Asteroid Astrometry from CCD Images

November 18, 2014

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Abstract

astrometry for asteroid name..etc

1. Introduction

(Some motivation for studying small bodies??) Distance is one of the most important but hard to measure quantities in astronomy. Due to the —- of ;ength scales, astronomical distances can only be inferred from the method of parallax rather than directly measured by physical —– . Parallax —- since historical times— space missions such as Gaia (Lindegren et al.) and Hipparcos (Perryman et al.) dedicated to precision astrometry of millioins of sources. Distance is necessary for deriving the Keplerian orbital elements, luminosity, true motion, size and mass of the astronomical object for detailed studies of origin, structure and evolutionary history of our Galaxy.

. In this experiment, we —— from 3 different epochs (October 16,17,20, 2013) to track the

Since we – , e need to

2. Data Collection and Reduction

2.1. Lick Observatory Nickel Telescope

robotic telescope (30-inch) with a moderate field of view (20 arc minute diameter/2k x 2k CCD)

2.2. Systematic Corrections

The purpose of bias,dark, flat correction is to make the intensity of the image approximately linearly proportional to the number of astronomical photons arriving at the detector.

Bias subtraction. Bias frames are images taken with zero integration time and used to subtract the digital offset at zero level. Common practice include taking a scalar value representative of the bias frame dataset (i.e. mean or median) and subtracting that from the original image. For the simple averaging method, the more bias frames we have the better, since as the number of bias frames we have increase, the standard deviation of the mean falls off as $1/\sqrt{N}$, thereby decreasing the uncertainty on the obtained mean value. We applied the method of indiscriminate rejection described by Chromey (2010) which removes the largest pixel value of the bias frame, then take the median on the remaining values. This method rejects the largest statistical outlier which is included when we simply take the mean for correction. Another method of bias correction is to set extra clock cycles so that the CCD reading continues for an additional 32 pixels after the physical image has been read out. Such use of overscan pixels can be advantageous over bias frames for bias correction in the case when the bias is changing over time since each image has its own bias correction taken immediately after the exposure. We corrected an image by taking the median of the 32 pixel. The median is chosen over the mean so that anomalies such as cosmic ray events or nearby radio activity are rejected. Even though we experimented with all the methods described above, since the goal of this lab is to conduct astrometric measurements using these images rather than detailed photometric study, we have simply chosen the more computationally trivial method of bias frame subtraction to apply to every image.

Dark and Flat Correction. Dark frames are long-exposure images taken with the shutters closed to calibrate for dark responses. We conduct dark correction by subtracting from a single dark frame with exposure time of 300 seconds. Since the CCD for Nickel Telescope is cooled by the LN2 dewar to a relatively low temperature, the thermal production of electrons is much lower than in room temperature. Therefore dark frames will not significantly improve the data quality. We took the dark and flat frames for the I,R filters taken on October 16, 2013 and applied them to the other data on the other days since dark and flat frames were not taken for the observation at the other dates. Using the twilight sky as a source of uniform illumination, the flat field images calibrate the pixel-by-pixel variation of each pixel. The exposure time was short and adjusted for every frame to accommodate for the sky's changing brightness. Every pixel on the 1024-by-1024 image is processed by :

$$\frac{\text{image} - \text{dark}}{\text{flat}} \times \text{median(flat)} \tag{1}$$

As shown in the equation, we first dark-subtract from the image so that the intensity is linearly proportional to the number of photon counts then we divide by the flat in order to scale for the pixel-by-pixel variations. By plotting the pixel histogram of the image, we see that most of the pixel values are very small relative to our original image data at around 0.005 (@@@@UNITSSS). In order to preserve the comprehensible data ranges of the original intensity values after subtraction, the image is normalized by a scalar value that is representative of the magnitude of the flat field data. The median value is chosen because unlike the mean its value is not skewed due to the bad pixel and columns present on the Nickel Telescope.

