# Starlight, Photoelectrons, & Centroids

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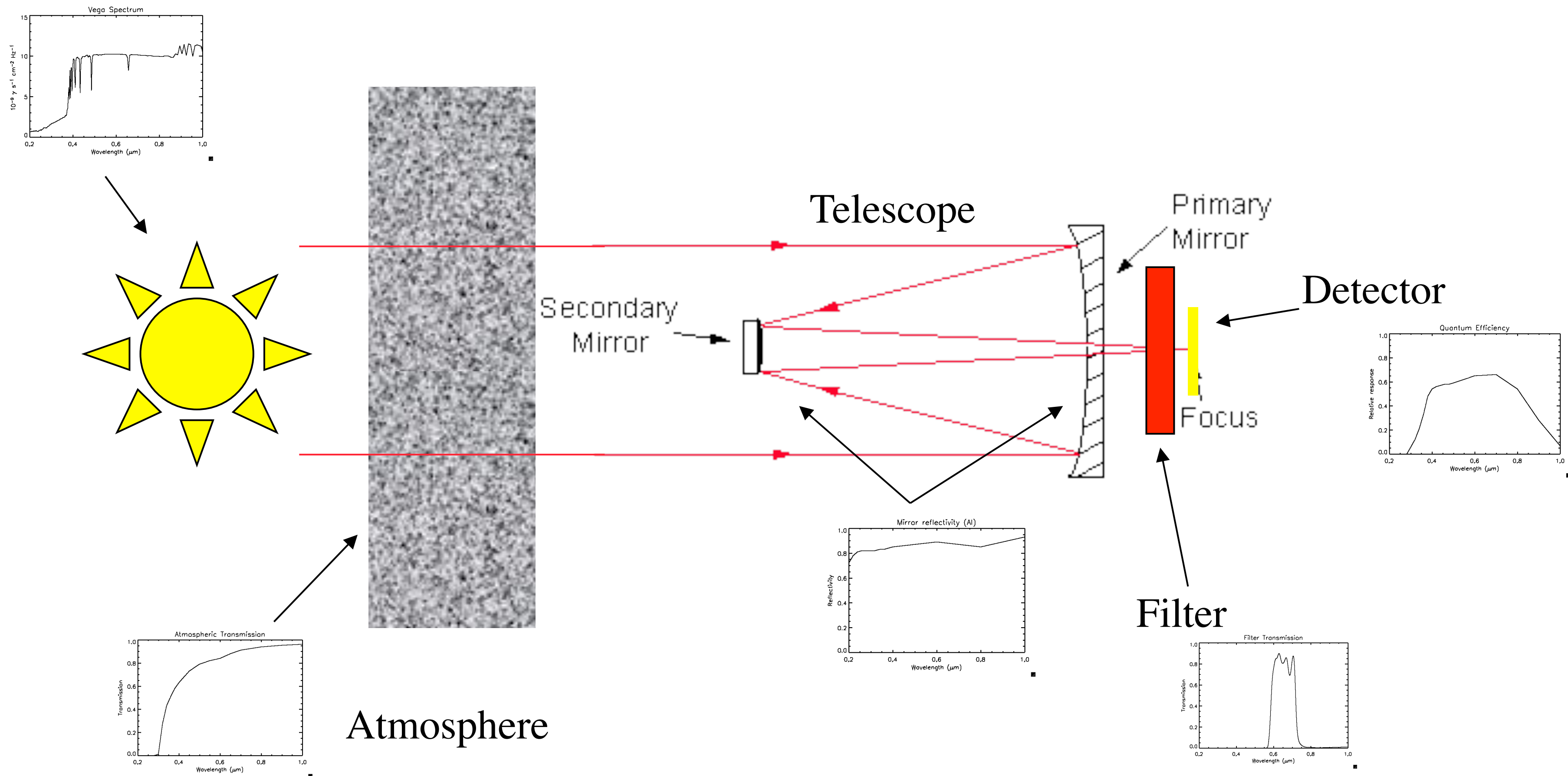
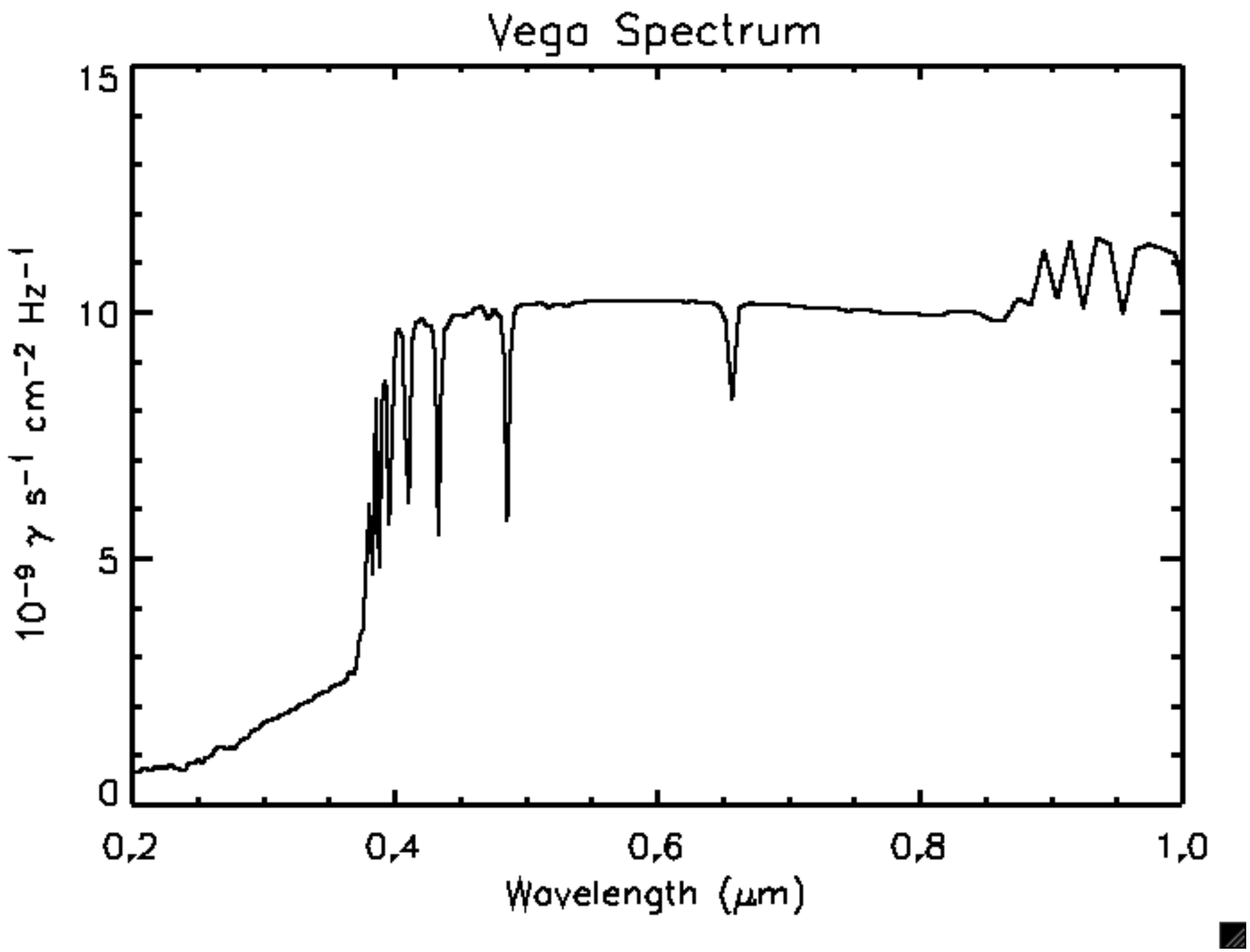
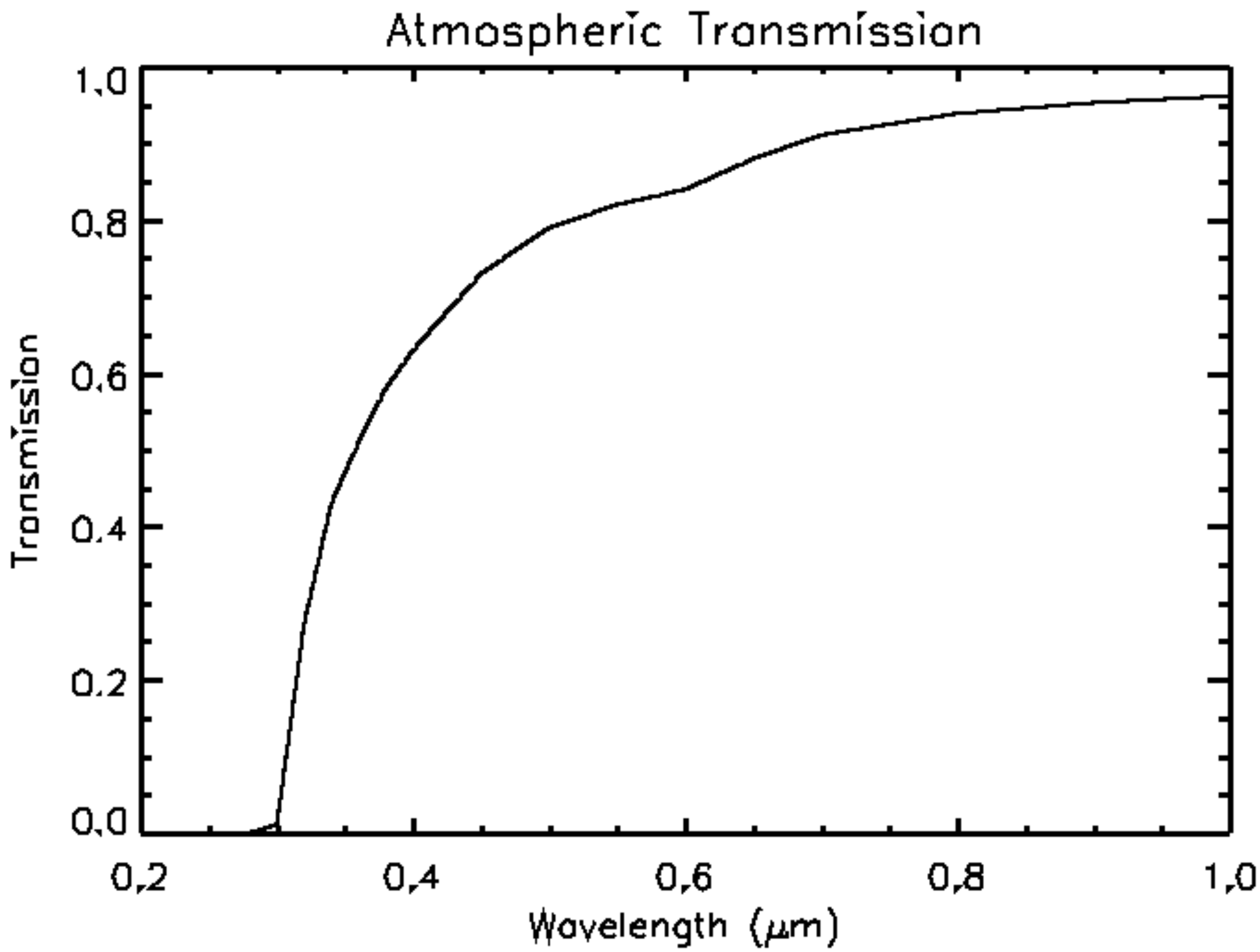
# Step 1: The Photon Path



