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Höhere Mathematik II
Sommersemester 2020

2. Übungsblatt

Aufgabe 9

a) i)

$$\begin{aligned}
\int_0^3 \left(e^{3x} - \sqrt[3]{e^x}\right) dx &= \int_0^3 \left(e^{3x} - e^{\frac{x}{3}}\right) dx \\
&= \int_0^3 e^{3x} dx - \int_0^3 e^{\frac{x}{3}} dx \\
&= \left[\frac{1}{3}e^{3x} + C\right]_0^3 - \left[3e^{\frac{x}{3}} + D\right]_0^3 \\
&= \left(\frac{e^9}{3} + C - \frac{e^0}{3} - C\right) - \left(3e + D - 3e^0 - D\right) \\
&= \frac{e^9}{3} - \frac{1}{3} - 3e + 3 \\
&= \frac{e^9 - 9e + 8}{3}
\end{aligned}$$

ii)

$$\begin{aligned}
\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cot(x)\ln(\sin x)dx &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\ln(\sin x)dx \\
u := \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)} \Rightarrow \; &= \int_{\sin\frac{\pi}{6}}^{\sin\frac{\pi}{3}} \frac{\cos x}{\sin x}\ln(\sin x)\frac{du}{\cos x} \\
&= \int_{\sin\frac{\pi}{6}}^{\sin\frac{\pi}{3}} \frac{1}{u}\ln(u)du \\
\int g(x)g'(x)dx \overset{KR}{=} \frac{1}{2}\left(g(x)\right)^2 + C \Rightarrow \; &= \frac{1}{2}\left(\ln(u)\right)^2 + C|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}
\end{aligned}$$

Aufgabe 10