Karlsruhe Institute of Technology

SAT Solving with distributed local search

Master Thesis of

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Abstract

Stochastic local search (SLS) is an elementary technique for solving combinational problems. Probsat is an algorithm paradigm of the simplest SLS solvers for Boolean Satisfiability Problem (SAT), in which the decisions only based on the probability distribution. In the first section of this paper, we introduce an efficient Probsat heuristic. We experimentally evaluate and analyze the performance of our solver in a combination of different techniques, including simulated annealing and WalkSAT. With the approach of formula partition, we introduce a parallel version of our solver in the second section. The parallelism improves the Efficiency of the solver. Using different random generator and other parameter settings in solving the sub-formula can bring further improvement in performance to our parallel solver.

Zusammenfassung

Stochastische lokale Suche (SLS) stellt eine elementare Technik zur Lösung von komplizierten kombinatorischen Problemen dar. Probsat ist einer der einfachsten SLS-Solver für das Erfüllbarkeitsproblem der Aussagenlogik (SAT), bei dem die Entscheidungen nur auf der Wahrscheinlichkeitsverteilung basieren. Im ersten Teil dieser Arbeit stellen wir eine effiziente Probsat-basierte Heuristik vor. Die Leistung unseres Algorithmus in einer Kombination verschiedener Techniken, einschließlich simulierter Abkühlung und WalkSAT wurde auch experimentell bewertet und analysiert. Mit dem Ansatz der Formelpartition wird im zweiten Teil eine parallele Version unseres Algorithmus eingeführt, die die Effizienz des Lösers verbessert. Die flexible Parametereinstellungen bei der Lösung der Teil-formeln kann eine weitere Verbesserung unseres Algorithmus bringen.

Contents

1 Introduction

1.1 Problem/Motivation

The ***propositional satisfiability problem*** (***SAT***) is the first proven NP-complete problem [1]. The problem is to determine whether an assignment of Boolean values to variables in a Boolean formula such that the expression evaluates to true. Hard combinational problems can be resolved with appropriate Encoding as a sat problem. The SAT problem has many applications in computer science like chip model checking [2], software verification [3] or in automated planning and scheduling in artificial intelligence [4]. Formula partition is one of the promising approaches in DPLL-like solvers [5]. By giving the order to the variables according to a good formula partition, the search gets a relatively balanced decision tree. But formula partition is rarely used in a local search for the SAT problem. How to combine the formula partition with local search, will the local search benefit from the partitioning, if the formula partitioning can guide a parallel local search, are still open questions.

1.2 Content

The SAT problem, as a well-known NP-complete problem, has received a great deal of attention and different local search heuristics have been developed. This paper is a survey on the stochastic local search on SAT problem with a guide of formula partition.
In section 1, we summarize the formal concept and introduces techniques used in this paper. One class of the most straightforward but efficient stochastic local search algorithms Probsat is the algorithm basic in our paper. Probsat was proposed in 2012 by Adrian Balint and Uwe Schoening [6]. Section 2 describes our Probsat algorithm and discusses our attempts to improve the original algorithm. By experimentally evaluation and comparison, some techniques turned out to be more efficient than the simple Probsat search. With the partition of variables and its corresponding formulas, the problem can be separated into two subproblems of similar size. In section 3, we search the potential benefit of formula partition in a parallel search. Section 4 describes the details in experiments and several empiric results mentioned in section 2 and section 3. Section 5 concludes the paper with further works.

1.3 Definitions and Notations

Propositional Satisfiability Problem
A variable with two possible logical values *TRUE* or *False* is a ***propositional variable***, which will be referred to as ***variable*** in this paper. A ***literal*** is an atomic formula in propositional logic. A literal can either be a ***positive literal*** v as the variable v or a ***negative literal*** $\bar{v}$ as negation of v. A ***clause*** is a disjunction of literals. A formula in conjunctive normal form (CNF) is a conjunction of clauses. We refer it as ***CNF-formula*** or simply as ***formula*** in this paper. An ***assignment*** a as a function a: $V \rightarrow \{True, False\}$ assigns the truth value to each variable v in the formula. We say the assignment satisfies a formula if the truth value of the formula with this assignment turns out to be true. Specifically, an assignment satisfies a clause, if one literal in the clause with value *True* in this assignment. A formula is a satisfying formula if one assignment exists satisfies all its clauses. We say an assignment a satisfying assignment if it satisfies the formula. Otherwise, we say there are conflicts in some clauses with this assignment, or some clauses are unsatisfying clauses with this assignment. The SAT

$$F = (v_1 \vee \bar{v_3}) \wedge (v_2 \vee v_1 \vee \bar{v_1})$$
$$Vars(F) = \{v_1, v_2, v_3\}$$
$$numVs(F) = |Vars(F)| = 3$$
$$Lits(F) = \{v_1, \bar{v_1}, v_2, v_3, \bar{v_3}\}$$
$$Cls(F) = \{C_1, C_2\}$$
$$numCs(F) = |Cls| = 2$$
$$C_1 = \{v_1, \bar{v_3}\}$$
$$C_2 = \{v_2, v_3, \bar{v_1}\}$$

$A(v_1) = True$, $A(v_2) = False$, $A(v_3) = True$,
A is an assignment satisfies F.

$\hat{A}(v_1) = True$, $\hat{A}(v_2) = False$, $\hat{A}(v_3) = False$,
$\hat{A}$ is an assignment with conflict in C_2.

problem is to determine whether a satisfying assignment exists for the given formula. If so, we denote the formula a ***satisfiable formula***.

Here is an example of SAT problem:

Set
A set is a container of unique elements. A set of 3 objects a, b, c is written as a, b, c. The size of a set is the number of elements in the set.

Local Search
For instance I of a hard combinational Problem P, there is a set of solutions S. According to the constraints of the problem, an object function (score or cost) Γ is used to evaluate the candidate solutions. The Goal of the local search is to find the solution of minimum cost (or the solution with the maximal score).
A local search starts with an initial complete solution. According to some heuristic, the local search makes local changes to its current solution iteratively, hence the name ***local search***. Starts from an initial solution, the search will evaluate the solutions which can be reached by applying a local change to the current solution and choose one of the neighbor solutions with local optimization. The search applies local moves until the optimal solution is reached, or in some cases, a generally good solution is reached. Local search is widely used in hard combinational problems such as the traveling salesman problem [13] and the graph coloring problem [14].

Local Search in SAT Problem
In the Boolean satisfiability problem, a local search operates primarily as follows: The search start from a randomly generated assignment as the initial solution. If this current assignment satisfied the formula, the search stops with success. Otherwise, a variable is chosen depends on some criterion. This selection is called ***pickVar***. By change the assignment of the selected variable v, a neighbor assignment of our current solution A is reached in next step, which is also called ***flipp***(A, v). A local search will move in the space of the assignments by making the variable flipping until a satisfying assignment is reached by the search.
The heuristic used for the flipping variable selection $pickVar$ is based on some scores of the variables in the current assignment. Consider the assignment $\hat{A}$ reached by taking a flip of

the variable in the current assignment A. The number of clauses satisfied in A,but not in $\hat{A}$ is called the ***breakcout*** of the local move from A to $\hat{A}$. Accordingly, the number of clauses, which become satisfying because of the flipping, is the ***makecount***. The number of newly satisfying clauses (*makecount*) minus the number of newly unsatisfying clauses (*breakcount*), which is denoted as ***diffscore***, represents the local improvement of the corresponding flipping. Apart from this, other aspects like the repetition number of one flip or the number of occurrences of the variables can be considered in a selection heuristic. An example is the unit propagation embedded local solver *EagleUp*, which prefers flipping of variables with the highest number of occurrences in a formula to creates new unit clauses sooner. To get local improvement effectively, man can only consider variables in unsat clauses for the flipping selection. This process is called a ***focused local search*** and commonly used.

Algorithm 1: Focused Local Search

`input` : A CNF Formula F
`parameter`: *Timeout*
`output` : a satisfying assignment A
$A \leftarrow$ random generated assignment A;
`while` ($\exists$ *unsatisfied clause* $\wedge$ *Timeout does not occur*) `do`
  $c \leftarrow$ random selected unsatisfied clause ;
  $x \leftarrow pickVar(A, c)$
  $A \leftarrow flip(A, x)$;

By choosing the variable with best score in $pickVal$, the seach will get greedy local improvemnet. The initial hope of the local search is that through iterative greedy local improvement the optimal global solution can be found. The typical problem of the local search is that the greedy local searches be trapped in local unattractive local optimal solution. To avoid this , some random flips are picked or even a worse solution will be chosen for the next step (***uphill moves***). There are some techniques following used in local search to avoid getting stuck in local optimum.

Stochastic Local Search (SLS)
The stochastic local search will use the probability distribution of the scores of candidate solutions instead of the static decision. For the candidate moves, the probability of being chosen $p(\Gamma(s))$ corresponds to the score $\Gamma(s)$ of the solution s. In this way, the advantage a move is, the probability of choosing it as the next step is higher. This randomization will avoid the stuck of the search in a local minimum and decrease the misguiding of the heuristic in specific situations.

Statistical Local Search
Tabu search is created by Fred W. Glover in 1986 [15] and formalized in 1989. For recognize the loop in a suboptimal region, the search trace is recorded in the process by mark the recently reached neighboring assignments as tabu. The tabu moves will not be touched in the further search to discourage getting stuck in a region.

Simulated Annealing
Simulated Annealing is an approach of local search solver to difficult combinational optimization problems proposed by Kirkpatrick, Gelatt, and Vecchi [7]. This approach is inspired by the metallic process annealing of shaping the material by heating and then slowly cooling the material. This approach works as a local optimization algorithm guided by a controlling parameter ***temperature***. By high temperature, an uphill move is allowed with high probability while only small steps are allowed in low temperature. The temperature is varying according

to the score of the current situation. For a current solution with a nearly optimal score, the temperature is near zero. For an unattractive local extreme with a poor score, the active search is tending to make uphill moves in high temperature.

WalkSAT

WalkSAT is a focused random local search strategy to solve SAT problem, which is originally introduces in 1994 [8]

. WALKsat may ignore the greedy flipping and flip a random variable in chosen unsatisfied clause wit probability p . By introducing these "uphill noises", the WalkSAT combines greedy local search and random walk to get an effective and robust random solver.

Algorithm 2: pickVar in WalkSAT

```
input    : current assignment A, unsatisfied clause c
parameter: probability p
output   : a variable x in c for flipping
for v in c do
    Evaluate v with function Γ(A, v);
with probability p: x ← v with maximum Γ(A, v);
with probability 1 − p: x ← randomly selected v in c.
```

The Probsat

Probsat is a class of SLS sat solver, which was introduced in 2012 by Adrian Balint and Uwe Schoening [6]. In a probsat solver, the score of a candidate flip is solely based on the make and break score. The paradigm is as follows: At first, a completely random assignment is set as the initial assignment. The algorithm performs local moves by flip a variable in a random chosen unsatisfying clause and stops as soon as there are no unsatisfied clauses exists, which means a satisfying assignment is found. The probability $p(v)$ of flipping the variable v in the chosen clause proportina to the score of v, which is calculated in a function $/Gamma(v, A)$ based on break score of v in the current assignment A [1] The idea behind this selection heuristic is to give the advantageous flipping relative high score, but the other flipping with small score has chance to be chosen. There are two kinds of score functions are considered in the paper of Adrian Balint:

$\Gamma(v, A) = (c_b)^{-break(v,A)}$ (break-only-exp-function)
$\Gamma(v, A) = (\epsilon + break(v, A)^{-c_b})$ (break-only-poly-function)

The pseudo code of a typical Probsat is shown below:

Algorithm 3: pickVar in probSAT

```
input    : current assignment A, unsatisfied clause c
output   : a variable x in c for flipping
for v in c do
    Evaluate v with function Γ(A, v);
x ← randomly selected variable v in c with probability p(v) = Γ(A,v) / Σ_{u∈c} Γ(A,u);
```

Formula partition

To introduce our parallel SAT solver with formula partitioning, we use a hypergraph represen-

[1] As mentioned in the probsat paper, it turns out in experiments that the influence of make is rather weak in selection functions, so the one parameter functions depends on *breakScore* can also lead to an efficient algorithm.

tation of SAT problem.
A hypergraph $G = (V, H)$ is a generalized graph, in which an hyperedge $h \in H$ is a non-empty subset of the vertices set V. For a SAT Formula F, its hypergraph representation $G(F) = (Vars(F), Cls(F))$ consists of $numVs$ vertices and $numCs$ hyperedges. Each vertex corresponds to a variable in F, and a hyperedge refers to a clause, which connects the variables in this clause.

Here is an example:
$F = \underbrace{(v_1 \vee v_2 \vee \bar{v_3})}_{C_1} \wedge \underbrace{(v_1 \vee v_3 \vee \bar{v_4})}_{C_2} \wedge \underbrace{(v_5 \vee v_6 \vee \bar{v_8})}_{C_3} \wedge \underbrace{(v_6 \vee v_7 \vee \bar{v_8})}_{C_4} \wedge \underbrace{(v_3 v \vee \bar{v_6})}_{C_5} \wedge \underbrace{(v_4 \vee v_5)}_{C_6}$
$G(F)$:

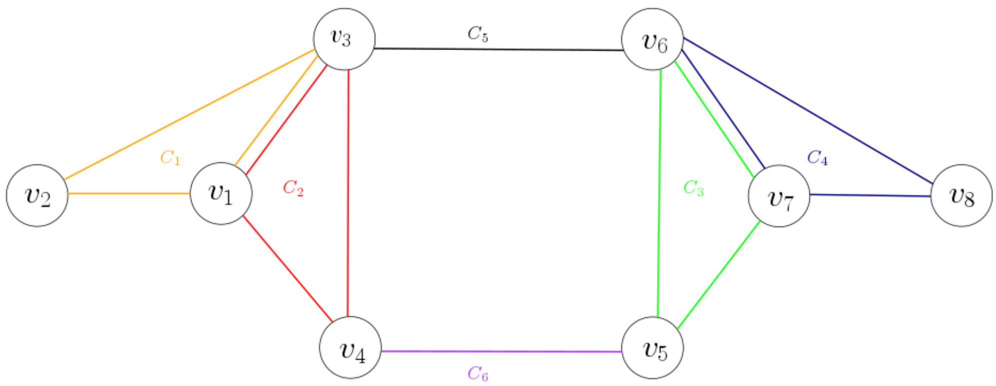


Figure 1: In a hypergraph, a hyperedge is a set of vertices. In our example, the vertices v_3 and v_4 are both in hyperedge $\{v_1, v_3, v_4\}$ and $\{v_1, v_2, v_3\}$, so they are connected twice. In our hypergraph representation of SAT problem, a hyperedge contains the vertices of the corresponding clauses. Another variant is a hypergraph representation, in which each literal refers to one vertex, and a hyperedge contains all the literals of the corresponding clause.

