Please upload your solution to Problem 3 to canvas for marking after the workshop.

Problem 1

The position of a particle moving along an x-axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine:
(a) the position,

(b) the velocity, and

(c) the acceleration of the particle at $t = 4$ s

Problem 2

A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

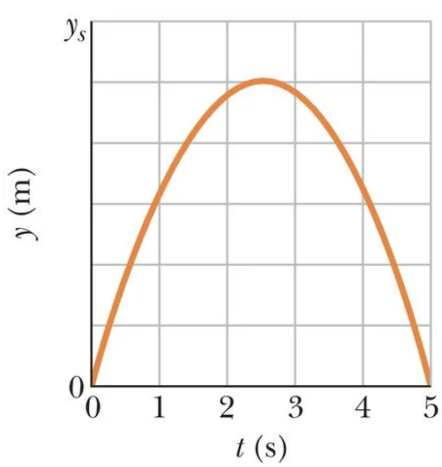
Problem 3

Two particles move along an x axis. The position of particle 1 is given by $x_1 = 6.00t^2 + 3.00t + 2.00$; the acceleration of particle 2 is given by $a_2 = -8.00t$ and, at $t = 0$, its velocity is $v_2 = 20ms^{-1}$. When the velocities of the particles match, what is their velocity?

Problem 4

A ball is shot vertically upward from the surface of another planet. A plot of y versus t for the ball is shown in the figure below, where y is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure?s vertical scaling is set by $y_s = 30.0$m. What are the magnitudes of :
(a) the free-fall acceleration on the planet and

(b) the initial velocity of the ball?



Problem 1

The position of a particle moving along an x-axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine:

(a) the position,

(b) the velocity, and

(c) the acceleration of the particle at $t = 4$ s

$$\left.\begin{aligned} &t = 4s \\ &x = 12t^2 - 2t^3 \end{aligned}\right\} \text{given}$$

a) $x = 12(4^2) - 2(4^3) = 64\,m$

b) $v = \frac{\partial x}{\partial t} = 24t - 6t^2$

$= 24(4) - 6(4^2) = 0\ ms^{-1}$

c) $a = \frac{\partial v}{\partial t} = 24 - 12t$

$= 24 - 12(4) = -24\ ms^{-2}$

HINTS

Velocity is $\frac{\partial x}{\partial t}$, Acceleration $\frac{\partial v}{\partial t}$

Must differentiate first, then sub in $t = 4s$

Problem 2

A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

① What are we explicitly given?

$t_{tower} = 1.5\,s$

$t_{max} = t_{tower} + 1\,s$

② What else do we know?

HINTS :)

$a = -9.81\ ms^{-2}$: gravity

$v_{max} = 0\ ms^{-1}$: rock must momentarily stop at max height.

③ SUVAT :

@ max		@ tower
$s = ?$		$s = ?$ *
$u = ?$	$=$	$u = ?$
$v = 0$		$v = ?$
$a = -9.81$	$=$	$a = -9.81$
$t = 2.5$		$t = 1.5$

First find for u@max :

$[u = v - at]$

$u = v_{max} - a\,t_{max} = 0 - (-9.81)(2.5) = 24.5\ ms^{-1}$

which is also u@tower. (INITIAL VELOCITY FIXED)

so, $s@tower = u\,t_{tower} + \frac{1}{2} a\, t_{tower}^2$ $[s = ut + \frac{1}{2}at^2]$

$= (24.5)(1.5) + \frac{1}{2}(-9.81)(1.5^2)$

$= 25.8\ m$

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(1) $\left.\begin{aligned} x_1 &= 6t^2 + 3t + 2 \\ a_2 &= -8t \\ u_2 &= 20 \end{aligned}\right\}$ given

We can't use SUVAT because a is NOT CONSTANT
$a = a(t)$

(2) $v_2 = -4t^2 + c$ $\left(\int a_2\, \partial t\right)$ and when $t=0$, $v_2 = u_2$, so

$v_2 = -4t^2 + 20$

$v_1 = 12t + 3$ $\left(\frac{\partial x_1}{\partial t}\right)$

when $v_2 = v_1$ $\therefore$ $20 - 4t^2 = 12t + 3$ $\therefore$ $4t^2 + 12t - 17 = 0$

QUADRATIC FORMULA $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

HINTS :)

has positive root $t = 1.05s$

$\therefore$ $v_2 = 20 - 4(1.05)^2 = 15.6\ ms^{-1}$

$v_1 = 12(1.05) + 3 = \boxed{15.6\ ms^{-1}}$

Problem 4

A ball is shot vertically upward from the surface of another planet. A plot of y versus t for the ball is shown in the figure below, where y is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0$m. What are the magnitudes of :

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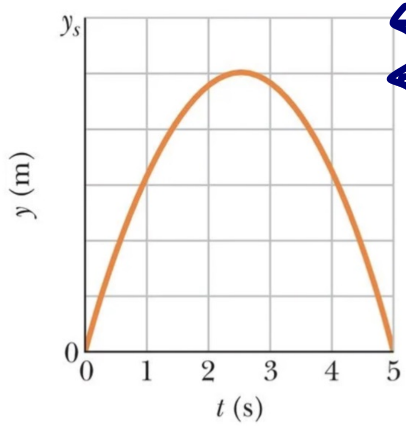

30

25

$S_{max} = 25$

$t_{max} = 2.5$ (from graph) } given

HINTS :)

$V_{max} = 0$: ball must momentarily stop at max height

$$S_{max} = \cancel{V_{max}}^{=0} t_{max} + \frac{1}{2} a t_{max}^2$$ ($s = vt + \frac{1}{2}at^2$ eqn of motion)

$$\therefore |a| = \left| \frac{2 S_{max}}{t_{max}^2} \right| = \left| \frac{50}{2.5^2} \right| = \boxed{8 \, ms^{-2}}$$ (note $a = -8 \, ms^{-2}$)

Now $u = V_M - a t_M$ ($u = v - at$ eqn of motion)

$$\therefore |u| = |(-8)(2.5)| = \boxed{20 \, ms^{-1}}$$