# A CCD Noise Model

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## 1 Signal

The CCD in the spectrometer employs the photoelectric effect to measure the intensity of light, $I_\nu$, at frequency $\nu$. The number of photoelectrons in a given pixel, $N_{pe}$, per exposure depends on the integration time, $\Delta t$, the solid subtended by a pixel, $\Delta\Omega$, and the spectral band-pass, $\Delta\nu$, covered by that pixel,

$$N_{pe} = \eta I_\nu \cos\theta \Delta\nu \Delta t \Delta\Omega / h\nu \ , \tag{1}$$

where $\theta$ is the angle between pixel normal and the incident beam, $\eta$ is the quantum efficiency ($\eta$ = 1 for a perfect pixel), and $h$ is Planck's constant.

Each pixel also generates a "dark current", $i_d$, even when there is no illumination, so that the total accumulated number of electrons is $N = N_{pe} + N_d$, where $N_d = i_d \Delta t$.

During readout of the CCD pixel the accumulated electric charge, $Ne$, is deposited on a capacitor of capacitance $C$, thereby generating a voltage

$$V_{pe} = Ne/C \ , \tag{2}$$

where $e$ is the charge of the electron. Typical values of $C$ are a few pF; one electron ($e$=1.60 × $10^{-19}$ C) on 1 pF generates a signal of 160 nV.

The signal generated by the USB-2000 spectrometer is a number returned by a digital circuit that converts voltage to a 12-bit number (0-4095): this number is directly proportional to the voltage. By convention, the signal from the analog to digital converter (ADC) is measured in *ADU*s (analog to digital units). The voltage measuring circuit has a constant of proportionality, $g$, with units of *ADU* per volt. Thus, the number that ends up in your data file is

$$ADU = g\, Ne/C + ADU_0 \ . \tag{3}$$

The quantity $ADU_0$ is an offset or bias—a non-zero count that is returned even when the number of photoelectrons is zero (this offset is a practical detail introduced into the circuit because noise fluctuations about zero signal can be positive or negative, but the ADC can only generate positive numbers.) The combined quantity $ge/C$ has units of *ADU* per electron. For convenience, this quantity is often known simply as the gain.

We can use our knowledge of Poisson statistics applied to counting photoelectrons to deduce the gain factor in Eq. (3). First, notice that Eq. (3) implies that the signal in *ADU* depends on the number of photoelectrons and the bias value, i.e.,

$$ADU = ADU\left(N, ADU_0\right) . \tag{4}$$

To find the error in the measured signal we can apply the fundamental formula for error propagation, which states that the variance in some quantity $f$, which is a function of $u$, $v$, $w$, … is

$$\sigma_f^2 = \left(\frac{\partial f}{\partial u}\right)^2 \sigma_u^2 + \left(\frac{\partial f}{\partial v}\right)^2 \sigma_v^2 + \left(\frac{\partial f}{\partial w}\right)^2 \sigma_w^2 + \dots , \tag{5}$$

where we have assumed that $u$, $v$, $w$, … are independent quantities with zero covariance.

## 2 Error propagation

Two types of noise contribute to the standard deviation of $ADU$ measured, or $\sigma_{ADU}$. By applying the law of error propagation, Eq. (5), to Eq. (3) we find

$$\sigma_{ADU}^2 = \left(\frac{\partial ADU}{\partial N}\right)^2 \sigma_N^2 + \left(\frac{\partial ADU}{\partial ADU_0}\right)^2 \sigma_0^2 , \tag{6}$$

again assuming zero covariance between these two noise sources. The first partial derivative (see Eq. (3)) is just the gain, $ge/C$, while the second is unity. The variance $\sigma^2_N$ in the first term is the Poisson noise, for which we know that $\sigma^2_N = N$. In general, there is some noise associated with each measurement, $\sigma_0$, which is known as the read noise. Thus, Eq. (6) simplifies to

$$\sigma_{ADU}^2 = \left(\frac{ge}{C}\right)^2 N + \sigma_0^2 . \tag{7}$$

By substituting $geN/C = ADU - ADU_0$ from Eq. (3) we have

$$\sigma_{ADU}^2 = \frac{ge}{C}\left(ADU - ADU_0\right) + \sigma_0^2 . \tag{8}$$

Thus, a plot of the variance, $\sigma^2_{ADU}$, versus the mean bias subtracted signal, $ADU - ADU_0$, should be a straight line with slope equal to $ge/C$ and intercept of read noise squared. An example of applying Eq. (8) to a Sony ILX511 1× 2048 pixel CCD array is shown in Figure 1.

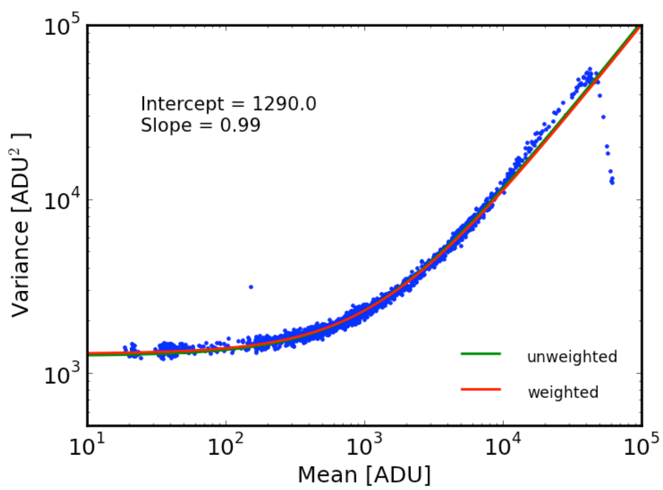


**Figure 1: : Variance/mean plot for a Sony ILX511 1 × 2048 pixel CCD array. These data are derived from 1000 dark-subtracted readouts. The mean and variance for 2048 pixel signals (dark subtracted) is plotted here as a point. The lines are straight-line fits corresponding a noise model consisting of constant read noise**

**and Poisson noise for pixels with mean < 10,000 ADU. The intercept gives the read noise and the slope gives the conversion from ADU to photoelectrons. Some non-linearity and excess noise is evident above 10,000 ADU.**

# 3 A few notes

As we noted above, the analog-to-digital converter is configured so that even when the input voltage is zero, the output number is non-zero. This offset is known as the bias. This offset or bias can be is measured by taking zero integration time ($\Delta t = 0$) exposures. This is not possible with the USB 2000 because there is no shutter; however, the shortest integration time (3 ms) is sufficiently short that no significant dark charge is accumulated.

Thus, one option when measuring the gain and read noise is to use a bright source that gives good signal to noise in short exposures (say a few tens of ms). Under these circumstances the dark probably is negligible and the quantity $ADU_0$ could be measured by turning off the light and repeating the experiment with the same exposure time.

If you use this on/off approach is the bias subtraction is perfect? Is $ADU_0$ is known with perfect precision?

Can you devise a scheme that does not require you to assume that the dark current is negligible?