Formula partition is a promising way to improve the SAT problems solving. Two partitioning are investigated in some works. One is to split the variables, which is used in this paper, and another one is to separate the clauses. For the algorithms with the technique of decision tree like DPLL, the formula partition can guide the order of decisions. For the local search, there is nearly research of the use of formula partition in local search. Based on this hypergraph represenation, we describe in this paper the formula partition with the notations in graph partition. For a hypergraph $G = (V, H)$, a (2-way) partitioning is to seperate the verices set in two disjoint subsets. A good partition is to seperate V in P_0 and P_1, which are of relative same size and only few hyperedges containing both verticse in P_0 and vertices in P_1. Based on a vertices partitioning, the hyperedges are seperated in three disjoint subsets H_0, H_1 and I. $H1$ contains the edeges connecting verticse in P_0, H_1 are edges in P_1. The **intersection** I are the edges containing vertices in P_0 and vertices in P_1.In our example formula above, a minimum cost balanced partition is $P_0 = \{v_1, v_2, v_3, v_4\}$ and $P_1 = \{v_5, v_6, v_7, v_8\}$. In this partition, $H_0 = \{C_1, C_2\}$, $H_1 = \{C_4, C_5\}$ and the intersection $I = \{C_5, C_6\}$

1.4 The Competitors

Our heuristic is based on the probSAT paradigm. To evaluate the performance of our algorithm, we compare our heustic with the original ProbSAT. Another random SAT solver used for a comparison with our algorithm is yalSAT, which is the champion in random track category in SAT competition 2017 [9].

probSAT [2]

The authors of the original Paper implement the ProbSAT. We compare our Solver with this original code [3].

In this original code, there are two implementation variants available. In the incremental approach, the breakScores of variables are calculated in the initialization phase and only updated in the further search. The other straightforward approach is to compute breakScores of the variables in consideration of flipping. This method is called non-incremental approach in original paper. As suggested in Experiments, we take the non-incremental approach for the 3SAT problems and incremental method for 5SAT and 7SAT to get optimal results of the probSAT solver.

The parameters of ProbSAT in our Experiments have been set as suggested in the original paper:

kSAT[a]	score Γ	c_b	ϵ	variants
3SAT	break-only-poly	2.06	0.9	non-incrementel
5SAT	break-only-exp	3.7	-	incremental
7SAT	break-only-exp	5.4	-	incremental

Table 1: Parameter setting for competitor probSAT

[a] k is the maximum length of the clause

yalSAT [4]

We use the version 03 submitted to the 2017 SAT competition of the yalSAT solver in our experiments. Armin Biere implements it as a reimplementation with extensions of probSAT. With the implementation of different variants of probSAT, the yalSAT uses a different variant of probSAT randomly in the restart of a round of search. In our comparison, we use the default settings of the yalSAT with specific seeds.

[2] https://github.com/adrianopolus/probSAT

[3] Using same parameter settings our implementation gets similar performance to the original code

[4] https://baldur.iti.kit.edu/sat-competition-2017/solvers/random/

2 Our local Solver

Our algorithm is a typical focused SLS algorithm, which solves the SAT problem with the basic shema:

Algorithm 4: Our Local Search

```
input     : A CNF Formula F
parameter: Timeout
output    : a satisfying assignment A
A ← initAssign(F)
while (∃ unsatisfied clause ∧ Timeout does not occur) do
    c ← pickCla(A) ;
    x ← pickVar(A, c)
    A ← flip(A, x);
```

In the following, we will describe the methods used in our local search.

2.1 initAssign(F)

In our algorithm, we have three variants to make assignment initialization. One is the *RandomInit* which is the random initiation like in the original probsat suggests. Two alternatives to this random assignment are with the consideration of number of literal occurrences. with the method *BiasInit* we assign $True$ to a variable if the number of occurrences of its positive literal is more than its negative literal. Otherwise, a variable is assigned initially with $False$. *Bias* − *RandomInit* combines the two initializations above, in which the assignment is generated bias randomly based on the occurrences of literals. In Experiment 1 in Section 4 (see 4.7.1) we compare these three alternatives based on the probsat algorithm. Our local search uses *RandomInit* for 3SAT problems and *BiasInit* for other problems.

2.2 pickCla(A)

$numT(c)$, the number of $True$ values in each clause c, are counted in Initilization phase and maintained in further search. The unsatisfying clauses will be cached in a set $UNSAT$. During the local flipping, these numbers will be updated when the flipping variable is in the clauses. Comparing to the $numT$, the $UNSAT$ is updated lazily. After Flipping, if the $numT$ of one clause is decreased to zero, it will be added in the $UNSAT$. To select an unsat clause in $pickCla(A)$, man needs to select a clause from the $UNSAT$ and Ocheck if it is still unsat with its $numT$ is zero. Otherwise, if the chosen clause c with $numT(c)$ as zero, it will be removed from the $UNSAT$ set. This step pickCla(A) will be repeated until one unsatisfied clause is found or the $UNSAT$ set is empty, which means the current Assignment A is a satisfying assignment.

2.3 pickVar(A,c)

Inspired by probSAT and walkSAT, Our pickVar combines the random walk and stochastic selection. We analyze experimentally the following variants for $pickVar$.

1. Varinat:COMBINE

In observation of the experiments of the probSAT, this stochastic search bases its selection on a random heuristic. Even the search is very close to a satisfying assignment, and the probability of the critical flipping is exceptionally high, it is possible that the stochastic search make uphill moves and leave then the region of the global minimum. To prevent this besides the stochastic way, we pick greedy flip with zero *breakScore* with a certain probability p. With probability $1-p$, we choose the variable for flipping using the probSAT heristic.

Algorithm 5: COMBINE

`input` : current assignment A, unsatisfied clause c
`parameter`: probability p
`output` : a variable x in c for flipping

greedyVs $\leftarrow \emptyset$;
`for` *all v in c* `do`
  `if` (*break(A,v)= 0* $\wedge$ $Permit(v)$) `then`
    greedyVs = greedVs + $\{v\}$
with probability p: $x \leftarrow$ randomly selected variable $v \in$ greedyVs;
with probability $1-p$: $x \leftarrow$ randomly selected variable v in c with probability $\frac{\Gamma(A,v)}{\sum_{u \in c} \Gamma(A,u)}$;

2. Varinat: WALK

Instead of using a constant probability p to choose between a greedy Literal without clause break and the random Literal using probSAT flipping directly, we see a list, called statistic list S to record how many times each variable is chosen for flipping. To avoid cycling, we see the variable v_i with a high value of S[i] to be disadvantages for flipping. After selecting a variable using the ProbSAT stochastic distribution, we make the choice randomly according to the statistic values of these two variables.

Algorithm 6: WALK

`input` : current assignment A, unsatisfied clause c
`parameter`: probability p
`output` : a variable x in c for flipping

greedyVs $\leftarrow \emptyset$;
`for` *all v in c* `do`
  `if` (*break(A,v)= 0* $\wedge$ $Permit(v)$) `then`
    greedyVs = greedVs + $\{v\}$
$greedyV \leftarrow$ randomly selected variable $v \in$ greedyVs ;
$randomV \leftarrow$ randomly selected variable v in c with probability $\frac{\Gamma(A,v)}{\sum_{u \in c} \Gamma(A,u)}$;
with probability $p = \alpha \times \frac{s(greedyV)}{s(greedyV)+s(randomV)}$: $x \leftarrow$ randomV;
with probability $1-p$: $x \leftarrow$ greedyV;

3. Varinat: GreedyBreak

Compared to find the greedy Literals with zero breakScores, the calculation of the decay function Γ values and get a random literal according to its distribution takes the most part in the whole search. In this variant *greedybreaking*, we search greedy Literal with small statistic value. Hier, we define a literal is a permitted greedy Literal if its break value is zero and its statistic value is under some Limit. If permitted greedy variables exist, we choose one randomly for flipping. Otherweise, we pick random Literal using probSAT heuristic. To set the limit based on the search history, we compare two functions in our experiment. In the first approach "Average", the limit is set statistic to $\alpha * \frac{numFs}{numVs}$.
In another approach "Random-Flip,"we select randomly a value r in $[0, numFs]$. For each greedy Literal, we check if its statistic value is smaller than $\alpha * r$.

Algorithm 7: TieBreak

`input` : current assignment A, unsatisfied clause c
`parameter`: probability p
`output` : a variable x in c for flipping

greedyVs $\leftarrow \emptyset$;
for *all v in c* **do**
  if (*break(A,v)= 0* $\wedge$ $Permit(v)$) **then**
    greedyVs = greedVs + $\{v\}$
if (*greedyVs is not empty*) **then**
  x $\leftarrow$ selected variable v in $greedyVs$ at random
else
  x $\leftarrow$ randomly selected variable v in c with probability $\frac{\Gamma(A,v)}{\sum_{u \in c} \Gamma(A,u)}$;

2.4 pickVar(A,c) with Simulated Annealing

Simulated Annealing is a technique, whose combination with WalkSAT is extensively studied. Besides the dynamic noises introduced above, we use simulated annealing to improve our three suggestions of $pickVal$.That is, we define the α as a function depending on the quality of current assignment q instead of using a static parameter in the whole process. To define the *temperature*, we have two variants: Global and Local.

Global:
In the process of search, the number of unsatisfied clauses is shown in $unsatN$. In this traditional variant *Global*, we use this number to define the quality of the current solution.
Local:
As the name suggested, in the *Local* variant, we measure the quality of the current assignment focused on the chosen clause/ The quality $q(A, c)$ is equal to the number of greedy Literals in current clause.

$$q_{Global}(A) = unsatN(A).$$
$$q_{local}(A) = |v|vinc \wedge breaks(v) == 0|.$$
$$\alpha = (c_b)^{-q)}$$

2.5 Data structures

Occurrences

In the process of initialization, the numbers of occurrences of one variable will be compared. In our implementation, we use a list to count and record these occurrences numbers. This list with size of $2 * numClauses$ is denoted as Occurrences List OL. For the variable with index i, the $OL[2i]$ is the number of literal vs occurrences; The $OL[2i+1]$ is for its negative occurrences.

Literals

Local search is a search where only small changes are made in each step. In our Situation, only the clauses include the flipping variables are involved in the flipping step. The most time in our solver is spent to update the $numTs$ of these involving clauses. To find the involving clauses of one Variable, two $2D$ Array $posL$ and $negL$is made to record the clauses of positive nad negative literals. For variable v_i, the posL[i] record the indexes of clauses containing the positive literal v_i. Tthe ones with negative literal $-v_i$ are in by $egL[i]$. To implement the flipping of the variable v_i, we update the $numTs$ of clauses with indexes in $posL[i]$ and $negL[i]$.

LookUp

The most time used in the search is the repeated calculation of tthe polynomial or exponential decay function Γ. With this observation in our experiments, we calculate the $\Gamma(x)$ with x from 0 to $0.5{*}numCs$ and keep the valuse in a list $LookUp$. In our implementation, we use this Table lookup to get the values instead of the reputation of time-consuming exponential operation.

Solution

A Solution in our implementation includes the boolean Assignment and three other structures to record information about the current assignment. The Solution is computed after assignment initialization and is updated during each flipping :

Name	Structure	Size	Meaning
$Assignment$	list	numVs	boolean assignment to variables
$NumTs$	list	numCs	number of $True$ values in each clause
$NumUnsat$	natural number	-	the number of unsatisfied clauses
$UNSAT$	set	-	indexes of unsatisfied clauses [a]

Table 2: Parameter setting for competitor probSAT

[a]This UNSAT is updated in flipping phase lazily by only adding new unsatisfied clauses and remove the clause chosen in $pickCla$.

3 Our Parallel Algorithm

3.1 1st Approach: The pure portfolio approach

Section 2 introduces swpSAT to solve the SAT problem. The result of the algorithm is a legal assignment of the boolean formula. In the observation of the experiments, which pseudo-random generator in use to make random values affect the performance. For each graph, there is one random generator that is most suitable in the aspect of the search path of this combination. So trying different random generator will improve the performance (See experiment). In the pure portfolio version of our algorithm, the agents run the swpSAT with different random generation policy. After a satisfying assignment found by an agent, the search will stop. This approach improved the performance compared to our sequential local search.

Algorithm 8: Focused Local Search

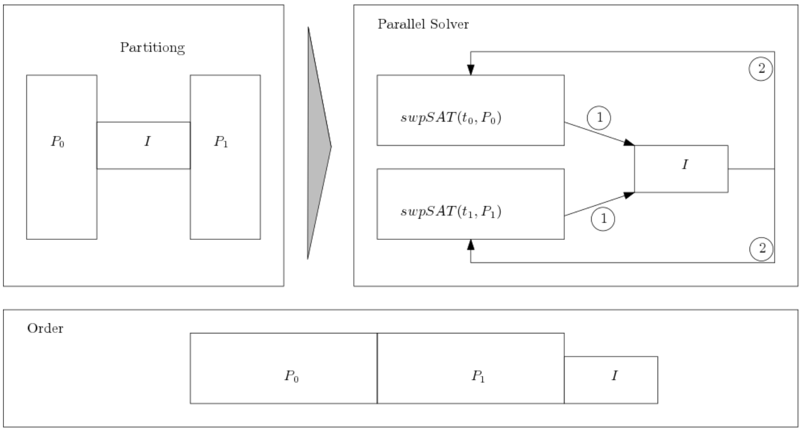
`input` : A CNF Formula F
`parameter`: $Timeout$,number of Processors n_p
`output` : a satisfying assignment A

$sat \leftarrow False$
$A \leftarrow initAssign(F)$ `for` *each* $Processor_i$ *for* $i \in \{0, .., n_p - 1\}$ `do`
  $A_i \leftarrow A$ `while` $(!sat \wedge !Timeout)$ `do`
    swpSAT(P)
    $sat \leftarrow True$
    $A \leftarrow A_i$

Ideally, This pure portfolio approach can find a satisfying assignment for a formula as fast as the minimum time used in the one-thread local search. But the threads do the same jobs but not split the whole task. In the following part, we will introduce some partitioning-based solving techniques to speed up the local search process. With a good formula partitions, the search tries to split independent jobs in threads.

