Mechanics & Relativity

Dr Lily Asquith (Lily)

Week 2

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Submit your workshop problem for marking by noon Friday

Y1 : Submitting Your Work For Marking

1. Identify the problem you have been asked to upload the solution for. For M&R, this is indicated with red text and a box around the question:

Problem 3

Two particles move along an x axis. The position of particle 1 is given by $x_1 = 6.00t^2 + 3.00t + 2.00$; the acceleration of particle 2 is given by $a_2 = -8.00t$ and, at $t = 0$, its velocity is $v_2 = 20ms^{-1}$. When the velocities of the particles match, what is their velocity?

For MM1, it is indicated with a key symbol.

3. Suppose that $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = \mathbf{j} + 3\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{k}$. Verify that

(a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$,

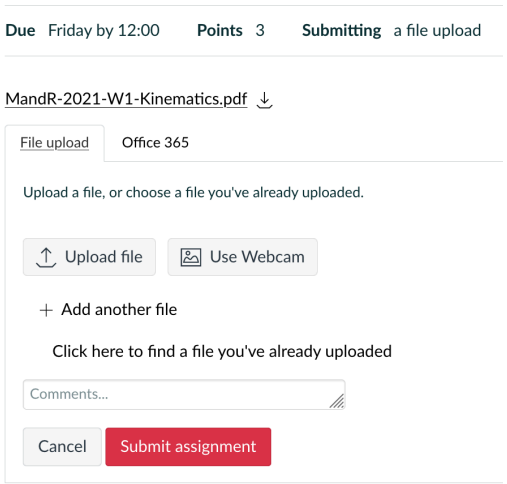
(b) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$,

(c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$.

2. Take a photo of your full worked solution for this problem (only this one).

3. Upload the file to canvas before 12 noon on Friday.

The upload will automatically be associated with you – no need to put your name or candidate number on it.



0 : Nothing uploaded

1 : Something uploaded, but nothing that makes sense. If they have uploaded a blank page for example, or if they have written the question but not attempted an answer. Or, if they uploaded just the answer, with no working.

2 : An attempt has been made to solve the question, but it is not complete. For example (for the first M&R question), they might have done the integral right but not solved the quadratic, or vice versa.

3 . The correct answer is given.

4. Await your grade. You will be awarded a mark out of 3 (within the following week, but probably very quickly.

The workshop problems (1 per week for 10 weeks) = 10% of your final mark

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Adaptive practice

There are three adaptive practice assignments for this week: 2.1, 2.2, 2.3.

The adaptive practice quizzes (3 per week for 11 weeks) = 10% of your final mark.

But more importantly, they are exactly what you need to do to get a grasp of the topics covered in this module.

The reason for this module is to prepare you for what is next.

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Vectors

This week's topics:

2.1 Intro to vectors for physics

2.2 Addition

2.3 Multiplication

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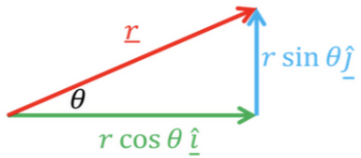
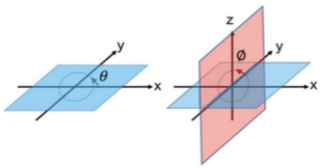
Key formulae for the toilet door

	1.1 Unit vector notation	$\underline{\mathbf{a}} = a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}$ $\lvert\underline{\mathbf{a}}\rvert = a = \sqrt{a_x^2 + a_y^2 + a_z^2}$ $\lvert\hat{\boldsymbol{i}}\rvert = 1$ 
	1.2 Components	$a_x = a\cos\theta$ $a_y = a\sin\theta$ $a_z = a\sin\phi$ $\theta = \arctan\frac{a_y}{a_x}$ $\phi = \arctan\frac{a_z}{a_y}$ 
	1.3 Addition / subtraction	$\underline{\mathbf{a}} + \underline{\mathbf{b}} = \underline{\mathbf{b}} + \underline{\mathbf{a}} = (a_x + b_x)\hat{\boldsymbol{i}} + (a_y + b_y)\hat{\boldsymbol{j}} + (a_z + b_z)\hat{\boldsymbol{k}}$
	1.4 The dot product	$\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = ab\cos\gamma$ $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = (a_x)(b_x) + (a_y)(b_y) + (a_z)(b_z)$
	1.5 The cross product	$\underline{\mathbf{a}} \times \underline{\mathbf{b}} = ab\sin\lambda\,\hat{\boldsymbol{n}}$ $\underline{\mathbf{a}} \times \underline{\mathbf{b}} = ((a_y)(b_z) - (a_z)(b_y))\,\hat{\boldsymbol{i}} + ((a_z)(b_x) - (a_x)(b_z))\,\hat{\boldsymbol{j}} + ((a_x)(b_y) - (a_y)(b_x))\,\hat{\boldsymbol{k}}.$ $\underline{\mathbf{a}} \times \underline{\mathbf{b}} = \begin{pmatrix} a_y & a_z \\ b_y & b_z \end{pmatrix}\hat{\boldsymbol{i}} + \begin{pmatrix} a_x & a_z \\ b_x & b_z \end{pmatrix}\hat{\boldsymbol{j}} + \begin{pmatrix} a_x & a_y \\ b_x & b_y \end{pmatrix}\hat{\boldsymbol{k}}$

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Why Vectors?

