

Giancoli Ch17: Temperature, Thermal Expansion, and Ideal Gas Law

Thermodynamics : the description of processes in terms of macroscopic quantities.

State variables: Quantities that can be used to describe the state of a system.

1 Atomic Theory of Matter

- unified atomic mass unit (u) defined as $1/12$ of ^{12}C
- Brownian motion
- Example: Calculate distance between atoms. given mass and density: Solve by basic unit conversion and $1 \times 1 \times 1$ box assumption.

2 Temperature

- properties that change with temperature (expansion, electrical resistance,color,light emission)
- Liquid level thermometer
- Bimetallic thermometer:¹ Consist of two metal strip (with different coefficient of expansion) welded together. At any temperature, their expansion rate differs \rightarrow strip (often in shape of coil) bends
- Electric thermometer : measure change in electrical properties under a certain temperature (e.g resistance thermometers, thermistors, thermocouples) \rightarrow very precise
- Celsius =centigrade (“hundred step”)scale
- One way to define temperature is to assign arbitrary values to two readily reproducible temperature
- Corresponding temperature $0^\circ\text{C} = 32^\circ\text{F}$
- ² $\therefore 1^\circ\text{F} = 5/9^\circ\text{C}$
- \therefore different expansion rates of different liquid, scale may be different.
- \therefore use Constant-volume gas thermometer

3 ^{0th} law of thermodynamics

**If A and B are each in thermal equilibrium with C
then they are also in thermal equilibrium with each other**

where thermal equilibrium is defined as:

System A & B are in thermal equilibrium if there is no **net** change between the two system.

¹We use this method instead of using a single strip because the expansion due to normal thermal changes are too small to be accurately detectable

²A change in 5°C = a change in 9°F

3.1 Pseudo-code notation

This can be more precisely put in pseudo-code looking notations³

$$\text{If}(A \rightleftharpoons C) \text{and}(B \rightleftharpoons C) : \quad (1)$$

$$\text{then } A \rightleftharpoons C \quad \text{where } A, B, C \text{ are systems} \quad (2)$$

4 Thermal Expansion

4.1 Linear Expansion

$$\Delta l = \alpha \Delta T l_0$$

The coefficient of expansion changes slightly with temperature change, but that change is negligible. It is largely determined by the material.

4.1.1 Example: Expansion of ring

The textbook makes the argument that you can hypothetically fill in the hole then then take it out. My worry is that you will have some sort of force that is exerted by the presence of the inner filling that wouldn't have existed if the inner filling wasn't there. I was thinking that half of the shell of the ring (concentrically) would expand and half of the shell (inner shell) would expand, making it seem like contracting. This model is incorrect because since we are considering a "thin" ring, in theory, there is only linear expansion no horizontal ones. (so the textbook approach of filling in the hole works) Therefore to achieve a longer length (i.e. circumference), our only approach is to increase the radius \therefore expansion.

Linear expansion has no meaning for fluid since no fixed shape.

4.2 Volume Expansion

$$\Delta V = \beta V_0 \Delta T$$

Volume of β is often $\approx 3\alpha$ for all isotropic (same in all direction) object.

This can be easily derived mathematically using linear expansion in all 3 dimensions (l, w, h) and truncating the trailing terms.

4.3 Application: Water

- Unlike most materials, water below 4 °C $\xrightarrow{\text{heated}}$ decrease in volume
- Act as normal volume expansion when above 4 °C
- Application: Fish under cold water survive \therefore only top part frozen (ice floats)

5 Thermal Stress

$$\Delta l = \frac{1}{E} \frac{F}{A} l_0 \text{ from previous section, we also know that:}$$

$$\Delta l = \alpha l_0 \Delta T \text{ equating the two, we get : } \alpha \Delta T = \frac{F}{A} \frac{1}{E}$$

$$\text{stress} = \alpha E \Delta T = \frac{F}{A} \text{ where } E: \text{Young's Modulus}$$

³Transitive notation; http://en.wikipedia.org/wiki/Zeroth_law_of_thermodynamics#Zeroth_law_as_equivalence_relation

6 The Gas Laws and Absolute Temperature

- Assumptions of an Ideal Gas:

- equilibrium states** of a system: variables that describe the system is isotropic and invariant over time.
- not too dense ,pressure not too high (≈ 1 atm)
- temperature not close to liquefaction (i.e. boiling) point

- Experimentally found **Boyle's law**:

$$V \propto \frac{1}{P} \rightarrow P\text{-}V \text{ graph is inverse}^4$$

\therefore if constant T, then $PV = \text{constant}$.

