Student Information

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Answer 1

a.

p	q	$p \wedge q$	$\neg p$	$\neg q$	$(\neg p) \vee (\neg q)$	$(p \wedge q) \iff (\neg p) \vee (\neg q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

The statement is a contradiction because the statement is false in every occasion.

b.

1. $p \implies ((q \vee \neg q) \implies (p \wedge q))$
2. $q \vee (\neg q) \equiv T$ Negation Laws
3. $T \implies (p \wedge q) \equiv F \vee (p \wedge q)$ Table 7 Line 1
4. $F \vee (p \wedge q) \equiv p \wedge q$ Identity Laws
5. $p \implies p \wedge q \equiv (p \implies p) \wedge (p \implies q)$ Table 7 Line 6
6. $(p \implies p) \equiv (\neg p) \vee p$ Table 7 Line 1
7. $(\neg p) \vee p \equiv T$ Negation Laws
8. $T \wedge (p \implies q) \equiv p \implies q$ Identity Laws
9. $p \implies q \equiv (\neg p) \vee p$ Table 7 Line 1

Answer 2

a) $\forall x \exists y \; W(x, y)$
b) $\forall x \exists y \; \neg F(x, y)$
c) $\forall x \; [W(x, P) \implies A(Ali, x)]$
d) $\exists y \; (W(Busra, y) \wedge (\exists y \; F(Tubitak, y))$
e) $\exists x \exists y \exists z \; (S(x, y) \wedge S(x, z)) \wedge (y \; ! = z)$
f) $\exists x \exists y \exists z \; (W(x, y) \wedge W(z, y)) \implies (x = z)$
g) $\exists x \exists y \exists z \; (W(x, y) \wedge W(z, y) \wedge (x \; ! = z)) \wedge (\forall t \; (W(q, y) \implies (x = q \; or \; z = q))$

Answer 3

1	$p \implies q$	*premise*
2	$(q \wedge (\neg r) \implies s$	*premise*
3	$\neg s$	*premise*
4	p	*assumption*
5	q	$\implies e\ 1, 4$
6	$\neg r$	*assumption*
7	$q \wedge (\neg r)$	$\wedge$ i $5, 6$
8	s	$\implies$ e $2, 7$
9	$\perp$	$\neg$e $3, 8$
10	r	$\neg e\ 6 - 9$
11	$p \implies r$	$\implies i\ 4, 10$

Answer 4

Ayse: We went to park (p)
Baris: : If we played hide and seek, then we did not eat candy. ($s \implies (\neg q)$)
Can: If we went to park, then we both ate candy and played games. ($p \implies (q \wedge r)$)
Duygu: If we played games, then we played hide and seek. ($r \implies s$)

1	p	*premise*
2	$p \implies (q \wedge r)$	*premise*
3	$r \implies s$	*premise*
4	q	*assumption*
5	$q \wedge r$	$\implies e\ 2, 1$
6	r	$\wedge e\ 5, 4$
7	s	$\implies e\ 3, 6$
8	$s \implies (\neg q)$	*assumption*
9	$\neg q$	$\implies$ e $8, 7$
10	$\perp$	$\neg$i $4, 9$
11	$\neg(s \implies (\neg q))$	$\neg i\ 8 - 10$

We assumed Baris says the truth in the eighth step. However, by applying natural deduction rules we obtained that Baris lies because we ended up getting the opposite of what Baris says.

Answer 5

1		$\forall x\ (P(x) \implies (Q(x) \implies R(x)))$	*premise*
2		$\exists x\ (P(x))$	*premise*
3		$\forall x\ (\neg R(x))$	*premise*
4	x_0	$P(x_0)$	*assumption*
5		$P(x_0) \implies (Q(x_0) \implies R(x_0))$	$\forall e$ 1
6		$\neg R(x_0)$	$\forall e$ 3
7		$Q(x_0) \implies R(x_0)$	$\implies e$ 5, 4
8		$Q(x_0)$	*assumption*
9		$R(x_0)$	$\implies$ e 7, 8
10		$\bot$	$\neg$e 6, 9
11		$\neg Q(x_0)$	$\neg i$ 8 – 10
12		$\exists x\ (\neg Q(x_0))$	$\exists i$ 11
13		$\exists x\ (\neg Q(x))$	$\exists e$ 2, 4-12