2006) show that the estimates $\widehat{F}_t$ can be treated as though they were observed in the subsequent forecasting regression. All returns and spreads are expressed in logs (i.e., the log of the gross return or spread), are displayed in percent (i.e., multiplied by 100), and are annualized by multiplying by 12. That is, if x is the original return or spread, we transform to $1200 \times \log(1 + x/100)$. Federal Reserve data are annualized by default and are therefore not re-annualized. Note that this annualization implies that the annualized standard deviation (volatility) is equal to the data standard deviation divided by $\sqrt{12}$. The data series used in this dataset are listed below by data source. Additional details on data transformations are given below the table.

We convert monthly data to quarterly by using either the beginning-of-quarter or end-of-quarter values. The decision to use beginning-of-quarter or end-of-quarter depends on the survey deadline of a particular forecast date. If the survey deadline is known to be in the middle of the second month of quarter t, then it is conceivable that the forecasters would have information about the first month of quarter t. Therefore, we use the first month of that quarter's values. Alternatively, a few anomalous observations have unknown survey deadlines (e.g., the SPF deadlines for 1990Q1). In such cases, we allow only information up to quarter $t-1$ to enter the model. Thus, we use the last month of the previous quarter's values in these cases. Let $X_{i,t}$ denote variable i observed at time t after, e.g., logarithm and differencing transformation, and let $X^A_{i,t}$ be the actual (untransformed) series. Let $\Delta = (1 - L)$ with $LX_{i,t} = X_{i,t-1}$. There are six possible transformations with the following codes:

$$1 \text{ Code } lv : X_{i,t} = X^A_{i,t}$$

$$2 \text{ Code } \Delta lv : X_{i,t} = X^A_{i,t} - X^A_{i,t-1}$$

$$3 \text{ Code } \Delta^2 lv : X_{i,t} = \Delta^2 X^A_{i,t}$$

$$4 \text{ Code } ln : X_{i,t} = \log(X^A_{i,t})$$

$$5 \text{ Code } \Delta ln : X_{i,t} = \log(X^A_{i,t}) - \log(X^A_{i,t-1})$$

$$6 \text{ Code } \Delta^2 ln : X_{i,t} = \Delta^2 \log(X^A_{i,t})$$

$$7 \text{ Code } \Delta lv/lv : X_{i,t} = \frac{X^A_{i,t} - X^A_{i,t-1}}{X^A_{i,t-1}}$$

Table A.13: List of Financial Dataset Variables

No.	Short Name	Source	Tran	Description
		Group 1: Prices, Yields, Dividends		
1	D_log(DIV)	CRSP	Δln	1 log D_t, see additional details below
2	D_log(P)	CRSP	Δln	1 log P_t, see additional details below
3	D_DIVreinvest	CRSP	Δln	1 log $D_t^{re,*}$, see additional details below
4	D_Preinvest	CRSP	Δln	1 log $P_t^{re,*}$, see additional details below
5	d-p	CRSP	ln	log $D_t - P_t$, see additional details below
		Group 2: Equity Risk Factors		
6	R15-R11	Kenneth French	lv	(Small, High) minus (Small, Low) sorted on (size, book-to-market)
7	Mkt-RF	Kenneth French	lv	Market excess return
8	SMB	Kenneth French	lv	Small Minus Big, sorted on size
9	HML	Kenneth French	lv	High Minus Low, sorted on book-to-market
10	UMD	Kenneth French	lv	Up Minus Down, sorted on momentum
		Group 3: Industries		
11	Agric	Kenneth French	lv	Agric industry portfolio
12	Food	Kenneth French	lv	Food industry portfolio
13	Beer	Kenneth French	lv	Beer industry portfolio
14	Smoke	Kenneth French	lv	Smoke industry portfolio
15	Toys	Kenneth French	lv	Toys industry portfolio
16	Fun	Kenneth French	lv	Fun industry portfolio
17	Books	Kenneth French	lv	Books industry portfolio
18	Hshld	Kenneth French	lv	Hshld industry portfolio
19	Clths	Kenneth French	lv	Clths industry portfolio
20	MedEq	Kenneth French	lv	MedEq industry portfolio
21	Drugs	Kenneth French	lv	Drugs industry portfolio
22	Chems	Kenneth French	lv	Chems industry portfolio
23	Rubbr	Kenneth French	lv	Rubbr industry portfolio
24	Txtls	Kenneth French	lv	Txtls industry portfolio
25	BldMt	Kenneth French	lv	BldMt industry portfolio
26	Cnstr	Kenneth French	lv	Cnstr industry portfolio
27	Steel	Kenneth French	lv	Steel industry portfolio
28	Mach	Kenneth French	lv	Mach industry portfolio
29	ElcEq	Kenneth French	lv	ElcEq industry portfolio
30	Autos	Kenneth French	lv	Autos industry portfolio
31	Aero	Kenneth French	lv	Aero industry portfolio
32	Ships	Kenneth French	lv	Ships industry portfolio
33	Mines	Kenneth French	lv	Mines industry portfolio
34	Coal	Kenneth French	lv	Coal industry portfolio
35	Oil	Kenneth French	lv	Oil industry portfolio
36	Util	Kenneth French	lv	Util industry portfolio
37	Telcm	Kenneth French	lv	Telcm industry portfolio
38	PerSv	Kenneth French	lv	PerSv industry portfolio
39	BusSv	Kenneth French	lv	BusSv industry portfolio
40	Hardw	Kenneth French	lv	Hardw industry portfolio
41	Chips	Kenneth French	lv	Chips industry portfolio
42	LabEq	Kenneth French	lv	LabEq industry portfolio
43	Paper	Kenneth French	lv	Paper industry portfolio
44	Boxes	Kenneth French	lv	Boxes industry portfolio
45	Trans	Kenneth French	lv	Trans industry portfolio
46	Whlsl	Kenneth French	lv	Whlsl industry portfolio
47	Rtail	Kenneth French	lv	Rtail industry portfolio
48	Meals	Kenneth French	lv	Meals industry portfolio
49	Banks	Kenneth French	lv	Banks industry portfolio
50	Insur	Kenneth French	lv	Insur industry portfolio
51	RlEst	Kenneth French	lv	RlEst industry portfolio
52	Fin	Kenneth French	lv	Fin industry portfolio
53	Other	Kenneth French	lv	Other industry portfolio
		Group 4: Size/BM		
54	1_2	Kenneth French	lv	(1, 2) portfolio sorted on (size, book-to-market)
55	1_4	Kenneth French	lv	(1, 4) portfolio sorted on (size, book-to-market)
56	1_5	Kenneth French	lv	(1, 5) portfolio sorted on (size, book-to-market)
57	1_6	Kenneth French	lv	(1, 6) portfolio sorted on (size, book-to-market)
58	1_7	Kenneth French	lv	(1, 7) portfolio sorted on (size, book-to-market)
59	1_8	Kenneth French	lv	(1, 8) portfolio sorted on (size, book-to-market)
60	1_9	Kenneth French	lv	(1, 9) portfolio sorted on (size, book-to-market)
61	1_high	Kenneth French	lv	(1, high) portfolio sorted on (size, book-to-market)
62	2_low	Kenneth French	lv	(2, low) portfolio sorted on (size, book-to-market)
63	2_2	Kenneth French	lv	(2, 2) portfolio sorted on (size, book-to-market)
64	2_3	Kenneth French	lv	(2, 3) portfolio sorted on (size, book-to-market)
65	2_4	Kenneth French	lv	(2, 4) portfolio sorted on (size, book-to-market)
66	2_5	Kenneth French	lv	(2, 5) portfolio sorted on (size, book-to-market)
67	2_6	Kenneth French	lv	(2, 6) portfolio sorted on (size, book-to-market)

