Student Information

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Answer 1

a)

The time taken to process and send the response is uniformly distributed which means the density is constant. The pdf of uniform distribution is:
$\frac{1}{b-a}$, for $a \leq t \leq b$

Since A and B are independent, we can find the joint density function ($f(t_A, t_B)$) by simply taking the product of $f(t_A)$ and $f(t_B)$.
$f(t_A, t_B) = f(t_A) * f(t_B)$
$f(t_A, t_B) = \frac{1}{100} * \frac{1}{100} = 0.0001$

We can find the e joint cdf like finding the joint pdf. Take the product of $F(t_A)$ and $F(t_B)$. Joint cdf is the integral taken version of joint pdf. In this case our separate cdf's will be in the form of:

$\frac{x}{b-a}$, for $a \leq x \leq b$
So, $F(t_A, t_B) = F(t_A) * F(t_B) = \frac{t_A}{100} * \frac{t_B}{100} = \frac{t_A * t_B}{10000}$

b)

We will use the joint cdf function to find the probabilities.
− For the probability that server A processes the packet and sends a response within the first 30 milliseconds:

$$P(0 \leq t_A \leq 30) = \frac{30-0}{100} = 0.3$$

− For the probability that server B takes between 40 and 60 milliseconds to process the packet and send a response:

$$P(40 \leq t_A \leq 60) = \frac{60-40}{100} = 0.2$$

Since these events are independent, in order to find the probability that server A processes the packet and sends a response within the first 30 milliseconds, while server B takes between 40 and

60 milliseconds to process the packet and send a response, we simply need to take the product of the probabilities that we have found which is equal to 0.3 * 0.2 = 0.06.

c)

The realition between T_A and T_B is $T_A - T_B \leq 10$. Than the probability that we want to find is:
$P(T_A - T_B \leq 10) = \iint f(t_A, t_B) dt_A \, dt_B$

In order to take the integral correctly, we need to separate the interval to 2 parts.
– While $0 \leq t_B \leq 90$ then $0 \leq t_A \leq t_B + 10$
– While $90 < t_B \leq 100$ then $0 \leq t_A \leq 100$

After this step, our equation looks like:
$P(T_A - T_B \leq 10) = \int_0^{90} \int_0^{T_B+10} f(t_A, t_B) dt_A \, dt_B + \int_{90}^{100} \int_0^{100} f(t_A, t_B) dt_A \, dt_B$

The result of this equation is $\dfrac{99}{200} * \dfrac{10}{100} = \dfrac{119}{200}$

d)

The realition between T_A and T_B is $|T_A - T_B| \leq 20$. Than the probability that we want to find is:
$P(|T_A - T_B| \leq 20) = \iint f(t_A, t_B) dt_A \, dt_B$

This time in order to take the integral correctly, we need to separate the interval to 3 parts.
– While $20 \leq t_B \leq 80$ then $t_B - 20 \leq t_A \leq t_B + 20$
– While $0 \leq t_B < 20$ then $0 \leq t_A \leq t_B + 20$
– While $80 \leq t_B < 100$ then $t_B - 20 \leq t_A \leq t_B + 100$

After this step, our equation looks like:
$P(|T_A - T_B| \leq 20) = \int_{20}^{80} \int_{T_B-20}^{T_B+20} f(t_A, t_B) dt_A \, dt_B + \int_0^{20} \int_0^{T_B+20} f(t_A, t_B) dt_A \, dt_B +$
$\int_{80}^{100} \int_{T_B-20}^{100} f(t_A, t_B) dt_A \, dt_B$

$$P(|T_A - T_B| \leq 20) = \frac{6}{25} + \frac{3}{50} + \frac{3}{50} = \frac{9}{25} = 0.36$$

Answer 2

a)

The question is a normal approximation to binomial because we know that p = 0.6, n = 150 and q = 0.4. Each Bernoulli Trial in this binomial distribution question has a mean which is equal to 0.6 and a standard deviation which is $\sqrt{p * q} = 0.49$.

So, $\mu = 0.6$ and $\sigma = 0.49$ for each Bernoulli Trial.

Since $n \geq 30$ we can use normal approximation.
The probability that is asked changes to:
$P(X > 0.65) \rightarrow= P(S_n \geq 0.65 * 150) = P(S_n \geq 97.5) = P(S_n > 97)$.
We must apply continuity correction in order to approximate appropriately. (Which is $P(S_n \geq 97.5)$ for continuous case.)

$$P(S_n \geq 97.5) = P(\frac{S_n - n * \mu}{\sigma * \sqrt{n}} \geq \frac{97.5 - 150 * 0.6}{0.49 * \sqrt{150}})$$

Then $P(S_n \geq 97.5) = P(Z > 1.25) = 1$ - $\Phi(1.25) = 1$ - $0.8944 = 0.1056$

b)

This question is same as the previous one. This time p = 0.1(mean), and standard deviation = $\sqrt{p * q} = 0.3$
So, $\mu = 0.1$ and $\sigma = 0.3$ for each Bernoulli Trial.