3.2 2nd Approach: Solve Order of clauses

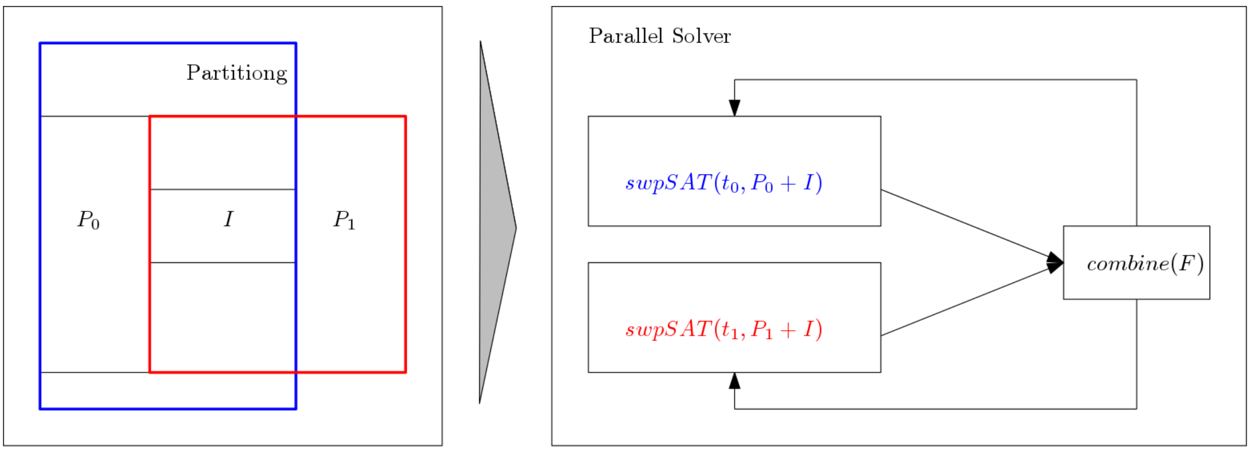
With the information of the formula partitioning, the first idea is to speed up our local search is to make the flipping in both partitioning set parallel. It is possible because the clauses in the different partitions do not share the same variables. The schema of this approach is following: The slave thread t_0 execute the swpSAT with conflict in clauses in H_0. The slave thread t_1 deals with H_1 in parallel. Then the assignments of the corresponding partitioning set of each agent are written in the shared memory. The master thread check then the conflicts in intersection I and make flips based on the assignment in shared memory. Handle with conflicts in the intersection will lead to some flips in both partitioning set and so will make conflicts in H_1 and H_2. Then the both processes will start again to handle conflict in partitions individually. This process will continue until there is no conflicts in the whole problem.



We did an experiment to test the performance of this approach (see Experiment in section). It shows, however a worse performance compared to the original swpSAT especially in the aspect of the number of solved problems. According to our observation in the implementation, we think the possible reason is the following: Its sequential version is precisely our swpSAT solver with priorities of unsatisfied clauses. Instead of choosing an unsatisfied clause in whole formula, this approach solves first $P1$ and then $P2$ (or inverse) and at last the Intersection I. So there is no reason f an improved performance in this parallel search. Worsely, because the priority of the clauses in search, the flip works are done in slave threads are destroyed in solving the intersection part, even we take the breaks score in partitioning sets in the count. There are more local minimums are visited in the search path. These "traps," which can be avoided by an independent chosen clause with variables in another partitioning set, search much slower.

3.3 3nd Approach: Assignment combination

In consideration of the failure in Approach 2, in this approach we make the slave threads solve intersection part besides a partition set. First, each slave thread finds a satisfying sub-assignment for the corresponding partitioning set and also the intersection part. Not like the approach 2, here two slave threads may have some different assignments to variables in the boundary of the partitioning. Then the master threads may deal with these differences. That is, we combine these two sub-assignment such that we can get a satisfying assignment or an assignment with only a few conflicts. Then the slave threads will adjust this assignment by make flips in the part of their charge (including the intersection part).



For the combination of two sub-assignments, there are several cases:
1. The assignments of one variable are different in both sub-solutions. This variable is not the critical one for a sub-assignment, so we can assign it as the value in the other sub-assignment.
2. If the variable in consideration is critical in both sub-assignment, there are some different policies. The simple one is to assign the variable randomly. We call this policy further random-Combine. Another one is to assign the variable as the value suggested by its charging thread. In another word, we combine the assignment according to the partitioning. This policy will not break any clauses in partition set. The slave thread will deal with conflicts in intersection individually in the next round. This policy refers as partition-based Combine. In experiment, we compare these two policies. Both policies can bring a good performance compared with our swpSAT solver. The possible reason is

3.4 4th Approach: Initialization with a guide of formula partitioning

Initialization with a guide of formula partitioning After the analysis of the failures in the 2 Approaches above, we came up with this approach, in which the formula partition information is only used to get an initial solution, and the further search in the whole problem is like our he pure portfolio approach. This approach intends to use the partitioning information to get a good initial solution. A local search from this initial solution with only a few conflicts in the intersections can reduce the search space and prevent long-term cycling in the search. The statistic information shared between the agents encourage the further search the flipping of non critical variables in clauses.

Algorithm 9: Focused Local Search

```
input    : A CNF Formula F
parameter: Timeout
output   : a satisfying assignment A
sat_0 ← False
sat_1 ← False
sat ← False
A ← initAssign(F)
foreach (Processor_i for i ∈ {0,1}) do
    A_i ← A
    while (!sat_i ∧ !Timeout) do
        swpSAT(P_i)
        sat_i ← True
        A[j] ← A_i[j] for v_j ∈ P_i
    while (!sat_{1−i} ∧ !Timeout) do
        swpSAT(P_{1−i})
        sat_{1−i} ← True
        A[j] ← A_i1 − i[j] for v_j ∈ P_{1−i}
    A_i ← A
    while (!sat ∧ !Timeout) do
        swpSAT(F)
        sat ← True
        A ← A_i
```

4 Evaluation

4.1 DIMACS standard format

All the benchmark CNF formula used in experiments are encoded in the DIMACS standard format [?]. This format is used to test and compare SAT solver in SAT competition. A DIMACS file contains the description of an instance using three types of lines[5]:

1. Comment line: Comment lines give information about the graph for human readers, like the author of the file or the seed used in generation. A comment line starts with a lower-case character c and will be ignored by programs:

c *this is an example of the comment line*

2. Problem line: The problem line appears exactly once in each DIMACS format file. The problem line is signified by a lower-case character p. For a formula with nV variables and nC clauses ,the problem line in its DIMACS file is:

p cnf nV nC

3. Clause Descriptor: An clause $\{v_1, v_2, , v_n\}$ in the graph is described in an edge Descriptor:

e v_1 v_2 v_n

Here is the DIMACS format of the formula $F = (v_1 \vee \bar{v_3}) \wedge (v_2 \vee v_1 \vee \bar{v_1})$:

c simple_v_3_c_2.cnf
p cnf 3 2
1 -3 0
2 3 -1 0

4.2 Benchmarks

The benchmark instances used in experiments are the 180 uniform instances (unif) in random benchmark categories in SAT competition 2017 [10]. In an unif problem file, all the clause have the same length. The suffix 'k' denotes the length of clauses. The r indicates the clause-to-variable ratio. The c and v are for the number of clauses and variables, while s is for the seed used in the generation process. Without flitering, there are at least 60 (33%) problems form our 180 benchmark collections are unsatisfiable.

4.3 Used plots and tables

the results of the following experiments are all shown in comparision table and illusatrated in cactus plot.

Comparison Table
See Table 4 for example.

[5] Only clause unweighted simple instances are tested in our experiments. For other descriptors and details of the DIMACS format

A comparison table shows the different results of algorithms. The first column contains the clause length k. The UNIF benchmark has 3 different sizes: 3SAT, 5SAT, and 7SAT. For a kSAT instance, each clause contains k variables. The fields of a comparison table in the following columns corresponds to a penalized runtime for the whole kSAT set, which assigns a runtime of two times the time limit for an unsolved instance. Because the UNIF benchmark graphs without filtering contain a part of unsatisfied problems, we assign penalized time only to problems which can not be solved by any solvers in our whole experiment. There are 61 3SAT instances, 89 5SAT instances and 66 7SAT instances found satisfied in our experiments. Besides the runtime score, the number of solved problems are in parentheses. The best results in comparison are in bold.

Cactus Plot
See Figure **??** for an example.
A cactus plot shows the performance of different algorithms. The y-axis shows the time in second used to solve the benchmark graphs. The y-axis is for the number of solved problems by a certain time. Each algorithm corresponds to a curve in different colors. The point (u, v) on a curve means by v seconds the corresponding algorithm have solved u problems.

4.4 Random seeds used in Experiments

To make our experiments results reproducible and robust, we repeat our tests with three specific seeds. We produce the seeds in experiments as follows: First, we use the sum of characters of the name of the solver to seed the pseudo-random generator in c++. Then we use this reinitialized generator to produce three random values, which are the seeds used later in our experiments.

solver	name	1.seed	2.seed	3.seed
probSAT	probsat	1988822874	338954226	858910419
yalSAT	yalsat	1851831967	280788293	1956345180
our local solver	local	1962042455	1112841915	566263966
our parallel solver	parallel	1749729997	68910537	473644167

Table 3: Parameter setting for competitor probSAT

4.4.1 Soft- and Hardware

The single-threaded experiments were run on computers that had Two Intel Xeon E5-2683 v4 processors (2.1 GHz 2x16-core + 2x16-HTcore) and 512GB RAM. The machine ran the 64-bit version of Ubuntu 14.04.5 LTS. The multi-threaded experiments were run on computers that had Two AMD(Ryzen +1800) processors (2009.230 MHz) with 32GB DDR4 RAM. The machine ran the 64-bit version of Ubuntu 16.04 LTS.

4.5 Parameter Settings in Experiment

The $TimeOut$ is set to 5Minutes in the experiments of local searches. For the probSAT heuristic, our local search uses the values for c_b and $/epsilon$ suggested in the probSAT paper. The *tolerence* τ used in the experiments is generated with the help of the algorithm parameter optimization tool SMAC [29] (sequential model-based algorithm configuration). SMAC ran our algorithms on LARGE problems in the UNIF category in SAT 2012 (75% of the instances training instances, 25% as test instances) using different $\tau \in [0, 10]$[6] and randomly generated seeds. With the help of SMAC.

k	$WALK$	$WALK_Local$	$WALK_Global$	$Average$	$Average_Local$	$Average_Global$
3	1	1	10	1	2	0.5
5	0.5	1	1	1	2	2
7	1	0.5	2	1	2	0.5

k	RF	RF_Local	RF_Global	$swpSAT$	α	SA
3	1	0.5	0.5	*Average*	1	–
5	0.5	2	9.5	*Average*	2	*Local*
7	0.5	1	0.5	*Average*	2	*Local*

4.6 Benchmark Generation

To combine the hypergraph partitioning and SAT local solver, we try to get a relatively balanced graph partition with small intersection for the benchmark SAT problems. On UNIF benchmark, we try to get graph partitioning using some partitioning algorithms (KaHypar and hmetis). Since the uniform random generation of this benchmark, these problems are without a real-world-like structure. Even with a high imbalance like 0.3 and high tolerance of intersection size (50% of edge sizes), the partitioning algorithms take more time than our SAT solver for more than 50% of our UNIF benchmarks. To investigate the local search on graphs with proper partitioning, we generate our benchmark COMBINE using the UNIF benchmark instances. As the name COMBINE suggested, we combine two UNIF benchmark instances in one SAT problem and make for the new generated problem with two disjoint subproblems an intersection. To create a satisfiable formula, we build the intersection based on a pair of random chosen satisfying assignments of the two UNIF problems in combination. To get satisfying assignments of UNIF benchmarks, we run different solvers in SAT competitions including CSCCSat, DCCASat, score2SAT, probSAT, and yalSAT. We do not use our local search to collect satisfying assignments. Otherwise, it is possible that after solve the two partitioning sets individually using our local solver, the current assignment is the same or a similar one used for the intersection generation.

We combine the UNIF problems in consideration of real-world uses. We combine 4 pairs of instances in 3SAT, 5SAT problems 7SAT problems. Besides that, we consider five combinations between 3SAT and 5 SAT instances and three combination of 5SAT and 7SAT instances. We combine problem in similar vertices size, which corresponds to balanced partitioning in the structure. We also combine one massive problem with a small instance of an imbalanced partitioning. For the intersection generation part, we generate clauses with three vertices in different partitioning sets.

We first choose vertices of two partition sets for intersection generation randomly. Here we consider also the balanced intersection and imbalanced intersection. In a balanced intersection, same proportion of vertices in both partitioning sets are chosen to generate the intersection. In an imbalanced intersection, the portions are not the same. To control the size of the intersection, we also count the number of the clauses in an intersection, if the proposition of

[6] Because of high time consume in parameter optimation, we solely compare τ as a natural number form One to Ten.

the intersection clauses in the clauses number in whole combined problems is upper a specified limit, the generation of intersection is stopped.

