Construction heuristics for the static multi-vehicle electric bike-repositioning problem

Abstract

Keywords: Bike-sharing; Bike-repositioning; Pick-up and delivery routing

1 Introduction

2 Mathematical formulation

The e-bike repositioning problem considers a set of stations, multiple repositioning vehicles, a depot, usable e-bikes (e-bikes with a sufficient battery level for users to cycle to any station), and not usable (e-bikes with an insufficient battery level for users to cycle to some stations). Each station has no charging facility for the battery of an e-bike, and is characterized by its capacity for e-bikes, initial numbers of usable and not usable e-bikes, and a penalty function. The penalty function represents the cost of the expected shortage of e-bikes and lockers incurred by users in the station during the next day as a function of the numbers of usable and not usable e-bikes at the station after the repositioning is carried out. The repositioning vehicles are used to transport e-bikes and batteries between stations and the depot. These vehicles have limited capacity for e-bikes and batteries. These vehicles start from and end at the depot and operate within a given time constraint. They are allowed to go back to the depot multiple times for loading or unloading e-bikes and for loading fully charged batteries and unloading batteries with an insufficient energy level to the depot. The depot is assumed to have very large capacities for e-bikes and batteries and no demand. It also has unlimited numbers of usable e-bikes and fully charged batteries. For the ease of formulation, the depot is modeled by both a node (which is both the starting and ending depot and is called the depot from now on) and multiple other nodes (called dummy depots) so that the depot and dummy depots are visited at most once. Each station is modeled by a node and is allowed to be visited at most once. At each visited station, not usable e-bikes can become usable by swapping their batteries with fully charged ones on repositioning vehicles. Alternatively, not usable e-bikes there can be loaded to vehicles and transport back to the depot or dummy depots if their vehicle capacities are allowed. The objective is to determine the route of each repositioning vehicle, the quantities of usable and not usable e-bikes loaded onto or unloaded from each repositioning vehicle at each visited node, and the number of battery swaps at each visited station using the fully charged batteries from each vehicle to minimize the sum of penalty cost at each station.

dummy depots: The intermediate visit?

2.1 Notations

Sets

$\mathcal{N}$	Set of stations
$\mathcal{N}_1$	Set of stations and dummy depots
$\mathcal{N}_0$	Set of nodes, including the stations, depot, and dummy depots
$\mathcal{D}$	Set of the depot and dummy depots
$\mathcal{V}$	Set of vehicles

Indices

i, j Indices of nodes

v Index of vehicle (or route)

Parameters

$s_i^{0\mathrm{U}}$ Initial number of usable e-bikes (e-bikes with a sufficient battery level) at station i

$s_i^{0\mathrm{N}}$ Initial number of non-usable e-bikes (e-bikes with an insufficient battery level) at station i

c_i Capacity of station i

k_1 Vehicle capacity for e-bikes

k_2 Vehicle capacity for batteries

T Repositioning time

L Time required to load an e-bike from a station onto a vehicle

U Time required to unload an e-bike from a vehicle to a station

H_1 Time required to perform a battery swap

H_2 Time required to load a fully charged battery at the depot

H_3 Time required to unload a fully charged battery at the depot

M A very large number

t_{ij} Travel time from node i to node j

Decision variables

x_{ijv} Binary variable that equals one if vehicle v travels directly from node i to node j, and zero otherwise

y_{ijv}^{U} Number of usable e-bikes on vehicle v when it travels directly from node i to node j

y_{ijv}^{N} Number of non-usable e-bikes on vehicle v when it travels directly from node i to node j

y_{ijv}^{B} Number of fully charged batteries on vehicle v when it travels directly from node i to node j

$y_{ijv}^{\mathrm{B}'}$ Number of batteries with an insufficient energy level on vehicle v when it travels directly from node i to node j

q_{iv} Auxiliary variable associated with node i used for the sub-tour elimination constraints

y_{iv}^{UP} Number of usable e-bikes loaded onto vehicle v at node i

y_{iv}^{UD} Number of usable e-bikes unloaded from vehicle v at node i

y_{iv}^{NP} Number of non-usable e-bikes loaded onto vehicle v at node i

y_{iv}^{ND} Number of non-usable e-bikes unloaded from vehicle v at node i only for depot/dummy depot

y_{dv}^{PB} Number of fully charged batteries loaded onto vehicle v at depot or dummy depot d

$y_{dv}^{\mathrm{DB}'}$ Number of insufficient energy batteries unloaded onto from vehicle v at depot or dummy depot d

y_{iv}^{B} Number of battery swaps at node i using fully charged batteries from vehicle v, which is the number of fully charged batteries unloaded from that vehicle at that node

s_i^{U} Number of usable e-bikes at station i at the end of the repositioning operation

s_i^{N} Number of non-usable e-bikes at station i at the end of the repositioning operation

Function

$f_i(s_i^{\mathrm{U}}, s_i^{\mathrm{N}})$ A convex penalty function for station i defined over s_i^U and s_i^N

2.2 Arc-indexed formulation

This section presents the arc-indexed formulation.

Minimize total penalty cost

$$\min \sum_{i \in \mathcal{N}} f_i(s_i^{\mathrm{U}}, s_i^{\mathrm{N}}) \tag{1}$$

Final number of the stations

$$\text{s. t. } s_i^{\mathrm{U}} = s_i^{0\mathrm{U}} - \sum_{v \in \mathcal{V}} (y_{iv}^{\mathrm{UP}} - y_{iv}^{\mathrm{UD}} - y_{iv}^{\mathrm{B}}) \qquad \forall i \in \mathcal{N} \tag{2}$$

$$s_i^{\mathrm{N}} = s_i^{0\mathrm{N}} - \sum_{v \in \mathcal{V}} (y_{iv}^{\mathrm{NP}} + y_{iv}^{\mathrm{B}}) \qquad \forall i \in \mathcal{N} \tag{3}$$

$$y_{iv}^{\mathrm{UP}} - y_{iv}^{\mathrm{UD}} = \sum_{j \in \mathcal{N}_0, j \neq i} y_{ijv}^{\mathrm{U}} - \sum_{j \in \mathcal{N}_0, j \neq i} y_{jiv}^{\mathrm{U}} \qquad \forall i \in \mathcal{N}_0, \forall v \in \mathcal{V} \tag{4}$$

$$y_{iv}^{\mathrm{NP}} = \sum_{j \in \mathcal{N}_0, j \neq i} y_{ijv}^{\mathrm{N}} - \sum_{j \in \mathcal{N}_0, j \neq i} y_{jiv}^{\mathrm{N}} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{5}$$

