Mechanics & Relativity

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Week 7

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Familiar Angular Variables

Angular displacement: $\theta = \frac{s}{r}$

Angular velocity: $\omega = \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$

Angular (Tangential) acceleration: $\alpha = \frac{d\omega}{dt} = \frac{1}{r}\frac{d^2s}{dt^2} = \frac{a}{r}$

Angular (Radial) acceleration: $a_c = -\frac{v^2}{r}$

Period: $T = \frac{s}{v} = \frac{\theta}{\omega} = \frac{2\pi}{\omega}$

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Familiar Angular Variables

The angular position of a point on a rotating wheel is given by
$\theta = 2.0\ [\text{rad}] + 4.0\ [\text{rad s}^{-2}]t^2 + 2.0\ [\text{rad s}^{-3}]t^3$.

(a) At t = 0, what is the point's angular position?

(b) At t = 0, what is its angular velocity?

(c) What is its angular velocity at t = 4.0 s?

(d) Calculate its angular acceleration at t = 2.0 s.

(e) Is its angular acceleration constant?

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Angular suvat

For constant angular (tangential) acceleration $\alpha = \frac{a}{r}$:

$v = u + at \rightarrow \omega = \omega_0 + \alpha t$

$s = ut + \frac{1}{2}at^2 \rightarrow \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Angular suvat

The angular speed α of an automobile engine is increased at a constant rate from 1200 rev/min to 3000 rev/min in 12 s.

(a) What is its angular acceleration in revolutions per minute-squared?

(b) How many revolutions does the engine make during this 12 s interval?

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New Angular Variables

Rotational Inertia (angular mass): $I = \int r^2 dm$

Torque (angular force): $\tau = r \times F = I\alpha$

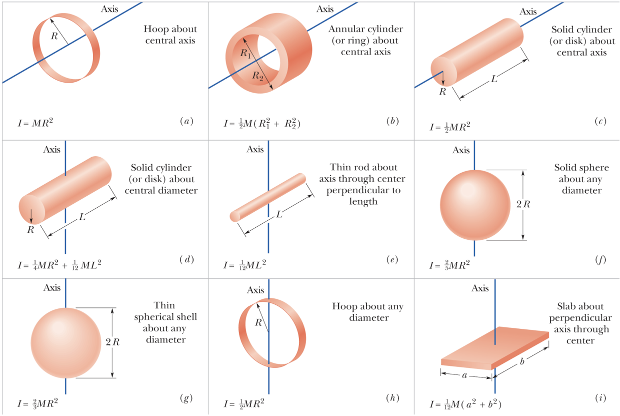
Rotational Work : $W = \int \tau d\theta$

Rotational KE : $KE = \frac{1}{2} I\omega^2$

Angular momentum : $L = r \times p$

Rotational Inertia

Rotational Inertia (angular mass): $I = \int r^2 dm$



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Torque

Torque (angular force): $\tau = r \times F = I\alpha$

The length of a bicycle pedal arm is 0.152 m, and a downward force of 111 N is applied to the pedal by the rider. What is the magnitude of the torque about the pedal arm's pivot when the arm is at angle
(a) 30°

(b) 90°

(c) 180° with the vertical?

Rotational KE

Rotational KE : $KE = \frac{1}{2} I\omega^2$

A thin rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. It is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with angular speed 4.0 rad/s. Neglecting friction and air resistance, find (a) the rod's kinetic energy at its lowest position and (b) how far above that position the center of mass rises.

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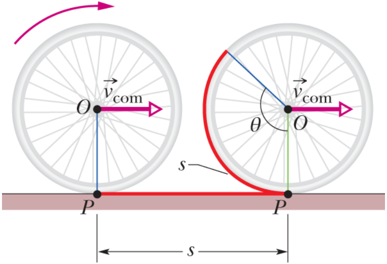
Rolling: translational KE

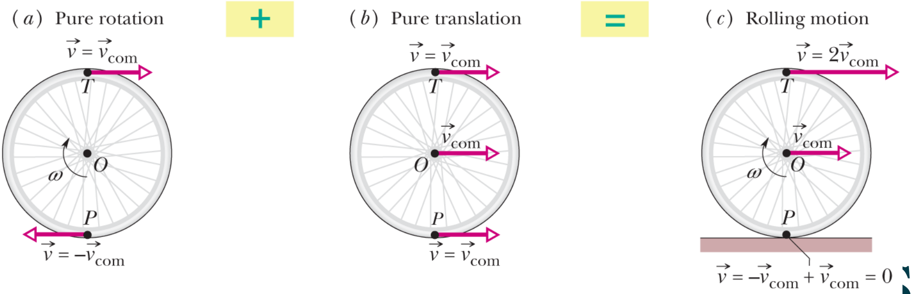
We have KE due to the translational movement of the centre of mass of the rolling object: $KE_{tran} = \frac{1}{2}mv_{com}^2$



Rolling: rotational + translational

We also have KE due to the rotational movement of the centre of mass of the rolling object:

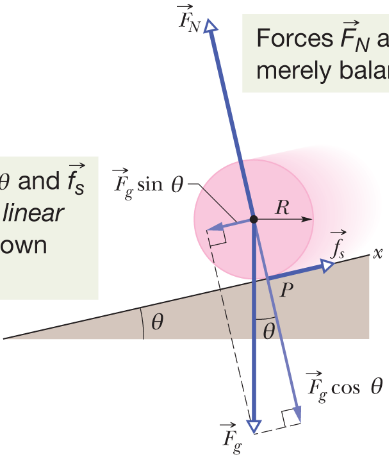
$$KE_{tot} = KE_{tran} + KE_{rot} = \frac{1}{2} m v_{com}^2 + \frac{1}{2} I \omega^2$$



Rolling

Forces $\vec{F}_N$ and $\vec{F}_g \cos\theta$ merely balance.

Forces $\vec{F}_g \sin\theta$ and $\vec{f}_s$ determine the *linear* acceleration down the ramp.



The torque due to $\vec{f}_s$ determines the *angular* acceleration around the com.

Angular Acceleration Problem

A wheel is initially rotating at 30 rad/s and has a constant angular acceleration. After 8.0 s, it has rotated through 90 rev. What is its angular acceleration?

Rolling Problem

A car traveling at 80.0 km/h has tires of 75.0 cm diameter.

(a) What is the angular speed of the tires about their axles?

(b) If the car is brought to a stop uniformly in 30.0 complete turns of the tires (without skidding), what is the magnitude of the angular acceleration of the wheels?

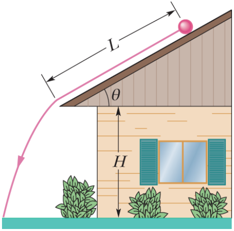
(c) How far does the car move during the braking?

Rolling Problem

A solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at angle $\theta = 30^\circ$.

(a) What is the angular speed of the cylinder about its centre as it leaves the roof?

(b) The roof's edge is at height H = 5.0 m. How far horizontally from the roof's edge does the cylinder hit the level ground?



Angular Momentum

Angular momentum : $L = r \times p$

Angular Momentum Problem

A 2.0 kg particle-like object moves in a plane with velocity components $v_x = 30$ ms^{-1} and $v_y = 60$ ms^{-1} as it passes through the point with (x, y) coordinates of (3.0, -4.0) m. Just then, in unit-vector notation, what is its angular momentum relative to

(a) the origin and

(b) the point located at (-2.0, -2.0) m?