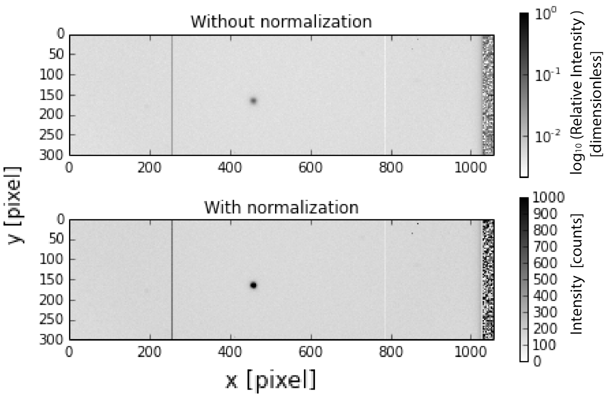

Figure 1: Without normalization, the image requires a logarithmic stretch to distinguish the asteroid and the background since it has to accommodate for the range of decimal values resulted from the division. With normalization, the image can be viewed on a linear scale with a lower intensity limit of 0 and upper limit of 1000.

2.3. *Centroid-finding*

intentionally adjust the Vmag lower cut than Vmag observed in NASA's small object dynamic so that it is able to identify the asteroid as a star. In order to determine the positions of the stars on the image, we need to find the centroid position with the weight as the First, we set a threshold which was empirically adjusted to yield just enough stars for pattern recognition at around 5000 (UNITSS!!!!). This identifies Since bad columns, corners, and borders often show up as completely dark pixels [1], it is mistaken as a whole row of stars , we have to manually mask the pixels by zeroing the intensity value at the selected pixel. Since this is a fairly trivial case where the dat-

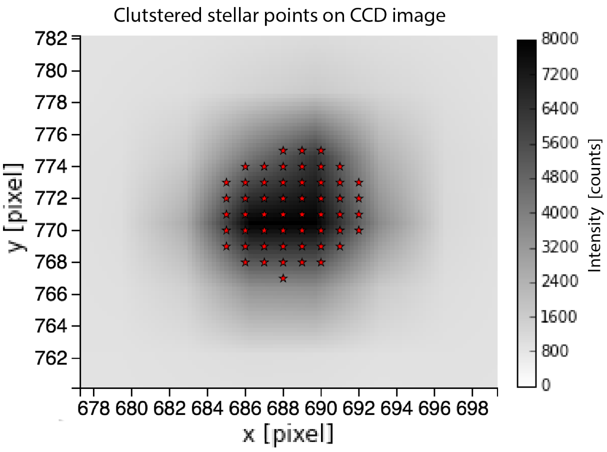

Figure 2: CENTROID MANY STUFF

apoints for each cluster is far apart, we use the more conceptually simplistic but computational intensive method of distance estimation instead of other clustering algorithm in the Machine Learning literature. We loop through list of points above same threshold intensity and for every point we compute its pixel distance from another point in the same list. If the separation distance between two datapoints is less than 20 pixel, then it is clustered into the same group. Each cluster represent a star. This threshold intensity is intentionally chosen so that the asteroid can be mistakenly identified as a star. (can we convert USNO (???)) Now we have a list of pixel positions and intensity values of the datapoints that makes up a star, in order to get a representative value from cluster information, we compute centroid by Eq. 2 (From lab handout) :

$$\langle x \rangle = \frac{\sum_i x_i I_i}{\sum_i I_i} \tag{2}$$

[1]Since image color scheme is inverted, the darkest pixel mean highest intensity.

