

AUTOMATICALLY VISUALIZING MATHEMATICS

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MOTIVATION

A well-chosen figure can be a tremendous help in understanding mathematical concepts. The production of a good illustration often requires extensive experience, and remarkable effort. To make such process easier, we are building PENROSE, a system that automatically produces professional-quality mathematical illustrations from high-level descriptions of mathematical objects.

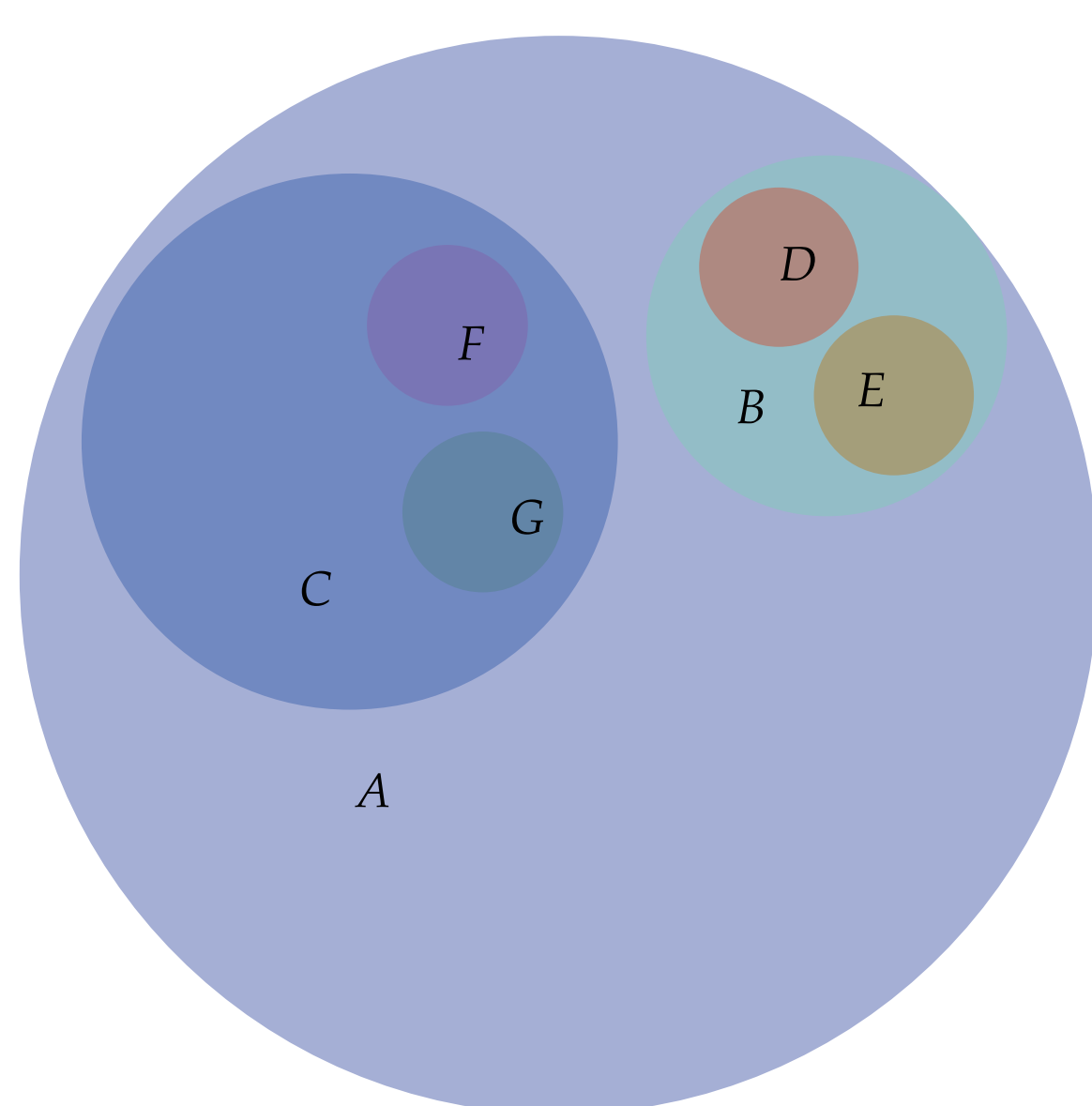
PENROSE provides two **extensible domain specific languages**: SUBSTANCE and STYLE. Similar to using HTML and CSS in the web, a Penrose user would write mathematical notations in SUBSTANCE, and use STYLE to specify the details about visualization.

