Lab IV: DOPPLER MEASUREMENT of SOLAR ROTATION

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This lab involves observations of the sun. Avoid looking directly at the sun or even its reflections. Never look at the sun through a telescope!

1 What I am Supposed to be Doing?

Here's a list of the major steps in this lab:

1. Read this lab handout, familiarize yourself with the other documents on the class web page related to this experiment, make a list of the principal steps, and draw up a plan to execute this lab;
2. Compute the spectral resolution of the spectrometer from the nominal properties of the instrument (cf. Table 1);
3. Learn how to log into the remote desktop that runs the spectrometer;
4. Collect spectra of the continuum lamp (quartz-halogen), the diode laser, and the Ne lamp;
5. Identify the emission lines of the laser and the Ne lamp, identify the echelle orders, and measure the wavelength scale and spectral resolution;
6. Obtain a drift scan of the solar disk and measure the Doppler shift as a function of time; and
7. Deduce the rotation velocity of the sun and find the radius and hence measure the astronomical unit.
8. Your lab report is due at 6PM, on the last day of instruction (2014/12/12.) Please submit your report electronically as a PDF filename "firstname.lastname.lab4.pdf".

2 Introduction

Berkeley astronomer Geoff Marcy records the spectra of stars and measures their radial velocity using the Doppler shift. He uses repeated measurements to search for periodic changes in velocity that reveal the presence of orbiting planets. In the non-relativistic limit, the Doppler shift ($v \ll c$), $\delta\lambda$, measures the radial component of the star's velocity

$$\mathbf{v} \cdot \hat{\mathbf{x}} = v_r = \frac{\delta\lambda}{\lambda}\, c, \tag{1}$$

where $\mathbf{v}$ is the velocity of the star, $\hat{\mathbf{x}}$ is a unit vector towards the star, and c is the speed of light.

Since Marcy measures the Doppler shift of the star and not the planet, the amplitude of the signal is small. For two masses, m_1 and m_2, in a circular orbit of semimajor axis, a, the orbital velocity is

$$v_1 = \left[\frac{G m_2^2}{a(m_1 + m_2)}\right]^{1/2}, \tag{2}$$

or inserting numerical values for a 1 solar mass star and a Jupiter-mass planet,

$$v_{star} = 12.7 \left(\frac{a}{5\ \mathrm{AU}}\right)^{-1/2} \left(\frac{M_{planet}}{M_{Jupiter}}\right)\ \mathrm{m\ s^{-1}}. \tag{3}$$

Since the speed of light is 3×10^8 m s^{-1}, you might expect that a spectral resolving power, R, of

$$R = \frac{\lambda}{\Delta\lambda} = \frac{c}{v} \simeq 2 \times 10^7 \tag{4}$$

would be required to detect planets. Marcy's group has recently detected Earth like planets (1 $M_{Jupiter}$= 316 $M_{\oplus}$) planets (Fig. 1). The velocity precision needed to find these objects is about 0.1 m s^{-1}.

The spectrographs used for the Doppler search have a resolving power of about 50,000 or a corresponding velocity precision of 6 km s^{-1}. How is it possible to detect planets with such crude measurements? The answer is that planet hunters observe their target stars at very high signal-to-noise and record many hundreds of narrow stellar absorption features (see Fig. 2). The combination yields a precision much better than 6 km s^{-1}.

In this lab we will use similar techniques, not to measure the Doppler shift of other stars, but that of our own sun.

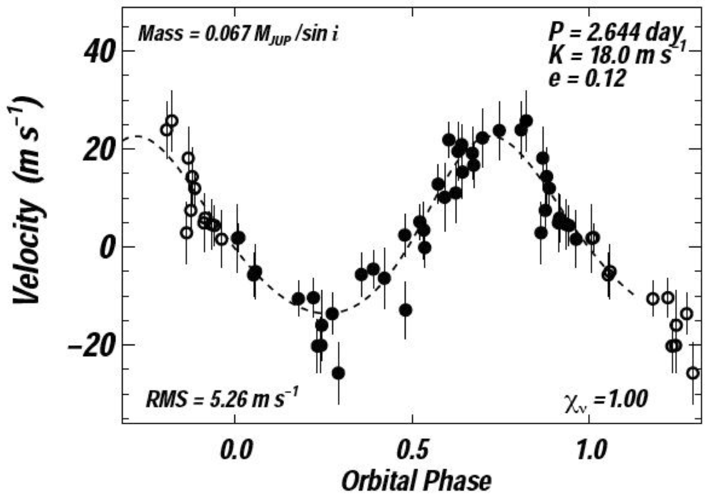

Figure 1: The detection of a Neptune-mass (20 $M_\oplus$) planet by Geoff Marcy's group. Measured velocities vs. orbital phase are plotted for GJ 436 (filled dots), with repeated points shown as open circles. The dotted line is the radial velocity curve from the best-fit orbital solution, $P = 2.644$ days, $e = 0.12$, $M \sin i = 0.067$ M_{JUP}. The RMS of the residuals to this fit is 5.26 m s^{-1} with a reduced $\chi^2 =$ 1.00.

