A New *In Situ* Procedure for Measuring the Dielectric Properties of Low Permittivity Materials

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***Abstract*—In situ measurements of the dielectric properties of relatively low, complex permittivity materials such as rocks and fatty tissues have recently become of significant interest in industrial and medical applications. The presently available *in vivo* probes although found adequate for measuring the dielectric properties of high-permittivity materials, such as high-dielectric tissues [1], [2], were not suitable for accurately measuring the properties of low-permittivity materials. For low-permittivity substances, the measurement procedure and the accuracy of the obtained results were found to significantly improve when the length of the center conductor (h) of the coaxial transmission line probe [1], [2] was further extended into the material under test. With such an extension, the approximate analysis used in the previous measurements [1], [2] were invalid, and hence a new measurement procedure was developed. This new procedure has the following features:**

1) it utilizes a rigorous expression for the input impedance of the *in situ* probe which accurately accounts for the radiation resistance of the probe for larger values of (h/λ);

2) the dielectric parameters of the sample under test were determined by comparing the measured and calculated values of the input impedance using an iterative two-dimensional (error surface) complex zero-finding routine.

In this paper, the new measurement procedure is described, results of the error (uncertainty) analysis are presented, and the accuracy of the measurement procedure is illustrated by presenting dielectric results for known lossy (octyl alcohol) and lossless (heptane) low-permittivity materials.

I. Introduction

IN SITU measurement of the dielectric properties of materials has recently received considerable interest. The *in situ* measurement methods have the advantage of maintaining the sample integrity and also of avoiding the tedious process of preparing samples. Obtained dielectric properties, on the other hand, are useful in many applications. For example, in the biological research area these measurements provide valuable information in evaluating the RF absorption characteristics of various tissues [3], and in the detection of pathophysiological processes [4] and diseased tissues [5]. In the oil industry, the *in situ* dielectric properties measurement of rocks was found useful in providing valuable information on the water saturation and the hydrocarbon content of rocks [6]. The relative insensitivity of the dielectric properties at the microwave frequencies to the salinity of rocks prompted the recent development of the electromagnetic propagation tools for well logging [7], [8].

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There are two presently available *in vivo* probes which have been developed for measuring the dielectric properties of tissue [1], [2]. Both probes are basically made of sections of coaxial transmission lines with [1] and without [2] an extended center conductor which is immersed in the material under test. Upon the utilization of these probes for measuring the dielectric properties of low, complex permittivity materials such as rocks, we found the accuracy of the obtained results were rather limited [9]. This may be due to the fact that in the absence of high-dielectric test samples, the fringing fields at the end of the coaxial probe were not actually concentrated in the material under test, and the measurement procedure was hence very sensitive to the specific construction of the probe (e.g., with or without Teflon plug) and contact problems at the interfaces. Furthermore, for the rock applications, it is desirable to change the effective sample volume from that in the immediate vicinity of the probe, to a much larger volume representing the average properties of the formation. This may be accomplished by further extending the center conductor of the coaxial probe into the formation and varying the frequency so h/λ would be $h/\lambda << 0.1$ for localized measurements, and $0.1 < h/\lambda < 0.25$ for average measurements over a large sample volume. Once again, in this case, the available theories for the *in vivo* probes [1], [2] are not applicable for $h/\lambda \geq 0.1$, and hence a new measurement procedure is required.

In this paper, we present a new *in situ* procedure for measuring the dielectric properties of low-permittivity materials. The method has the following two main features:

1) it utilizes the rigorous expression developed by Wu [10] for the input impedance of the coaxial probe when immersed in the material under test. The method of analysis is, therefore, not limited to small values of h/λ;
2) because of the complexity of the appropriate mathematical expression used in this case, it was found necessary to utilize an iterative two-dimensional complex zero-finding routine to determine the dielectric parameters of interest.

Results of the error analysis calculations and experimental measurements are presented.

II. Theory

The *in situ* dielectric probe is basically a monopole antenna immersed in the material under test as shown in

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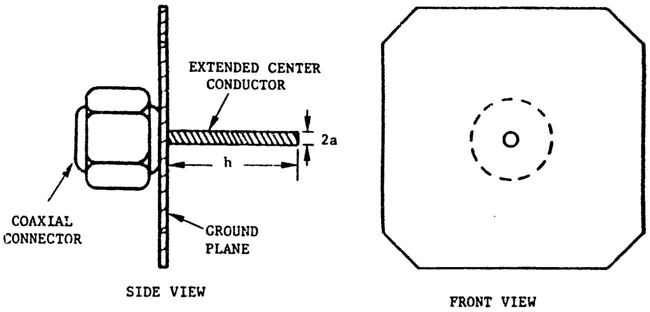

Fig. 1. The monopole probe. The center conductor of a coaxial line is extended in the dielectric material under test.