No.	Short Name	Source	Tran	Description
68	2_7	Kenneth French	*lv*	(2, 7) portfolio sorted on (size, book-to-market)
69	2_8	Kenneth French	*lv*	(2, 8) portfolio sorted on (size, book-to-market)
70	2_9	Kenneth French	*lv*	(2, 9) portfolio sorted on (size, book-to-market)
71	2_high	Kenneth French	*lv*	(2, high) portfolio sorted on (size, book-to-market)
72	3_low	Kenneth French	*lv*	(3, low) portfolio sorted on (size, book-to-market)
73	3_2	Kenneth French	*lv*	(3, 2) portfolio sorted on (size, book-to-market)
74	3_3	Kenneth French	*lv*	(3, 3) portfolio sorted on (size, book-to-market)
75	3_4	Kenneth French	*lv*	(3, 4) portfolio sorted on (size, book-to-market)
76	3_5	Kenneth French	*lv*	(3, 5) portfolio sorted on (size, book-to-market)
77	3_6	Kenneth French	*lv*	(3, 6) portfolio sorted on (size, book-to-market)
78	3_7	Kenneth French	*lv*	(3, 7) portfolio sorted on (size, book-to-market)
79	3_8	Kenneth French	*lv*	(3, 8) portfolio sorted on (size, book-to-market)
80	3_9	Kenneth French	*lv*	(3, 9) portfolio sorted on (size, book-to-market)
81	3_high	Kenneth French	*lv*	(3, high) portfolio sorted on (size, book-to-market)
82	4_low	Kenneth French	*lv*	(4, low) portfolio sorted on (size, book-to-market)
83	4_2	Kenneth French	*lv*	(4, 2) portfolio sorted on (size, book-to-market)
84	4_3	Kenneth French	*lv*	(4, 3) portfolio sorted on (size, book-to-market)
85	4_4	Kenneth French	*lv*	(4, 4) portfolio sorted on (size, book-to-market)
86	4_5	Kenneth French	*lv*	(4, 5) portfolio sorted on (size, book-to-market)
87	4_6	Kenneth French	*lv*	(4, 6) portfolio sorted on (size, book-to-market)
88	4_7	Kenneth French	*lv*	(4, 7) portfolio sorted on (size, book-to-market)
89	4_8	Kenneth French	*lv*	(4, 8) portfolio sorted on (size, book-to-market)
90	4_9	Kenneth French	*lv*	(4, 9) portfolio sorted on (size, book-to-market)
91	4_high	Kenneth French	*lv*	(4, high) portfolio sorted on (size, book-to-market)
92	5_low	Kenneth French	*lv*	(5, low) portfolio sorted on (size, book-to-market)
93	5_2	Kenneth French	*lv*	(5, 2) portfolio sorted on (size, book-to-market)
94	5_3	Kenneth French	*lv*	(5, 3) portfolio sorted on (size, book-to-market)
95	5_4	Kenneth French	*lv*	(5, 4) portfolio sorted on (size, book-to-market)
96	5_5	Kenneth French	*lv*	(5, 5) portfolio sorted on (size, book-to-market)
97	5_6	Kenneth French	*lv*	(5, 6) portfolio sorted on (size, book-to-market)
98	5_7	Kenneth French	*lv*	(5, 7) portfolio sorted on (size, book-to-market)
99	5_8	Kenneth French	*lv*	(5, 8) portfolio sorted on (size, book-to-market)
100	5_9	Kenneth French	*lv*	(5, 9) portfolio sorted on (size, book-to-market)
101	5_high	Kenneth French	*lv*	(5, high) portfolio sorted on (size, book-to-market)
102	6_low	Kenneth French	*lv*	(6, low) portfolio sorted on (size, book-to-market)
103	6_2	Kenneth French	*lv*	(6, 2) portfolio sorted on (size, book-to-market)
104	6_3	Kenneth French	*lv*	(6, 3) portfolio sorted on (size, book-to-market)
105	6_4	Kenneth French	*lv*	(6, 4) portfolio sorted on (size, book-to-market)
106	6_5	Kenneth French	*lv*	(6, 5) portfolio sorted on (size, book-to-market)
107	6_6	Kenneth French	*lv*	(6, 6) portfolio sorted on (size, book-to-market)
108	6_7	Kenneth French	*lv*	(6, 7) portfolio sorted on (size, book-to-market)
109	6_8	Kenneth French	*lv*	(6, 8) portfolio sorted on (size, book-to-market)
110	6_9	Kenneth French	*lv*	(6, 9) portfolio sorted on (size, book-to-market)
111	6_high	Kenneth French	*lv*	(6, high) portfolio sorted on (size, book-to-market)
112	7_low	Kenneth French	*lv*	(7, low) portfolio sorted on (size, book-to-market)
113	7_2	Kenneth French	*lv*	(7, 2) portfolio sorted on (size, book-to-market)
114	7_3	Kenneth French	*lv*	(7, 3) portfolio sorted on (size, book-to-market)
115	7_4	Kenneth French	*lv*	(7, 4) portfolio sorted on (size, book-to-market)
116	7_5	Kenneth French	*lv*	(7, 5) portfolio sorted on (size, book-to-market)
117	7_6	Kenneth French	*lv*	(7, 6) portfolio sorted on (size, book-to-market)
118	7_7	Kenneth French	*lv*	(7, 7) portfolio sorted on (size, book-to-market)
119	7_8	Kenneth French	*lv*	(7, 8) portfolio sorted on (size, book-to-market)
120	7_9	Kenneth French	*lv*	(7, 9) portfolio sorted on (size, book-to-market)
121	8_low	Kenneth French	*lv*	(8, low) portfolio sorted on (size, book-to-market)
122	8_2	Kenneth French	*lv*	(8, 2) portfolio sorted on (size, book-to-market)
123	8_3	Kenneth French	*lv*	(8, 3) portfolio sorted on (size, book-to-market)
124	8_4	Kenneth French	*lv*	(8, 4) portfolio sorted on (size, book-to-market)
125	8_5	Kenneth French	*lv*	(8, 5) portfolio sorted on (size, book-to-market)
126	8_6	Kenneth French	*lv*	(8, 6) portfolio sorted on (size, book-to-market)
127	8_7	Kenneth French	*lv*	(8, 7) portfolio sorted on (size, book-to-market)
128	8_8	Kenneth French	*lv*	(8, 8) portfolio sorted on (size, book-to-market)
129	8_9	Kenneth French	*lv*	(8, 9) portfolio sorted on (size, book-to-market)
130	8_high	Kenneth French	*lv*	(8, high) portfolio sorted on (size, book-to-market)
131	9_low	Kenneth French	*lv*	(9, low) portfolio sorted on (size, book-to-market)
132	9_2	Kenneth French	*lv*	(9, 2) portfolio sorted on (size, book-to-market)
133	9_3	Kenneth French	*lv*	(9, 3) portfolio sorted on (size, book-to-market)
134	9_4	Kenneth French	*lv*	(9, 4) portfolio sorted on (size, book-to-market)
135	9_5	Kenneth French	*lv*	(9, 5) portfolio sorted on (size, book-to-market)
136	9_6	Kenneth French	*lv*	(9, 6) portfolio sorted on (size, book-to-market)
137	9_7	Kenneth French	*lv*	(9, 7) portfolio sorted on (size, book-to-market)
138	9_8	Kenneth French	*lv*	(9, 8) portfolio sorted on (size, book-to-market)
139	9_high	Kenneth French	*lv*	(9, high) portfolio sorted on (size, book-to-market)

No.	Short Name	Source	Tran	Description
140	10_low	Kenneth French	*lv*	(10, low) portfolio sorted on (size, book-to-market)
141	10_2	Kenneth French	*lv*	(10, 2) portfolio sorted on (size, book-to-market)
142	10_3	Kenneth French	*lv*	(10, 3) portfolio sorted on (size, book-to-market)
143	10_4	Kenneth French	*lv*	(10, 4) portfolio sorted on (size, book-to-market)
144	10_5	Kenneth French	*lv*	(10, 5) portfolio sorted on (size, book-to-market)
145	10_6	Kenneth French	*lv*	(10, 6) portfolio sorted on (size, book-to-market)
146	10_7	Kenneth French	*lv*	(10, 7) portfolio sorted on (size, book-to-market)
147	VXO	Fred MD	*lv*	VXOCLS

CRSP Data Details Value-weighted price and dividend data were obtained from the Center for Research in Security Prices (CRSP, Center for Research in Security Prices (1926–2022)). From the Annual Update data, we obtain the monthly value-weighted return series `vwretd` (with dividends) and `vwretx` (excluding dividends). These series have the interpretations: $VWRET_t = \frac{P_{t+1}+D_{t+1}}{P_t}$, $VWRETX_t = \frac{P_{t+1}}{P_t}$. From these series, a normalized price series P_t can be constructed recursively as: $P_0 = 1$, $P_t = P_{t-1} \times VWRETX_{t-1}$. A dividend series can then be constructed using: $D_t = P_{t-1} \times (VWRET_{t-1} - VWRETX_{t-1})$. In order to remove seasonality of dividend payments from the data, instead of D_t we use the series: $\bar{D}_t = \frac{1}{12}\sum_{j=0}^{11} D_{t-j}$, i.e., the moving average over the entire year. For the price and dividend series under "reinvestment," we calculate the price under reinvestment, P_t^{re}, as the normalized value of the market portfolio under reinvestment of dividends, using the recursion: $P_0^{re} = 1$, $P_t^{re} = P_{t-1} \times VWRET_{t-1}$. Similarly, we can define dividends under reinvestment, D_t^{re}, as the total dividend payments on this portfolio (the number of "shares" of which have increased over time) using: $D_t^{re} = P_{t-1}^{re} \times (VWRET_{t-1} - VWRETX_{t-1})$. As before, we can remove seasonality by using: $\bar{D}_t^{re} = \frac{1}{12}\sum_{j=0}^{11} D_{t-j}^{re}$. Five data series are constructed from the CRSP data as follows: `D_log(DIV)`: $\Delta\log(\bar{D}_t)$; `D_log(P)`: $\Delta\log(P_t)$; `D_DIVreinvest`: $\Delta\log(\bar{D}_t^{re})$; `D_Preinvest`: $\Delta\log(P_t^{re})$; `d-p`: $\log(\bar{D}_t) - \log(P_t)$.