3 Measuring the Radius of the Sun

An elementary problem in astronomy is to measure the diameter of the sun. If you know the solar radius, $R_\odot$, the it is trivial to compute the earth-sun distance knowing the angular diameter of the sun.

The radius of the sun can be measured by comparing the observed solar rotational period, T_{rot}, with the rotation speed of the solar surface. The rotational speed, v_{rot}, is

$$v_{rot} = \frac{2\pi R_\odot}{T_{rot}}. \tag{5}$$

The rotation period can be established by noting the motion of sun spots, and at the solar equator the period is about 25 days. The rotation speed can be inferred from the Doppler shift of material in the solar photosphere at the solar limb (the edge of the sun.)

Fortunately, at optical wavelengths the solar photosphere exhibits strong, narrow absorption features that can be used to measure the Doppler shift. Fig. 2 shows the spectrum of the sun at high spectral resolution.

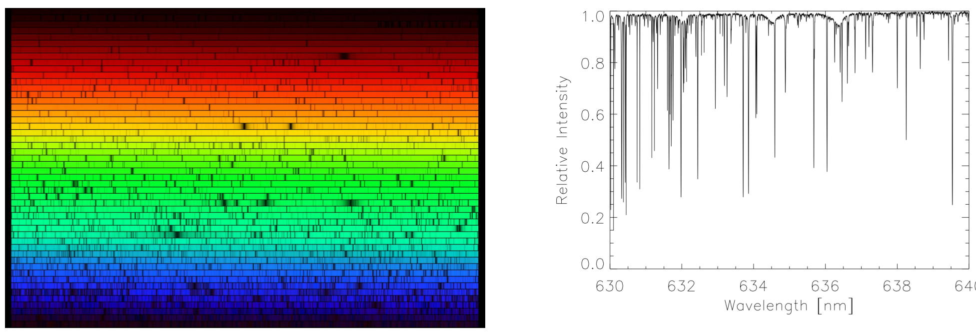

Figure 2: *Left:* A high resolution solar spectrum. This image was created from spectra obtained with the Fourier Transform Spectrometer at the National Solar Observatory on Kitt Peak, AZ. (see `/www.noao.eduimage_gallery/html/im0600.html`) Wavelength increases from left to right along each strip, and from bottom to top. Each slice covers 6 nm, for a range of 400 to 700 nm. *Right:* A small section (10 nm) of the solar spectrum at high spectral resolution ($\lambda/\delta\lambda \simeq 10^5$) from the McMath-Pierce 1-meter Fourier Transform Spectrometer. Numerous narrow absorption lines are present which make it possible to measure the Doppler shift of of stars to high precision.

Data from the solar spectral atlas used to make Fig. 2 are available from the National Solar Observatories Digital Library at:

`http://diglib.nso.edu/ftp.html`

It is the presence of these narrow absorption features that make possible the indirect detection of exoplanets; we will use them to establish the rotation speed of the sun.

From other astronomical methods, e.g., transits of Venus, we can estimate that the radius of the sun is approximately 700,000 km, in which case the rotation speed at the equator, using Eq. (5) must be about 2 km/s. The non-relativistic Doppler shift $\delta\lambda$ is

$$\delta\lambda = \frac{v}{c}\lambda_0, \tag{6}$$

where λ_0 is the rest-wavelength. Given that the speed of light is approximately 300,000 km/s, this order of magnitude estimate suggests that we need a spectral resolving power of

$$R = \frac{\lambda}{\delta\lambda} = \frac{c}{v_{rot}} \simeq 150,000 \tag{7}$$

to detect the Doppler shift due to solar rotation. You might suspect that even higher spectral resolution may be necessary, since the Doppler shift is only sensitive to the line of sight component of velocity. The full amplitude of the Doppler effect is seen only exactly at the limb of the sun and when the earth lies in the equatorial plane of the sun. Generally, the Doppler shift will be $v_{rot} \sin\theta \cos\phi$, where θ and ϕ are the angles shown in Fig. 3.

3.1 Spectral Resolving Power

There is a general result in spectroscopy which states that the limiting spectral resolving power of an instrument is

$$R = \frac{\lambda}{\delta\lambda} = mN, \tag{8}$$

where m is the order of interference, and N is the number of interfering beams. Eq. 8 follows from the uncertainty principle of quantum mechanics.

Suppose we are using a diffraction grating in first order, i.e., $m = 1$ with 100 grooves/mm. By "first order" we mean that the grating is oriented so that there is a one wavelength delay between adjacent diffracted beams. In this case the grating would have to be $150,000/100$ mm $= 1.5$ m long to achieve a spectral resolution equivalent to a 2 km/s Doppler shift! While meter-long path lengths may be practical for large, expensive research instruments, e.g. Fig. 4, they are certainly not an option for an undergraduate astronomy lab.