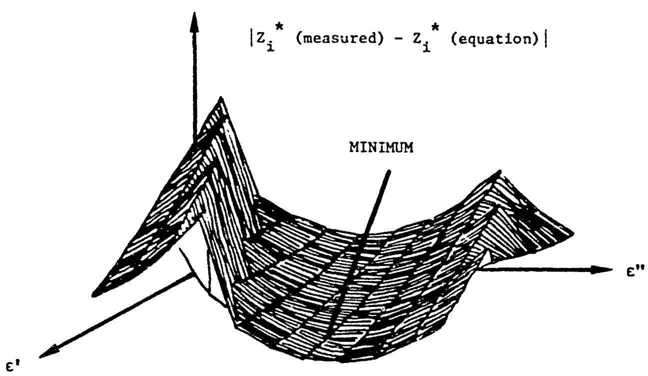

Fig. 2. Schematic diagram illustrating the two-dimensional error surface that results from the complex zero-finding routine ZANLYT.

Fig. 1. As indicated earlier, for dielectric measurements of low-permittivity material, we found it necessary to extend the center conductor further into the material under test. An expression for the input impedance of the dielectric probe which is basically a monopole antenna immersed in the material under test is given by [10]

$$Z_i^* = \frac{\omega\mu_0}{j2k(S + CU)} \tag{1}$$

where the functions k, S, C, and U depend on the measurement frequency, the length h, and diameter a of the extended center conductor of the coaxial probe, and on the dielectric properties of the medium ϵ' and ϵ''. Detailed expressions for these functions, as well as the approximations used are given in Appendix A. Because of the complexity of the expression in (1), it was not possible to directly derive the complex permittivity parameters from the measured input impedance. Instead, we utilized an iterative procedure which calculates the permittivity parameters by minimizing the difference between the measured and calculated values of the input impedance of the probe. The computer program is known as a complex zero-finding routine [11]. It is available in the International Mathematics and Statistics Library (IMSL) under the name ZANLYT. The iterative routine results in a two-dimensional error surface such as that shown in Fig. 2, where the minimum indicates the most appropriate value of ϵ' and ϵ'' that satisfy the measured value of the input impedance. In other words, the minimum in Fig. 2 is obtained when the values of ϵ' and ϵ'' used in the input impedance equation provide a value of Z_i^* which is in agreement with the measured value. Initial guesses of the value of the unknown complex dielectric constant are used in the zero-finding routine to avoid divergence of the solution. Up to ten initial guesses can be used in order to span a larger error surface of Fig. 2. The zero-finding routine then focuses on the initial guesses that produce the lowest minima. This avoids the possibility of finding only a local minimum instead of the actual surface minimum.

III. Experimental Measurements and Results

The experimental procedure for measuring the input impedance of the dielectric probe utilizes the standard HP Microwave Network Analyzer (HP 8410B). Special effort should be made to maintain good contact between the dielectric probe and the material under test. Also, the use of a ground plane of a radius approximately equal to 12 cm was found necessary to fine tune the measured values of the input impedance. Otherwise, the measurement procedure is standard and very similar to that described in detail by others in earlier publications [1], [2].

Before making experimental measurements, we performed uncertainty calculations to determine the sensitivity of the complex permittivity parameter to errors and uncertainties in the measurement procedure. Since the input impedance determination, on the other hand, is based on the measurement of the magnitude and phase of the reflection coefficient, the uncertainty analysis calculations involved changing the magnitude and the phase of the complex reflection coefficient by predetermined amounts 5 percent, 10 percent, etc., determining the resulting input impedance value and then using the zero-finding routine to finally calculate the errors in the dielectric parameters $\Delta\epsilon'$ and $\Delta\epsilon''$. The obtained results shown in Figs. 3–6, therefore, include the additional errors in determining the dielectric properties as a result of using the zero-finding technique. The results shown in Figs. 3–6 show the relative stability of the measurement system and its reasonable tolerance to reasonable measurement errors (less than 10 percent in $\Delta\Gamma$ and less than 5 percent in $\Delta\phi$) in the complex reflection coefficient.