The components of a vector

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A pinch of trigonometry

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Which way is up?

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Unit vector notation

Worked example 1 (Ch 3 problem 1)

What are (a) the x component and (b) the y component of a vector in the xy plane if its direction is 250° counterclockwise from the positive direction of the x axis and its magnitude is 7.3 m?

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Worked example 2 (Ch 3 problem 7)

Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude 7 m, 1 m, and 5 m.

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Adding vectors

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Worked example 3

Two vectors are given:

What is $\underline{a} + \underline{b}$?

What is $\underline{a} - \underline{b}$?

What is the magnitude of $(\underline{a} + \underline{b})$?

What is the angle of $\underline{a}$ wrt the positive $\hat{\boldsymbol{i}}$ direction?

What is the angle between $\underline{a}$ and $\underline{b}$?

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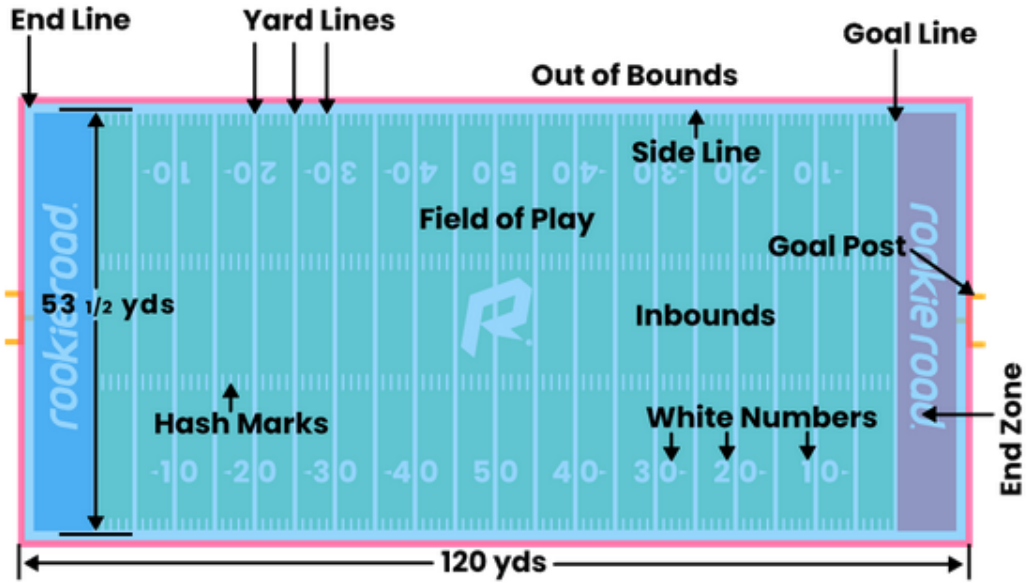
Adaptive practice quiz question from 2.1

An advertising blimp is directly over a football stadium. The copilot is watching the game. At t = 0 s, the copilot sees the following: A running back and a defensive back are both at the same yard line, with the running back near the sideline and the defensive back 5.00 m closer to midfield. The defensive back starts at rest, while the running back is running at 5.80 m/s parallel to the sideline. If the defensive back has 5.50 seconds to catch the running back just before the goal line and travels in a straight line, what is the magnitude of acceleration that he must have in order to make the tackle? _____ m/s^2



Football Field Diagram

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Vector Multiplication

The dot product

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Worked Example 4 (Ch 3 problem 39)

Vector $\underline{a}$ has a magnitude of 6.00 units, vector $\underline{b}$ has a magnitude of 7.00 units, and $\underline{a} \cdot \underline{b}$ has a value of 14.0. What is the angle between the directions of $\underline{a}$ and $\underline{b}$?

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The cross product : spanner example

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The cross product : graphically

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The cross product : Lily's cartoon matrices

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Recap soh-cah-toa

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Recap unit vector notation, components

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Recap vector multiplication

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Problem solving tools

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Problem 1: What makes sense?