The approach we take is analogous to that used in the Doppler searches for exoplanets. The reflex Doppler motion due to an unseen planet is a few meters a second, but high resolution spectrometers used to detect Doppler planets achieve only $R \simeq 50,000$. The meter per second precision is accomplished by determining line positions at modest spectral resolution but at very high signal to noise. Many lines are used simultaneously; thus the ultimate Doppler precision is

$$\delta v \simeq \frac{1}{SNR} \frac{1}{\sqrt{M}} \frac{c}{R}, \tag{9}$$

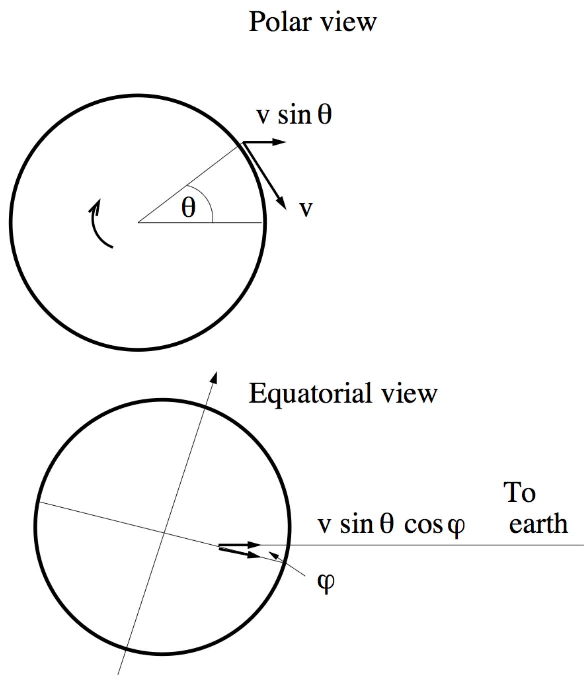

Figure 3: The amplitude of the solar rotation at the equator is v_{rot}. The full Doppler amplitude is observed only at the limb, and elsewhere is reduced by a factor of $\sin\theta$ when the earth lies in the plane of the solar equator. The earth lies in the solar equatorial plane on June 5 and December 7. Otherwise there is an angle, φ between the direction of rotation and the line of sight, and the observed velocity is $v_{rot}\sin\theta\cos\varphi$.

where SNR is the signal-to-noise in the measurement of individual line positions and M is the number of lines measured. If $R \simeq 50,000$, $SNR = 100$ and $M = 100$, the raw spectrometer resolution of $\delta v = 300,000/50,000$ km/s $= 6$ km/s becomes 6 m/s!

To see the plausibility of this argument consider the measurement of a line feature in a spectrum using two pixels, with positions labeled $x_- = -1$ and $x_+ = +1$. If the number of photons detected in x_+ is N_+ and the number of photons detected in pixel x_- is N_- then the center of light is

$$\langle x \rangle = \sum x_i N_i / \sum N_i = \frac{N_+ - N_-}{N_+ + N_-}, \tag{10}$$

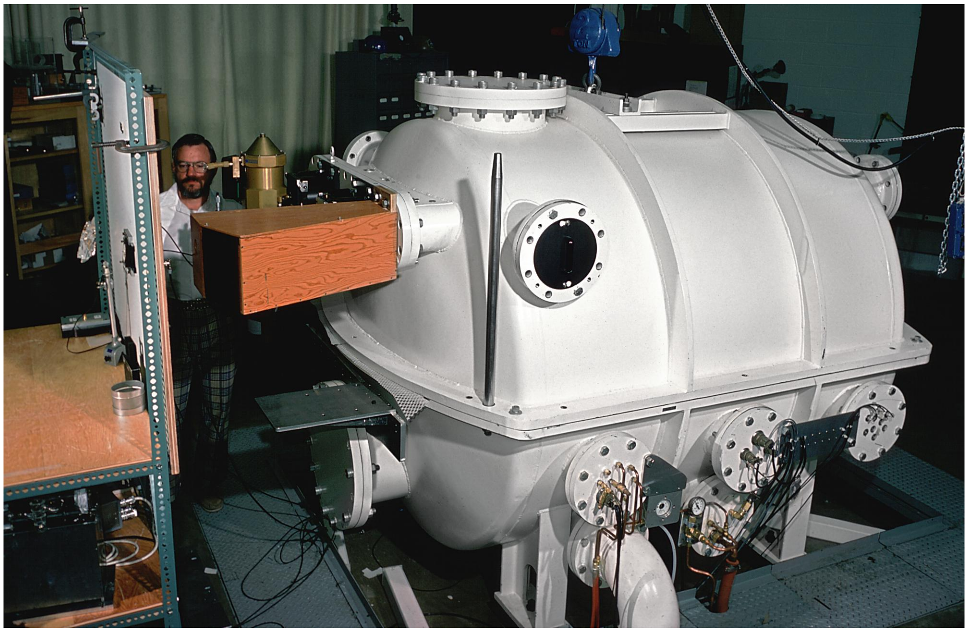

Figure 4: The McMath-Pierce 1-meter Fourier Transform Spectrometer at Kitt Peak National Observatory. This "folded" Michelson interferometer is housed in a vacuum vessel, and is used to obtain very high resolution spectra of the sun, e.g., the spectrum shown in Fig. 2. As the name implies the maximum path length is 1 meter and hence this instrument is capable of achieving a spectral resolution in excess of 10^6 at visible wavelengths.

