Curvature Estimation based on Distance Conversion of a 3D image

Yukihiro Nishikawa* and Suguru Kikuchi

Kyoto Institute of Technology, 1 Matsugasaki Hashigami-cho, Sakyo-ku, Kyoto 606-8585, Japan

*email: kiro@kit.ac.jp

Surface curvatures has been of importance in understanding the complicated network structures. Nevertheless, it is not simple to measure the curvatures in the experimental systems. We has proposed two methods to measure the curvatures from the 3d images [1,2], and one more is also proposed by Lopez-Barron et al.[3]. All methods concerns the surface rendering process, and hence the "hard" binarization is necessary in advance to the surface rendering. Binarization usually accompanies *delicate* noise reduction. Therefore the less-noise 3d image is necessary for measuring accurate curvatures. Instead of that, we propose a new method based on distance conversion, which also needs the binarization but is rather insensitive to the noises. Furthermore, the method is theoretically applicable to other physical measurements, such as BET.

In the Parallel Surface Method (PSM)[1], the area of the parallelly-constructed surface (say, parallel surface) at distance t, S(t), is related to the mean (H) and Gaussian (K) curvatures as,

$$S(t) = S(0)(1 + 2Ht + Kt^2). \qquad (1)$$

Immediately, we can obtain the integration form of eq. (1) as,

$$V(t) = \int_0^t S(t)dt = S(0)\left(t + Ht + \frac{K}{3}t^3\right), \qquad (2)$$

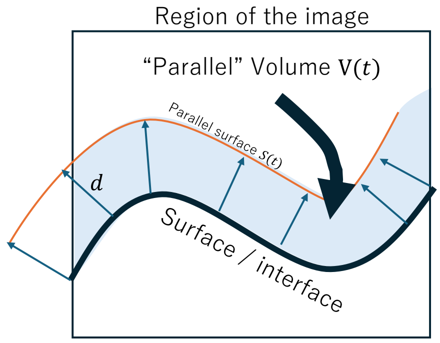

Figure 1 Schematic illustration of $V(t)$

where $V(t)$ is the volume between the original surface ($t = 0$) and the parallel surface at distance t. Figure 1 shows the schematic illustration of $V(t)$. For simplicity fig. 1 shows the 2D case. In $V(t)$, the distance to the original surface is less than t. In the digital images, the distance can be estimated via distance conversion [4]. Then now we obtain a new method to measure the surface curvatures for 3d images: (i) binarizing the image, (ii) applying the distance conversion, (iii) counting the voxels less than t and (iv) fitting thus obtained $V(t)$ with eq. (2).

Since the distance conversion is slightly insensitive to the image noises, the new method is expected to be more robust than other methods. $V(t)$ is essentially the volume of the layer of thickness t on the original surface, which can be obtained besides the 3d image processing. BET measurement is the one, in which an inert gas is condensed on the surface under high pressure. The thickness of the condensed gas layer is considered to be constant everywhere on the surface which is exactly same definition as $V(t)$.

In the 3d image analysis version, we found that the method needs the stochastic correction. As seen fig. 1, some parts of the parallel surface often protrude the region of the image where are also necessary for $V(t)$ but are not counted in the distance conversion. To correct this situation, we estimated the probability of $V(t)$ outside the image are estimated and used in the fitting by eq. (2). The new method is critically examined by using model images including gyroid.

[1] Y. Nishikawa, H. Jinnai, T. Koga, T. Hashimoto, S. T. Hyde, Langmuir, **14**, 1242 (1998)
[2] Y. Nishikawa, T. Koga, T. Hashimoto, H. Jinnai, Langmuir, **17**, 3254 (2001).
[3] C.R. Lopez-Barron, C. W. Macosko. Langmuir **25** 9392 (2009).
[4] T. Saito, J. Toriwaki, Pat. Rec., 27 1551 (1994).