Discrete Gyroid Structures: Defect-Driven Tiling and Analogies with Zeolite Frameworks

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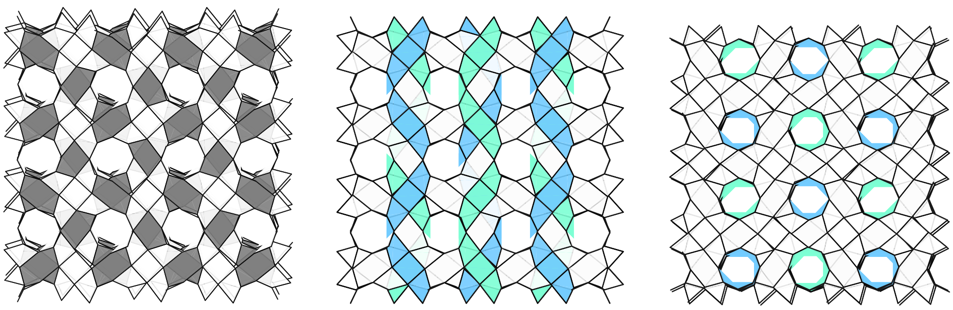
This presentation explores the connection between discrete gyroid structures with defects and the framework of zeolite structures. Both structures exhibit complex topologies and periodic characteristics that are of interest in materials science.

Gyroidal structures, discovered by Alan Schoen in 1970, are known for their continuous, non-self-intersecting surfaces that divide space into two interpenetrating labyrinths. These structures are notable for their minimal surface properties, making them an intriguing subject of study in both theoretical and applied sciences. Beyond their continuous form, gyroidal structures can also be discretized. This process involves transforming the smooth surface into a network of discrete points or tiles, which allows for easier computational modeling and practical application.

Our approach to discretizing gyroidal structures starts from regular tiling (planar tiling). By introducing defects into these regular tilings, we can describe the gyroidal surface in a more controlled manner. This method allows us to connect the polygons used in tiling with the Euler-Poincaré formula, providing a quantitative relationship between the number of polygons and the genus of the surface. Regular tiling typically involves three types of polygons: triangles, squares, and hexagons. Among these, triangles and hexagons are dual to each other, and the use of hexagons has been previously discussed in the context of Chern-Chuang's work [1]. Therefore, our focus in this study is on square tiling.

To ensure that the tiling can adhere to a curved surface, it is necessary to use squares along with defect hexagons. This combination ensures that each vertex in the tiling is tetravalent (4-degree). Moreover, zeolites have a silicon atom framework where each silicon atom is connected to four oxygen atoms, forming a tetrahedral network. This tetravalent connectivity leads to a wide variety of zeolite structures, many of which share similarities with gyroidal structures such as BSV and ANA, demonstrating their relevance and applicability.



(a) (b) (c)

Figure 1 Structure diagram of zeolite BSV. (a) highlighting hexagonal defects. (b) highlighting ribbons formed by squares, colored to distinguish left- and right-handed helices. (c) Top view of (b) showing the hollow space formed by helices.

[1] C. Chuang, B. Y. Jin, W. C. Wei, C. C. Tsoo. Proceedings of Bridges: Mathematical Connections in Art, Music, and Science, pp. 503-506 (2012). 10.13140/2.1.4284.4807.