Since $n \geq 30$ we can use normal approximation.
The probability that is asked changes to:
$P(X > 0.65) \rightarrow= P(S_n \leq 0.15 * 150) = P(S_n \leq 22.5) = P(S_n < 23)$.
We must apply continuity correction in order to approximate appropriately. (Which is $P(S_n \leq 22.5)$ for continuous case.)

$$P(S_n \leq 22.5) = P(\frac{S_n - n * \mu}{\sigma * \sqrt{n}} \leq \frac{22.5 - 150 * 0.1}{0.3 * \sqrt{150}})$$

Then $P(S_n \leq 22.5) = P(Z < 2.04) = = \Phi(2.04) = 0.9793$

Answer 3

The question is $P(170 \leq X \leq 180)$
The mean (μ) is 175.
The standard deviation (σ) is 7.

We are going to standardize both 170, X and 180.
Standardizing formula = $z = \frac{x - \mu}{\sigma}$

Let's standardize 170, X and 180.
$-$ X $\rightarrow z$
$-$ 170 $\rightarrow \frac{170 - 175}{7} =$ -0.71

$$- 180 \rightarrow \frac{180 - 175}{7} = 0.71$$

Then $P(170 \leq X \leq 180) = P(-0.71 \leq z \leq 0.71) = \Phi(0.71)$ - (1 - $\Phi(0.71)) = 0.5222$

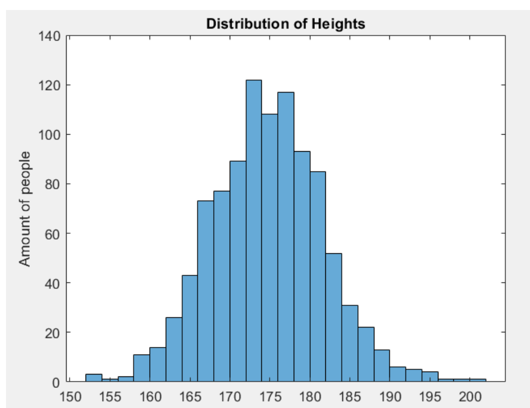
Answer 4

a)

```
%% My Code
part_a()

function [] = part_a()
    mean = 175;
    std_deviation = 7;
    N = 1000;
    heights_db = normrnd(mean, std_deviation, N, 1);

    figure
    histogram(heights_db);
    xlabel('Height (cm)', 'FontSize', 11);
    ylabel('Amount of people', 'FontSize', 11);
    title('Distribution of Heights')
end
```



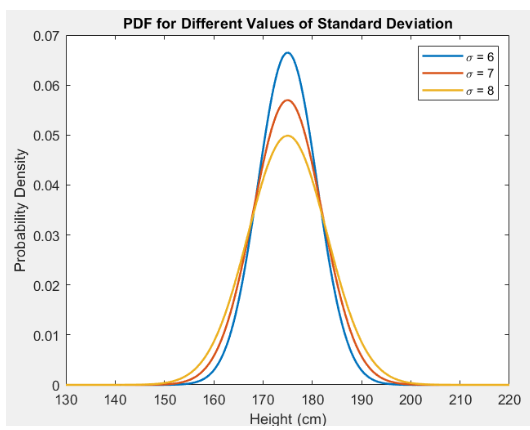
(a) Since the mean is 175 cm, the distribution of heights gathered around 175 cm on the histogram.

b)

```matlab
%% My Code
part_b()

function [] = part_b()
    x = linspace(130, 220, 1000);
    mean = 175;
    pdf6 = normpdf(x, mean, 6);
    pdf7 = normpdf(x, mean, 7);
    pdf8 = normpdf(x, mean, 8);

    figure
    plot(x, pdf6, 'LineWidth', 1.5)
    hold on
    plot(x, pdf7, 'LineWidth', 1.5)
    plot(x, pdf8, 'LineWidth', 1.5)
    xlabel('Height (cm)', 'FontSize', 11)
    ylabel('Probability Density', 'FontSize', 11)
    title('PDF for Different Values of Standard Deviation', '
       FontSize', 11);
    legend('\sigma = 6', '\sigma = 7', '\sigma = 8')
end
```



(a) As the standard deviation increases, we observe that the distribution becomes sharper around the mean since std.deviation is as known as scale parameter too.

c)

```
%% My Code
part_c()

function [] = part_c()
    % Define variables
    mean = 175;
    std_dev = 7;
    sample_size = 150;

    l_bound = 170;
    u_bound = 180;

    N = 1000;
    proportion_vec = [0.45, 0.50, 0.55];
    num_props = length(proportion_vec);
    success_counts = zeros(1, num_props);

    % Simulate experiments
    for i = 1:N
        sample_data = normrnd(mean, std_dev, sample_size, 1);

        % Check how many samples lie within the bounds
        num_within_bounds = sum(sample_data > l_bound & sample_data
            < u_bound);

        % Check if proportion of samples within bounds exceeds
           threshold
        for j = 1:num_props
            if num_within_bounds / sample_size >= proportion_vec(j)
                success_counts(j) = success_counts(j) + 1;
            end
        end
    end

    % Compute probabilities and print results
    success_probs = success_counts / N;
    for j = 1:num_props
        disp(['Probability of at least %' num2str(proportion_vec(j)
           *100) ' of adults with heights between 170 and 180: '
           num2str(success_probs(j))]);
    end
end
```

```
Probability of at least %45 of adults with heights between 170 and 180: 0.973
Probability of at least %50 of adults with heights between 170 and 180: 0.765
Probability of at least %55 of adults with heights between 170 and 180: 0.282
fx >>
```

(a) We observe that, as the proportion of samples in [170cm,180cm] becomes greater, the probability decreases dramatically.