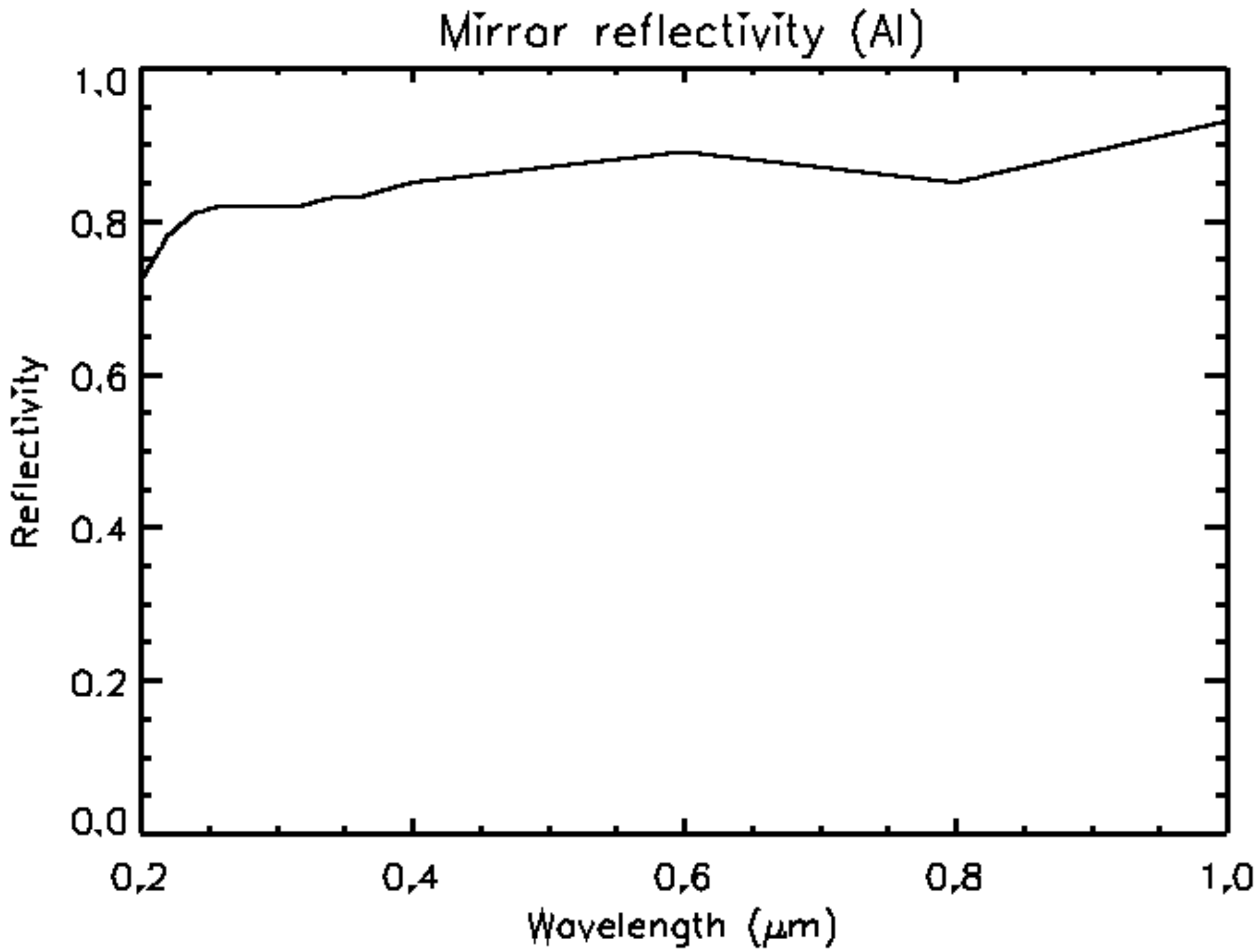
# Spectrum of Vega



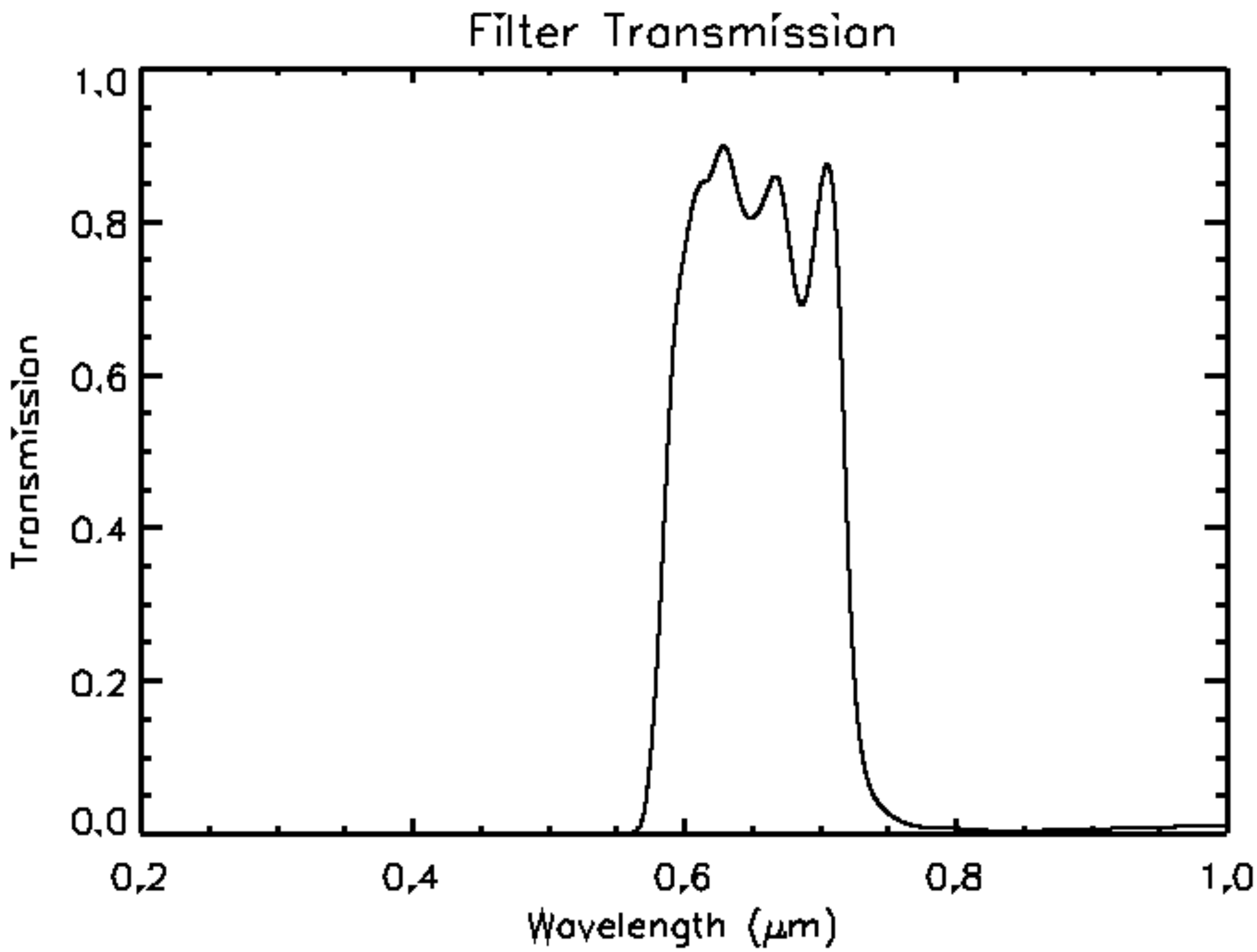
# Scattering & Absorption by the Earth's Atmosphere



