Please upload your solution to Problem 3 to canvas for marking after the workshop.

Problem 1

Write an expression for a displacement vector $\underline{\mathbf{r}}$ which is in the x, y plane, has length 1.9 cm, and is at an angle 71° from the x-axis.

Problem 2

Vector $\underline{\alpha}$, which is directed along an x-axis, is to be added to vector $\underline{\beta}$, which has a magnitude of 7 m. The sum is a third vector that is directed along the y-axis, with a magnitude that is 3 times that of $\underline{\alpha}$. What is that magnitude of $\underline{\alpha}$?

Problem 3

A vector product $\underline{\mathbf{P}} = a\underline{\mathbf{B}} \times \underline{\mathbf{C}}$, where $a = 2$, $\underline{\mathbf{B}} = 2\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}} + 6\hat{\boldsymbol{k}}$ and $\underline{\mathbf{C}} = 4\hat{\boldsymbol{\imath}} - 20\hat{\boldsymbol{\jmath}} + 12\hat{\boldsymbol{k}}$.

What is $\underline{\mathbf{P}}$ in unit vector notation ?

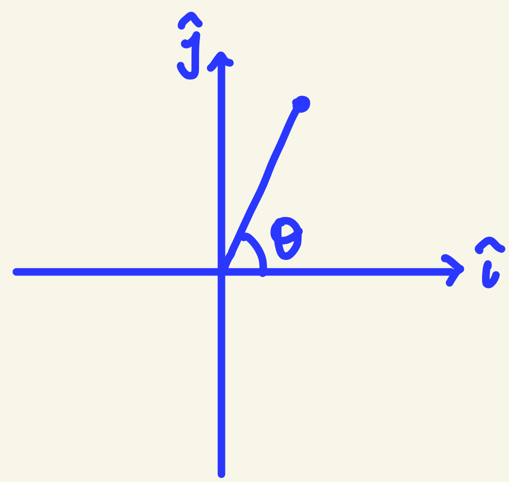
Problem 1

Write an expression for a displacement vector $\underline{\mathbf{r}}$ which is in the x, y plane, has length 1.9 cm, and is at an angle $71°$ from the x-axis.

$\underline{r} = r_x \hat{i} + r_y \hat{j}$ [is in the x, y plane]

$|\underline{r}| = 1.9\,cm$ [length = magnitude]

$\theta = 71°$



$$r_x = r\cos\theta$$
$$= (1.9\,cm)\cos(71°)$$
$$= 0.62\,cm$$

$$r_y = r\sin\theta$$
$$= (1.9\,cm)\sin(71°)$$
$$= 1.8\,cm$$

$$\boxed{\underline{r} = 0.62\,cm\ \hat{i} + 1.8\,cm\ \hat{j}}$$

Problem 2

Vector $\underline{\alpha}$, which is directed along an x-axis, is to be added to vector $\underline{\beta}$, which has a magnitude of 7 m. The sum is a third vector that is directed along the y-axis, with a magnitude that is 3 times that of $\underline{\alpha}$. What is that magnitude of $\underline{\alpha}$?

$$\underline{\alpha} = \alpha_x \hat{i} = \alpha \hat{i}$$

$$|\underline{\beta}| = 7m$$

$$\underline{\gamma} = \underline{\alpha} + \underline{\beta} = \gamma_y \hat{j} = \gamma \hat{j}$$

$$\gamma = 3\alpha$$

$$\underline{\gamma} = \underbrace{(\alpha_x + \beta_x)}_{\text{must} = 0} \hat{i} + \underbrace{(\alpha_y + \beta_y)}_{\text{must} = \gamma} \hat{j} = \gamma \hat{j}$$

$$\alpha_x + \beta_x = 0 \quad \therefore \beta_x = -\alpha_x = -\alpha$$

$$\alpha_y + \beta_y = \gamma \quad \text{and} \quad \alpha_y = 0 \quad \therefore \beta_y = \gamma = 3\alpha$$

$$\underline{\beta} = -\alpha \hat{i} + 3\alpha \hat{j}$$

$$|\underline{\beta}| = \sqrt{(-\alpha)^2 + (3\alpha)^2} = \sqrt{10\alpha^2} = 7m$$

So, $$\boxed{\alpha = \frac{7\,m}{\sqrt{10}} = 2m}$$

Problem 3

A vector product $\underline{\mathbf{P}} = a\underline{\mathbf{B}} \times \underline{\mathbf{C}}$, where $a = 2$, $\underline{\mathbf{B}} = 2\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} + 6\hat{\boldsymbol{k}}$ and $\underline{\mathbf{C}} = 4\hat{\boldsymbol{i}} - 20\hat{\boldsymbol{j}} + 12\hat{\boldsymbol{k}}$.
What is $\underline{\mathbf{P}}$ in unit vector notation ?

$$\underline{P} = a\underline{B} \times \underline{C} = (a\underline{B}) \times \underline{C}$$

$$a\underline{B} = 2 \times (2\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= 4\hat{i} + 8\hat{j} + 12\hat{k}$$

$$a\underline{B} \times \underline{C} = (4\hat{i} + 8\hat{j} + 12\hat{k}) \otimes (4\hat{i} - 20\hat{j} + 12\hat{k})$$

$$= \begin{array}{c} \begin{matrix} y & z \end{matrix} \\ \begin{pmatrix} 8 & 12 \\ -20 & 12 \end{pmatrix} \end{array} \hat{i} + \begin{array}{c} \begin{matrix} z & x \end{matrix} \\ \begin{pmatrix} 12 & 4 \\ 12 & 4 \end{pmatrix} \end{array} \hat{j} + \begin{array}{c} \begin{matrix} x & y \end{matrix} \\ \begin{pmatrix} 4 & 8 \\ 4 & -20 \end{pmatrix} \end{array} \hat{k}$$

$$= [(8 \times 12) - (12 \times -20)]\hat{i} + \cancelto{=0}{[(12 \times 4) - (4 \times 12)]}\,\hat{j}$$

$$+ [(4 \times -20) - (8 \times 4)]\,\hat{k}$$

$$\boxed{\underline{P} = 336\hat{i} - 112\hat{k}}$$