All non-usable e-bikes are unloaded at depot

$$y_{dv}^{\mathrm{ND}} = \sum_{j \in \mathcal{N}_0, j \neq d} y_{jdv}^{\mathrm{N}} \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{6}$$

No non-usable e-bikes when the vehicle leaves the depot

$$\sum_{j \in \mathcal{N}_0, j \neq d} y_{djv}^{\mathrm{N}} = 0 \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{7}$$

$$y_{dv}^{\mathrm{PB}} = \sum_{j \in \mathcal{N}_0, j \neq d} y_{djv}^{\mathrm{B}} \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{8}$$

$$\sum_{j \in \mathcal{N}_0, j \neq d} y_{djv}^{\mathrm{B}'} = 0 \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{9}$$

$$-y_{iv}^{\mathrm{B}} = \sum_{j \in \mathcal{N}_0, j \neq i} y_{ijv}^{\mathrm{B}} - \sum_{j \in \mathcal{N}_0, j \neq i} y_{jiv}^{\mathrm{B}} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{10}$$

$$y_{iv}^{\mathrm{B}} = \sum_{j \in \mathcal{N}_0, j \neq i} y_{ijv}^{\mathrm{B}'} - \sum_{j \in \mathcal{N}_0, j \neq i} y_{jiv}^{\mathrm{B}'} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{11}$$

$$y_{dv}^{\mathrm{DB}'} = \sum_{j \in \mathcal{N}_0, j \neq d} y_{jdv}^{\mathrm{B}'} \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{12}$$

$$\sum_{j \in \mathcal{N}_0, j \neq d} y_{jdv}^{\mathrm{B}} = 0 \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{13}$$

k1, k2: vehicle capacity

$$y_{ijv}^{\mathrm{U}} + y_{ijv}^{\mathrm{N}} \leq k_1 \times x_{ijv} \qquad \forall i, j \in \mathcal{N}_0, i \neq j, \forall v \in \mathcal{V} \tag{14}$$

$$y_{ijv}^{\mathrm{B}} + y_{ijv}^{\mathrm{B}'} \leq k_2 \times x_{ijv} \qquad \forall i, j \in \mathcal{N}_0, i \neq j, \forall v \in \mathcal{V} \tag{15}$$

How about the last stop to the depot?, should be N1?

$$\sum_{j \in \mathcal{N}_0, j \neq i} x_{ijv} = \sum_{j \in \mathcal{N}_0, j \neq i} x_{jiv} \qquad \forall i \in \mathcal{N}_0, \forall v \in \mathcal{V} \tag{16}$$

How about touring? 2->3, 3->4, 4-> cannot visit 2 again, but cannot capture by this formula

$$\sum_{j \in \mathcal{N}_0, j \neq i} x_{ijv} \leq 1 \qquad \forall i \in \mathcal{N}_1, \forall v \in \mathcal{V} \tag{17}$$

$$\sum_{v \in \mathcal{V}} y_{iv}^{\mathrm{UP}} \leq s_i^{0\mathrm{U}} + \sum_{v \in \mathcal{V}} y_{iv}^{\mathrm{B}} \qquad \forall i \in \mathcal{N} \tag{18}$$

$$\sum_{v \in \mathcal{V}} y_{iv}^{\mathrm{NP}} \leq s_i^{0\mathrm{N}} - \sum_{v \in \mathcal{V}} y_{iv}^{\mathrm{B}} \qquad \forall i \in \mathcal{N} \tag{19}$$

$$\sum_{v\in\mathcal{V}} y_{iv}^{\mathrm{UD}} \le c_i - (s_i^{0\mathrm{U}} + s_i^{0\mathrm{N}}) \qquad \forall i \in \mathcal{N} \tag{20}$$

$$\sum_{i\in\mathcal{N}_0} (y_{iv}^{\mathrm{UP}} - y_{iv}^{\mathrm{UD}}) = 0 \qquad \forall v \in \mathcal{V} \tag{21}$$

$$\sum_{i\in\mathcal{N}_1} y_{iv}^{\mathrm{B}} = \sum_{d\in\mathcal{D}} y_{dv}^{\mathrm{PB}} \qquad \forall v \in \mathcal{V} \tag{22}$$

$$y_{iv}^{\mathrm{B}} \le \sum_{j\in\mathcal{N}_0\backslash\mathcal{D}, j\neq i} y_{jiv}^{\mathrm{B}} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{23}$$

Should have no D here?

$$y_{iv}^{\mathrm{B}} \le s_i^{0\mathrm{N}} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{24}$$

$$y_{dv}^{\mathrm{PB}} \le k_2 \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{25}$$

Total time

$$\sum_{i\in\mathcal{N}_0} (Ly_{iv}^{\mathrm{UP}} + Uy_{iv}^{\mathrm{UD}}) + \sum_{i\in\mathcal{N}} (H_1 y_{iv}^{\mathrm{B}} + Ly_{iv}^{\mathrm{NP}}) + \sum_{d\in\mathcal{D}} (H_2 y_{dv}^{\mathrm{PB}} + H_3 y_{dv}^{\mathrm{DB}'} + Uy_{mv}^{\mathrm{ND}}) + \sum_{i,j\in\mathcal{N}_0: i\neq j} t_{ij} x_{ijv} \le T \qquad \forall v \in \mathcal{V} \tag{26}$$

mv^?