Problem	Intersection
k3-r3.94.cnf-k3-r4.04.cnf-0.1-0.1-0.1.cnfP	1.04%
k3-r3.96.cnf-k3-r4.06.cnf-0.1-0.1-0.2.cnfP	1.00%
k3-r3.98.cnf-k3-r4.0.cnf-0.1-0.1-0.4.cnfP	1.03%
k7-r55.0.cnf-k7-r56.0.cnf-0.1-0.2-0.2.cnfP	15.53%
k7-r55.0.cnf-k7-r56.0.cnf-0.1-0.4-0.1.cnfP	6.84%
k7-r55.0.cnf-k7-r56.0.cnf-0.2-0.2-0.2.cnfP	12.04%
k7-r55.0.cnf-k7-r56.0.cnf-0.2-0.4-0.1.cnfP	7.53%
k7-r55.0.cnf-k7-r56.0.cnf-0.4-0.1-0.4.cnfP	6.38%
k7-r55.0.cnf-k7-r56.0.cnf-0.4-0.2-0.1.cnfP	7.35%
k7-r55.0.cnf-k7-r56.0.cnf-0.4-0.4-0.2.cnfP	6.11%
k7-r57.0.cnf-k7-r60.0.cnf-0.1-0.1-0.2.cnfP	16.67%
k7-r57.0.cnf-k7-r60.0.cnf-0.1-0.1-0.4.cnfP	25.21%
k7-r57.0.cnf-k7-r60.0.cnf-0.1-0.2-0.2.cnfP	14.77%
k7-r57.0.cnf-k7-r60.0.cnf-0.1-0.4-0.1.cnfP	6.46%
k7-r57.0.cnf-k7-r60.0.cnf-0.2-0.1-0.2.cnfP	13.92%
k7-r57.0.cnf-k7-r60.0.cnf-0.2-0.2-0.4.cnfP	11.40%
k7-r57.0.cnf-k7-r60.0.cnf-0.2-0.4-0.2.cnfP	7.08%
k7-r57.0.cnf-k7-r60.0.cnf-0.4-0.1-0.4.cnfP	5.88%
k7-r57.0.cnf-k7-r60.0.cnf-0.4-0.2-0.4.cnfP	6.86%
k7-r57.0.cnf-k7-r60.0.cnf-0.4-0.4-0.4.cnfP	5.81%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.1-0.1.cnfP	9.09%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.1-0.2.cnfP	16.67%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.1-0.4.cnfP	25.07%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.2-0.1.cnfP	9.09%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.2-0.4.cnfP	14.68%
k7-r58.0.cnf-k7-r62.0.cnf-0.1-0.4-0.4.cnfP	6.36%
k7-r58.0.cnf-k7-r62.0.cnf-0.2-0.1-0.1.cnfP	9.09%
k7-r58.0.cnf-k7-r62.0.cnf-0.2-0.1-0.2.cnfP	13.96%
k7-r58.0.cnf-k7-r62.0.cnf-0.2-0.2-0.2.cnfP	11.58%
k7-r58.0.cnf-k7-r62.0.cnf-0.2-0.4-0.2.cnfP	6.97%
k7-r58.0.cnf-k7-r62.0.cnf-0.4-0.1-0.4.cnfP	5.98%
k7-r58.0.cnf-k7-r62.0.cnf-0.4-0.4-0.4.cnfP	5.60%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.1-0.1-0.4.cnfP	1.30%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.1-0.2-0.2.cnfP	2.52%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.1-0.4-0.2.cnfP	4.88%

Problem	Intersection
k3-r3.92.cnf-k3-r3.88.cnf-0.2-0.1-0.4.cnfP	0.38%
k3-r3.92.cnf-k3-r3.88.cnf-0.2-0.4-0.1.cnfP	0.13%
k3-r3.94.cnf-k3-r4.04.cnf-0.1-0.2-0.1.cnfP	0.36%
k3-r3.94.cnf-k3-r4.04.cnf-0.1-0.4-0.4.cnfP	0.11%
k3-r3.94.cnf-k3-r4.04.cnf-0.2-0.1-0.2.cnfP	0.37%
k3-r3.94.cnf-k3-r4.04.cnf-0.2-0.2-0.1.cnfP	0.27%
k3-r3.94.cnf-k3-r4.04.cnf-0.4-0.2-0.4.cnfP	0.11%
k3-r3.94.cnf-k3-r4.04.cnf-0.4-0.4-0.4.cnfP	0.09%
k3-r3.96.cnf-k3-r4.06.cnf-0.1-0.2-0.2.cnfP	0.37%
k3-r3.96.cnf-k3-r4.06.cnf-0.1-0.4-0.1.cnfP	0.11%
k3-r3.96.cnf-k3-r4.06.cnf-0.2-0.1-0.2.cnfP	0.35%
k3-r3.96.cnf-k3-r4.06.cnf-0.2-0.2-0.2.cnfP	0.27%
k3-r3.96.cnf-k3-r4.06.cnf-0.2-0.4-0.2.cnfP	0.13%
k3-r3.96.cnf-k3-r4.06.cnf-0.4-0.1-0.2.cnfP	0.10%
k3-r3.96.cnf-k3-r4.06.cnf-0.8-0.8-0.05.cnfP	0.06%
k3-r3.98.cnf-k3-r4.0.cnf-0.1-0.2-0.1.cnfP	0.38%
k3-r4.267-v11000.cnf-k5-r16.2.cnf-0.8-0.8-0.05.cnfP	0.52%
k3-r4.267-v11200.cnf-k5-r16.8.cnf-0.8-0.8-0.05.cnfP	0.49%
k3-r4.267-v11600.cnf-k5-r17.0.cnf-0.8-0.8-0.05.cnfP	0.52%
k3-r4.267-v7400.cnf-k5-r17.2.cnf-0.8-0.8-0.05.cnfP	0.34%
k3-r4.267-v9600.cnf-k5-r17.4.cnf-0.8-0.8-0.05.cnfP	0.42%
k5-r21.117-v200.cnf-k5-r16.0.cnf-0.1-0.1-0.2.cnfP	0.04%
k5-r21.117-v200.cnf-k5-r16.0.cnf-0.2-0.1-0.2.cnfP	0.09%
k5-r21.117-v200.cnf-k5-r16.0.cnf-0.4-0.1-0.4.cnfP	0.17%
k5-r21.117-v220.cnf-k5-r17.6.cnf-0.8-0.8-0.05.cnfP	0.01%
k5-r21.117-v280.cnf-k5-r16.4.cnf-0.1-0.1-0.4.cnfP	0.06%
k5-r21.117-v280.cnf-k5-r16.4.cnf-0.2-0.1-0.1.cnfP	0.12%
k5-r21.117-v280.cnf-k5-r16.4.cnf-0.4-0.1-0.1.cnfP	0.23%
k5-r21.117-v290.cnf-k5-r16.6.cnf-0.1-0.1-0.2.cnfP	0.06%
k5-r21.117-v290.cnf-k5-r16.6.cnf-0.2-0.1-0.4.cnfP	0.12%
k5-r21.117-v290.cnf-k5-r16.6.cnf-0.4-0.1-0.1.cnfP	0.24%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.2-0.1-0.4.cnfP	0.21%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.2-0.2-0.4.cnfP	0.41%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.2-0.4-0.2.cnfP	0.80%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.4-0.1-0.4.cnfP	0.04%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.4-0.2-0.4.cnfP	0.09%
k7-r59.0.cnf-k7-r87.79-v90.cnf-0.4-0.4-0.2.cnfP	0.18%
k7-r87.79-v102.cnf-k5-r17.8.cnf-0.8-0.8-0.05.cnfP	0.00%
k7-r87.79-v106.cnf-k5-r18.0.cnf-0.8-0.8-0.05.cnfP	0.01%
k7-r87.79-v110.cnf-k5-r18.2.cnf-0.8-0.8-0.05.cnfP	0.01%

4.7 Experiments

4.7.1 Experiment 1: initAssign(F)

Experiment 1 compares three strategies of initialization in our solver. The *biasInit* suggestion is assign variables based on occurrences of their literals. It assigns True to variables whose positive literal occurs more than its negative literal. Another alternative *randomInit* is to build a coloring randomly. In a combination of these two variants *randomBiasInit*, the boolean value is assigned to variables based on bias randomly on literals occurrences. the probability to assign True to variable v_i is $\frac{posOccurences[i]}{posOccurences[i]+negOccurences[i]}$.

k	*RandomInit*	*BiasInit*	*Bias-RandomInit*
3	9221.9 (**55**)	9157.76 (54)	**9078.27**(**55**)
5	7143.9 (82)	**4351.09**(**87**)	4582.54 (**87**)
7	6238.51(60)	**5421.9** (60)	6310.7(60)

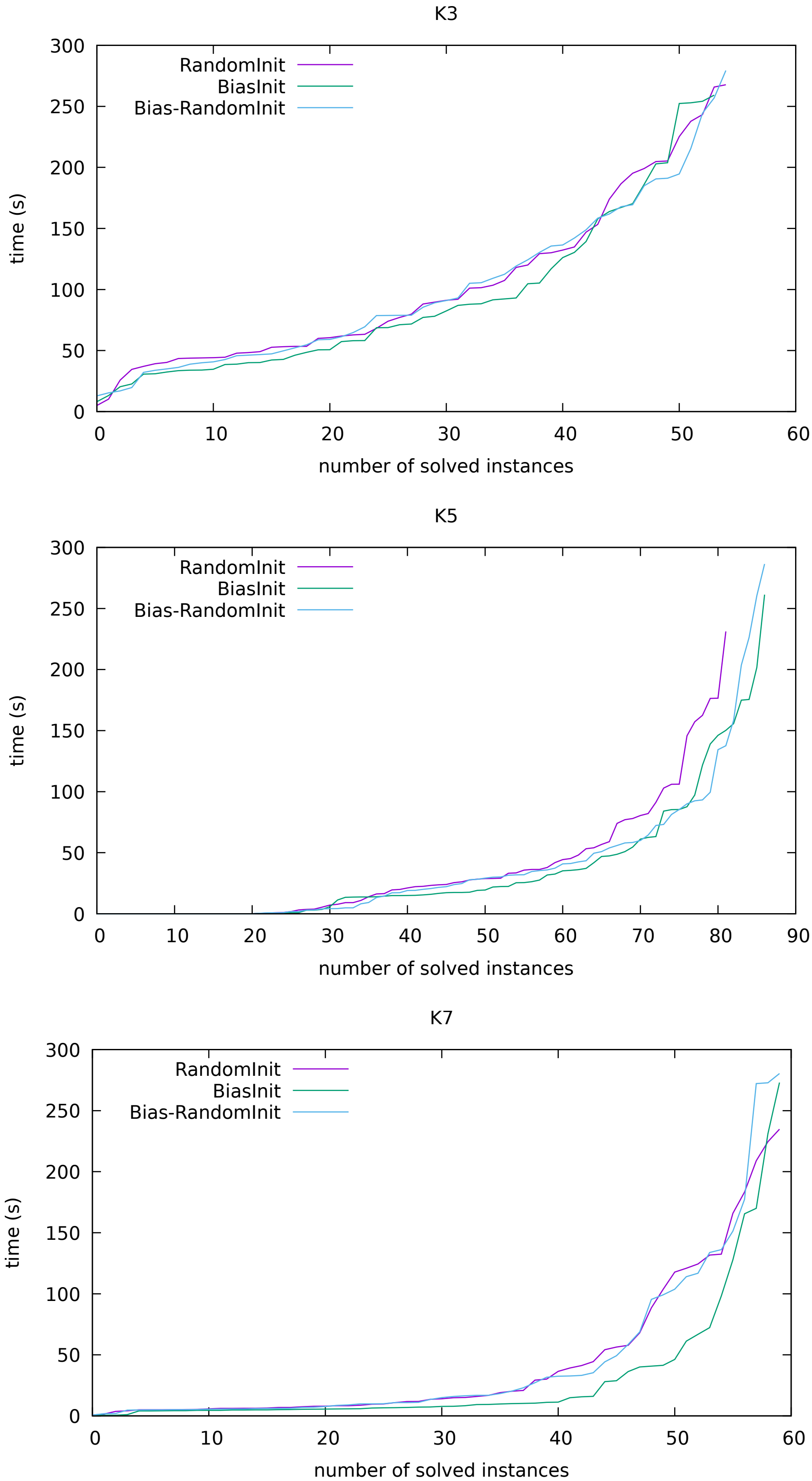


Figure 2: 3SAT 5SAT and 7SAT instances.

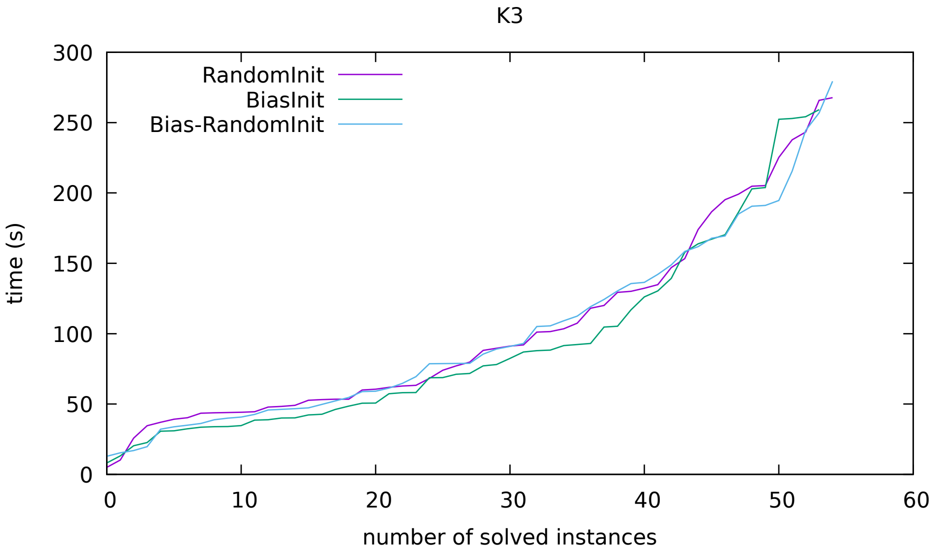


Figure 3: Three suggestions have very similar performance. In our solver, we use *RanomInit* for $3SAT$ as initiliazation method because of its simplify.