Kenneth French Data Details The following data are obtained from the data library of Kenneth French's Dartmouth website (French (1926–2022)):

- Fama/French Factors: From this dataset we obtain the series `RF`, `Mkt-RF`, `SMB`, and `HML`.
- 25 Portfolios Formed on Size and Book-to-Market (5 x 5): From this dataset we obtain the series `R15-R11`, which is the return spread between the (small, high book-to-market) and (small, low book-to-market) portfolios.
- Momentum Factor (Mom): From this dataset we obtain the series `UMD`, which is equal to the momentum factor.
- 49 Industry Portfolios: From this dataset we use all value-weighted series, excluding any series that have missing observations from January 1960 onward. This yields the series `Agric` through `Other`. The omitted series are `Soda`, `Hlth`, `FabPr`, `Guns`, `Gold`, and `Softw`.
- 100 Portfolios Formed on Size and Book-to-Market: From this dataset we use all value-weighted series, excluding any series that have missing observations from January 1960 onward. This yields variables with names `X_Y`, where X denotes the size index $(1, 2, \ldots, 10)$ and Y denotes the book-to-market index $(\text{Low}, 2, 3, \ldots, 8, 9, \text{High})$. The omitted series are `1_low`, `1_3`, `7_high`, `9_9`, `10_8`, `10_9`, and `10_high`.

VXO Data Details VXO data is obtained from the Monthly Database for Macroeconomic Research (FRED-MD, McCracken (2015–2022)).

D.2.6 Daily Financial Data

Daily Data and construction of daily factors These data are used in the machine learning forecasts. The daily financial series in this data set are from the daily financial dataset used in Andreou et al. (2013). I create a smaller daily database which is a subset of the large cross-section of 991 daily series in their dataset. Our dataset covers five classes of financial assets: (i) the Commodities class; (ii) the Corporate Risk category; (iii) the Equities class; (iv) the Foreign Exchange Rates class and (v) the Government Securities. The dataset includes up to 87 daily predictors in a daily frequency from 23-Oct-1959 to 24-Oct-2021 (14852 trading days) from the above five categories of financial assets. I remove series with fewer than ten years of data and time periods with no variables observed, which occurs for some series in the early part of the sample. For those years, I have less than 87 series. There are 39 commodity variables which include commodity indices, prices and futures, 16 corporate risk series, 9 equity series which include major US stock market indices and the 500 Implied Volatility, 16 government securities which include the federal funds rate, government treasury bills of securities from three months to ten years, and 7 foreign exchange variables which include the individual foreign exchange rates of major five US trading partners and two effective exchange rate. I choose these daily predictors because they are proposed in the literature as good predictors of economic growth.

I construct daily financial factors in a quarterly frequency in two steps. First, I use these daily financial time series to form factors at a daily frequency. The raw data used to form factors are always transformed to achieve stationarity and standardized before performing factor estimation (see generic description below). I re-estimate factors at each date in the sample recursively over time using the entire history of data available in real time prior to each out-of-sample forecast. In the second step, I convert these daily financial indicators to quarterly weighted variables to form quarterly factors by selecting an optimal weighting scheme according to the method described below (see the weighting scheme section). The data series used in this dataset are listed below in Table A.14 by data source. The tables also list the transformation applied to each variable to make them stationary before generating factors. The transformations used to stationarize a time series are the same as those explained in the section "Monthly financial factor data".

Table A.14: List of Daily Financial Dataset Variables

No.	Short Name	Source	Tran	Description
			Group 1: Commodities	
1	GSIZSPT	Data Stream	Δln	S&P GSCI Zinc Spot - PRICE INDEX
2	GSSBSPT	Data Stream	Δln	S&P GSCI Sugar Spot - PRICE INDEX
3	GSSOSPT	Data Stream	Δln	S&P GSCI Soybeans Spot - PRICE INDEX
4	GSSISPT	Data Stream	Δln	S&P GSCI Silver Spot - PRICE INDEX
5	GSIKSPT	Data Stream	Δln	S&P GSCI Nickel Spot - PRICE INDEX
6	GSLCSPT	Data Stream	Δln	S&P GSCI Live Cattle Spot - PRICE INDEX
7	GSLHSPT	Data Stream	Δln	S&P GSCI Lean Hogs Index Spot - PRICE INDEX
8	GSILSPT	Data Stream	Δln	S&P GSCI Lead Spot - PRICE INDEX
9	GSGCSPT	Data Stream	Δln	S&P GSCI Gold Spot - PRICE INDEX
10	GSCTSPT	Data Stream	Δln	S&P GSCI Cotton Spot - PRICE INDEX
11	GSKCSPT	Data Stream	Δln	S&P GSCI Coffee Spot - PRICE INDEX
12	GSCCSPT	Data Stream	Δln	S&P GSCI Cocoa Index Spot - PRICE INDEX
13	GSIASPT	Data Stream	Δln	S&P GSCI Aluminum Spot - PRICE INDEX
14	SGWTSPT	Data Stream	Δln	S&P GSCI All Wheat Spot - PRICE INDEX
15	EIAEBRT	Data Stream	Δln	Europe Brent Spot FOB U$/BBL Daily
16	CRUDOIL	Data Stream	Δln	Crude Oil-WTI Spot Cushing U$/BBL - MID PRICE
17	LTICASH	Data Stream	Δln	LME-Tin 99.85% Cash U$/MT
18	CWFCS00	Data Stream	Δln	CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE
19	CCFCS00	Data Stream	Δln	CBT-CORN COMP. CONTINUOUS - SETT. PRICE
20	CSYCS00	Data Stream	Δln	CBT-SOYBEANS COMP. CONT. - SETT. PRICE
21	NCTCS20	Data Stream	Δln	CSCE-COTTON #2 CONT.2ND FUT - SETT. PRICE
22	NSBCS00	Data Stream	Δln	CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE
23	NKCCS00	Data Stream	Δln	CSCE-COFFEE C CONTINUOUS - SETT. PRICE
24	NCCCS00	Data Stream	Δln	CSCE-COCOA CONTINUOUS - SETT. PRICE
25	CZLCS00	Data Stream	Δln	ECBOT-SOYBEAN OIL CONTINUOUS - SETT. PRICE
26	COFC01	Data Stream	Δln	CBT-OATS COMP. TRc1 - SETT. PRICE
27	CLDCS00	Data Stream	Δln	CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE
28	CLGC01	Data Stream	Δln	CME-LEAN HOGS COMP. TRc1 - SETT. PRICE
29	NGCCS00	Data Stream	Δln	CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE
30	LAH3MTH	Data Stream	Δln	LME-Aluminium 99.7% 3 Months U$/MT
31	LED3MTH	Data Stream	Δln	LME-Lead 3 Months U$/MT
32	LNI3MTH	Data Stream	Δln	LME-Nickel 3 Months U$/MT
33	LTI3MTH	Data Stream	Δln	LME-Tin 99.85% 3 Months U$/MT
34	PLNYD	www.macrotrends.net	Δln	Platinum Cash Price (U$ per troy ounce)
35	XPDD	www.macrotrends.net	Δln	Palladium (U$ per troy ounce)
36	CUS2D	www.macrotrends.net	Δln	Corn Spot Price (U$/Bushel)
37	SoybOil	www.macrotrends.net	Δln	Soybean Oil Price (U$/Pound)
38	OATSD	www.macrotrends.net	Δln	Oat Spot Price (US$/Bushel)
39	WTIOilFut	US EIA	Δln	Light Sweet Crude Oil Futures Price: 1St Expiring Contract Settlement ($/Bbl)
			Group 2: Equities	
40	S&PCOMP	Data Stream	Δln	S&P 500 COMPOSITE - PRICE INDEX
41	ISPCS00	Data Stream	Δln	CME-S&P 500 INDEX CONTINUOUS - SETT. PRICE
42	SP5EIND	Data Stream	Δln	S&P500 ES INDUSTRIALS - PRICE INDEX
43	DJINDUS	Data Stream	Δln	DOW JONES INDUSTRIALS - PRICE INDEX
44	CYMCS00	Data Stream	Δln	CBT-MINI DOW JONES CONTINUOUS - SETT. PRICE
45	NASCOMP	Data Stream	Δln	NASDAQ COMPOSITE - PRICE INDEX
46	NASA100	Data Stream	Δln	NASDAQ 100 - PRICE INDEX
47	CBOEVIX	Data Stream	lv	CBOE SPX VOLATILITY VIX (NEW) - PRICE INDEX
48	S&P500toVIX	Data Stream	Δln	S&P500/VIX
			Group 3: Corporate Risk	
49	LIBOR	FRED	Δlv	Overnight London Interbank Offered Rate (%)
50	1MLIBOR	FRED	Δlv	1-Month London Interbank Offered Rate (%)
51	3MLIBOR	FRED	Δlv	3-Month London Interbank Offered Rate (%)
52	6MLIBOR	FRED	Δlv	6-Month London Interbank Offered Rate (%)
53	1YLIBOR	FRED	Δlv	One-Year London Interbank Offered Rate (%)
54	1MEuro-FF	FRED	lv	1-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
55	3MEuro-FF	FRED	lv	3-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds

