CS 464 – Introduction to Machine Learning

Homework 2

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QUESTIONS

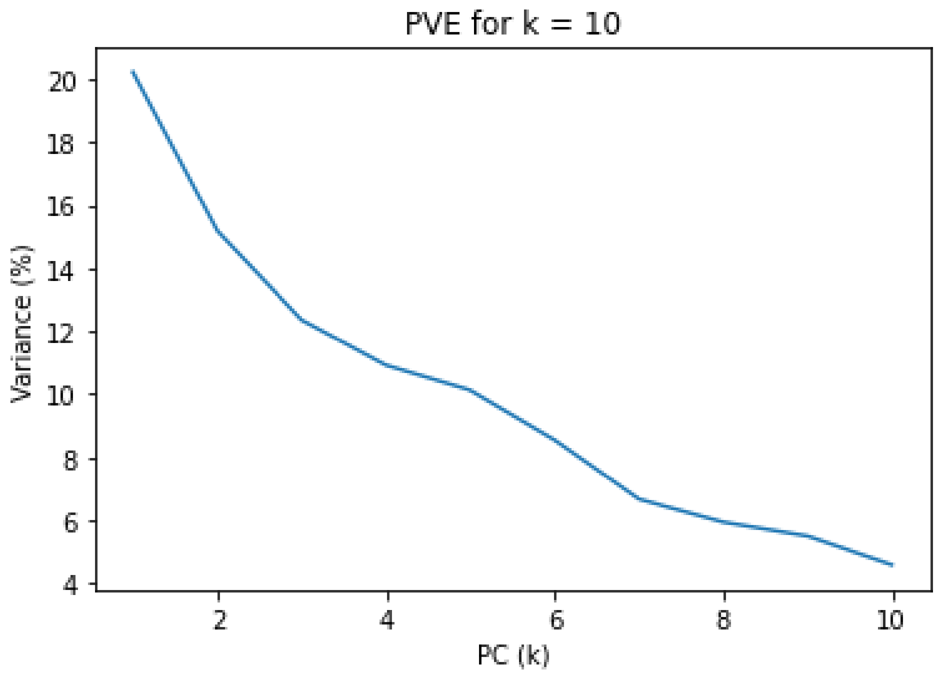
1. PCA & Digits

(1.1)

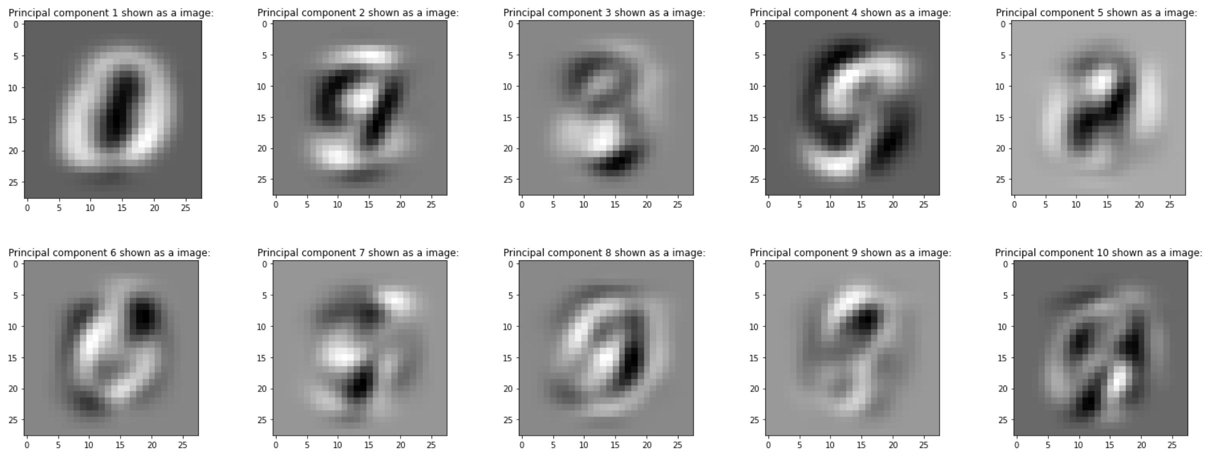
The PVE for k = 10 is given as in the following graph. It can be said that the variance decreases as the principal component analysis is made for more principal components. Since principal components are basically the eigenvectors of the covariance matrix of the dataset, eigenvectors sorted with ascending order with respect to their eigenvalues will have greater variance due to the proportion of variance captured by kth PC given in the equation below.

$$PV_k = \frac{\lambda_k}{\sum_i \lambda_i}$$

Where λ_i corresponds to an eigenvalue of the covariance matrix, and λ_k is the eigenvalue of the kth eigenvalue. Hence, the eigenvector with an eigenvalue of larger index will have hence less variance because it has a lesser value considering that eigenvalues had been sorted in ascending order. When all the proportion of variances are plotted, the following proportion of variance explained (PVE) plot is obtained:

