

Please upload your solution to Problem 3 to canvas for marking after the workshop.

Problem 1

The position of a particle moving along an x -axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine:

- (a) the position,
- (b) the velocity, and
- (c) the acceleration of the particle at $t = 4$ s

Problem 2

A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

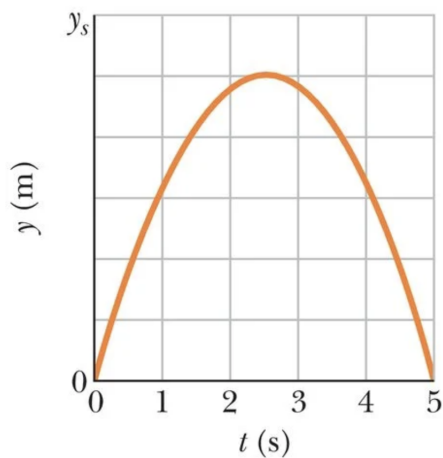
Problem 3

Two particles move along an x axis. The position of particle 1 is given by $x_1 = 6.00t^2 + 3.00t + 2.00$; the acceleration of particle 2 is given by $a_2 = -8.00t$ and, at $t = 0$, its velocity is $v_2 = 20\text{ms}^{-1}$. When the velocities of the particles match, what is their velocity?

Problem 4

A ball is shot vertically upward from the surface of another planet. A plot of y versus t for the ball is shown in the figure below, where y is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0\text{m}$. What are the magnitudes of :

- (a) the free-fall acceleration on the planet and
- (b) the initial velocity of the ball?



Problem 1

The position of a particle moving along an x -axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. Determine:

(a) the position,

(b) the velocity, and

(c) the acceleration of the particle at $t = 4$ s

$$\left. \begin{array}{l} t = 4 \text{ s} \\ x = 12t^2 - 2t^3 \end{array} \right\} \text{ given}$$

$$\text{a) } x = 12(4^2) - 2(4^3) = 64 \text{ m}$$

$$\begin{aligned} \text{b) } v &= \frac{\partial x}{\partial t} = 24t - 6t^2 \\ &= 24(4) - 6(4^2) = 0 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{c) } a &= \frac{\partial v}{\partial t} = 24 - 12t \\ &= 24 - 12(4) = -24 \text{ ms}^{-2} \end{aligned}$$

HINTS

Velocity is $\frac{\partial x}{\partial t}$, Acceleration $\frac{\partial v}{\partial t}$

Must differentiate first,

then sub in $t = 4$ s

Problem 2

A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

① What are we explicitly given?

$$t_{\text{tower}} = 1.5 \text{ s}$$

$$t_{\text{max}} = t_{\text{tower}} + 1 \text{ s}$$

② What else do we know?

$$a = -9.81 \text{ ms}^{-2} : \text{gravity}$$

$$v_{\text{max}} = 0 \text{ ms}^{-1} : \text{rock must momentarily stop at max height.}$$

HINTS



③ SUVAT :	@ max		@ tower
	$s = ?$	\equiv	$s = ? *$
	$u = ?$	\equiv	$u = ?$
	$v = 0$		$v = ?$
	$a = -9.81$	\equiv	$a = -9.81$
	$t = 2.5$		$t = 1.5$

First find for u @Max:

$$u = v_{\text{max}} - a t_{\text{max}} = 0 - (-9.81)(2.5) = 24.5 \text{ ms}^{-1}$$

$$[u = v - at]$$

which is also u @tower. (INITIAL VELOCITY FIXED)

$$\text{so, } s_{\text{@tower}} = u t_{\text{tower}} + \frac{1}{2} a t_{\text{tower}}^2 \quad [s = ut + \frac{1}{2} at^2]$$

$$= (24.5)(1.5) + \frac{1}{2}(9.81)(1.5^2)$$

$$= \underline{25.8 \text{ m}}$$

Problem 3

Two particles move along an x axis. The position of particle 1 is given by $x_1 = 6.00t^2 + 3.00t + 2.00$; the acceleration of particle 2 is given by $a_2 = -8.00t$ and, at $t = 0$, its velocity is $v_2 = 20\text{ m s}^{-1}$. When the velocities of the particles match, what is their velocity?

$$\textcircled{1} \left. \begin{aligned} x_1 &= 6t^2 + 3t + 2 \\ a_2 &= -8t \\ v_2 &= 20 \end{aligned} \right\} \text{given}$$

$$\textcircled{2} \quad v_2 = v_2 + a_2 t = 20 - 8t^2 \quad (v = v + at \text{ eqn. of motion})$$
$$v_1 = 12t + 3 \quad \left(\frac{dx_1}{dt} \right)$$

$$\text{when } v_2 = v_1 \quad \therefore \quad 20 - 8t^2 = 12t + 3 \quad \therefore \quad 8t^2 + 12t - 17 = 0$$

$$\text{QUADRATIC FORMULA} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HINTS ☺

has positive root $t = 0.889 \dots \text{ s}$

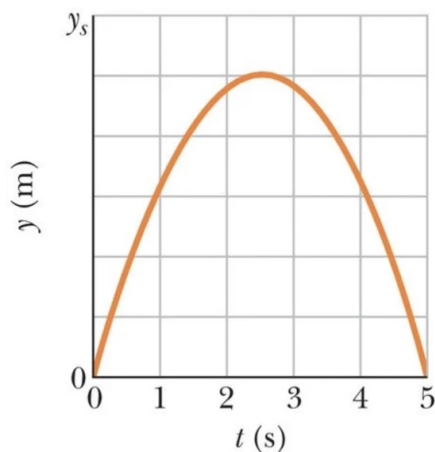
$$\therefore v_2 = 20 - 8(0.889^2) = 13.7 \text{ m s}^{-1}$$

$$v_1 = 12(0.889) + 3 = \boxed{13.7 \text{ m s}^{-1}}$$

Problem 4

A ball is shot vertically upward from the surface of another planet. A plot of y versus t for the ball is shown in the figure below, where y is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0\text{m}$. What are the magnitudes of :

- (a) the free-fall acceleration on the planet and
- (b) the initial velocity of the ball?



$$\left. \begin{array}{l} S_{\max} = 30 \\ t_{\max} = 2.5 \text{ (from graph)} \end{array} \right\} \text{ given}$$

HINTS:

$v_{\max} = 0$: ball must momentarily stop at max height

$$S_{\max} = v_{\max} t_{\max} + \frac{1}{2} a t_{\max}^2$$

$(s = vt + \frac{1}{2}at^2 \text{ eqn of motion})$

$$\therefore |a| = \left| \frac{2S_{\max}}{t_{\max}^2} \right| = \left| \frac{60}{2.5^2} \right| = \boxed{9.6 \text{ ms}^{-2}} \quad (\text{note } a = -9.6)$$

$$\text{Now } u = v_m - at_m \quad (u = v - at \text{ eqn of motion})$$

$$\therefore |u| = (9.6)(2.5) = \boxed{24 \text{ ms}^{-1}}$$