$$q_{jv} \ge q_{iv} + 1 - M(1 - x_{ijv}) \qquad \forall i \in \mathcal{N}_0, j \in \mathcal{N}_1, i \neq j, \forall v \in \mathcal{V} \tag{27}$$

$$x_{ijv} \in \{0, 1\} \qquad \forall i, j \in \mathcal{N}_0 : i \neq j, \forall v \in \mathcal{V} \tag{28}$$

$$y_{iv}^{\mathrm{UD}} \ge 0, y_{iv}^{\mathrm{UP}} \ge 0, \text{integer} \qquad \forall i \in \mathcal{N}_0, \forall v \in \mathcal{V} \tag{29}$$

$$y_{iv}^{\mathrm{B}} \ge 0, y_{iv}^{\mathrm{NP}} \ge 0, \text{integer} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{30}$$

$$y_{dv}^{\mathrm{PB}} \ge 0, y_{dv}^{\mathrm{DB}'} \ge 0, y_{dv}^{\mathrm{ND}} \ge 0, \text{integer} \qquad \forall d \in \mathcal{D}, \forall v \in \mathcal{V} \tag{31}$$

$$y_{ijv}^{\mathrm{U}} \ge 0, y_{ijv}^{\mathrm{N}} \ge 0, y_{ijv}^{\mathrm{B}} \ge 0, y_{ijv}^{\mathrm{B}'} \ge 0 \qquad \forall i, j \in \mathcal{N}_0 : i \neq j, \forall v \in \mathcal{V} \tag{32}$$

$$s_i^{\mathrm{N}} \ge 0, s_i^{\mathrm{U}} \ge 0 \qquad \forall i \in \mathcal{N} \tag{33}$$

$$q_{iv} \ge 0 \qquad \forall i \in \mathcal{N}_0, \forall v \in \mathcal{V} \tag{34}$$

The objective (1) is to minimize the total penalty cost. Equations (2)-(3) define the final numbers of usable and non-usable e-bikes at each station. For usable e-bikes, the final number at a station equals the initial number minus the number of usable e-bikes loaded onto each vehicle from that station, plus the sum of the number of usable e-bikes unloaded from each vehicle and the number of non-usable e-bikes with their batteries replaced by fully charged batteries. For non-usable e-bikes, the final number at a station equals the initial number minus the sum of the number of non-usable e-bikes loaded onto each vehicle from that station and the number of non-usable e-bikes involving battery swaps. Equations (4) requires that the number of usable e-bikes loaded onto or unloaded from a vehicle at a node equals the difference between the vehicle load before and after visiting that node. Equations (5) are similar to Equations (4) except that it is for non-usable e-bikes at stations. Equations (6) ensure that all non-usable e-bikes on a vehicle are unloaded at a dummy depot or the depot whenever the vehicle visits it. Equations (7) require that there are no non-usable e-bikes on a vehicle when the vehicle leaves a dummy depot or the depot Equations (8) state that all fully charged batteries on a vehicle departing from a dummy depot or the depot are loaded from that depot. Equations (9) require that there are no batteries with an insufficient energy level on a vehicle when the vehicle leaves the depot or dummy depot.

Equations (10) state that the number of battery swaps at a station equals the difference in the number of fully charged batteries on a vehicle before and after the vehicle visits that station. Equations (11) ensure that the number of battery swaps at a station equals the difference in the number of batteries without sufficient energy levels on a vehicle after and before the vehicle visits that station. Equations (12) require that all batteries with insufficient energy levels on a vehicle are unloaded at a dummy depot or the depot whenever the vehicle visits it. Equations (13) ensure that the number of fully charged batteries on a vehicle is zero when the vehicle is back to the depot or a dummy depot. Constraints (14) and (15) ensure that the e-bike load and the battery load on each vehicle cannot be greater than the corresponding vehicle capacities. Equations (16) make sure that if a vehicle visits a station, it must leave that station. Constraints (17) ensure that each vehicle can visit a station at most once. Constraints (18) require that the total pick-up quantity of usable e-bikes at each node is not larger than the number of usable e-bikes available at that node, including those initially available and those non-usable e-bikes performing battery swaps. Constraints (19) ensure that the total pick-up quantity of non-usable e-bikes at each node is not larger than the number of non-usable e-bikes available at that node after battery swaps are performed. Constraints (20) require that the total drop-off quantities of usable e-bikes at each node are not larger than the remaining capacity of that node. Constraints (21) ensure that all usable e-bikes loaded onto each vehicle are eventually unloaded. Constraints (22) make sure that the total number of battery swaps performed at all stations equals the total number of fully charged batteries picked up from the depot or a dummy depot. Constraints (23) restrict that the number of battery swaps at a station using the batteries from an arrived vehicle is not greater than the number of fully charged batteries on that vehicle. Constraints (24) limit that the number of battery swaps at a station using the batteries from an arrived vehicle is not greater than the initial number of non-usable e-bike at that station. Constraints (25) provide an upper bound, which is the vehicle capacity for batteries, on the number of fully charged batteries loaded onto a vehicle from the depot or a dummy depot. Constraints (26) restrict the sum of the loading and unloading times of e-bikes, the total time spent on battery swaps, the loading and unloading times of batteries, and the travel time of each vehicle not exceeding the repositioning time available. Constraints (27) are the sub-tour elimination constraints (see Miller et al., 1960). Constraints (28)-(34) are domain constraints.

In order to speed up computation speed, Raviv et al. (2013) added the following:

$$\sum_{j \in \mathcal{N}_1} x_{0jv} \geq 1 \qquad \forall v \in \mathcal{V} \tag{35}$$

$$y_{iv}^{\mathrm{UD}} \leq \min(c_i - (s_i^{0\mathrm{U}} + s_i^{0\mathrm{N}}), k_1) \sum_{j \in \mathcal{N}_0} x_{ijv} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{36}$$

$$y_{iv}^{\mathrm{UP}} + y_{iv}^{\mathrm{UD}} + y_{iv}^{\mathrm{B}} + y_{iv}^{\mathrm{NP}} \geq \sum_{j \in \mathcal{N}_0} x_{ijv} \qquad \forall i \in \mathcal{N}, \forall v \in \mathcal{V} \tag{37}$$

$$\sum_{j \in \mathcal{N}_1} j \cdot x_{0jv} \leq \sum_{j \in \mathcal{N}_1} j \cdot x_{0,j,v+1} \qquad \forall v \in \mathcal{V} \tag{38}$$

Constraints (35) ensure that each vehicle departs from the depot at least one. Constraints (36) further tighten the solution space for unloading quantities of usable e-bikes of each vehicle at a station respectively by including vehicle capacity for e-bikes and the remaining station capacity, and conditioning the quantities only for cases in which the corresponding vehicle visits that station. Constraints (37) tighten the solution space by ensuring that each vehicle enters a station must have a loading or an unloading e-bike activity or perform a battery swap. Constraints (38) are the break symmetry constraints.