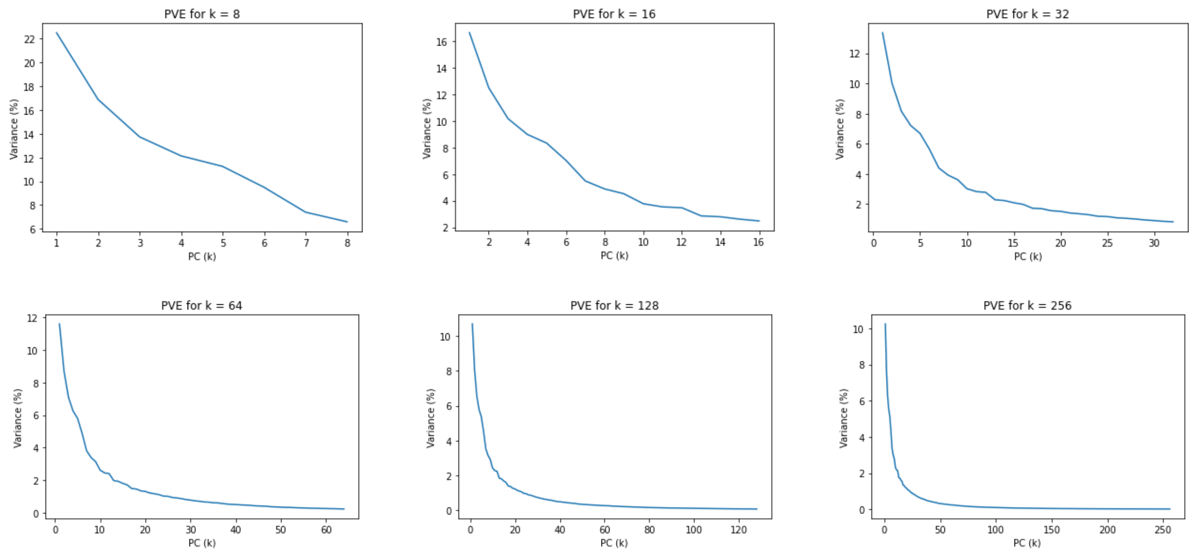


Each principle component can be reshaped in a 28 by 28 matrix. Since the dataset contains 28x28 pixel images of different digits, such a principle component would correspond to a deconstructed version of the key features of any digit. Since this PCA was done for all digit images in the dataset, the eigenvectors would correspond to the general of structure of a digit's image. These eigenvectors correspond to images as seen below. These images do not make sense at first glance, but is still useful information when reconstructing a digit, especially if PCA is done for a single, meaningful image.



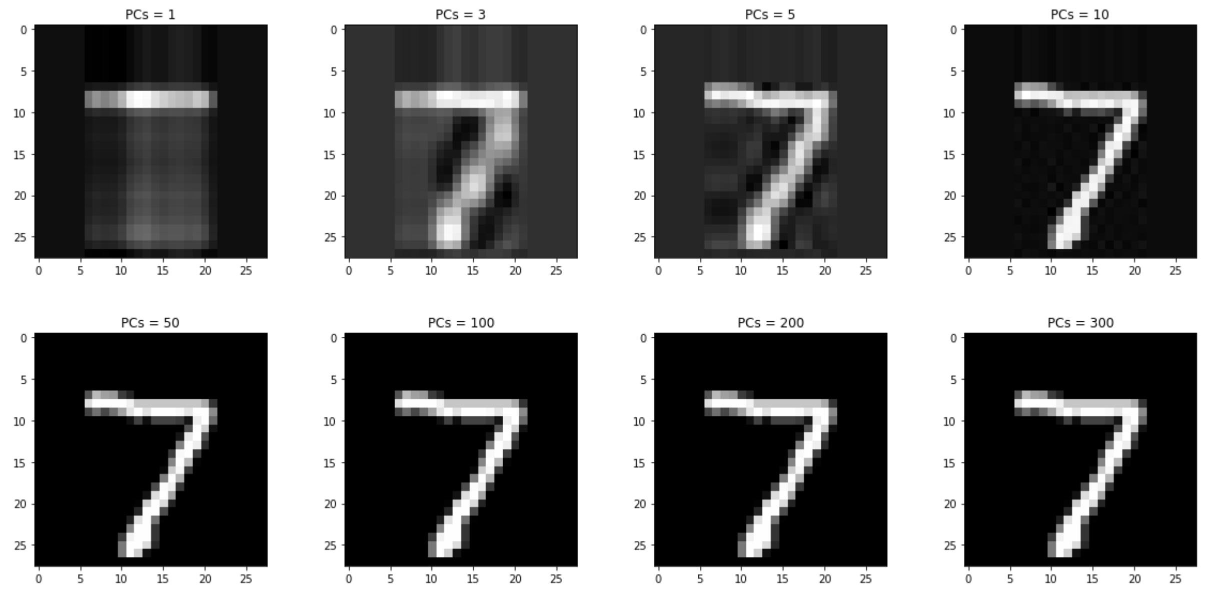
(1.2)

