CSE - 015: Homework 3

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1 Knights and Knaves:

Recall the Knights and Knaves puzzle we saw in class. In that fictional world there are two types of people. Knights - who always tell the truth, and Knaves - who always lie. It is impossible to distinguish them by appearance, but only by the truth of their statements.

- One day a traveller was wandering around the island of Knights and Knaves, when he encountered two local inhabitants, P and Q. The traveller asked: "Is any of you a knave?". P replied: "At least one of us is a knave".

Case	P	Q
1	Knight	Knight
2	Knave	Knight
3	Knight	Knave
4	Knave	Knave

  - Can you find out what P and Q are? If so, what are they? If not, explain why not, and what other information you would need to know.
    - Knave: always tells lie
    - Knight: always tells truth
    - P's statement is false for the case 1 and True for the cases 2,3 and 4. If P is a Knight then what he says would be true, that means case 1 can't happen but case 3 can. If P is a Knave, whatever he would say must be False, meaning neither case 2 nor case 4 can happen as Knave always lie. Thus the only possible case is Row 3 and therefore P is a Knight and Q is a Knave.

- Later on, the traveller met two other locals, A and B. He asked whether either of them is a knight. A replied: "If B is a knave, then I am a knave too".

Case	P	Q
1	Knight	Knight
2	Knave	Knight
3	Knight	Knave
4	Knave	Knave

  - What are A and B?
    - Knave: always tells lie

    - Knight: always tells truth
    - A's statement is false for the case 1 and True for the cases 2,3 and 4. If P is a Knight then what he says would be true, that means case 1 and case 3 can't happen. If P is a knave then what he says would be False, this means case 4 can't happen. The only possibility is that A is a Knave and B is a Knight, i.e, case 2 is the only possible scenario.

2 Logical Identities:

Simplify the following propositions. Show all steps of your solutions.

- $-(p \to (q \to p))$
  - According to the Implication Law: $p \to q \equiv \neg q \vee p$. so, $\neg(p \to (\neg q \vee p)$
  - Again using Implication Law: $\neg(\neg p \vee (\neg q \vee p))$
  - According to De Morgans's Law: $\neg(a \vee b) \equiv \neg a \wedge \neg b$, $\neg p \equiv a$ *and* $(\neg q \vee p) \equiv b$
    - $\neg(\neg p) \wedge \neg(\neg q \vee p)$
  - Again using De Morgans's Law: $\neg(a \vee b) \equiv \neg a \wedge \neg b$,
    - $\neg q$ is a
    - p is b
    - $\neg(\neg p) \wedge \neg(\neg q) \wedge \neg p$
  - According to double negation law: $\neg(\neg p) \equiv p$
    - $p \wedge q \wedge \neg p$
    - $p \wedge \neg p \wedge q$
  - $p \wedge \neg p \equiv F$, thus
  - $FALSE \wedge q \equiv q$
- $-p((p \wedge q) \to (q \vee p))$
  - Let $(p \wedge q)$ be a, $(q \vee p)$ be b
  - According to Implication Law, $p \to q \equiv \neg p \vee q$
    - $\neg a \vee b$
  - Substituting the values of a and b we have,
    - $\neg(p \wedge q) \vee (q \vee p)$
  - According to De Morgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
    - $\neg p \vee \neg q \vee (q \vee p)$ ——— 1
  - According to associative law: $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - Rearranging 1,
    - $(\neg p \vee p) \vee (\neg q \vee q)$
  - According to negation laws, $p \vee \neg p \equiv T$
  - $TRUE \vee TRUE \equiv TRUE$

3 Logical Equivalences

Determine whether or not the following pairs of propositions are equivalent. Show all steps.

- $p \to (q \to r)$ and $(p \wedge q) \to r$
  - Answer: $p \to (q \to r) \not\equiv (p \wedge q) \to r$

q	r	p	$q \to r$	$p \to (q \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

and

p	q	r	$p \wedge q$	$(p \wedge q) \to r$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

- $p \to (q \to r)$ and $(p \to q) \to r$
  - Answer: $p \to (q \to r) \not\equiv (p \to q) \to r$

q	r	p	$q \to r$	$p \to (q \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

and

p	q	r	$p \to q$	$(p \to q) \to r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

4 Logical Consequence

- 1.

  Jimmy is smart

  Smart People are rich

  ———————————-

  Jimmy is Rich

  Answer: Valid: Jimmy is rich
  Jimmy is smart

  Jimmy then smart

  Jimmy $\to$ smart
  Smart People are rich

  Smart then rich

  Smart $\to$ rich

  Jimmy $\to$ rich $\equiv$ Jimmy is rich

- 2.

  Islands are surrounded by water

  Puerto Rico is surrounded by water

  ———————————————

  Puerto Rico is an island

  Answer: Not Valid: We cannot say Puerto Rico is an island is always true.
  Islands are surrounded by water

  Islands $\to$ surrounded by water

Puerto Rico is surrounded by water

Puerto Rico $\rightarrow$ surrounded by water

Puerto Rico is not always an island