

Assignment 4

R-2.8 Illustrate the performance of the selection-sort algorithm on the following input sequence (22, 15, 26, 44, 10, 3, 9, 13, 29, 25).

22	15	26	44	10	3	9	13	29	25
3	15	26	44	10	22	9	13	29	25
3	9	26	44	10	22	15	13	29	25
3	9	10	44	26	22	15	13	29	25
3	9	10	13	26	22	15	44	29	25
3	9	10	13	15	22	26	44	29	25
3	9	10	13	15	22	26	44	29	25
3	9	10	13	15	22	25	44	29	25
3	9	10	13	15	22	25	26	29	44
3	9	10	13	15	22	25	26	29	44
3	9	10	13	15	22	25	26	29	44

R-2.9 Illustrate the performance of the insertion-sort algorithm on the input sequence of the previous problem.

22	15	26	44	10	3	9	13	29	25
15	22	26	44	10	3	9	13	29	25
15	22	26	44	10	3	9	13	29	25
15	22	26	44	10	3	9	13	29	25
10	15	22	26	44	3	9	13	29	25
3	10	15	22	26	44	9	13	29	25
3	9	10	15	22	26	44	13	29	25
3	9	10	13	15	22	26	44	29	25
3	9	10	13	15	22	26	29	44	25
3	9	10	13	15	22	25	26	29	44

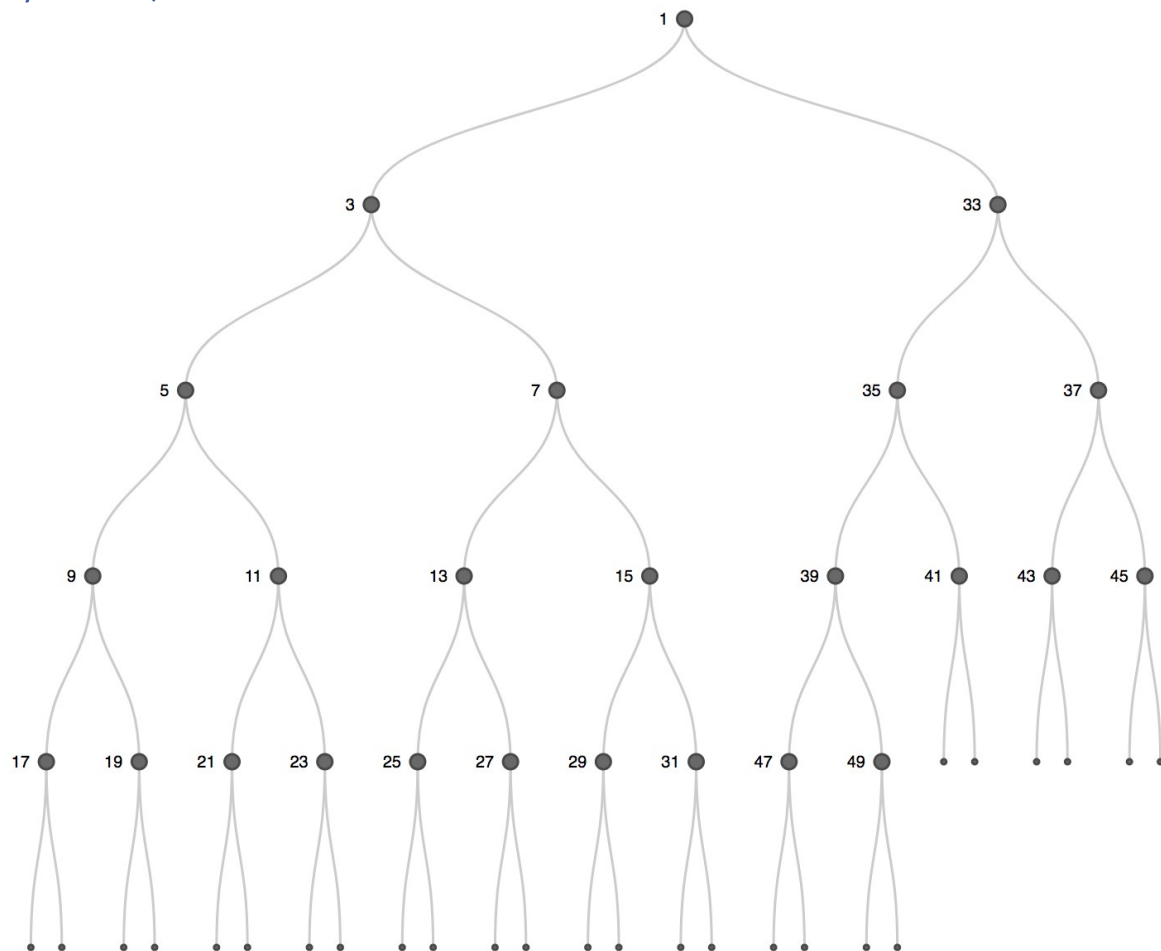
R-2.10 Give an example of a worst-case sequence with n elements for insertion-sort runs in $\Omega(n^2)$ time on such a sequence.

