

20기 정규세션

ToBig's 19기 강의자 하주찬

Neural Network Basic

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Unit 01 | Perceptron

Unit 02 | Feed forward & Backpropagation

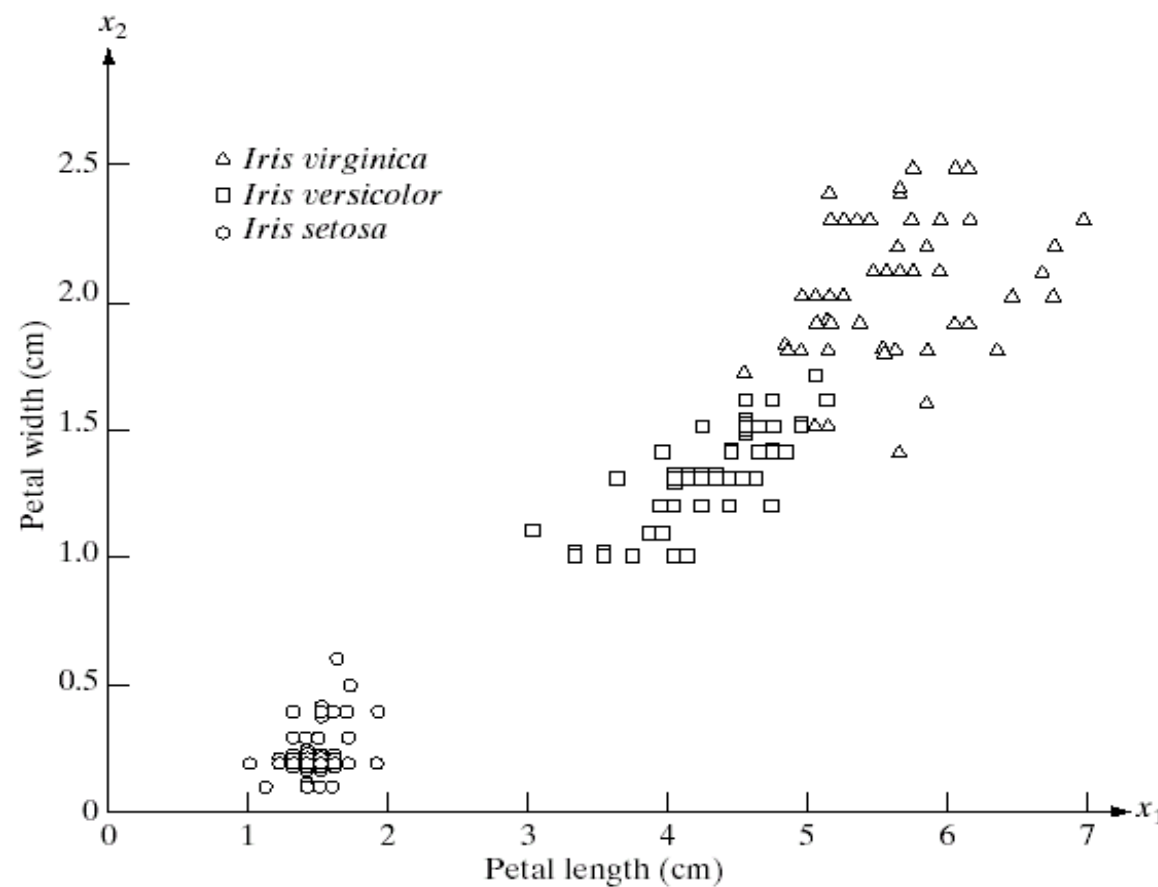
01 | Perceptron

Unit 01 | Perceptron

Why we need Perceptron?

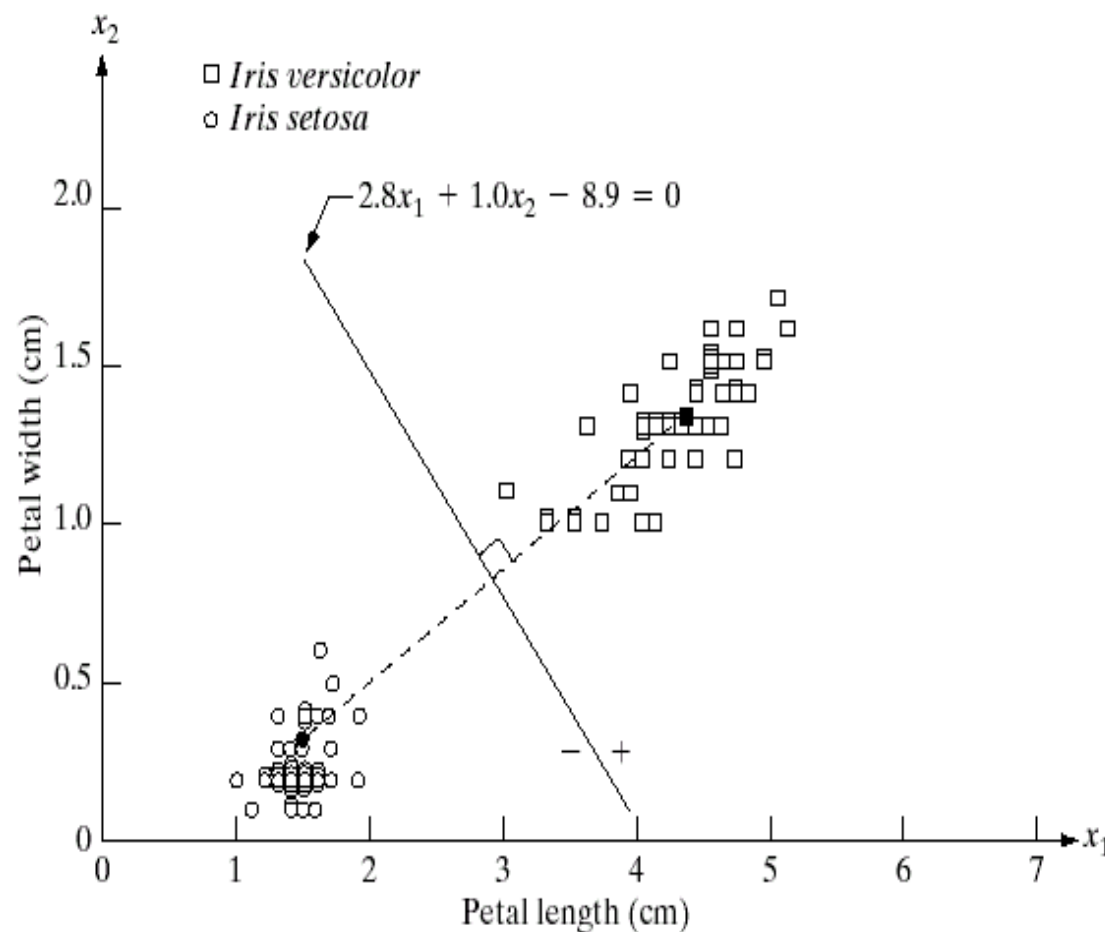
FIGURE 12.1

Three types of iris flowers described by two measurements.



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Why we need Perceptron?

**FIGURE 12.6**

Decision boundary of minimum distance classifier for the classes of *Iris versicolor* and *Iris setosa*. The dark dot and square are the means.

Unit 01 | Perceptron

Minimum distance classifier

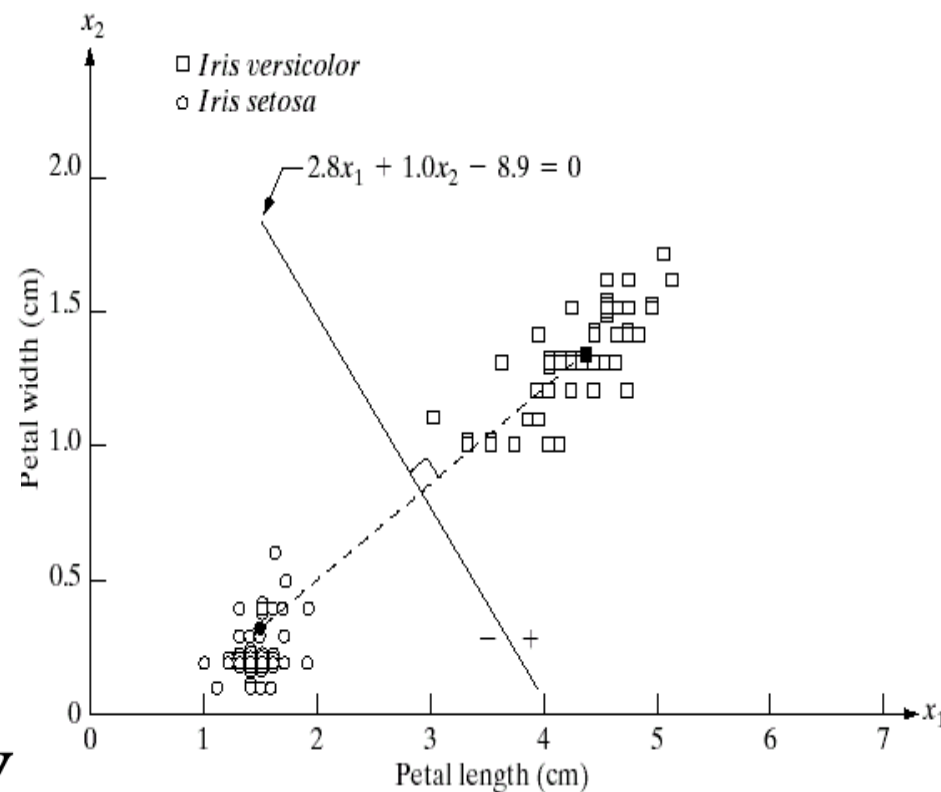
- Mean vector of that class

- $$m_j = \frac{1}{N_j} \sum_{X \in \omega_j} X, \quad j = 1, 2, \dots, W$$

- Distance metric:

- Euclidean distance

- $$D_j(X) = \|X - m_j\|, \quad j = 1, 2, \dots, W$$

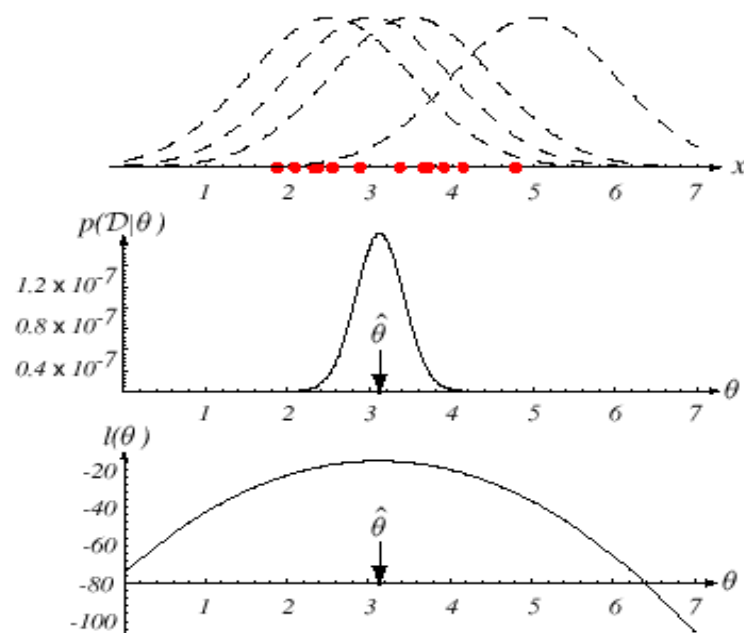
**FIGURE 12.6**

Decision boundary of minimum distance classifier for the classes of *Iris versicolor* and *Iris setosa*. The dark dot and square are the means.

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Optimal statistical classifier

$$P(\omega_i | \mathbf{x}) > P(\omega_j | \mathbf{x}), \forall j \neq i$$



Maximum likelihood estimation

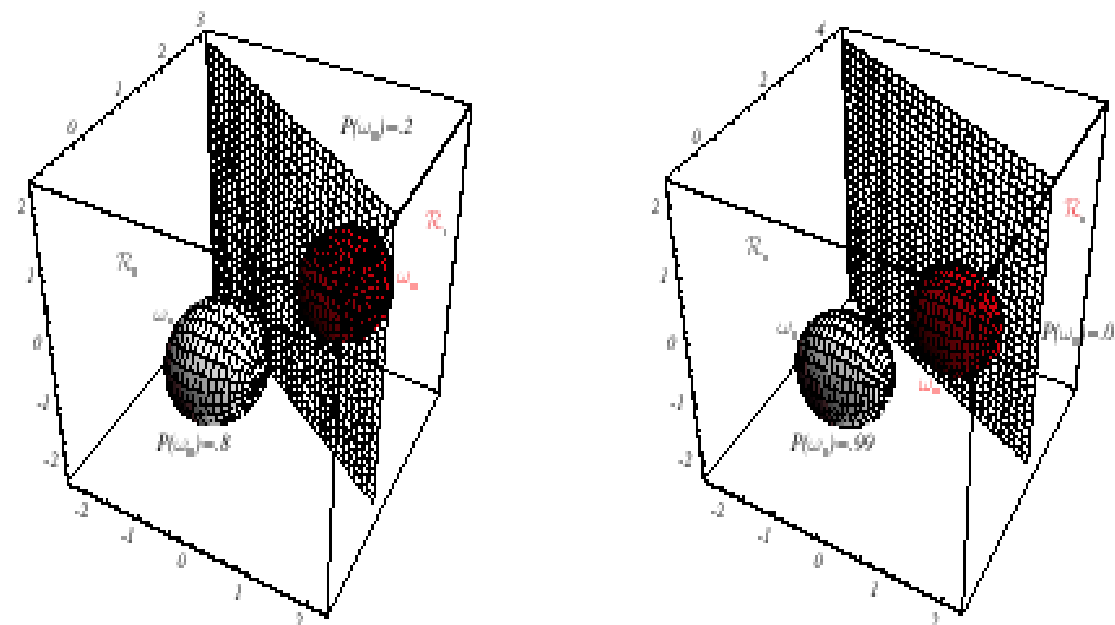


FIGURE 2.11. As the priors are changed, the decision boundary shifts; for sufficiently disparate priors the boundary will not lie between the means of these one-, two- and three-dimensional spherical Gaussian distributions. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

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Linear discriminant function

$$g(\mathbf{x}) = \mathbf{W}^T \mathbf{x} + w_0$$

W: weight vector, **w**₀: the bias

$$\bullet \mathbf{x} = \mathbf{x}_p + \frac{r \cdot \mathbf{w}}{\|\mathbf{w}\|},$$