and $-1 \leq \langle x \rangle \leq 1$. By error propagation, the error in the $\langle x \rangle$ is

$$\sigma_x^2 = \frac{N_-}{(N_+ + N_-)^2} + \frac{N_+}{(N_+ + N_-)^2}, \tag{11}$$

or

$$\sigma_x = \frac{1}{\sqrt{N}}, \tag{12}$$

where $N = N_- + N_+$ is the total number of photons counted. The additional precision implied by the factor of $\sqrt{M}$ is justified by noting that if we have M independent measurements from M different lines, the standard error is reduced by the square root of the number of measurements.

Stated succinctly, the core problem to be solved in this lab is

how to measure the centroid of an absorption line to a small fraction of a pixel and combine this measurement for multiple lines.

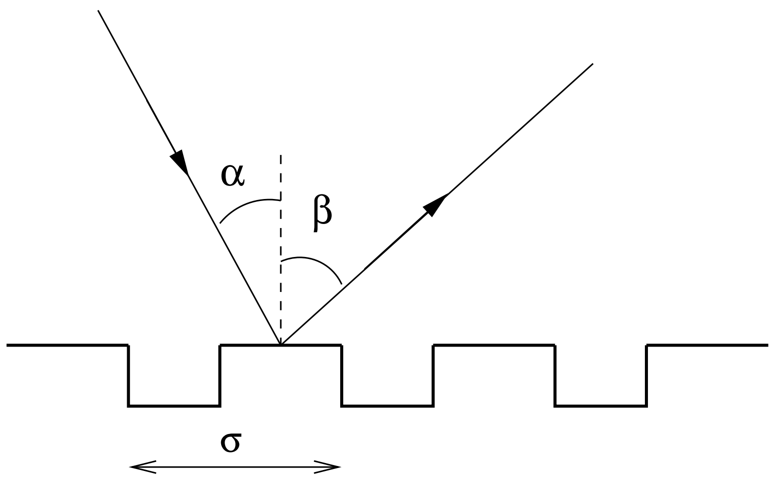

Figure 5: The angles of incidence, α, and diffraction, β, are shown for a simple grating with groove spacing σ. Angles are measured from the dashed vertical line, which represents the grating normal. In this example α and β are both positive.

4 Diffraction Gratings & Spectrometers

The solar spectrometer uses a diffraction grating, which is exceptionally delicate. Under no circumstances touch the grating. The grating cannot be cleaned, so it is imperative to keep dust and other contaminants off the grating. Always wear gloves when making any adjustments on the instrument.

4.1 Basic Relations

The heart of our solar spectrometer is a diffraction grating. The starting point for this section is the grating equation

$$\frac{m\lambda}{\sigma} = \sin\alpha + \sin\beta \tag{13}$$

where α and β are the angles of incidence and diffraction, m is the order of interference, λ is the wavelength, and σ is the spacing of the grooves in the diffracting element. Equation (13) states the condition for constructive interference. The + sign is appropriate for a reflection grating using the sign convention illustrated in Fig. 5.

The free spectral range, $\Delta\lambda$, is the difference between the wavelength for two lines in adjacent orders (m and $m+1$) that show up at the same value of β, assuming fixed α and σ. Thus,

$$m(\lambda + \Delta\lambda) = \sigma(\sin\alpha + \sin\beta) \quad (14)$$

$$(m+1)\lambda = \sigma(\sin\alpha + \sin\beta) \quad (15)$$

$$\Delta\lambda = \lambda/m. \quad (16)$$

This circumstance is depicted in Fig. 6. As a consequence, a narrow band "order sorting" filter with band pass smaller than $\Delta\lambda$, or a second diffraction grating with grooves oriented perpendicular to primary grating, must be used to isolate or separate the various orders of the primary grating.