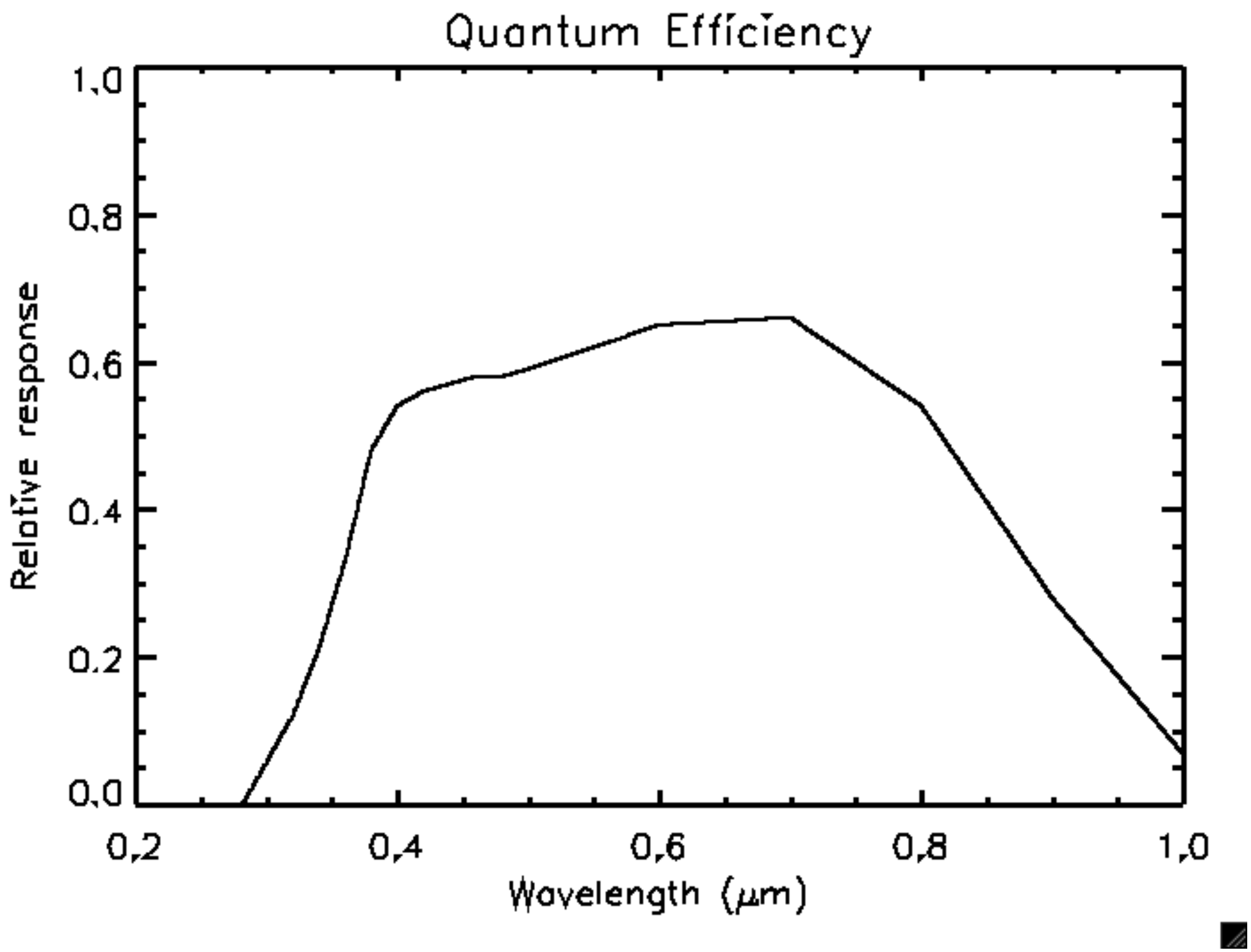
# Mirror Reflectivity



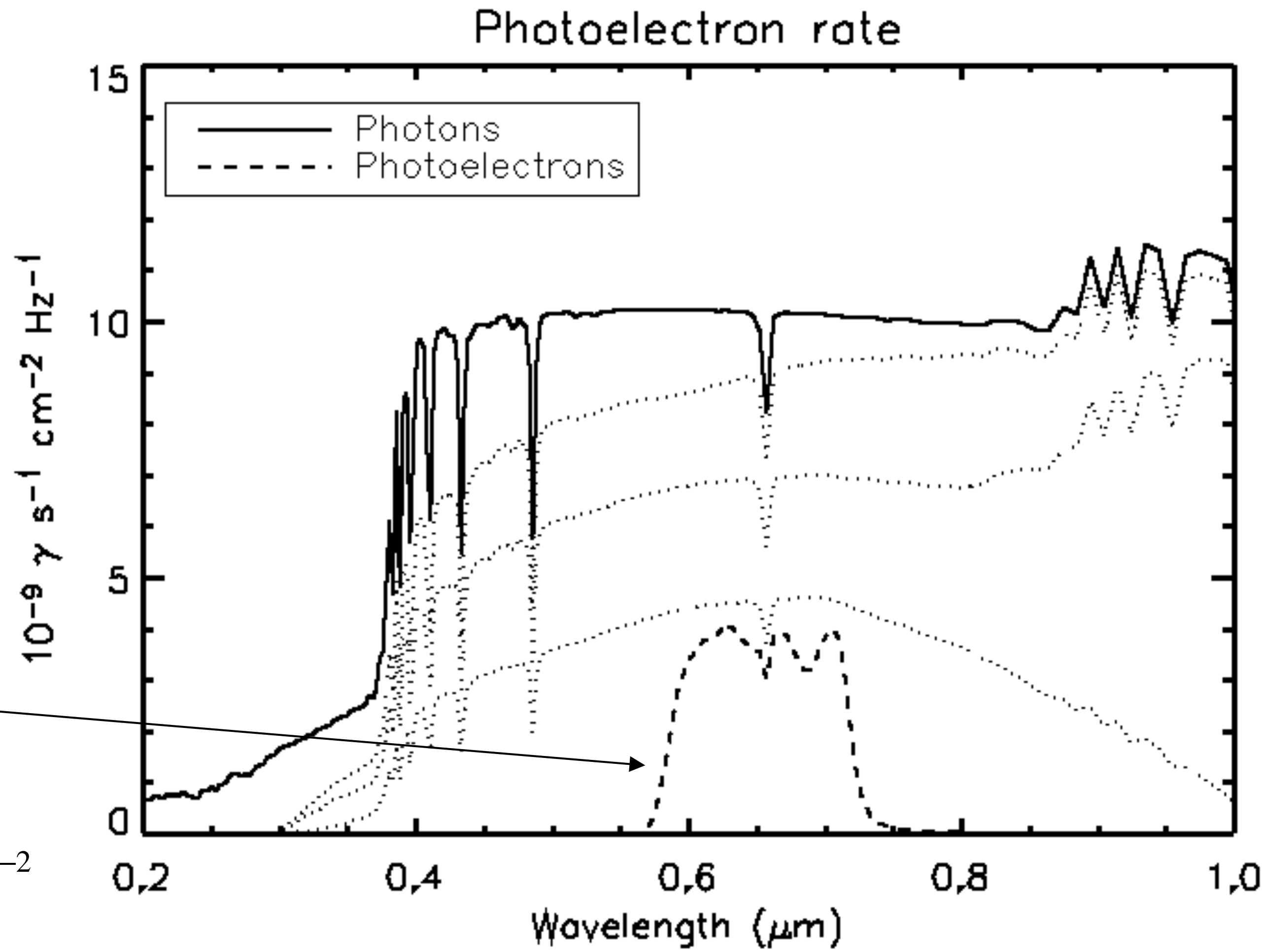
# Optical Filter



# Detector Efficiency



# System Throughput




$$\int_{\nu_1}^{\nu_2} \frac{\eta_\nu F_\nu}{h\nu} d\nu =$$

$$5.88 \times 10^{10}\, \gamma\ s^{-1}\, cm^{-2}$$

# Step 2: Systematic Errors

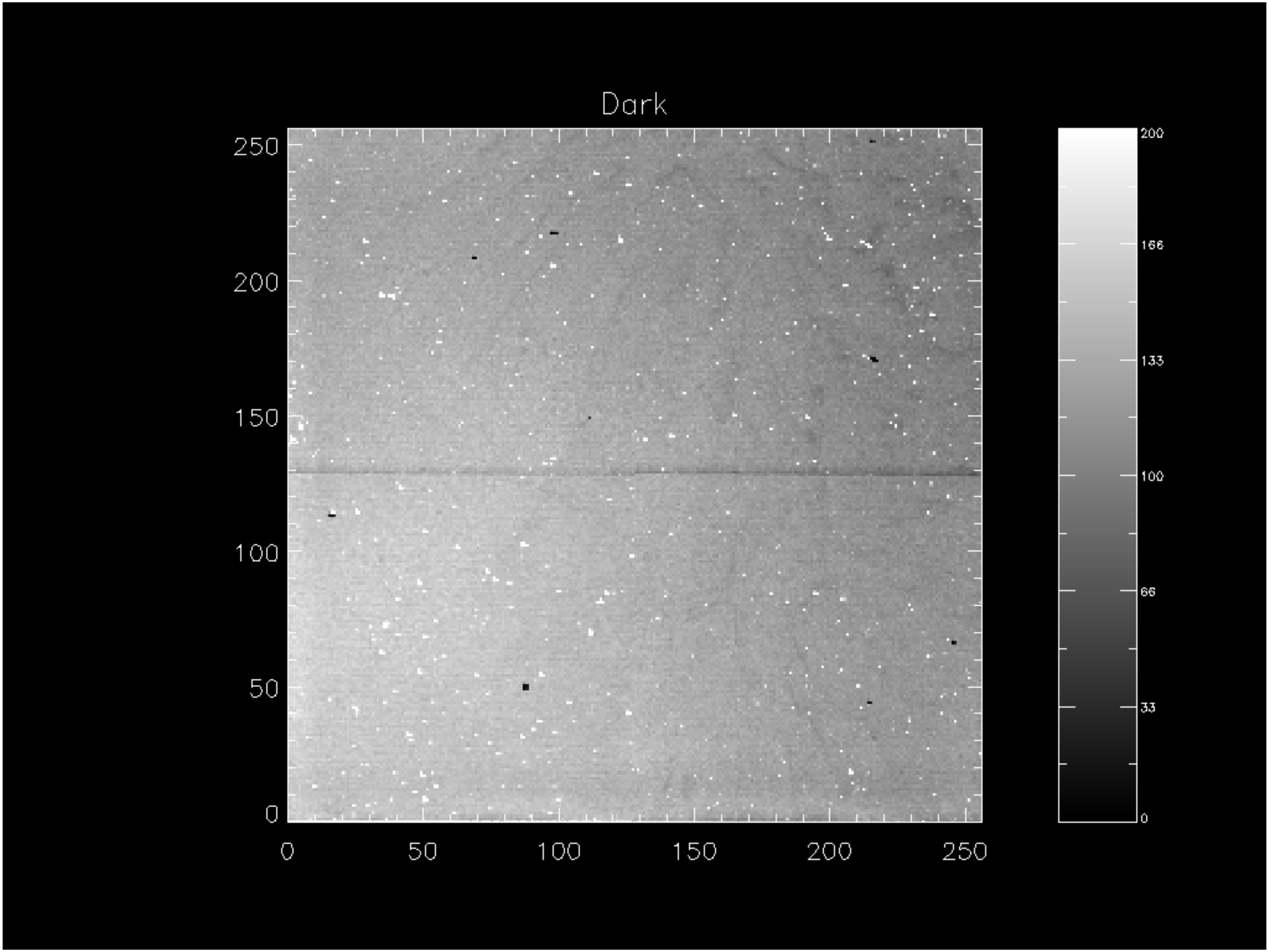
- Imaging detectors suffer from a number of errors that must be corrected before the data can be used for photometry
- Goal is to make the ADUs from the `FITS` files proportional to the brightness of the astronomical source

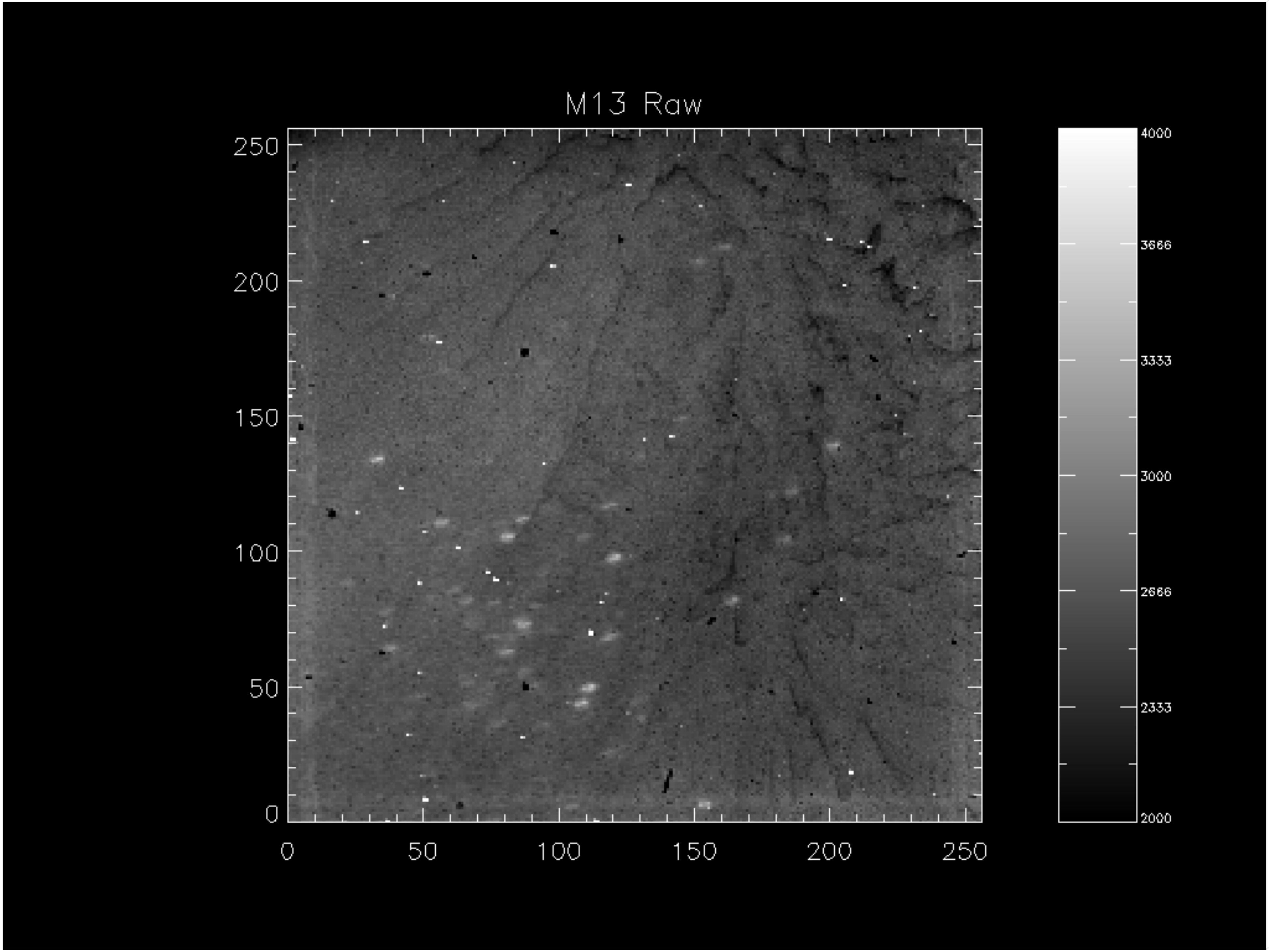
# Bias & Dark Current

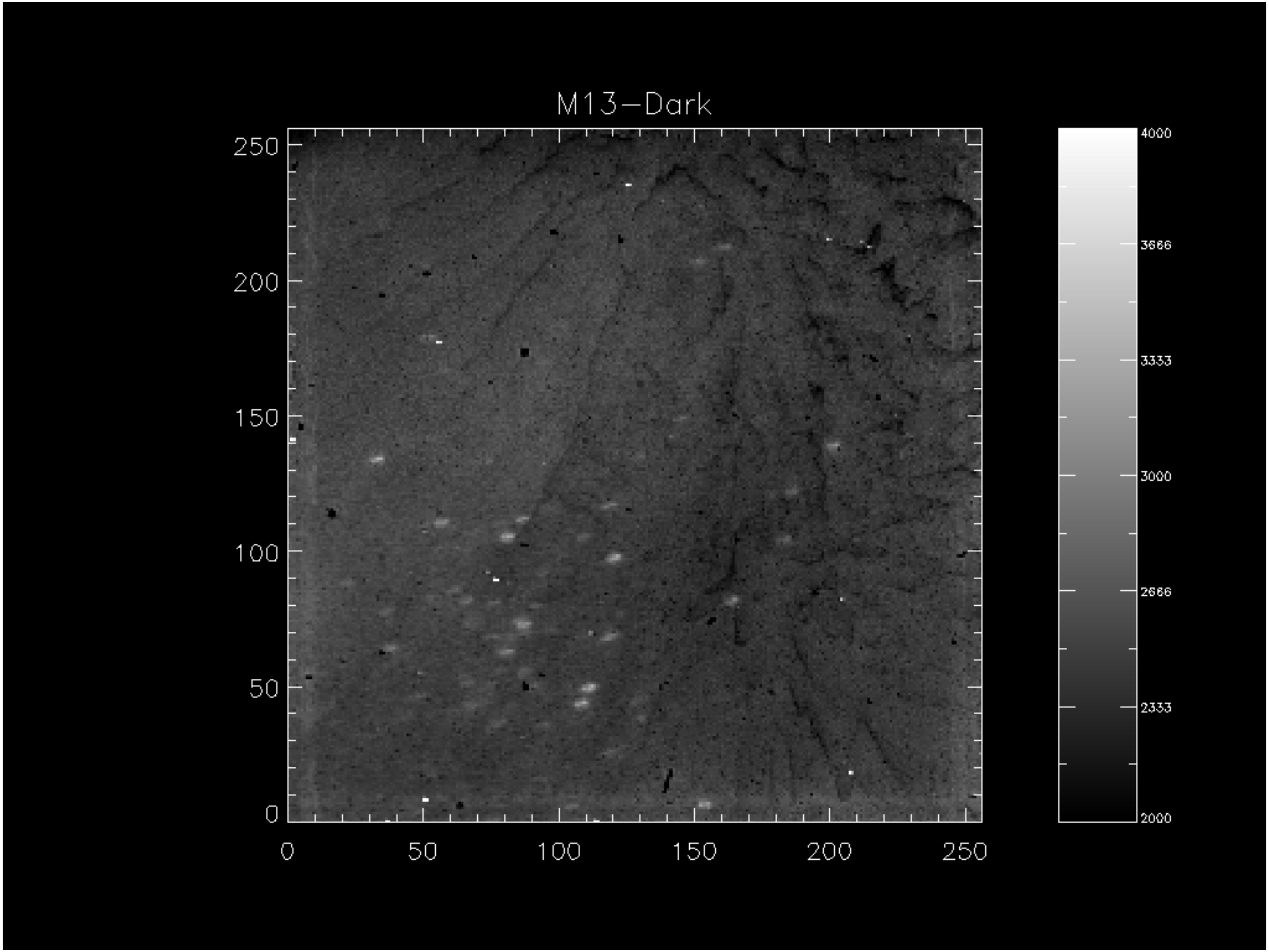
- Even a zero second exposure gives non-zero ADU
  - Dark current masquerades as real signal
  - Dark current & bias (constants DC offset) can be removed either by subtracting
    1. A dark frame of the same exposure time as the science image—takes care of bias too, or
    2. An image of blank sky—takes care of bias & dark, and also subtracts the sky brightness! (can be hard to find blank sky)

