KNOWING THE INFORMED PLAYER'S PAYOFFS AND SIMPLE PLAY IN REPEATED GAMES

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Abstract

We revisit the classic model of two-player repeated games with undiscounted utility, observable actions, and one-sided incomplete information, and further assume the informed player has state-independent preferences. We show the informed player can attain a payoff in equilibrium if and only if she can attain it in the simple class of equilibria first studied by Aumann, Maschler and Stearns (1968), in which information is only revealed in the game's initial stages. This sufficiency result does not extend to the uninformed player's equilibrium payoff set.

Following Aumann and Maschler (1966),[1] a rich literature ensued studying infinitely repeated, undiscounted games with incomplete information. The focus of this paper is on the two-player (non-zero-sum) environment in which only one player is informed about the state of the world, and players observe only

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[1]See the preface of Aumann and Maschler (1995) and the foreword in Mertens, Sorin and Zamir (2015) for historiographical accounts.

each other's actions in each stage of the repeated game. Aumann, Maschler and Stearns (1968) characterize payoff vectors (i.e., payoffs of both informed and uninformed players) of a particular class of equilibria, which we call *AMS equilibria*. In these equilibria, the informed player uses an initial finite number of stages to communicate information about the state to the uninformed player, with no additional information being conveyed for the rest of the game. Although simple, AMS equilibria entail a loss of generality. Hart (1985) provides a full characterization of all equilibrium payoff vectors using bimartingale processes—a type of martingale process studied in detail in Aumann and Hart (1986) that represents much more sophisticated communication and coordination between the players. Out of concern about the interpretability of an infinite-horizon model in which an uninformed player might never learn his own payoff, Shalev (1994) considers the "known-own payoff" case in which the *uninformed* player's payoff is state independent. He shows the payoff vectors attainable in fully revealing AMS equilibria characterize all possible equilibrium payoff vectors. Cripps and Thomas (2003) introduce discounting to the known-own payoff case and show that, when the informed player is arbitrarily patient relative to the uniformed player, the characterization of the informed player's payoffs is essentially the same as in the undiscounted case. They also study the case with equal discount rates for the players. Their result, and subsequent generalizations by Pęski (2008; 2014), demonstrate that the set of equilibrium payoff vectors in the undiscounted case is typically a strict subset of the limit set of equilibrium payoff vectors in the discounted case with equal discount rates.

Forges (1990) shows that tools developed by Aumann, Maschler and Stearns (1968) and Hart (1985) in the context of repeated games can also be applied to games of pure communication, that is, non-repeated games in which players are endowed with explicit forms of communication.[2] Aumann and Hart (2003) charac-

[2]See Forges (2020) for a survey that details the connections between the repeated-games literature and the communication literature.

terize the equilibrium payoff vectors in long cheap-talk games using bimartingale processes. Lipnowski and Ravid (2020), inter alia, show that when an informed sender has state-independent preferences, the sender does not benefit from extra rounds of pre-play communication (although the uninformed receiver can benefit from such extra rounds).

The current paper specializes the repeated games studied by Aumann, Maschler and Stearns (1968) and Hart (1985) to the case in which the informed player's preferences are state independent. Using results from Hart (1985), Aumann and Hart (1986), and Lipnowski and Ravid (2020), we show AMS equilibria are sufficient for attaining all of the informed player's equilibrium payoffs. We also provide two examples demonstrating AMS equilibria are insufficient for capturing the entire set of equilibrium payoff vectors.

