Mechanics & Relativity

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28-30 September 2021 (Week 1)

Kinematics

This week's topics:

1.1 Displacement, Velocity & Acceleration

1.2 Equations of motion (SUVAT)

1.3 Reading graphs

Quiz 1.1 Velocity & Acceleration

A rocket ship has a position given by $x = bt^2$ meters. In order for it to reach a velocity of 3,000 m/s in 40,000 meters, what must be the value of b? ______ m/s^2

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Quiz 1.1 Velocity & Acceleration

Captain Kirk is chasing a Gorn and sees the Gorn will reach safety in 250m, as measured from Kirk's location. Kirk is running at 5 m/s while the Gorn is running at 4 m/s and the Gorn has a 90m head start. Scotty brings the transporter back online and can transport Kirk instantly to catch the Gorn. How far must Kirk be transported to catch the Gorn before it reaches safety? Assume the transport is instant.

- 50m
- 40m
- 30m
- 80m

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Quiz 1.1 Velocity & Acceleration

An object has an acceleration given by $a = 12 - 6t \ m/s^2$. If it begins with an initial velocity of 5 m/s at x = 0 meters, what is its position when it stops momentarily?

- -18 m
- 53 m
- 18 m
- -53 m

suvat

s : for position
u : for (magnitude of) **initial** velocity (aka initial speed)
v : for (magnitude of) velocity (aka speed)
a : for (magnitude of) acceleration
t : for time

We will consider situations where displacement s and velocity v are functions of time t.

We cannot use suvat for situations where the acceleration is not constant.

It is convenient to memorise certain formulae for calculating one of these things when others are known, such as

$\underline{v} = \underline{u} + \underline{a}t$

But where does this come from?

How do we know v= u + at ?

The first equation of motion for translational motion is derived starting from the definition for acceleration as the change in velocity with time:

$$\underline{a} = \frac{d}{dt}\underline{v} \rightarrow \int \underline{a}dt = \int \frac{d}{dt}\underline{v}dt \rightarrow \underline{a}t + \underline{c} = \underline{v}$$

We can write $\int \underline{a}dt = \underline{a}t$ because a is constant: not a function of time.

We find the constant of integration $\underline{c}$ by setting $t = 0 \rightarrow \underline{c} = \underline{v}_0$.

This gives us $\underline{a}t + \underline{v}_0 = \underline{v}$, which is often reorganised as

$\underline{v} = \underline{u} + \underline{a}t$ **1.1** where we are now using $\underline{v}_0 \rightarrow \underline{u}$ for the **initial velocity**.

This equation contains no $\underline{s}$, so we use it when $\underline{s}$ is unknown and unwanted

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How about $s = ut + \frac{1}{2}at^2$?

The second equation of motion is derived starting from the definition for velocity:

$$\underline{v} = \frac{d}{dt}\underline{s} \rightarrow \int \underline{v}dt = \int \frac{d}{dt}\underline{s}dt$$

$$v = \frac{ds}{dt} \rightarrow \int vdt = \int \frac{ds}{dt}dt$$

Now we have to substitute for $\underline{v} = \underline{u} + \underline{a}t$ (suvat 1) in the LHS because unlike $\underline{a}$ (which has to be constant for these equations to work) $\underline{v}$ **can be a function of time**:

$$\int (\underline{u} + \underline{a}t)dt = \int \frac{d}{dt}\underline{s}dt \rightarrow \underline{u}t + \frac{1}{2}\underline{a}t^2 + \underline{c} = \underline{s}$$

We find the constant of integration $\underline{c}$ by setting $t = 0 \rightarrow \underline{c} = \underline{s}_0$

This gives us $\underline{u}t + \frac{1}{2}\underline{a}t^2 + \underline{s}_0 = \underline{s}$, which is often reorganised as

$$\boxed{\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2} \quad 1.2$$

where we are now using $\underline{s}_0 = 0$ for the initial position.

