CSE - 015: Homework 5

Jaime Rivera

April 3, 2019

1 Complexity Analysis

Derive a complexity function for the following algorithms:

```
def doNothing(someList):
        return False
```

- Answer: O(1)

```
def doSomething(someList):
        if len(someList) == 0:
           return 0
        else if len(list == 1):
           return 1
        else:
           return doSomething(someList[1:])
```

- Answer: O(n)

```
def doSomethingElse(someList):
        n = len(someList)
        for i in range (n):
               for j in range(n):
                     if someList[i] > someList[j]
                        temp = someList[i]
                        someList[i] = someList[j]
                        someList[j] = temp
             return someList
```

- Answer: $O(n^2)$

2 Order of Complexity

Prove the following:

f(n) = O(g(n)) if there exists a positive integer n_0 and a positive constant c, such that f(n) <= $c * g(n) \bigvee$ n >= n_0

- $f(n) = 3n + 2 \in O(n)$
  - Answer: Clearly, for c = 4 and n >= 2, we have
    0 <= f(n) <= 4n
    Hence, f(n) = O(n)
- $g(n) = 7 \in O(1)$
  - Answer: Clearly, for c = 8 and n >= 0, we have
    0 <= f(n) <= 8*1
    Hence, f(n) = O(1)
- $h(n) = n^2 + 2n + 4 \in O(n^2)$
  - Answer: Clearly, for c = 3 and n >= 3, we have
    0 <= f(n) <= $3 * n^2$
    Hence, $f(n) = O(n^2)$

3 Mathematical Induction

Use mathematical induction to show that the following results hold for all positive integers:

- $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
  - Answer:
    for n = 1:
    LHS = 1
    RHS =$\frac{1(1+1)}{2} = \frac{2}{2} = 1$
    so, LHS = RHS for n = 1
    lets assume LHS = RHS for n = k
    so, $1 + 2 + ... + k = \frac{k(k+1)}{2}$
    for n = k + 1:
    LHS = 1+2+ ...+k + (k+1)
    =$\frac{k(k+1)}{2} + (k + 1)$
    =$k(k + 1) + \frac{2(k+1)}{2}$
    =$\frac{((k+1)(k+2))}{2}$
    =RHS
    so, LHS = RHS for n = k +1, given that LHS = RHS for n = k
    hence proved by induction
- $2 + 2^2 + 2^3 + 2^4 + ... + 2^n = 2^{n+1} - 2$

- Answer:

  for n = 1:

  LHS = 2
  RHS = $2^{1+1} - 2 = 2^2 - 2 = 4 - 2 = 2$
  so, LHS = RHS for n = 1
  Let's assume LHS = RHS for n = k:
  so, $2 + 2^2 + 2^3 + ... + 2^k = 2^{k+1} - 2$
  for n = k + 1:
  LHS = so, $2 + 2^2 + 2^3 + ... + 2^k + 2^{k+1}$
  $=2^{k+1} - 2 + 2^{k+1}$
  $=2 * (2^{k+1}) - 2$
  $=2^{k+2} - 2$
  =RHS
  so, LHS = RHS for n = k +1, given that LHS = RHS for n = k
  hence proved by induction