No.	Short Name	Source	Tran	Description
56	6MEuro-FF	FRED	lv	6-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
57	APFNF-AANF	Data Stream	lv	1-Month A2/P2/F2 Nonfinancial Commercial Paper (NCP) (% P. A.) minus 1-Month Aa NCP (% P.A.)
58	APFNF-AAF	Data Stream	lv	1-Month A2/P2/F2 NCP (% P.A.) minus 1-Month Aa Financial Commercial Paper (% P.A.)
59	TED	Data Stream, FRED	lv	3Month Tbill minus 3-Month London Interbank Offered Rate (%)
60	MAaa-10YTB	Data Stream	lv	Moody Seasoned Aaa Corporate Bond Yield (% P.A.) minus Y10-Tbond
61	MBaa-10YTB	Data Stream	lv	Moody Seasoned Baa Corporate Bond Yield (% P.A.) minus Y10-Tbond
62	MLA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: A Rated: Effective Yield (%) minus Y10-Tbond
63	MLAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aa Rated: Effective Yield (%) minus Y10-Tbond
64	MLAAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aaa Rated: Effective Yield (%) minus Y10-Tbond
				Group 4: Treasuries
65	FRFEDFD	Data Stream	Δlv	US FED FUNDS EFF RATE (D) - MIDDLE RATE
66	FRTBS3M	Data Stream	Δlv	US T-BILL SEC MARKET 3 MONTH (D) - MIDDLE RATE
67	FRTBS6M	Data Stream	Δlv	US T-BILL SEC MARKET 6 MONTH (D) - MIDDLE RATE
68	FRTCM1Y	Data Stream	Δlv	US TREASURY CONST MAT 1 YEAR (D) - MIDDLE RATE
69	FRTCM10	Data Stream	Δlv	US TREASURY CONST MAT 10 YEAR (D) - MIDDLE RATE
70	6MTB-FF	Data Stream	lv	6-month treasury bill market bid yield at constant maturity (%) minus Fed Funds
71	1YTB-FF	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus Fed Funds
72	10YTB-FF	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus Fed Funds
73	6MTB-3MTB	Data Stream	lv	6-month treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
74	1YTB-3MTB	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
75	10YTB-3MTB	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus 3M-Tbills
76	BKEVEN05	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 5-year (%)
77	BKEVEN10	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 10-year (%)
78	BKEVEN1F4	FRB	lv	BKEVEN1F4
79	BKEVEN1F9	FRB	lv	BKEVEN1F9
80	BKEVEN5F5	FRB	lv	US Inflation compensation: coupon equivalent forward rate: 5-10 years (%)
				Group 5: Foreign Exchange (FX)
81	US_CWBN	Data Stream	Δln	US NOMINAL DOLLAR BROAD INDEX - EXCHANGE INDEX
82	US_CWMN	Data Stream	Δln	US NOMINAL DOLLAR MAJOR CURR INDEX - EXCHANGE INDEX
83	US_CSFR2	Data Stream	Δln	CANADIAN $ TO US $ NOON NY - EXCHANGE RATE
84	EU_USFR2	Data Stream	Δln	EURO TO US$ NOON NY - EXCHANGE RATE
85	US_YFR2	Data Stream	Δln	JAPANESE YEN TO US $ NOON NY - EXCHANGE RATE
86	US_SFFR2	Data Stream	Δln	SWISS FRANC TO US $ NOON NY - EXCHANGE RATE
87	US_UKFR2	Data Stream	Δln	UK POUND TO US $ NOON NY - EXCHANGE RATE

From Daily to Quarterly Factors: Weighting Schemes After we obtain daily financial factors $G_{D,t}$, we use weighting schemes proposed in the literature on Mixed Data Sampling (MIDAS) regressions to form quarterly factors, denoted $G^Q_{D,t}$. Let G^D_t denote a factor in daily frequency formed from the daily financial dataset, and let G^Q_t denote a quarterly aggregate of the corresponding daily factor time series. Let $G^D_{ND-j,d_t,t}$ denote the value of a daily factor on the j-th day counting backwards from the survey deadline d_t in quarter t. Hence, the day d_t of quarter t corresponds to $j = 0$, so the daily factor on the survey deadline is $G^D_{ND,d_t,t}$. For simplicity, we suppress the subscript d_t, writing $G^D_{ND-j,t}$.

We compute the quarterly aggregate of a daily financial factor as a weighted average of observations over the ND business days before the survey deadline. This means that the forecaster's information set includes daily financial data up to the previous ND business days before the survey deadline. The quarterly factor G^Q_t is defined as:

$$G^Q_t(w) = \sum_{j=1}^{ND} w_j \times G^D_{ND-j,t}$$

where w_j is a weight. We consider the following three types of weighting schemes to convert daily factor observations to quarterly aggregates. Each weighting scheme weights information by some function of the number of days prior to the survey deadline.

1. $w_i = 1$ for $i = 1$ and $w_i = 0$ otherwise. This weighting scheme places all weight on the data from the last business day before the survey deadline and zero weight on any data prior to that day.

2. $w_i = \delta^i / \sum_{j=1}^{ND} \delta^j$, where we consider a range of δ values with $\delta \in \{0.1, 0.2, 0.3, 0.7, 0.8, 0.9, 1.0\}$. The smaller the δ, the more rapidly information prior to the survey deadline is down-weighted. This down-weighting is progressive but not non-monotonic. The case $\delta = 1$ corresponds to a simple average of observations across all days.

3. The third parameterization uses two parameters $\theta = (\theta_1, \theta_2)'$ and allows for non-monotonic weighting of past information. The weights are defined as:

$$w(i; \theta_1, \theta_2) = \frac{f\left(\frac{i}{ND}; \theta_1, \theta_2\right)}{\sum_{j=1}^{ND} f\left(\frac{j}{ND}; \theta_1, \theta_2\right)}$$

   where $f(x; a, b) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$, and $\Gamma(a)$ is the gamma function $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x}\, dx$. The weights $w(i; \theta_1, \theta_2)$ are the Beta polynomial MIDAS weights of Ghysels et al. (2007), based on the Beta function. This weighting scheme is flexible enough to generate a wide range of possible shapes with only two parameters.

We consider these possible weighting schemes and choose the optimal weighting scheme w^* from 24 candidate weighting schemes for each daily financial factor G_t^D by minimizing the sum of squared residuals in a regression of $y_{j,t+h}$ on G_t^Q:

$$y_{j,t+h} = \alpha + \beta \times G_t^Q(w) + u_{t+h}$$

This procedure is conducted in real time using recursive regressions. We re-estimate the weights at each date in the sample recursively over time, using the entire history of data available in real time prior to each out-of-sample forecast. We assume that $ND = 14$, which implies that forecasters use daily information from at most the past two weeks before the survey deadline. This process is repeated for each daily financial factor in $G_{D,t}$ to form quarterly factors $G_{D,t}^Q$.