3 The algorithm

In this section, we present several construction heuristics for solving the multi-vehicle electric bike-repositioning problem. These are sequential heuristics where one route is constructed at a time. The main ideas of these heuristics are as follows: At each iteration, a station is chosen at a time to be visited by the vehicle. This selection process is continued until it is not possible to extend the route any further due to the stated constraints. The whole process is repeated for all vehicle routes. For each station, there are four decisions that have to be made: 1) the number of bikes with their batteries replaced (only non-usable bikes will have their batteries replaced); 2) the number of non-usable bikes (i.e., bikes with batteries of at most 30% of the capacity remaining) to collect; 3) the number of usable bikes to collect; 4) the number of usable bikes to drop off. Dealing with the non-usable bikes has a higher priority than dealing with usable bikes. This is due to the value of penalty function will improve greatly when the number of non-usable bikes is reduced. Hence, the decisions are made in the order listed above. Regarding 3) and 4), for each station, at most one of these will have a positive value. The main differences between the construction heuristics lie in the selection strategy on how a station is chosen to be included in a route as well as how the number of battery replacements is determined. Details of these construction heuristics can be found in Sections 3.2-3.5.

In order to improve the solutions, we present several simple improvement procedures in Section **??**.

3.1 Solution representation

A solution x is made up of a set of vehicle routes and a set of values denoting the number of battery swaps, as well as loading and unloading quantities, where each route v is represented as $(i_0, i_1, \ldots, i_{n_v})$, $i_0 = i_{n_v} = 0$ and $i_1, \ldots, i_{n_v - 1} \in \mathcal{N}_0$. A node is denoted as i_h, where $h = 1, \ldots, n_v - 1$ is the placement of i_h in route v. For each node i_h, there are the following values associated with it: $y^{\mathrm{B}}_{i_h v}$, $y^{\mathrm{NP}}_{i_h v}$, and $y^{\mathrm{UP}}_{i_h v}$ or $y^{\mathrm{UD}}_{i_h v}$. Solution x is evaluated by an evaluation function $z(x) = \sum_{i \in \mathcal{N}} f_i(s^{\mathrm{U}}_i, s^{\mathrm{N}}_i)$, where s^{U}_i and s^{N}_i are defined by (2) and (3).

3.2 Construction heuristic 1

The vehicles will not have the same route?

Route v is extended by first including the closest non-visited station g (in terms of shortest travel time). However, station g may be visited by other routes. Then, the number of non-usable bikes with their batteries replaced (y^{B}_{gv}) is determined by several factors: the current number of non-usable bikes at the station ($\bar{s}^{\mathrm{N}}_g$)[1], the number of fully-charged batteries on board leaving for the station ($y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v}$), and the remaining time of the repositioning operation (τ). With this, the number of non-usable bikes is reduced, while the number of usable bikes is increased. If there are still non-usable bikes left, then the number of non-usable bikes to collect (y^{NP}_{gv}) is determined by
why s bar?
$\bar{s}^{\mathrm{N}}_g$, the residual vehicle capacity, and τ. In order to determine whether it is necessary to perform a collection or a drop-off of usable bikes and the quantity of the bikes, we need to determine the
Why the ideal level of usable bikes is related to this?
ideal level of the usable bikes given the value of $\bar{s}^{\mathrm{N}}_g$ (μ). The number of usable bikes to collect is determined by the number of bikes to collect until reaching μ, residual vehicle capacity, and τ. Finally, the number of bikes to drop off is determined by the number of bikes to drop off until reaching μ and the vehicle load of usable bikes. This process is repeated until it is not possible to add any more stations to the route without violating any constraints.

Due to the fact that a full load of batteries is assigned to the vehicle, it may be necessary to adjust this load as it makes no sense to return to the depot with one or several fully-charged batteries (lines 61-65). After returning the vehicle to the depot, it may be necessary for the same vehicle to leave the depot again if there still exist at least one station that is not at its optimal state (which is defined by the number of bikes present at the station). Depending on specific situations, the vehicle may be loaded with ω fully-charged batteries (Algorithm 1, lines 70-72), the vehicle may be empty so it can collect some bikes (Algorithm 1, lines 73-75), or the vehicle may be loaded with $\bar{\omega}$ usable bikes so they can be dropped off at one or several stations (Algorithm 1, lines 76-80). The algorithm continues with extending the route by selecting a station as described in the above paragraph. The algorithmic framework of this heuristic is shown in Algorithm 1.

3.3 Construction heuristic 2

The way y^{B}_{gv} is calculated in Algorithm 1 may lead to too many usable bikes at the station.
補救
This means that extra work is needed to pick up these bikes. A remedy to this is to restrict the number of batteries that can be swapped by taking into account the optimal inventory level of the usable bikes. With this restriction, the number of swaps will reduce, the number of unusable bikes to collect will increase, and the number of usable bikes to collect will reduce. Hence, line 11 in Algorithm 1 is replaced with $y^{\mathrm{B}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_g, y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v}, \max\{0, \hat{\mu} - \bar{s}^{\mathrm{U}}_g\}, \lfloor \tau/(H_1+H_2+H_3) \rfloor\}$
pick the best e which gives smallest penalty
with $\hat{\mu} = \arg\min_{e \in \Lambda} f_g(e, 0)$, where $\Lambda = \{0, \dots, c_g\}$. Another necessary modification is applied to line 66 in Algorithm 1 regarding the number of batteries to load the vehicle. We do not want

[1] similarly, $\bar{s}^{\mathrm{U}}_g$ denotes the current number of usable bikes at the station