SUBSTANCE & STYLE LANGUAGE

```
Set A, B, C, D, E, F, G
Subset B A
Subset C A
Subset D B
Subset E B
```

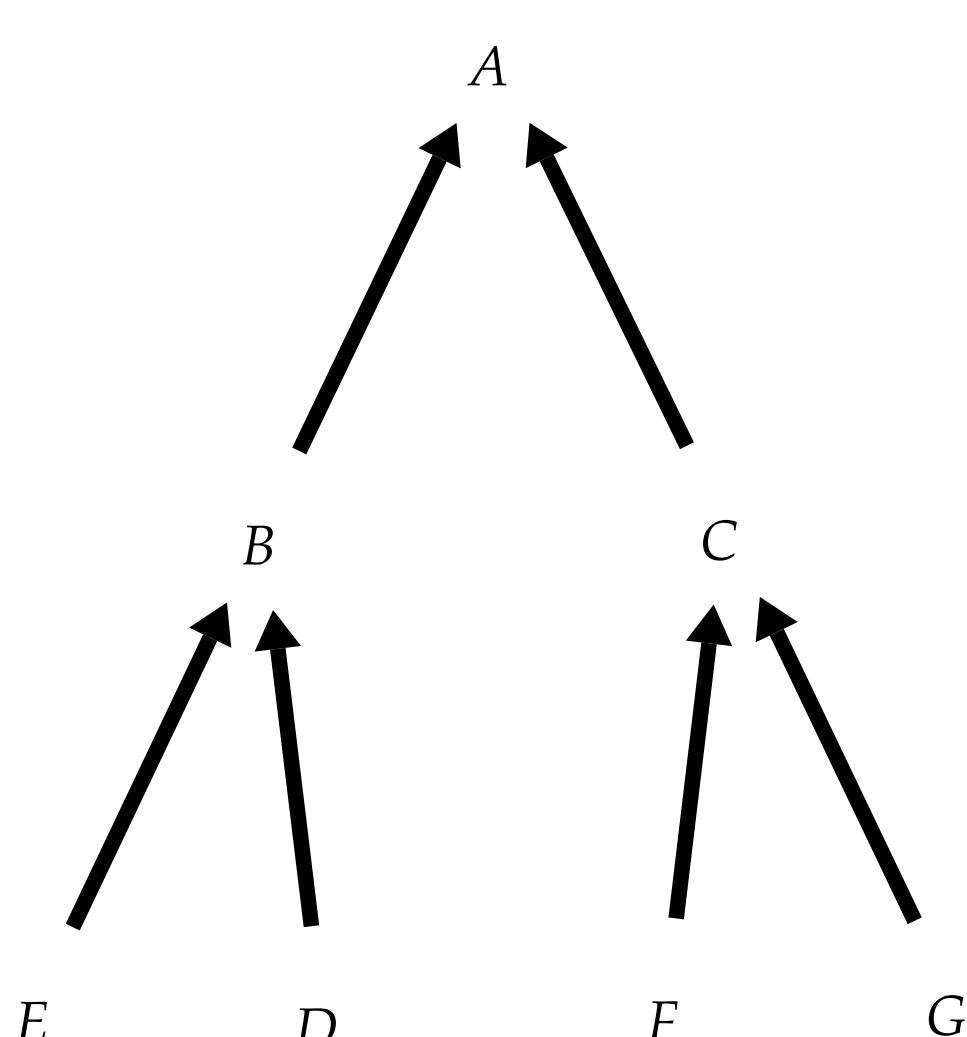
```
Subset F C
Subset G C
NoIntersect E D
NoIntersect F G
NoIntersect B C
```

A Substance program of 3 layers of subset relations



Style #1: Venn Diagram

```
Set x {
  shape = Circle { }
  ensure x contains x.label
}
NoIntersect x y {
  ensure x nonOverlapping y
}
Subset x y {
  ensure y contains x
  ensure x smallerThan y
  ensure y.label outsideOf x
}
```