The accuracy of the procedure was finally evaluated by measuring the complex permittivity of two known dielectric liquids. The two liquids measured were octyl alcohol [12] $C_8H_{17}OH$ and heptane [13] C_7H_{16}, both at 25°C. The obtained impedance and complex permittivity results are shown in Figs. 7–10. From these figures, it is clear that the obtained results correspond very well with the data available in the literature. In all cases, the errors were always found to be less than ± 5 percent.

It may be worth mentioning the factors which affected the frequency band over which the results were reported. The lower frequency limit was due to maintaining the validity of the input impedance expression in [10] which required $2\pi h/\lambda$, where λ is the wavelength in the material under test, to be larger than or equal to one. Therefore, the lower frequencies used in Figs. 7–10 were chosen so

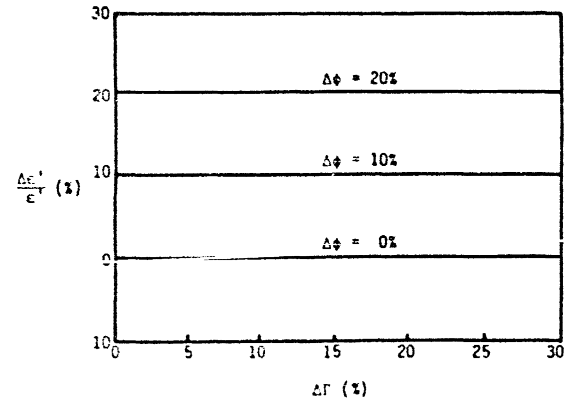

Fig. 3. Results of the error analysis in ϵ' due to uncertainties $\Delta\Gamma$ percent and $\Delta\phi$ percent in the magnitude and phase of the reflection coefficient. $\epsilon^* = 3.0 + j3.0$ and frequency $f = 400.0$ MHz.

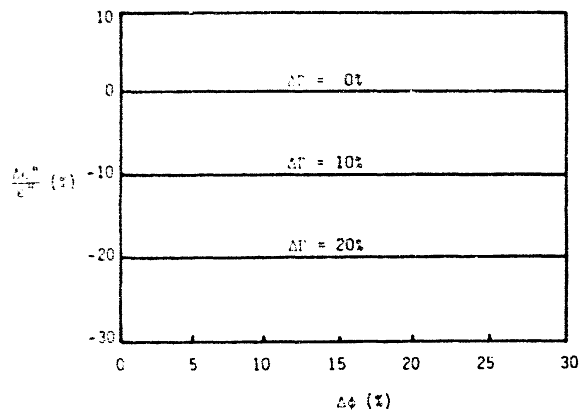

Fig. 4. Results of the error analysis in ϵ'' due to uncertainties $\Delta\Gamma$ percent and $\Delta\phi$ percent in the magnitude and phase of the reflection coefficient. $\epsilon^* = 3.0 + j3.0$ and frequency $f = 400.0$ MHz.

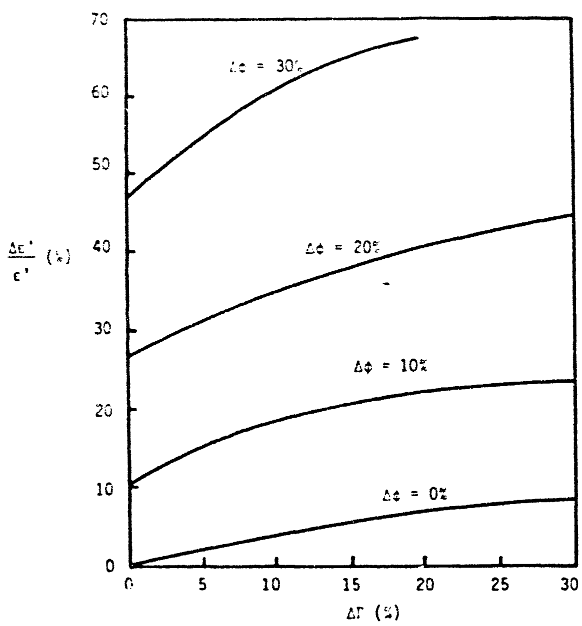

Fig. 5. Results of the error analysis in ϵ' due to uncertainties $\Delta\Gamma$ percent and $\Delta\phi$ percent in the magnitude and phase of the reflection coefficient. $\epsilon^* = 6.0 + j6.0$ and frequency $f = 400.0$ MHz.