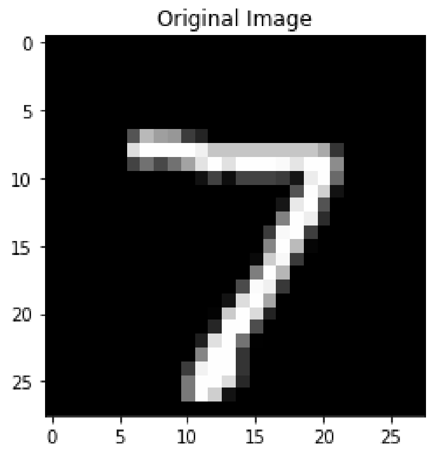
The PVE for the corresponding principle component numbers are given in the following results. It is clearly seen as the number of principal components increase, the proportional variance decreases.



(1.3)

As explained in (1.1), the principle components, or rather eigenvalues, represent the general features of an image, which when dot multiplied with the reduced image, can reconstruct the original image. It is expected that the larger the principle component number, the more accurate results will be obtained. This is clearly seen if we take the example of a single digit image of the dataset. The following results are obtained with PCA analysis of a digit with different number of principle components. As expected, since the variance decreases almost to 0 after reaching a certain point, the image gets reconstructed more accurately, but naturally the bigger the number of principle components, the reconstructed image will be more complex and have a larger dimension. In order to reduce the image without losing too much of the original shape, one can use 10 or 50 principle components, as these images closely represent the original one, despite being reduced in dimension.





2. Linear and Polynomial

(2.1)

The matrix y corresponds to the matrix labels, ground truths or actual values of the data in the training dataset, expressed as below. Assume that there are n different data with their labels specified.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix}$$

The matrix X corresponds to the modified matrix of features of the data in the training dataset, expressed as below. Assume that there are n different data with m features each. We add the row of 1's because since this is linear regression, there needs to be a constant value, which can be only obtained if 1's are used.

$$X = \begin{bmatrix} 1 & x_{1,1} & \cdots & x_{1,m} \\ 1 & x_{2,1} & \cdots & x_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n,1} & \cdots & x_{n,m} \end{bmatrix}$$

As for the β value, it will be used to represent the trained coefficient for the linear regression model. Assuming that we had m different features, the β matrix will represent the coefficients of the linear regression model in the mth dimension.

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_m \end{bmatrix}$$

Hence, the linear regression model's loss can be defined as, where y is the actual values, and $X\beta$ is the predicted value as a multivariate regression model, given in open form as well.

$$J_n = \|y - X\beta\|^2 = (y - X\beta)^T(y - X\beta)$$
$$= y^Ty - 2\beta^TX^Ty + \beta^TX^TX\beta$$

The aim is to minimize this loss. Minimizing this loss will give the general closed form solution for the multivariate regression model, which can be achieved by taking the partial derivative of J_n with respect to β, and setting it to 0. The calculations are given below. Note that some of the derivative relationships for matrices and their transform were used in the calculations, but is not specified here.

$$\frac{\partial J_n}{\partial \beta} = 0 - \frac{\partial}{\partial \beta}(2\beta^TX^Ty) - \frac{\partial}{\partial \beta}(\beta^TX^TX\beta) = 0$$

$$-2X^T(-y + X\beta) = 0$$

From this equation, if we find the value of β with respect to the other values, we obtain the trained coefficients for the multinomial linear regression, which is the solution we seek.

$$\hat{\beta} = (X^TX)^{-1}X^Ty$$

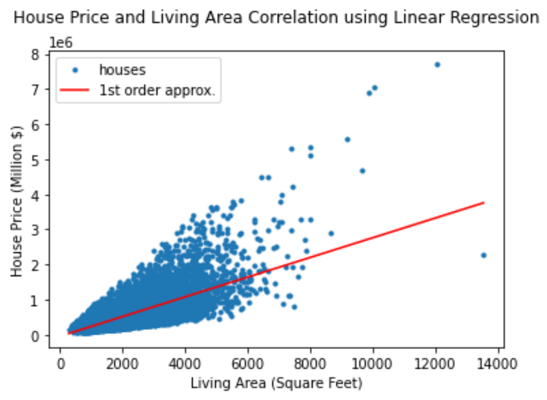
(2.2)

For the given dataset, the rank was found to be 5, which means that $X^T X$ is 5 by 5 dimensional matrix. Since we have one column corresponding to 0th dimension, this tells that the best suited linear regression model is 4 dimensional. This makes sense as there are 4 different features associated to a datum.