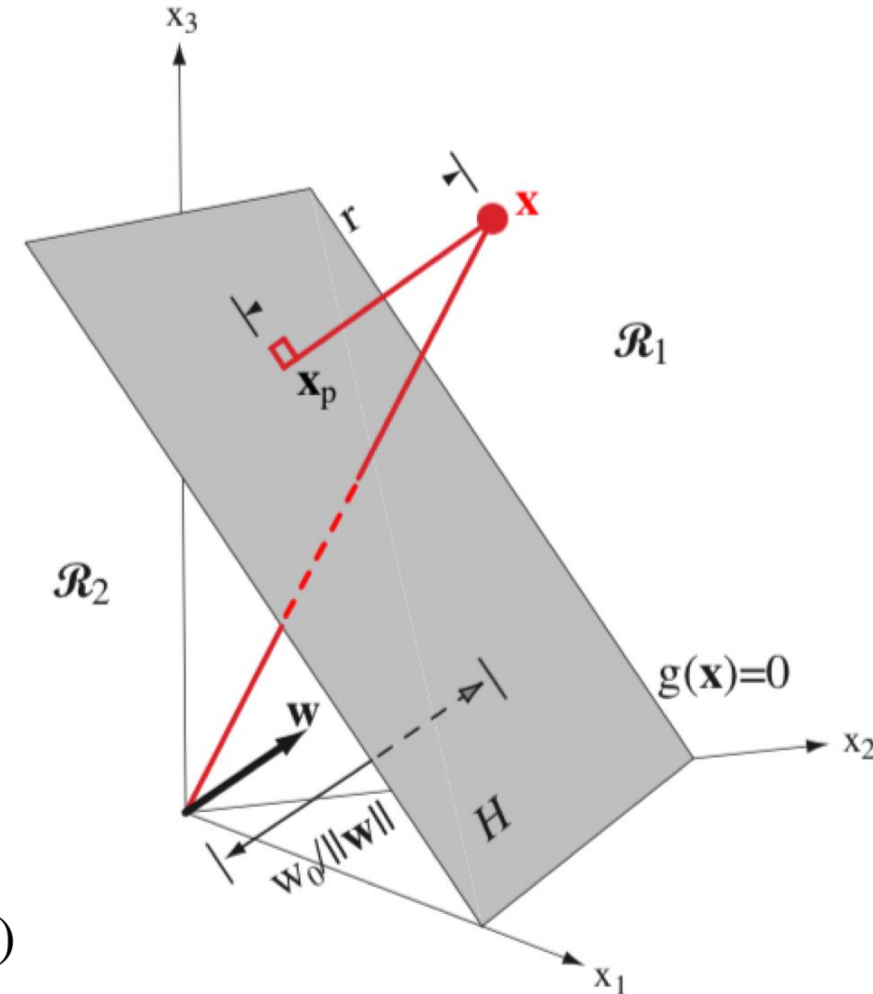
• since **w** is collinear with $\mathbf{x} - \mathbf{x}_p$ and

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = 1$$

$$\bullet g(\mathbf{x}) = \mathbf{w}^t \mathbf{x} + w_0 = r \|\mathbf{w}\|,$$

• since $g(\mathbf{x}_p) = 0$ and $\mathbf{w}^t \cdot \mathbf{w} = \|\mathbf{w}\|^2$

$$\bullet r = \frac{g(\mathbf{x})}{\|\mathbf{w}\|} = d(\mathbf{x}, H)$$



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Perceptron

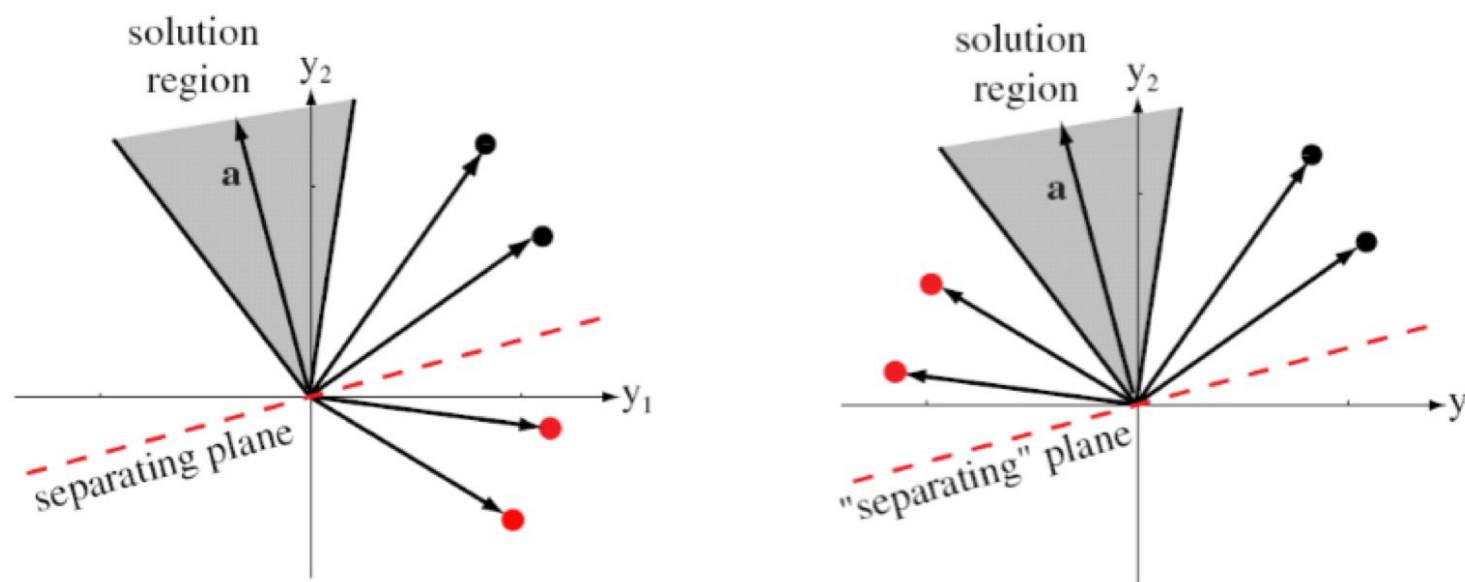
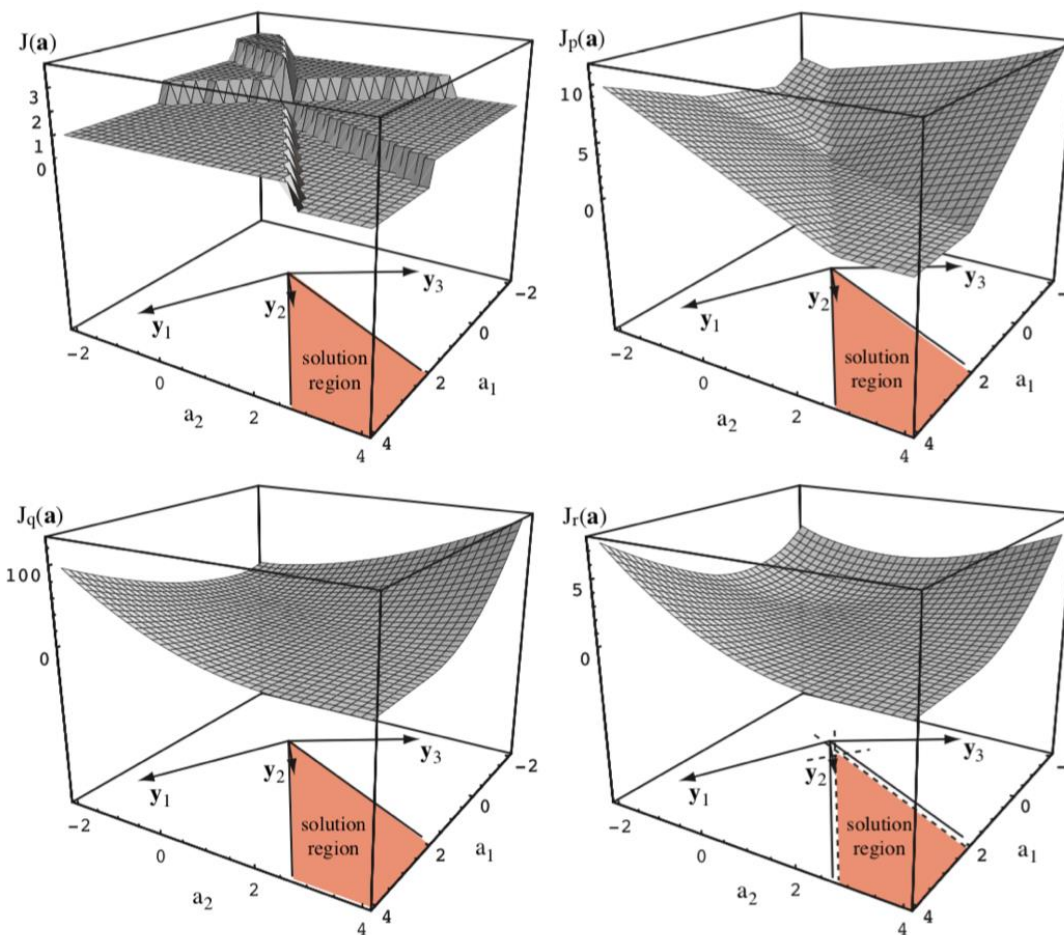


Figure 5.8: Four training samples (black for ω_1 , red for ω_2) and the solution region in feature space. The figure on the left shows the raw data; the solution vectors leads to a plane that separates the patterns from the two categories. In the figure on the right, the red points have been “normalized” — i.e., changed in sign. Now the solution vector leads to a plane that places all “normalized” points on the same side.

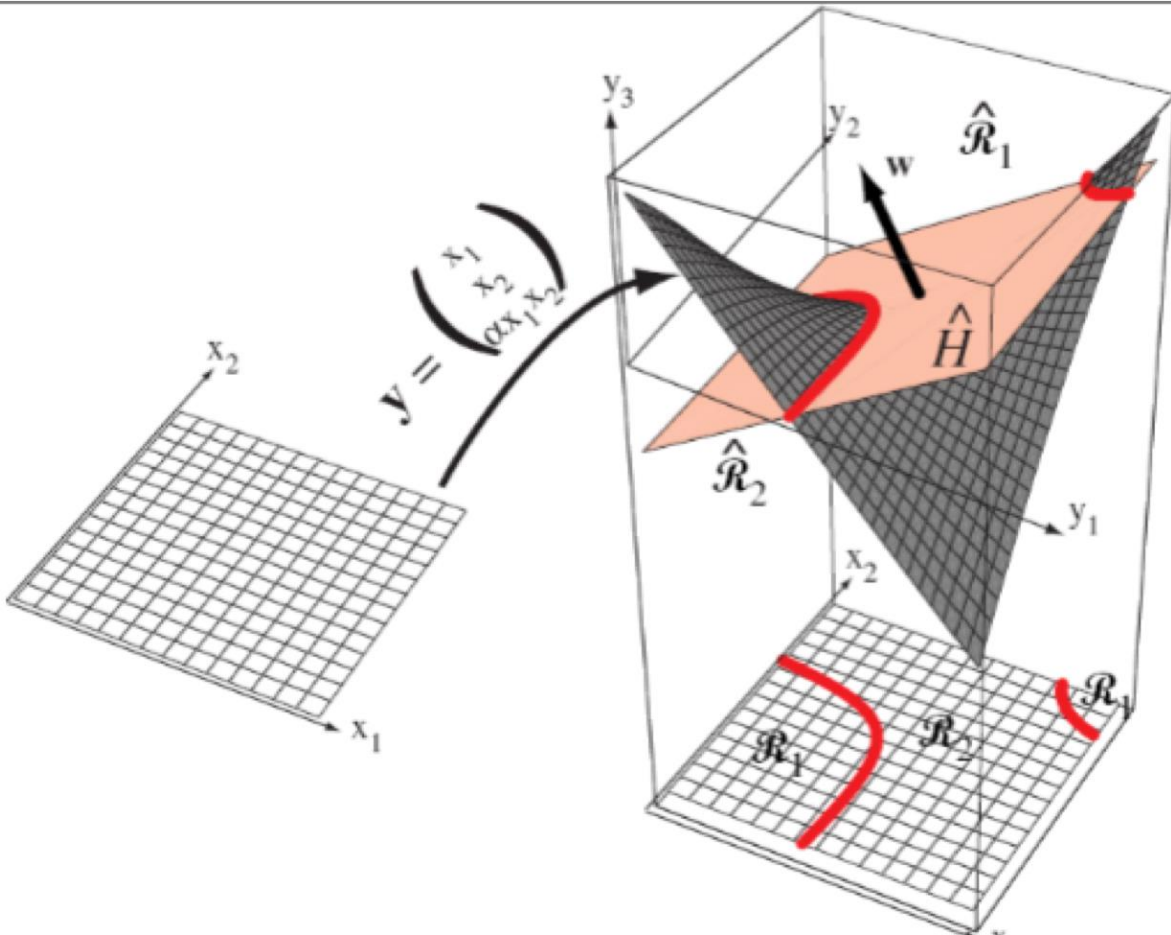
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Perceptron



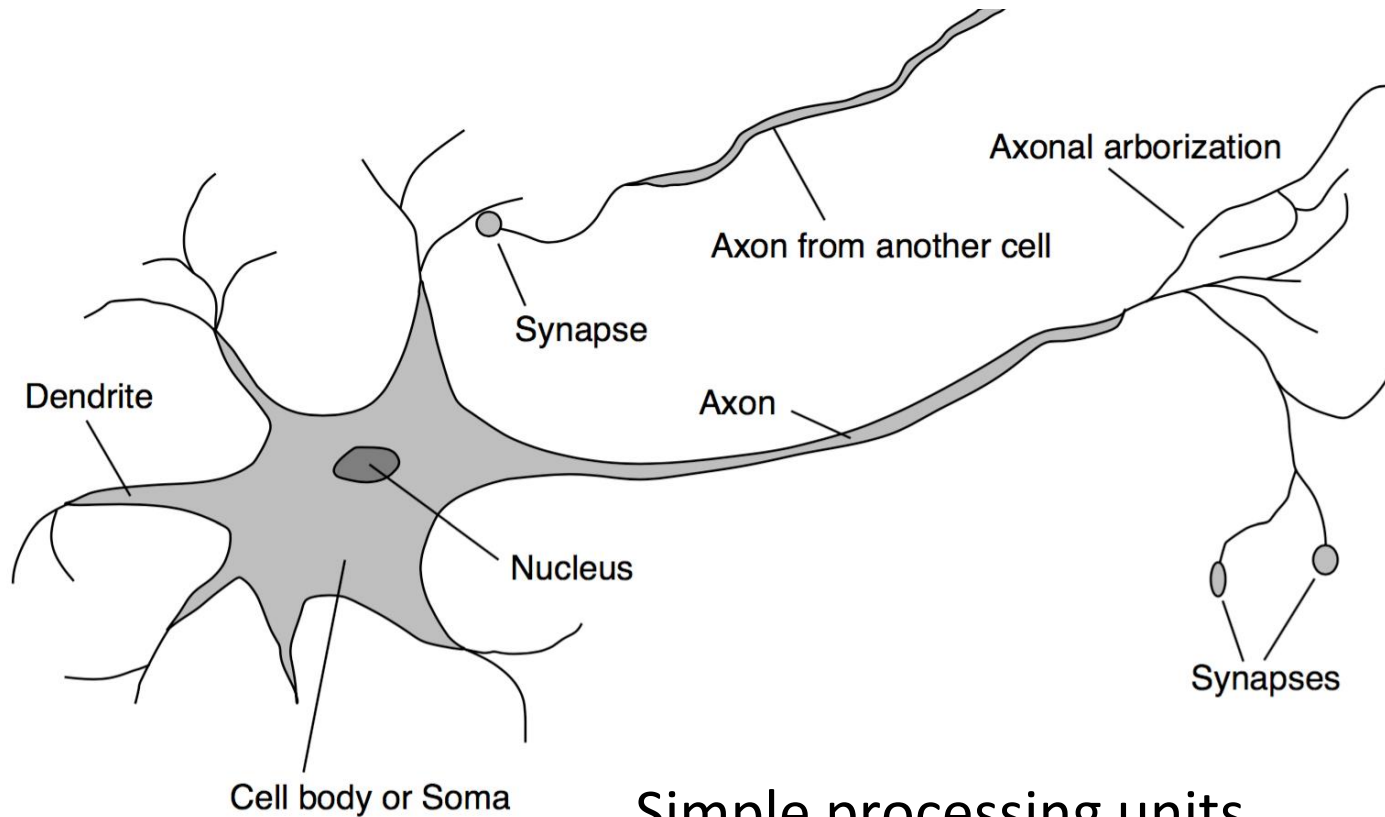
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Perceptron



$$g(\mathbf{x}) = \sum_{i=1}^n w_{ii} \mathbf{x}_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^n w_{ij} \mathbf{x}_i \mathbf{x}_j + \sum_{i=1}^n w_i \mathbf{x}_i + w_{n+1}$$