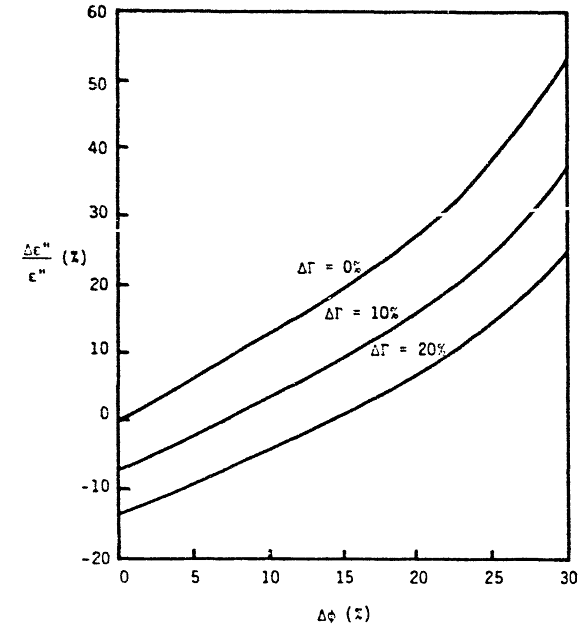

Fig. 6. Results of the error analysis in ϵ'' due to uncertainties $\Delta\Gamma$ percent and $\Delta\phi$ percent in the magnitude and phase of the reflection coefficient. $\epsilon^* = 6.0 + j6.0$ and frequency $f = 400.0$ MHz.

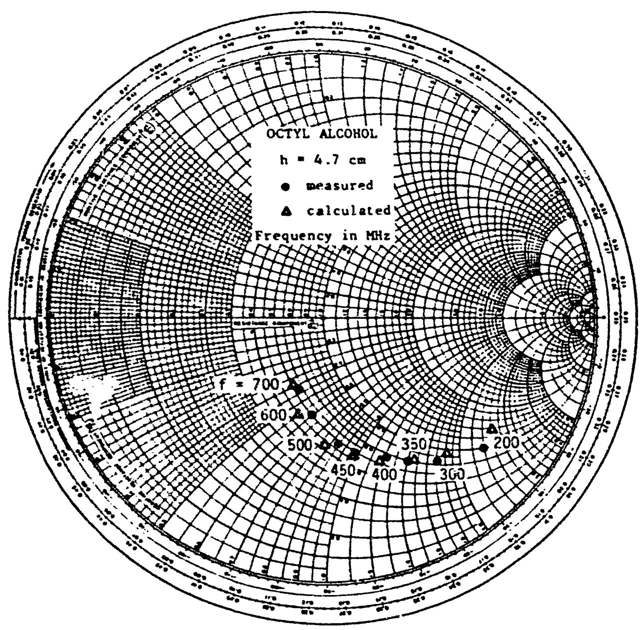

Fig. 7. Comparison of measured and calculated impedances of the dielectric probe at various frequencies when immersed in octyl alcohol.

that $2\pi h/\lambda = 1$ for specific extended length of the center conductor h. The upper frequency, on the other hand, was related to maintaining $2\pi h/\lambda$ sufficiently small, (e.g., h/λ smaller than 0.25) so as to reasonably contain the radiation within a test sample of reasonable size. In all cases, the sample size was maintained large enough so that reflections at sample boundaries may be neglected. Naturally, this is not an inherent limitation of the method when used with true *in situ* measurements. As a matter of fact, the

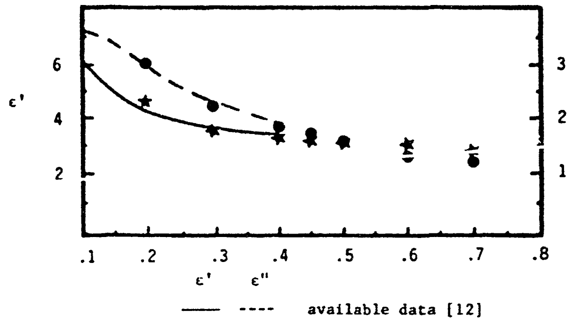

Fig. 8. The measured complex permittivity $\epsilon^* = \epsilon' + j\epsilon''$ of octyl alcohol as a function of frequency (GHz).

