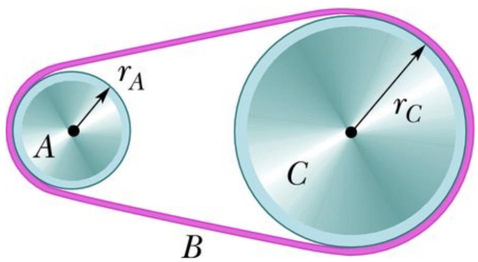
Please upload your solution to Problem 3 to canvas for marking after the workshop.

Problem 1

What is the angular speed (in radians per second) of the ~~following~~ components of the minute hand of a smoothly running analog watch?

Problem 2

In the figure below, wheel A of radius $r_A = 10$ cm is coupled by belt B to wheel C of radius $r_C = 25$ cm. The angular speed of wheel A is increased from rest at a constant rate of 1.6 rads^{-2}. Find the time needed for wheel C to reach an angular speed of 100 rev/min, assuming the belt does not slip.



Problem 3

A 0.400 kg ball is shot directly upward at initial speed 40.0 ms^{-1}. What is its angular momentum about P, 2.00 m horizontally from the launch point, when the ball is
(a) at maximum height?
(b) halfway back to the ground?
What is the torque on the ball about P due to the gravitational force when the ball is
(c) at maximum height?
(d) halfway back to the ground?

Problem 4

A uniform solid sphere rolls down an incline.
(a) What must be the incline angle if the linear acceleration of the centre of the sphere is to have a magnitude of $0.10g$?
(b) If a frictionless block were to slide down the incline at that angle, would its acceleration magnitude be more than, less than, or equal to $0.10g$? Why?

Want more practice?

Further problems on Angular Variables: Chapter 10.1-10.4
Further problems on Inertia & Torque: Chapter 10.5-10.8
Further problems on Rolling & Angular Momentum: Chapter 11

Problem 1

What is the angular speed (in radians per second) of the ~~following~~ components of the minute hand of a smoothly running analog watch?

MINUTE HAND : MOVES $\frac{1}{60}$ of 2π radians in 60 seconds

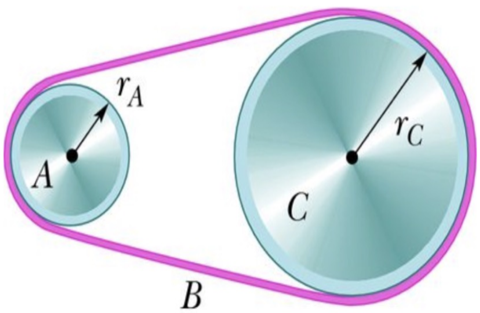
$$\Delta\theta = \frac{2\pi}{60} \text{ rad} = 0.1047\ldots \text{ rad}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{0.1047\ldots \text{ rad}}{60 \text{ s}} = 0.001745\ldots \text{ rad s}^{-1}$$

$$\boxed{\omega = 1.75 \times 10^{-3} \text{ rad s}^{-1}}$$

Problem 2

In the figure below, wheel A of radius $r_A = 10$ cm is coupled by belt B to wheel C of radius $r_C = 25$ cm. The angular speed of wheel A is increased from rest at a constant rate of 1.6 rads^{-2}. Find the time needed for wheel C to reach an angular speed of 100 rev/min, assuming the belt does not slip.



To get one full rev in wheel C we need to provide a linear displacement $S = 2\pi r_c$.

We get this from turning wheel A a distance $S = 2\pi r_c = k 2\pi r_A$. So $K = \frac{r_c}{r_A} = \frac{25}{10} = 2.5$.

For 100 revs in C, we need 250 revs in A.

250 revs in A per min = 250 revs per 60 s

$$\omega_A = \frac{250}{60} \text{rev } s^{-1} = \left(\frac{250}{60}\right) \times (2\pi \text{ rad})\ s^{-1} = 26.18\ldots \text{ rad } s^{-1}$$

SUVAT (α = const.) *

$\theta = (250 \text{ revs}) \times (2\pi \text{ rad})$

$\omega_0 = 0$

$\omega = 26.18.. \text{ rad } s^{-1}$

$\alpha = 1.6 \text{ rad } s^{-2}$

t

$\omega = \omega_0 + \alpha t$

$t = \frac{\omega - \omega_0}{\alpha}$

$= \frac{26.18\ldots \text{ rad } s^{-1}}{1.6 \text{ rad } s^{-2}} = \boxed{16s}$

Problem 3

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(a) at maximum height?
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~~Problem 4~~

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = rmv\sin\theta_{rv}\,\hat{n}$$

a) at max height $v = 0$, so $\vec{L} = 0$

b) only force is gravity ∴ const. acceleration ∴ SUVAT

GOING UP:
$S_y = ?$
$U_y = 40\ ms^{-1}$
$V_y = 0$
$a_y = -10\ ms^{-2}$
$t = ?$

$V_y = U_y + a_y t \rightarrow 0 = 40 - 10t$ ∴ $\boxed{t = 4s}$ TIME FOR MAX HEIGHT.