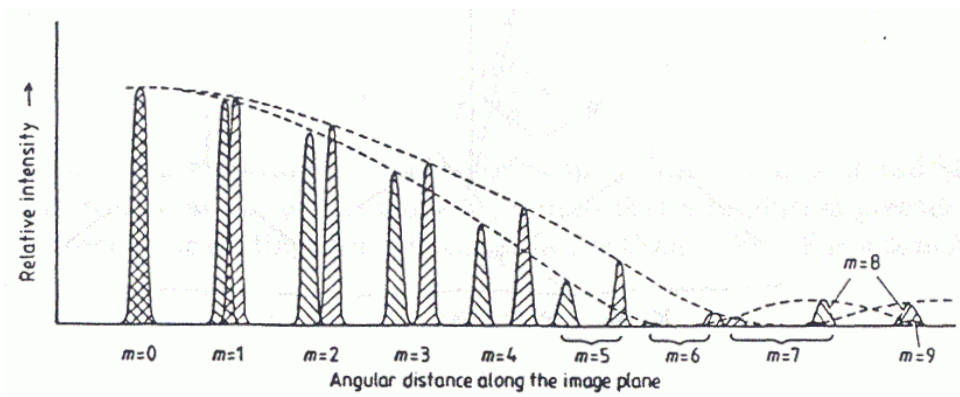

Figure 6: Spectral lines overlapping in adjacent orders of interference. A grating spectrograph is usable over only a small wavelength range, $\simeq \Delta\lambda$ known as the free spectral range. A portion of the image for a source with two emission lines is depicted. Orders 6 and 7 overlap so that the long wavelength line of $m = 6$ starts to blend into the short wavelength line of $m = 7$. The dotted lines show the envelope of the grating response function for a unblazed grating.

For a square groove shape, the energy is diffracted into the orders $m = 0, \pm1, \pm2, \pm3...$, with most of the energy going into $m = 0$ and successively less energy in $m = \pm 1$ and higher orders. Using any one order yields low efficiency. Choosing a suitable shape for the periodic structure makes the directions of the constructive interference and specular reflection from the grating facets coincide for a given wavelength and order; a technique known as blazing. The incident and diffracted beams satisfy the rules of geometric optics,

$$\alpha = \delta + \theta, \tag{17}$$
$$\beta = \delta - \theta, \tag{18}$$

where δ is the blaze angle, 2θ is the full angle between the incident and diffracted beams. The order of diffraction on blaze is m,

$$m = 2\left(\frac{\sigma}{\lambda}\right)\sin\delta\cos\theta. \tag{19}$$

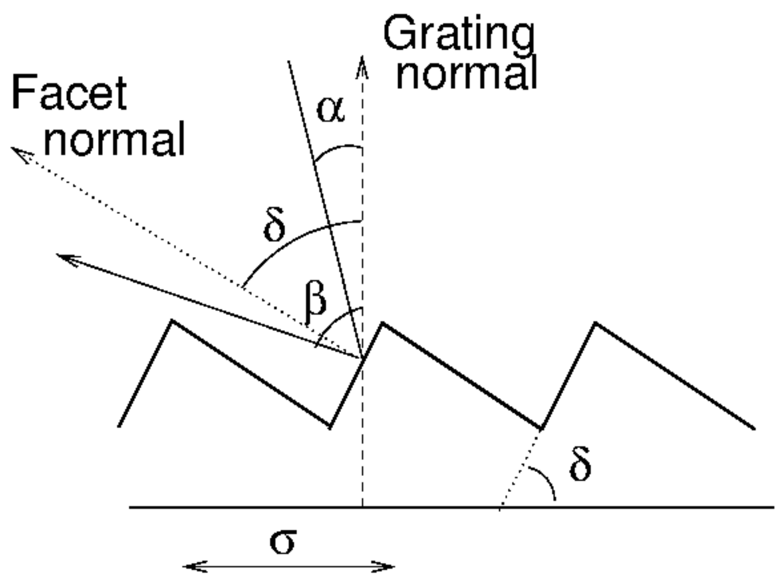

Figure 7: A grating with angled grooves can be used to direct the diffracted light preferentially into one order. The blaze angle, δ, is measured from grating normal to the facet normal.

4.2 Spectral Resolution

The angular dispersion is

$$\left(\frac{\partial\beta}{\partial\lambda}\right)_{m,\alpha} = \frac{m}{\sigma\cos\beta} \tag{20}$$

The spectral resolution is

$$\delta\lambda = \left(\frac{\partial\lambda}{\partial\beta}\right)_{m,\alpha} \delta\beta, \tag{21}$$

where $\delta\beta$ is the spread of angles of diffraction for a monochromatic source. If the source has finite angular size, $\delta\alpha$, then

$$\delta\lambda = \left(\frac{\partial\lambda}{\partial\beta}\right)_{m,\alpha} \left(\frac{\partial\beta}{\partial\alpha}\right)_{m,\lambda} \delta\alpha \tag{22}$$

