Mechanics & Relativity

Topic 1: Kinematics

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28-30 September 2021 (Week 1)

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Kinematics

This week's topics:

1.1 Displacement, Velocity & Acceleration

1.2 Equations of motion (SUVAT)

1.3 Reading graphs

Notation

s : for position (or sometimes x, or y, or r...)
u : for (magnitude of) initial velocity (aka initial speed)
v : for (magnitude of) velocity (aka speed)
a : for (magnitude of) acceleration
t : for time (is time a vector?)

Δ : means *'change in'*

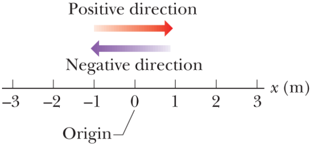
δ or d : means *'teeny weeny change in'*

Standard units of displacement in space and time are metres and seconds, unless otherwise stated.

Displacement

To begin to talk about where something is, let alone where it is headed, we need three things:

1. A coordinate system, *eg* cartesian coordinates (x, y, z).
2. A reference point: **the origin**.
3. A positive direction.



We can then define the displacement as **the change in position:**

$$\Delta s = s_f - s_i \quad (1.1)$$

Poll everywhere checkpoint

Here are three pairs of initial and final positions: $[s_i, s_f]$ along an x axis. Which pairs give a *negative displacement* Δs?

(a) [-3 m, +5 m]
(b) [-3 m, -7 m]
(c) [7 m, -3 m]

Use your phone to go to: pollev.com/ilovephysics and select the option:
A : a & b give negative displacements
B : a & c
C : b & c

Don't panic, these polls are always anonymous!

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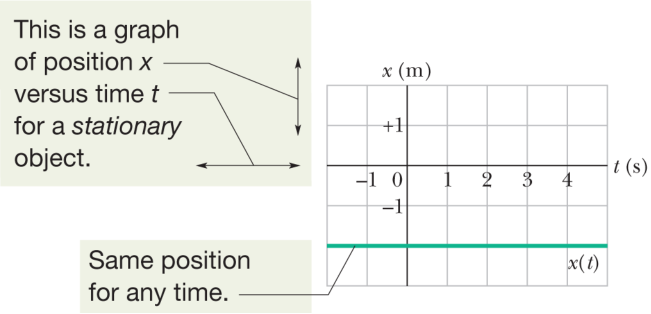
Displacement

An object may be motionless in space, but it will always move through time.

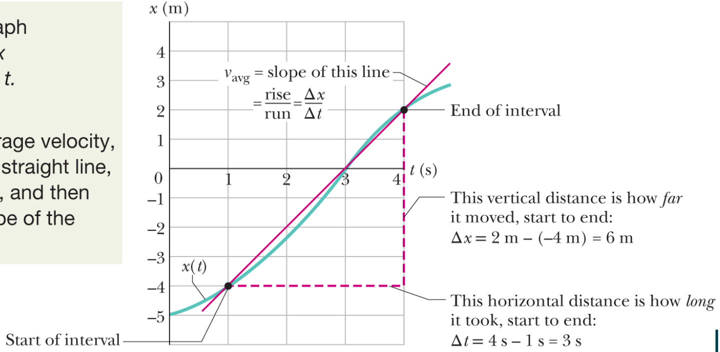


Average velocity

To find the average velocity, we divide the total distance by the total time.

This is a graph of position x versus time t.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



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Solving a problem of this sort

You drive along a straight road for 8.4 km at 70 km/h, at which point your car runs out of petrol and stops. Over the next 30 min, you walk another 2.0 km along the road to a petrol station.

What are your overall displacement, time taken, and average speed from the beginning of your drive to your arrival at the station?

Instantaneous velocity

We can define the velocity (and acceleration) either as average or as instantaneous.

The average velocity and acceleration over a period of time given by Δt is:

$$\underline{v}_{avg} = \frac{\Delta \underline{s}}{\Delta t}; \quad \underline{a}_{avg} = \frac{\Delta \underline{v}}{\Delta t} \qquad 1.2$$

The instantaneous velocity and acceleration at an exact moment in time is:

$$\underline{v} = \frac{\delta}{\delta t}\underline{s}; \quad \underline{a} = \frac{\delta}{\delta t}\underline{v} = \frac{\delta^2}{\delta t^2}\underline{s} \qquad 1.3$$

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A bit more notation / reminder of calculus...