3. Astrometry

Using stellar positions to calibrate the image

3.1. Pixel to Standard Coordinate conversion

Since most reference stars are distant point sources (?), it is convenient to treat them as projected position onto a sphere centered at the observer. To match up and compare the stars on the CCD and the USNO stars in celestial coordinate, we must plot the USNO stars to pixel coordinates in the same coordinate system. In order to do so, we must first project celestial coordinates (α, δ) into standard Cartesian coordinates (X,Y) using Eq. 4 and 4 (From lab handout) :

$$X = -\frac{\cos(\delta)\sin(\alpha - \alpha_0)}{\cos(\delta_0)\cos(\delta)\cos(\alpha - \alpha_0) + \sin(\delta)\sin(\delta_0)} \tag{3}$$

$$Y = -\frac{\sin(\delta_0)\cos(\delta)\cos(\alpha - \alpha_0) - \cos(\delta_0)\sin(\delta))}{(\cos(\delta_0)\cos(\delta)\cos(\alpha - \alpha_0) + \sin(\delta)\sin(\delta_0)} \tag{4}$$

where (α, δ) are the (RA,DEC) for the USNO stars and (α_0, δ_0) are the center of the imaging field (i.e. the RA, DEC used to submit the USNO query[2]) Then we use Eq.5 and 6 (From lab handout) to scale and offset the standard coordinate so that they are in pixel coordinate (x,y) of Nickel's CCD camera.

$$x = f\left(\frac{X}{p}\right) + x_0 \tag{5}$$

$$y = f\left(\frac{Y}{p}\right) + y_0 \tag{6}$$

[2]We were unable to use the RA, DEC given in the FITS header since it did not refer to the same sky location as what is shown on Aladin. Instead, we had to visually identify the star patterns in nearby region and set the center of the field position returned by Aladin and use those as our (α_0, δ_0)

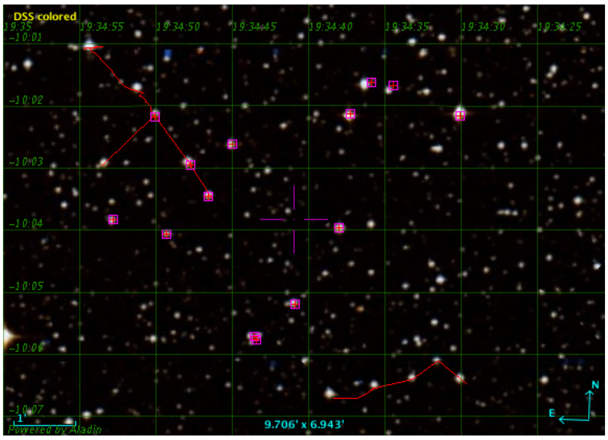

Figure 3: Matching starfield pattern for October 16, 2013 with Aladin Sky Atlas, annotated to assist with pattern recognition. Center crosshair position approximates the RA,DEC used in the USNO query.

3.2. *Least Square*

3.2.1. *Theory*

The simple relation (Eq.5 and 6) does not factor in the possible rotation between CCD and standard Cartesian coordinate system. The goal of using least square is to attain the value of the plate constants in the first two rows of the transformation matrix **T** described by Eq. 7 (From Lab Handout). The value of the plate constants a_{ij} resulted from a combination of scale, shear and rotation operations on the input image. **T** provides a linear mapping between pixel space **x**=(x,y,1) and standard Cartesian $\mathbb{R}^3$ space **X**=(X,Y,1).

$$\mathbf{T} = \begin{pmatrix} (f/p)a_{11} & (f/p)a_{12} & x_0 \\ (f/p)a_{21} & (f/p)a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

where $(x_0, y_0)$are the telescope's pointing offset and f/p=561000 $\frac{\text{pixels}}{\text{radian}}$.

