Latent Topic Analysis

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The goal of proablistic topic models is to discover latent topics in texts. Each *topic* is simply a distribution of words.

Probablistic Latent Semantic Analysis (PLSA, or refered to as PLSI in some texts)

Assume that words in a document D is generated by sampling words independnetly from a mixture of k topic models $p(w|\theta_i)$, the probability of generating word w in document D is the product of the probability that a topic is in the document and the probability that a word is in the topic, summing this over all possible topics.

$$p_D(w) = \sum_{i=1}^{k} p(i|D)p(w|\theta_i) \quad (1)$$

We know that each document is generated by drawing independently from a multinomial distribution, so the probability of the document (document likelihood) is the product over the probabilities of all words (This is simmilar to how the query likelihood model is formulated):

$$p(D) = \prod_{w \in V} p_D(w)^{c(w,D)}$$

So taking log on both sides, we can get our log-likelihood:

$$logp(D) \sum_{w \in V} c(w, D) logp_D(w)$$

Plugging in Eq.1 we get :

$$logp(D) = \sum_{w \in V} c(w, D) log \sum_{i=1}^{k} p(i|D)p(w|\theta_i)$$

Now since our data is some big corpus of text C containing multiple documents $D_1 \ldots . D_{|C|}$, we simply sum up all the log-likelihoods for each docuemnt:

$$logp(C|\Lambda) = \sum_{D \in C} \sum_{w \in V} c(w, D) log \sum_{i=1}^{k} p(i|D) p(w|\theta_i)$$

- where Λ is a set of all parameters including k switches that controls the mixture of multinomial distribution ($p(i|D)$, or sometimes written as $\pi_{d,i}$, topic selection probability for each document), we also have k unknown topics distributions . - The goal of the EM algorithm is to solve for :

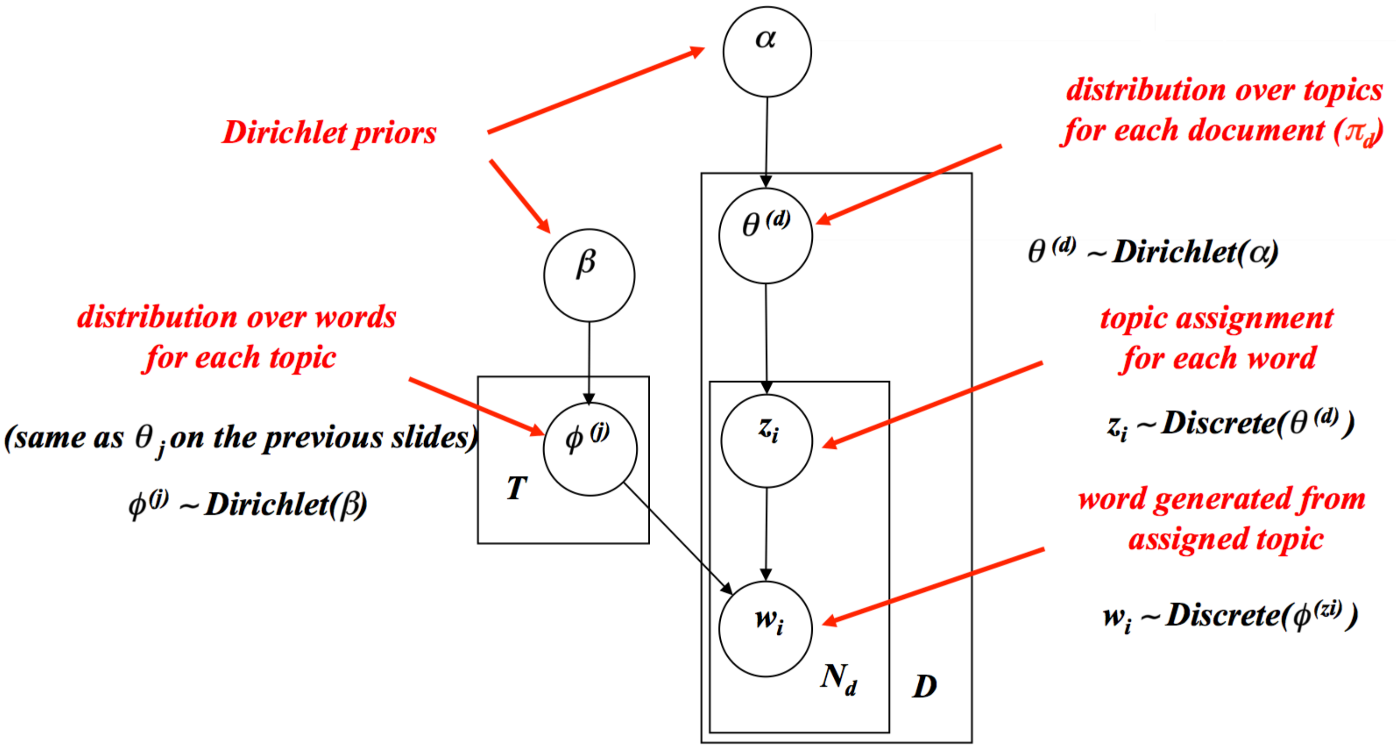
$$\hat{\Lambda} = argmaxp(C|\Lambda)$$

- But the problem with PLSA is that there are too many parameter to estimate π switches, so there are many local maxes where EM can be easily trapped. - One way that we could solve this is to impose a prior on the parameters (i.e. a higher level generative model that tells you how what the value of each parameter should be), we chose that prior to be a Dirichlet distribution. - The Dirichlet distribution is controlled by an α parameter, which could be thought of as adding pseudo counts to a word to encourage that the topic θ_i assigns a higher probability to that word. - Another problem with PLSA is that it is <u>not generative</u>, i.e. can not predict topic distribution for unseen documents.

Latent Dirchlet Allocation (LDA):

- LDA resolves both of these problems :
  - generative model where each new document can be generated by drawing from the Dirichlet with our trained parameters
  - because the Dirchlet now "summarizes" our topic selection probability parameters (π_d, i), we now have less parameters (k-1 rather than 2k-1).

- At a high level, what LDA does is that it first draws from a Dirichlet distribution of topics (controlled by parameter α) for the whole corpus. Each topic in that distribution (θ) has its own word distribution for document M which is a multinomial distribution.
- So we first pick a topic (θ_i) for the document D from our topic distribution ($\sim Dir(\alpha)$). Then in order to generate the words inside the document, we pick a topic assignment for each word (z_i), then using the word distribution for that topic ($\sim Dir(\beta_i)$), we can then pick a word (w_i).



References:

- Zhai,ChengXiang, Statistical Language Models for Information Retrieval (2009) pg 87-93