$S_y = U_y t + \frac{1}{2}a_y t^2 \rightarrow S_y = (40)(4) + (0.5)(-10)(4^2)$
$= 160 - 80$

∴ $\boxed{S_y = 80m}$ MAX HEIGHT

∴ $\boxed{S_{1/2} = 40m}$ HALF MAX HEIGHT

GOING DOWN:
$S_y = -40m$
$U_y = 0$
$V_y = ?$
$a_y = -10\ ms^{-2}$
$t = ?$

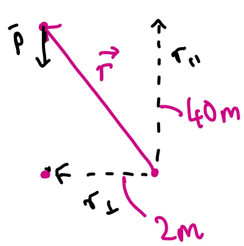
$S_y = U_y t + \frac{1}{2}a_y t^2 \rightarrow -40 = 0 + (0.5)(-10)t^2$

∴ $t = \sqrt{\frac{-40\ m}{-5\ ms^{-2}}} = \sqrt{8}\ s$ ∴ $\boxed{t_{1/2} = 2.8\ s}$

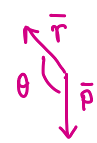
$V_y = U_y + a_y t$
$= 0 + (-10)(2.8)$ ∴ $V_y = -28\ ms^{-1}$ ∴ $\boxed{V_{\frac{1}{2}} = -28\ ms^{-1}}$

So we find $\bar{p} = m\bar{v} = (0.4)(-28) = -11.2\ kg\,ms^{-1}$ $\quad \vec{p} = -11.2\ kg\,ms^{-1}\ \hat{j}$

NEXT PIECE: WHAT IS $r\sin\theta_{rp}\ \hat{n}$?



$r\sin\theta_{rp} = r_\perp = -2m$



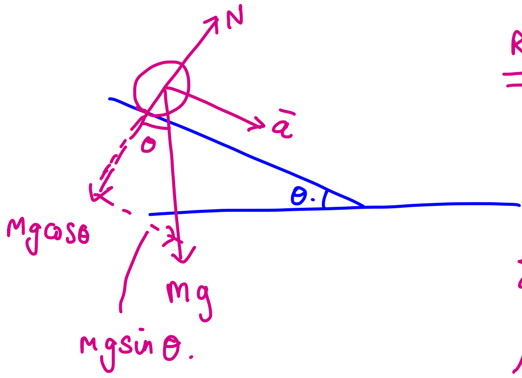
∴ $\vec{L} = (-11.2\ kg\,ms^{-1}\ \hat{j})(-2m)$
$= 22.4\ kg\,ms^{-1}\ \odot$ ← out of page because ↻ = +ve.

c) $\bar{\tau} = \vec{r} \times \vec{F}$
$= rF\sin\theta_{rF}\ \hat{n}$
$= r_\perp F\ \hat{n} = (-2m)(0.4\ kg)(-9.8\ ms^{-2}) = \boxed{7.84\ Nm\ \odot}$

d) $\boxed{\text{Same as c}}$

Problem 4

A uniform solid sphere rolls down an incline.
(a) What must be the incline angle if the linear acceleration of the centre of the sphere is to have a magnitude of $0.10g$?
(b) If a frictionless block were to slide down the incline at that angle, would its acceleration magnitude be more than, less than, or equal to $0.10g$? Why?



ROLLING means static frictional force is enough to overcome $mg\sin\theta$.

$$\mu_s N > mg\sin\theta$$

$$\Sigma \bar{F} = m\bar{a} = mg\sin\theta - \mu_s N$$

$$\mu_s N = mg\sin\theta - ma$$

TORQUE $\bar{\tau} = \bar{r} \times \bar{F} = r_\perp F$; $\Sigma\tau = \tau_F$: only friction contributes to the torque, because GRAVITY acts on CENTRE-OF-MASS.

$$\tau = r\mu_s N = mgr\sin\theta - mar$$

And $\tau = I\alpha = \frac{2}{5}mr^2\alpha = \frac{2}{5}mr^2\frac{a}{r}$

LOOKED UP $I_{sphere} = \frac{2}{5}Mr^2$

STUDENTS KNOW TO DO THIS!