Unit 01 | Perceptron



Inspired by interconnected neurons
in biological systems

Simple processing units

- Each unit receives a number of real-valued inputs
- Each unit produces a single real-valued output

Unit 01 | Perceptron

Perceptron

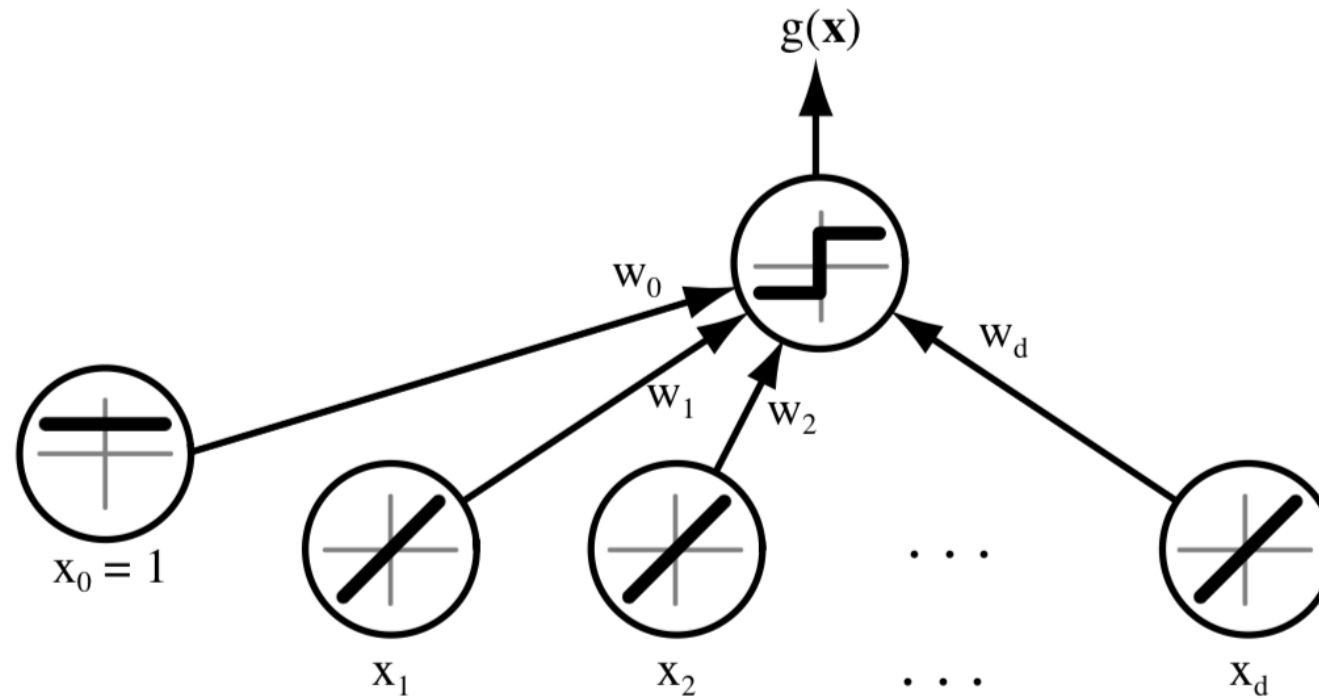
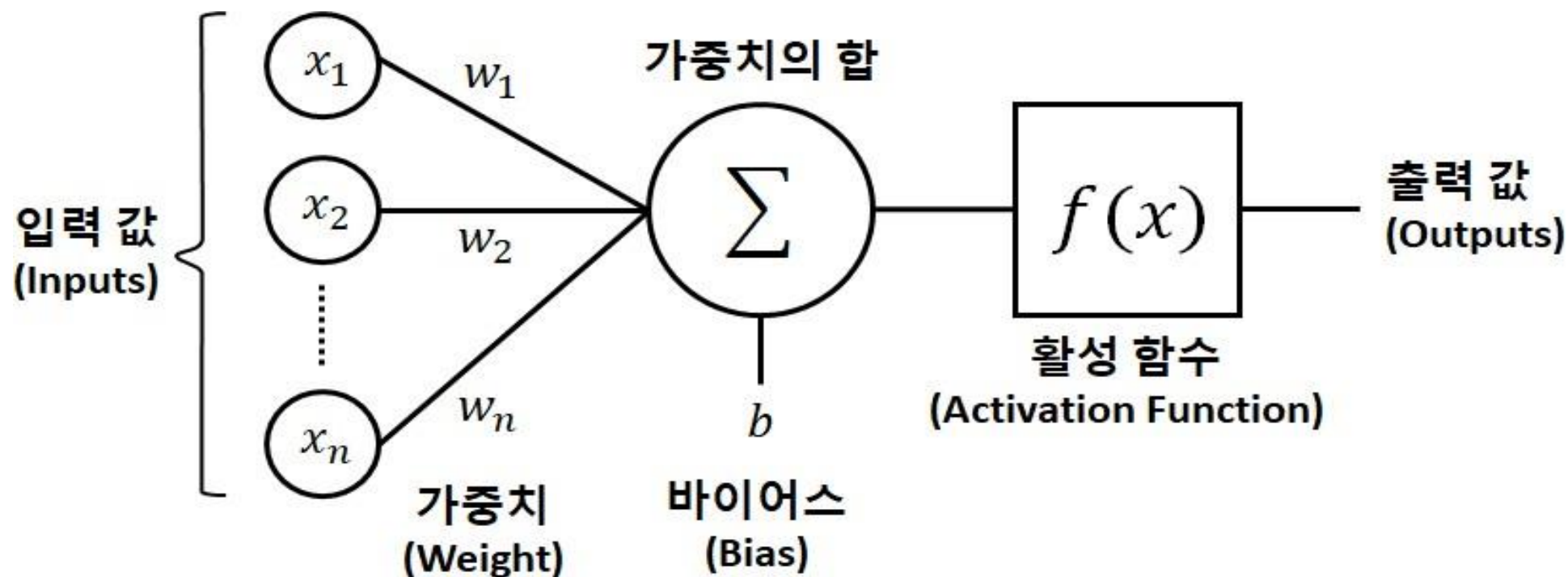


Figure 5.1: A simple linear classifier having d input units, each corresponding to the values of the components of an input vector. Each input feature value x_i is multiplied by its corresponding weight w_i ; the output unit sums all these products and emits a $+1$ if $\mathbf{w}^t \mathbf{x} + w_0 > 0$ or a -1 otherwise.

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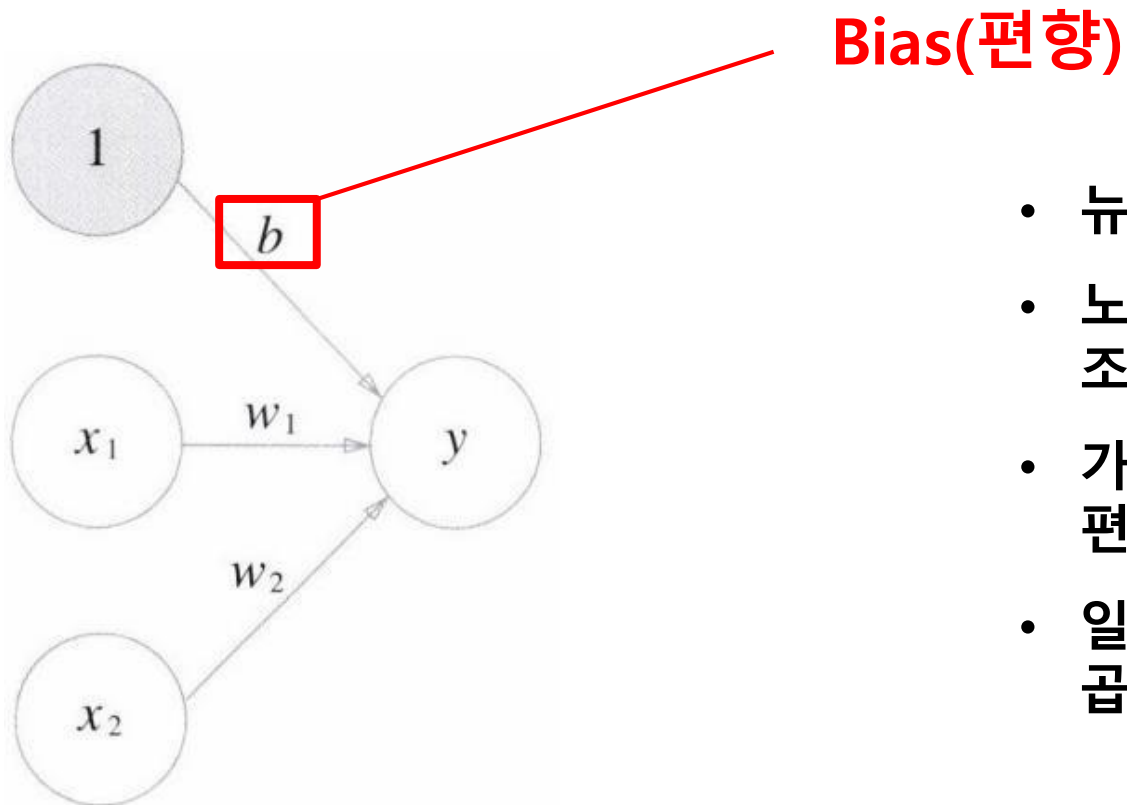
Perceptron



- 가중치(weight) : 각각의 입력에 대해 중요도를 부여 하는 수치

Unit 01 | Perceptron

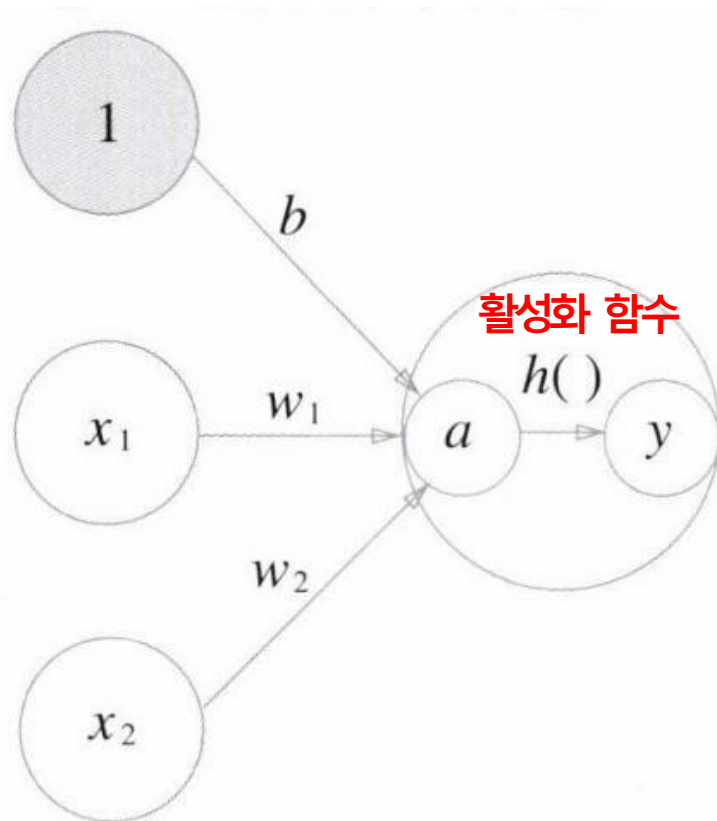
Perceptron



- 뉴런이 얼마나 쉽게 활성화 되느냐를 제어
- 노드의 민감도를 조정하거나 활성화를 조정하는 역할
- 가중치 만으로 세밀한 조정이 되지 않을 시 편향을 주어 조정이 가능하다.
- 일반적으로 입력값을 1로 고정하고 편향 b 를 곱한 변수로 표현한다.

Unit 01 | Perceptron

활성화 함수

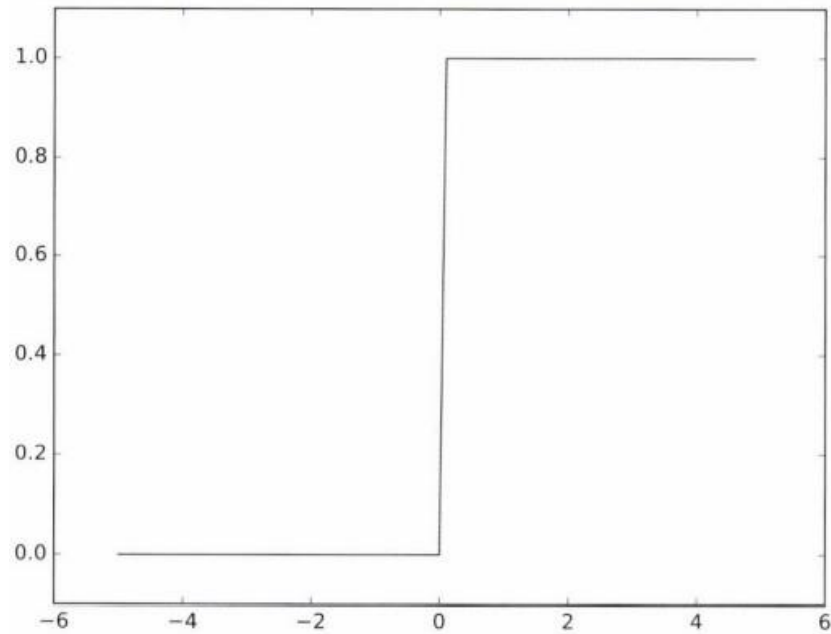


Activation Function (활성화 함수)

- 입력신호의 총합을 출력 신호로 변환하는 함수
- 활성화 함수로는 Step function, Relu, Sigmoid, tanh 등 여러 함수 존재