$\frac{ds}{dt}$: the differential of position With Respect To (wrt) time.

$\frac{d^2s}{dt^2}$: the second differential of position wrt time.

Example & Notation:

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Checkpoint

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$):

(1) $x = 3t - 2$

(2) $x = -4t^2 - 2$

(3) $x = \frac{2}{t^2}$

(4) $x = -2$

(a) In which situation(s) is the velocity v of the particle constant?

(b) In which is v in the negative x direction?

Acceleration

Acceleration usually means 'speeding up' in normal conversation. In physics it also means 'slowing down'.

If something has a changing speed, then its acceleration is non-zero

Which of these positions as a function of time correspond to constant acceleration?

$x = 4t^3 - 55$:

$x = 4t^2 - 55$:

$x = 4t - 55$:

$x = 4/t - 55$:

$x = 4/t^2 - 55$:

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Before next lecture

Retry the pre-lecture quiz 1.1 Velocity and Acceleration, if you like.

Attempt the pre-lecture quiz for 1.2 Equations of Motion.

See you tomorrow morning for lecture 1.2

suvat

s : for position
u : for (magnitude of) **initial** velocity (aka initial speed)
v : for (magnitude of) velocity (aka speed)
a : for (magnitude of) acceleration
t : for time

We will consider situations where displacement s and velocity v are functions of time t.

We cannot use suvat for situations where the acceleration is not constant.

It is convenient to memorise certain formulae for calculating one of these things when others are known, such as

$\underline{v} = \underline{u} + \underline{a}t$

But where does this come from?

How do we know v= u + at ?

The first equation of motion for translational motion is derived starting from the definition for acceleration as the change in velocity with time:

$$\underline{a} = \frac{d}{dt}\underline{v} \rightarrow \int \underline{a}dt = \int \frac{d}{dt}\underline{v}dt \rightarrow \underline{a}t + \underline{c} = \underline{v}$$

We can write $\int \underline{a}dt = \underline{a}t$ because a is constant: not a function of time.

We find the constant of integration $\underline{c}$ by setting $t = 0 \rightarrow \underline{c} = \underline{v}_0$.

This gives us $\underline{a}t + \underline{v}_0 = \underline{v}$, which is often reorganised as

$\underline{v} = \underline{u} + \underline{a}t$ **1.4** where we are now using $\underline{v}_0 \rightarrow \underline{u}$ for the **initial velocity**.

This equation contains no $\underline{s}$, so we use it when $\underline{s}$ is unknown and unwanted

How about $s = ut + \frac{1}{2}at^2$?

The second equation of motion is derived starting from the definition for velocity:

$$\underline{v} = \frac{d}{dt}\underline{s} \rightarrow \int \underline{v}dt = \int \frac{d}{dt}\underline{s}dt$$

Now we have to substitute for $\underline{v} = \underline{u} + \underline{a}t$ (suvat 1) in the LHS because unlike $\underline{a}$ (which has to be constant for these equations to work) $\underline{v}$ **can be a function of time**:

$$\int (\underline{u} + \underline{a}t)dt = \int \frac{d}{dt}\underline{s}dt \rightarrow \underline{u}t + \frac{1}{2}\underline{a}t^2 + \underline{c} = \underline{s}$$

We find the constant of integration $\underline{c}$ by setting $t = 0 \rightarrow \underline{c} = \underline{s}_0$

This gives us $\underline{u}t + \frac{1}{2}\underline{a}t^2 + \underline{s}_0 = \underline{s}$, which is often reorganised as

$\boxed{\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2}$ **1.5** where we are now using $\underline{s}_0 = 0$ for the initial position.