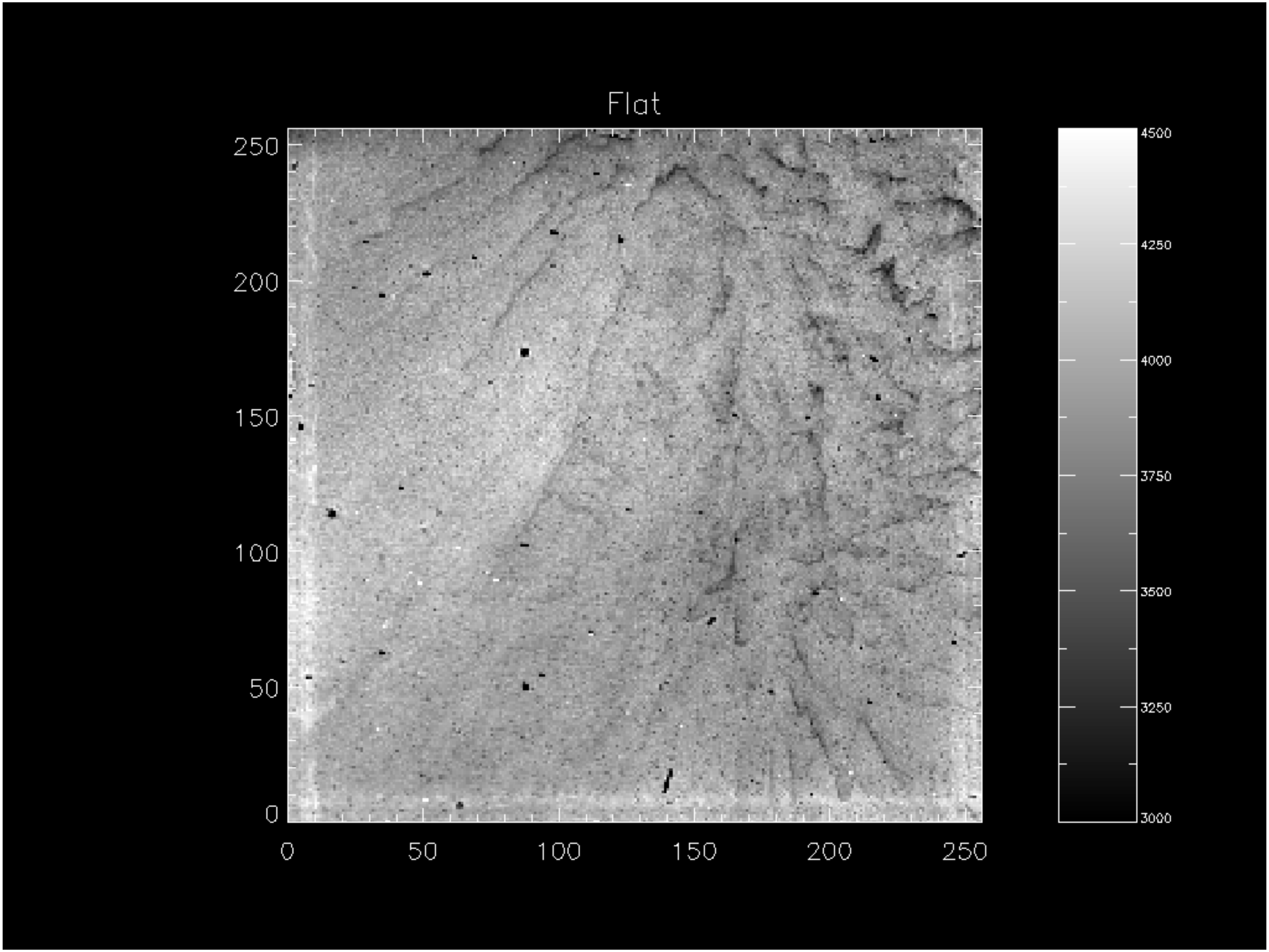
# Relative Pixel Gain *a.k.a.* Flat Field

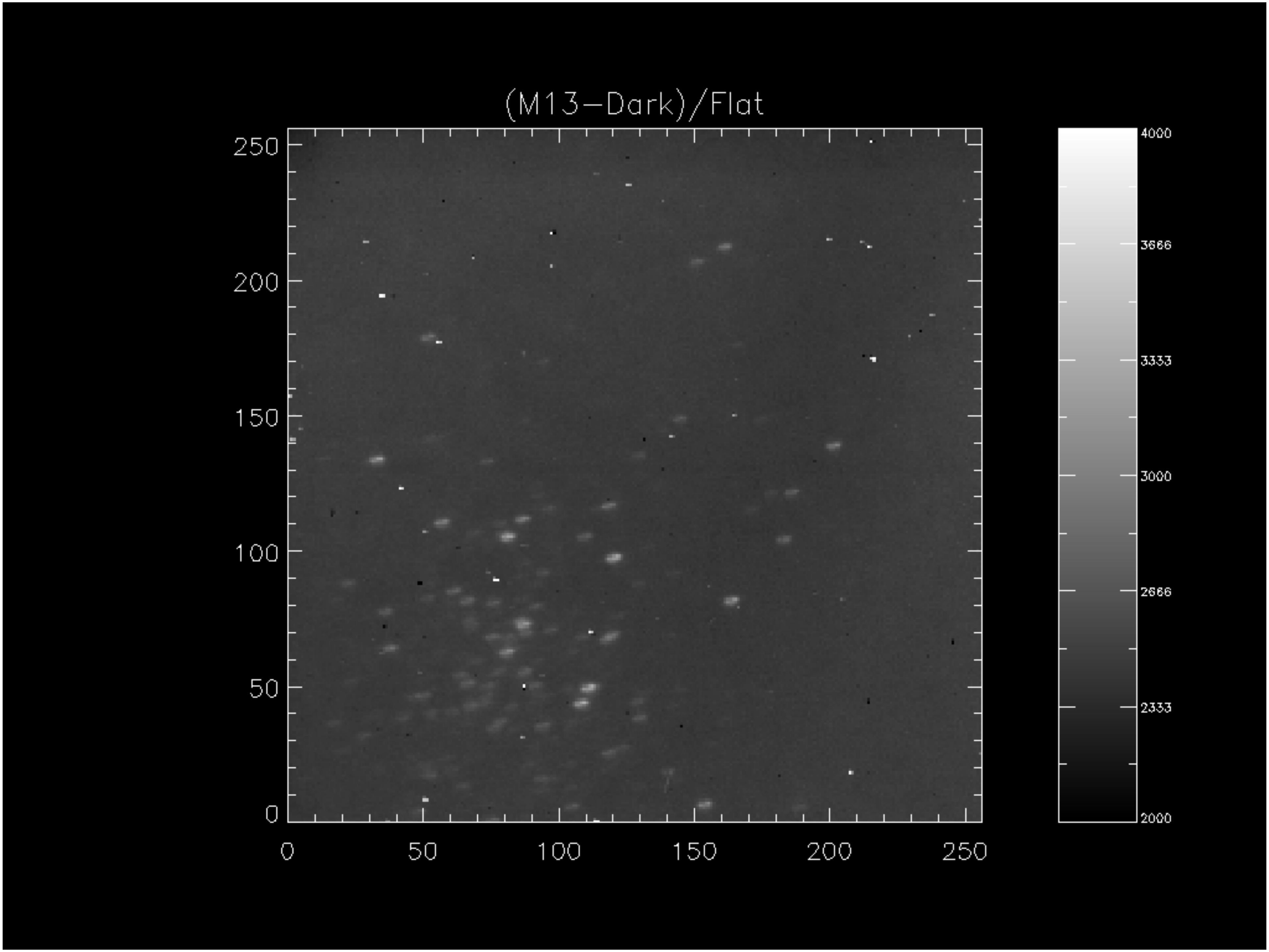
- Every pixel in the detector array has a slightly different response to light
  - Some pixels are more efficient than others
- Need to correct for pixel-to-pixel variations by constructing a flat field
  - Make a flat field by observing a uniform source, e.g., the twilight sky
  - ***Divide dark-subtracted images by the flat field***

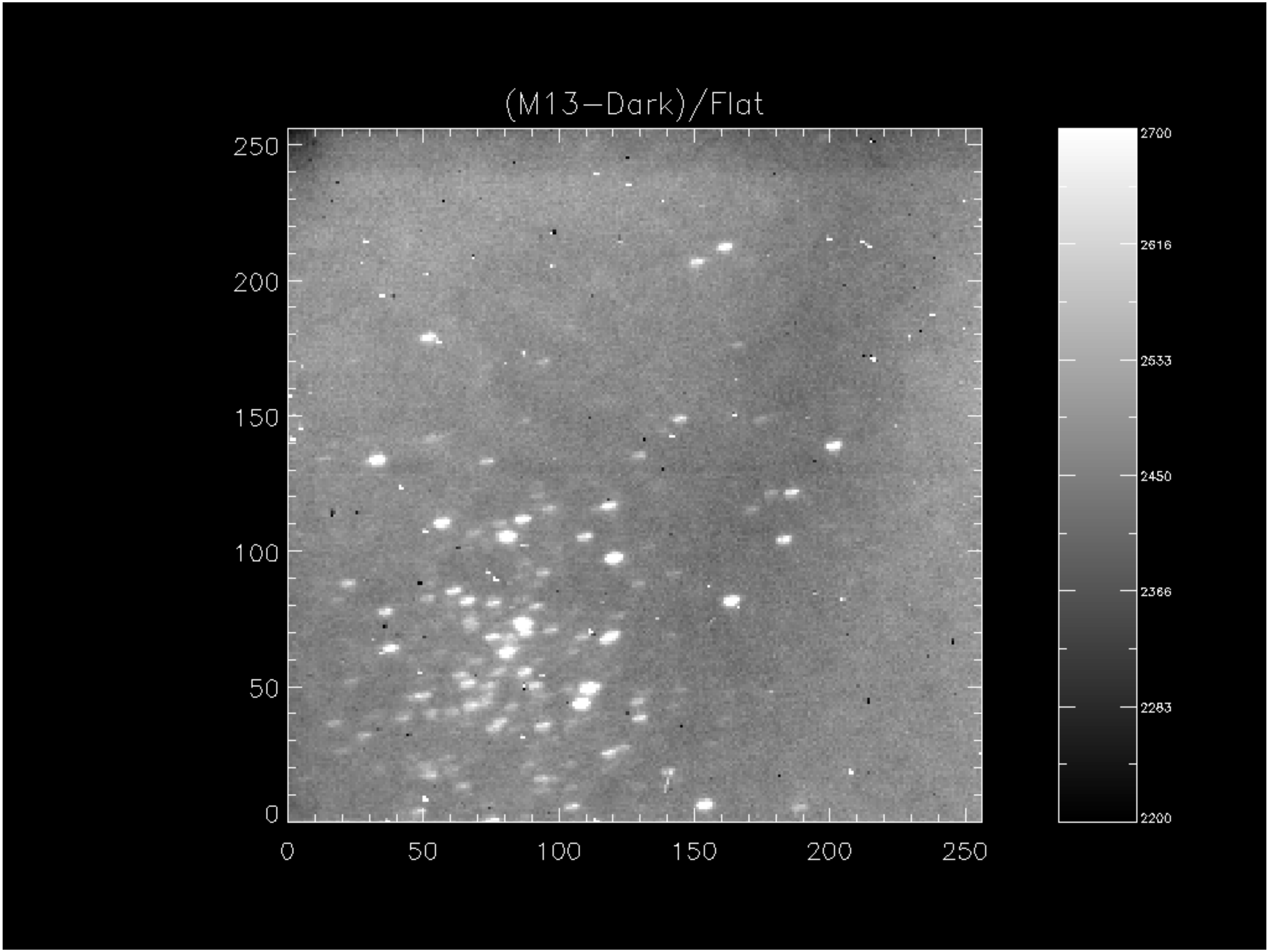

Dark
0
50
100
150
200
250
0
50
100
150
200
250
0
33
66
100
133
166
200


M13 Raw
250
200
150
100
50
0
0
50
100
150
200
250
4000
3666
3333
3000
2666
2333
2000


M13–Dark
0
50
100
150
200
250
4000
3666
3333
3000
2666
2333
2000


Flat
0
50
100
150
200
250
4500
4250
4000
3750
3500
3250
3000


(M13−Dark)/Flat
250
200
150
100
50
0
0
50
100
150
200
250
4000
3666
3333
3000
2666
2333
2000



(M13−Dark)/Flat
250
200
150
100
50
0
0
50
100
150
200
250
2700
2616
2533
2450
2366
2283
2200