(2.3)

Trained Beta Coefficients for 1st Order Linear Regression are:
b0 = -43580.743094474354
b1 = 280.6235678974484
The Mean Square Error for 1st Order LR is: 201200997618.10632

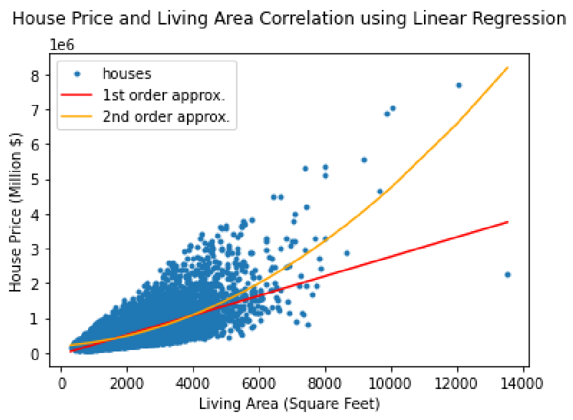
This regression model shows that there are many houses accumulating in the price range of 0-1 million dollars and to 6000 square feet living area, as the slope is not too large.



(2.4)

Trained Beta Coefficients for 2nd Order Linear Regression are:
b0 = 199222.27930548566
b1 = 67.99409468579292
b2 = 0.038581260937201815
The Mean Square Error for 2nd order LR is: 206577201240.17664

This regression model shows again that there are many houses accumulating in the price range of 0-1 million dollars and to 6000 square feet living area, however the error is higher in this model, hence we can conclude that it disregards some frequent instances out of the above mentioned range, causing an accumulation in error.



3. Logistic Regression

(3.1)

```
Confusion Matrix for learning rate = 1e-05 :
Number of True Positives:  83
Number of True Negatives:  1100
Number of False Positives:  2
Number of False Negatives:  15
Accuracy is  98.58333333333333  %


Confusion Matrix for learning rate = 0.0001 :
Number of True Positives:  83
Number of True Negatives:  1100
Number of False Positives:  2
Number of False Negatives:  15
Accuracy is  98.58333333333333  %


Confusion Matrix for learning rate = 0.001 :
Number of True Positives:  84
Number of True Negatives:  1099
Number of False Positives:  3
Number of False Negatives:  14
Accuracy is  98.58333333333333  %


Confusion Matrix for learning rate = 0.01 :
Number of True Positives:  88
Number of True Negatives:  1091
Number of False Positives:  11
Number of False Negatives:  10
Accuracy is  98.25  %

Confusion Matrix for learning rate = 0.1 :
Number of True Positives:  89
Number of True Negatives:  1071
Number of False Positives:  31
Number of False Negatives:  9
Accuracy is  96.66666666666667  %
```

It seems that the learning rate = 0.001 gives the best results, as the accuracy and precision rates are as high as small rates, and computationally less expensive than the small rates, and there is no decrease in precision like in the larger rates.

=> Performance Metrics for Full Batch Gradient Ascent are:

```
Number of True Positives:  84
Number of True Negatives:  1099
Number of False Positives:  3
Number of False Negatives:  14
Accuracy is 98.58333333333333 %
Precision is 96.55172413793103 %
Recall is 85.71428571428571 %
Negative Predictive Value is 98.74213836477988 %
False Positive Rate is 0.27223230490018147 %
False Discovery Rate is 3.4482758620689653 %
F1 measure is 0.908108108108108
F1 measure is 0.9655172413793103
```

Total time elapsed for full batch gradient ascent is 0.3534 seconds. Full batch analysis is usually faster, however it may be prone to errors as it might have gotten stuck in a local extremum. Mini batch and stochastic may also be used, but they are normally computationally more expensive.

(3.2)

Performing the Mini Patch Gradient Ascent for Batch Size = 100, the following Performance Metrics are obtained:

```
Number of True Positives:  84
Number of True Negatives:  1099
Number of False Positives:  3
Number of False Negatives:  14
Accuracy is 98.58333333333333 %
Precision is 96.55172413793103 %
Recall is 85.71428571428571 %
Negative Predictive Value is 98.74213836477988 %
False Positive Rate is 0.27223230490018147 %
False Discovery Rate is 3.4482758620689653 %
F1 measure is 0.908108108108108
F1 measure is 0.9655172413793103
```

Total time elapsed for mini batch gradient ascent is 11.6631 seconds

The results received here are the same as the full batch gradient ascent, so it seems that the results are accurate. However note that these computations were done for 1000 iterations. When less iterations were selected, (n=10), it seemed that mini batch gradient ascent was more effective. So the same results can be obtained in the batch gradient ascent with less iteration.

=> Performing the Stochastic Gradient Ascent, the following Performance Metrics are obtained:

```
Number of True Positives:  84
Number of True Negatives:  1099
Number of False Positives:  3
Number of False Negatives:  14
Accuracy is 98.58333333333333 %
Precision is 96.55172413793103 %
Recall is 85.71428571428571 %
Negative Predictive Value is 98.74213836477988 %
False Positive Rate is 0.27223230490018147 %
False Discovery Rate is 3.4482758620689653 %
F1 measure is 0.908108108108108
F1 measure is 0.9655172413793103

Total time elapsed for stochastic gradient ascent is 124.3753 seconds
```

The result here is again the same, however it is too computationally expensive, given that it took over 2 minutes to complete. Mini-batch should be preferred compared to this method. Note that stochastic gradient ascent is the same thing as mini-batch gradient ascent with batch size =1, so the same function was used by altering the batch size to 1.