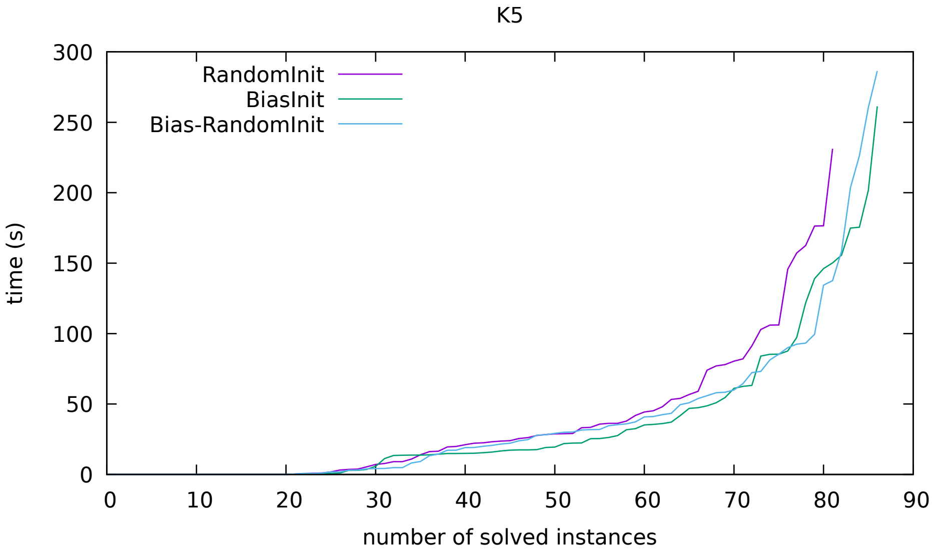


Figure 4: Two bias suggestions show advantages especially for huge 5SAT instances.

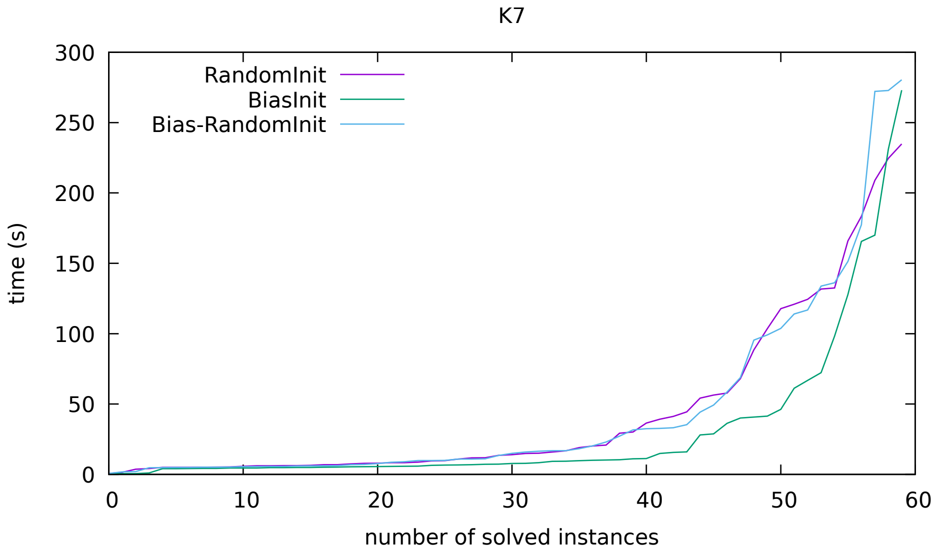


Figure 5: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.2 Experiment 2: pickVar(F)

With the comparison of three Variant of $pickVal$ with the one in probSAT, our suggestions are faster and solve more instances in K3 and K5. For K7, there are no noticeable differences in results.

k	$probSAT$	$WALK$	$Average$	$Random - Flip$
3	9221.9 (55)	7430.12 (57)	**6161.11**(**61**)	8362.42 (55)
5	7143.9 (82)	4433.05 (87)	**3308.16**(**89**)	4052.47(87)
7	6238.51(**60**)	6358.76(**60**)	6525.597(59)	**5800.46**(**60**)

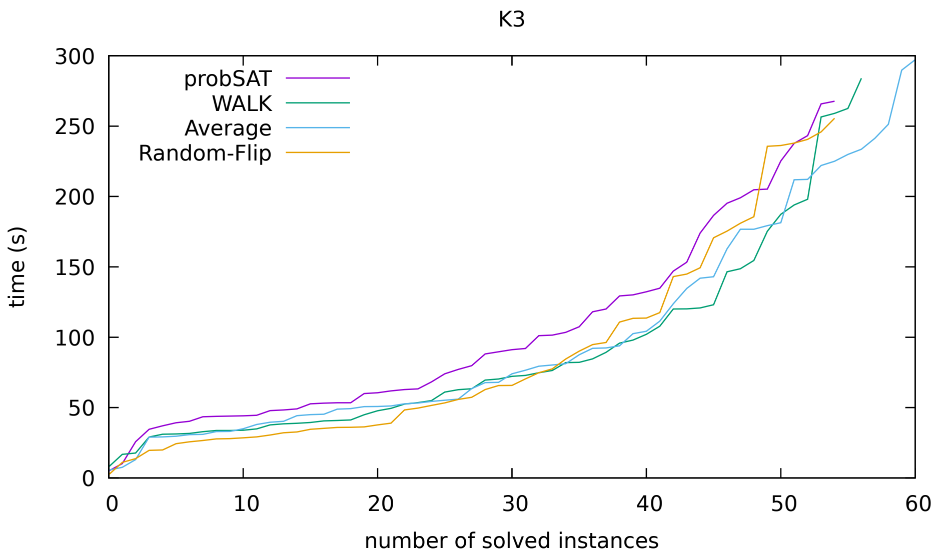


Figure 6: For k3, our suggestions have better performances. The $WALK$ and $Average$ can solve more problems. The best one is the $Average$ (with $\alpha = 1$), which solves 10% more problems.

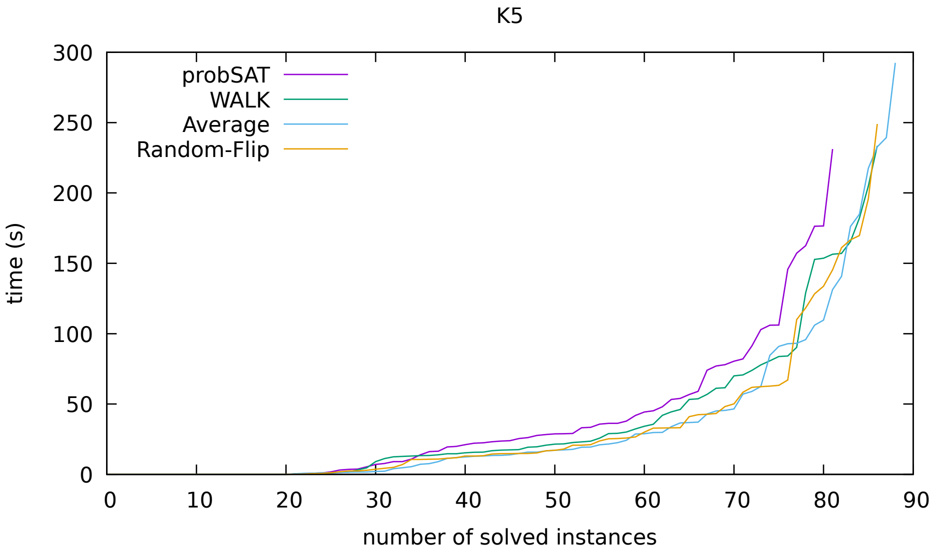


Figure 7: The improvement through our suggestions are big and stable. They solve more problems efficiently. The *Average* (with $\alpha = 1$) can solve 8% more problems than the probSAT with only 46% time (in PAR-2 schema).

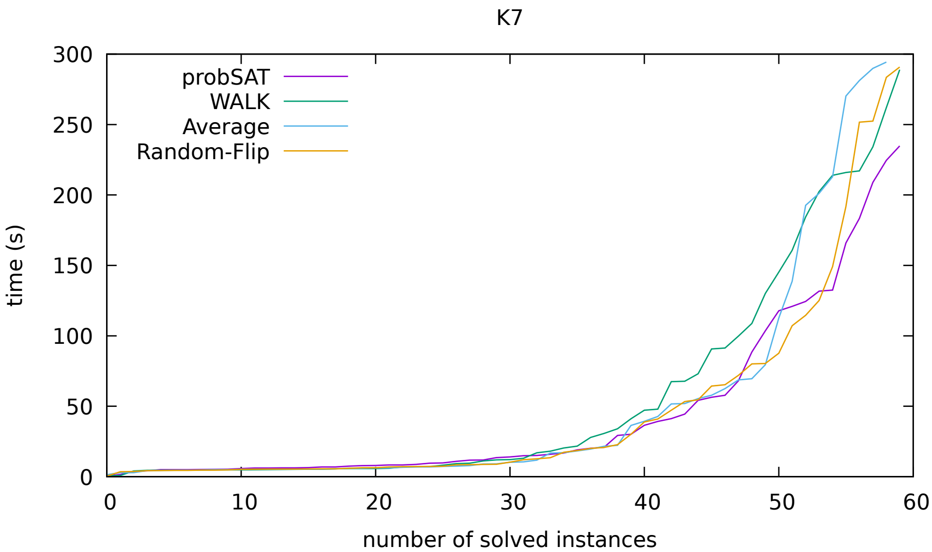


Figure 8: For $7SAT$ problems, the $WALK$ and $Average$ have generally worse inperformance.The $Random - Flip$ can solve some number of problems as the original $probSAT$ and has shown littel improvement in performance within 125 Seconds.

4.7.3 Experiment 3: WALK with Simulated Annealing

k	$WALK$	$WALK - Local$	$WALK - Global$
3	**7430.12(57)**	8346.76(56)	9023.96(56)
5	4433.05(87)	3330.61(89)	**3117.45(89)**
7	6358.76(60)	**5409.67(61)**	6566.06(59)

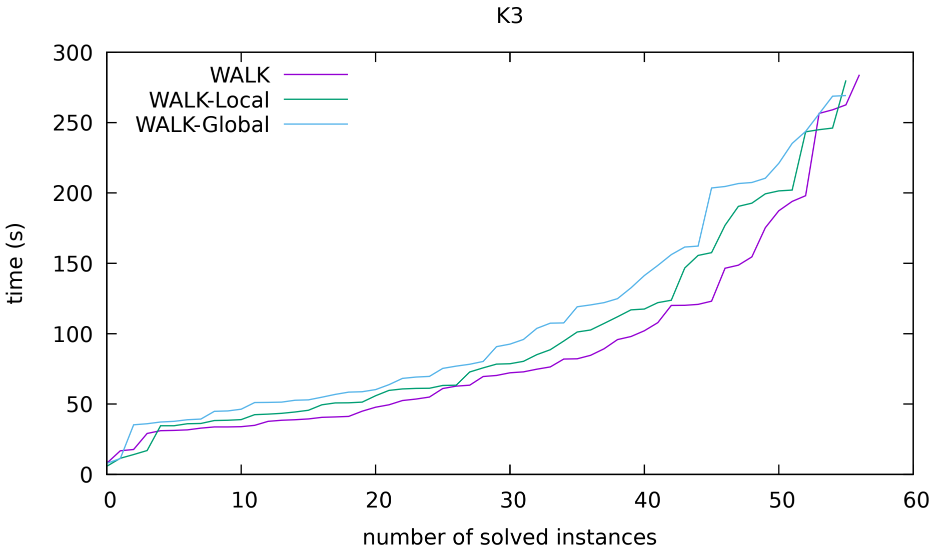


Figure 9: The combination with simulated Annealing show worse efficiency than the original $WALK$. They solve one less problems.

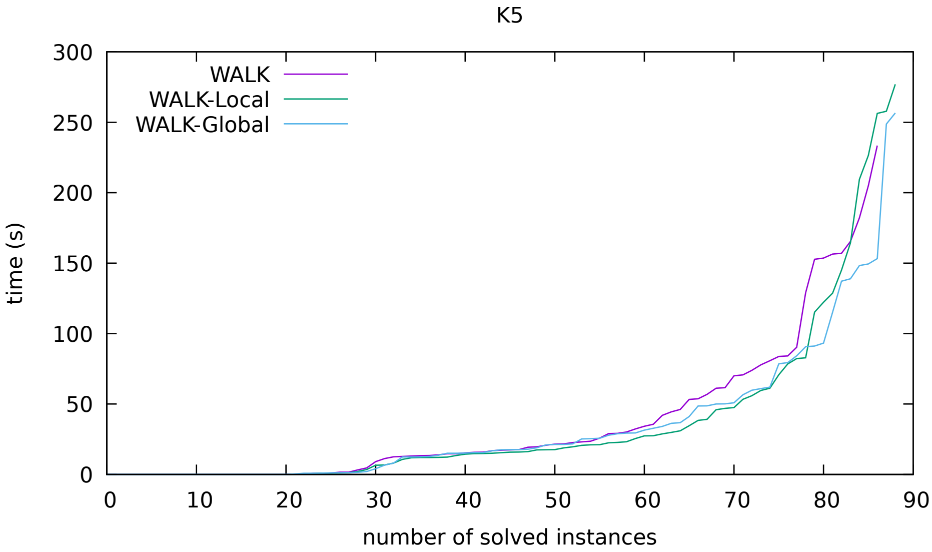


Figure 10: Two bias suggestions show advantages especially for huge instances.

