# CSE - 015: Homework 4

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## 1 Mathematical Proofs:

Prove (or disprove) the following results, showing all steps of your argument.

- 1. The sum of two odd integers is even.
  - sum of two odd integers is even.
  - even numbers can be represented by = 2n.
  - odd numbers can be represented by = 2n+1.
  - sum of two odd numbers.
  - $= (2n + 1) + (2n + 2)$.
  - $= (4n + 2) = 2(n + 1)$.
  - It is multiples of 2, so it is divided by 2, therefore it is even.
  - Even: [number divided by 2 and reminder will be '0'].
  - ODD: [number divided by 2 and reminder will be '1'].
- 2. The sum of two even integers is even.
  - sum of two even integers is even
  - even number = 2n
  - sum of two even numbers = 2n+2n
  - =4n
  - =2(2n)
  - It is multiple of 2. So it is even
- 3. The square of an even number is even.
  - even number = 2n
  - square of even number = $(2n)(2n)$
  - $= 4n^2$
  - $= 2(2n^2)$
  - It is a multiple of 2, since it can be divided by 2 it is even.
- 4. The product of two odd integers is odd.

  - odd number = 2n+1
  - product of two odd numbers
  - $= (2n+1)(2n+1)$
  - $= 4n^2 + 4n + 1$
  - $= 2(2n^2 + 2n) + 1$ ①
  - If we divide ① then reminder is always (1) so it is odd.

- 5. If $n^3 + 5$ is odd then n is even, for any $n \in Z$
  - Proof by contraposition
  - The contrapositive is "If n is odd, then n3 + 5 is even for any $n \in Z$ ." Assume that n is odd. We can now write n = 2k + 1 for some integer k. Then $n^3 + 5 = (2k+1)^3 + 5 = 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$. Thus $n^3 + 5$ is two times some integer, so it is even by the definition of an even integer.

- 6. If $3n + 2$ is even then n is even, for any $n \in Z$
  - odd x odd = odd
  - odd + even = odd
  - Thus, if n was odd, then:
  - $3n + 2$ = odd x odd + even = odd + even = odd
  - We know that $3n + 2$ is even for any $n \in Z$
  - n can't be odd

- 7. The sum of a rational number and an irrational number is irrational.
  - Proof by Contradiction
  - Assume:
  - rational + irrational = rational
  - rational = a/b form
  - $a/b + x = m/n$
  - $x = m/n - a/b$
  - $= \frac{mb-an}{nb}$ = it is in p/q form
  - x is irrational (not in p/q form)
  - rational + irrational = rational

- 8. The product of two irrational numbers is irrational.
  - Proof:
  - two irrational numbers a,b
  - $a * b = c$
  - $1/n * n = 1 -> rational$
  - $a * b = c$
  - $n * n = n^2 -> irrational$
  - Some times it is rational. Some time is is irrational. It depends on numbers $a, b$.

## 2 Basic Counting Principles:

Answer the following questions. Show all steps of your solutions.

- 1. How many different three-letter initials can people have?
  - In any three letter initial the word has to be among the 26 letter alphabet (a-z).
  - The total number of three letter initials $= 26 * 26 * 26 = 17576$.
- 2. How many different arrangements of the English alphabet are there?
  - Since they are 26 letters in the English alphabet, so they can be arranged in 26! arrangements.
- 3. There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
  - Select any one of the 18 students(mathematics majors) and any one of the 325 (computer science majors).
  - So total number of ways in which representatives can be selected is 18*325=5850.
- 4. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
  - Each sex total number of types of shirts $12 * 3 = 36$ for each color there are three sizes).
  - The total number of types of shirts for both sex will be $36 * 2 = 72$.
- 5. A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student answers every question?
  - Each question can be answered in 4 ways so total number of ways in which all questions can be answered is
  - $4^{10} = 1048576$
- 6. Suppose we have the same multiple choice test as described in question 5, but we relax the assumption that the student has to answer all questions. In other words, how many ways are there for a student answer the questions on the test if the student can leave answers blank?
  - Let student answers i questions these i questions can be selected from 10 questions is 10Ci and after selecting these i questions can be answered in 4i ways so total number number of ways in which student can answer these i questions is....
  - $C_i * 4^i$ and i varies from 0 to 10 ......(student can select no question to answer or can answer all the questions)
  - Total number of ways will be equal to:
    $(x + y)^n = \sum_{i=0}^{\infty} C_i^n * x^{n-i} * y^i$

    $\sum_{i=0}^{i=10} C_i^{10} * 4^i = \sum_{i=0}^{i=10} C_i^{10} * 4^i * 1^{10-i} = (1 + 4)^{10} = 5^{10}$