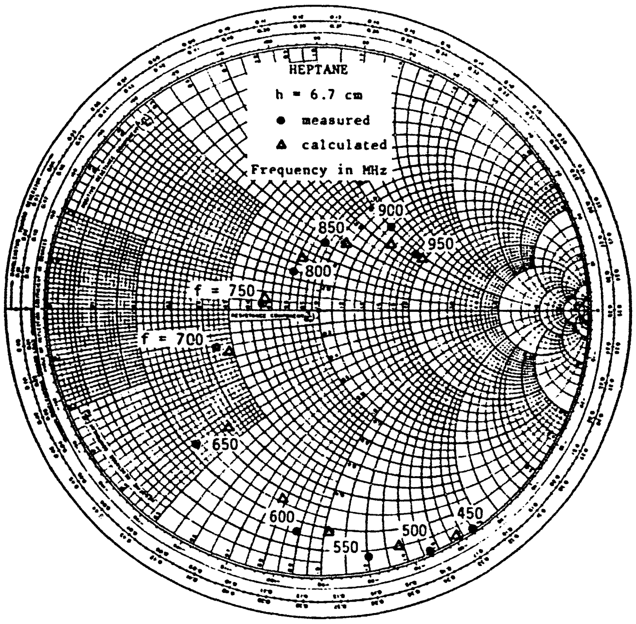

Fig. 9. Comparison of measured and calculated impedances of the dielectric probe at various frequencies when immersed in heptane.

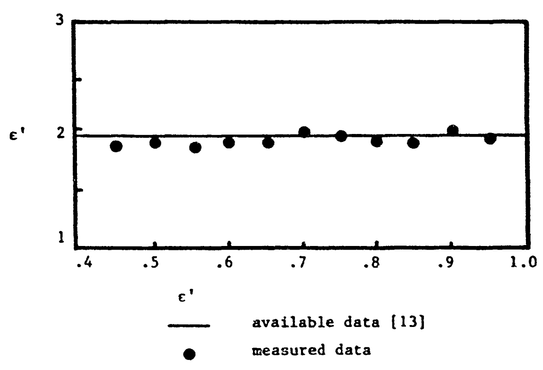

Fig. 10. The measured complex permittivity $\epsilon^* = \epsilon' + j\epsilon''$ of heptane as a function of frequency (GHz).

possibility of changing the volume of the measured test sample by changing the frequency in an *in situ* dielectric measurement is a very desirable characteristic in applications of interest to the oil industry.

IV. Discussion and Conclusions

A novel *in situ* procedure for measuring the dielectric properties of low, complex permittivity materials has been developed. The dielectric probe is basically an open-ended coaxial line with an extended center conductor immersed in the material under test. The measurement technique utilizes a rigorous expression for the input impedance of the monopole probe and a complex zero-finding routine to determine the dielectric parameters of the material under test. Results of the uncertainty analysis were presented together with experimental results for lossless and lossy low-permittivity materials. The complex permittivity results were within ± 5 percent of the data available in the literature. The upper-frequency limit of the probe is related to maintaining the volume of the test sample, a reasonable one to avoid errors due to extending the radiation beyond the sample boundaries. With true *in situ* measurements, this upper-frequency limit may be of little importance. The low-frequency limit, on the other hand, is related to the validity of the input impedance expression which requires $2\pi h/\lambda$ to be equal to or larger than unity [10]. This probe and the described measurement procedure provides for the first time the capability of changing the effective volume of the sample under test by simply varying the measurement frequency and/or the length of the extended center conductor. The limitations on the previously used analysis procedure [1], [2], and specifically the restrictions on the length of the extended portion of the center conductor of the dielectric probe, have made the previously reported measurements particularly sensitive to the dielectric properties in the immediate neighborhood of the probe. Efforts to extend the *in situ* measurements to true on-line characterization of rock formation, as well as to utilize the time-domain techniques for broadband characterization of the dielectric material under test from single measurement [12] are presently underway.

Appendix A

The approximations made are:

$$(a/h) << 1 \quad (a/\lambda) << 1 \quad \beta h > 1 \quad 0 \le (\sigma/\omega\epsilon) \le \infty.$$

The complete equation is

$$Z_i^* \text{ (equation)} = \frac{\omega\mu_0}{j2k(S + CU)} \text{ ohms}$$

where

$$C = -\frac{1}{2}\frac{(2T - T')\sin(kh) - (2S - S')\cos(kh)}{T'\cos(kh) + S'\sin(kh)}$$

$$U = -j(A_2 - A_3).$$

The following quantities are involved in the definition of C and U:

$$\gamma = 0.57722, \quad \gamma' = 1.6449$$

$$\Omega_0 = \ln(\lambda/a) - \ln \pi - \gamma, \quad \Omega_0' = \Omega_0 - \ln 2$$