$$\mathbf{a} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_N \end{bmatrix} \quad \mathbf{B} = \begin{pmatrix} (f/p)X_1 & (f/p)Y_1 & 1 \\ (f/p)X_2 & (f/p)Y_2 & 1 \\ \vdots & \vdots & \vdots \\ (f/p)X_N & (f/p)Y_1 & 1 \end{pmatrix} \tag{8}$$

$$\mathbf{c} = \begin{pmatrix} a_{11} & a_{12} & x_0 \end{pmatrix} \tag{9}$$

The offset value is adjusted after conducting least square using first the standard (x_0, y_0) first identifying matching pairs of stars from USNO and CCD image, then calculate the distance between them using the Pythagoras theorem. The distance is an offset that can be added to all the datapoints to precisely match the two datasets, or simlarly substriute for the initial (x_0, y_0) with the addition. In order to compute the plate constants to construct the **T** matrix Constructs a vector —- of all the centroid $\langle x \rangle$ of all the stars on the CCD image [3]. By minimizing χ^2=$||\mathbf{a} - \mathbf{Bc}||^2$,

This is similarly conducted on y by replacing a_x with a vector constructed with all the computed centroid $\langle y \rangle$ and we obtain c_y = The purpose of using the least square method is to find a conversion matrix between the CCD-based pixel coordinate to standard coordinate. The standard coordinates can then be projected to celestial coordinates based on the Eq. (LABEL!!!): (TYPE EQUATION HERE)

3.2.2. *Practice*

The actual —— is the reverse steps of how the theory can be intuitively explained.

Since the model we used for least square is a linear —- with its intercept set as zero, we don't minimize the offset parameter. parameter has no offset (translation). We need to manually —. Alternatively we could have also come up with a model that also minimized the offset. We find that the final projected image in celestial coordinates is horizontally flipped compared to the the our CCD image (INCLUDE FIG??) and Aladin Sky Atlas (also CCD image) 3.

3.3. *Ephemeris information*

Position vector can be obtained from the HORIZONS Web-Interface heliocentric vector to Earth This could be verified with the

[3]This list should include the asteroid, which is intentionally misidentified as a star since the same procedure is conducted in the same manner

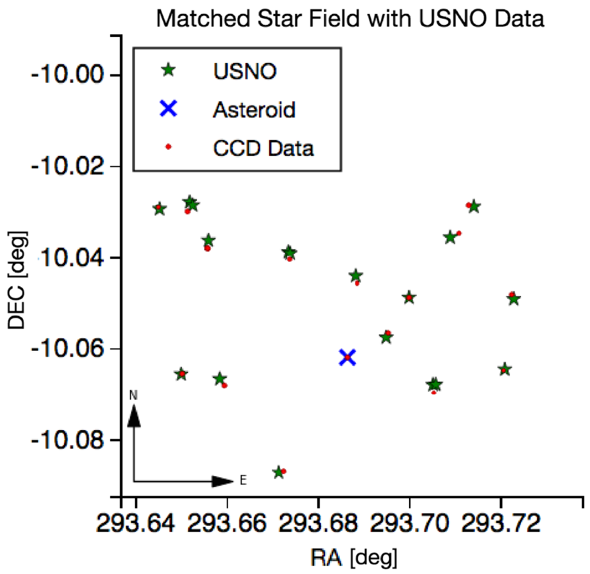

Figure 4: Matching starfield with USNO-B1 of stars with Vmag¿ (——), October 16, 2013. The asteroid is easily identified as it shows up on the CCD image but not on the USNO-B1 catalog.

vector from earth to asteroid. JPL horizon by default returns the ecliptic coordinate, Make sure to chose equtorial coordinate . This would affect the values for y and z components of the position (and velocity) vector returned since in the equitorial system the plane is around the equator

xy-plane: plane of the Earth's mean equator at the reference epoch x-axis : out along ascending node of instantaneous plane of the Earth's orbit and the Earth's mean equator at the reference epoch z-axis : along the Earth mean north pole at the reference epoch We conduct the all procedures mentioned in the above sections to the night of Oct 16, 17, 20, and 21. Only 3 epochs are necessary for computing the distance. Following Eq.9 (Stewart, pg.174) , we chose the 3 of the closest nights (16,17,20) in order to minimize the interval that we are computing our time derivative over to get as close to $\lim_{h\to 0}$ of the derivative definition as possible.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad (10)$$

where h is an infinitesimal interval of x. Before proceeding to using the positions of the asteroids at 3 epochs to find asteroid's distance, we plotted the coordinates of the asteroid as a sanity check as shown in Fig. 5.