(3.3)

There is a possibility that the given datasets might be uneven; that is, for a binary classification example like this, for example, there might be an unbalanced number of True and False values in the dataset, which may affect the training process negatively. The accuracy, precision and recall measure unfortunately may give misleading results, telling that the accuracy is very high, but in truth, the model might have become too rigid, and when exposed to newer test data, the model fail may give bad results. The F1 and F2 measures may be more informative regarding this, as it allows us to weigh the precision and recall, and allows us to receive a measure that considers both these facts, so basically they might be useful when determining if there is balance between recall and precision, and by what factor. False positive rate, FPR, determines how much data that is actually negative has been classified incorrectly among all actually negative values. On the other hand, FPR may be used with recall to determine the Receiver Operating Characteristic curve of the data. This might be helpful in determining how conservative or liberal our classifier might be; which is a measure of determining how evenly distributed the TP, TN, FN and FP values are. Negative Predictive Value, NPV, may be useful if we want to determine how accurately we can determine that a negative result is actually correct among all negatively classified objects. This is an important measure as if this is too low, we'd be classifying data instances that are actually positive as negative, and for this example, and these people would be able to get away with tax fraud. False Discovery Rate is also another important metric, because it determines how much data we've classified as positive among those who are actually negative, among all data that has been predicted as positive. This would mean that, if this value is too high, for this example, we'd be accusing the wrong people of tax fraud. This is actually the compliment probability of precision.

CODES

Initialization for all Questions

```
#------------------------------INITIALIZATION----------------------------------

from google.colab import drive
drive.mount('/content/gdrive')

!ls /content/gdrive/My\ Drive/Dora/Bilkent/CS464/HW2 # Use YOUR OWN DIRECTORY!!


import os
import csv
import math
import random
import operator
import pdb
import time

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.image as mpimg

from timeit import default_timer as timer
from pylab import imshow,title,figure,show, gray


np.random.seed(123)

root = '/content/gdrive/My Drive/Dora/Bilkent/CS464/HW2'
```

Code for Question 1

```
#------------------------------QUESTION 1-------------------------------------

#-------------------------Question 1.1----------------------------

print("\n------------------------- QUESTION 1 ------------------------\n" )
digits_csv = os.path.join(root, 'digits.csv')

#read the csv files with the first column being the header
pd_digits = pd.read_csv(digits_csv, header=0)
pd_digits_new = pd_digits.drop(columns = "label")
digits = pd_digits_new.to_numpy()

#uncomment the option below to see all the columns clearly.
#pd.set_option('display.max_columns', 28)
pd.reset_option('display.max_columns')

#PCA as a function, returns eigenvlaues, principle component matrix, and the reduced matrix
def PCA(data, k_pcomponents) :

    #computing eigenvalues and eigenvectors of the covariance matrix
    data_meaned = (data - np.mean(data.T,axis=1)).T # subtract the mean (along columns)
    eigenvalues, eigenvectors = np.linalg.eigh(np.cov(data_meaned))

    p = np.size(eigenvectors,axis=1)
    sorted_index = np.argsort(eigenvalues)[::-1] #sort eigenvalue indexes in ascending order

    #sorting the eigenvectors according to the sorted eigenvalue indexes
    eigenvectors = eigenvectors[:,sorted_index]  # sorting eigenvectors
    eigenvalues = eigenvalues[sorted_index] # sorting eigenvalues
    if k_pcomponents < p and k_pcomponents >= 0:
        eigenvectors = eigenvectors[:,range(k_pcomponents)] #obtaining PCs
        eigenvalues = eigenvalues[range(k_pcomponents)]

    data_reduced = np.dot(eigenvectors.T, data_meaned) #projectioning data into the reduced space
    return eigenvalues, eigenvectors, data_reduced

def PVE(principal_eigenvalues) :

    sum_eigenvalues = principal_eigenvalues.sum(axis=0)
    variance_matrix = principal_eigenvalues/sum_eigenvalues
    #each column in variance_matrix corresponds to the PVE of a PC
    return variance_matrix

def Reshape28x28(principal_components, k_pcomponents) :

    principal_components = principal_components.T
    reshaped_matrix = []

    for k in range(k_pcomponents) :
        square_component = principal_components[k].reshape(28, 28)
        reshaped_matrix.append(square_component)

    return reshaped_matrix


k_pcomponents = 10
digits_eigenvalues, digits_pcomponents , digits_reduced = PCA(digits, k_pcomponents)
digits_variance = PVE(digits_eigenvalues)
reshaped_pcomponents = Reshape28x28(digits_pcomponents, k_pcomponents)

print("\nPrincipal Components Matrix, where columns refer to each principle component:")
display(pd.DataFrame(digits_pcomponents)) #transform PC's into database
print("\nReduced Matrix")
display(pd.DataFrame(digits_reduced))
print("\nPrincipal Values Explained Matrix, rows refer to each principle component's PVE:")
display(pd.DataFrame(digits_variance))
print()

#this for loop is for visualizing each principal component as a 28 by 28 matrix.
for k in range(k_pcomponents) :
    print("\nPrincipal component {} reshaped as a 28x28 matrix: ".format(k+1))
    display(pd.DataFrame(reshaped_pcomponents[k]))

print()



myarr = []
for j in range(k_pcomponents):
  myarr.append(j+1)

plt.plot( myarr , digits_variance.transpose()*100)
plt.title("PVE for k = {}".format(k_pcomponents))
plt.ylabel('Variance (%)')
plt.xlabel('PC (k)')
plt.show()

#displaying eigenvectors of the dataset
for k in range(k_pcomponents) :


    figure()
    imshow(reshaped_pcomponents[k])
    title("Principal component {} shown as a image: ".format(k+1))


show()



#-------------------------Question 1.2-----------------------------

k_matrix = [8, 16, 32, 64, 128, 256]
variance = [0]*6

for i in range(len(k_matrix)):
    peigens, pcomponents , reduced = PCA(digits, k_matrix[i])
    variance[i] = PVE(peigens)
    num = k_matrix[i]
    arr = []
    for j in range(num):
        arr.append(j+1)

    print()
    plt.plot( arr , variance[i].transpose()*100)
    plt.title("PVE for k = {}".format(num))
    plt.ylabel('Variance (%)')
    plt.xlabel('PC (k)')
    plt.show()

#-------------------------Question 1.3----------------------------


print("\nImage Reconstructions are: \n")


first_image = digits[0].reshape(28, 28)
k_pcomponents = [1,3,5,10,50,100,200,300]


def Reconstruction(image28x28, k_pcomponent):


    eigenvalues, pcomponents , reduced = PCA(image28x28, k_pcomponent) # image reconstruction
    recon = np.dot(pcomponents, reduced).T + np.mean(image28x28,axis=0)


    return recon


first_digit = digits[0].reshape(28, 28)


for k_pcomponent in k_pcomponents:


    recon_image = Reconstruction(first_digit, k_pcomponent)
    figure()
    imshow(recon_image)
    title('PCs = '+str(k_pcomponent))


figure()
imshow(first_digit)
title('Original Image')
show()
```