1 Model

Our model is a two-player repeated game with undiscounted utility, one-sided incomplete information, and observable actions (as studied by Aumann, Maschler and Stearns (1968) and Hart (1985)), specialized to the case in which the informed player has state-independent preferences. Formally, the game has two players: one informed (player 1) and one uninformed (player 2). The game begins with a realization of a payoff-relevant random state θ from a finite set Θ (with at least two elements) according to a full-support distribution $\mu_0 \in \Delta\Theta$.[3] Then, player 1 observes the realization of θ, and the players subsequently play the stage game infinitely many times. In each period $t \in \mathbb{N}$, each player $j \in \{1,2\}$ chooses an action from a finite set A_j (with at least two elements) simultaneously, and the

[3]We adopt the following notational conventions throughout the paper. Given a finite set X, let ΔX denote the set of all probability measures over X. Given a real-valued function f on some convex space Z, let $\text{vex}\, f$ denote the convexification of f (i.e., pointwise largest convex function on Z that does not exceed f). Given a correspondence $V : X \rightrightarrows Y$, let $\text{gr}(V)$ denote its graph $\{(y,x) :\ x \in X,\ y \in V(x)\}$.

stage payoff is given by $u_1 : A \to \mathbb{R}$ and $u_2 : A \times \Theta \to \mathbb{R}$ for players 1 and 2, respectively, where $A := A_1 \times A_2$. At the end of each period, players observe the period's chosen action profile but not the resulting payoffs.

Let $\sigma_1 : \mathcal{H} \times \Theta \to \Delta A_1$ denote player 1's strategy and let $\sigma_2 : \mathcal{H} \to \Delta A_2$ denote player 2's strategy, where $\mathcal{H} := \bigcup_{t=0}^{\infty} A^t$ is the set of public histories. A strategy profile $\sigma = (\sigma_1, \sigma_2)$ and a belief $\mu \in \Delta\Theta$ induce a unique probability measure $\mathbb{P}_{\sigma,\mu}$ on $\Omega := A^\infty \times \Theta$; let $\mathbb{E}_{\sigma,\mu}$ denote the expectation operator with respect to $\mathbb{P}_{\sigma,\mu}$.[4] Define expectations of players 1 and 2's payoffs up to and including stage t, respectively, as

$$v_1^t(\sigma) := \mathbb{E}_{\sigma,\mu_0}\left[\frac{1}{t}\sum_{s=1}^{t} u_1(a_t)\right],\ v_2^t(\sigma) := \mathbb{E}_{\sigma,\mu_0}\left[\frac{1}{t}\sum_{s=1}^{t} u_2(a_t,\theta)\right]. \tag{1.1}$$

Following Aumann and Maschler (1968),[5] a strategy profile σ is an *equilibrium* if:[6]

$$\liminf_{t\to\infty} v_1^t(\sigma) \geq \limsup_{t\to\infty} \sup_{\sigma_1'} v_1^t(\sigma_1', \sigma_2) \text{ and } \liminf_{t\to\infty} v_2^t(\sigma) \geq \limsup_{t\to\infty} \sup_{\sigma_2'} v_2^t(\sigma_1, \sigma_2'). \tag{1.2}$$

The payoffs for players 1 and 2 associated with an equilibrium σ are their respective limit payoffs (which are assured to exist given (1.2)):

$$v_1(\sigma) := \lim_{t\to\infty} v_1^t(\sigma) \text{ and } v_2(\sigma) := \lim_{t\to\infty} v_2^t(\sigma). \tag{1.3}$$

[4]For each $t \in \mathbb{N}$, let $\mathscr{H}^t$ be the finite algebra generated by the discrete algebra on A^t, and let $\mathscr{H}^\infty$ denote the product σ-algebra on A^∞. Then, $\mathbb{P}_{\sigma,\mu}$ is a probability measure on the measurable space $(\Omega, \mathscr{H}^\infty \otimes 2^\Theta)$, which is uniquely defined by the Kolmogorov extension theorem.

[5]Aumann, Maschler and Stearns (1968) and Hart (1985) refer to σ that satisfies (1.2) as a uniform equilibrium. The associated payoffs $(v_1(\sigma), v_2(\sigma))$ of a (uniform) equilibrium σ (see (1.3)) can be achieved in an ϵ-equilibrium of some finitely repeated game; that is, for any $\epsilon > 0$, there exists $T_\epsilon \in \mathbb{N}$ such that, for all $t > T_\epsilon$, $v_1^t(\sigma', \sigma_2) \leq v_1(\sigma) + \epsilon$ for all σ_1' and $v_2^t(\sigma_1, \sigma_2') \leq v_2(\sigma) + \epsilon$ for all σ_2'. Hart (1985) defines an equilibrium in which T_ϵ may depend on σ and shows the set of payoffs of such equilibria and the set of payoffs of the equilibrium defined above coincide (Proposition 2.1.4).