D.2.7 LDA Data

The LDA data are used as inputs into the machine learning forecasts. The database for our Latent Dirichlet Allocation (LDA) analysis contains around one million articles published in *Wall Street Journal* between January 1984 to June 2022. The current vintage of the results reported here is based a randomly selected sub-sample of 200,000 articles over the same period, one-fifth size of the entire database. The sample selection procedures follows Bybee et al. (2021). First, I remove all articles prior to January 1984 and after June 2022 and exclude articles published in weekends. Second, I exclude articles with subject tags associated with obviously non-economic content such as sports. Third, I exclude articles with the certain headline patterns, such as those associated with data tables or those corresponding to regular sports, leisure, or books columns. I filter the articles using the same list of exclusions provided by Bybee et al. (2021). Last, I exclude articles with less than 100 words.

Processing of texts The processing of the texts can be summarized into five steps:

1. Tokenization: parse each article's text into a white-space-separated word list retaining the article's word ordering.

2. I drop all non-alphabetical characters and set the remaining characters to lower-case, remove words with less than 3 letters, and remove common stop words and URL-based terms. I use a standard list of stop words from the Python library *gensim.parsing.preprocessing.*

3. Lemmatization and Stemming: lemmatization returns the original form of a word using external dictionary *Textblob.Word* in Python and based on the context of the word. For instance, as a verb, "went" is converted to"go". Stemming usually refers to a heuristic process that remove the trailing letters at the end of the words, such as from "assesses" to "assess', and "really" to "real". I use the Python library *Textblob.Word* to implement the lemmatization and *SnowballStemmer* for the stemming. The results are not very sensitive to the particular Python packages being used.

4. From the first three steps, I obtain a list of uni-grams which are a list of singular words. For example, "united" and "states" are uni-grams from "united states". From the list of uni-grams, I generate a set of bi-grams as all pairs of (ordered) adjacent uni-grams. For example, "united states" together is one bi-gram. I then exclude uni-grams and bi-grams appearing in less than 0.1% of articles.

5. Last, I convert an article's word list into a vector of counts for each uni-gram and bi-gram. For example, the vector of counts $[5, 7, 2]$ corresponds to the number of times the words $["federal", "reserve", "bank"]$ appear in the article.

The LDA Model The LDA model Blei et al. (2003) essentially achieves substantial dimension reduction of the word distribution of each article using the following assumptions. I assume a factor structure on the vectors of word counts. Each factor is a topic and each article is a parametric distribution of topics, specified as follows,

$$\underbrace{\overbrace{w_i}^{V\times 1}}_{\text{word dist of article } i} \sim \text{Mult}\left(\underbrace{\overbrace{\Phi'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\theta_i}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_i}_{\#\text{ of words}}\right)$$

where Mult is the multinomial distribution. In the above equation, w_i is a vector of word counts of each unique term (uni-gram or bi-gram) in article i, whose size is equal to the number of unique terms V. K is the number of factors in article i. In the estimation, I assume $K = 180$ following Bybee et al. (2021). Φ is a matrix sized $K \times V$, whose kth row and vth column is equal to the probability of the unique term v showing up in topic k. θ_i stores the weights of all k topics contained in article i, which sum up to one. Dimension reduction is achieved as long as $K << V$ (the number of topics are significantly smaller than the number of unique terms). More specifically, it reduces the dimension from $T \times V$ to $T \times K$ (the size of θ) + $K \times V$ (the size of Φ).

Real-time news factors. I also generate real-time news factors for each month t starting from January 1991. In theory, I could train the LDA model using each real-time monthly vintage but it is computationally challenging. Instead, I simplify the procedure by training the LDA model using quarterly vintages t, $t + 3$, $t + 6$, etc, and use the LDA model parameters estimated at t to filter news paper articles within the quarter and generate news factors for those months. More specifically, given every article's word distribution $w_{i,t+s}$,for $s = 0, 1, 2$, and the estimated real-time topic-word distribution parameters $\hat{\Phi}_t$ using articles till date t, one can obtain the filtered topic distribution of each article $\hat{\theta}_{i,t+s}$, as follows,

$$\underbrace{\overbrace{w_{i,t+s}}^{V\times 1}}_{\text{word dist of article } i \text{ at time } t+s} \sim \text{Mult}\left(\underbrace{\overbrace{\hat{\Phi}'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\hat{\theta}_{i,t+s}}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_{i,t+s}}_{\#\text{ of words}}\right).$$

LDA Estimation I use the built-in LDA model estimation toolbox in the Python library `https://pypi.org/project/gensim/`Gensim to implement the model estimation. The model requires following initial inputs and parameters and it is estimated using Bayesian methods. In theory, maximum-likelihood estimation is possible but it is computationally challenging.

1. I create a document-term matrix $\mathbf{W}$ as a collection of w_i for all articles i in the sample. The number of rows in $\mathbf{W}$ is equal to the number of articles in our sample and the number of columns in $\mathbf{W}$ is equal to the number of unique uni-gram and bi-grams (after being filtered) across all articles. The matrix $\mathbf{W}$ is used as an input for the LDA model estimation. I then follow Bybee et al. (2021) and set the number of topics K to be 180. The authors used Bayesian criteria to find 180 to be an optimal number of topics.

2. In the Python library Gensim, the key parameters of the LDA estim are α and β.With a higher value of α, the documents are composed of more topics. With a higher values of β, each topic contains more terms (uni- or bi-grams). In the implementations, I do not impose any explicit restrictions on initial values of those parameters and set them to be "auto". These two parameters, alongside Φ' and $\{\theta_i\}_i$, are estimated by the toolbox from Python library `https://pypi.org/project/gensim/`Gensim.

Real-time LDA Factors With the estimated topic weights $\theta_{i,t}$ of each article i from the LDA model, I fruther construct time series of the overall news attention to each topic, or a news factor. The value of the topic k at time t is the average weights of topic k of all articles published at t, specified as follows,

$$F_{k,t} = \frac{\sum_i \hat{\theta}_{i,k,t}}{\#\text{ of articles at } t}$$

for all topics k.

D.2.8 Machine Variables to Be forecast

Returns and price growth When evaluating the MSE ratio of the machine relative to that of a benchmark survey, we use the machine forecast for the return or price growth measure that most closely corresponds to the concept that survey respondents are asked to predict:

1. CFO survey asks respondents about their expectations for the S&P 500 return over the next 12 months. Following Nagel and Xu (2021), we interpret the survey to be asking about $r^d_{t,t+12}$, the one-year CRSP value-weighted return (including dividends) from the current survey month to the same month one year ahead.

2. Gallup/UBS survey respondents report the return (including dividends) they expect on their own portfolio one year ahead. We interpret the survey to be asking about $r^d_{t,t+12}$, the one-year CRSP value-weighted return(including dividends) from the current survey month to the same month one year ahead.

3. Livingston survey respondents provide 12-month ahead forecasts of the S&P 500 index. We convert the level forecast to price growth forecast by taking the log difference between the 12-month ahead level forecast and the nowcast of the S&P 500 index for the current survey month. Therefore, we interpret the survey to be asking about the one-year price growth in the S&P 500 index.

4. Bloomberg Consensus Forecasts asks survey respondents about the end-of-year closing value of the S&P 500 index. We interpret the survey to be asking about the h-month price growth in the S&P 500 index. The horizon of the forecast changes depending on when in the year the panelists are answering the survey.

5. Michigan Survey of Consumers (SOC) asks respondents about their perceived probability that an investment in a diversified stock fund would increase in value in the year ahead. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

6. Conference Board (CB) survey asks respondents about their categorical belief on whether they expect stock prices to increase, decrease, or stay the same over the next year. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

Earnings growth (IBES "Street" Earnings) For earnings growth forecasts, we use a quarterly S&P 500 total earnings series based on IBES street earnings per share (EPS), as described above. Street earnings exclude discontinued operations, extraordinary charges, and other non-operating items, making them better aligned with the earnings measure targeted by survey respondents. We convert EPS to total earnings using the S&P 500 index divisor and use the resulting quarterly series directly, prior to any monthly interpolation, since the machine learning algorithm operates at a quarterly frequency. The IBES street earnings series spans 1983Q4 to 2021Q4.

