Main points

1. The Nature of Uncertainty
Life contains uncertain events. To navigate them, we think ahead about all possible outcomes before making decisions. Probability is the tool that helps turn uncertainty into something we can analyze.

2. Evaluating Risk
We judge whether a risk is worth taking by asking questions such as:
– What is the chance of success?
– What is the chance of failure?
This pushes us to quantify uncertainty instead of guessing.

3. Why Study Probability?
To estimate how likely each outcome is. Probability and statistics help us predict patterns, compare risks, and choose more rational decisions.

4. What Is Probability?
Probability is the numerical measure of how likely an event is to occur. It represents the "chance" or "likelihood" of a specific outcome.

5. What Is an Event?
An event is a specific outcome or a group of outcomes.
Examples:
– Getting heads when flipping a coin.
– Rolling a four on a six-sided die.

6. Comparing Events
We judge whether an event is "likely" or "unlikely" by measuring probability. Comparing probabilities shows us which event is more likely.

7. Numerical Expression of Probability
Probabilities can be written as fractions, percentages, or real numbers between 0 and 1.
– Probability = 1 → event is certain.
– Probability = 0 → event is impossible.
0 < P(A) < 1: The event is uncertain, and a higher value means a higher likelihood.

8. Basic Probability Formula
For an event A:
P(A) = (number of favorable outcomes) / (total number of possible outcomes)

"Favorable" means outcomes we are interested in.
"Sample space" is the full set of all possible outcomes.

Examples:
– Coin flip: P(Heads) = 1/2
– Rolling a 4 on a fair die: P(4) = 1/6
– Rolling a number divisible by 3 (3 or 6): P = 2/6 = 1/3

 # Why the probability is between 0 and 1 ?

because probability is a proportion, and proportions are always between 0 and 1.

You already have the perfect tool to see why: your basic probability formula.

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P(A) = (Number of Favorable Outcomes) / (Total Number of Possible Outcomes)
```

1. **The Numerator (Top Number):** This is a count of the outcomes you're interested in. It can't be negative, and at its very worst, it can be **0** (if none of the outcomes are the one you want).
2. **The Denominator (Bottom Number):** This is the count of *all* possible outcomes. It must always be **at least 1** (you can't have an experiment with zero possible outcomes).
3. **The Division (The Fraction):** You are dividing a smaller number (or an equal one) by a larger number (or an equal one). This is the key.

- **9. Independent Events**

Two events are independent if the occurrence of one does not affect the probability of the other.

The probability that **both** independent events A *and* B occur is the *product* of their individual probabilities.

$P(A \cap B) = P(A) \times P(B)$

Example:
P(Ace and Spade) = P(Ace) × P(Spade) — if drawn with replacement.

- `P(Ace) = 4/52 = 1/13`
- `P(Spade) = 13/52 = 1/4`
- `P(Ace and Spade) = (1/13) * (1/4) = 1/52`

What you studied today (summary list)
– Concept of uncertainty
– Evaluating risk
– Motivation for studying probability
– Definition of probability
– Definition of an event
– Likely vs unlikely events
– Probability values (0 to 1)
– Basic formula for P(A)
– Sample space
– Classic examples with coins and dice
– Independent events

A clean, permanent summary you can keep

Probability — Essential Overview

Probability quantifies uncertainty. It measures how likely an event is to occur. An event is a specific outcome or a group of outcomes from a sample space.

Probabilities are expressed as numbers between 0 and 1. A probability of 1 means the event is certain; a probability of 0 means impossible. Higher values reflect higher likelihood.

The foundational rule is:
P(A) = favorable outcomes / total possible outcomes
Example: Rolling a 4 on a fair six-sided die → 1/6.

Independent events do not influence each other. For independent events A and B:
$P(A \cap B) = P(A) \times P(B)$

Probability helps evaluate risk, compare outcomes, and make better decisions under uncertainty.

Concept	What it is	Formula / Key Idea	Example
Probability (P(A))	Likelihood of an event A.	A number between 0 and 1.	P(Rain) = 0.3 means a 30% chance.
Sample Space	All possible outcomes.	A set.	Coin: {H, T}. Die: {1,2,3,4,5,6}
Event	The outcome(s) we care about.	A subset of the sample space.	Rolling an even number: {2,4,6}
Basic Formula	How to calculate P(A).	`P(A) = (Favorable Outcomes) / (Total Outcomes)`	P(Heads) = 1/2
Independent Events	One doesn't affect the other.	`P(A and B) = P(A) * P(B)`	Flipping a coin twice.
Impossible Event	Will never happen.	`P(A) = 0`	Rolling a 7 on a standard die.
Certain Event	Will always happen.	`P(A) = 1`	The sun will rise tomorrow.