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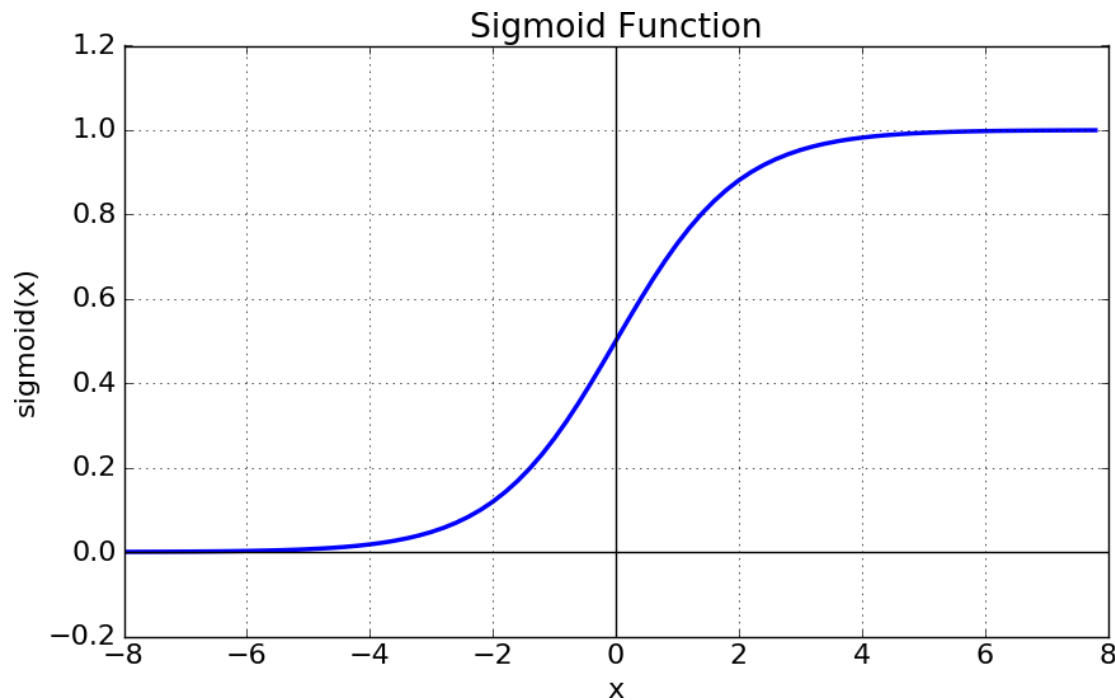
활성화 함수 – Step function



- 출력이 0 또는 1
- 활성화할지 말지 여부만 반환

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활성화 함수 – Sigmoid

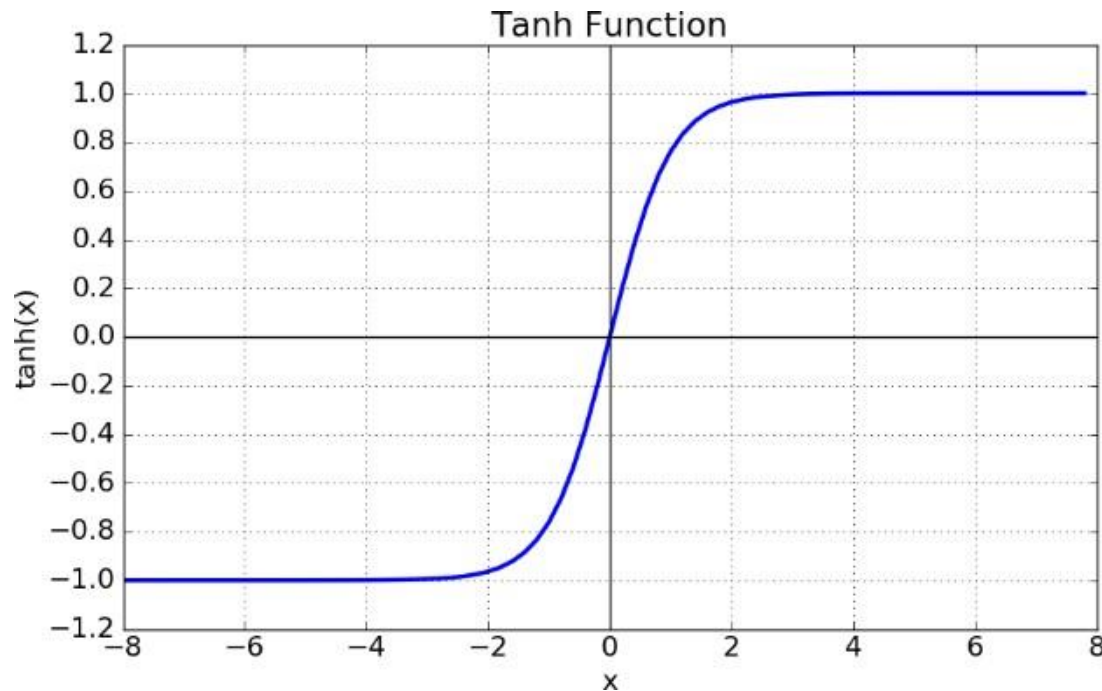


$$h(x) = \frac{1}{1 + \exp(-x)}$$

- 0에서 1 사이의 값 출력
- 활성화 여부가 아닌, 활성화 정도를 반환
- 1에 가까울수록 많이 활성화됐다는 뜻

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활성화 함수 - Tanh



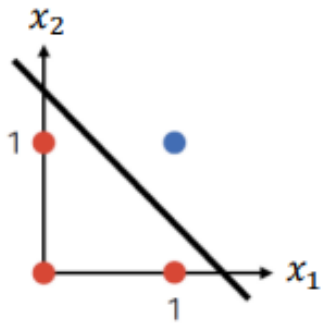
$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- -1에서 1 사이의 값 출력
- 시그모이드 함수보다 범위가 넓어 출력값의 변화폭이 더 크고 이로 인해 기울기 소실 증상이 적음.

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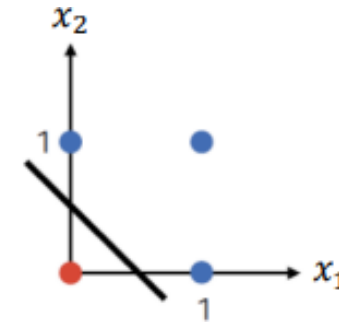
Perceptron 연산

AND



x_1	x_2	y
1	1	1
1	0	0
0	1	0
0	0	0

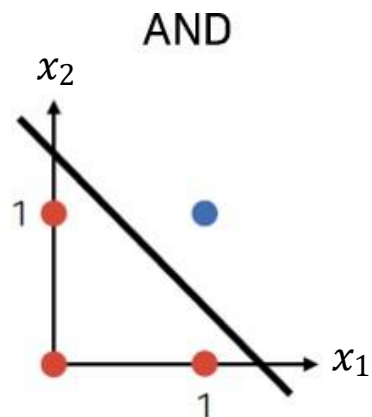
OR



x_1	x_2	y
1	1	1
1	0	1
0	1	1
0	0	0

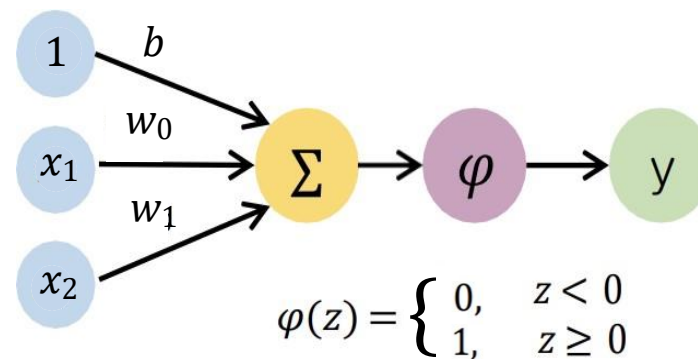
Unit 01 | Perceptron

Perceptron 연산



$$w_0 = 1.0, w_1 = 1.0, b = -1.5$$

x_1	x_2	S	y
0	0	-1.5	0
0	1	-0.5	0
1	0	-0.5	0
1	1	0.5	1



$$\varphi(w_1x_1 + w_2x_2 + w_3) = y$$

- ① $(1.0 \times 0) + (1.0 \times 0) + (-1.5) = -1.5$
 $\varphi((1.0 \times 0) + (1.0 \times 0) + (-1.5)) = 0$
- ② $(1.0 \times 0) + (1.0 \times 1) + (-1.5) = -0.5$
 $\varphi((1.0 \times 0) + (1.0 \times 1) + (-1.5)) = 0$
- ③ $(1.0 \times 1) + (1.0 \times 0) + (-1.5) = -0.5$
 $\varphi((1.0 \times 1) + (1.0 \times 0) + (-1.5)) = 0$
- ④ $(1.0 \times 1) + (1.0 \times 1) + (-1.5) = 0.5$
 $\varphi((1.0 \times 1) + (1.0 \times 1) + (-1.5)) = 1$

Unit 01 | Perceptron

Perceptron convergence theorem

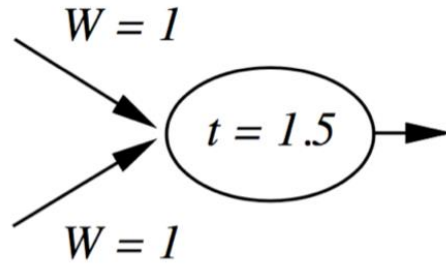
- If training samples are linearly separable, then the sequence of weight vectors given by Algorithm 4 (Fixed-increment single-sample Perceptron) will terminate at a solution vector

Algorithm 4 (Fixed-increment single-sample Perceptron)

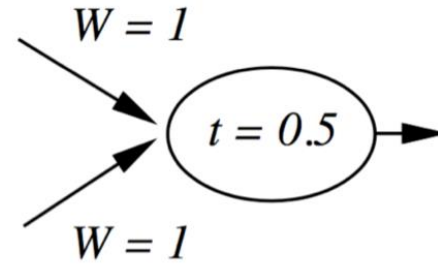
```
1 begin initialize  $a, k = 0$   
2       do  $k \leftarrow (k + 1) \bmod n$   
3       if  $y_k$  is misclassified by  $a$  then  $a \leftarrow a + y_k$   
4       until all patterns properly classified  
5   return  $a$   
6 end
```