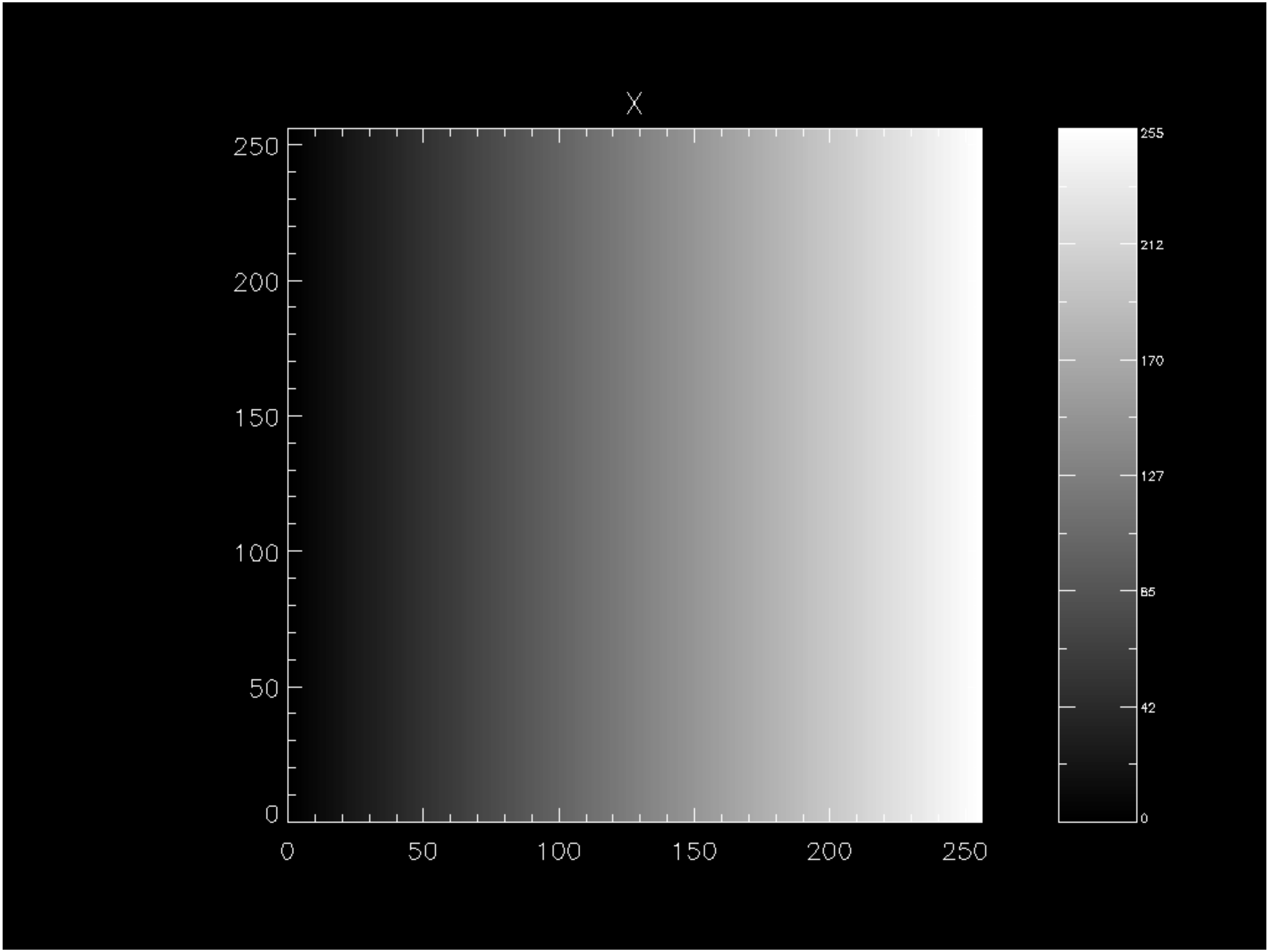
# Moments

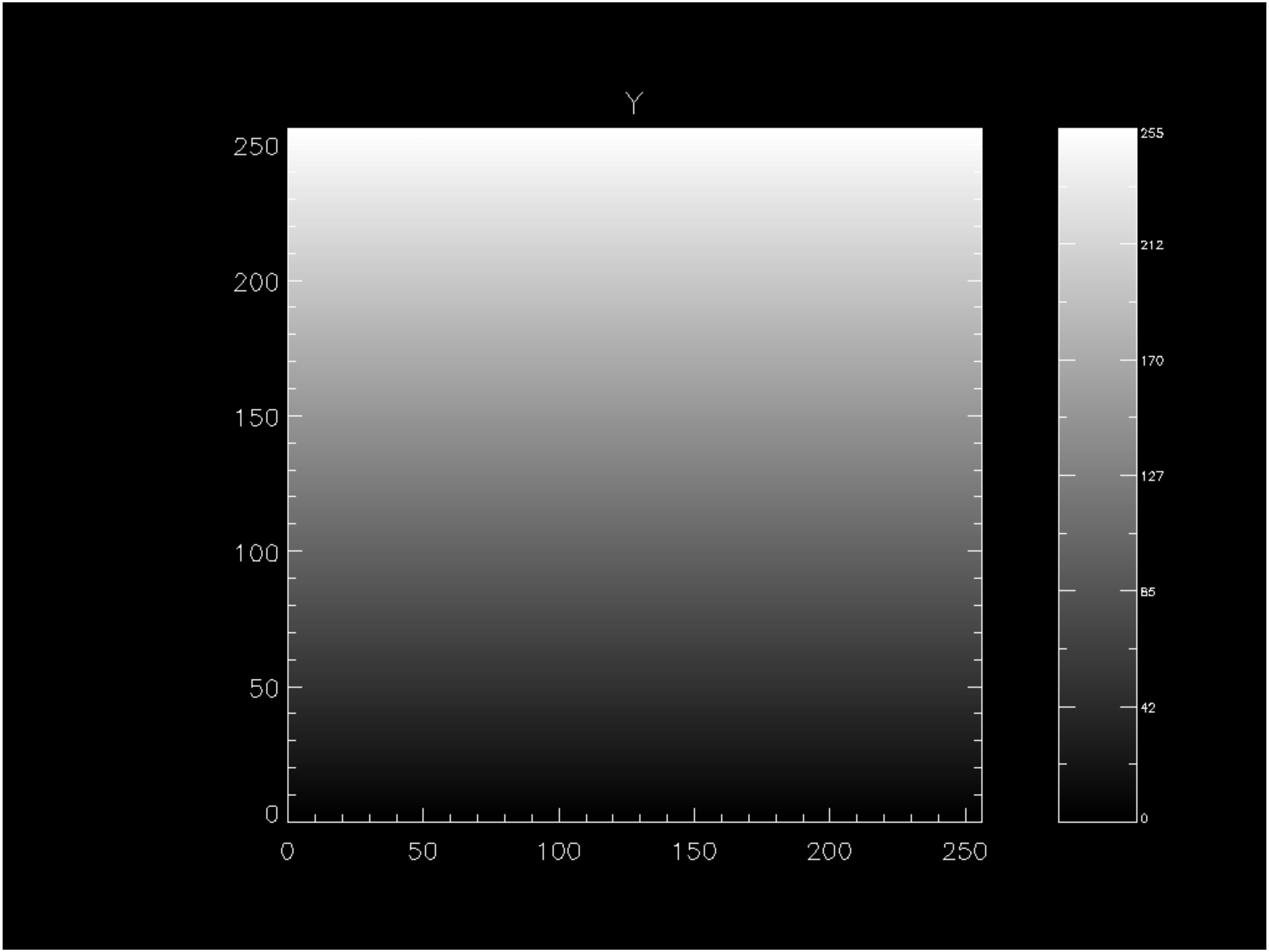
- For each star we can construct moments of its light distribution
  - The first moment is

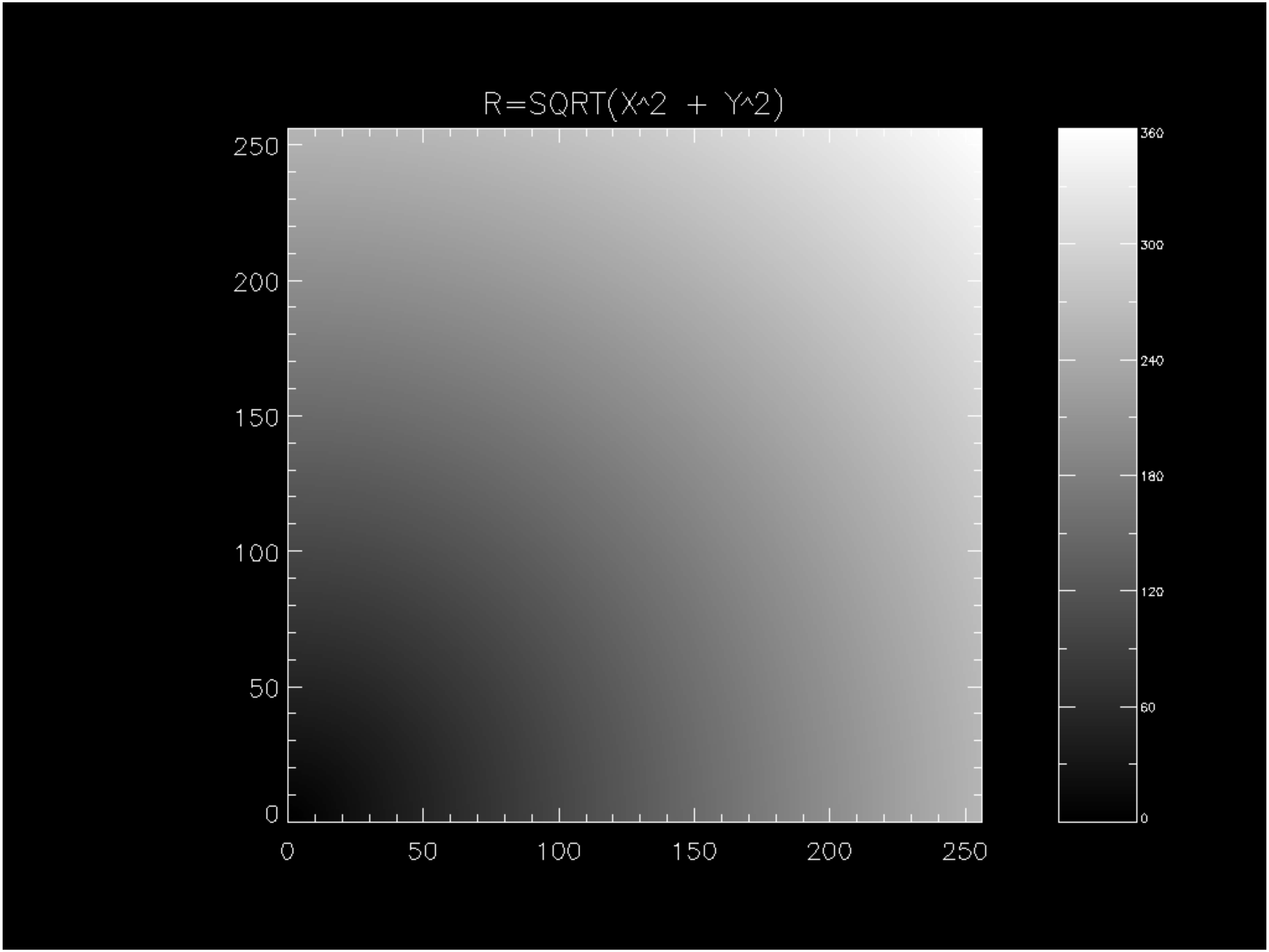
$$\langle x \rangle = \sum_i x_i I_i \Big/ \sum_i I_i$$

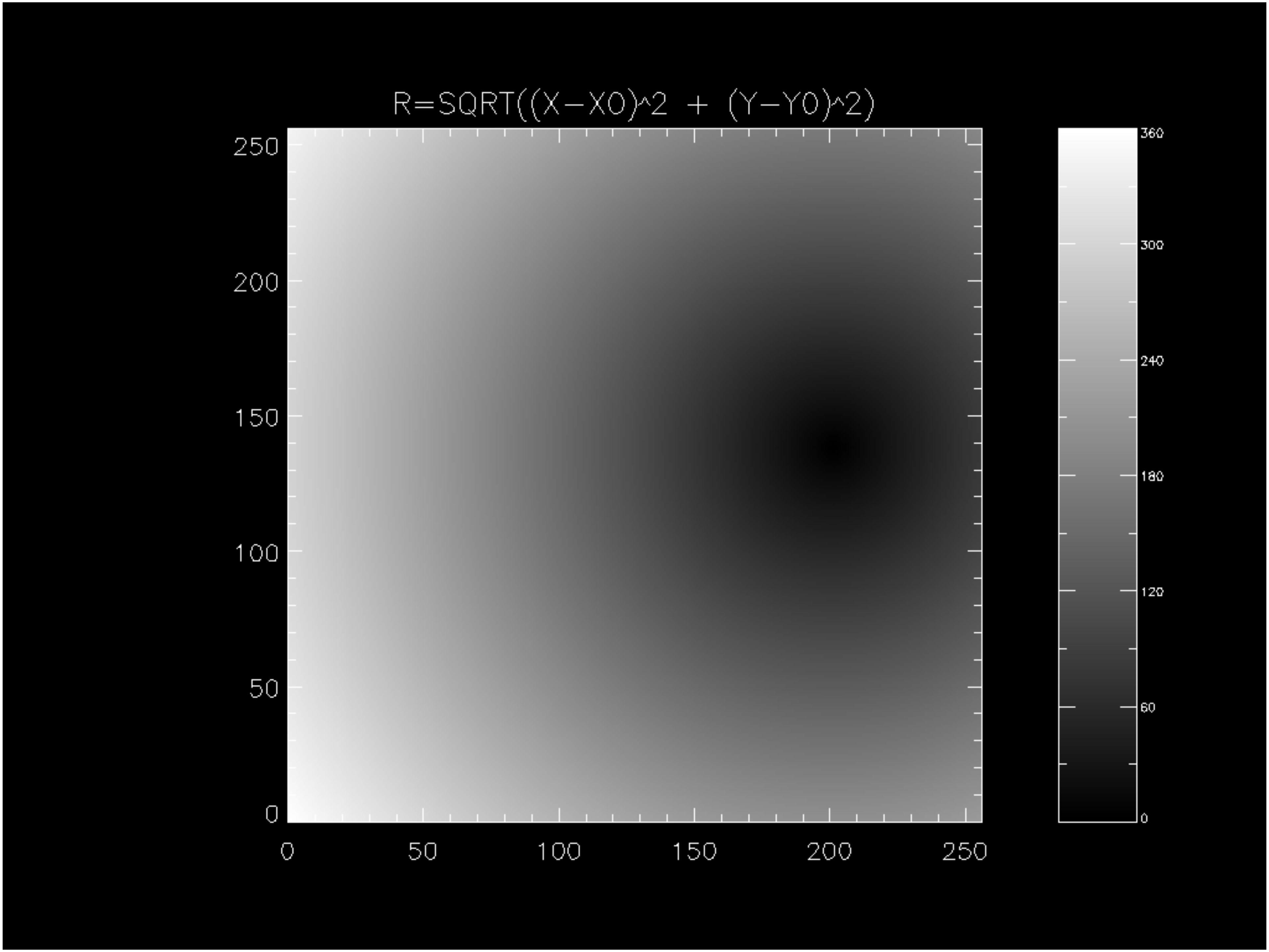
# How Bright is that Star?