The term

$$\left(\frac{\partial\beta}{\partial\alpha}\right)_{m,\lambda} = \frac{\cos\alpha}{\cos\beta} \tag{23}$$

is known as the anamorphic magnification. The spectral resolving power, $R = \lambda/\delta\lambda$, is given by

$$R = \frac{m\lambda}{\sigma\ \cos\alpha\ \delta\alpha} \tag{24}$$

Thus the resolution is limited by the angular diameter of the source illuminating the spectrometer. In our case the the source is a nominal $d_{fib} = 50$ μm diameter optical fiber, which feeds a collimating optic of focal length, $f_{col} = 100$ mm, thus

$$\delta\alpha = \frac{d_{fiber}}{f_{col}} \tag{25}$$

5 Understanding the Spectrometer

A labeled photograph of the spectrometer is shown in Fig. 9. One of your first tasks is to understand the properties and parameters of the spectrograph. Derive formulae for the following properties and based on the information listed in Table 1 compute the following for a wavelength of 632.8 nm:

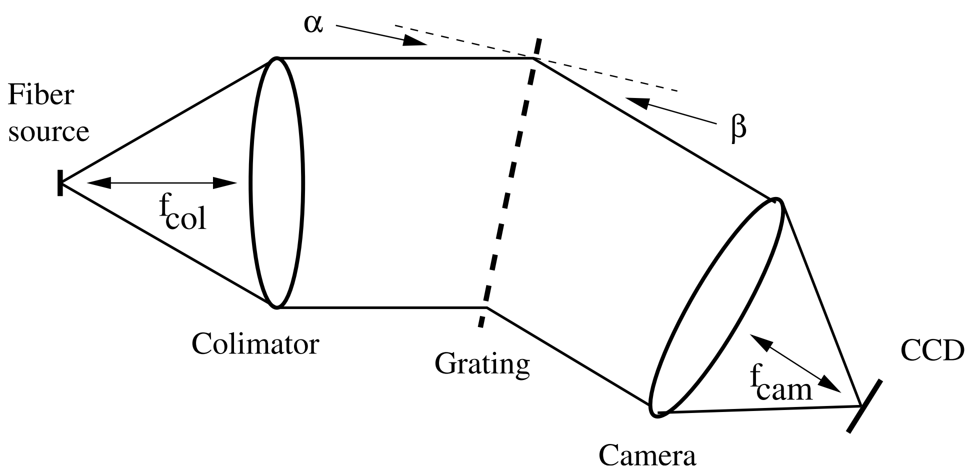

Figure 8: Schematic layout of a spectrometer using a transmission grating. A fiber source illuminates a collimator lens which produces a parallel beam on a dispersing element (a transmission grating in this case). The diffracted light from the grating is collected by a camera and focused onto a CCD. The spectrometer makes an image of the fiber source on the CCD. The finite size of the fiber determines the spectral resolution of the system.

- Echelle order of interference on blaze (m)
- Echelle free spectral range for this order [nm]
- Minimum and maximum wavelengths of this echelle order [nm]
- Angular spread of echelle order [deg]
- Magnification (including the echelle grating anamorphic term)
- Spectral resolution element[1] [pixels]
- Spectral dispersion of the echelle grating [nm/pixel]
- Spectral resolving power [$R = \lambda/\delta\lambda$]
- Spectral resolution [km/s]
- Width of the echelle order on the CCD [mm]

[1] i.e., the size of the image of the fiber on the CCD in pixels.

- Width of the echelle order on the CCD [pixels]

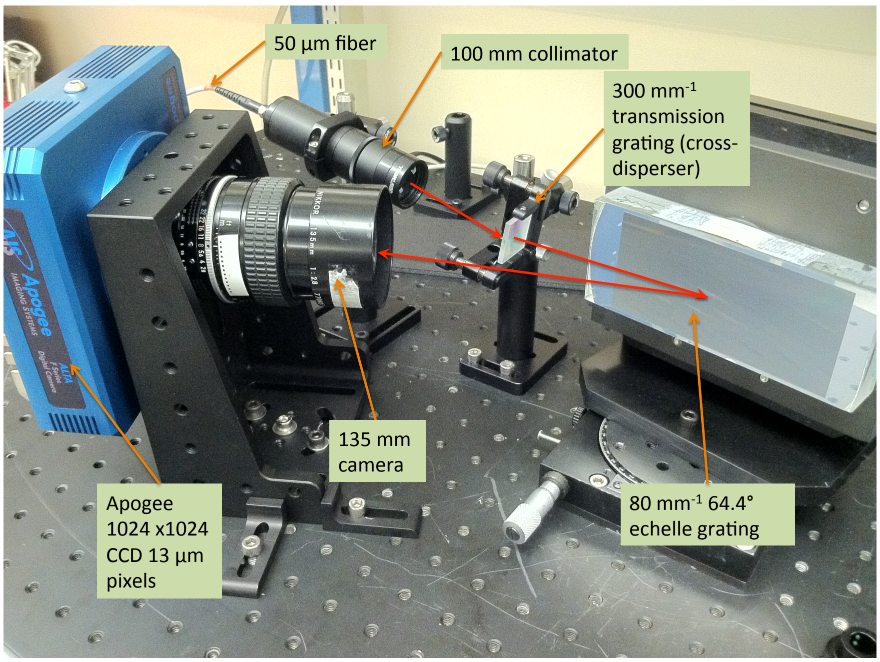

Figure 9: A photograph of the cross-dispersed echelle spectrometer. The principal components are labeled.