$$A_1 = \ln\left[1 + j\frac{\pi}{\Omega_0'}\right] + \frac{\pi^2}{12}\left[\frac{1}{(\Omega_0' - \ln 2)^2} - \frac{1}{(\Omega_0' - \ln 2 + j\pi)^2}\right]$$

$$\Omega_2 = 2\Omega_0' + \ln(2kh) + \gamma - j\frac{\pi}{2}, \quad \Omega_2' = \Omega_2 + \ln 2$$

$$\Omega_3 = \Omega_2 + j2\pi, \quad \Omega_3' = \Omega_2' + j2\pi$$

$$A_2 = \ln(\Omega_3/\Omega_2) + \tfrac{1}{2}\gamma'[\Omega_2^{-2} - \Omega_3^{-2}]$$

$$A_2' = \ln(\Omega_3'/\Omega_2') + \tfrac{1}{2}\gamma'[\Omega_2'^{-2} - \Omega_3'^{-2}]$$

$$A_3 = -j\frac{1}{2kh}e^{2jkh}\left[\frac{1}{\Omega_2} - \frac{1}{\Omega_3}\right]$$

$$A_3' = -j\frac{1}{4kh}e^{4jkh}\left[\frac{1}{\Omega_2'} - \frac{1}{\Omega_3'}\right]$$

$$S = \tfrac{1}{2}(-A_1 + A_2 + A_3), \quad S' = \tfrac{1}{2}(-A_1 + A_2' + A_3')$$

$$T = j\tfrac{1}{2}(-A_1 - A_2 + A_3), \quad T' = j\tfrac{1}{2}(-A_1 - A_2' + A_3').$$

The propagation constant of the medium is given by:

$$k^1 = \beta + j\alpha = \omega\left[\mu\left(\epsilon' - j\frac{\sigma}{\omega}\right)\right]^{1/2}$$

[1]In (16) of [10], ω was missing. The above expression for k is the correct one.

where

$$\sigma = \omega\epsilon''.$$

References

[1] E. C. Burdette, F. L. Cain, and J. Seals, "*In-vivo* probe measurement technique for determining dielectric properties at VHF through microwave frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 414-427, 1980.
[2] T. W. Athey, M. A. S...chly, and S. S. Stuchly, "Measurement of radio-frequency permittivity of biological tissue with an open-ended coaxial line: Part I," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 82-86, 1982.
[3] H. P. Schwan and K. R. Foster, "RF-field interactions with biological systems: Electrical properties and biophysical mechanisms," *Proc. IEEE*, vol. 68, pp. 104-113, 1980.
[4] L. E. Larsen, J. H. Jacobi, and A. K. Krey, "Preliminary observations with an electromagnetic method for the noninvasive analysis of cell suspension physiology and induced pathophysiology," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 581-595, 1978.
[5] E. C. Burdette, J. Seals, R. L. Magin, and S. P. Auda, "Dielectric properties of normal and neoplastic animal and human tissues determined from *in vivo* probe measurements," *Cancer Research*, in press.
[6] M. F. Iskander and J. B. DuBow, "Time- and frequency-domain technique for measuring the dielectric properties of rocks: A review," *J. Microwave Power*, vol. 18, pp. 55-74, 1983.
[7] R. Freedman and J. P. Vogiatzis, "Theory of microwave dielectric constant logging using the electromagnetic wave propagation method," *Geophysics*, vol. 44, pp. 969-986, 1979.
[8] M. F. Iskander, S. L. Rattlingourd, and J. Oomrigar, "A new electromagnetic propagation tool for well logging," *SPE Conf. Rep.*, SPE 13189, 1984.
[9] M. F. Iskander and S. C. Olson, "An *in-vivo* probe for measuring the dielectric properties of low-permittivity tissue," *6th BEMS Conf. Rep.*, 1984.
[10] D. W. Gooch, C. W. Harrison, Jr., R. W. P. King, and T. T. Wu, "Impedances of long antennas in air and in dissipative media," *J. Res. Nat. Bur. Stand.*, vol. 67D, pp. 355-360, 1963.
[11] D. E. Muller, "A method for solving algebraic equations using an automatic computer," *Mathematical Tables and Aids to Computation*, vol. 10, pp. 208-215, 1956.
[12] M. F. Iskander and S. S. Stuchly, "A time-domain technique for measurement of the dielectric properties of biological substances," *IEEE Trans. Instrum. Meas.*, vol. 21, pp. 55-74, 1972.
[13] A. Von Hippel, *Dielectric Materials and Applications*. Cambridge, MA: MIT Press, 1954.