[6]Unlike under more general payoff environments (e.g., Simon et al., 1995; Shalev, 1994), equilibrium existence is immediate; e.g., players could employ, i.i.d. across histories, a mixed-strategy equilibrium from the complete-information static game with payoffs u_1 and $\int_\Theta u_2(\cdot,\theta)\mathrm{d}\mu_0(\theta)$.

A vector $\vec{s} = (s_1, s_2) \in \mathbb{R}^2$ is an *equilibrium payoff vector* if an equilibrium σ exists such that $\vec{s} = (v_1(\sigma), v_2(\sigma))$. For $j \in \{1, 2\}$, $s_j \in \mathbb{R}$ is an *equilibrium Pj-payoff* if an equilibrium σ exists for which $s_j = v_j(\sigma)$.

2 Result

Aumann, Maschler and Stearns (1968) characterize equilibrium payoff vectors that can be induced by a simple type strategy profiles in which player 1's behavior in the initial stages is used to communicate information about θ to player 2, and the players subsequently coordinate their actions with no further information being revealed. We call an equilibrium σ an *AMS equilibrium* if some $\ell \in \mathbb{N}$ exists such that (i) σ_1 does not condition on θ for any on-path history after stage ℓ; and (ii) players ignore player 2's behavior in the first ℓ stages.[7] We refer to an equilibrium payoff vector associated with an AMS equilibrium as an *AMS-equilibrium payoff vector*. Finally, for $j \in \{1, 2\}$, $s_j \in \mathbb{R}$ is an *AMS-equilibrium Pj-payoff* if some AMS equilibrium σ exists such that $s_j = v_j(\sigma)$.

Aumann, Maschler and Stearns (1968) provide examples of equilibrium payoff vectors that cannot be AMS-equilibrium payoff vectors. Hart (1985) subsequently provides a characterization of all equilibrium payoffs via strategy profiles that allow players to engage in more sophisticated communication and coordination than in Aumann, Maschler and Stearns (1968). Our main result is that, when player 1's preference is state independent, the additional sophistication allowed under Hart (1985) is unnecessary from player 1's perspective. In particular, simple communication-coordination strategy profiles that induce AMS equilibria are sufficient. We formally state the result below.

[7]Formally, an equilibrium σ is an AMS equilibrium if, for any $t \in \mathbb{Z}_+$ with $t \geq \ell$, any pair of public histories $h, h' \in A^t$, and any pair of states $\theta, \theta' \in \Theta$, we have (i) $\sigma_1(h, \theta) = \sigma_1(h, \theta')$, and (ii) if h and h' differ only in the first ℓ periods of player 2's play, then $\sigma_1(h, \theta) = \sigma_1(h', \theta)$ and $\sigma_2(h) = \sigma_2(h')$.

Proposition 1. *Any equilibrium P1-payoff is an AMS-equilibrium P1-payoff.*

The proof applies Hart's (1985) characterization of equilibrium payoff vectors via bimartingales, showing player 1's payoff induced by such a martingale satisfies the conditions for an AMS equilibrium when player 1's payoff is state independent.

We first introduce a correspondence yielding the set of payoffs that are feasible and individually rational given any belief $\mu \in \Delta\Theta$ as defined by Hart (1985). Letting $\overline{u}$ be a bound on the players' possible payoff magnitudes and $\mathcal{R} := [-\overline{u}, \overline{u}] \subseteq \mathbb{R}$, define

$$F^* : \Delta\Theta \rightrightarrows \mathcal{R}^2 \tag{2.1}$$

$$\mu \mapsto \{(s_1, s_2) \in F(\mu) : \ s_1 \geq \underline{u}_1,\ s_2 \geq \operatorname{vex} \underline{u}_2(\mu)\}, \tag{2.2}$$