5.1 Calibration

The spectrometer has many degrees of freedom. For example, if you change the focus of the collimation optics, you have to go through a complex alignment procedure to get the beam back into collimation. There are also x/y alignments and rotations (θ_x/θ_y) for each lens, which in general should not be manipulated. If the image quality is poor ($FWHM > 3$ pixels) the most likely necessary adjustment is the focus on the Nikon camera lens (see Fig. 9) on the APOGEE CCD. Ask an instructor if you need access to NCH 241 to change this setting.

Table 1: Nominal Spectrometer Properties

Property	Value
Echelle grating groove density ($1/\sigma$)	80 mm^{-1}
Echelle blaze angle (θ_b)	64.4°
Fiber diameter (d_{fiber})	50 μm
Collimator focal length (f_{col})	100 mm
Camera focal length (f_{cam})	135 mm
Apogee CCD pixel size	13 μm
$2\theta^a$	22°

[a]Full angle between incident and diffracted beam

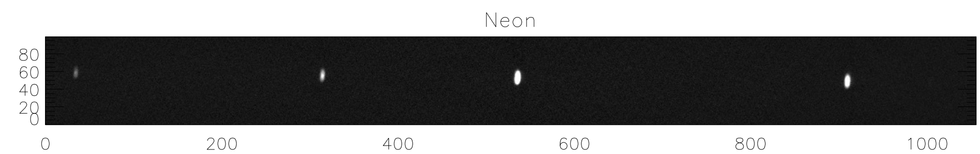

Figure 10: A small section of spectrum of the Ne calibration lamp. The white spots correspond to individual emission lines. The dispersion direction, i.e., wavelength, is along rows. Note that the x-axis displays the entire width of the spectrum, but the y-axis spans only 100 pixels. You can speed the data acquisition process by windowing the APOGEE CCD before you read out data.

The two adjustments that may want consider making are the rotation angle of the echelle grating table (rotation about an axis parallel to the grating grooves) and orientation of the cross disperser. Since the free spectral range of the echelle grating is small ($\lambda/m = 20$ nm) accurate alignment of the echelle is critical, and it may be necessary to rotate this grating to center the spectral orders on the CCD.

The cross disperser only has a coarse mechanical adjustment, and is fixed at an angle of 11° and should not need to be moved. We also have three narrow (10 nm) filters centered a 633, 590 and 488 nm that if necessary can be used to isolate individual echelle orders. The tilt of echelle orders relative to the CCD depends on m, and only one order of diffraction will run approximately parallel to rows. This rotation can be adjusted if necessary.

The first step in acquiring astronomical data is to calibrate the wavelength scale, measure the spectral dispersion (nm per pixel) and estimate the spectral resolution. This can be done by focusing on a single echelle order and obtaining a spectrum of the Ne lamp. The Ne lamp unit sits on the side of the optical bench, and feeds the spectrograph input via a short orange optical fiber. **Note: Optical fibers are delicate! Do not bend them sharply, step on them or pinch them.**

You will only need short exposures, 1–10 s. If you do not see any emission lines it may be that the neon lamp control is not working, the calibration fiber is not connected, or the lens cap is on the camera lens. You can check this by illuminating the fiber with quartz-halogen lamp. When you turn on the quartz-halogen lamp you should see continuous streaks of light running approximately along rows, with the brightest section in the middle of the CCD array. Again, you will only need a short exposure of a few seconds. If you don't see any light ask for help! If all else fails we can use the 635 nm diode laser to align the spectrometer and find a fiducial point in the spectrum.

Fig. 10 shows an example of the Ne spectrum near 633 nm. You should see a series of bright spots running along CCD rows. The spectrum spans the full width of the APOGEE detector, but is only a few tens of pixels high. You can speed up data acquisition by setting a window in the y-direction of about 100 pixels.

Your next step is to identify the Ne line spots. You can do this using the on-line Ne atlas at Kitt Peak National Observatory to identify the lines :

`http://www.noao.edu/kpno/specatlas/henear/henear.html`

An ASCII table of these data is at:

`http://www.noao.edu/kpno/specatlas/henear/henearhres.dat`

Note that both these lists include lines from species other than Ne. The best reference for the wavelengths is NIST.

Use the Ne data to establish the wavelength scale of the spectrometer, show that the dispersion ($d\lambda/d$pixel) is approximately linear. Do this by plotting wavelength as a function of pixel value. Estimate the spectral resolution of the spectrometer in nm and km/s from the measured dispersion and the size of the images of the fiber. Compare your result with Fig. 11. Note that the dispersion direction (increasing wavelength) is not oriented exactly along rows. If the FWHM of your Ne lines is larger than 3-4 pixels, something is amiss.