This equation contains no $\underline{v}$, so we use it when $\underline{v}$ is unknown and unwanted

Eliminating The Unknowns : *I don't know a !*

Eliminate a by rearranging [1]: Sub into [2]:

$\boxed{v = u + at \ [1]}$

$\boxed{s = ut + \frac{1}{2}at^2 \ [2]}$

$$s = ut + \frac{1}{2}[(v-u)/t]t^2$$
$$= ut + \frac{1}{2}[(v-u)]t$$

$\boxed{s = \frac{1}{2}(v+u)t}$ 1.3. contains no a, so we use it when a is unknown

Eliminating The Unknowns : *I don't know t !*

Eliminate t by rearranging [1]: $t = (v - u)/a$

Sub into [2]:

$$\begin{aligned} s &= u[(v-u)/a] + \frac{1}{2}a[(v-u)/a]^2 \\ &= uv/a - u^2/a + \frac{1}{2}a(v^2 + u^2 - 2uv)/a^2 \\ &= uv/a - u^2/a + \frac{1}{2}v^2/a + \frac{1}{2}u^2/a - \frac{1}{2}2uv)/a \\ &= -u^2/a + \frac{1}{2}v^2/a + \frac{1}{2}u^2/a \end{aligned}$$

$\boxed{v = u + at \ [1]}$

$\boxed{s = ut + \frac{1}{2}at^2 \ [2]}$

$\boxed{s = \frac{1}{2}(v^2 + u^2)/a}$ 1.4

contains no t, so we use it when t is unknown

Eliminating The Unknowns : *I don't know u !*

Eliminate u by rearranging [1]: $u = v - at$

Sub into [2]:

$$\begin{aligned} s &= [v - at]t + \frac{1}{2}at^2 \\ &= vt - at^2 + \frac{1}{2}at^2 \end{aligned}$$

$\boxed{v = u + at \ [1]}$

$\boxed{s = ut + \frac{1}{2}at^2 \ [2]}$

$\boxed{s = vt - \frac{1}{2}at^2}$ **1.5**

contains no u, so we use it when u is unknown

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Freefall Acceleration

We will talk about gravity later. For now:

Acceleration due to gravity $g = -9.81ms^{-2}$

This is an approximation: g actually varies. But we treat it as a constant, and it is **constant wrt to time**.

For problems on and near the Earth's surface, we can use g for a in suvat: handy.

The fastest creature on earth is supposedly the Peregrine Falcon, which can 'fly' (actually dive) at 389 kmh^{-1}.

What is this in ms^{-1}?

$389km/h = 3.89 \times 10^2 km/h = 3.89 \times 10^5 m/h = 3.89 \times 10^5 m/3600s = 108m/s$

Freefall Acceleration

How fast could I 'fly' if dropped from a height of 1km?

$u = 0$ ms^{-1}
$s = 10^3$ m
$a = -9.81$ ms^{-2}
Which variable is both unknown and unwanted?

$s = \frac{1}{2}(v^2 + u^2)/a$
$2as = v^2 + u^2$
$v = \sqrt{2as - u^2}$
$|v| = \sqrt{2 * 9.81 * 1000} = 140m/s$
So why am I not listed on wikipedia as the fastest creature on Earth?

If there is no air resistance

In the absence of any external forces, all objects fall towards the centre of the planet with $a = g = 9.81\ \mathrm{ms}^{-2}$.

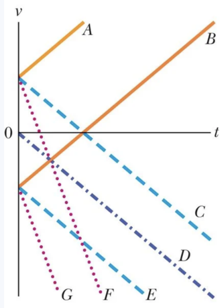
This can seem *counter-intuitive* because it is something we can almost never observe.

We are all accelerating at g right now. The normal force between us and Earth is matching that acceleration in the opposite direction.

Could I accelerate at g forever, if there were no atmosphere or other stars, planets?

Poll Everywhere Checkpoint

You are standing on a bridge with two eggs. You drop one, and you throw the other directly downwards.



Use your phone to go to: pollev.com/ilovephysics

(a) Which line best describes the motion of the dropped egg?

(b) Which line best describes the motion of the thrown egg?

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Before next lecture

Retry the pre-lecture quiz 1.2 Equations of Motion, if you like.

Attempt the pre-lecture quiz for 1.3 Graphical Analysis.