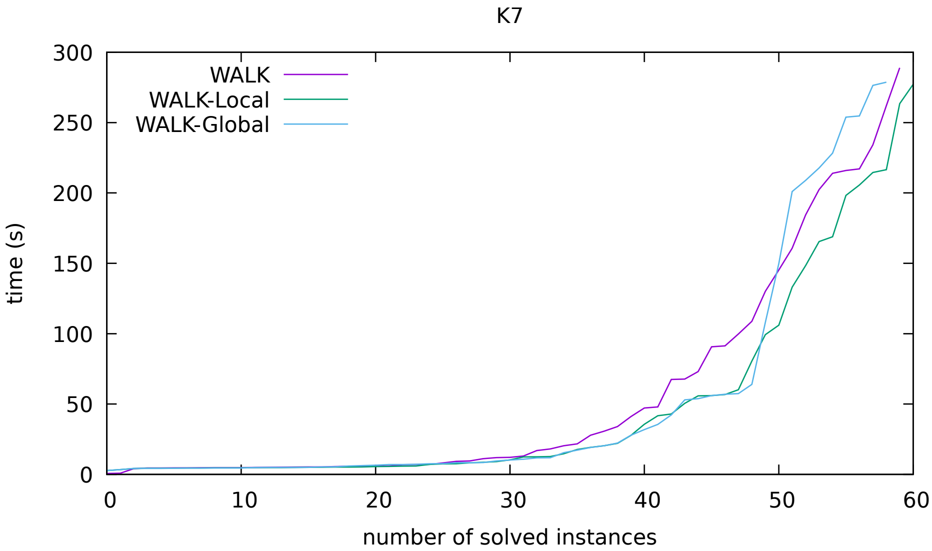


Figure 11: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.4 Experiment 4: Average with Simulated Annealing

k	$Average$	$Average - Local$	$Average - Global$
3	**6161.11**(**61**)	9254.18(53)	9870.25(53)
5	3308.16(89)	**2793.32**(89)	2939.74(89)
7	6525.59(59)	**3829.95**(**65**)	5738.2(61)

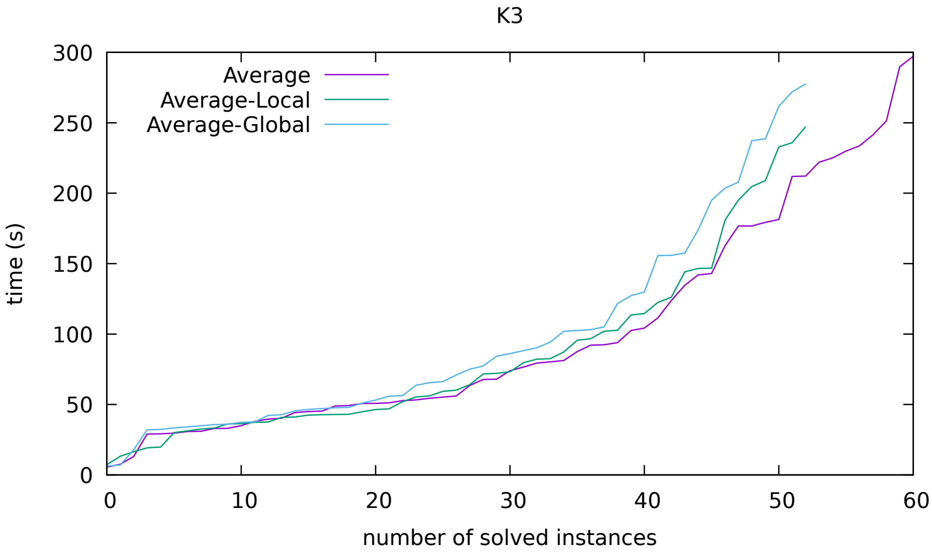


Figure 12: The performances of these three algorithms are guiet semiliar for small problems within 50 Seconds. The original Average can solve 15% problems.

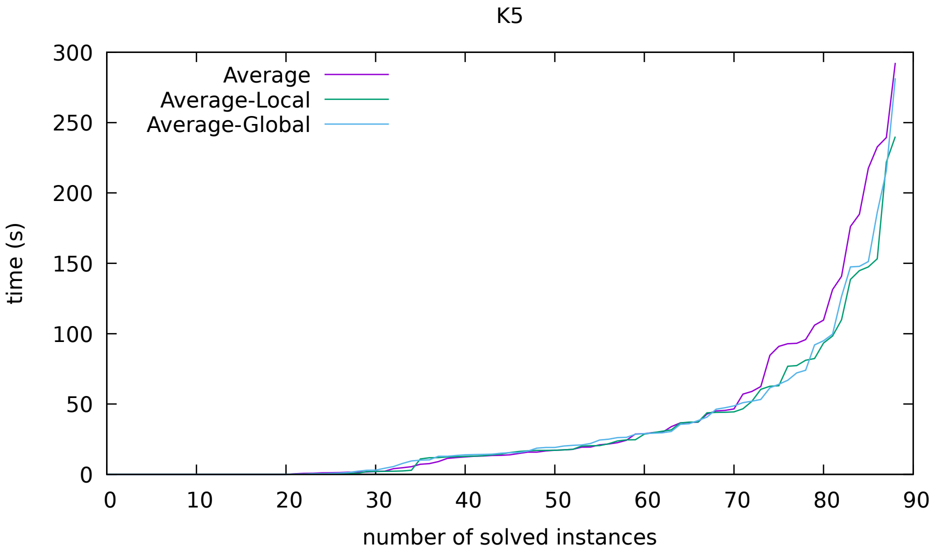


Figure 13: There are not big differences in *Average* with different combination of simulated annealing. They have shown small Advantages in solving huge problems.

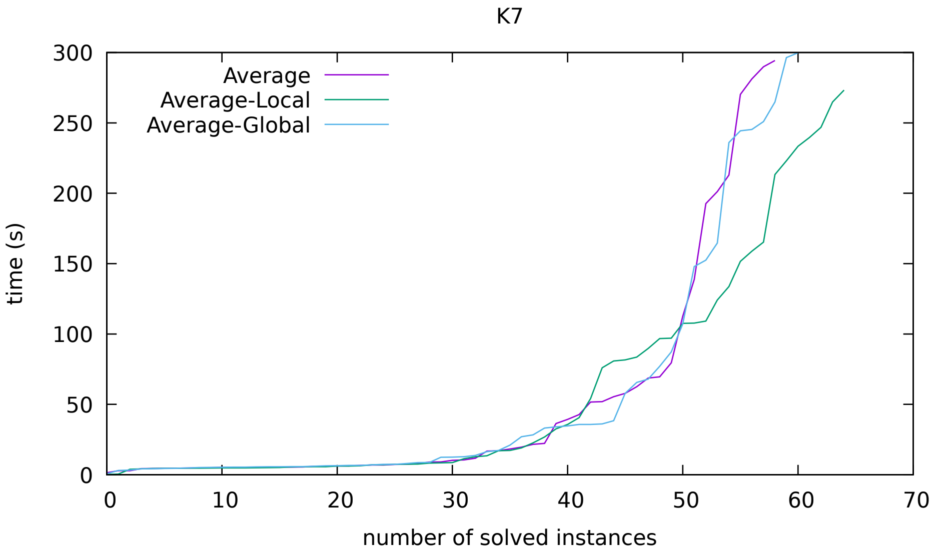


Figure 14: For $7SAT$ problems, the *Local* version can solve 6 problems more than the original *Average*. It has also shown its efficiency in solving huge $7SAT$ problems. The *Local* (with $\alpha = 2$) can solve 10% more problems than the probSAT within less than 60% time (in PAR-2 schema).

4.7.5 Experiment 5: Random-Flip with Simulated Annealing

k	$Random - Flip$	$Random - Flip - Local$	$Random - Flip - Global$
3	8362.42(55)	8409.8(55)	**7308.01(58)**
5	4052.47(87)	4132.07(87)	**4003.06(88)**
7	5800.46(60)	6792.23(59)	**4903.61(60)**

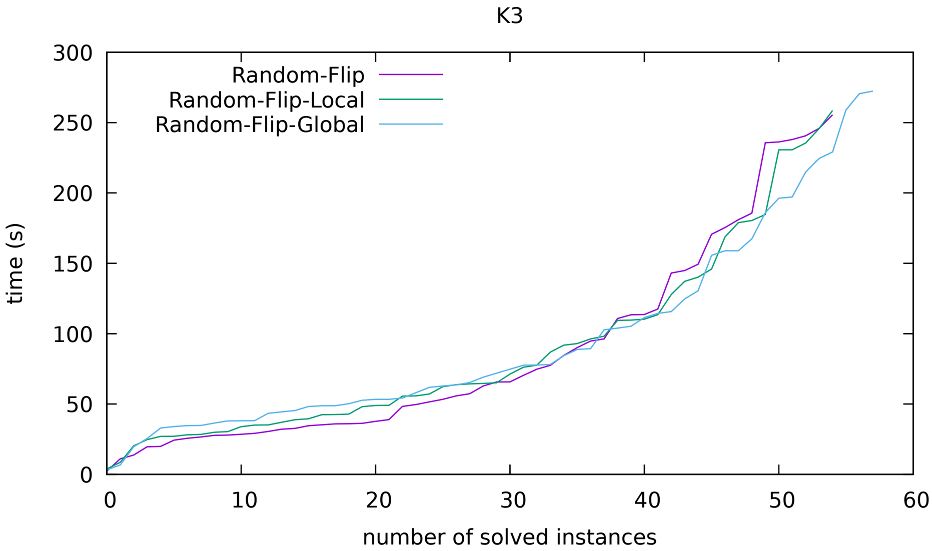


Figure 15: Three suggestions have very similar performance.

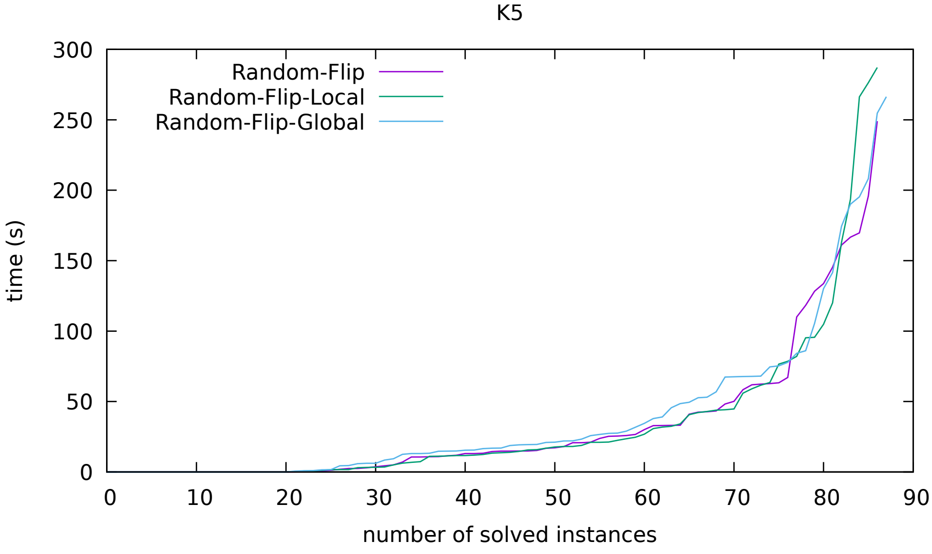


Figure 16: Two bias suggestions show advantages especially for huge instances.

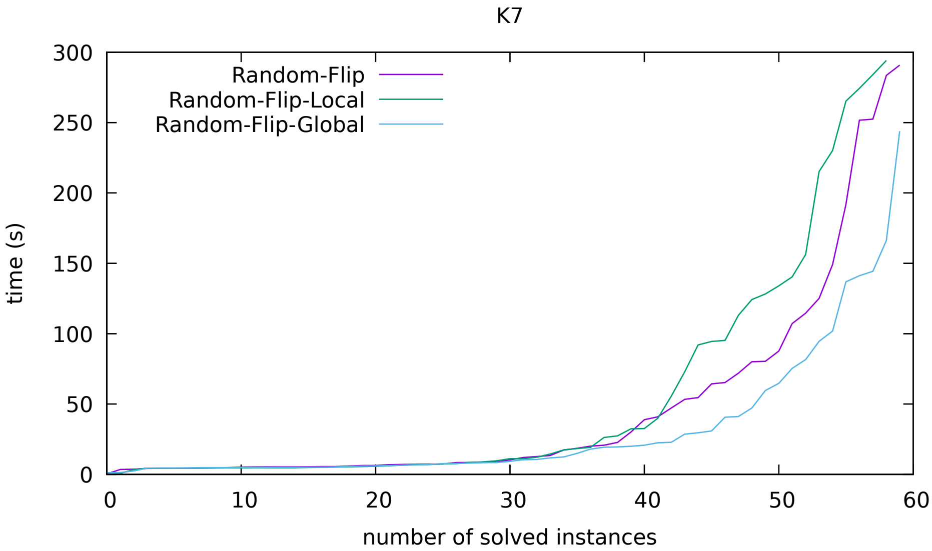


Figure 17: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

k	*Average*	*AverageLocal*	*AverageGlobal*	*RF*	*RFLocal*	*RFGlobal*
3	5.43%	2.43%	5.43%	9.98%	10.01%	3.13%
5	4.07%	0.35%	0.00%	9.18%	9.17%	0.33%
7	2.28%	0.03%	0.00%	4.30%	4.14%	0.05%

4.7.6 Experiment 6: probSAT vs WALK.

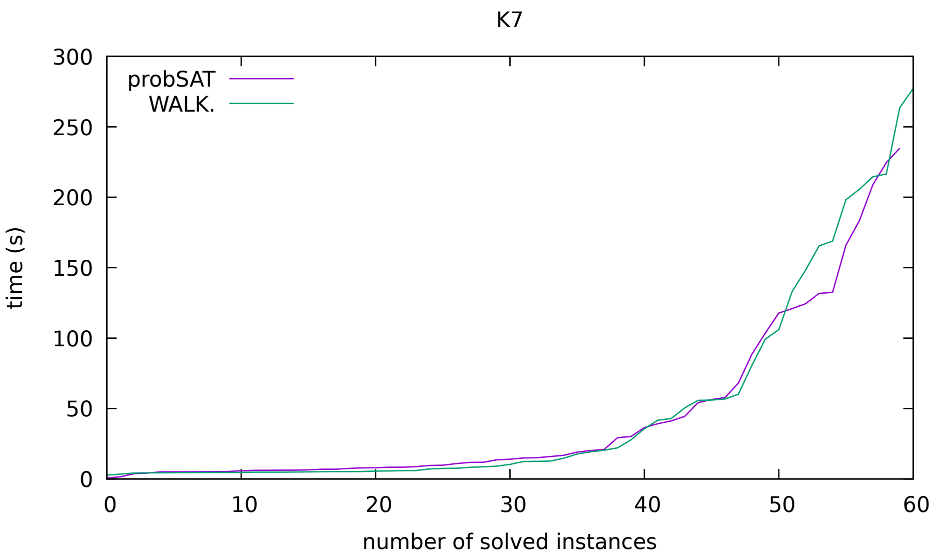


Figure 18: Three suggestions have very similar performance.