Unit 01 | Perceptron

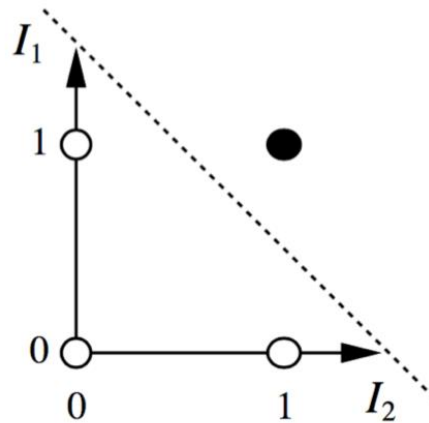
Linearly separable



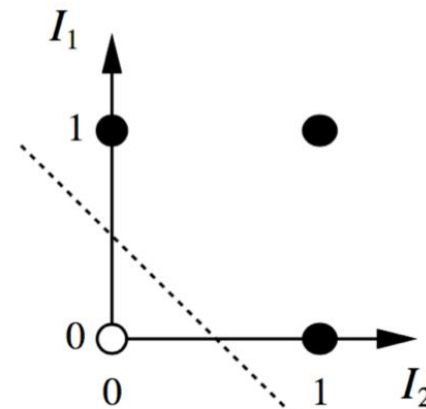
AND



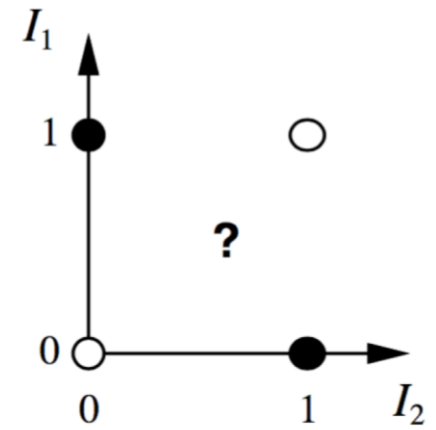
OR



(a) I_1 and I_2



(b) I_1 or I_2

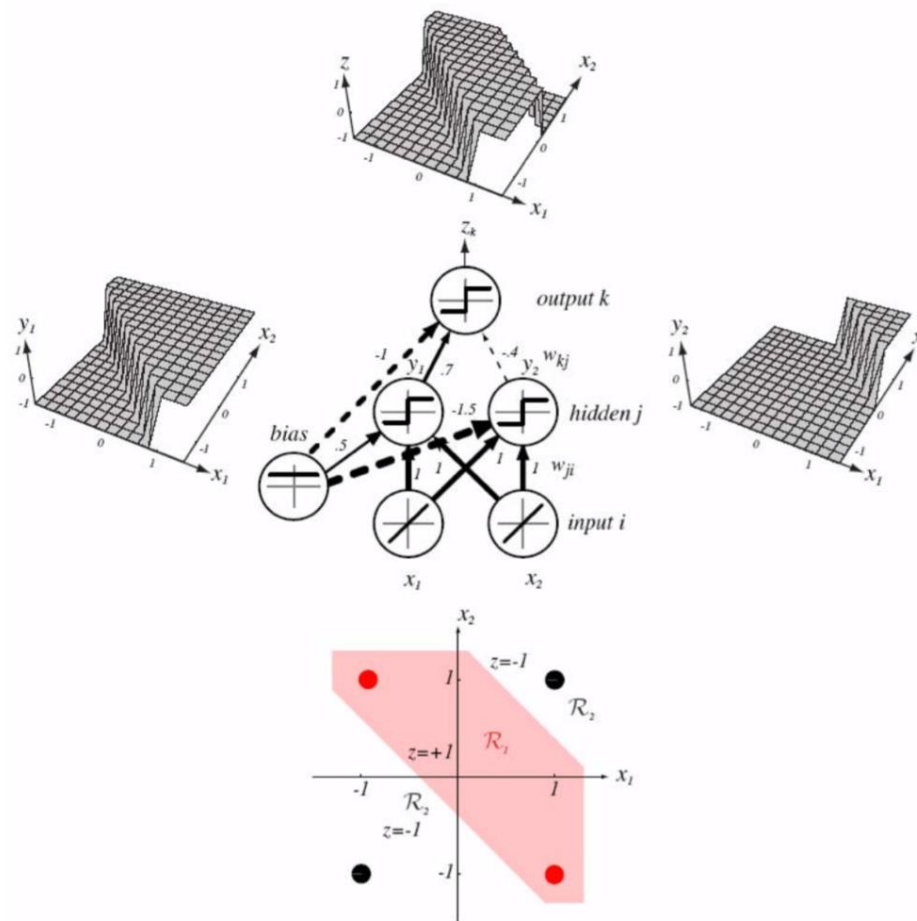


(c) I_1 xor I_2

Unit 01 | Perceptron

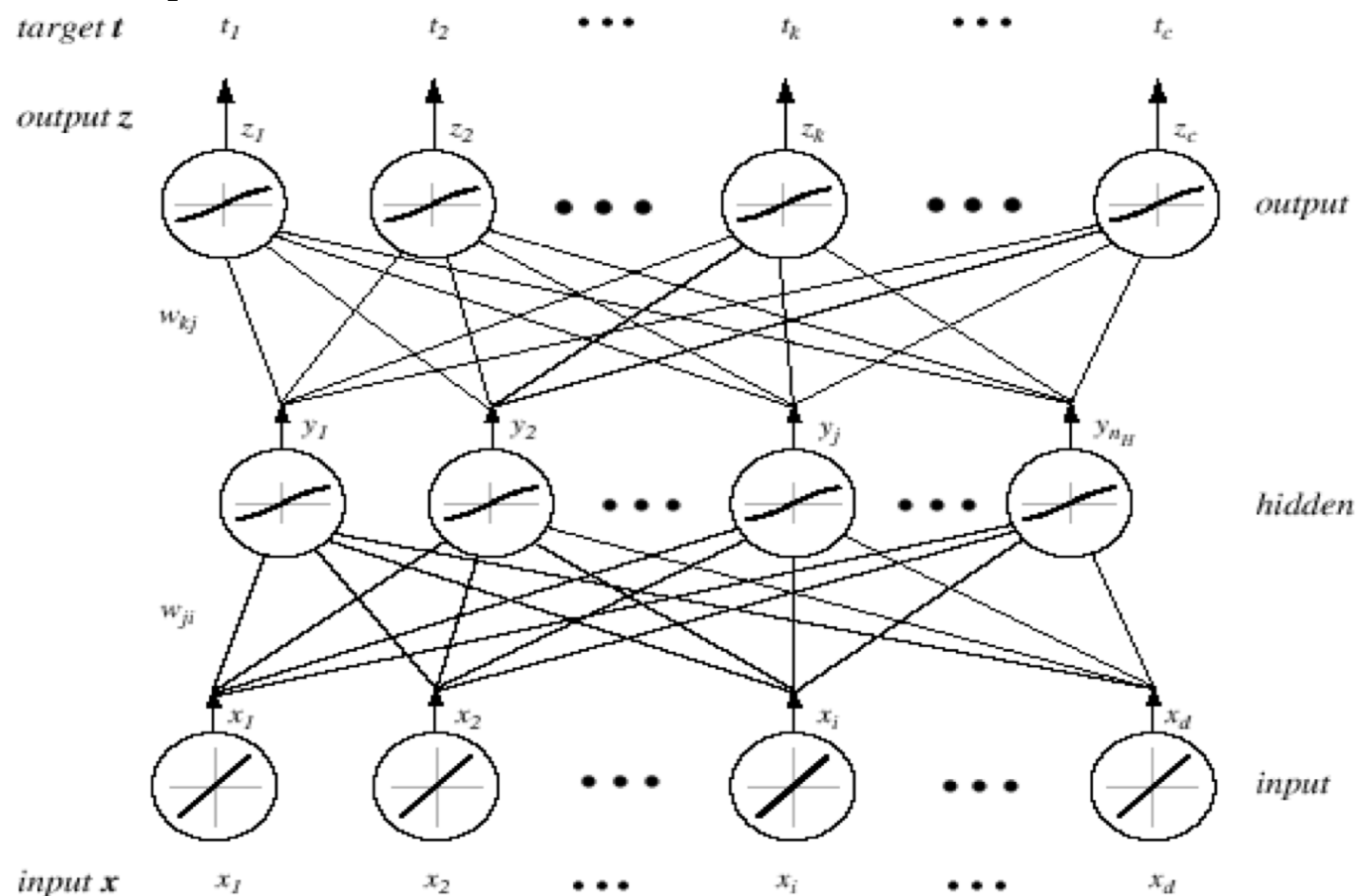
Linearly separable

Input (x)	Hidden (y)	Output (z)
(1,1)	(1,1)	-1
(1,-1)	(1,-1)	1
(-1,1)	(1,-1)	1
(-1,-1)	(-1,-1)	-1



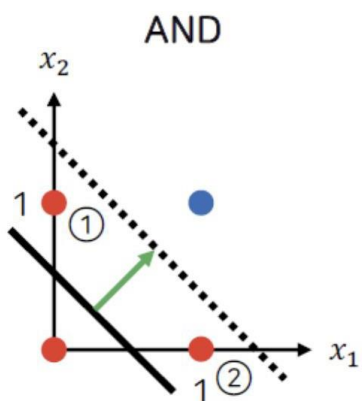
Unit 01 | Perceptron

Multi-layer perceptron



Unit 01 | Perceptron

Perceptron 학습



$$w_1 = 0.55, w_2 = 0.55, b = -0.65$$

x_1	x_2	o	y
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

①

②

$$w_i \leftarrow w_i + \eta(y - o)x_i \quad \eta = 0.05$$

$$\begin{aligned} b &\leftarrow b + 0.05(0 - 1) \times 1 \\ \textcircled{1} \quad w_1 &\leftarrow w_1 + 0.05(0 - 1) \times 0 \\ w_2 &\leftarrow w_2 + 0.05(0 - 1) \times 1 \end{aligned}$$

$$\begin{aligned} b &\leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7 \\ w_1 &\leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55 \\ w_2 &\leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5 \end{aligned}$$

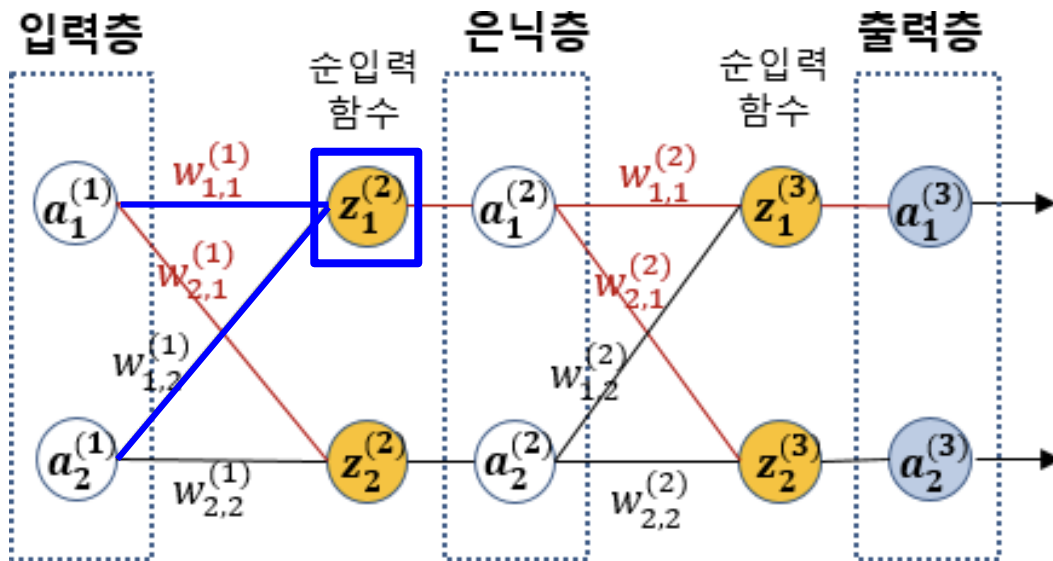
$$\begin{aligned} b &\leftarrow b + 0.05(0 - 1) \times 1 \\ \textcircled{2} \quad w_1 &\leftarrow w_1 + 0.05(0 - 1) \times 1 \\ w_2 &\leftarrow w_2 + 0.05(0 - 1) \times 0 \end{aligned}$$

$$\begin{aligned} b &\leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7 \\ w_1 &\leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5 \\ w_2 &\leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55 \end{aligned}$$

02 | Feedforward & Backpropagation

Unit 02 | Backpropagation

순전파(Feedforward)



$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

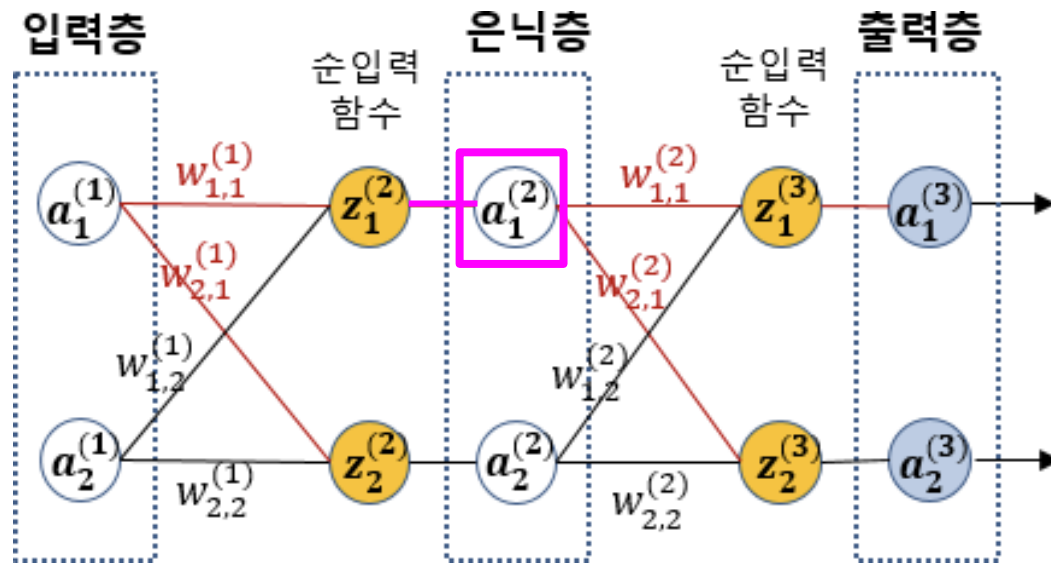
$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$

Unit 02 | Backpropagation

순전파(Feedforward)



$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

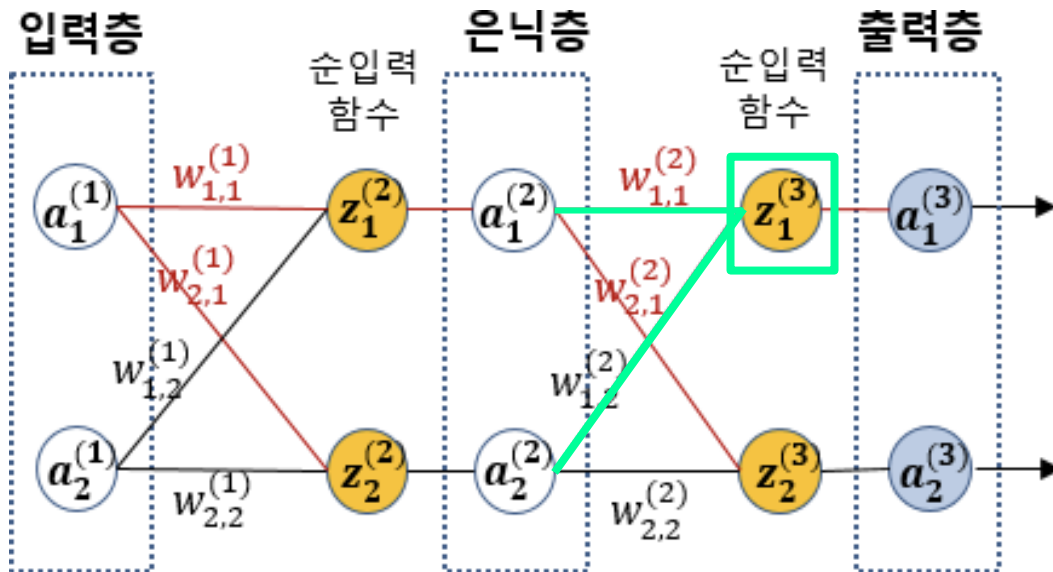
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$$a_1^{(3)} = \phi(z_1^{(3)})$$

Unit 02 | Backpropagation

순전파(Feedforward)



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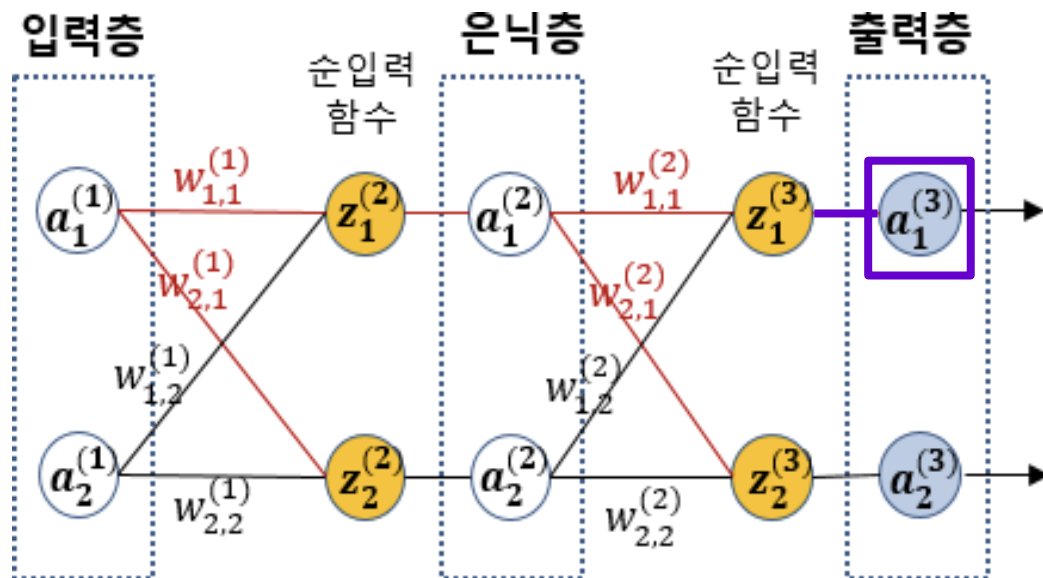
$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$

Unit 02 | Backpropagation

순전파(Feedforward)



$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

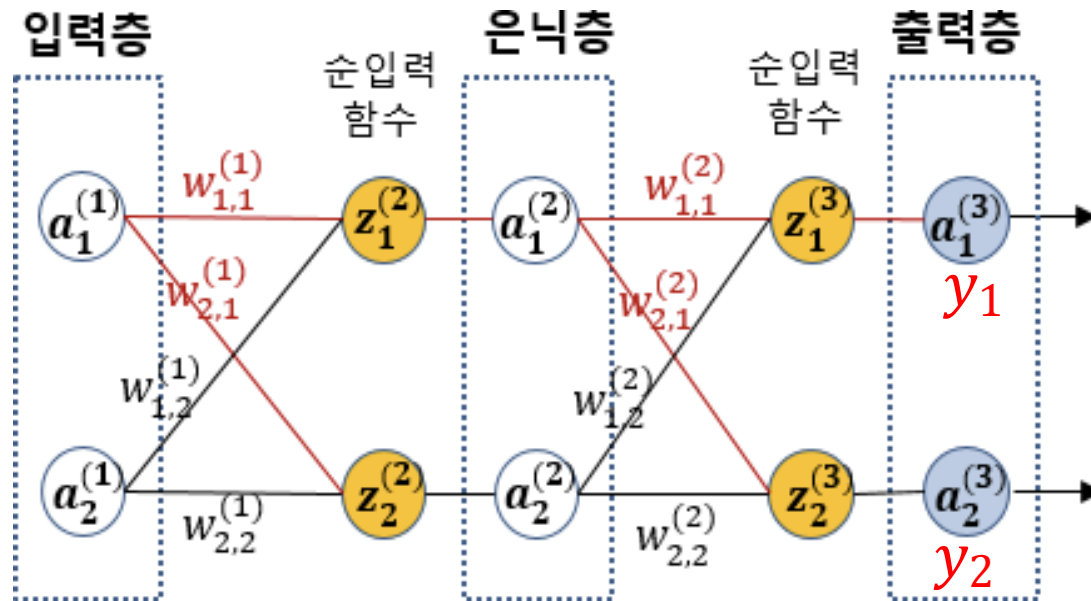
$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$