This equation contains no $\underline{v}$, so we use it when $\underline{v}$ is unknown and unwanted

Eliminating The Unknowns : *I don't know a !*

Eliminate a by rearranging [1]: $a = (v - u)/t$

Sub into [2]:

$$\boxed{v = u + at \; [1]}$$

$$\boxed{s = ut + \tfrac{1}{2}at^2 \; [2]}$$

$$\begin{aligned} s &= ut + \frac{1}{2}[(v - u)/t]t^2 \\ &= ut + \frac{1}{2}[(v - u)]t \end{aligned}$$

$s = \frac{1}{2}(v + u)t$ 1.6 . contains no a, so we use it when a is unknown

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Eliminating The Unknowns : *I don't know t !*

Eliminate t by rearranging [1]: $t = (v - u)/a$

Sub into [2]:

$$
\begin{aligned}
s &= u[(v-u)/a] + \frac{1}{2}a[(v-u)/a]^2 \\
&= uv/a - u^2/a + \frac{1}{2}a(v^2 + u^2 - 2uv)/a^2 \\
&= uv/a - u^2/a + \frac{1}{2}v^2/a + \frac{1}{2}u^2/a - \frac{1}{2}2uv)/a \\
&= -u^2/a + \frac{1}{2}v^2/a + \frac{1}{2}u^2/a
\end{aligned}
$$

$\boxed{v = u + at \ [1]}$

$\boxed{s = ut + \frac{1}{2}at^2 \ [2]}$

$s = \frac{1}{2}(v^2 + u^2)/a$ **1.7**

contains no t, so we use it when t is unknown

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Eliminating The Unknowns : *I don't know u !*

Eliminate u by rearranging [1]: $u = v - at$

Sub into [2]:

$$s = [v - at]t + \frac{1}{2}at^2$$
$$= vt - at^2 + \frac{1}{2}at^2$$

$$\boxed{v = u + at \ [1]}$$

$$\boxed{s = ut + \tfrac{1}{2}at^2 \ [2]}$$

$s = vt - \frac{1}{2}at^2$ **1.8**

contains no u, so we use it when u is unknown

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Freefall Acceleration

We will talk about gravity later. For now:

Acceleration due to gravity $g = -9.81 ms^{-2}$

This is an approximation: g actually varies. But we treat it as a constant, and it is **constant wrt to time**.

For problems on and near the Earth's surface, we can use g for a in suvat: handy.

The fastest creature on earth is supposedly the Peregrine Falcon, which can 'fly' (actually dive) at 389 kmh^{-1}.

What is this in ms^{-1}?

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Freefall Acceleration

How fast could I ‘fly’ if dropped from a height of 1km?
$u = 0$ ms^{-1}
$s = 10^3$ m
$a = -9.81$ ms^{-2}
Which variable is both unknown and unwanted?

$s = \frac{1}{2}(v^2 + u^2)/a$
$2as = v^2 + u^2$
$v = \sqrt{2as - u^2}$
$|v| = \sqrt{2 * 9.81 * 1000} = 140 m/s$
So why am I not listed on wikipedia as the fastest creature on Earth?

Solving a problem of this sort

A car traveling 56.0 $\mathrm{kmh^{-1}}$ is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later.

(a) What is the magnitude of the car's constant acceleration before impact?

(b) How fast is the car traveling at impact?

If there is no air resistance

In the absence of any external forces, all objects fall towards the centre of the planet with $a = g = 9.81$ ms^{-2}.

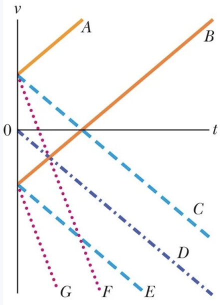
This can seem *counter-intuitive* because it is something we can almost never observe.

We are all accelerating at g right now. The normal force of the Earth is matching that acceleration in the opposite direction.

Could I accelerate at g forever, if there were no atmosphere or other stars, planets?

Poll Everywhere Checkpoint

You are standing on a bridge with two eggs. You drop one, and you throw the other directly downwards.



Use your phone to go to: pollev.com/ilovephysics

(a) Which line best describes the motion of the dropped egg?

(b) Which line best describes the motion of the thrown egg?

Before next lecture

Retry the pre-lecture quiz 1.2 Equations of Motion, if you like.

Attempt the pre-lecture quiz for 1.3 Graphical Analysis.