- The light from a star is spread over several pixels
- How do we sum the light to get a measure of the total signal from the star?
  1. Identify the location of the star
  2. Select the associated pixels by making a mask
  3. Sum up the light
     – Subtract the sky background if necessary

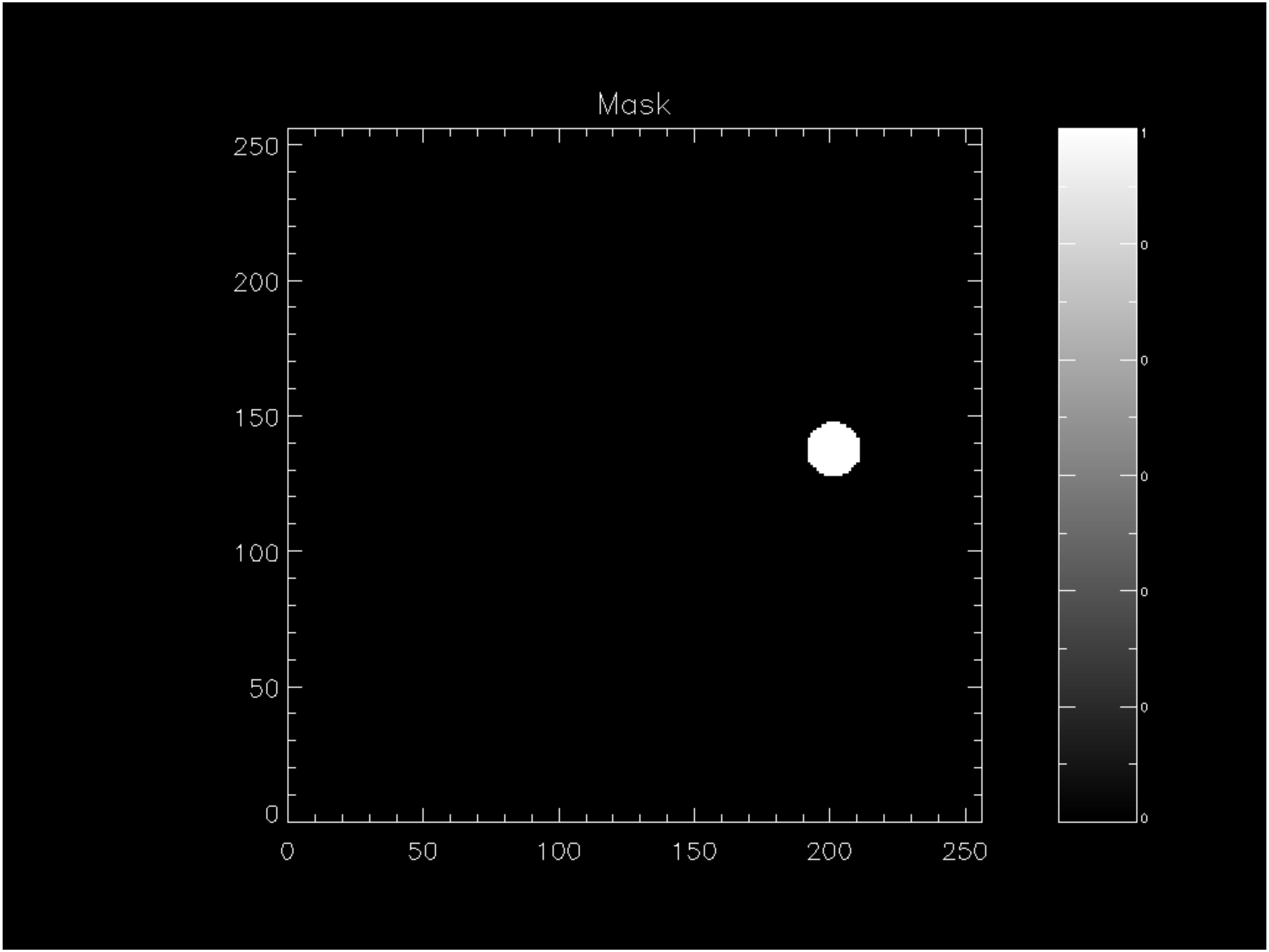

X
250
200
150
100
50
0
0
50
100
150
200
250
255
212
170
127
85
42
0

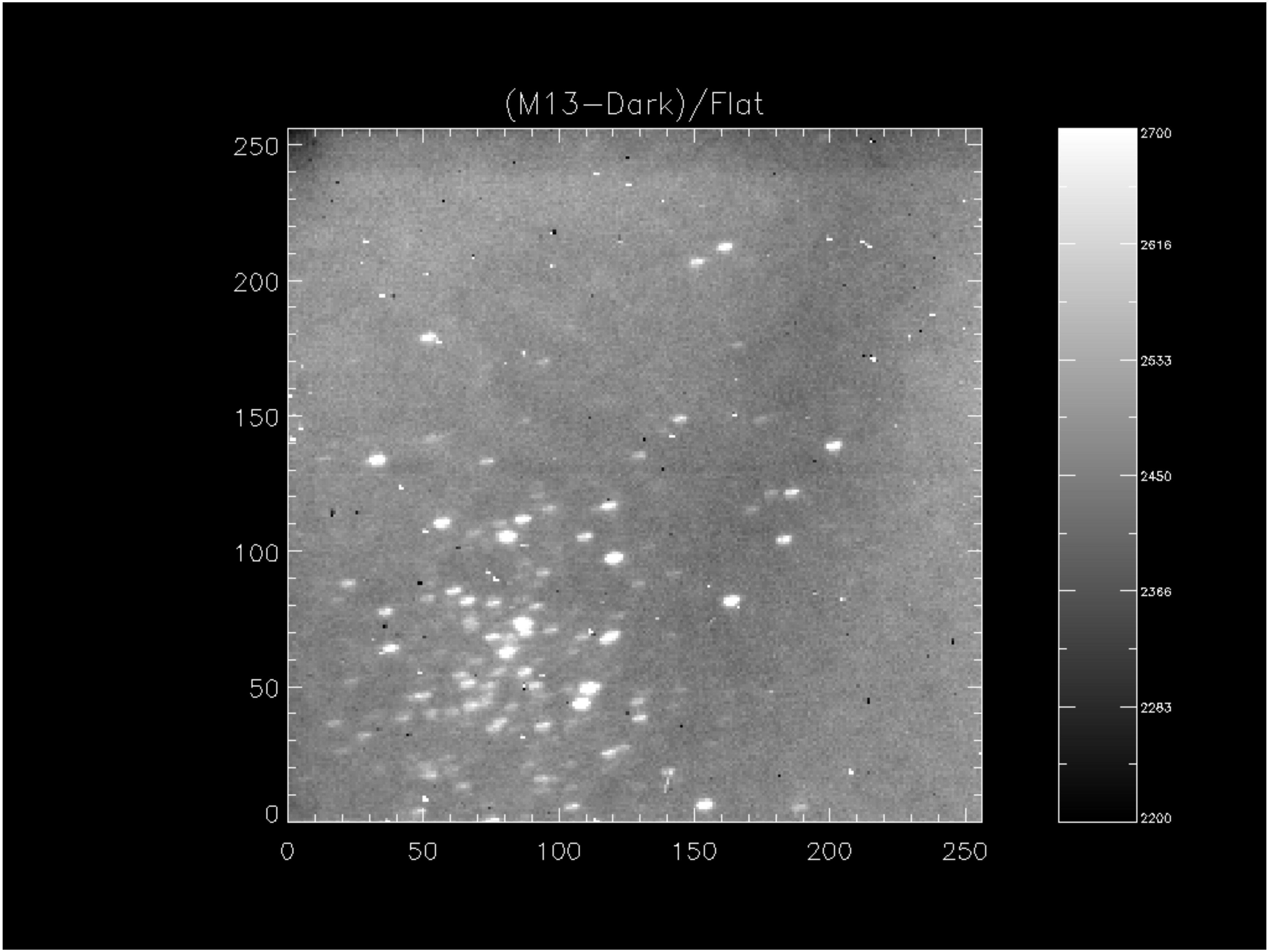

Y
0
50
100
150
200
250
0
50
100
150
200
250
255
212
170
127
85
42
0


R=SQRT(X^2 + Y^2)
0
50
100
150
200
250
0
60
120
180
240
300
360

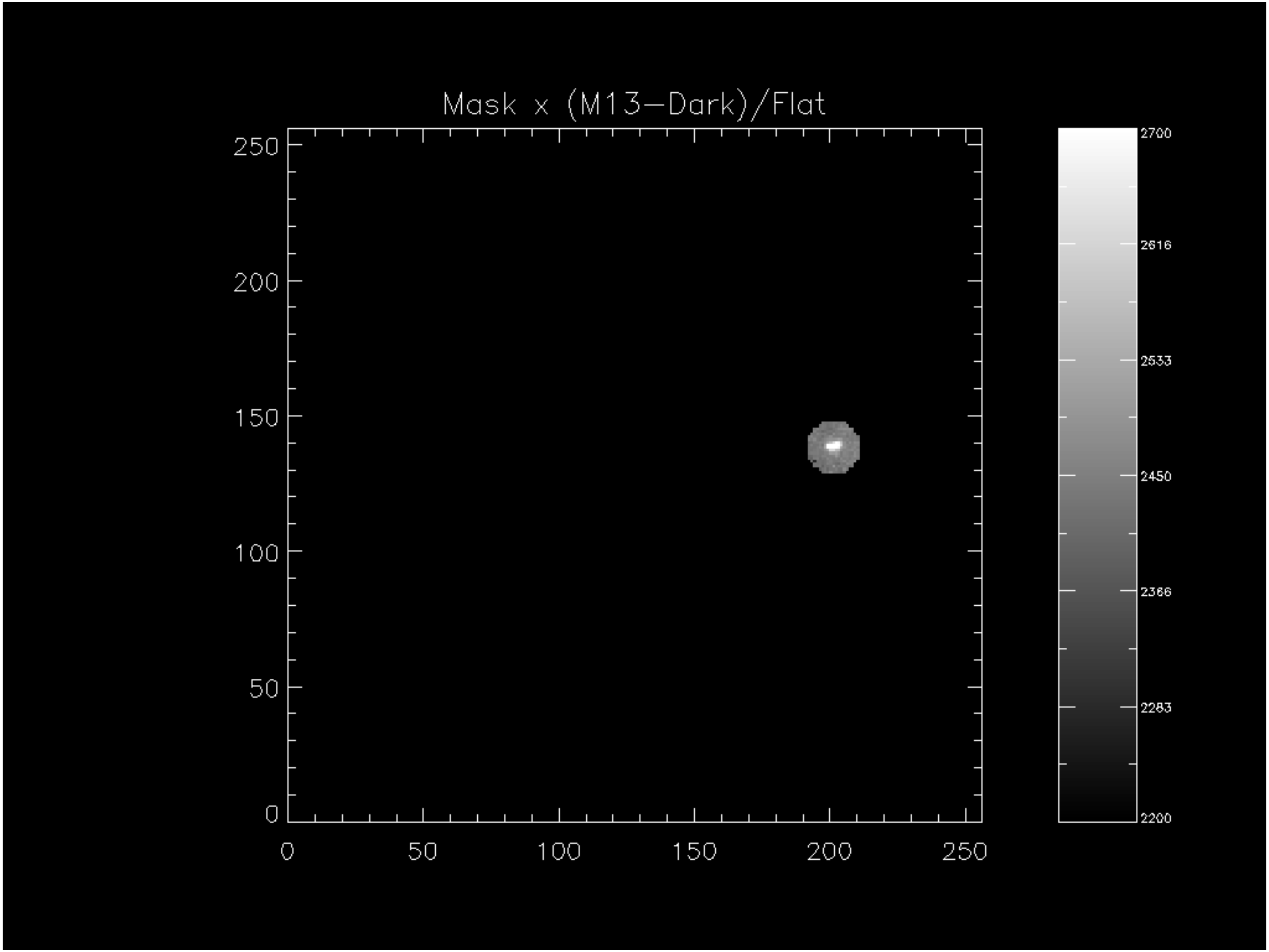


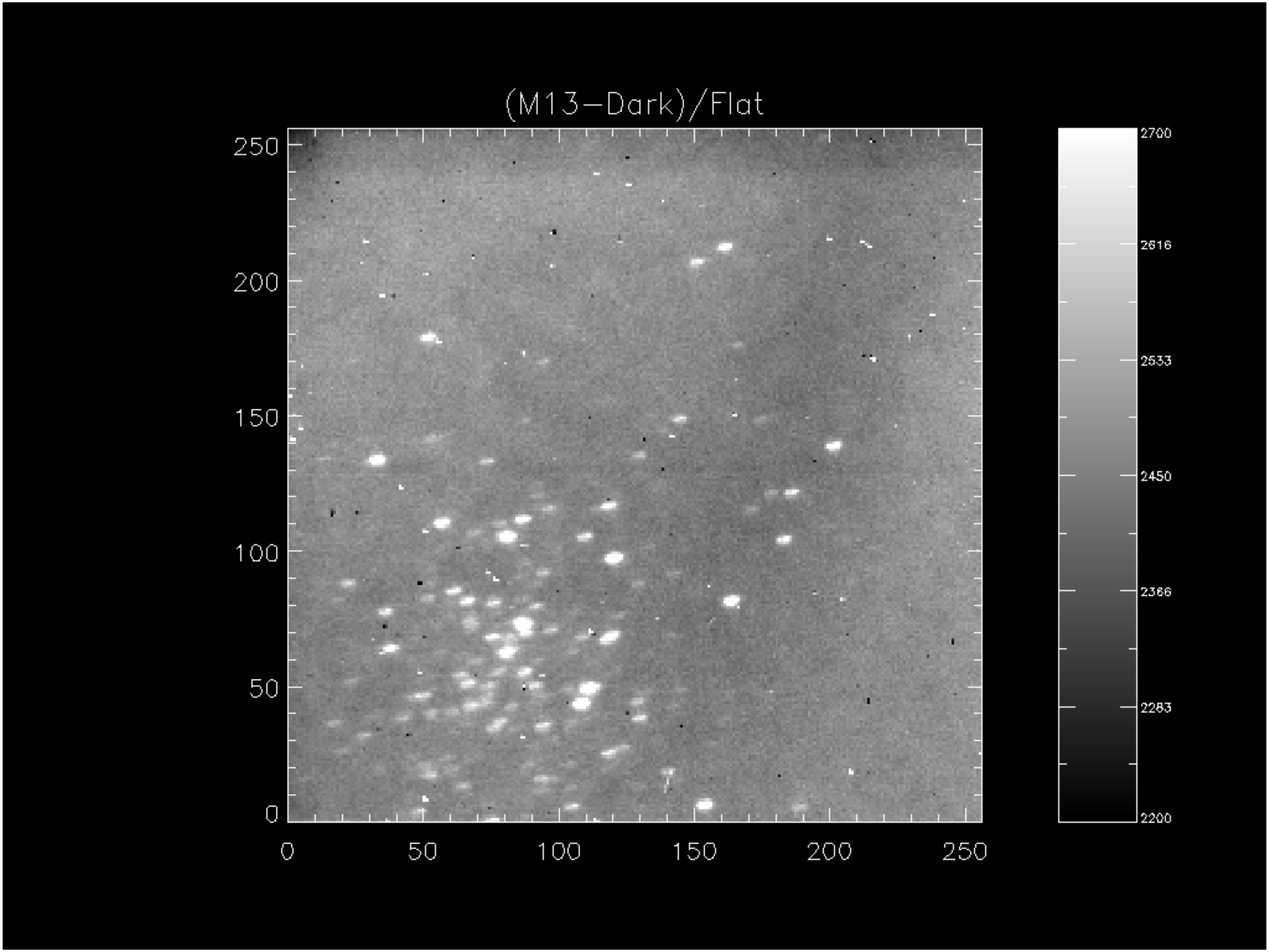
R=SQRT((X-X0)^2 + (Y-Y0)^2)
0
50
100
150
200
250
360
300
240
180
120
60
0