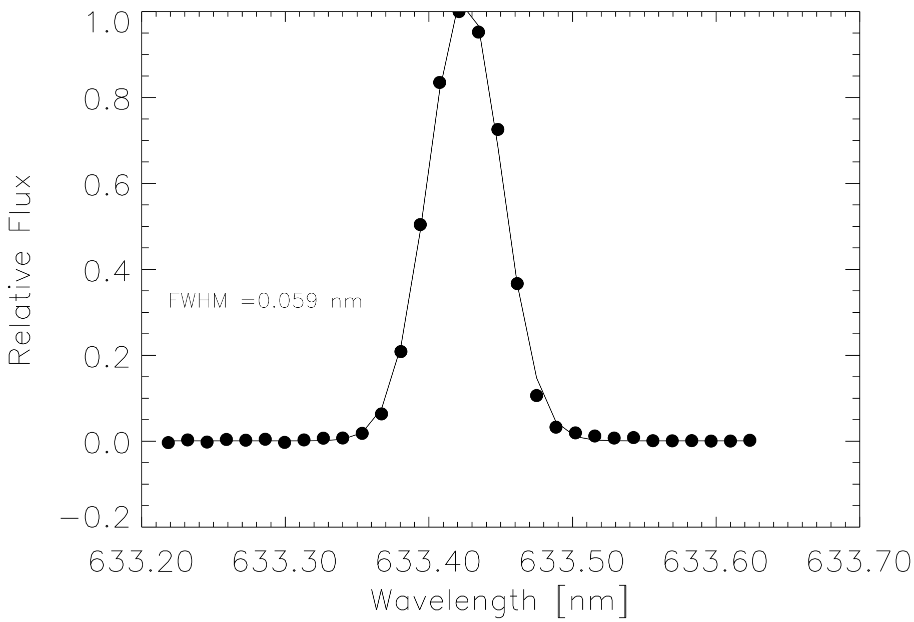

Figure 11: A small section of the Ne lamp spectrum showing a single emission line. A Gaussian profile has been fit to the line showing that the FWHM is about 0.06 nm.

5.2 Solar Observing

Never look at the sun through any optical instrument! Do not look through the eyepiece or the finding telescope. You must not set up the solar telescope without the aperture mask, and the first time you use the solar telescope you must do it with Graham.

Install the 4-inch Newtonian in the NCH 541 observing platform. You will need to align the mount so that the polar axis of the telescope is oriented parallel to the spin axis of the earth. You can align the telescope N/S by using the holes drilled in the concrete. and the equatorial wedge on the telescope can be adjusted, knowing that the latitude of Berkeley is 37°. Make sure that the telescope is balanced, so that the telescope pointing does not sag over time.

Once the telescope is aligned, retrieve and install the orange 50 μm fiber and attach it to the telescope projection screen. Point the telescope to the declination of the sun and slew the telescope in hour angle until you have an image of the solar disk. Check the telescope focus.

To reasonable accuracy ($\pm 0.^\circ 3$) the declination of the sun can be approximated by

$$\delta_\odot \simeq -23.^\circ 44 \cos\left[2\pi\left(\frac{n+10}{365.24}\right)\right], \tag{26}$$

where $-23.^\circ 44$ is the obliquity of the ecliptic, and n is the day of the year.

Refine the pointing in declination so that an hour-angle scan moves the telescope across the solar diameter, which can be identified as the broadest dimension of the sun.

Take a test exposure with the solar disk on the fiber and choose an exposure time that gives plenty of counts ($20,000 < ADU < 50,000$), but does not saturate.

Point the telescope a few minutes in hour angle to the west of the sun and begin taking CCD frames in continuous readout mode. The rotation of the earth will scan the solar disk over the fiber at a rate of 15 arc seconds per second of time (when the sun is on the celestial equator ($\delta = 0$). To minimize the observing time overhead window down the readout box so that only a few hundred pixels in the y-direction are read out. Continue taking exposures until the western limb has passed over the fiber for about one more minute.

5.3 Measure the Doppler Shift

You now have about 50 to 100 spectra. The early spectra will contain only scattered sunlight. The spectra where the sun is directly over the fiber should be apparent. You need to figure out how to correct your spectra for scattered light and to extract flux as a function of position. You can't use a single row of data because this will throw away valuable photons, and because the dispersion direction is not oriented exactly along rows.

Once you have extracted these "1-d" spectra you can use the method of cross-correlation to search for the Doppler shift from one spectrum to the next. This cross-correlation is more tricky than you have experienced before, because you need to find a fractional pixel offset between spectra!

When you have mastered measuring fractional pixels shifts, you should see a systematic drift in line positions between the E and W limbs. Plot the Doppler shift as a function of time (taken from the FITS header) and demonstrate this shift.

Compute the coordinates of the observed position on the sun as a function of time and evaluate the geometric projection factor depicted in Fig. 3. Show how your predicted velocity variation as a function of time compares with your observations. Compute the radius of the sun and quote your error bars.