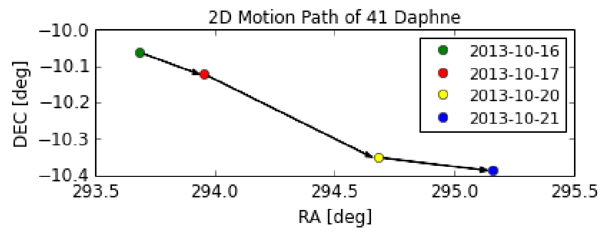

Figure 5: The minor deviation from the straight motion path could be explained by the asteroid's motion in the other directions. As shown in the JPL emphemeris velocity vectors, even though the V_y is much larger than V_x and V_z, V_x and V_z are still non-zero. Since the 3D orbital of the asteroid are suppressed onto the 2D celestial coordinates, ——– (?)

3.4. Iterative solution

The system of equation converges because in order to solve it you must have talk about ρ...[4] By conducting this procedure 1000

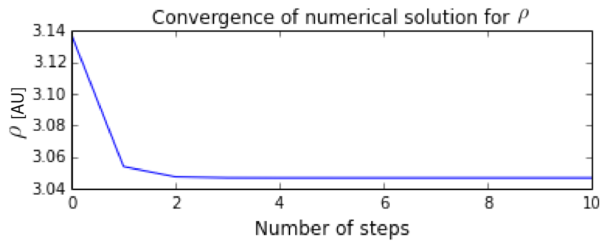

Figure 6: Convergence

times, we find that the solution takes an average of 5 steps to converge[5] Since the vector $\rho\vec{s}$

[4]Since unit vectors are by normalized by its own magnitude, it is by definition dimensionless. Therefore, to construct physical units in the Earth-Asteroid vector, ρ must have units of AU.

[5]We arbitrarily define a solution as "converging" when the difference between solutions at consecutive steps is less than 1.0×10^{-5}. It is interesting to note that each step approximately bring closer to the actual solution by an order of magnitude.

subtends from the Earth to asteroid, the norm of $\rho\vec{s}$ yields the distance of 3.047 AU from Earth to 41 Daphne on the night of October 17, 2013, which has a 19% percent difference from the values returned by NASA Solar System Bodies web tools. The possible source of error contributing to the deviation from the nominal value is discussed in the next section.

4. Error and Uncertainty

4.1. Centroid Error

In order to find the error on the value of ρ that scales the Earth-to-asteroid distance, we must first determine the error on the centroid values used to find ρ. By assuming that the error on each pixel is uncorrelated, we can apply the method of error propagation on Eq. 2 and obtain:

$$\sigma^2_{<x>} = \sum_i \left(\frac{\partial \langle x \rangle}{\partial I_i} \right)^2 \qquad (11)$$

Computing $\frac{\partial \langle x \rangle}{\partial I_i}$ and plugging it back into Eq. 10:

$$\sigma^2_{\langle x \rangle} = \frac{\sum_j I_j (x_j - \langle x \rangle)^2}{\left(\sum_i I_i \right)^2} \qquad (12)$$

Using the list of clustered star position as described in Sec. 2.3 The same method is also used to calculate the error on ¡y¿ However this obtained results is in units of pixels, we must convert this error to units of degree as shown in Fig.7

4.2. Monte Carlo

Since we do not have an analytical solution to solve for ρ we can not compute the partial derivatives used in the error propagation equation. Instead we use the Monte Carlo method to approximate the error on our obtained value of ρ given the centroid error of the asteroid's celestial position. First, we construct two Gaussian nosie distribution with standard deviation as the x and y centroid error

