Mechanics & Relativity

Dr Lily Asquith (Lily)

28-30 September 2021 (Week 1)

UNIVERSITY OF SUSSEX

Kinematics

This week's topics:

1.1 Displacement, Velocity & Acceleration

1.2 Equations of motion (SUVAT)

1.3 Reading graphs

Notation

S : scalar

$\underset{\sim}{s}$ or $\bar{s}$ or $\vec{s}$: vector

s : for position (or sometimes x, or y, or r...)
u : for (magnitude of) initial velocity (aka initial speed) v_0
v : for (magnitude of) velocity (aka speed)
a : for (magnitude of) acceleration
t : for time (is time a vector?)

Δ : means *'change in'* "Delta"

δ or d : means *'teeny weeny change in'* "d"

Standard units of displacement in space and time are metres and seconds, unless otherwise stated.

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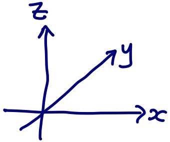
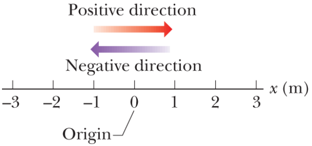
Displacement

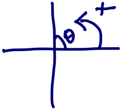
To begin to talk about where something is, let alone where it is headed, we need three things:

Rene Descartes

1. A coordinate system, *eg* cartesian coordinates (x, y, z).
2. A reference point: **the origin**.
3. A positive direction.







We can then define the displacement as **the change in position:**

$$\Delta s = s_f - s_i \quad (1.1)$$

S_f : final

S_i : initial

Poll everywhere checkpoint

Here are three pairs of initial and final positions: $[s_i, s_f]$ along an x axis. Which pairs give a *negative displacement* Δs?

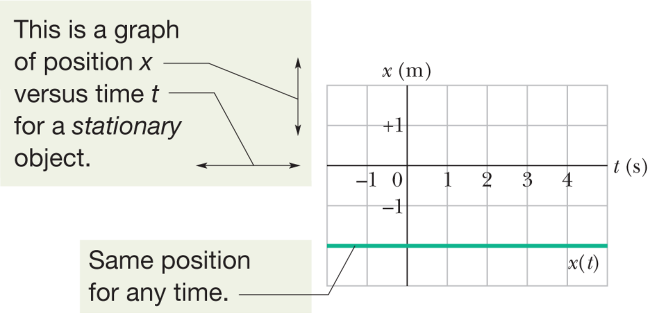
(a) [-3 m, +5 m]
(b) [-3 m, -7 m]
(c) [7 m, -3 m]

Use your phone to go to: pollev.com/ilovephysics and select the option:
A : a & b give negative displacements
B : a & c
C : b & c

Don't panic, these polls are always anonymous!

Displacement

An object may be motionless in space, but it will always move through time.

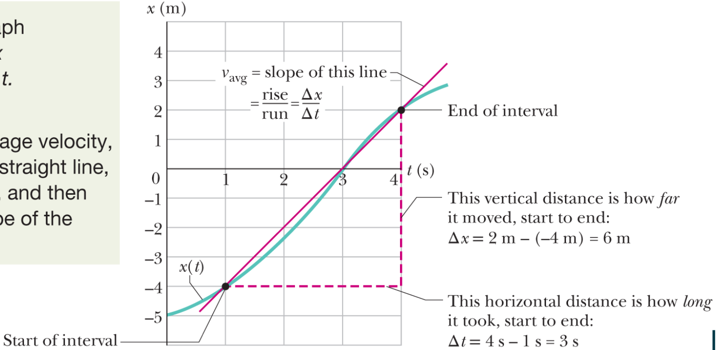


Average velocity

To find the average velocity, we divide the total distance by the total time.

This is a graph of position x versus time t.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



Solving a problem of this sort

$v = \frac{s}{t}$

You drive along a straight road for 8.4 km at 70 km/h, at which point your car runs out of petrol and stops. Over the next 30 min, you walk another 2.0 km along the road to a petrol station.

What are your overall displacement, time taken, and average speed from the beginning of your drive to your arrival at the station?

A

$s = 8.4$ km

$v = 70$ kmh^{-1}

$t = 0.12$

$70\ \text{km}h^{-1} = \frac{8.4\ \text{km}}{t}$

$t = \frac{8.4}{70} = 0.12h$

B

$s = 2$ km

$v = 4$

$t = 30$ min $= 0.5$ hours

$v = \frac{2}{0.5} = 4$ km/h

$S_{TOT} = 10.4$ km

$t_{TOT} = 0.62\ h$

$V_{TOT} = \frac{S_{TOT}}{T_{TOT}}$

$= \frac{10.4\ \text{km}}{0.62\ h}$

$= 16.8\ \text{km}h^{-1}$

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Instantaneous velocity

We can define the velocity (and acceleration) either as average or as instantaneous.

The average velocity and acceleration over a period of time given by Δt is:

$$\underline{v}_{avg} = \frac{\Delta \underline{s}}{\Delta t}; \quad \underline{a}_{avg} = \frac{\Delta \underline{v}}{\Delta t} \qquad 1.2$$

The instantaneous velocity and acceleration at an exact moment in time is:

$$\underline{v} = \frac{\delta}{\delta t}\underline{s}; \quad \underline{a} = \frac{\delta}{\delta t}\underline{v} = \frac{\delta^2}{\delta t^2}\underline{s} \qquad 1.3$$

$$v = \frac{\partial s}{\partial t} = \dot{s}$$

$$a = \frac{\partial^2 s}{\partial t^2} = \ddot{s}$$

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A bit more notation / reminder of calculus...

$\frac{ds}{dt}$: the differential of position With Respect To (wrt) time.

$\frac{d^2s}{dt^2}$: the second differential of position wrt time.

Example & Notation:

$$x = 3t^2 + 4t + 6$$

$$\dot{x} = \frac{\partial x}{\partial t} = 6t + 4$$

$$\ddot{x} = \frac{\partial^2 x}{\partial t^2} = 6$$

Checkpoint

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $x > 0$):

(1) $x = 3t - 2$ $\dot{x} = 3$

(2) $x = -4t^2 - 2$ $\dot{x} = -8t$

(3) $x = \frac{2}{t^2}$ $x = 2t^{-2}$ $\therefore$ $\dot{x} = -4t^{-3}$

(4) $x = -2$ $\dot{x} = 0$

(a) In which situation(s) is the velocity v of the particle constant?

(b) In which is v in the negative x direction?

Acceleration

Acceleration usually means 'speeding up' in normal conversation. In physics it also means 'slowing down'.

If something has a changing speed, then its acceleration is non-zero

Which of these positions as a function of time correspond to constant acceleration?

$x = 4t^3 - 55$: $\quad \dot{x} = 12t^2 \quad \ddot{x} = 24t$

$x = 4t^2 - 55$: $\quad \dot{x} = 8t \quad \ddot{x} = 8$

$x = 4t - 55$: $\quad \dot{x} = 4 \quad \ddot{x} = 0$

$x = 4/t - 55$: $\quad \dot{x} = -4t^{-2} \quad \ddot{x} = 8t^{-3}$

$x = 4/t^2 - 55$: $\quad \dot{x} = -8t^{-3} \quad \ddot{x} = 24t^{-4}$

Before next lecture

Retry the pre-lecture quiz 1.1 Velocity and Acceleration, if you like.

Attempt the pre-lecture quiz for 1.2 Equations of Motion.

See you tomorrow morning for lecture 1.2