Unit 02 | Backpropagation

손실함수(Cost Function)



$$\text{MSE} = \frac{1}{2N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

$$J_1 = \frac{1}{2} (a_1^{(3)} - y_1)^2$$

$$J_2 = \frac{1}{2} (a_2^{(3)} - y_2)^2$$

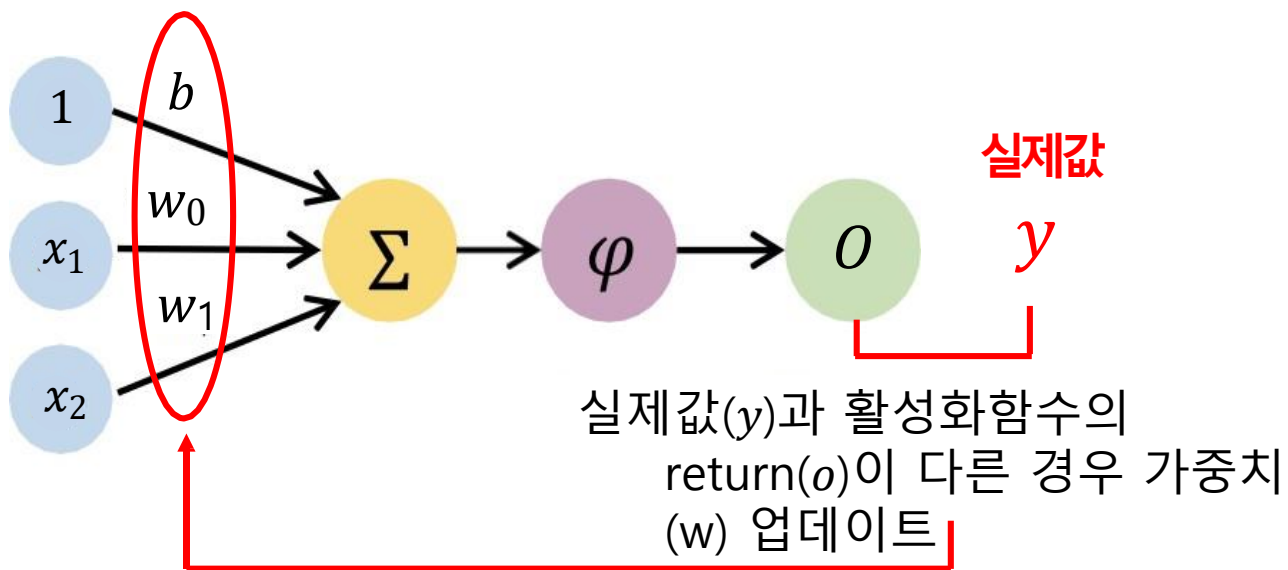
Unit 02 | Backpropagation

역전파(Backpropagation)

순전파(Feedforward) 알고리즘에서 발생한 오차를 줄이기 위해
새로운 가중치를 업데이트하고, 새로운 가중치로 다시 학습하는 과정

Unit 02 | Backpropagation

역전파(Backpropagation)



가중치 조정 식

$$w_i \leftarrow w_i + \eta(y - o)x_i$$



학습률(learning rate)

너무 작으면 학습 속도가 매우 느리고 너무 크면 가중치를 미세하게 조정하지 못하기 때문에 최적의 가중치를 찾기 어려움

Unit 02 | Backpropagation

편미분

다변수함수의 특정 변수를 제외한 나머지 변수를 상수로 생각하여 미분

$$z = f(x, y) = x^2 + xy + y^2$$

$$\frac{\partial z}{\partial x} = 2x + y, \quad \frac{\partial z}{\partial y} = 2y + x$$

$$\Delta f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (2x + y, 2y + x)$$

Unit 02 | Backpropagation

Chain Rule

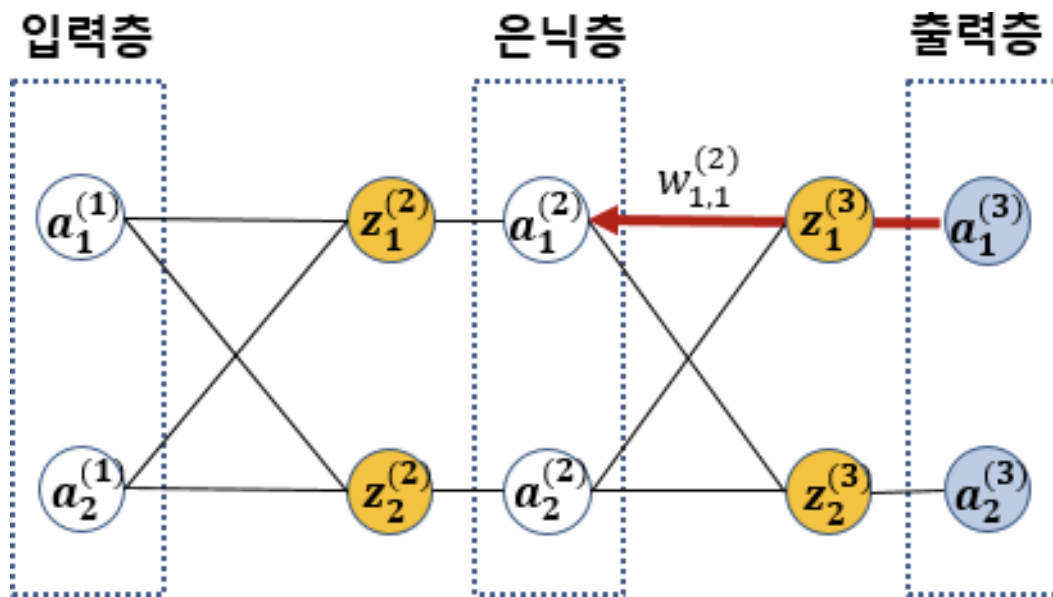
연쇄 법칙, 합성 함수를 미분할 때의 계산 공식

$$f(g(x))' = f'(g(x))g'(x)$$

$$y = f(u), u = g(x) \text{ 일 때, } \frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x} \text{ 성립}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

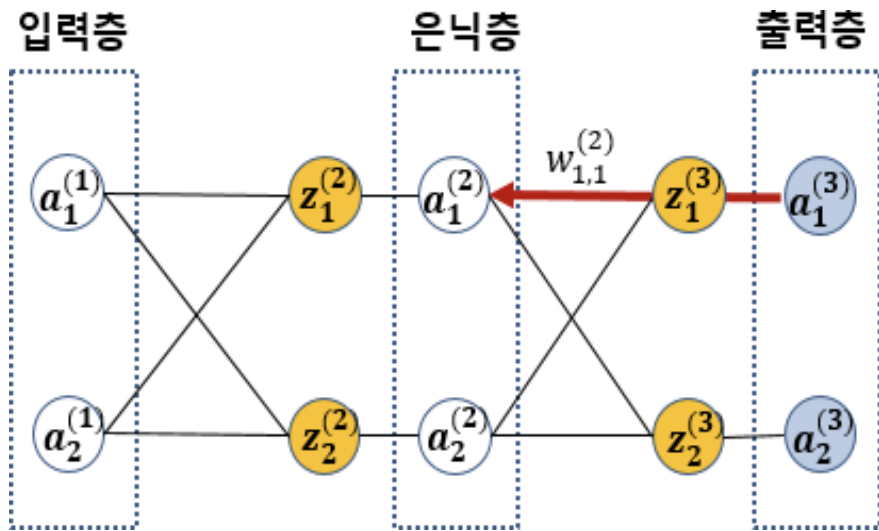
$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}}$$

역전파의 출발노드인 $a_1^{(3)}$ 의 J_{total} 은 J_1

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



손실함수

$$J_1$$

$$J_2$$

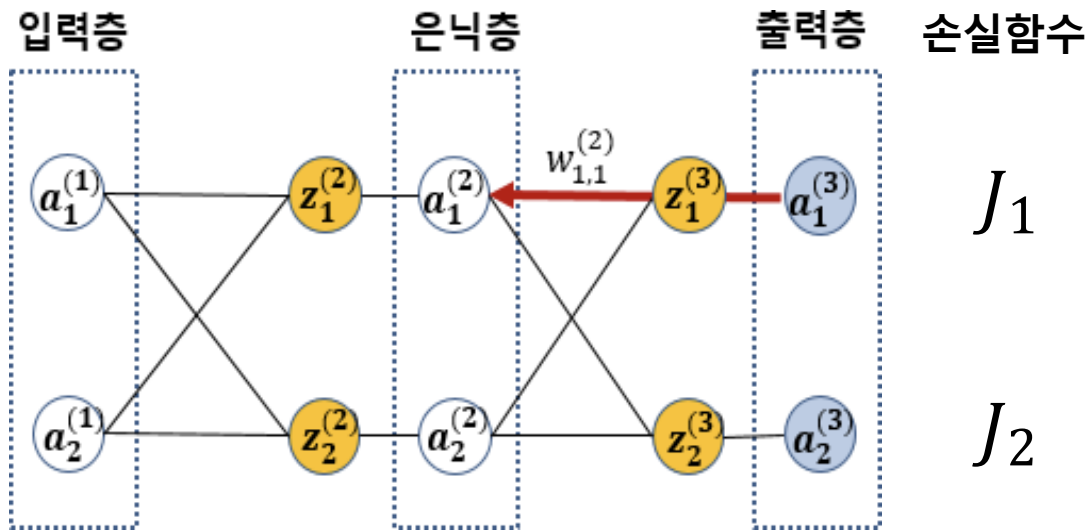
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \underbrace{-\frac{\partial J_1}{\partial a_1^{(3)}}}_{\textcircled{1}} \times \underbrace{\frac{\partial a_1^{(3)}}{\partial z_1^{(3)}}}_{\textcircled{2}} \times \underbrace{\frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}}_{\textcircled{3}}$$

참고: $J_1 = \frac{1}{2} (a_1^{(3)} - y_1)^2$

$$\textcircled{1} \quad \frac{\partial J_1}{\partial a_1^{(3)}} = \frac{1}{2} \frac{\partial}{\partial a_1^{(3)}} (a_1^{(3)} - y_1)^2 = (a_1^{(3)} - y_1)$$

Unit 02 | Backpropagation

역전파(Backpropagation)



$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \underbrace{-\frac{\partial J_1}{\partial a_1^{(3)}}}_{\textcircled{1}} \times \underbrace{\frac{\partial a_1^{(3)}}{\partial z_1^{(3)}}}_{\textcircled{2}} \times \underbrace{\frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}}_{\textcircled{3}}$$

참고: $a_1^{(3)} = \phi(z_1^{(3)})$ $\sigma'(x) = \frac{\partial}{\partial x} \frac{1}{1+e^{-x}}$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

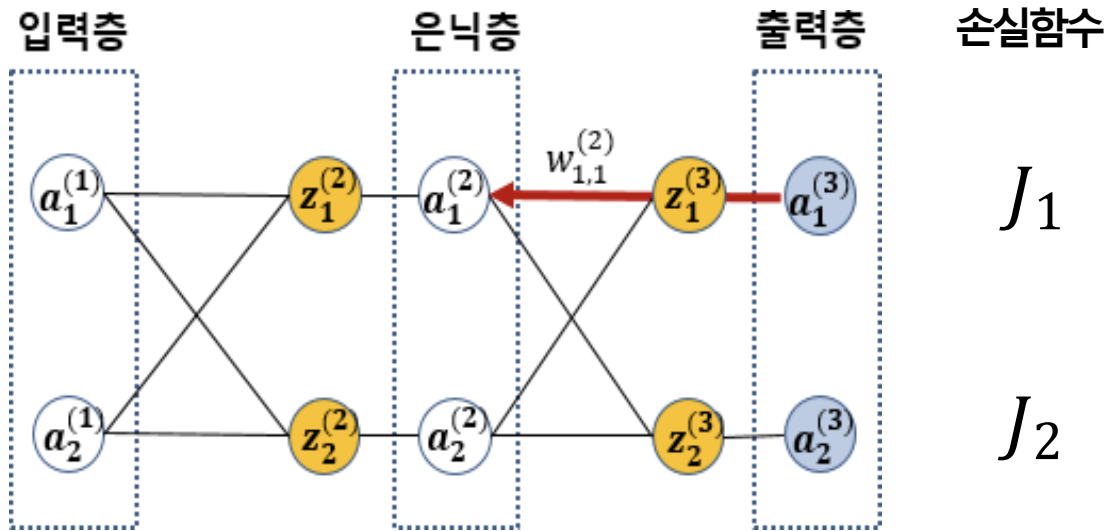
$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \sigma(x)(1 - \sigma(x))$$

② $\frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} = \phi(z_1^{(3)}) (1 - \phi(z_1^{(3)})) = a_1^{(3)} (1 - a_1^{(3)})$

Unit 02 | Backpropagation

역전파(Backpropagation)



$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \underbrace{-\frac{\partial J_1}{\partial a_1^{(3)}}}_{\textcircled{1}} \times \underbrace{\frac{\partial a_1^{(3)}}{\partial z_1^{(3)}}}_{\textcircled{2}} \times \underbrace{\frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}}_{\textcircled{3}}$$

참고: $z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$

$\textcircled{3} \quad \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}} = a_1^{(2)}$

Unit 02 | Backpropagation

역전파(Backpropagation)

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \underbrace{\frac{\partial J_1}{\partial a_1^{(3)}}}_{\textcircled{1}} \times \underbrace{\frac{\partial a_1^{(3)}}{\partial z_1^{(3)}}}_{\textcircled{2}} \times \underbrace{\frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}}_{\textcircled{3}} = \underbrace{(a_1^{(3)} - y_1)}_{\textcircled{1}} \times \underbrace{a_1^{(3)}(1 - a_1^{(3)})}_{\textcircled{2}} \times \underbrace{a_1^{(2)}}_{\textcircled{3}}$$

$$\delta_1^{(3)} = \frac{\partial J_1}{\partial z_1^{(3)}} = (a_1^{(3)} - y_1) \times a_1^{(3)}(1 - a_1^{(3)})$$

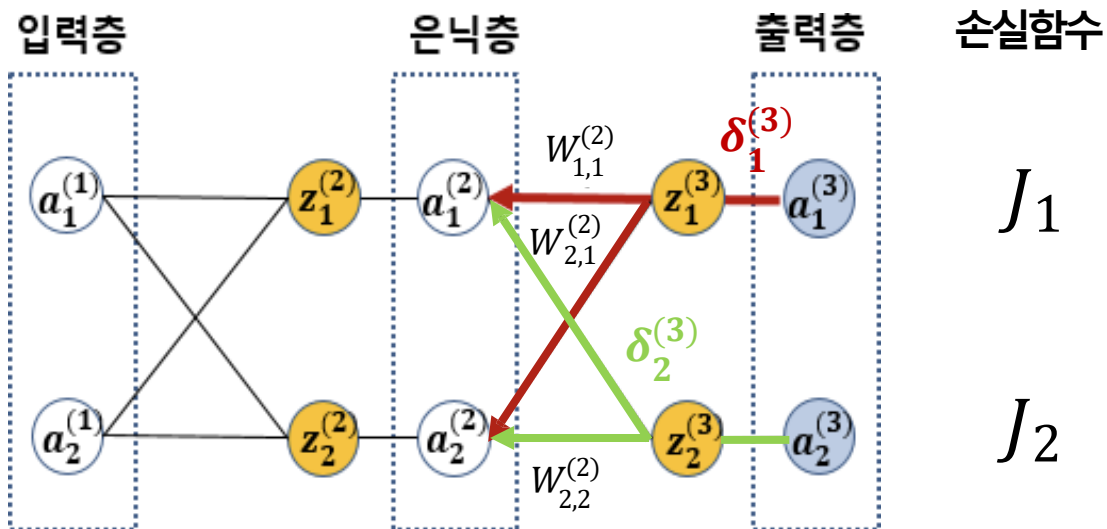
$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \boxed{w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)

$$\delta_1^{(3)} = -\frac{\partial J_1}{\partial z_1^{(3)}} = (a_1^{(3)} - y_1) \times a_1^{(3)}(1 - a_1^{(3)})$$