For Long-Term Growth (LTG) forecasts, IBES defines LTG as the "expected annual increase in operating earnings over the company's next full business cycle. These forecasts refer to a period of between three to five years." We compare survey responses of LTG against machine forecasts under alternative interpretations of LTG. First, we consider machine forecasts of annual five-year forward growth, i.e., annual earnings growth from four to five years ahead (Bianchi et al. (2024b)). Second, we consider machine forecasts of annualized 5-year growth, i.e., annual earnings growth from current quarter to five years ahead, following the interpretation in Bordalo et al. (2019). Third, we consider machine forecasts of annualized earnings growth from one to 10 years ahead, following the interpretation in Nagel and Xu (2021)

Inflation We construct forecasts of annual inflation defined as $\pi_{t+4,t} = \ln\left(\frac{PGDP_{t+4}}{PGDP_t}\right)$, where $PGDP_t$ is the quarterly level of the chain-weighted GDP price index. Following Coibion and Gorodnichenko (2015), we use the vintage of inflation data that is available four quarters after the period being forecast.

D.2.9 Economic Names of Factors

Macro, Financial, Daily Factors Any labeling of the factors is imperfect because each is influenced to some degree by all the variables in the large dataset, and the orthogonalization means that no one of them will correspond exactly to a precise economic concept like output or unemployment. Following Ludvigson and Ng (2007), we relate the factors to the underlying variables in the large dataset. For each time period in our evaluation sample, we compute the marginal R^2 from regressions of each of the individual series in the panel dataset onto each factor, one at a time. Each series $\tilde{x}_{it}$ is assigned the group name in the data appendix tables naming all series, e.g., non-farm payrolls are part of the Employment group (EMP). If series $\tilde{x}_{it}$ has the highest average marginal R^2 over all evaluation periods for factor G_{kt}, we label G_{kt} according to the group to which $\tilde{x}_{it}$ belongs, e.g., G_{kt} is an Employment factor. We further normalize the sign of each factor so that an increase in the factor indicates an increase in $\tilde{x}_{it}$. Thus, in the example above, an increase in G_{kt} would indicate a rise in non-farm payrolls. Table A.15 reports the series with largest average marginal R^2 for each factor of each large dataset.

LDA Factors We follow Bybee et al. (2021) in assigning economic names to the Latent Dirichlet Allocation (LDA) factors. The kth LDA factor $F_{k,t}$ at period t is defined as the average attention weight $\theta_{i,k,t}$ allocated to topic k across all articles published during the period. A topic is a probability distribution over words. Formally, the kth topic is a V-dimensional vector $\phi_{k,t}$ in the kth row of the topic-word distribution $\Phi_t = [\phi_{1,t}, \ldots, \phi_{K,t}]'$, where K is the total number of topics and V is the number of unique words in the corpus. Since the parameters $\theta_{i,k,t}$ and $\phi_{k,t}$ are estimated recursively

Table A.15: Economic Interpretation of the Factors

	Series with Largest R^2	
	Macro Factors	Label
$G_{1,M,t}$	Nonfarm Payrolls	Macro Factor: Employment
$G_{2,M,t}$	Interest paid by consumers	Macro Factor: Money and Credit
$G_{3,M,t}$	Agg. Weekly hours - Service-producing	Macro Factor: Employment
$G_{4,M,t}$	Agg. Weekly hours - Good-producing	Macro Factor: Employment
$G_{5,M,t}$	Nonborrowed Reserves	Macro Factor: Money and Credit
$G_{6,M,t}$	Housing Starts	Macro Factor: Housing
$G_{7,M,t}$	Change in private inventories	Macro Factor: Orders and Investment
$G_{8,M,t}$	PCE: Service	Macro Factor: Consumption
	Financial Factors	Label
$G_{1,F,t}$	$D\_log(P)$	Financial Factor: Prices, Yield, Dividends
$G_{2,F,t}$	SMB	Financial Factor: Equity Risk Factors
$G_{3,F,t}$	HML	Financial Factor: Equity Risk Factors
$G_{4,F,t}$	$R15\_R11$	Financial Factor: Equity Risk Factors
$G_{5,F,t}$	$D\_DIVreinvest$	Financial Factor: Prices, Yield, Dividends
$G_{6,F,t}$	Smoke	Financial Factor: Industries
$G_{7,F,t}$	UMD	Financial Factor: Equity Risk Factors
$G_{8,F,t}$	Telcm	Financial Factor: Industries
	Daily Factors	Label
$G_{1,D,t}$	ECBOT-SOYBEAN OIL	Daily Factor: Commodities
$G_{2,D,t}$	A Rated minus Y10 Tbond	Daily Factor: Corporate Risk
$G_{3,D,t}$	6-month US T-bill	Daily Factor: Treasuries
$G_{4,D,t}$	6-month treasury bill minus 3M-Tbills	Daily Factor: Treasuries
$G_{5,D,t}$	CBT-MINI DOW JONES	Daily Factor: Equities
$G_{6,D,t}$	Corn	Daily Factor: Commodities
$G_{7,D,t}$	APFNF-AAF	Daily Factor: Corporate Risk
$G_{8,D,t}$	US nominal dollar broad index	Daily Factor: FX

Note: This table reports the series with largest marginal R^2 for the factor specified in the first column. The marginal R^2 is computed from regressions of each of the individual series onto the factor, one at a time, for the time period that the factor shows up as relevant for the median bias.

over real-time quarterly vintages of news articles, the estimates may vary over the training samples and are likewise denoted with a t subscript. To summarize the economic content of the topic that prevailed the most consistently across time, we select the key word that is expected to occur with the highest average probability across our testing subsample of interest, i.e., the largest element in the V-dimensional vector $\frac{1}{T}\sum_{t=1}^{T}\phi_{k,t}$ where t indexes the time periods of the evaluation sample of length T. We use Table 6 of Bybee et al. (2021) to map the top key word for each topic to their topic label. The authors have manually assigned a label to each topic based on their reading of the key terms list. We also use the same table to categorize each topic label into broader meta-topics. For example, the key word "clinton" has the highest average probability of occurring under topic $k = 1$ across an testing subsample of 2005Q1 to 2021Q4. Therefore, we label $F_{1,t}$ according to the label for which "clinton" belongs to, which is the topic label "Clintons" that falls under a broader meta-topic label "Political Leaders."

D.2.10 Machine Input Data: Predictor Variables

The vector $\mathbf{Z}_{jt} \equiv \left(y_{j,t}, \hat{\mathbf{G}}_t', \mathbf{W}_{jt}'\right)'$ is an $r = 1 + r_G + r_W$ vector which collects the data at time t with

$$\mathcal{Z}_{jt} \equiv \left(y_{j,t}, ..., y_{j,t-p_y}, \hat{\mathbf{G}}_t', ..., \hat{\mathbf{G}}_{t-p_G}', \mathbf{W}_{jt}', ..., \mathbf{W}_{jt-p_W}'\right)'$$

a vector of contemporaneous and lagged values of $\mathbf{Z}_{jt}$, where p_y, p_G, p_W denote the total number of lags of $y_{j,t}$, $\hat{\mathbf{G}}_t'$, $\mathbf{W}_{jt}'$, respectively. The predictors below are listed as elements of $y_{j,t}, \hat{\mathbf{G}}_{jt}'$, or $\mathbf{W}_{jt}'$ for variables.

Stock return and price growth predictor variables and specifications For y_j equal to CRSP value-weighted returns or S&P 500 price index growth, we first predict the one-year log stock return or price growth that is expected to occur h quarters into the future from time $t+h-4$ to $t+h$, i.e., $\mathbb{E}_t[r_{t+h-4,t+h}]$. For horizons longer than one year, since the h-quarter long horizon return is the sum of one-year returns between time t to $t+h$, we first forecast the forward one-year returns separately and then add the components together to get machine forecasts of h-quarter long horizon returns. The forecasting model considers the following variables. Lags of the dependent variable:

1. y_{t-1}, y_{t-2} one and two quarter lagged stock returns or price growth.