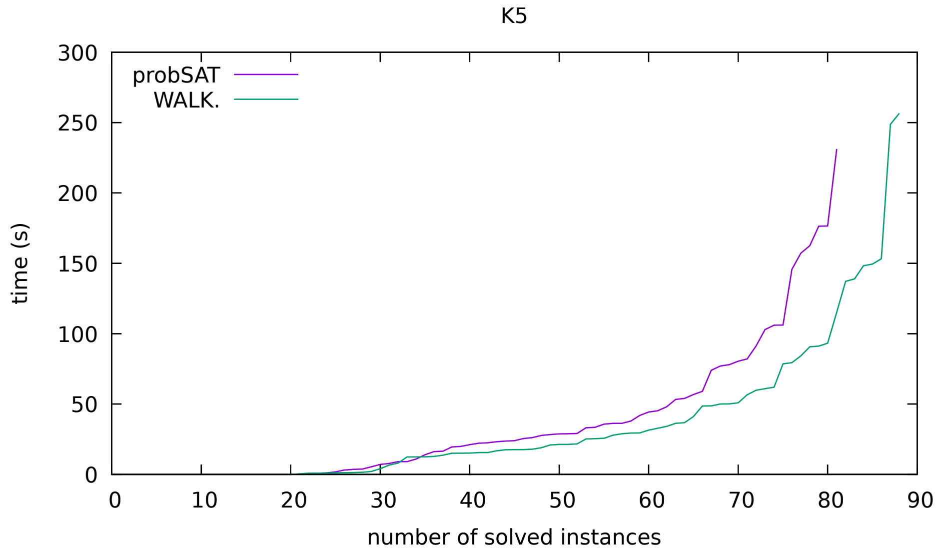


Figure 19: Two bias suggestions show advantages especially for huge instances.

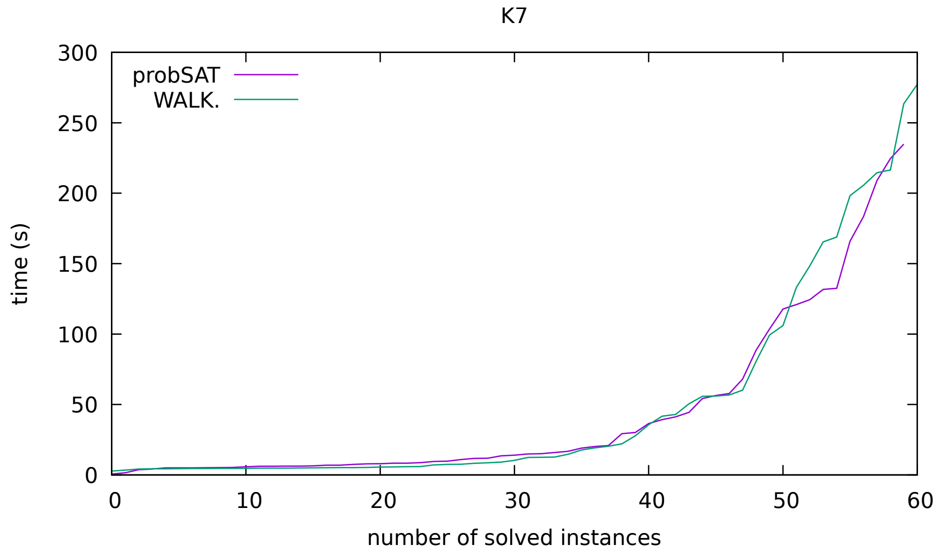


Figure 20: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.7 Experiment 7: probSAT vs Average.

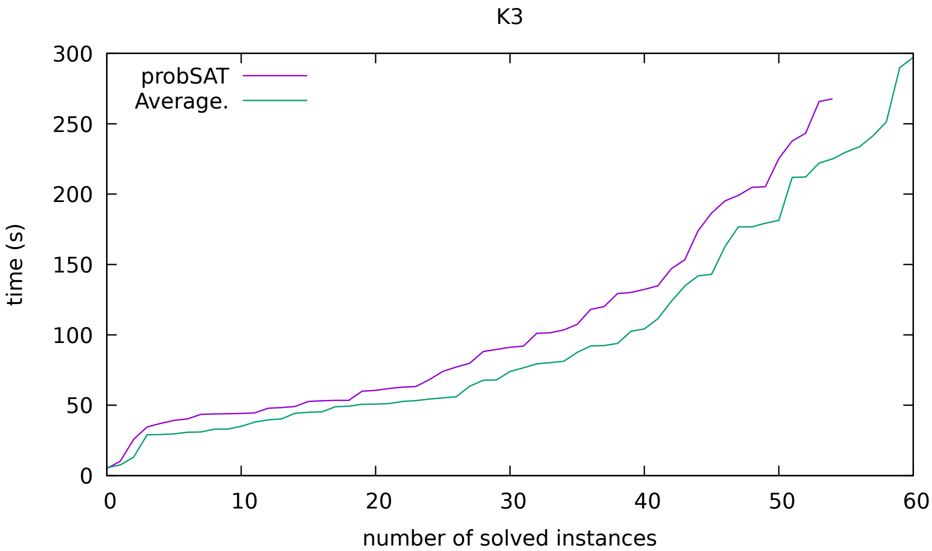


Figure 21: Three suggestions have very similar performance.

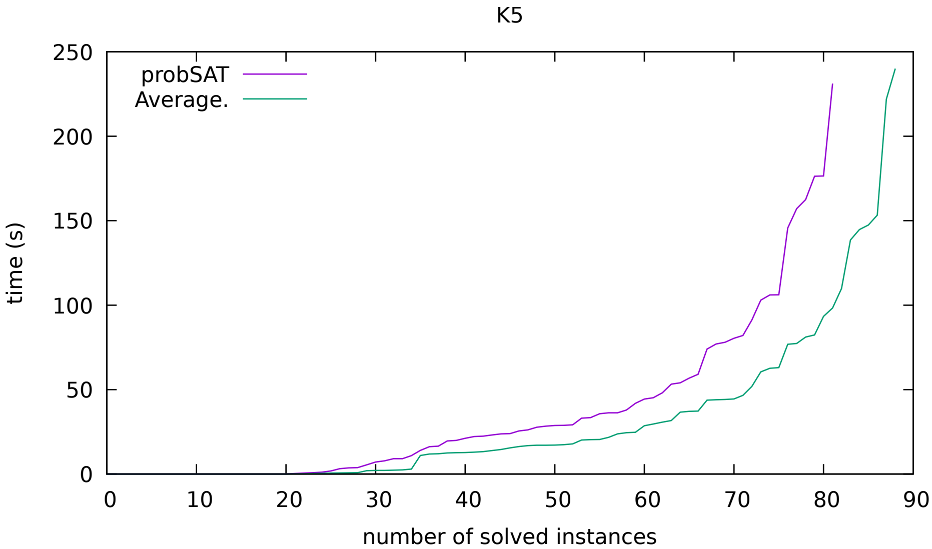


Figure 22: Two bias suggestions show advantages especially for huge instances.

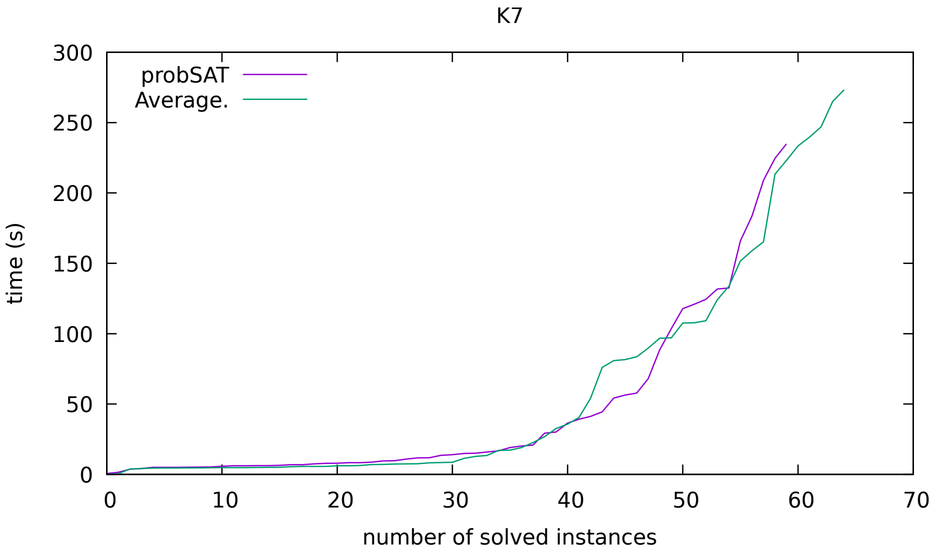


Figure 23: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.8 Experiment 8:probSAT vs Random-Flip.

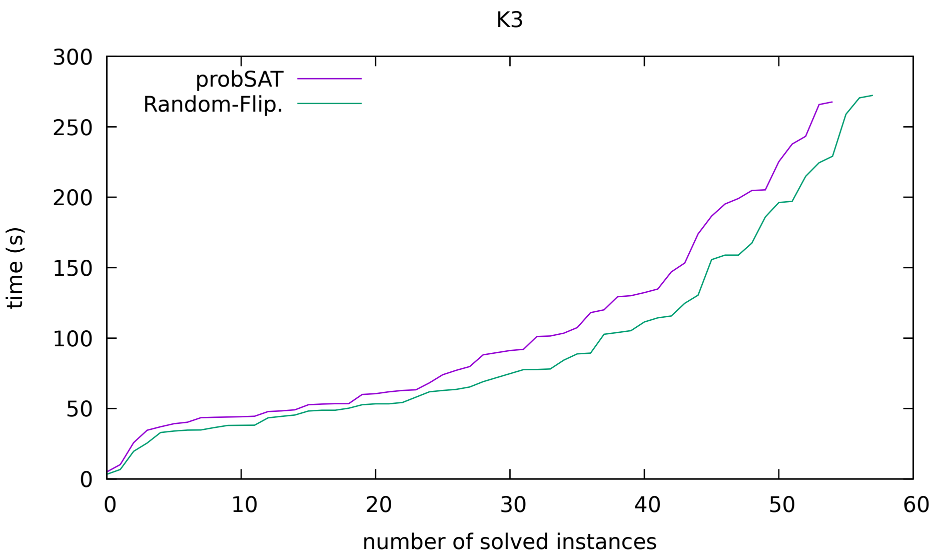


Figure 24: Three suggestions have very similar performance.

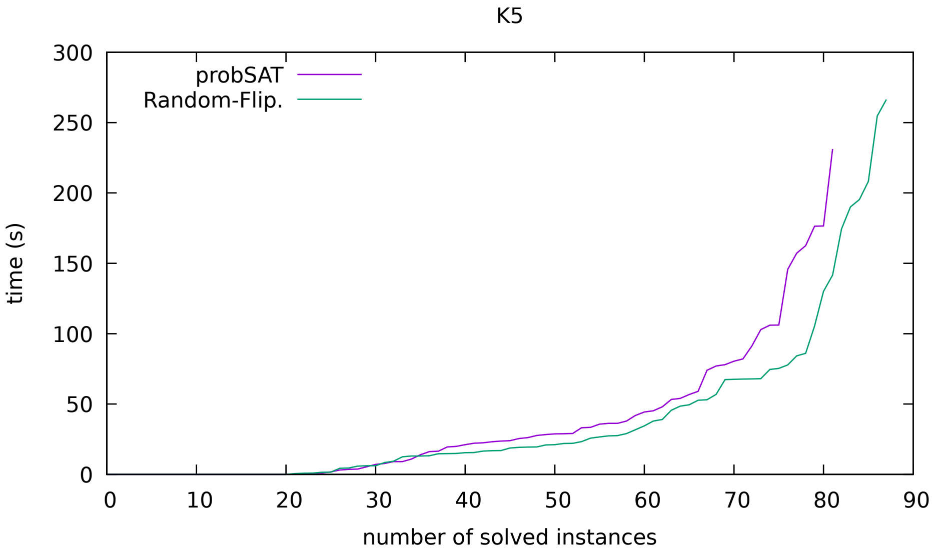


Figure 25: Two bias suggestions show advantages especially for huge instances.

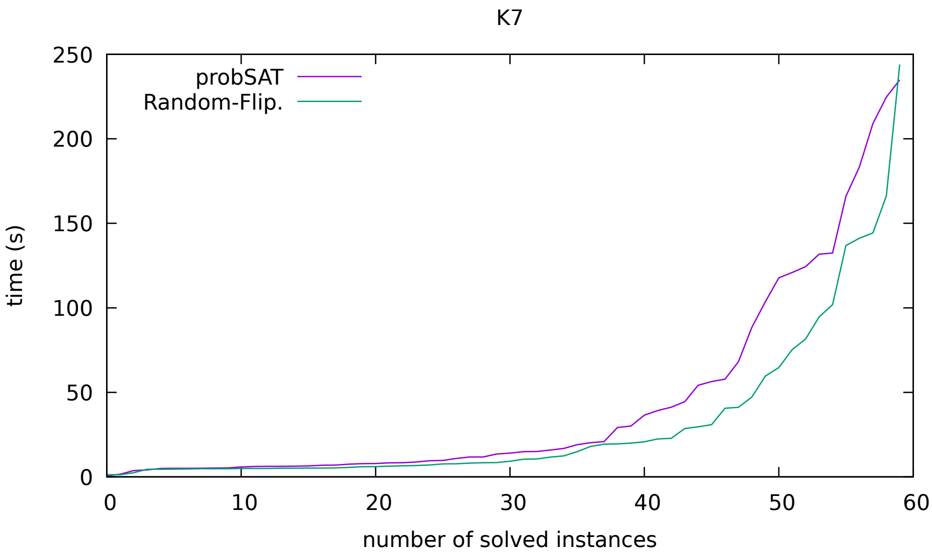


Figure 26: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.9 Experiment 9: 2017-UNIF Comparision

k	$probSAT$	$yalSAT$	$WALK$	$Average/swpSAT$	$Random-Flip$
3	9221.9(55)	17062.35(41)	7430.12(57)	**6161.11(61)**	7308.01(58)
5	7143.9(82)	5676.63(85)	3330.61(89)	**2939.74(89)**	4003.06(88)
7	6238.51(60)	10063.4(54)	5409.67(61)	**3829.95(65)**	4903.61(60)

k	$probSAT$	$WALK$	$Average/swpSAT$	$Random-Flip$
3	954426	946662	**1012761**	988477
5	389453	**443172**	414765	423364
7	248025	237387	**248028**	221107

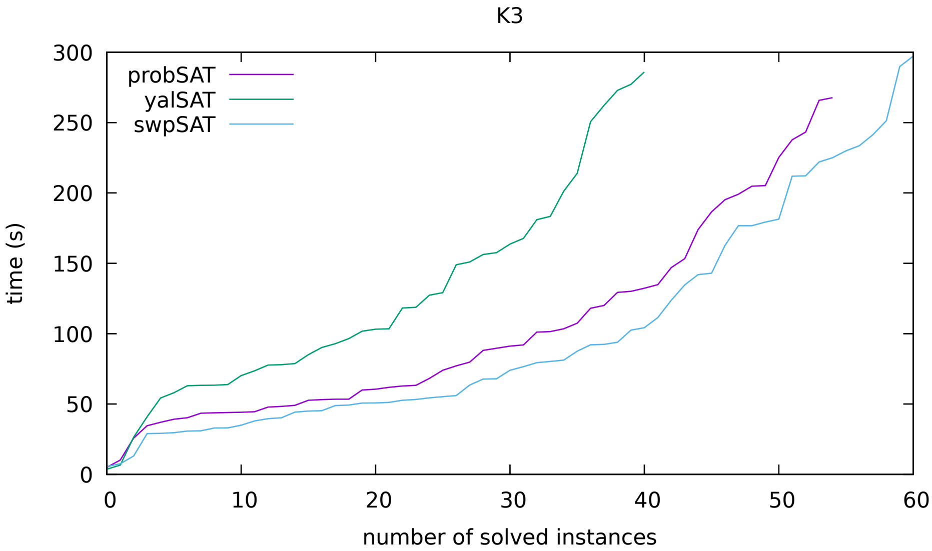


Figure 27: Three suggestions have very similar performance.