13 Which of the following are correct (meaningful) vector expressions? What is wrong with any incorrect expression?

(a) $\vec{A} \cdot \left(\vec{B} \cdot \vec{C}\right)$

(b) $\vec{A} \times \left(\vec{B} \cdot \vec{C}\right)$

(c) $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right)$

(d) $\vec{A} \times \left(\vec{B} \times \vec{C}\right)$

(e) $\vec{A} + \left(\vec{B} \cdot \vec{C}\right)$

(f) $\vec{A} + \left(\vec{B} \times \vec{C}\right)$

(g) $5 + \vec{A}$

(h) $5 + \left(\vec{B} \cdot \vec{C}\right)$

(i) $5 + \left(\vec{B} \times \vec{C}\right)$

(j) $\left(\vec{A} \cdot \vec{B}\right) + \left(\vec{B} \times \vec{C}\right)$

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Problem 2: The Dreaded Cube

•••32 In Fig. 3-31, a cube of edge length a sits with one corner at the origin of an xyz coordinate system. A *body diagonal* is a line that extends from one corner to another through the center. In unit-vector notation, what is the body diagonal that extends from the corner at (a) coordinates (0, 0, 0), (b) coordinates (a, 0, 0), (c) coordinates (0, a, 0), and (d) coordinates (a, a, 0)? (e) Determine the angles that the body diagonals make with the adjacent edges. (f) Determine the length of the body diagonals in terms of a.

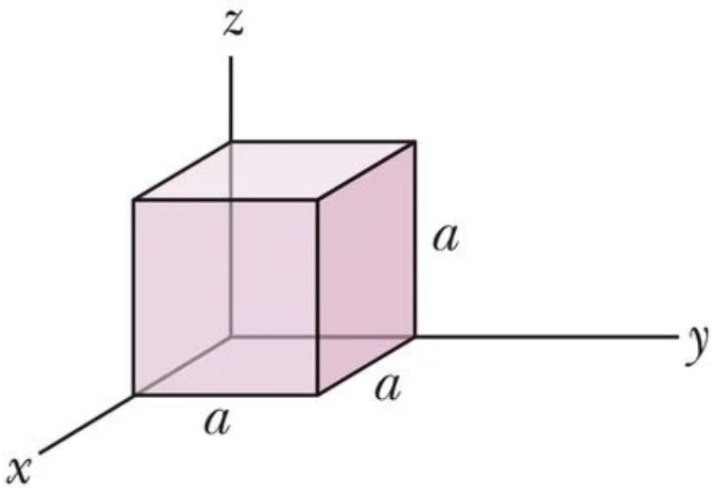

Figure 3-31 Problem 32.

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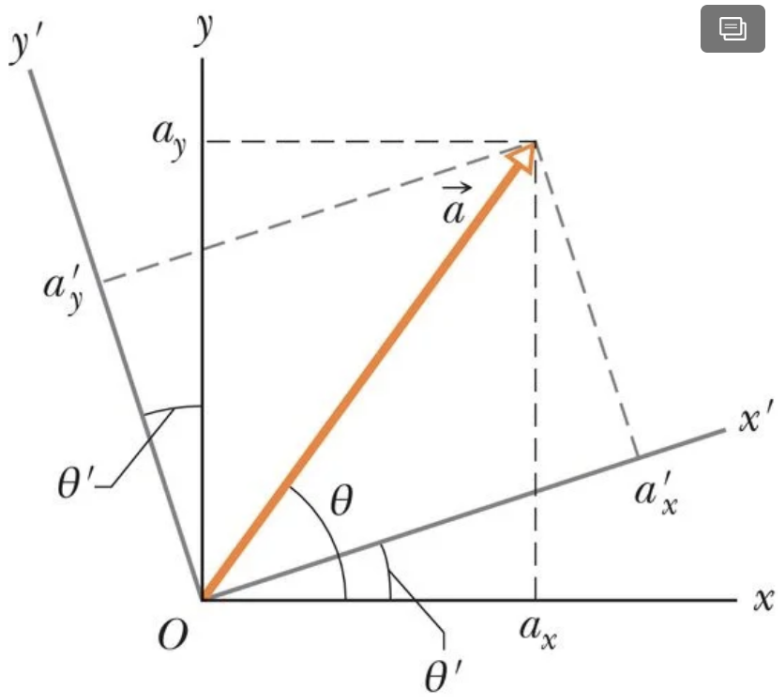
Problem 3: Messing with the coordinates

••31 In Fig. 3-30, a vector $\vec{a}$ with a magnitude of 17.0 m is directed at angle $\theta = 56.0°$ counterclockwise from the $+x$ axis. What are the components (a) a_x and (b) a_y of the vector? A second coordinate system is inclined by angle $\theta' = 18.0°$ with respect to the first. What are the components (c) a'_x and (d) a'_y in this primed coordinate system?



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Problem 4: The 'Furious Product'

78 What is the magnitude of $\vec{a} \times \left(\vec{b} \times \vec{a} \right)$ if $a = 3.90$, $b = 2.70$, and the angle between the two vectors is 63.0°?

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