Table A.16: Economic Interpretation of LDA Factors

Factor	Meta-Topic	Topic	**Factor**	Meta-Topic	Topic
$LDA_{1,t}$	Politics	Clintons	$LDA_{26,t}$	Fin Mkts	Trading
$LDA_{2,t}$	Fin Mkts	Intl exchanges	$LDA_{27,t}$	Fin Mkts	Trading
$LDA_{3,t}$	Industry	Couriers	$LDA_{28,t}$	Banks	Mortgages
$LDA_{4,t}$	Fin Mkts	Options/VIX	$LDA_{29,t}$	Activism	Futures/indices
$LDA_{5,t}$	Fin Mkts	FX/metals	$LDA_{30,t}$	Banks	NPLs
$LDA_{6,t}$	Asset Mgrs	Mutual funds	$LDA_{31,t}$	Fin Mkts	Payouts
$LDA_{7,t}$	Fin Mkts	Exchanges	$LDA_{32,t}$	Govt	Public/private
$LDA_{8,t}$	Fin Mkts	FX/metals	$LDA_{33,t}$	Banks	Mortgages
$LDA_{9,t}$	Fin Mkts	Intl exchanges	$LDA_{34,t}$	Fin Mkts	Exchanges
$LDA_{10,t}$	Asset Mgrs	Mutual funds	$LDA_{35,t}$	Fin Mkts	IPOs
$LDA_{11,t}$	Fin Mkts	Trading	$LDA_{36,t}$	Banks	Mortgages
$LDA_{12,t}$	Activism	Futures/indices	$LDA_{37,t}$	Fin Mkts	Trading
$LDA_{13,t}$	Fin Mkts	Trading	$LDA_{38,t}$	Activism	Futures/indices
$LDA_{14,t}$	Transport	Automotive	$LDA_{39,t}$	Fin Mkts	Exchanges
$LDA_{15,t}$	Banks	Mortgages	$LDA_{40,t}$	Banks	NPLs
$LDA_{16,t}$	Govt	Public/private	$LDA_{41,t}$	Asset Mgrs	Mutual funds
$LDA_{17,t}$	Transport	Automotive	$LDA_{42,t}$	Fin Mkts	Trading
$LDA_{18,t}$	Banks	Mortgages	$LDA_{43,t}$	Banks	Mortgages
$LDA_{19,t}$	Transport	Airlines	$LDA_{44,t}$	Fin Mkts	Trading
$LDA_{20,t}$	Fin Mkts	Trading	$LDA_{45,t}$	Industry	Chemicals/paper
$LDA_{21,t}$	Transport	Automotive	$LDA_{46,t}$	Govt	Watchdogs
$LDA_{22,t}$	Fin Mkts	Exchanges	$LDA_{47,t}$	Mideast/Terror	Nuclear/NK
$LDA_{23,t}$	Transport	Airlines	$LDA_{48,t}$	Intl Affairs	UK
$LDA_{24,t}$	Fin Mkts	Trading	$LDA_{49,t}$	Activism	Futures/indices
$LDA_{25,t}$	Fin Mkts	Options/VIX	$LDA_{50,t}$	Fin Mkts	Exchanges

Notes: This table summarizes the economic interpretation of each LDA factor. Meta-topic and topic labels are based on keyword distributions following Bybee et al. (2021). Abbreviations: Fin Mkts = Financial Markets, Asset Mgrs = Asset Managers/I-Banks, FX = Currencies/Metals, NPLs = Nonperforming Loans, NK = North Korea, Govt = Government.

The factors in $\hat{\mathbf{G}}'_{jt}$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}'_{jt}$ include:

1. *LDA topics* $F_{k,t-j}$, for topic $k = 1, 2, ...50$ and $j = 0, 1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

2. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

3. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

4. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

5. *Long-term growth of earnings*: 5-year growth of the SP500 earnings per share.

6. *Short rates.* When forecasting returns or price growth, the machine controls for the current nominal short rate, $\ln(1+3MTB_t/100)$, imposing a unit coefficient. This is equivalent to forecasting the future return minus the current short rate.

The 92 macro series in $\mathcal{D}^M$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^f$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns. A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf. The 87 daily financial indicators in $\mathcal{D}^D$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

Earning growth predictor variables and specifications For y_t equal to S&P 500 log earning growth, we construct a forecasted value for y_t, denoted $\hat{y}_{t|t-h}$, based on information known up to time t using the following variables. Lags of the dependent variable:

1. y_{t-1}, y_{t-2} one and two quarter lagged earnings growth.

The factors in $\hat{\mathbf{G}}'_{jt}$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k=0,1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k=0,1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k=0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}'_{jt}$ include:

1. *LDA factors* $F_{k,t-j}$, for topic $k=1,2,...50$ and $j=0,1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

2. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

3. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

4. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

Inflation predictor variables For y_j equal to inflation, the forecasting model considers the following variables. Lags of the dependent variable:

1. $y_{t-1,t-h-1}$ one quarter lagged inflation.

The factors in $\hat{\mathbf{G}}'_{jt}$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}'_{jt}$ include:

1. $\mathbb{F}^{(i)}_{jt-k}[y_{jt+h-k}]$, lagged values of the ith type's forecast, where $k = 1, 2$

2. $\mathbb{F}^{(s\neq i)}_{jt-1}[y_{jt+h-1}]$, lagged values of other type's forecasts, $s \neq i$

3. $var_N\left(\mathbb{F}^{(\cdot)}_{t-1}[y_{jt+h-1}]\right)$, where $var_N(\cdot)$ denotes the cross-sectional variance of lagged survey forecasts

4. $skew_N\left(\mathbb{F}^{(\cdot)}_{t-1}[y_{jt+h-1}]\right)$, where $skew_N(\cdot)$ denotes the cross-sectional skewness of lagged survey forecasts

5. Trend inflation measured as $\overline{\pi}_{t-1} = \begin{cases} \rho\overline{\pi}_{t-2} + (1-\rho)\pi_{t-1}, \rho = 0.95 & \text{if } t < 1991\text{Q4} \\ \text{CPI10}_{t-1} & \text{if } t \geq 1991\text{Q4} \end{cases}$, where CPI10 is the median SPF forecast of annualized average inflation over the current and next nine years. Trend inflation is intended to capture long-run trends. When long-run forecasts of inflation are not available, as is the case pre-1991Q4, we us a moving average of past inflation.

6. $\dot{GDP}_{t-1}$ = detrended gross domestic product, defined as the residual from a regression of GDP_{t-1} on a constant and the four most recent values of GDP as of date $t-8$. See Hamilton (2018).

7. $\dot{EMP}_{t-1}$ = detrended employment, defined as the residual from a regression of EMP_{t-1} on a constant and the four most recent values of EMP as of date $t-8$. See Hamilton (2018).

8. $\mathbb{N}^{(i)}_t[\pi_{t,t-h}]$ = Nowcast as of time t of the ith percentile of inflation over the period $t-h$ to t.

D.3 Cross-Sectional Forecasts for Book-to-Market Portfolios

I construct machine learning forecasts of stock returns for five value-weighted portfolios sorted by book-to-market ratios. For each portfolio, I re-estimate the time-series Long Short-Term Memory (LSTM) model separately, applying the dynamic machine learning procedure described in Section D. The stock universe consists of all firms listed on the NYSE, AMEX, and NASDAQ with available IBES analyst coverage for one- and two-year ahead earnings expectations and long-term growth forecasts. Monthly total returns for these firms are obtained from CRSP. The sample spans March 1965 to December 2024.

To construct predictors, I follow the cross-sectional asset pricing literature and compile a broad set of stock-level characteristics. Specifically, I include 94 firm characteristics, of which 61 are updated annually, 13 quarterly, and 20 monthly. These characteristics span valuation ratios, profitability, investment, size, momentum, volatility, and other firm-level attributes, based on the definitions in Green et al. (2013). Book equity and operating profitability follow Fama and French (2015). I rank-transform each characteristic cross-sectionally within each month to the $[-1, 1]$ interval, as in Gu et al. (2020). I also include 74 industry dummies based on two-digit Standard Industrial Classification (SIC) codes. Table A.17 provides further details on these predictors. To avoid forward-looking bias, I apply realistic reporting lags: monthly characteristics are assumed available with a one-month delay, quarterly characteristics with at least a four-month delay, and annual characteristics with at least a six-month delay. Missing values are replaced with the cross-sectional median at each period.

Following Gu et al. (2020), I construct an expanded set of predictors that interact portfolio-level characteristics with aggregate macroeconomic state variables. Let $\mathcal{C}_{i,t}$ denote the vector of value-weighted portfolio characteristics for portfolio i, and let $\mathcal{X}_t$ denote the vector of aggregate predictors, which includes a constant and the same macroeconomic variables used to forecast aggregate returns, price growth, and earnings growth, respectively. The final predictor set for portfolio i at time t is given by $\mathcal{X}_{i,t} = \mathcal{X}_t \otimes \mathcal{C}_{i,t}$, where $\otimes$ denotes the Kronecker product. This structure generates interaction terms that capture how aggregate economic conditions influence the effect of portfolio-level characteristics on expected returns.