The worst-case sequence example is one in descending order such as **9 8 7 6 5 4 3 2 1** to be sorted to ascending order.

R-2.13 Suppose a binary tree T is implemented using a vector S , as described in Section 2.3.4. If n items are stored in S in sorted order, starting with index 1, is the tree T a heap? Justify your answer.

Yes, it is a heap because: $S[i] \geq S[i/2] \Rightarrow \text{key}(v) \geq \text{key}(\text{parent}(v))$.

R-2-18 Draw an example of a heap whose keys are all the odd numbers from 1 to 49 (with no repeats), such that the insertion of an item with key 32 would cause up-heap bubbling to proceed all the way up to a child of the root (replacing that child's key with 32).



C-2.32 Let T be a heap storing n keys. Give an efficient algorithm for reporting all the keys in T that are smaller than or equal to a given query key x (which is not necessarily in T). For example, given the heap on Figure 2.41 and query key $x=7$, the algorithm should report 4, 5, 6, 7. Note that the keys do not need to be reported in sorted order. Ideally, your algorithm should run in $O(k)$ time, where k is the number of keys reported.

Algorithm reportKey(T, x)

Input: A heap T , and query value x

Output: A list of keys smaller than or equal to x

returnList \leftarrow new List

If $\neg T.\text{isEmpty}()$

 reportKeyHelper($T, 1, x, \text{returnList}$)

return returnList

Algorithm reportKeyHelper($T, i, x, \text{returnList}$)

Input: A heap T , index of a node in the heap, the query value x , and the return list to contain reported values

if $T[i] \leq x$

 returnList.add($T[i]$)

```

left <- i * 2
if left < T.size()
    leftList <- reportKeyHelper(T, left, x, returnList)
right <- i * 2 + 1
if right < T.size()
    rightList <- reportKeyHelper(T, right, x, returnList)

```

Design an algorithm, `isPermutation(A, B)` that takes two sequences A and B and determines whether or not they are permutations of each other, i.e., same elements but possibly occurring in a different order. Hint: A and B may contain duplicates. What is the worst-case time complexity of your algorithm? Justify your answer

Algorithm `isPermutation(A, B)`

Input: 2 sequences to check if they are permutations of each other

Output: true if they are permutations of each other, false otherwise

If `A.size() != B.size()`

 return false

PQ <- new PriorityQueue with HeapSort implementation

`sortedA <- PQ.sort(A)`

`sortedB <- PQ.sort(B)`

`elementsA <- sortedA.elements()`

`elementsB <- sortedB.elements()`

while `elementsA.hasNext()`

`elementA <- elementsA.nextObject()`

`elementB <- elementsB.nextObject()`

 if `elementA != elementB`

 return false

return true

The worst-case complexity of this algorithm is $O(n \log n)$ because of the heap-sorting