So, $\cancel{m}g\cancel{r}\sin\theta - \cancel{m}a\cancel{r} = \frac{2}{5}\cancel{m}\cancel{r}a$

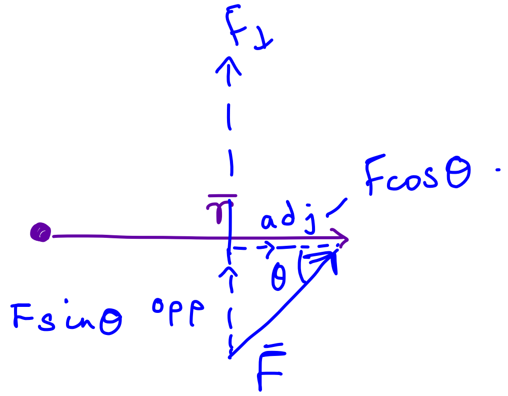
$g\sin\theta - a = \frac{2}{5}a \quad \therefore \quad g\sin\theta = \frac{7}{5}a \quad \therefore \quad \theta = \sin^{-1}\left(\frac{7a}{5g}\right)$

So, if $a = 0.1g$, $\theta = \sin^{-1}\left(\frac{0.7}{5}\right) = 0.14$ rad $= \boxed{8^\circ}$

b) BLOCK, NO FRIC. $\Sigma F = ma = mg\sin\theta$

From part a we know $a_{sphere} = \frac{5}{7}g\sin\theta$. Compare $a_{block} = g\sin\theta$, we see $\boxed{a_{block} > a_{sphere}}$

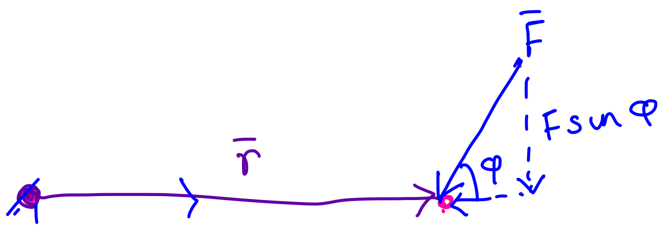
CROSS PRODUCTS $\bar{\tau} = \bar{r} \times \bar{F}$ and $\vec{L} = \bar{r} \times \bar{p}$



$$\bar{r} \times \bar{F} = rF\sin\theta\ \hat{n}$$
$$= rF_\perp\ \hat{n}.$$

$\odot$

"Lefty Loosey"

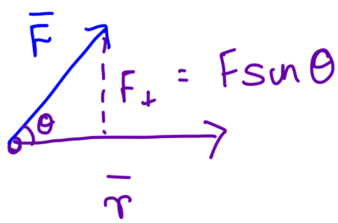


$$\bar{r} \times \bar{F} = rF\sin\varphi\ \hat{n}$$
$$= rF_\perp\ \hat{n}$$

$\otimes$

"Righty tighty"

LILY'S WAY



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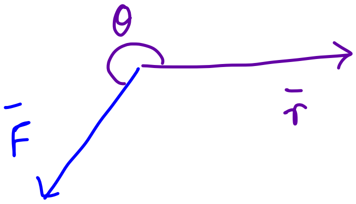
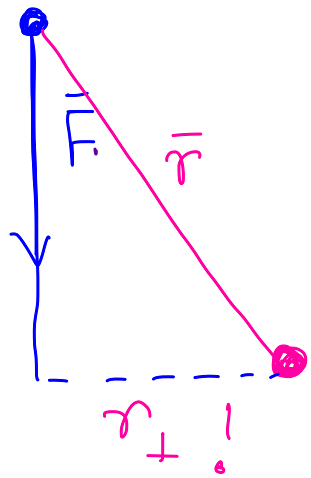
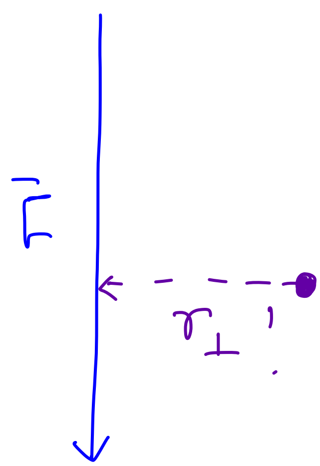


$\odot$ BUT now $\theta > 180$ SO $\otimes$

SO $\bar{r} \times \bar{F} = -rF_\perp\ \hat{n}.$





$F r_\perp$.