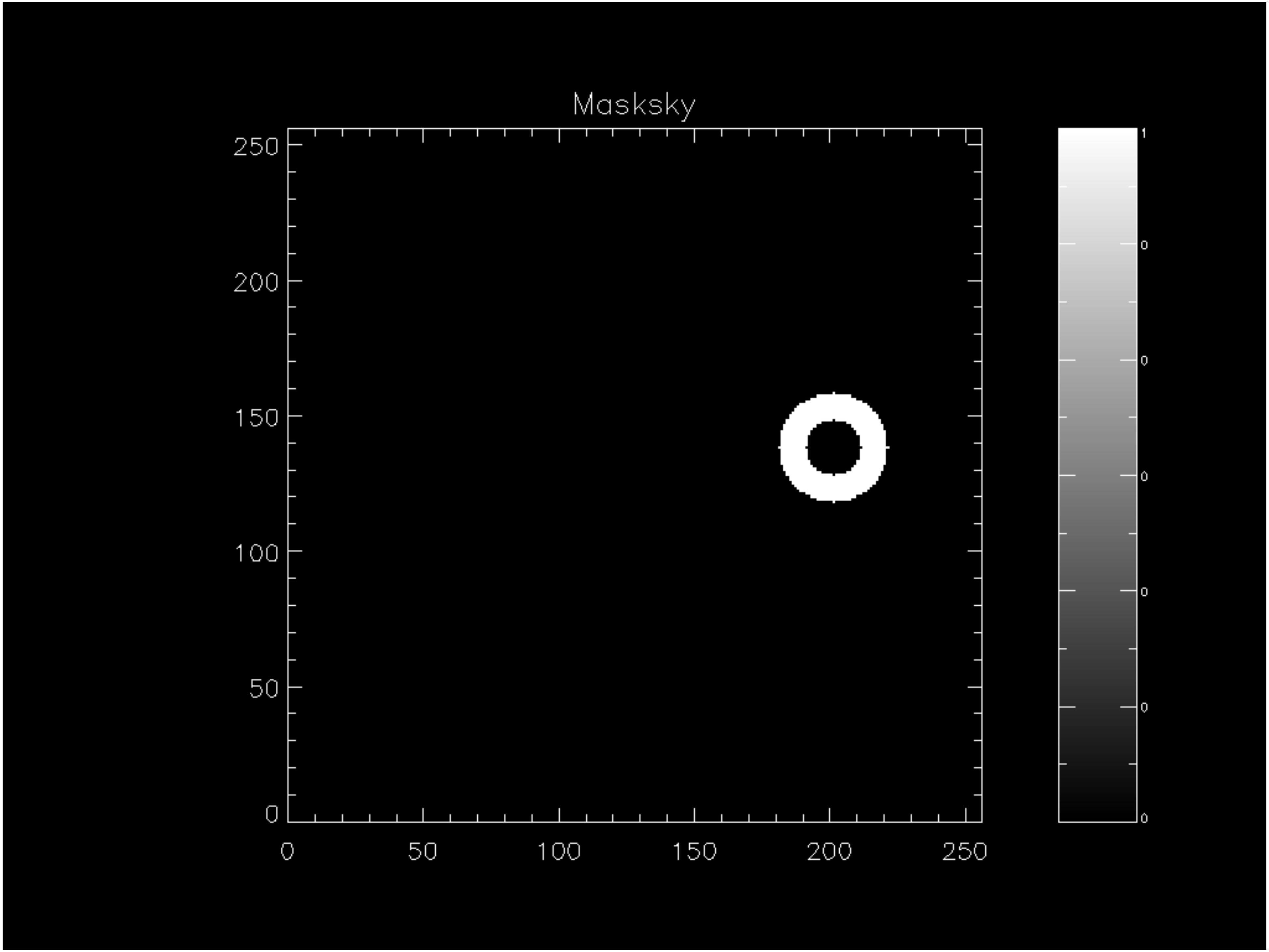

Mask
250
200
150
100
50
0
0
50
100
150
200
250
1
0
0
0
0
0
0

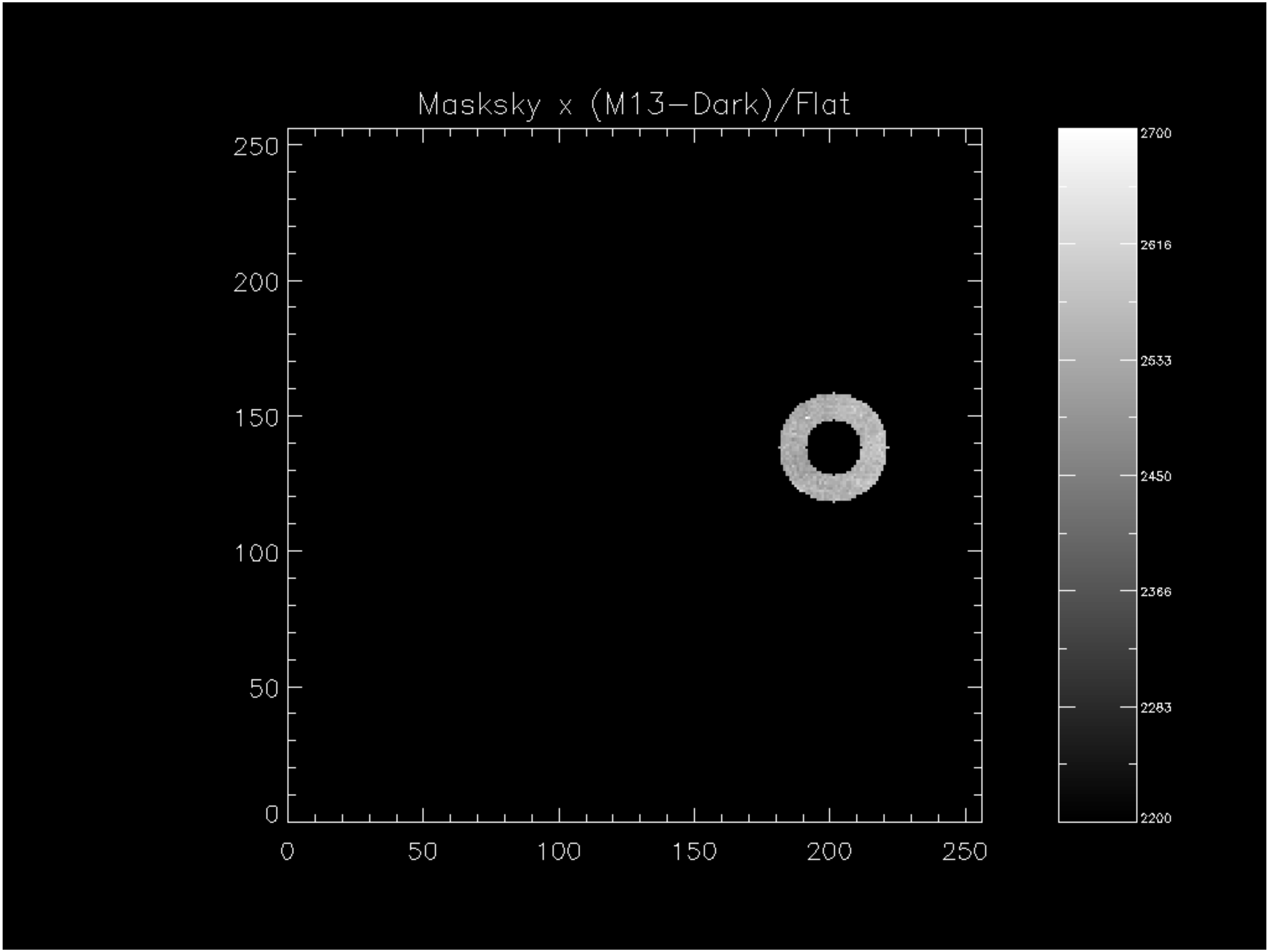


(M13–Dark)/Flat
0
50
100
150
200
250
2700
2616
2533
2450
2366
2283
2200


Mask x (M13−Dark)/Flat
0
50
100
150
200
250
2700
2616
2533
2450
2366
2283
2200


(M13−Dark)/Flat
0
50
100
150
200
250
2700
2616
2533
2450
2366
2283
2200


Masksky
0
50
100
150
200
250
0
50
100
150
200
250
1
0
0
0
0
0
0


Masksky x (M13−Dark)/Flat
0
50
100
150
200
250
2700
2616
2533
2450
2366
2283
2200

# Step 3: Modeling the Noise

- What is the SNR of a given observation?
- How do I choose and optimize the photometric parameters
  - Exposure time required?
  - Aperture diameter?
  - Location and size of sky annuli?

