Mechanics & Relativity

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Week 6 special (centre of mass)

Centre of Mass Position

We do physics using the centre of mass of objects.

$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$x_{com} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

$\underline{r}_{com} = x_{com}\hat{\boldsymbol{i}} + y_{com}\hat{\boldsymbol{j}} + z_{com}\hat{\boldsymbol{k}}$

$\underline{r}_{com} = \frac{\Sigma m_i \underline{r}_i}{\Sigma m_i}$

Generally:

$x_{com} = \frac{1}{M}\int x dm$

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Centre of Mass Position

The com position can (sometimes!) more usefully be written in terms of constant density ρ:

$\rho = \frac{M}{V} \therefore M = \rho V$

$x_{com} = \frac{1}{\rho V} \int x \rho dV$

$x_{com} = \frac{1}{V} \int x dV$

We will come back to this next week when we think about moments of inertia.

Centre of Mass Velocity, Acceleration

The centre of mass velocity & acceleration are defined in exactly the same way as the centre of mass position.

$$\underline{r}_{com} = x_{com}\hat{\boldsymbol{i}} + y_{com}\hat{\boldsymbol{j}} + z_{com}\hat{\boldsymbol{k}}$$
$$\underline{r}_{com} = \frac{\Sigma m_i \underline{r}_i}{\Sigma m_i}$$

$$\underline{v}_{com} = v_{x,com}\hat{\boldsymbol{i}} + v_{y,com}\hat{\boldsymbol{j}} + v_{z,com}\hat{\boldsymbol{k}}$$
$$\underline{v}_{com} = \frac{\Sigma m_i \underline{v}_i}{\Sigma m_i}$$

$$\underline{a}_{com} = a_{x,com}\hat{\boldsymbol{i}} + a_{y,com}\hat{\boldsymbol{j}} + a_{z,com}\hat{\boldsymbol{k}}$$
$$\underline{a}_{com} = \frac{\Sigma m_i \underline{a}_i}{\Sigma m_i}$$

Such that we can use N2:

$$\underline{F} = m\underline{a}_{com}$$

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