Integrating acceleration over time

We know that $a = \frac{dv}{dt}$

$$\int_{t_0}^{t} a dt = \int_{t_0}^{t} \frac{dv}{dt} dt$$
$$= \int_{t_0}^{t} dv$$
$$= v_t - v_{t0}$$

$$\int_{t_0}^{t} a dt = v_t - v_{t0} \qquad (1.9)$$

The integral of the acceleration over time gives the change in velocity

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Integrating velocity over time

Similarly $v = \frac{ds}{dt}$

$$\int_{t_0}^{t} v dt = \int_{t_0}^{t} \frac{ds}{dt} dt$$
$$= \int_{t_0}^{t} ds$$
$$= s_t - s_{t0}$$

$$\int_{t_0}^{t} v dt = s_t - s_{t0} \qquad 1.10$$

The integral of the velocity over time gives the change in position

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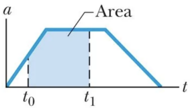
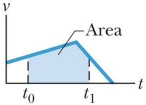
The area under a curve



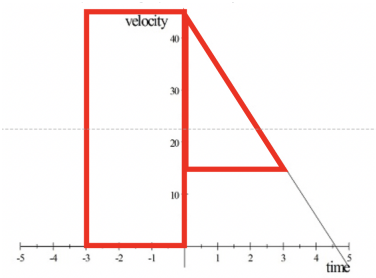
(a)

This area gives the change in velocity.

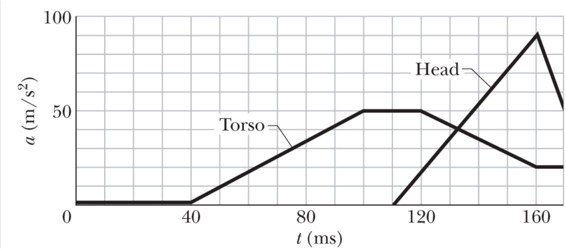


(b)

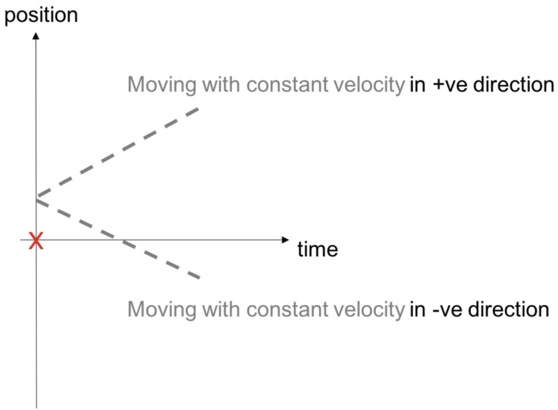
This area gives the change in position.



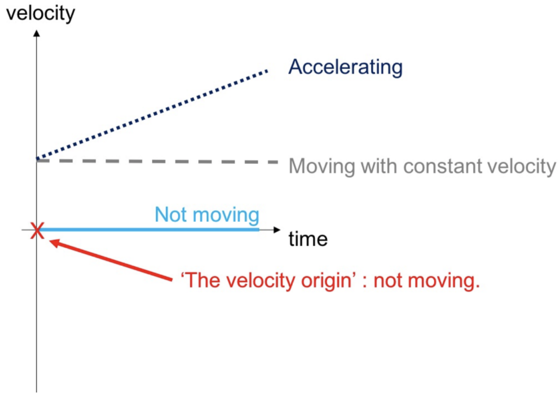
A example: whiplash curve



Tackling confusion with directions

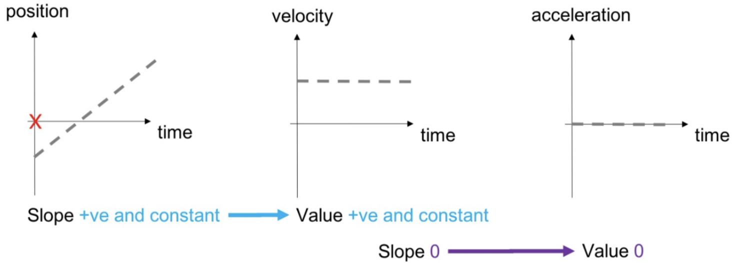


Zero velocity is constant velocity



Matching slopes to values

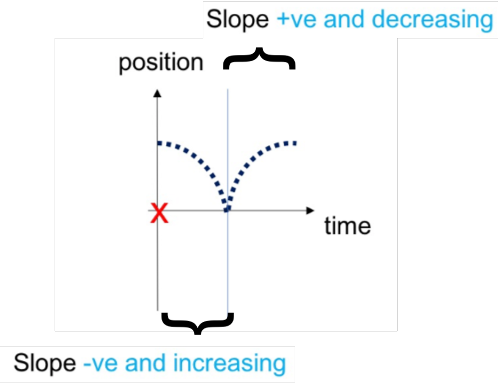
Example: *A fire engine passes me (x) at 50 mph*