Code for Question 2

```
#-------------------------------QUESTION 2-------------------------------------

print("\n------------------------- QUESTION 2 -------------------------\n" )
home_features = os.path.join(root, 'question-2-features.csv')
home_prices = os.path.join(root, 'question-2-labels.csv')

df_home_features = pd.read_csv(home_features, header=0)
df_home_prices = pd.read_csv(home_prices, header=0)

array_home_features = df_home_features.to_numpy()
array_home_prices = df_home_prices.to_numpy()
ones_array = np.array([np.ones(len(array_home_features[:]))])

array_concat_features = np.concatenate((ones_array.T, array_home_features), axis = 1)

xTx = np.matmul(array_concat_features.transpose(), array_concat_features)
yTy = np.matmul(array_home_prices.transpose(), array_home_prices)

rank_features = np.linalg.matrix_rank(xTx)
rank_labels = np.linalg.matrix_rank(yTy)
print('The rank of X transpose times X is: {}'.format(rank_features))

def SolveClosedForm(features, labels):
    first_matrix = np.matmul(features.transpose(), features)
    first_matrix = np.linalg.inv(first_matrix) # this calculates ((x^T)x)^(-
1), however it is a constant, so it cannot be inverted, since its the same
    second_matrix = np.matmul(features.transpose(), labels) # this calculates (x^T)y
    predicted_coeffs = np.matmul(first_matrix, second_matrix) # predicted beta set, since both are just integers, th
is operation is ok.
    return predicted_coeffs


#linear regression to fit into a first order equation
print("\n1ST ORDER LINEAR REGRESSION ANALYSIS:")

array_concat_sqftliving = array_concat_features[:,[0,1]]

beta_prediction = SolveClosedForm(array_concat_sqftliving, array_home_prices)
b0 = beta_prediction[0][0]
b1 = beta_prediction[1][0]
print("\nTrained Beta Coefficients for 1st Order Linear Regression are: \n b0 = {} \n b1 = {} \n ".format(b0, b1))

living_area = array_concat_sqftliving[:,1]
sorted_livingarea = np.sort(living_area)
predicted_prices = b0+b1*sorted_livingarea

plt.plot(living_area, array_home_prices, '.')
plt.plot(sorted_livingarea, predicted_prices, 'red')
plt.title("House Price and Living Area Correlation using Linear Regression\n")
plt.ylabel('House Price (Million $)')
plt.xlabel('Living Area (Square Feet)')
plt.legend(["houses", "1st order approx."])
plt.show()

MSE = np.square(np.subtract(array_home_prices, predicted_prices)).mean()

print('\nThe Mean Square Error for 1st Order LR is : {} \n'.format(MSE))



#linear regression to fit into a second order equation
print("\n2ND ORDER LINEAR REGRESSION ANALYSIS:")

array_sqftliving_squared = np.array([np.square(array_concat_sqftliving[:,1])])
array_concatsqr_sqftliving = np.concatenate((array_concat_sqftliving, array_sqftliving_squared.T), axis = 1)

sqrbeta_prediction = SolveClosedForm(array_concatsqr_sqftliving, array_home_prices)
b0 = sqrbeta_prediction[0][0]
b1 = sqrbeta_prediction[1][0]
b2 = sqrbeta_prediction[2][0]
print("\nTrained Beta Coefficients for 2nd Order Linear Regression are: \n b0 = {} \n b1 = {} \n b2 = {} \n ".format
(b0, b1, b2))

living_area_sqr = np.square(living_area)
sorted_livingareasqr = np.sort(living_area_sqr)
sqrpredicted_prices = b0 + b1*sorted_livingarea + b2*sorted_livingareasqr

plt.plot(living_area, array_home_prices, '.')
plt.plot(sorted_livingarea, predicted_prices,'red')
plt.plot(sorted_livingarea, sqrpredicted_prices, 'orange' )
plt.title("House Price and Living Area Correlation using Linear Regression\n")
plt.ylabel('House Price (Million $)')
plt.xlabel('Living Area (Square Feet)')
plt.legend(["houses", "1st order approx.", "2nd order approx."])
plt.show()

MSE = np.square(np.subtract(array_home_prices, sqrpredicted_prices)).mean()

print('\nThe Mean Square Error for 2nd order LR is : {}'.format(MSE))
```

