Sheet 3

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Jacobian G_t^x

$$G_t^x = I + \frac{\partial}{\partial (x, y, \theta)^T} \begin{pmatrix} \delta_{\text{trans}} \cos(\theta + \delta_{\text{rot1}}) \\ \delta_{\text{trans}} \sin(\theta + \delta_{\text{rot1}}) \\ \delta_{\text{rot1}} + \delta_{\text{rot2}} \end{pmatrix} = I + \begin{pmatrix} 0 & 0 & (1) \\ 0 & 0 & (2) \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} (1) &= \frac{\partial}{\partial \theta} \delta_{\text{trans}} \cos(\theta + \delta_{\text{rot1}}) = \delta_{\text{trans}} \frac{\partial}{\partial \theta} \cos(\theta + \delta_{\text{rot1}}) \\ &= \delta_{\text{trans}} \frac{\partial \cos}{\partial (\theta + \delta_{\text{rot1}})} \frac{\partial (\theta + \delta_{\text{rot1}})}{\partial \theta} = -\delta_{\text{trans}} \sin(\theta + \delta_{\text{rot1}}) \end{aligned}$$

$$(2) = \frac{\partial}{\partial \theta} \delta_{\text{trans}} \sin(\theta + \delta_{\text{rot1}}) = \delta_{\text{trans}} \cos(\theta + \delta_{\text{rot1}})$$

Jacobian ${}^{low}H_t^i$

$$\begin{aligned} \bar{\mu}_t &= \begin{pmatrix} x, & y, & \theta, & m_{j,x}, & m_{j,y} \end{pmatrix} \\ \delta &= \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} {}^{low}H_t^i &= \frac{\partial h(\bar{\mu}_t)}{\partial \bar{\mu}_t} = \frac{\partial}{\partial \bar{\mu}_t} \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix} = \frac{\partial}{\partial \bar{\mu}_t} \begin{pmatrix} \sqrt{\delta^T \delta} \\ \text{atan2}(\delta_x, \delta_y) - \bar{\mu}_{t,\theta} \end{pmatrix} \\ &= \frac{1}{\delta^T \delta} \begin{pmatrix} -\sqrt{\delta^T \delta}\ \delta_x & -\sqrt{\delta^T \delta}\ \delta_y & 0 & \sqrt{\delta^T \delta}\ \delta_x & \sqrt{\delta^T \delta}\ \delta_y \\ \delta_y & -\delta_x & -\delta^T \delta & -\delta_y & \delta_x \end{pmatrix} \end{aligned}$$