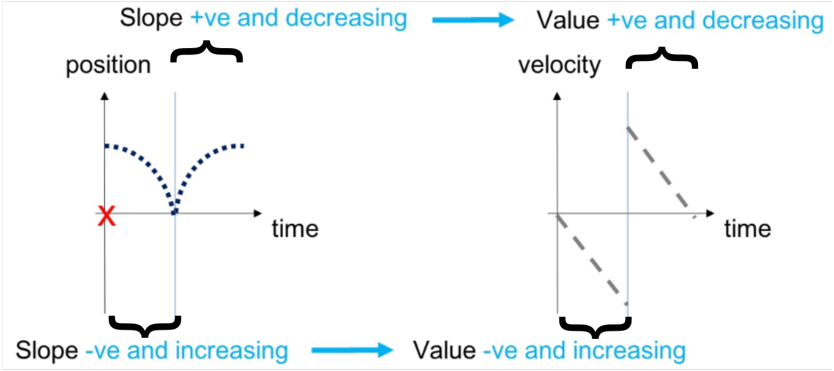
A bouncing ball

How would we draw the motion of a ball being dropped from height, reaching ground, and bouncing back up again?

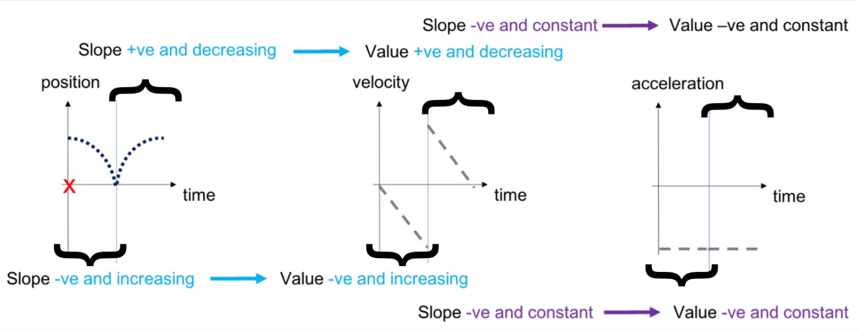
Divide & Conquer



Divide & Conquer

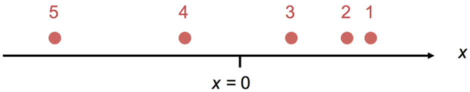


Divide & Conquer

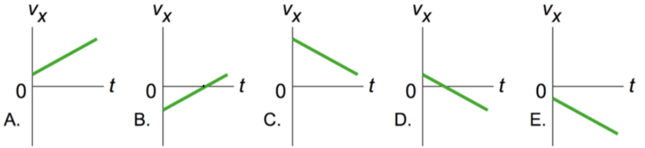


Poll Everywhere Checkpoint (pollev.com/ilovephysics)

This is a motion diagram of an object moving along the *x*-direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt starting at t=0 s.



Which of the following v_x against t graphs best matches the motion shown in the motion diagram?

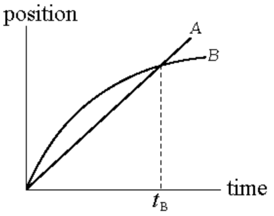


Reading graphs: additional problem

Concept Question: Instantaneous Velocity

The graph shows the position as a function of time for two trains running on parallel tracks. For times greater than $t = 0$, which of the following is true:

1. At time t_B, both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity at some time before t_B, .
4. Somewhere on the graph, both trains have the same acceleration.



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Preparing for next week's Kinematics Workshop

- The problems for the workshop are on Canvas.
- Have a go at the problems prior to your workshop, so you know in advance what you would like the DT's help with.
- Each workshop has a question marked out as the one you will be graded on.
- Upload your solution to this before the end of next week.

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Tips

- You will need to use the quadratic formula at least once, so remind yourself what that is and when it is useful.
- What information is given in the question? Write it down.
- What information is known but not given? Write it down.
- Underline or draw a box around your answer and take a photo of it (including working) and upload it to canvas.