Code for Question 3

```
#------------------------------QUESTION 3-------------------------------------

print("\n------------------------- QUESTION 3 -------------------------\n" )

train_features = os.path.join(root, 'question-3-features-train.csv')
train_labels = os.path.join(root, 'question-3-labels-train.csv')
test_features = os.path.join(root, 'question-3-features-test.csv')
test_labels = os.path.join(root, 'question-3-labels-test.csv')

df_train_features = pd.read_csv(train_features, header=0)
df_train_labels = pd.read_csv(train_labels, header=0)
df_test_features = pd.read_csv(test_features, header=0)
df_test_labels = pd.read_csv(test_labels, header=0)

concat_train = pd.concat([df_train_labels, df_train_features] , axis = 1)
concat_test =  pd.concat([df_test_labels, df_test_features] , axis = 1)

array_train_features = concat_train.to_numpy()[:,1:30]
array_train_labels = concat_train.to_numpy()[:,0]

def sigmoid(score):
    return 1/(1 + np.exp(-score))

def Performance(weights, test_features, test_label, best_one = False):

    w0 = weights[0]
    wi = weights[1:len(weights)]

    #this function is defined to compute the confusion matrix values for the ML models.
    prediction_matrix = []*len(test_features)
    true_positive = 0
    true_negative = 0
    false_negative = 0
    false_positive = 0

    for c in range(len(test_features)):
        score = w0 + np.sum(np.dot(test_features[c], wi))
        fraud_true = sigmoid(score)
        fraud_false = 1-sigmoid(score)

        if fraud_false > fraud_true:
            prediction = 0
        elif fraud_false < fraud_true:
            prediction = 1
        elif fraud_false == fraud_true:
            prediction = 0

        #for i in range(len(prediction_matrix)):
        if prediction == 1 and prediction == test_label[c] :
            true_positive += 1
        elif prediction == 0 and prediction == test_label[c] :
            true_negative += 1
        elif prediction == 1 and prediction != test_label[c] :
            false_positive += 1
        elif prediction == 0 and prediction != test_label[c] :
            false_negative += 1

    accuracy = (true_positive+true_negative)/(true_negative+true_positive+false_negative+false_positive)*100
    precision = true_positive / (true_positive + false_positive)*100
    recall = true_positive / (true_positive + false_negative)*100
    npv = true_negative / (true_negative + false_negative)*100
    fpr = false_positive / (true_negative + false_positive)*100
    fdr = false_positive / (true_positive + false_positive)*100
    f1 = 2*precision*recall/(precision + recall)/100
    beta = 2
    f2 = (beta^2+1)*precision*recall/((beta^2)*precision + recall)/100  #beta f-
measure with beta = 2, giving extra weight to recall

    print('Number of True Positives: ', true_positive)
    print('Number of True Negatives: ', true_negative)
    print('Number of False Positives: ', false_positive)
    print('Number of False Negatives: ', false_negative)
    print('Accuracy is ', accuracy, ' %')

    if best_one :
        print('Precision is ', precision, '%')
        print('Recall is ', recall, '%')
        print('Negative Predictive Value is ', npv, '%')
        print('False Positive Rate is ', fpr, '%')
        print('False Discovery Rate is ', fdr, '%')
        print('F1 measure is ', f1)
        print('F1 measure is ', f2)



def Normalize(input_array):
    normalized_array = np.zeros(len(input_array))
    minimum = min(input_array)
    maximum = max(input_array)

    for i in range(len(input_array)):
        normalized_array[i] = (input_array[i] - minimum)/(maximum - minimum)
    return normalized_array

def BatchGradAscent(features, labels, rate):

    weights = np.zeros(len(features[0])+1)
    w0 = weights[0]
    wi = weights[1:len(weights)]
    iterations = 1000

    for iteration in range(iterations):
        score = w0 + np.dot(features, wi)
        predictions = sigmoid(score)
        error = labels - predictions
        dgradient = np.dot(features.T, error)

        w0 += rate * np.sum(error)
