Exploring the Homogeneity of Disordered Minimal Surfaces

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Bicontinuous geometries, both ordered and amorphous, are commonly found in many soft matter systems. For ordered phases with crystalline symmetry, periodic bicontinuous minimal surfaces provide established models, including Schoen's Gyroid and Schwarz's Diamond surfaces. Conversely, a minimal surface model for amorphous phases has been lacking. Here, we study models for amorphous bicontinuous phases, such as sponge phases. We use the Surface Evolver [1], with a novel topology-stabilizing minimization routine, to generate discretized minimal surfaces from initially coarse polyhedral meshes. We demonstrate that minimal companion surfaces to the amorphous Diamond network exist, providing a structural model for sponge mesophases.

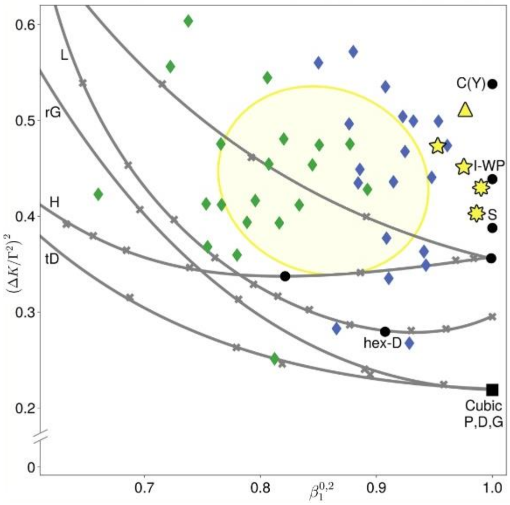

Figure 1 Curvature variance and isotropy of several amorphous diamond surface subsamples, compared to known surface families.

As per Hilbert's Embedding Theorem, the Gauss curvature of any curved minimal surface cannot be constant. An analysis of variances $(\Delta K/\Gamma^2)^2$ of the Gaussian curvature [2] of both periodic and aperiodic surfaces reveals that the amorphous Diamond minimal surface has no substantial, interior long-wavelength curvature variations (see Fig. 2). However, the total variance of Gauss curvature for all considered amorphous minimal surface models is significantly larger than that of the cubic Diamond and Gyroid surfaces (see Fig. 1). The functional form of the curvature distributions and the duality properties of the two labyrinthic components afford pertinent insight into the origin of curvature variations. We also consider these surfaces' isotropy indices $\beta_1^{0,2}$ [3]. We find that the amorphous Diamond surfaces are highly isotropic (see Fig. 1), typically producing $\beta_1^{0,2} > 0.95$ with $\beta_1^{0,2} = 1$ corresponding to completely isotropic surfaces.

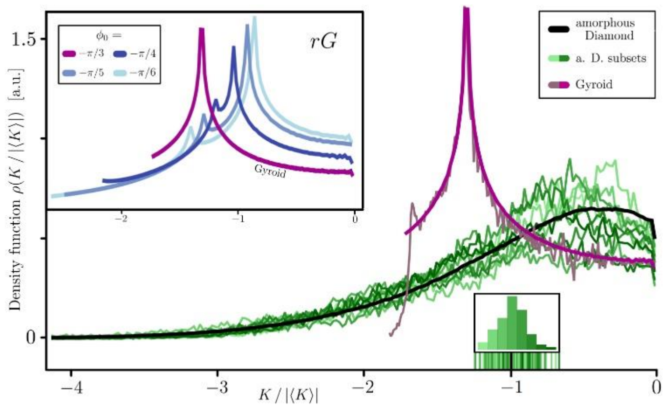

Figure 2 Curvature distributions of amorphous Diamond surface subsamples next to Gyroid deformation families.

Our work demonstrates the degree to which the curvature homogeneity of the cubic Gyroid and Diamond surfaces is superior to their entropy-favored amorphous counterparts. It thereby provides a general geometric result of relevance to bicontinuous structure formation in soft matter and biology across all length scales, relevant to both ordered and amorphous phases.

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