- A century later, Jacques Charles found that when pressure is constant, $\frac{V}{T} = \text{constant} \rightarrow V\text{-}T$ graph linear
- Guy-Lussac's law: at constant volume, $\frac{P}{T} = \text{constant} \rightarrow P \propto T$

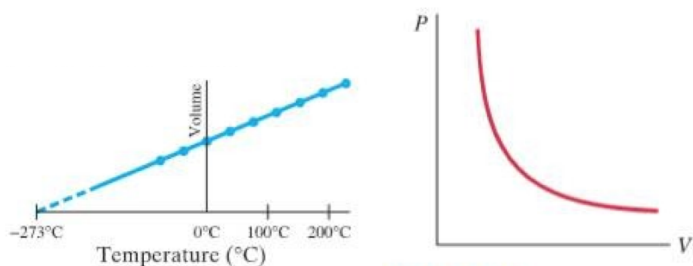


Figure 1: These graph always have x intercept at -273 degrees (0K), but more often the graph stops at liquefaction point (dotted) \therefore it is impossible to have negative volume

7 Ideal Gas Law

- By combining the three laws in the previous section, we have the general relation $PV \propto T$
- Consider also how the amount of gas present affects system
- think of a blowing a balloon, $V \propto n$ where n is the number of moles of gas ⁵
- we now have $PV \propto nT$
- To convert to an equality, we must introduce a constant of proportionality⁶
 $R = \text{gas constant} = 8.314 \frac{J}{mol \cdot K}$
- STP** : $T = 273$ K $P = 1.00$ atm $= 1.013 \times 10^5 N/m^2 = 101.3$ kPa
- using the ideal gas law, it is easy to see that STP volume for 1 mole of **any gas** is $22.4 m^3$ or Liters

⁴where P is the absolute pressure

⁵recall that number of moles can be found by $\frac{\text{mass (g)}}{\text{molar mass (gmol)}}$

⁶remember to use K for ideal gas law calculation and absolute pressure

8 Ideal Gas Law in Terms of Molecules: Avogadro's Number

- Avogadro's hypothesis: equal volumes of gas at same P and T contain equal number of molecules
- R is a constant for any gas \therefore hypothesis and ideal gas law
- There are $N_A = 6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}}$ number of molecules in one mole of any pure substance
- We can figure this relation by dimensional analysis:

$$N = \text{number of molecules} = \left[\frac{\text{molecules}}{\text{mol}} \cdot \text{mol} \right] = n \cdot N_A$$

- Rewriting Ideal Gas Law in terms of Boltzman constant:

$$k = \text{Boltzman Constant} = \frac{R}{N_A} = 7 \frac{8.31 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{mol} \cdot \text{K}}}{6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}}} = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$$
$$PV = nRT = \frac{N}{N_A} RT = NkT$$

9 Ideal Gas Temperature Scale – a Standard

- **ideal gas temperature scale:** universally-defined standard temperature measured by constant-volume gas thermometer ⁸
- Like the Celsius and Fahrenheit scale, we need to specify two fixed points in order to set the scale.
- So we choose P=0 at T=0K and the triple point of water.
- Triple point of water is 4.58 torr (610.62 pascals) and 0.01 °C; where water in solid, liquid, and gas states can coexist in equilibrium

- **Definition of the ideal gas temperature scale:**

For an ideal gas in a constant-volume gas thermometer, we measure the absolute (Kelvin) temperature as:

$$T = (273.16\text{K}) \lim_{P_{tp} \rightarrow 0} \frac{P}{P_{tp}} \quad \text{where } P_{tp} \text{ is the pressure of the gas at triple point}$$

- Define triple point as 273.16K (when $P=P_{tp}$)
- We use the limit \therefore the temperature function is only an approximation (changes with the type of gas used in the thermometer). From graph, we can see that values for $P_{tp} > 0$ varies and $P_{tp}=0$ for any type of gas. Therefore we take the limit of $P_{tp} \rightarrow 0$
- Using this equation (\therefore lim), the temperature is independent of the gas used. But make sure you use the gas with the right properties for measurement. ⁹

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⁸works on the principle of $P \propto T$ @ constant V

⁹For example, you can not measure the absolute temperature under 1K with He, \therefore Helium liquefies at 1K, lowest condensation point of all gases.