Construction and Classification of Hyperbolic Diagrams and their Triply Periodic Weavings & Polycatenanes via Mapping to the Gyroid

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This poster presents a framework for studying the topology of triply periodic tangles embedded in the thickened gyroid, referred to as ***G-TP tangles***, which play a key role in materials science and chemistry. G-TP tangles arise, for example, in self-assembled materials, where the relationship between structure and functions is influenced by the underlying topology. The thickened gyroid serves as a periodic scaffold, making it ideal for modeling molecular structures and polymeric systems.

Some G-TP tangles can be related to graphs on the hyperbolic plane, where hyperbolic tilings are mapped onto triply periodic minimal surfaces (TPMS) using the symmetries and periodicities inherent in hyperbolic space to embed entangled structures into the gyroid scaffold [1], as illustrated in Figure 1.

Figure 1. A tiling of the hyperbolic plane, its mapping a fundamental domain of the gyroid and its G-TP tangle (from [1]).

In this poster, we extend this work by presenting a method for constructing and classifying G-TP tangles. We begin by considering a graph on a genus 2 surface Σ. Using the *polygonal link method*, we transform this graph into a link diagram on Σ [2]. By selecting appropriate characteristic curves, the link diagram is unfolded into a tangle diagram in a hyperbolic octagon, which is then lifted into a periodic tangle diagram on the hyperbolic plane, as illustrated in Figure 2. The above method of [1] is then used to map this periodic diagram onto the gyroid. Finally, this diagram can be lifted to the thickened gyroid, resulting in a G-TP tangle embedded in Euclidean 3-space. This study can be generalized to other TPMS.

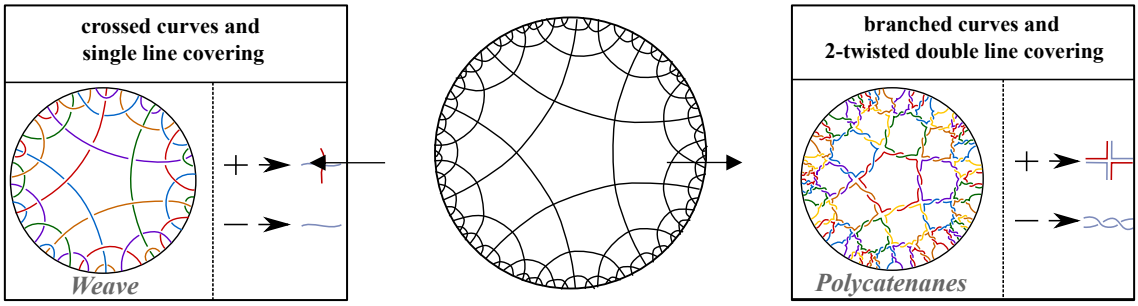

Figure 2. A periodic tiling of the hyperbolic plane to which two different polygonal link methods are applied [2].

To classify these G-TP tangles, we use the *extended Jones polynomial* for link diagrams on Σ representing unit cells of periodic hyperbolic diagrams, as defined in [3]. To account for global isotopies of the G-TP tangles, invariance under Dehn twists on Σ is considered, which correspond to volume-preserving shearing of the G-TP tangles, as well as the choice of unit cells of different volumes.

[1] M.E. Evans, V. Robins, S.T. Hyde, *Acta Crystallographica Section A*, **69**, 262–275 (2013).
[2] M. Fukuda, M. Kotani, S. Mahmoudi, *arXiv:2206.12168* (2024).
[3] S. Mahmoudi, arXiv: *2009.13896* (2024). (to appear in *Communications of the Korean Mathematical Society*)