$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$



같은 방식으로

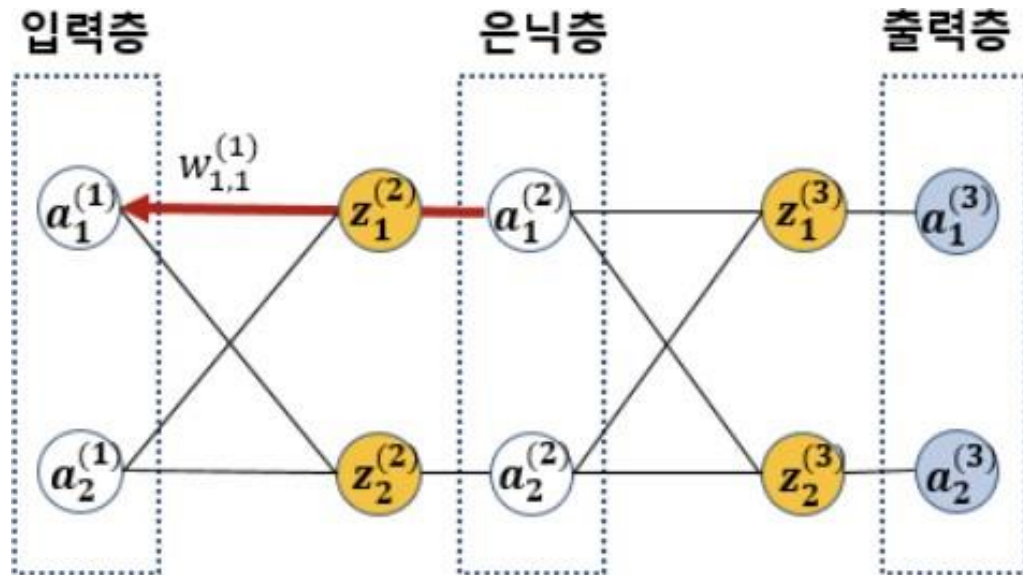
$$\delta_2^{(3)} = \frac{\partial J_2}{\partial z_2^{(3)}} = (a_2^{(3)} - y_2) \times a_2^{(3)}(1 - a_2^{(3)})$$

$$w_{2,1}^{(2)} = w_{2,1}^{(2)} - \delta_2^{(3)} a_1^{(2)}$$

$$w_{2,2}^{(2)} = w_{2,2}^{(2)} - \delta_2^{(3)} a_2^{(2)}$$

Unit 02 | Backpropagation

역전파(Backpropagation)

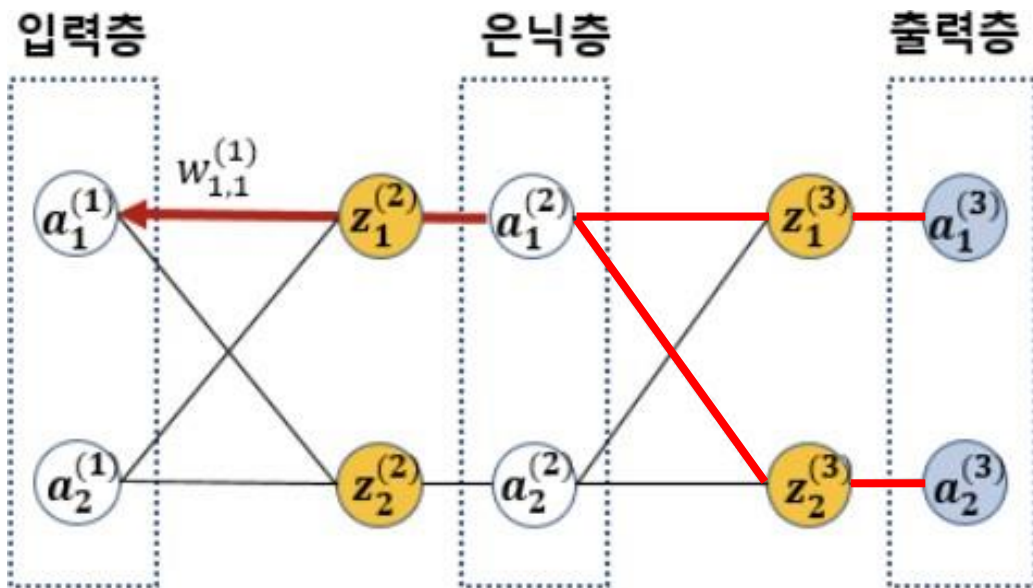


$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



손실함수

J_1

J_2

$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

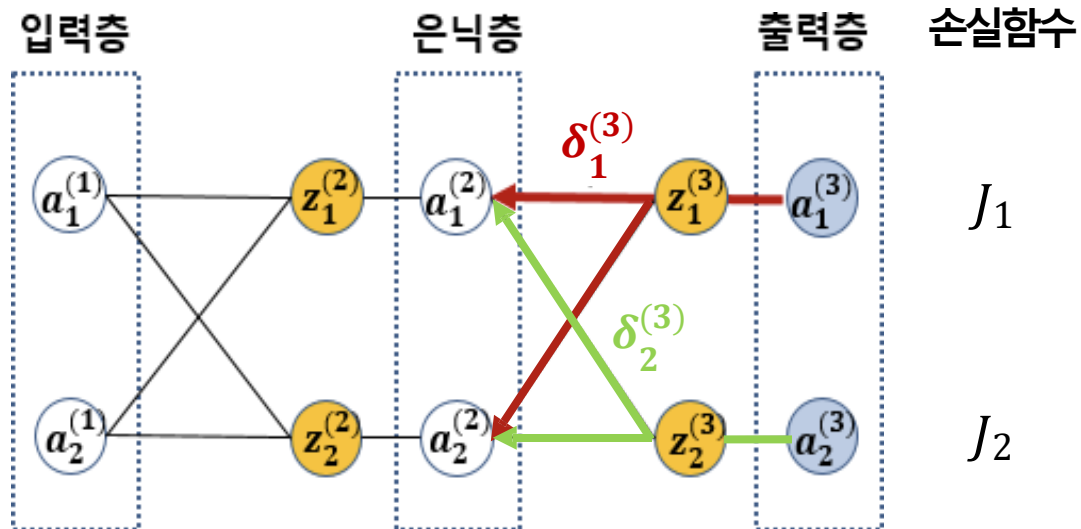
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$a_1^{(2)}$ 의 J_{total} 은 $J_1 + J_2$

$$\frac{\partial J_{total}}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_1^{(2)}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

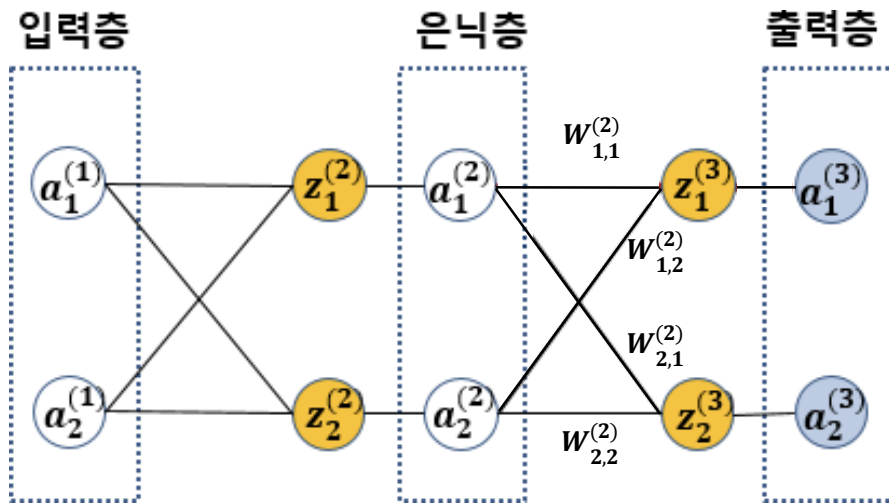
$$\delta_1^{(3)} = \frac{\partial J_1}{\partial z_1^{(3)}} = (a_1^{(3)} - y_1) \times a_1^{(3)} (1 - a_1^{(3)})$$

$$\delta_2^{(3)} = \frac{\partial J_2}{\partial z_2^{(3)}} = (a_2^{(3)} - y_2) \times a_2^{(3)} (1 - a_2^{(3)})$$

$$\begin{aligned} \frac{\partial J_{total}}{\partial a_1^{(2)}} &= \frac{\partial J_1}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_1^{(2)}} \\ &= \delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)} \end{aligned}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

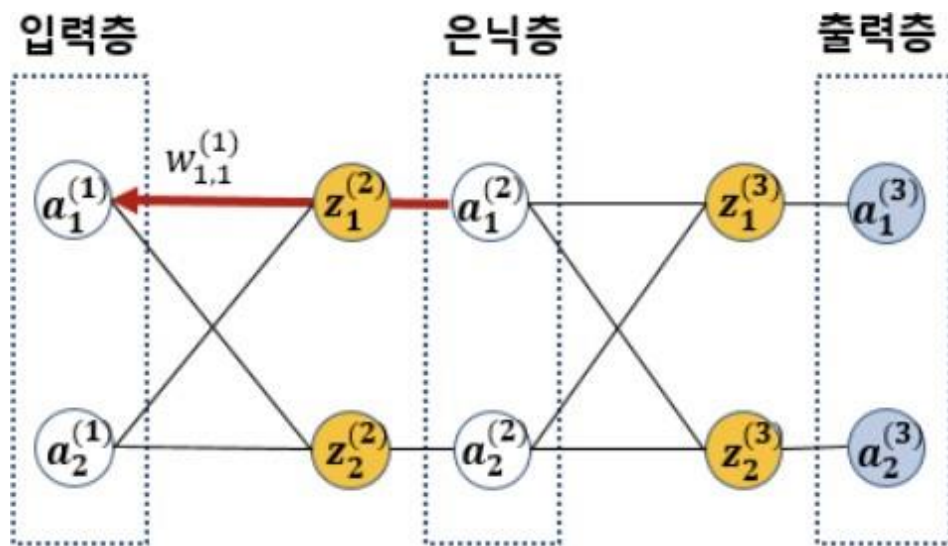
$$z_2^{(3)} = w_{2,1}^{(2)} a_1^{(2)} + w_{2,2}^{(2)} a_2^{(2)}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\begin{aligned} \frac{\partial J_{total}}{\partial a_1^{(2)}} &= \frac{\partial J_1}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_1^{(2)}} \\ &= \delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)} \end{aligned}$$

Unit 02 | Backpropagation

역전파(Backpropagation)

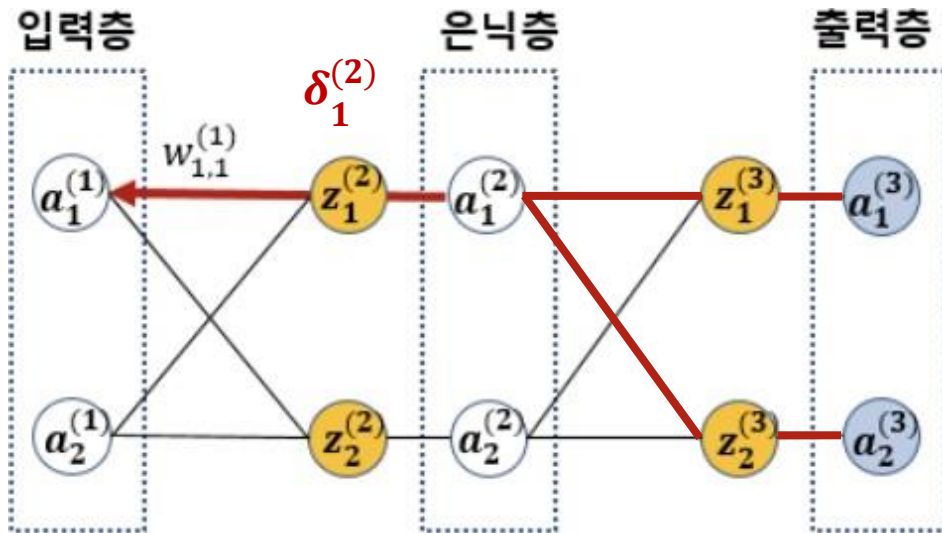


$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \underbrace{(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)})}_{\text{Hidden Layer Error}} \times \underbrace{a_1^{(2)} (1 - a_1^{(2)})}_{\text{Activation Function Derivative}} \times \underbrace{a_1^{(1)}}_{\text{Input Value}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)

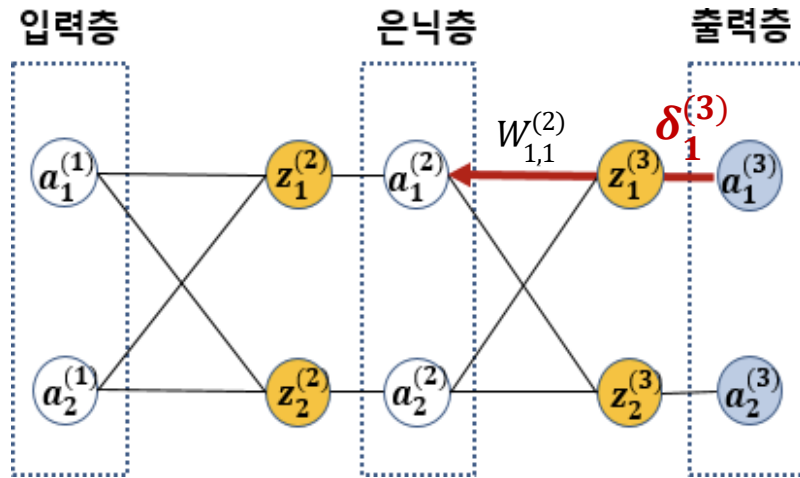


$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)} \right) \times a_1^{(2)} (1 - a_1^{(2)}) \times a_1^{(1)}}{\delta_1^{(2)} = \left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)} \right) \times a_1^{(2)} (1 - a_1^{(2)})}$$

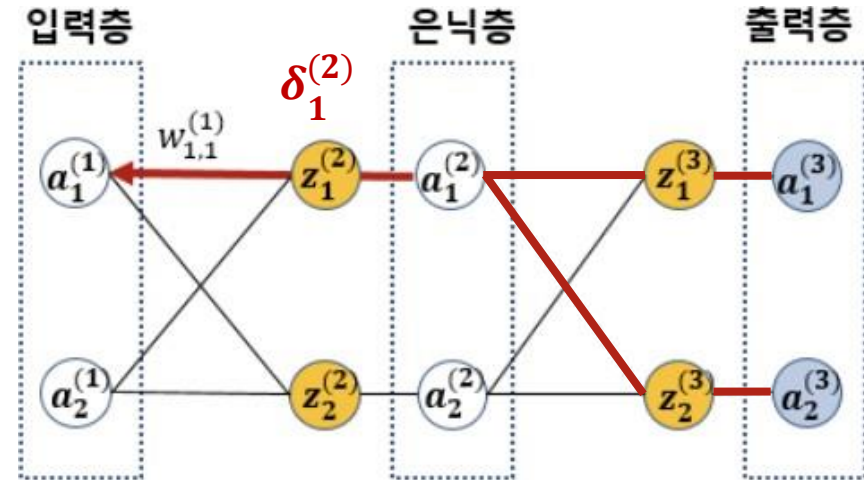
$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \boxed{w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}}$$