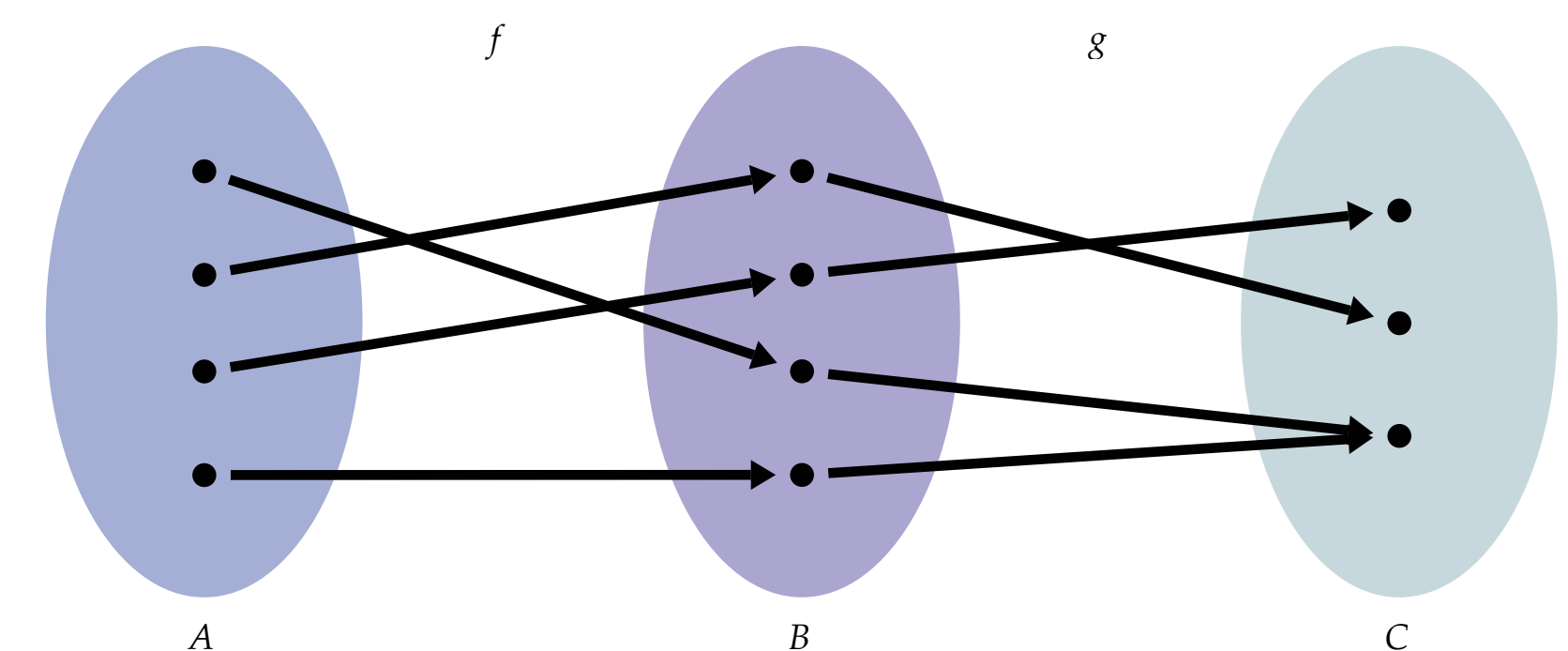
Style #2: Tree Diagram

```
Set x { shape = Text{ } }
Subset x y {
  encourage y onTop x
  encourage x sameX y
  shape = Arrow {
    start = x.shape
    end = y.shape
    label = None
  }
}
Set x, Set y {
  encourage x repel y
}
```

We designed a STYLE language that selects SUBSTANCE objects using **pattern matching**, declares geometries, and specifies **objectives** (encourage) or **constraints** (ensure). The STYLE files are **reusable** for all similar SUBSTANCE programs.