Algorithm 1 Construction heuristic 1

for $v = 1, \ldots, |\mathcal{V}|$ **do** What is the meaning of this initial route? Randomly generate?
Initialize the route as $(i_0, \ldots, i_{n_v})$.
Set $\tau = T$, $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = k_2$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$.
Set $h = 0$ and $\varrho = 0$
repeat h: set of visited nodes?
$g = \arg\min_{j \in \mathcal{N} \setminus \{h\}, j \notin v} t_{hj}$ Closest
Set $\bar{s}^{\mathrm{U}}_g = s^{0\mathrm{U}}_g + \sum_{k \in V} y^{\mathrm{UD}}_{gk} - \sum_{k \in V} y^{\mathrm{UP}}_{gk} + \sum_{k \in V} y^{\mathrm{B}}_{gk}$
Set $\bar{s}^{\mathrm{N}}_g = s^{0\mathrm{N}}_g - \sum_{k \in V} y^{\mathrm{NP}}_{gk} - \sum_{k \in V} y^{\mathrm{B}}_{gk}$
if $g \in \mathcal{N}$ **then**
remaining time after inserting g into the last station
Set $\tau_1 = \tau$ and $\tau = \tau - t_{i_{n_v-1} g} - t_{g i_{n_v}} + t_{i_{n_v-1} i_{n_v}}$
Set $y^{\mathrm{B}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_g, y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$
if $y^{\mathrm{B}}_{gv} > 0$ **then** Inserted g and do battery swapping
Node g is inserted between i_{n_v-1} and i_{n_v} in route v. Set $\tau = \tau - (H_1 + H_2 + H_3) y^{\mathrm{B}}_{gv}$.
Set $y^{\mathrm{B}}_{i_{n_v-1} g v} = y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{B}}_{g i_{n_v} v} = y^{\mathrm{B}}_{i_{n_v-1} g v} - y^{\mathrm{B}}_{gv}$.
Set $y^{\mathrm{B}'}_{i_{n_v-1} g v} = y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{B}'}_{g i_{n_v} v} = y^{\mathrm{B}'}_{i_{n_v-1} g v} + y^{\mathrm{B}}_{gv}$.
Set $\bar{s}^{\mathrm{N}}_g = \bar{s}^{\mathrm{N}}_g - y^{\mathrm{B}}_{gv}$ and $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g + y^{\mathrm{B}}_{gv}$.
end if
if $\bar{s}^{\mathrm{N}}_g > 0$ **then** Then, pick-up not usable bikes
Set $y^{\mathrm{NP}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_g, k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v}, \lfloor \tau/(L + U) \rfloor\}$
if $y^{\mathrm{NP}}_{gv} > 0$ **then**
if $y^{\mathrm{B}}_{gv} = 0$ **then**
Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
end if
Set $\tau = \tau - (L + U) y^{\mathrm{N}}_{gv}$.
Set $y^{\mathrm{N}}_{i_{n_v-1} g v} = y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{N}}_{g i_{n_v} v} = y^{\mathrm{N}}_{i_{n_v-1} g v} + y^{\mathrm{NP}}_{gv}$.
Set $\bar{s}^{\mathrm{N}}_g = \bar{s}^{\mathrm{N}}_g - y^{\mathrm{NP}}_{gv}$
end if
end if miu: optimal value
$\mu = \arg\min_{e \in \Lambda} f_g(e, \bar{s}^{\mathrm{N}}_g)$, where $\Lambda = \{0, \ldots, c_g - \bar{s}^{\mathrm{N}}_g\}$
if $\mu < \bar{s}^{\mathrm{U}}_g$ **then** if larger than optimal value, then pick up usable bike
Set $y^{\mathrm{UP}}_{gv} = \min\{\bar{s}^{\mathrm{U}}_g - \mu, k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v}, \lfloor \tau/(L + U) \rfloor\}$
if $y^{\mathrm{UP}}_{gv} > 0$ **then**
if $y^{\mathrm{B}}_{gv} = 0$ and $y^{\mathrm{NP}}_{gv} = 0$ **then**
Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
end if
Set $\tau = \tau - (L + U) y^{\mathrm{UP}}_{gv}$.
Set $y^{\mathrm{U}}_{i_{n_v-1} g v} = y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{U}}_{g i_{n_v} v} = y^{\mathrm{U}}_{i_{n_v-1} g v} + y^{\mathrm{UP}}_{gv}$.
Set $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g - y^{\mathrm{UP}}_{gv}$
end if if smaller than optimal value, then drop off usable bike
last visiting depot?
else if $\mu > \bar{s}^{\mathrm{U}}_g$ **then**
Set $y^{\mathrm{UD}}_{gv} = \min\{\mu - \bar{s}^{\mathrm{U}}_g, y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v}\}$ What is the meaning of this
γ = the position of the originating depot in route v node i_{n_v-1} belongs to
$\rho = \max_{\epsilon = \gamma, \ldots, n_v - 1}\{y^{U}_{i_\epsilon i_{\epsilon+1} v} + y^{N}_{i_\epsilon i_{\epsilon+1} v}\}$
if $y^{\mathrm{UD}}_{gv} < \mu - \bar{s}^{U}_g$ **then** row is to calculate the the max. number of bikes on the vehicle
$\delta = \min\{k_1 - \rho, \lfloor \tau/(L + U) \rfloor, \mu - \bar{s}^{U}_g - y^{UD}_{gv}\}$ between the time to visit the depot
$y^{UD}_{gv} = y^{UD}_{gv} + \delta$
end if