# How to Begin

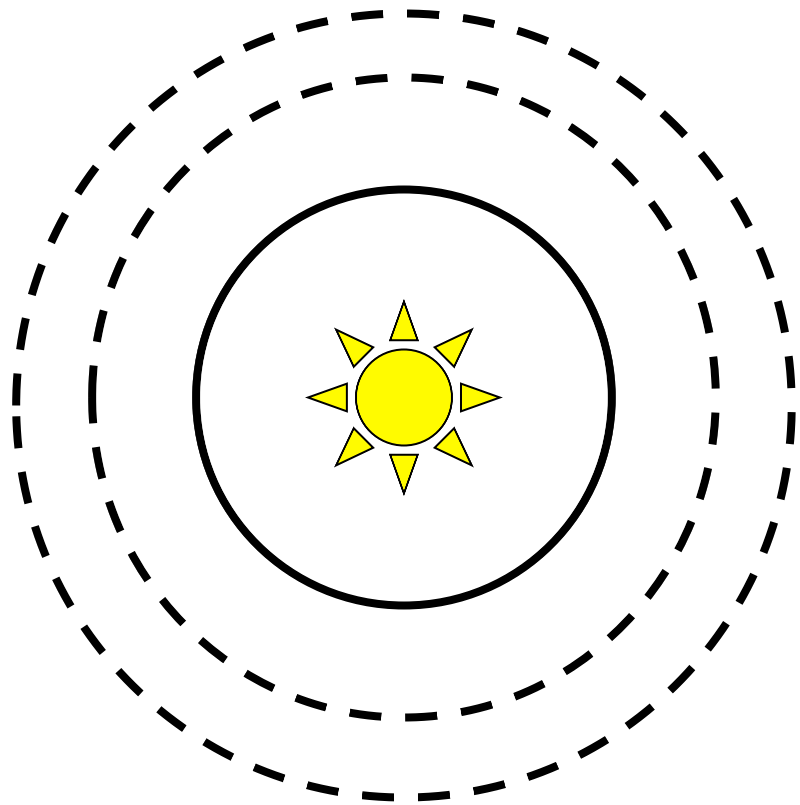
- Write down an expression for the signal and use error propagation to find the noise
  - Express results as signal-to-noise ratio vs. photometric parameters

# The Model

- The purpose is to ***estimate*** the noise contributions
  - Often getting the answer to within a factor of two is fine
  - Make simplifying assumptions—so long as you can justify them

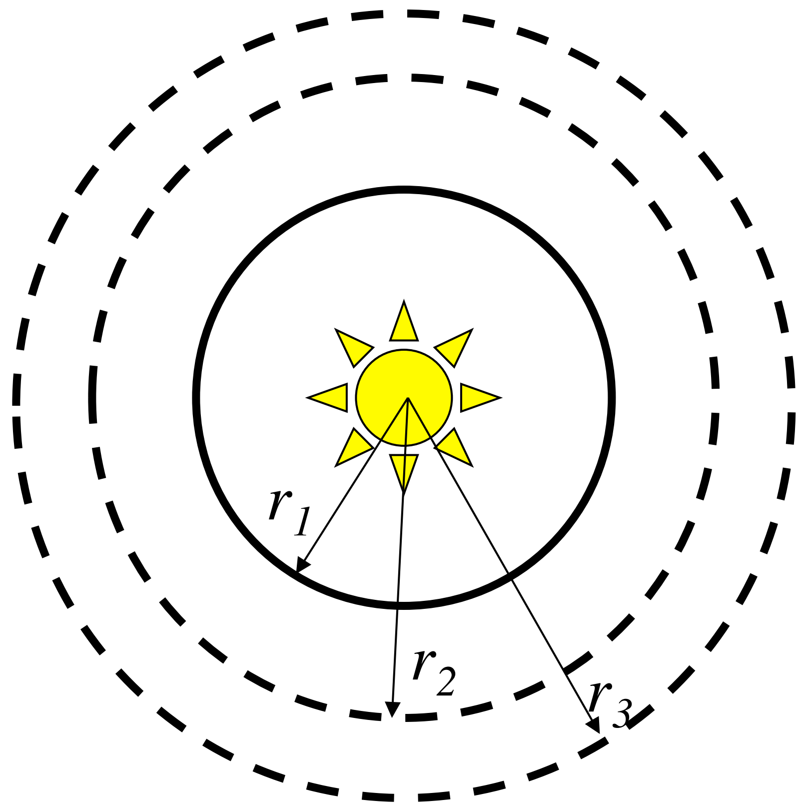
# A Photometric Model

- What parameters describe the measurement?



# A Photometric Model

- Star
  - Brightness
  - Center ($x_0, y_0$)
  - Width ($\sigma$)
- Sky background in annulus
  - $B$
- Detector
  - *QE*, readnoise, dark current
- Aperture sizes
  - $r_1, r_2, r_3$



# Photometric Model

- Write down an expression for the signal, $S_i$, in units of photoelectrons
  - In an individual pixel

$$S_i = F_i + B_i + Q_i + E_i$$

  - $F_i$ is the stellar signal = $f_i t$ at pixel $i$ [$e^-$]
    - Different for every pixel
  - $Q_i$ is the dark charge = $i_i t$ [$e^-$] in a given pixel
    - The dark current $i_i$ varies from pixel to pixel
    - For SNR model assume constant
  - $B_i$ is the sky background = $b_i t$ assumed uniform [$e^-$]
    - Varies from pixel to pixel, for SNR model assume constant
  - $E_i$ is the readout electronic offset or bias [$e^-$]
    - Varies from pixel to pixel, for SNR model assume constant

# The Stellar Signal

- The stellar signal is found by subtracting the background from $S_i$ and summing over the $N$ pixels that contain the star

$$F_i = S_i - \left(B_i + Q_i + E_i\right)$$

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1} S_i - \left(B_i + Q_i + E_i\right)$$

$$N_1 = \pi r_1^2$$

- Error in $F_N$ is due to noise in in the signal itself, $F_N$
- Noise due to dark charge, $Q_i$
- Noise from the background, $B$
- The read out noise $\sigma_{RO}$

# Noise Sources

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1}\left[ S_i - \underbrace{\left(B_i + Q_i + E_i\right)}_{Background} \right]$$

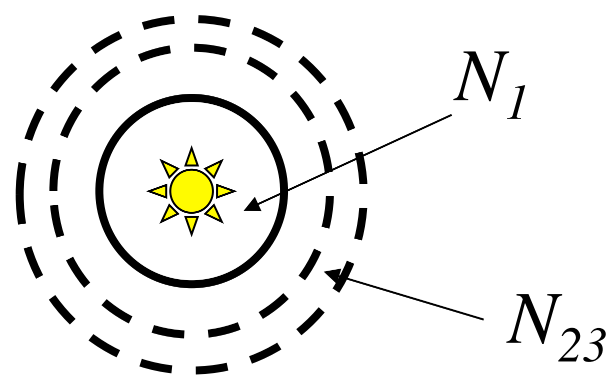
$\langle B \rangle =$ average sky/pixel & $\langle Q \rangle$ the average dark charge/pixel

$$\sigma_F^2 = \underbrace{F_N}_{\text{Poisson signal noise}} + \underbrace{N_1\left(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2\right)}_{\text{Poisson noise within } r_1} + \underbrace{N_1 \sigma_{Sky}^2}_{\sigma_{Sky} \text{ is the error in the sky measured between } r_2 \,\&\, r_3}$$

$$\sigma_{Sky}^2 = \left(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2\right) / N_{23}$$

Every pixel between $r_2$ & $r_3$ contributes to the accuracy of the sky measurement

$$\underbrace{N_1 = \pi r_1^2}_{Star}, \quad \underbrace{N_{23} = \pi r_3^2 - \pi r_2^2}_{Sky}$$



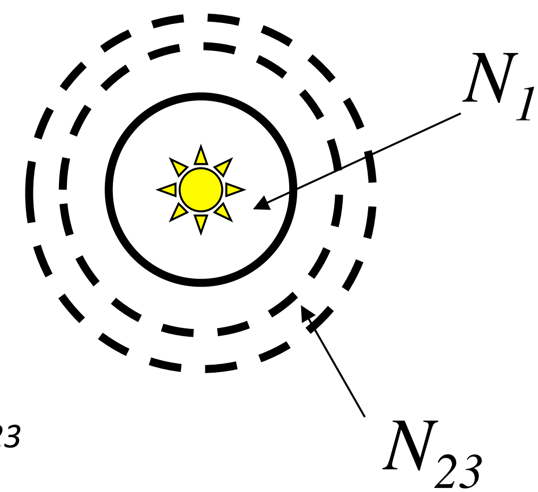
# Noise Sources

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1}\left[ S_i - \underbrace{\left(I_i + B_i + E_i\right)}_{Background} \right]$$

$\langle B \rangle =$ average sky/pixel & $\langle Q_d \rangle$ the average dark charge/pixel

$$\sigma_F^2 = \underbrace{F_N}_{\text{Poisson signal noise}} + \underbrace{N_1\left(\langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2\right)}_{\text{Poisson noise within } r_1} + \underbrace{N_1\left(\langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2\right)/N_{23}}_{\text{Poisson noise within } r_2<r<r_3}$$

$$\underbrace{N_1 = \pi r_1^2}_{Star}, \quad \underbrace{N_{23} = \pi r_3^2 - \pi r_2^2}_{Sky}$$




- How do we choose $r_1$, $r_2$, $r_3$?
  - Signal increases with $N_1$
  - Noise increases with $N_1$ and decreases with $N_{23}$

# Signal-to-Noise

$$SNR = \frac{\overbrace{F_N}^{Signal}}{\sqrt{\underbrace{F_N}_{Signal\,Noise} + \underbrace{N_1\left(\langle B\rangle + \langle Q\rangle + \sigma_{RO}^2\right)}_{\substack{Sky\,Dark\,\&\,RON \\ in\,star\,aperture}} + \underbrace{N_1\left(\langle B\rangle + \langle Q\rangle + \sigma_{RO}^2\right)/N_{23}}_{\substack{Sky\,Dark\,\&\,RON \\ in\,sky\,aperture}}}}$$

- How do we choose $r_1$, $r_2$, $r_3$?
  - Signal increases with $N_1$
  - Noise increases with $N_1$ and decreases with $N_{23}$

# An Example

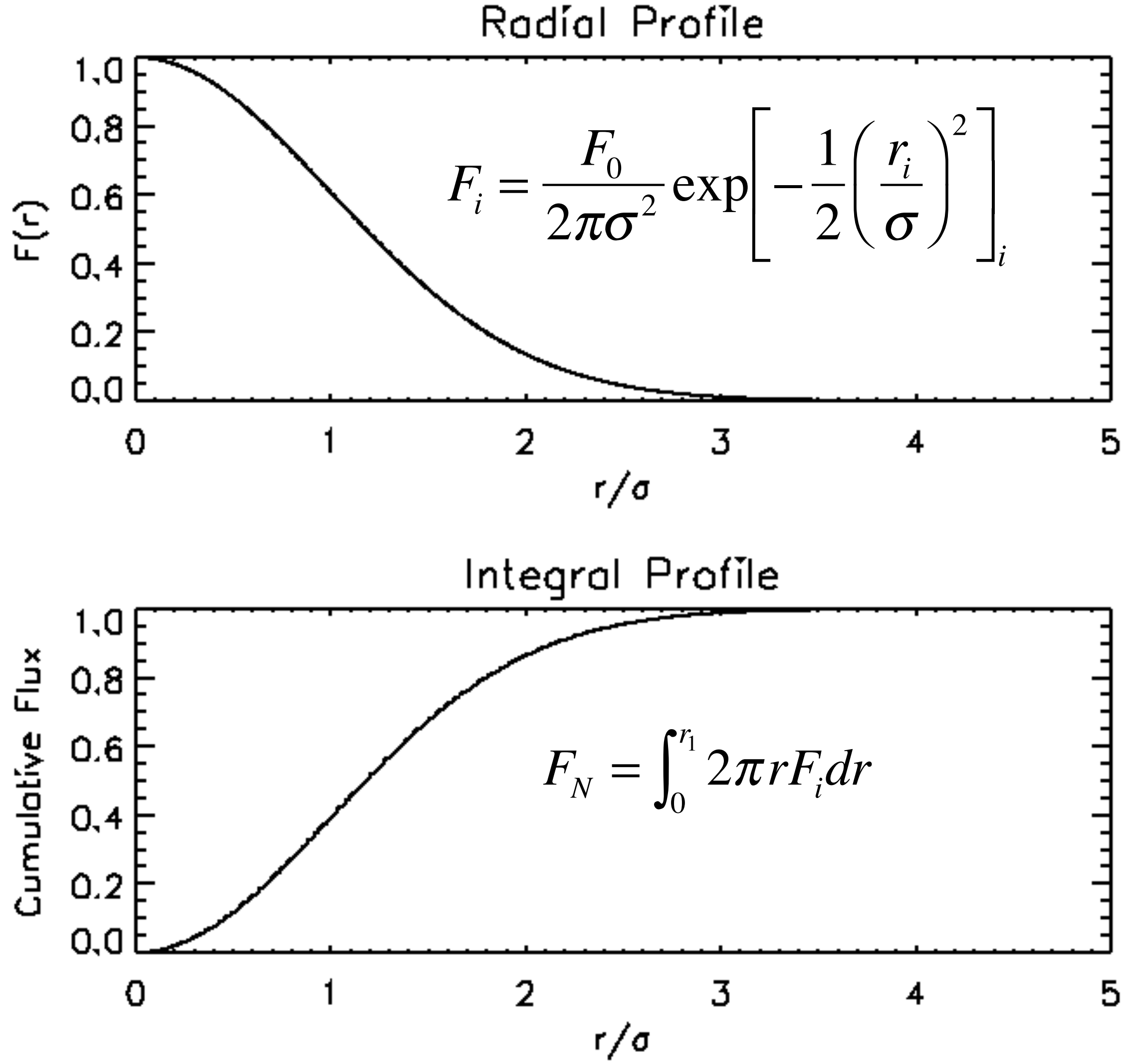
- Suppose the stellar signal (PSF) has a circular Gaussian shape

$$F_i = \frac{F_0}{2\pi\sigma^2}\exp\left[-\frac{1}{2}\left(\frac{r_i}{\sigma}\right)^2\right]_i, \quad r_i^2 = (x-x_0)^2 + (y-y_0)^2$$

$$F_N = \int_0^{r_1} 2\pi r F_i dr$$

  - This tells us how $F_N$ changes with aperture radius

# Star Profile & Integral



# SNR vs. $r_1$

- $F_0 = 100\ e^-$
- $B_i = 100\ e^-$
- $I_i = 0\ e^-$
- $\sigma_{RO} = 10\ e^-$ *rms*
- $N_{23} >> N_1$