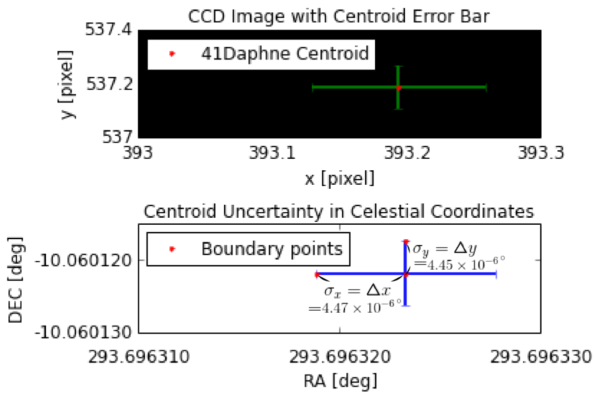

Figure 7: Since the pixel-to-celestial coordinate conversion procedure described in Sec. 3.2 transforms coordinates rather than interval values. We select two boundary points that defines where the x and y error bar subtends and convert these to celestial coordinates. Then subtracting boundary points from the asteroid's centroid position in celestial coordinates, we obtain the x and y uncertainty ranges (σ_x, σ_y) in units of degrees, which has a value of around 0.016 arcseconds.

that we obtained in Sec. 4.1. Then we draw a random value from each distribution to construct a noise vector for the x,y component with the noise on the z component set as zero.[6] The noise vector is added to $\vec{R}$ and this newly computed $\vec{R}$ is used in the iterative algorithm to compute the solution for ρ as described in Sec.3.4 with other parameters kept the same. The Gaussian distribution is centered at zero so that the random noise may be positive or negative. This ensures that the computed ρ solutions are not skewed to a higher value by the addition of the noise vector. By performing the Monte Carlo procedure on 100000 trials, the standard deviation of the list of computed ρ values yields the approximate error on the value of ρ. As shown by conducting this procedure on different number of trials from 10^3 to 10^6, the choice of using $1{\times}10^5$ trial is sufficiently large enough so that the standard deviation result does not vary by a

[6] We have no information about the error on z because it is not involved in the centroid-finidng calculations. Since the CCD image is 2D, we can only compute the centroid error along x and y component.

lot relative to a choice of $1{\times}10^6$ or more.

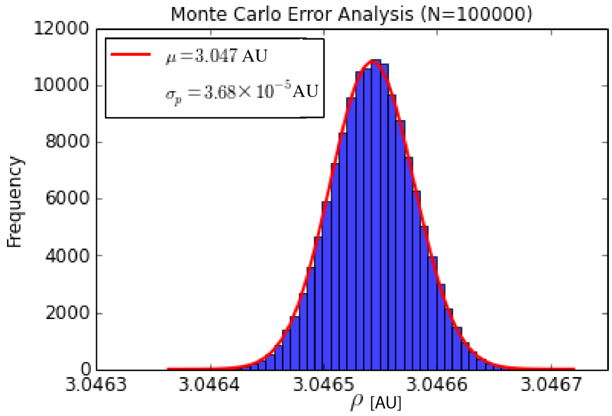

Figure 8: The standard deviation on this Gaussian distribution approximates the error on ρ to be 3.68×10^{-5} AU ($\approx$ 5505 km).

4.3. *Other possible error contribution*

The —- certainly doesn't add to the 0.48 AU deviation from the values returned by NASA's small body simulation (?) One possible error is on NASA's Derivatives of position approximation

velocity and acceleration is obtained h→0 we only did Monte Carlo on x y error and not on the z component added noise vector For this excercise, we only examined errors on the asteroid. However, uncertainty in the measurement of stellar position can also affect our asteroid position result since stellar positions are taken into consideration in the least square procedure. This propagates to uncertainty on the plate constant values we obtain which is used in the calculation for conversion from —- to —- coordinates. However this effect should be secondary to the ones we computed directly for the asteroid.

5. Conclusion

blah

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