if $y^{\mathrm{UD}}_{gv} > 0$ **then**
  if $y^{\mathrm{B}}_{gv} = 0$ and $y^{\mathrm{NP}}_{gv} = 0$ **then**
    Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
  end if
  if $\delta > 0$ **then**
    Set $y^{U}_{i_\epsilon i_{\epsilon+1} v} = y^{U}_{i_\epsilon i_{\epsilon+1} v} + \delta \ \forall \epsilon = \gamma, \dots, n_v - 1$
  end if
  Set $y^{\mathrm{U}}_{i_{n_v-1} g v} = y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{U}}_{g i_{n_v} v} = y^{\mathrm{U}}_{i_{n_v-1} g v} - y^{\mathrm{UD}}_{gv}$.
  Set $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g + y^{\mathrm{UD}}_{gv}$
end if
end if
if $y^{\mathrm{B}}_{gv} = 0$, $y^{\mathrm{NP}}_{gv} = 0$, $y^{\mathrm{UP}}_{gv} = 0$, and $y^{\mathrm{UD}}_{gv} = 0$ **then**
  Set $\tau = \tau_1$ and $\varrho = \varrho + 1$.
else
  Set $h = g$ and $\varrho = 0$
end if
else
  Set $\varrho = \varrho + 1$
What is this counting for?
end if
$\Phi = \{j \in \mathcal{N} : \hat{\mu} \neq \bar{s}^{\mathrm{U}}_j \text{ or } \bar{s}^{\mathrm{N}}_j \neq 0 \text{ where } \hat{\mu} = \arg\min_{e=0,\dots,c_j} f_j(e, 0)\}$
if $\varrho \geq |\Phi|$ **then** Number of Optimal station >= not optimal station?
  if v is not empty **then**
    if $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} > 0$ **then**
      $\gamma =$ the position of the originating depot in route v node i_{n_v-1} belongs to
      Set $y^{\mathrm{B}}_{i_\gamma i_{\gamma+1} v} = k_2 - y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}$ bring the correct amount of battery
      Set $y^{\mathrm{B}}_{i_\epsilon i_{\epsilon+1} v} = y^{\mathrm{B}}_{i_\epsilon i_{\epsilon+1} v} - y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}$ $\forall \epsilon$ where $\epsilon = \gamma + 1, \dots, n_v - 1$.
    end if
    $\omega = \min\{\sum_{j \in \mathcal{N}} \bar{s}^{\mathrm{N}}_j, k_2, \lfloor \tau / (H_1 + H_2 + H_3) \rfloor\}$
    Set $\Omega_1 = \{j \in \mathcal{N} : \mu > \bar{s}^{\mathrm{U}}_j, \mu = \arg\min_{e \in \Lambda} f_j(e, \bar{s}^{\mathrm{N}}_j)$, where $\Lambda = \{0, \dots, c_j - \bar{s}^{\mathrm{N}}_j\}\}$
    Set $\Omega_2 = \{j \in \mathcal{N} : \mu < \bar{s}^{\mathrm{U}}_j, \mu = \arg\min_{e \in \Lambda} f_j(e, \bar{s}^{\mathrm{N}}_j)$, where $\Lambda = \{0, \dots, c_i - \bar{s}^{\mathrm{N}}_i\}\}$
    Set $\rho_1 = \sum_{i \in \mathcal{N}} \max\{\mu - \bar{s}^{\mathrm{U}}_j, 0\}$ and $\rho_2 = \sum_{i \in \mathcal{N}} \max\{\bar{s}^{\mathrm{U}}_j - \mu, 0\}$
    Use up all batteries and no place for bikes
    if $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, and $\omega > 0$ **then**
      A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
      Set $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = \omega$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$, and $\varrho = 0$
    No not usable bikes now
    else if $k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v}$, $|\Omega_2| > 0$, and $\omega = 0$ **then**
      A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
      Set $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$, and $\varrho = 0$
    else if $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$ and $|\Omega_1| > 0$ **then**
      A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
      $\bar{\omega} = \min\{\max\{\rho_1 - \rho_2, 0\}, k_1, \lfloor \tau / (L + U) \rfloor\}$
      Set $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = \bar{\omega}$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$.
      Set $\tau = \tau - (L + U)\bar{\omega}$, and $\varrho = 0$
    else if $i_{n_v-1} = 0$ and $i_{n_v} = 0$ **then**
      Remove i_{n_v} from the route.
    end if
  else
    total excess usable bike-total deficit usable bike
    $\bar{\omega} = \min\{\max\{\rho_1 - \rho_2, 0\}, k_1, \lfloor \tau / (L + U) \rfloor\}$
    Set $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = \bar{\omega}$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$.
    Set $\tau = \tau - (L + U)\bar{\omega}$ and $\varrho = 0$.
  end if
end if
until no more nodes can be added without violating the constraints
end for

to load the vehicle with too many fully-charged batteries, hence, when making the decision it is necessary to take into account the optimal inventory level of the usable bikes as well. Line 66 is changed to the following: $\omega = \min\{\sum_{j\in\mathcal{N}} \min\{\bar{s}_j^{\mathrm{N}}, \max\{0, \hat{\mu} - \bar{s}_j^{\mathrm{U}}\}\}, k_2, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$. These are the only differences between these two heuristics.

3.4 Construction heuristic 3

This construction heuristic is also a sequential heuristic. The main difference between this and the one presented in Section 3.2 lies in the selection of a non-visited station g to be included in a route v. Instead of relying on travel time to make a decision, the heuristic uses the number of unusable bikes at a station as a decisive factor when deciding which station to extend the route with. The stations are first sorted in a non-increasing order according to their initial inventory levels of non-usable bikes, $s_g^{0\mathrm{N}}$, and stored in a vector W. The first non-visited station in W is chosen. The decisions regarding the number of battery swaps, the number of non-usable/usable bikes to collect or drop-off are done exactly in the same was as in Algorithm 1. A station is added at a time until it is no longer possible to extend the route any further without violating one or several constraints. may have a lot of touring trip?

Then an adjustment of the number of fully-charged batteries loaded on the vehicle may need to be done because it may save some operational time for other operations. After the vehicle is returned to the depot, it may leave the depot again due to the necessity to deal with the unusable bikes locked up at the stations. Depending on the situation, the vehicle will be loaded with ω fully-charged batteries (lines 62-64) or the vehicle will not be loaded with anything (lines 65-67). At this stage, loading the vehicle with usable bikes is not an option as the focus here is to reducing the number of unusable bikes. The heuristic then continues with extending the route by choosing the first non-visited station from W. This whole process continues until it is no longer possible to add more stations. At this stage, all non-usable bikes have now been collected from the stations.

The focus of the next stage is on the usable bikes only. A station g is selected from W, then the value of y_{gv}^{UP} or y_{gv}^{UD} needs to be decided. Notice that it is no longer necessary to deal with y_{gv}^{B} and y_{gv}^{NP} as $y_{gv}^{\mathrm{NP}} = 0$. When it is not possible to add more stations, the vehicle returns to the depot to empty the vehicle. The vehicle may leave the depot again empty (lines 91-94) or load it with usable bikes (lines 86-90). The steps of this heuristic are shown in Algorithm 2.

The differences between Algorithm 1 and Algorithm 2 are the following:

- the selection strategy: Algorithm 1 focuses on the shortest travel time, while Algorithm 2 focuses on the largest number of non-usable bikes when choosing a station.
- the overall strategy: Algorithm 2 is divided into two main stages, where the goal of the

first stage is mainly about getting rid of the unusable bikes from the stations whereas the second stage is about getting the inventory level of the usable bikes as close to the optimal state as possible. Algorithm 1 does not have this separation.