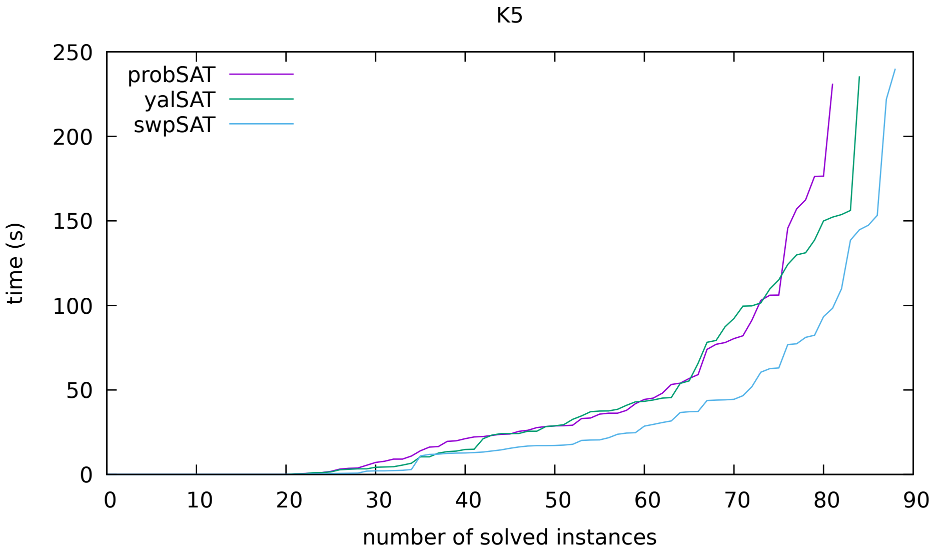


Figure 28: Two bias suggestions show advantages especially for huge instances.

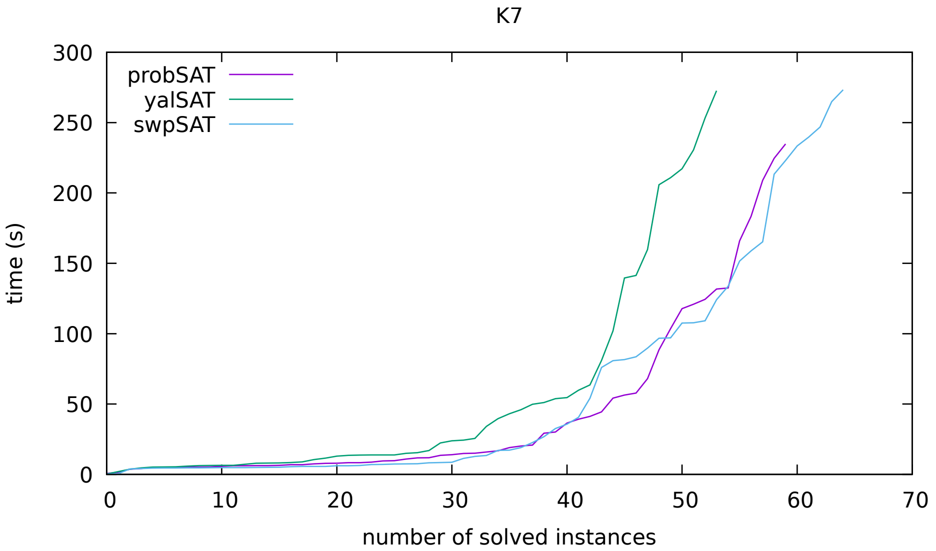


Figure 29: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

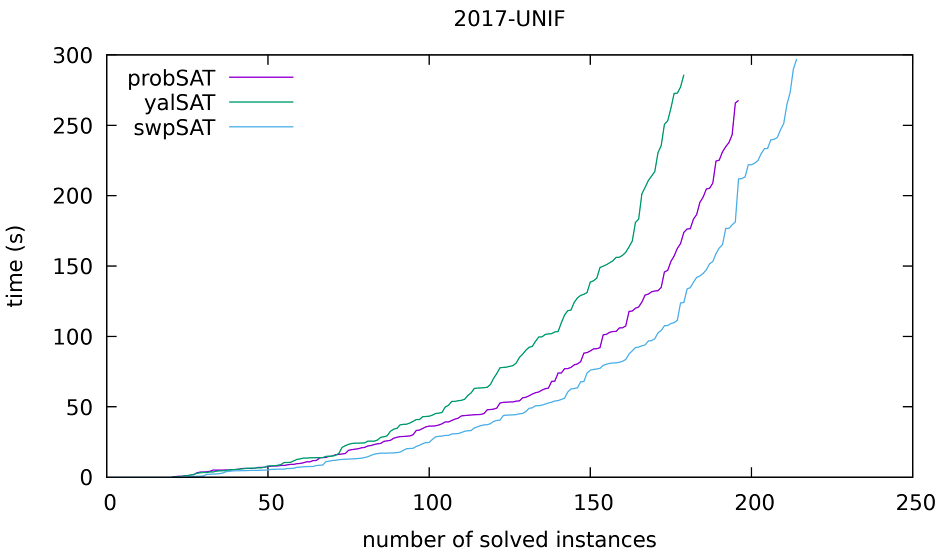


Figure 30: For 7SAT problems, the two random Initialization are similar in performance, while the bias initialization shows its efficiency.

4.7.10 Experiment 10: The pure portfolio approach

Our parallel implementation uses OpenMP to support shared memory multiprocessing. In our local *C++* Implementation, we use the function *rand()* in the standard library to generate pseudo-random integer. This function is not thread-safe. To implement a deterministic parallel implementation, we use *rand()* for in the first thread and use the thread safe random engine in *C++* for random value generation for other threads. Based on the observation in our Experiments, the sequential implementation with the simple *rand()* function has the best performance for the whole set. For most problems, some random value generators can search following a valid search path to a satisfying assignment. But there is no one random generator that are advantages for all the problems. This approach takes advantage of the performances differences of random generator. The threads execute the swpSAT with different random engine in parallel. If one thread find a satisfying assignment,the whole parallel search can stop. With our experiment, the parallel search gets a performance like the minimum runtime of one-thread local search with different random value engines.

4.7.11 Experiment 11: Initialization with a guide of formula partitioning

Solver	$3-3(big)$	$3-3(small)$	$3-5$	$5-5$	$5-7$
swpSAT	1861.73	7955.19	-	2748.13	-
FineInit	501.78 (26.95%)	1734.79 (21.81%)	-	621.47(22.61%)	-
Solver	$7-7$	BIG	SMALL	COMBINE	
swpSAT	3276.39	2794.15	15579.71	18373.86	
FineInit	376.06 (11.48%)	902.69(32.31%)	3099.02(19.89%)	4001.71(21.78%)	

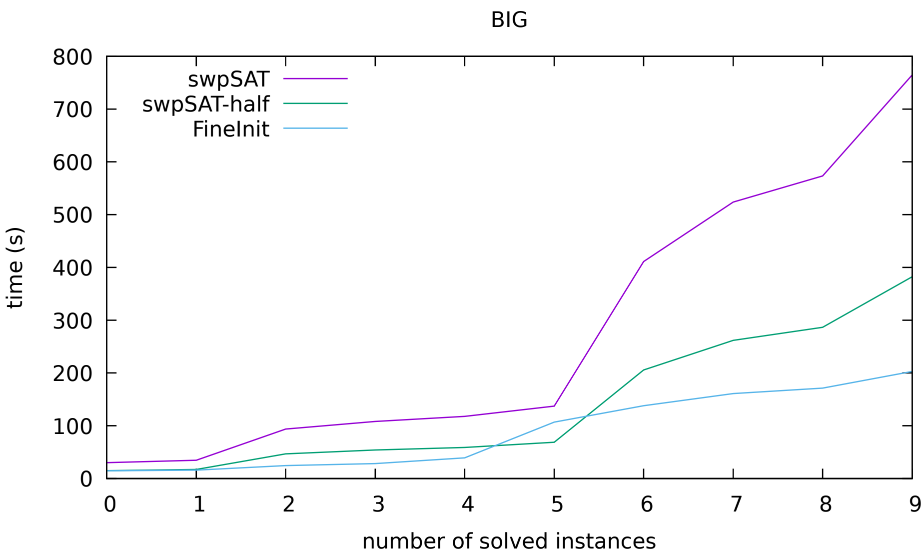


Figure 31:

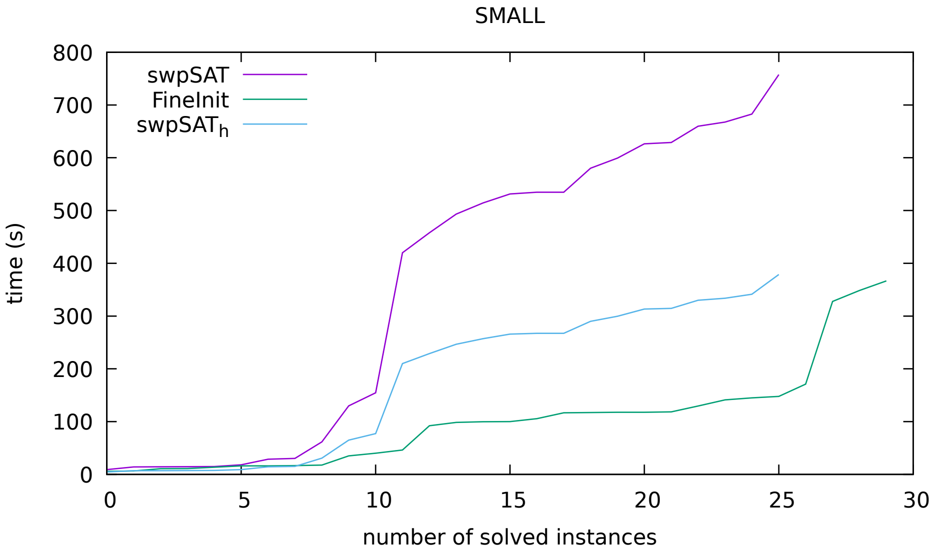


Figure 32:

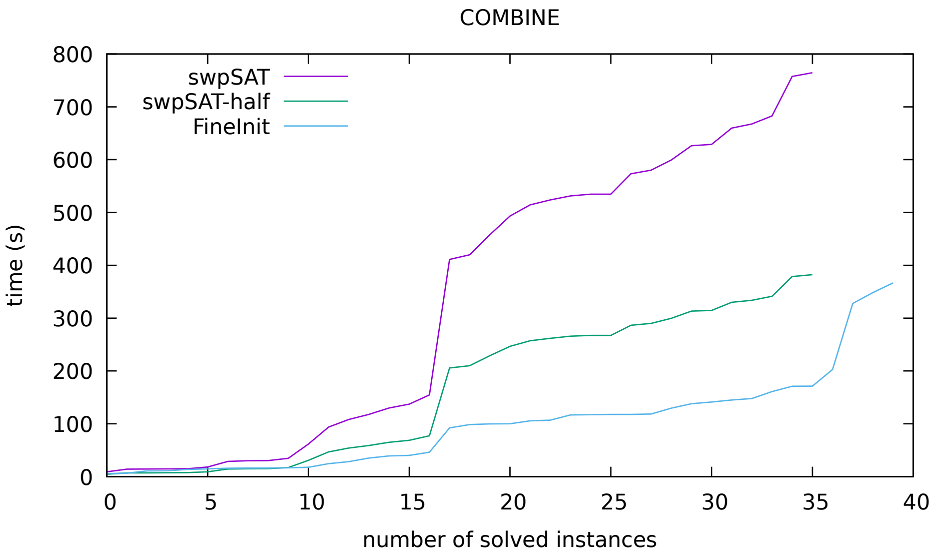


Figure 33:

5 Conclusion

Local search is a universally applicable approach to solve random SAT problems. Our paper presents a stochastic local search algorithm with the cooperation of WALKSAT and probSAT.

In section 3, we discussed the basic scheme of our sequential algorithm. In, we compare the original probSAT with a randomly generated initial solution and our version based on the occurrence of os literals in formel. Based on our experiment, our initialization is advantages n the number of solved problems and also in the execution time of the search. To get the advantage of the greedy algorithm and use the robustness of the stochastic process, we introduce the data structure statistic list to guide the decision between these two processes in step $pickVar()$. Here we propose some variants to make the distinction between greedy choice and a random choice. Generally, our local searches get better performance than the probSAT algorithm.
Based on the performance of these searches in kSAT problems, we get our swpSAT solver, which combines the advantages of the local searches.

In section 4, we make our swpSAT solver parallel with different approaches. Most problems get similar results with the different seed of one random generator. However, the formulas get different results with different random generation. With this fact, we make the parallel version of our local search, in which the agents run the search with different random generators and then take advantages of the suitable one.
In the following part of this section, we discussed the combination of formula partitioning and our parallel solver. After trying several approaches with failure, we found the way of using formula partitioning to make a fine initial solution save search time and furthermore solve more problems. Our experiments evaluate the hyperthesis that the formula partitioning information can guide the local search and improve the efficiency in the parallel search.

5.1 Further work

6 Bibliography

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