Table A.17: Details of Firm Characteristics

No.	Acronym	Characteristic	Authors	Source	Frq.
1	absacc	Absolute accruals	Bandyopadhyay, Huang, Wirjanto 2010	Compustat	Y
2	acc	Working capital accruals	Sloan 1996	Compustat	Y
3	aeavol	Abnormal earnings ann volume	Lerman, Livnat, Mendenhall 2007	Compustat/CRSP	Q
4	age	Years since first coverage	Jiang, Lee, Zhang 2005	Compustat	Y
5	agr	Asset growth	Cooper, Gulen, Schill 2008	Compustat	Y
6	baspread	Bid-ask spread	Amihud, Mendelson 1989	CRSP	M
7	beta	Beta	Fama, MacBeth 1973	CRSP	M
8	betasq	Beta squared	Fama, MacBeth 1973	CRSP	M
9	bm	Book-to-market	Rosenberg, Reid, Lanstein 1985	Compustat/CRSP	Y
10	bm_ia	Industry-adj book-to-market	Asness, Porter, Stevens 2000	Compustat/CRSP	Y
11	cash	Cash holdings	Palazzo 2012	Compustat	Q
12	cashdebt	Cash flow to debt	Ou, Penman 1989	Compustat	Y
13	cashpr	Cash productivity	Chandrashekar, Rao 2009	Compustat	Y
14	cfp	Cash flow to price ratio	Desai, Rajgopal, Venkatachalam 2004	Compustat	Y
15	cfp_ia	Industry-adj cash flow to price ratio	Asness, Porter, Stevens 2000	Compustat	Y
16	chatoia	Industry-adj chg asset turnover	Soliman 2008	Compustat	Y
17	chcsho	Chg shares outstanding	Pontiff, Woodgate 2008	Compustat	Y
18	chempia	Industry-adj chg employees	Asness, Porter, Stevens 1994	Compustat	Y
19	chinv	Chg inventory	Thomas, Zhang 2002	Compustat	Y
20	chmom	Chg 6-month momentum	Gettleman, Marks 2006	CRSP	M
21	chpmia	Industry-adj chg profit margin	Soliman 2008	Compustat	Y
22	chtx	Chg tax expense	Thomas, Zhang 2011	Compustat	Q
23	cinvest	Corporate investment	Titman, Wei, Xie 2004	Compustat	Q
24	convind	Convertible debt indicator	Valta 2016	Compustat	Y
25	currat	Current ratio	Ou, Penman 1989	Compustat	Y
26	depr	Depreciation over PP&E	Holthausen, Larcker 1992	Compustat	Y
27	divi	Dividend initiation	Michaely, Thaler, Womack 1995	Compustat	Y
28	divo	Dividend omission	Michaely, Thaler, Womack 1995	Compustat	Y
29	dolvol	Dollar trading volume	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
30	dy	Dividend-to-price ratio	Litzenberger, Ramaswamy 1982	Compustat	Y
31	ear	Earnings announcement return	Kishore, Brandt, Santa-Clara, Venkatachalam 2008	Compustat/CRSP	Q
32	egr	Gr common shareholder equity	Richardson, Sloan, Soliman, Tuna 2005	Compustat	Y
33	ep	Earnings-to-price ratio	Basu 1977	Compustat	Y
34	gma	Gross profitability	Novy-Marx 2013	Compustat	Y
35	grCAPX	Gr capex	Anderson, Garcia-Feijoo 2006	Compustat	Y
36	grltnoa	Gr long-term net operating assets	Fairfield, Whisenant, Yohn 2003	Compustat	Y
37	herf	Industry sales concentration	Hou, Robinson 2006	Compustat	Y
38	hire	Employee gr rate	Bazdresch, Belo, Lin 2014	Compustat	Y
39	idiovol	Idiosyncratic return volatility	Ali, Hwang, Trombley 2003	CRSP	M
40	ill	Illiquidity	Amihud 2002	CRSP	M
41	indmom	Industry momentum	Moskowitz, Grinblatt 1999	CRSP	M
42	invest	Capital expenditures and inventory	Chen, Zhang 2010	Compustat	Y
43	lev	Leverage	Bhandari 1988	Compustat	Y
44	lgr	Gr long-term debt	Richardson, Sloan, Soliman, Tuna 2005	Compustat	Y
45	maxret	Maximum daily return	Bali, Cakici, Whitelaw 2011	CRSP	M
46	mom12m	12-month momentum	Jegadeesh 1990	CRSP	M
47	mom1m	1-month momentum	Jegadeesh, Titman 1993	CRSP	M
48	mom36m	36-month momentum	Jegadeesh, Titman 1993	CRSP	M
49	mom6m	6-month momentum	Jegadeesh, Titman 1993	CRSP	M
50	ms	Financial statement score	Mohanram 2005	Compustat	Q
51	mvel1	Size	Banz 1981	CRSP	M
52	mve_ia	Industry-adj size	Asness, Porter, Stevens 2000	Compustat	Y
53	nincr	Number of earnings increases	Barth, Elliott, Finn 1999	Compustat	Q
54	operprof	Operating profitability	Fama, French 2015	Compustat	Y
55	orgcap	Organizational capital	Eisfeldt, Papanikolaou 2013	Compustat	Y
56	pchcapx_ia	Industry-adj % chg capex	Abarbanell, Bushee 1998	Compustat	Y
57	pchcurrat	% chg current ratio	Ou, Penman 1989	Compustat	Y
58	pchdepr	% chg depreciation	Holthausen, Larcker 1992	Compustat	Y

No.	Acronym	Firm Characteristic	Authors	Source	Freq.
59	pchgm pchsale	% chg gross margin - % chg sales	Abarbanell, Bushee 1998	Compustat	Y
60	pchquick	% chg quick ratio	Ou, Penman 1989	Compustat	Y
61	pchsale pchinvt	% chg sales - % chg inventory	Abarbanell, Bushee 1998	Compustat	Y
62	pchsale pchrect	% chg sales - % chg receivables	Abarbanell, Bushee 1998	Compustat	Y
63	pchsale pchxsga	% chg sales - % chg SG&A	Abarbanell, Bushee 1998	Compustat	Y
64	pchsaleinv	% chg sales-to-inventory	Ou, Penman 1989	Compustat	Y
65	pctacc	Percent accruals	Hafzalla, Lundholm, Van Winkle 2011	Compustat	Y
66	pricedelay	Price delay	Hou, Moskowitz 2005	CRSP	M
67	ps	Financial statement score	Piotroski 2000	Compustat	Y
68	quick	Quick ratio	Ou, Penman 1989	Compustat	Y
69	rd	R&D increase	Eberhart, Maxwell, Siddique 2004	Compustat	Y
70	rd_mve	R&D to market capitalization	Guo, Lev, Shi 2006	Compustat	Y
71	rd_sale	R&D to sales	Guo, Lev, Shi 2006	Compustat	Y
72	realestate	Real estate holdings	Tuzel 2010	Compustat	Y
73	retvol	Return volatility	Ang, Hodrick, Xing, Zhang 2006	CRSP	M
74	roaq	Return on assets	Balakrishnan, Bartov, Faurel 2010	Compustat	Q
75	roavol	Earnings volatility	Francis, LaFond, Olsson, Schipper 2004	Compustat	Q
76	roeq	Return on equity	Hou, Xue, Zhang 2015	Compustat	Q
77	roic	Return on invested capital	Brown, Rowe 2007	Compustat	Y
78	rsup	Revenue surprise	Kama 2009	Compustat	Q
79	salecash	Sales to cash	Ou, Penman 1989	Compustat	Y
80	saleinv	Sales to inventory	Ou, Penman 1989	Compustat	Y
81	salerec	Sales to receivables	Ou, Penman 1989	Compustat	Y
82	secured	Secured debt	Valta 2016	Compustat	Y
83	securedind	Secured debt indicator	Valta 2016	Compustat	Y
84	sgr	Sales gr	Lakonishok, Shleifer, Vishny 1994	Compustat	Y
85	sin	Sin stocks	Hong, Kacperczyk 2009	Compustat	Y
86	sp	Sales to price	Barbee, Mukherji, Raines 1996	Compustat	Y
87	std_dolvol	Volatility liquidity dollar volume	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
88	std_turn	Volatility liquidity share turnover	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
89	stdacc	Accrual volatility	Bandyopadhyay, Huang, Wirjanto 2010	Compustat	Q
90	stdcf	Cash flow volatility	Huang 2009	Compustat	Q
91	tang	Debt capacity / firm tangibility	Almeida, Campello 2007	Compustat	Y
92	tb	Tax income to book income	Lev, Nissim 2004	Compustat	Y
93	turn	Share turnover	Datar, Naik, Radcliffe 1998	CRSP	M
94	zerotrade	Zero trading days	Liu 2006	CRSP	M