3.5 Construction heuristic 4

Using the same arguments as with those presented in Section 3.3. Line 11 in Algorithm 2 is replaced with $y^{\mathrm{B}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_{g}, y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v}, \max\{0, \hat{\mu} - \bar{s}^{\mathrm{U}}_{g}\}, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$ with $\hat{\mu} = \arg\min_{e\in\Lambda} f_g(e, 0)$, where $\Lambda = \{0, \ldots, c_g\}$. Line 61 in Algorithm 2 is changed to the following: $\omega = \min\{\sum_{j\in\mathcal{N}} \min\{\bar{s}^{\mathrm{N}}_{j}, \max\{0, \hat{\mu} - \bar{s}^{\mathrm{U}}_{j}\}\}, k_2, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$.

4 Computational experiments

5 Conclusions

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Algorithm 2 `Construction heuristic 3`

Require: Let W be an ordered set of nodes and W_d the dth node of W.

Sort W such that $d > \bar{d} \Rightarrow s^{0\mathrm{N}}_{W_d} \geq s^{0\mathrm{N}}_{W_{\bar{d}}}$.
for $v = 1, \ldots, |\mathcal{V}|$ **do**
  Set $d = 1$. Initialize the route as $(i_0, \ldots, i_{n_v})$.
  Set $\tau = T$, $y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v} = k_2$, $y^{\mathrm{B}'}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} = 0$.
  repeat
    Set $g = W_d$
    Set $\bar{s}^{\mathrm{U}}_g = s^{0\mathrm{U}}_g + \sum_{k \in V} y^{\mathrm{UD}}_{gk} - \sum_{k \in V} y^{\mathrm{UP}}_{gk} + \sum_{k \in V} y^{\mathrm{B}}_{gk}$
    Set $\bar{s}^{\mathrm{N}}_g = s^{0\mathrm{N}}_g - \sum_{k \in V} y^{\mathrm{NP}}_{gk} - \sum_{k \in V} y^{\mathrm{B}}_{gk}$
    Set $\tau_1 = \tau$ and $\tau = \tau - t_{i_{n_v-1}g} - t_{gi_{n_v}} + t_{i_{n_v-1}i_{n_v}}$
    if $\bar{s}^{\mathrm{N}}_g > 0$ and $g \notin v$ **then**
      Set $y^{\mathrm{B}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_g, y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v}, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$
      if $y^{\mathrm{B}}_{gv} > 0$ **then**
        Node g is inserted between i_{n_v-1} and i_{n_v} in route v. Set $\tau = \tau - (H_1 + H_2 + H_3) y^{\mathrm{B}}_{gv}$.
        Set $y^{\mathrm{B}}_{i_{n_v-1}gv} = y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v}$ and $y^{\mathrm{B}}_{gi_{n_v}v} = y^{\mathrm{B}}_{i_{n_v-1}gv} - y^{\mathrm{B}}_{gv}$.
        Set $y^{\mathrm{B}'}_{i_{n_v-1}gv} = y^{\mathrm{B}'}_{i_{n_v-1}i_{n_v}v}$ and $y^{\mathrm{B}'}_{gi_{n_v}v} = y^{\mathrm{B}'}_{i_{n_v-1}gv} + y^{\mathrm{B}}_{gv}$.
        Set $\bar{s}^{\mathrm{N}}_g = \bar{s}^{\mathrm{N}}_g - y^{\mathrm{B}}_{gv}$ and $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g + y^{\mathrm{B}}_{gv}$.
      end if
      if $\bar{s}^{\mathrm{N}}_g > 0$ **then**
        Set $y^{\mathrm{NP}}_{gv} = \min\{\bar{s}^{\mathrm{N}}_g, k_1 - y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} - y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v}, \lfloor \tau/(L + U) \rfloor\}$
        if $y^{\mathrm{NP}}_{gv} > 0$ **then**
          if $y^{\mathrm{B}}_{gv} = 0$ **then**
            Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
          end if
          Set $\tau = \tau - (L + U) y^{\mathrm{N}}_{gv}$.
          Set $y^{\mathrm{N}}_{i_{n_v-1}gv} = y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v}$ and $y^{\mathrm{N}}_{gi_{n_v}v} = y^{\mathrm{N}}_{i_{n_v-1}gv} + y^{\mathrm{NP}}_{gv}$.
          Set $\bar{s}^{\mathrm{N}}_g = \bar{s}^{N}_g - y^{\mathrm{NP}}_{gv}$
        end if
      end if
      $\mu = \arg\min_{e \in \Lambda} f_g(e, \bar{s}^{\mathrm{N}}_g)$, where $\Lambda = \{0, \ldots, c_g - \bar{s}^{\mathrm{N}}_g\}$
      if $\mu < \bar{s}^{\mathrm{U}}_g$ **then**
        Set $y^{\mathrm{UP}}_{gv} = \min\{\bar{s}^{\mathrm{U}}_g - \mu, k_1 - y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} - y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v}, \lfloor \tau/(L + U) \rfloor\}$
        if $y^{\mathrm{UP}}_{gv} > 0$ **then**
          if $y^{\mathrm{B}}_{gv} = 0$ and $y^{\mathrm{NP}}_{gv} = 0$ **then**
            Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
          end if
          Set $\tau = \tau - (L + U) y^{\mathrm{UP}}_{gv}$.
          Set $y^{\mathrm{U}}_{i_{n_v-1}gv} = y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v}$ and $y^{\mathrm{U}}_{gi_{n_v}v} = y^{\mathrm{U}}_{i_{n_v-1}gv} + y^{\mathrm{UP}}_{gv}$.
          Set $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g - y^{\mathrm{UP}}_{gv}$
        end if
      else if $\mu > \bar{s}^{\mathrm{U}}_g$ **then**
        Set $y^{\mathrm{UD}}_{gv} = \min\{\mu - \bar{s}^{\mathrm{U}}_g, y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v}\}$
        $\gamma =$ the position of the originating depot in route v node i_{n_v-1} belongs to
        $\rho = \max_{\epsilon=\gamma,\ldots,n_v-1}\{y^{U}_{i_\epsilon i_{\epsilon+1}v} + y^{N}_{i_\epsilon i_{\epsilon+1}v}\}$
        if $y^{\mathrm{UD}}_{gv} < \mu - \bar{s}^{U}_g$ **then**
          $\delta = \min\{k_1 - \rho, \lfloor \tau/(L + U) \rfloor, \mu - \bar{s}^{U}_g - y^{UD}_{gv}\}$
          $y^{UD}_{gv} = y^{UD}_{gv} + \delta$
        end if