Unit 02 | Backpropagation

역전파(Backpropagation)



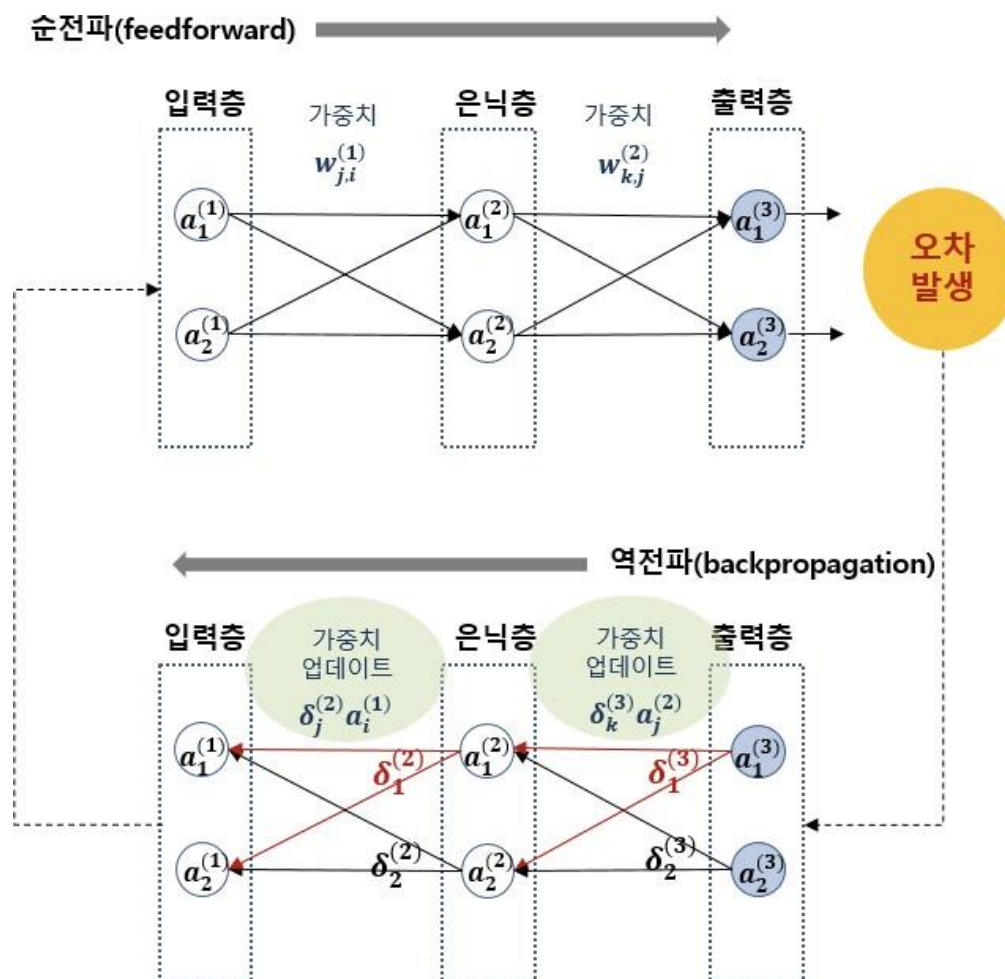
$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$



$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}$$

Unit 02 | Backpropagation

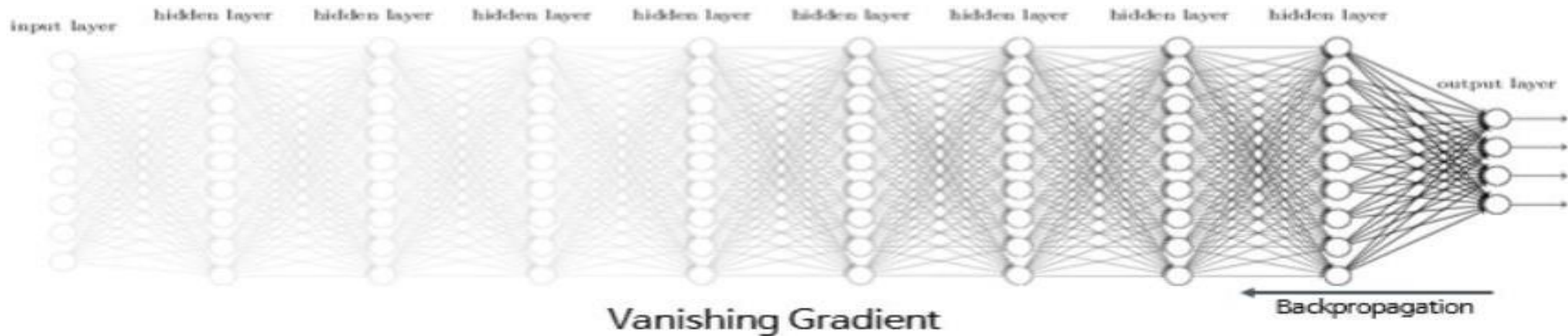
역전파 (Backpropagation)



Unit 02 | Backpropagation

Vanishing Gradient Problem

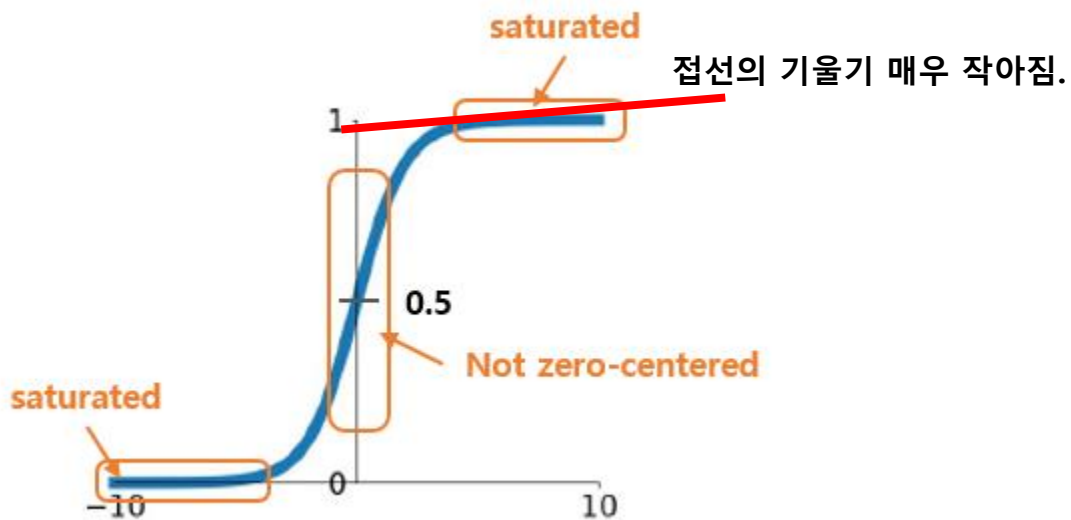
깊이가 깊은 심층신경망에서는 역전파 알고리즘이 입력층으로 전달됨에 따라
그래디언트가 점점 작아져 결국 가중치 매개변수가 업데이트 되지 않는 경우가 발생



Unit 02 | Backpropagation

Vanishing Gradient Problem - sigmoid

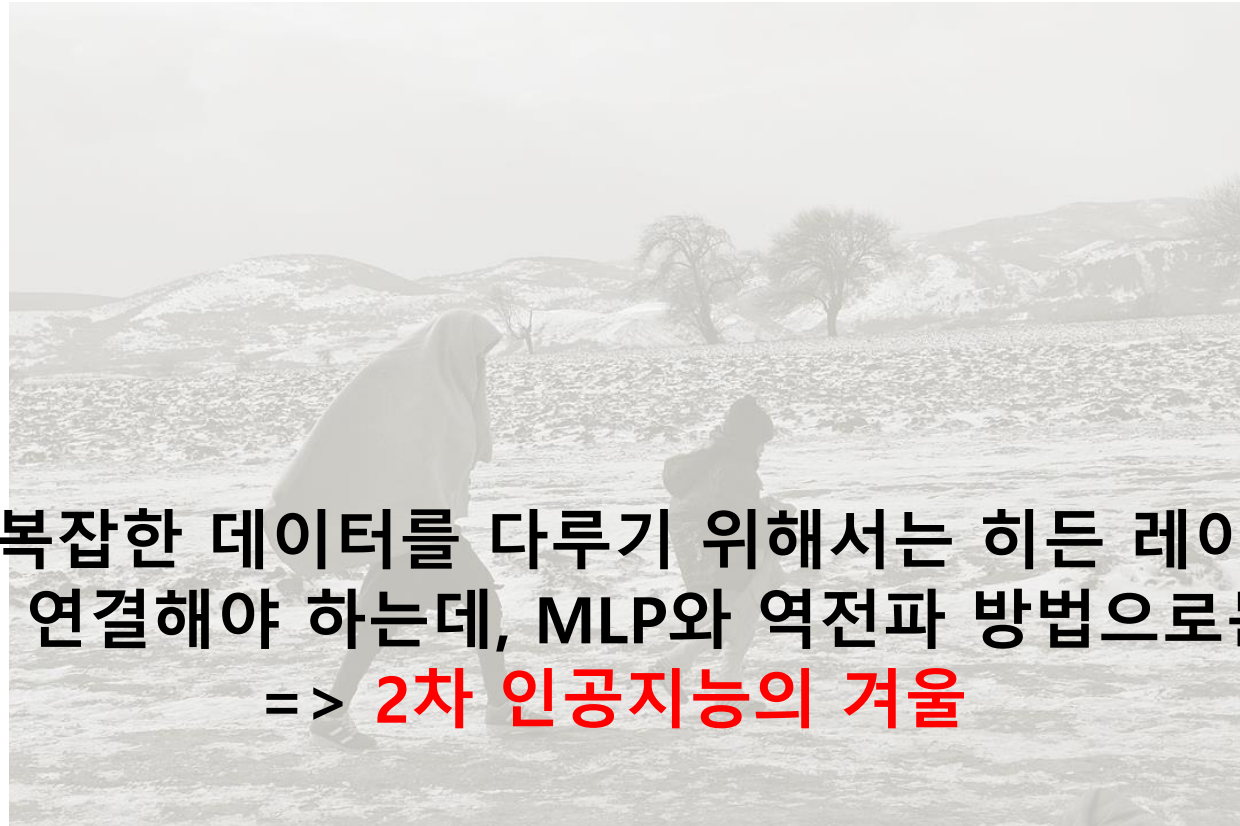
Sigmoid



- 기울기가 작아지는 좌우 부분은 미분하면 0이 됨
- 역전파를 이용하여 편미분할 때 $\frac{dj_{total}}{dw}$ 가 0이되어 가중치 업데이트가 없어지는 현상이 saturated 현상

Unit 02 | Backpropagation

Vanishing Gradient Problem



크고 복잡한 데이터를 다루기 위해서는 히든 레이어를
여러 개 연결해야 하는데, MLP와 역전파 방법으로는 한계
=> **2차 인공지능의 겨울**

Unit 02 | Backpropagation

활성화함수 - ReLU 함수

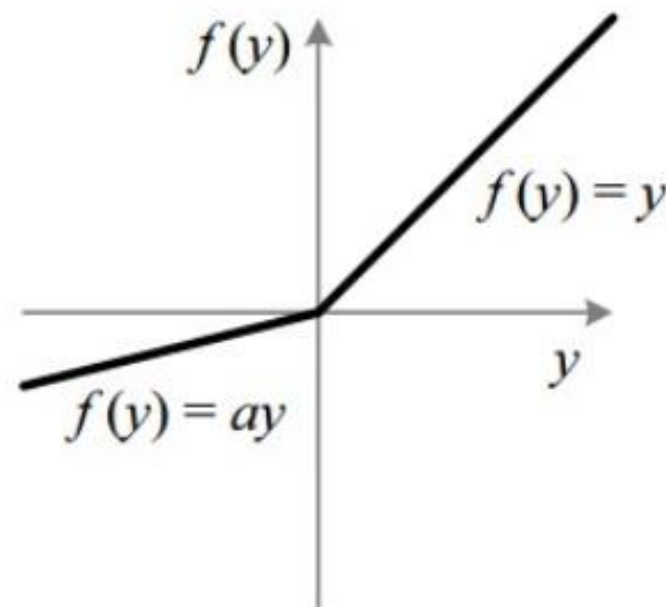
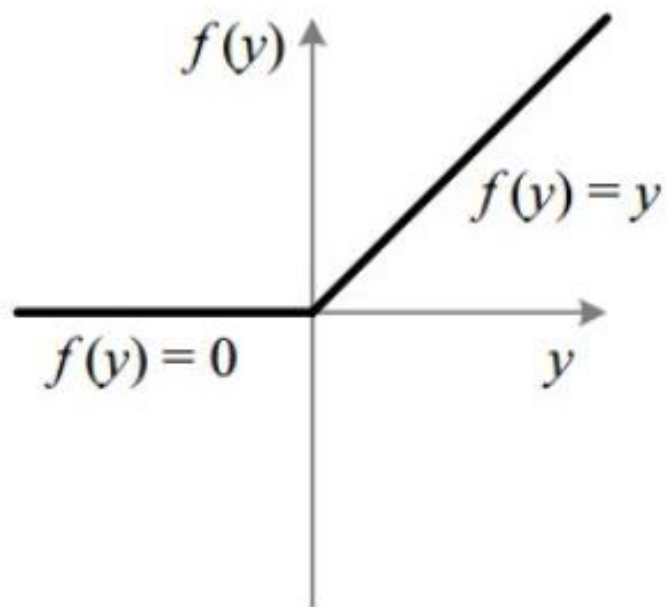


Fig : ReLU v/s Leaky ReLU

Unit | 과제

**“week3_NeuralNetworkBasic_assignment1.pdf” 파일의 문제들을
상세한 풀이과정과 함께 풀어주세요.**

**“week3_NeuralNetworkBasic_assignment.ipynb” 노트북 파일에서
코드 실습을 진행해 주세요.**

Reference

참고자료

- 투빅스 17기 김현태님 강의자료
- 밑바닥부터 시작하는 딥러닝
- 건국대학교 김학수 교수님 '기계학습' 강의
- <https://heeya-stupidbutstudying.tistory.com/38>
- 역전파: <https://m.blog.naver.com/samsjang/221033626685>
- <https://brunch.co.kr/@gdhan/6>
- <https://www.letr.ai/explore/story-20211105-1>
- <https://www.youtube.com/watch?v=bpBdSDEGVIA>
- [신경망의 기본 구조 \(velog.io\)](#)
- [텐서플로우 딥러닝 강의 12-2 - ReLU 활성화함수 - YouTube](#)
- [딥러닝 Neural Network AND 함수, XOR 문제 해결 방법 \(tistory.com\)](#)
- [퍼셉트론\(Perceptron\) \(tistory.com\)](#)
- [실체가 손에 잡히는 딥러닝\(3\) “이것만은 꼭 알아두자! 딥러닝의 꽃 - 가중치, 편향, 활성화 함수, 역전파” | Popit](#)
- [\[기계학습\] Neural Networks 1 \(velog.io\)](#)
- 활성화 함수 종류 : [\[ML\] 활성화 함수\(Activation Function\) 종류 정리 \(tistory.com\)](#)
- [Vanishing Gradient Problem\(기울기 소멸 문제\) – 창의 컴퓨팅\(Creative Computing\)](#)

Q & A

들어주셔서 감사합니다.