VISUALIZING MATHEMATICAL DEFINITIONS

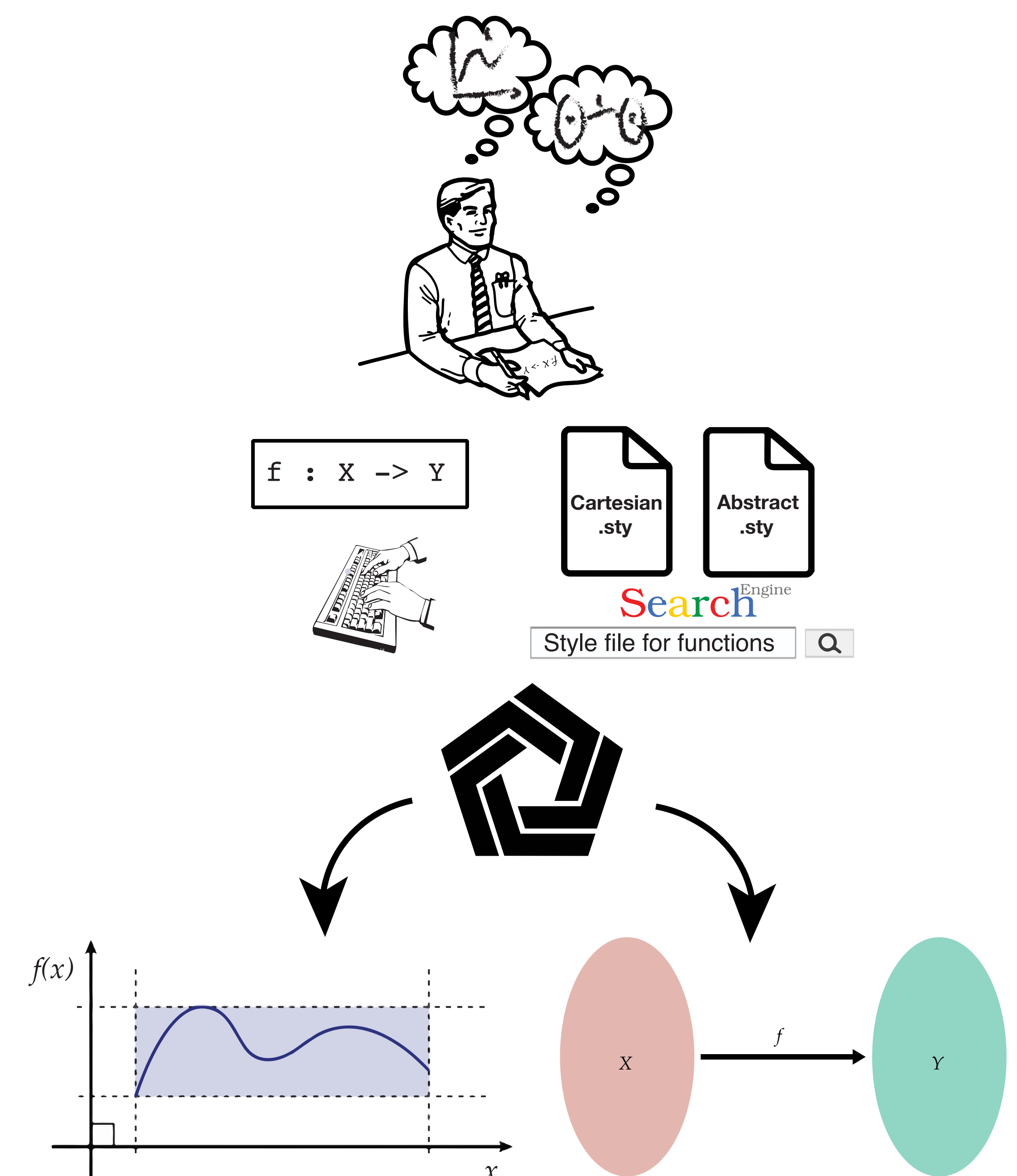
```
f: A -> B
g: B -> C
Definition Surjection(Map f, Set A, Set B):
  forall b : B | exists a : A | f(a) = b
Size A 4
Size B 4
Size C 3
Surjection(f, A, B)
Surjection(g, B, C)
```



Illustrating theorem: the composition of surjective functions is also surjective

To illustrate mathematical definitions, we often pick representative examples. PENROSE allows users to specify **mathematical definitions in first-order logic**, and **automatically generates instances** of the definition by integrating Alloy, an external SAT-solver.

MAKING DIAGRAMS WITH PENROSE



FUTURE WORK

- Generate figures in **general position**
- Currently using **Exterior Point Method** for layout optimization. Try other strategies.
- Extend Substance to other domains.
- Develop an mechanism for integration with external tools and programs written in general purpose languages
- Programmatic and Direct Manipulation: infer STYLE code from GUI actions.