        wi += rate * dgradient

    predicted_weights = np.append(w0, wi)

    return predicted_weights


# function to create a list containing mini-batches
def MiniBatches(features, labels, batch_size):
    mini_batches = []
    featuresdf = pd.DataFrame(features)
    labelsdf = pd.DataFrame(labels)
    concat_matrix = pd.concat([featuresdf, labelsdf], axis=1)
    data = concat_matrix.to_numpy()
    np.random.shuffle(data)
    n_minibatches = data.shape[0] // batch_size

    for i in range(n_minibatches + 1):
        mini_batch = data[i * batch_size:(i + 1)*batch_size, :]
        features_mini = mini_batch[:, :-1]
        labels_mini = mini_batch[:, -1].reshape((-1, 1))
        mini_batches.append((features_mini, labels_mini))

    if data.shape[0] % batch_size != 0:
        mini_batch = data[i * batch_size:data.shape[0]]
        features_mini = mini_batch[:, :-1]
        labels_mini = mini_batch[:, -1].reshape((-1, 1))
        mini_batches.append((features_mini, labels_mini))

    return mini_batches

# function to perform mini-batch gradient descent
def MiniGradientDescent(features, labels, rate, batch_size = 100):
    weights = np.random.normal(0, 0.01, len(features[0])+1)
    w0 = weights[0]
    wi = weights[1:len(weights)]
    iterations = 1000

    for iteration in range(iterations):
        mini_batches = MiniBatches(features, labels, batch_size)
        for mini_batch in mini_batches :
            features_mini, labels_mini = mini_batch
            score = w0 + np.dot(features_mini, wi)

            predictions = sigmoid(score)
            error = labels_mini.flatten() - predictions

            dgradient = np.dot(features_mini.T, error)

            w0 += rate * np.sum(error)
            wi += rate * dgradient

    predicted_weights = np.append(w0, wi)

    return predicted_weights

array_train_features[:, 28] = Normalize(array_train_features[:, 28])

log_scale = [1e-5, 1e-4, 1e-3, 1e-2, 1e-1]
trained_weights = [0]*len(log_scale)

array_test_labels = concat_test.to_numpy()[:,0]
array_test_features = concat_test.to_numpy()[:,1:30]
array_test_features[:, 28] = Normalize(array_test_features[:, 28])

tic = [0]*len(log_scale)
toc = [0]*len(log_scale)

for r in range(len(log_scale)):

    tic[r] = time.perf_counter()
    trained_weights[r] = BatchGradAscent(array_train_features, array_train_labels, log_scale[r])
    toc[r] = time.perf_counter()
    print('\nConfusion Matrix for learning rate = {} : '.format(log_scale[r]))
    Performance(trained_weights[r], array_test_features, array_test_labels)
    print()

print('\nIt seems that the learning rate = 0.001 gives the best results, as the accuracy and precision rates are as
high as small rates, and\nis less computationally expensive than the small rates, and there is no decrease in precis
ion like in the larger rates.')
print('\n=> Performance Metrics for Full Batch Gradient Ascent are:\n')
Performance(trained_weights[2], array_test_features, array_test_labels, best_one = True)
print(f"\nTotal time elapsed for full batch gradient ascent is {toc[2] - tic[2]:0.4f} seconds")

tic = time.perf_counter()
minibatch_weights = MiniGradientDescent(array_train_features, array_train_labels, log_scale[2])
toc = time.perf_counter()
print('\n => Performing the Mini Patch Gradient Ascent for Batch Size = 100, the following Performance Metrics are o
btained: \n')
Performance(minibatch_weights, array_test_features, array_test_labels, best_one = True)
print(f"\nTotal time elapsed for mini batch gradient ascent is {toc - tic:0.4f} seconds")

#stochastic gradient ascent is basically mini batch with batch size of 1.
tic = time.perf_counter()
stochastic_weights = MiniGradientDescent(array_train_features, array_train_labels, log_scale[2], batch_size=1)
toc = time.perf_counter()
print('\n=> Performing the Stochastic Gradient Ascent, the following Performance Metrics are obtained: \n')
Performance(stochastic_weights, array_test_features, array_test_labels, best_one = True)
print(f"\nTotal time elapsed for stochastic gradient ascent is {toc - tic:0.4f} seconds")
```