48: **if** $y^{\mathrm{UD}}_{gv} > 0$ **then**
49: **if** $y^{\mathrm{B}}_{gv} = 0$ and $y^{\mathrm{NP}}_{gv} = 0$ **then**
50: Node g is inserted between i_{n_v-1} and i_{n_v} in route v.
51: **end if**
52: **if** $\delta > 0$ **then**
53: Set $y^{U}_{i_\epsilon i_{\epsilon+1} v} = y^{U}_{i_\epsilon i_{\epsilon+1} v} + \delta \ \forall \epsilon = \gamma, \ldots, n_v - 1$
54: **end if**
55: Set $y^{\mathrm{U}}_{i_{n_v-1} g v} = y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v}$ and $y^{\mathrm{U}}_{g i_{n_v} v} = y^{\mathrm{U}}_{i_{n_v-1} g v} - y^{\mathrm{UD}}_{gv}$.
56: Set $\bar{s}^{\mathrm{U}}_g = \bar{s}^{\mathrm{U}}_g + y^{\mathrm{UD}}_{gv}$
57: **end if**
58: **end if**
59: **if** $y^{\mathrm{B}}_{gv} = 0$, $y^{\mathrm{NP}}_{gv} = 0$, $y^{\mathrm{UP}}_{gv} = 0$, and $y^{\mathrm{UD}}_{gv} = 0$ **then**
60: Set $\tau = \tau_1$.
61: **end if**
62: **end if**
63: Set $d = d + 1$
64: **if** $d > |W|$ **then**
65: **if** $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} > 0$ **then**
66: $\gamma =$ the position of the originating depot in route v node i_{n_v-1} belongs to
67: Set $y^{\mathrm{B}}_{i_\gamma i_{\gamma+1} v} = k_2 - y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}$.
68: Set $y^{\mathrm{B}}_{i_\epsilon i_{\epsilon+1} v} = y^{\mathrm{B}}_{i_\epsilon i_{\epsilon+1} v} - y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v}$ $\forall \epsilon$ where $\epsilon = \gamma + 1, \ldots, n_v - 1$.
69: **end if**
70: $\omega = \min\{\sum_{j \in \mathcal{N}} \bar{s}^{\mathrm{N}}_j, k_2, \lfloor \tau/(H_1 + H_2 + H_3) \rfloor\}$
71: **if** $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, and $\omega > 0$ **then**
72: A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
73: Set $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = \omega$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$, $d = 1$.
74: **else if** $k_1 - y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} - y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$ **then**
75: A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
76: Set $y^{\mathrm{B}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1} i_{n_v} v} = 0$, $y^{\mathrm{U}}_{i_{n_v-1} i_{n_v} v} = 0$, $d = 1$.
77: **else if** $i_{n_v-1} = 0$ and $i_{n_v} = 0$ **then**
78: Remove i_{n_v} from the route.
79: **end if**
80: **end if**
81: **until** no more nodes $g \in W$ can be added without violating the constraints or $d > |W|$
82: Set $d = 1$.
83: **repeat**
84: Set $g = W_d$
85: Set $\tau_1 = \tau$, $\tau = \tau - t_{i_{n_v-1} g} - t_{g i_{n_v}} + t_{i_{n_v-1} i_{n_v}}$, and $\chi = 0$
86: Same as lines 29-49.
87: **if** $y^{\mathrm{UP}}_{gv} = 0$ and $y^{\mathrm{UD}}_{gv} = 0$ **then**
88: Set $\tau = \tau_1$.
89: **end if**
90: Set $d = d + 1$.
91: **if** $d > |W|$ **then**
92: Set $\Omega_1 = \{j \in \mathcal{N} : \mu > \bar{s}^{\mathrm{U}}_j, \mu = \arg\min_{e \in \Lambda} f_j(e, \bar{s}^{\mathrm{N}}_j)$, where $\Lambda = \{0, \ldots, c_j - \bar{s}^{\mathrm{N}}_j\}\}$
93: Set $\Omega_2 = \{j \in \mathcal{N} : \mu < \bar{s}^{\mathrm{U}}_j, \mu = \arg\min_{e \in \Lambda} f_j(e, \bar{s}^{\mathrm{N}}_j)$, where $\Lambda = \{0, \ldots, c_i - \bar{s}^{\mathrm{N}}_i\}\}$
94: Set $\rho_1 = \sum_{j \in \mathcal{N}} \max\{\mu - \bar{s}^{\mathrm{U}}_j, 0\}$ and $\rho_2 = \sum_{j \in \mathcal{N}} \max\{\bar{s}^{\mathrm{U}}_j - \mu, 0\}$

if $y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} + y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v} = 0$ and $|\Omega_1| > 0$ **then**
A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
$\omega = \min\{\max\{\rho_1 - \rho_2, 0\}, k_1, \lfloor \tau/(L+U) \rfloor\}$
Set $y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} = \omega$, $y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1}i_{n_v}v} = 0$.
Set $\tau = \tau - (L+U)\omega$, $d = 1$, and $\chi = 0$.
else if $y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} > 0$, $|\Omega_1| > 0$, and $\chi = 0$ **then**
Set $\chi = 1$ and $d = 1$.
else if $k_1 - y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} - y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v} = 0$ and $|\Omega_2| > 0$ **then**
A depot node is inserted between i_{n_v-1} and i_{n_v} in route v.
Set $y^{\mathrm{U}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{N}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{B}}_{i_{n_v-1}i_{n_v}v} = 0$, $y^{\mathrm{B}'}_{i_{n_v-1}i_{n_v}v} = 0$.
Set $\tau = \tau - (L+U)\omega$, $d = 1$, and $\chi = 0$.
else if $i_{n_v-1} = 0$ and $i_{n_v} = 0$ **then**
Remove i_{n_v} from the route.
end if
end if
until no more nodes $g \in W$ can be added without violating the constraints or $d > |W|$
end for