

Assignment 2

R-2.1 Describe, using pseudo-code, implementations of the methods `insertBefore(p, e)`, `insertFirst(e)`, and `insertLast(e)` of the List ADT, assuming the list is implemented using a doubly-linked list.

Algorithm `insertBefore(p, e)`

```
Input: Position p, new element e
Output: New Position q
q <- new Position
q.element <- e
q.prev <- p.prev
q.next <- p
p.prev.next <- q
p.prev <- q
return q
```

Algorithm `insertFirst(e)`

```
Input: New element e
Output: New Position q
q <- new Position
q.element <- e
q.next <- header.next
q.prev <- header
header.next.prev <- q
header.next <- q
return q
```

Algorithm `insertLast(e)`

```
Input: New element e
Output: New Position q
q <- new Position
q.element <- e
q.next <- trailer
q.prev <- trailer.prev
trailer.prev.next <- q
trailer.prev <- q
return q
```

C-2.1 Describe, in pseudo-code, a link-hopping method for finding the middle node of a doubly linked list with header and trailer sentinels, and an odd number of real nodes between them. (Note: This method can only use link-hopping; it cannot use a counter.) What is the running time of this method?

Algorithm `findMiddleNode(L)`

```
Input: A doubly linked list L with an odd number of real nodes between header and trailer
Output: The middle node
p <- L.header
q <- L.trailer
while p != q do
    p = first.next
    q = last.prev
```

```
return p
```

The running time of this algorithm is $O(n/2)$

C-2.2 Describe, in pseudo-code, how to implement the queue ADT using two stacks. What is the running time of the enqueue() and dequeue() methods in this case?

Algorithm enqueue(e)

```
Input: Element e
stack1.push(e)
```

Algorithm dequeue()

```
Output: Element at the front of the queue
if stack1.isEmpty() and stack2.isEmpty() then
    throw EmptyQueueException
else
    if stack2.isEmpty() then
        while !stack1.isEmpty() do
            stack2.push(stack1.pop())
    p <- stack2.pop()
    return p
```

The running time for enqueue algorithm is $O(1)$

The running time for dequeue algorithm is $O(n)$

C-2.3 Describe how to implement the stack ADT using two queues. What is the running time of the push() and pop() methods in this case?

Algorithm push(e)

```
Input: Element e
queue1.enqueue(e)
```

Algorithm pop()

```
Output: Element at the top of the stack
if queue1.isEmpty() and queue2.isEmpty() then
    throw EmptyStackException
else
    for i <- 1 to queue1.size() - 1 do
        queue2.enqueue(queue1.dequeue())
    p <- queue1.dequeue()
    tmp <- queue1
    queue1 <- queue2
    queue2 <- tmp
    return p
```

The running time of push method is $O(1)$

The running time of pop method is $O(n)$

C-2-4 Describe a recursive algorithm for enumerating all permutations of the numbers {1, 2,..., n}. What is the running time of your method?

Algorithm swap(s, i, j)

```
Input: Sequence s, indexes i and j to swap data in s
Output: Sequence with data swapped
temp <- s[i]
s[i] <- s[j]
s[j] <- temp
```

Algorithm permute(s, i)

```
Input: Sequence s, and index i of a position in s whose value is 0 initially
Output: A sequence of permutations of s
permutations <- new Sequence
```

```

if i < s.size() - 1 then
    for j from i to s.size() do
        if j  $\neq$  i then
            swap(s, i, j)
            result <- permute(s, i + 1)
            for all permutation in result do
                permutations.insertLast(permutation)
            if j  $\neq$  i then
                swap(s, i, j)
        else
            permutations.insertLast(s)
    return permutations

```

C-2-5 Describe the structure and pseudo-code for an array-based implementation of the vector ADT that achieves $O(1)$ time for insertions and removals at rank 0, as well as insertions and removals at the end of the vector. Your implementation should also provide for a constant-time `elemAtRank` method.

Algorithm `elemAtRank(r)`

Input: Rank r

Output: Element at rank r

return $V[(f + r) \bmod N]$

Algorithm `insertAtRank(r, o)`

Input: Rank r , and object o to be inserted at rank r

If `size() = N - 1` then

 throw `FullVectorException`

start <- $(f + r) \bmod N$

if start = f do // insert at rank 0

$f \leftarrow (f - 1) \bmod N$

$V[f] \leftarrow o$

else if start = l then // insert at the end

$V[l] \leftarrow o$

$l \leftarrow (l + 1) \bmod N$

else then // insert somewhere between f and l

$i \leftarrow l$

 while $i \neq$ start do

$V[i] \leftarrow V[i - 1]$

$i \leftarrow i - 1$

 if $i < 0$ then

$i \leftarrow i + N$

$V[start] \leftarrow o$

$l \leftarrow (l + 1) \bmod N$

Algorithm `removeAtRank(r)`

Input: Rank r

Output: Removed element at rank r

If `isEmpty()` then

 throw `EmptyVectorException`

start <- $(f + r) \bmod N$

$o \leftarrow V[start]$

if start = f then // remove at rank 0

$f \leftarrow (f + 1) \bmod N$

else if start = $l - 1$ then // remove at the end

$l \leftarrow (l - 1) \bmod N$

```

else then                                     // remove somewhere between f and l
    i <- start
    end <- l - 1
    if end < 0 then
        end <- end + N
    while i ≠ end do
        next <- (i + 1) mod N
        V[i] <- V[next]
        i <- next
    return o

```