where

$$\begin{aligned} F(\mu) &: \Delta\Theta \rightrightarrows \mathcal{R}^2 \\ \mu &\mapsto \operatorname{co}\left\{\left(u_1(a), \int_\Theta u_2(a, \cdot)\, \mathrm{d}\mu\right) : \ a \in A\right\} \end{aligned}$$

is the correspondences that gives the set of feasible expected payoffs in the one-stage game from using a correlated state-independent strategy given a prior belief $\mu \in \Delta\Theta$; $\underline{u}_1 \in \mathbb{R}$ and $\underline{u}_2 : \Delta\Theta \to \mathbb{R}$ are the minmax values for players 1 and 2, respectively, in the one-stage game in which neither player observes the realization of θ with a common prior belief $\mu \in \Delta\Theta$; that is

$$\begin{aligned} \underline{u}_1 &:= \min_{\alpha_2 \in \Delta A_2} \max_{\alpha_1 \in \Delta A_1} \int_A u_1 \mathrm{d}(\alpha_1 \otimes \alpha_2), \\ \underline{u}_2(\mu) &:= \min_{\alpha_1 \in \Delta A_1} \max_{\alpha_2 \in \Delta A_2} \int_{A \times \Theta} u_2 \mathrm{d}(\alpha_1 \otimes \alpha_2 \otimes \mu). \end{aligned}$$

Define $F_1^* : \Delta\Theta \rightrightarrows \mathcal{R}$ as the projection of F^* to player 1's payoffs; that is

$$F_1^*(\mu) := \{s_1 \in \mathcal{R} : \exists s_2 \in \mathcal{R} \text{ with } (s_1, s_2) \in F^*(\mu)\}.$$

We first observe that F_1^* is a Kakutani correspondence (i.e., nonempty-valued, compact-valued, convex-valued, and upper hemicontinuous).

Lemma 1. *The correspondence F_1^* is a Kakutani correspondence.*

Proof. Let $\hat{F} : \Delta\Theta \rightrightarrows \mathcal{R}^2$ denote the correspondence $\mu \mapsto \{(u_1(a), \int_\Theta u_2(a, \cdot)\mathrm{d}\mu) : a \in A\}$. Observe that $\hat{F}$ is closed- and bounded-valued. Thus, $\hat{F}$ is compact-valued, and because its graph is closed (being a union of the graphs of finitely many continuous functions), $\hat{F}$ is upper hemicontinuous. As a convex hull of the real-valued correspondence $\hat{F}$, F is convex- and compact-valued and upper hemicontinuous. Define $\tilde{F} : \Delta\Theta \rightrightarrows \mathcal{R}^2$ as the individual rationality correspondence; that is, $\mu \mapsto \{(s_1, s_2) \in \mathcal{R}^2 : s_1 \geq \underline{u}_1, s_2 \geq \operatorname{vex} \underline{u}_2(\mu)\}$. Observe that $\tilde{F}$ is convex-, closed-, and bounded-valued (taking values in a bounded set $\mathcal{R}^2$). Moreover, $\tilde{F}$ is upper hemicontinuous because $\operatorname{vex} \underline{u}_2$ is lower semicontinuous. Because $\hat{F}$ is convex-valued with a closed graph, whereas $\tilde{F}$ is convex-valued with a compact graph, their intersection is convex-valued with a compact graph. Hence, F^* is convex-valued, compact-valued, and upper hemicontinuous. For any $\mu \in \Delta\Theta$, $F^*(\mu)$ contains the payoff vector associated with player 1 playing the minmax mixed strategy for u_1 and player 2 playing a minmax mixed strategy for $\int_\Theta u_2(\cdot, \theta)\mathrm{d}\mu(\theta)$, and so F^* is nonempty-valued. Therefore, F^* is a Kakutani correspondence. Because F_1^* is a projection of F^* to the first coordinate, which is a continuous transformation, F_1^* is a Kakutani correspondence. ■

The remainder of the proof centers around the following lemma that follows immediately from the Main Theorem in Hart (1985). To state it, define a *bimartingale* as a $\mathcal{R} \times \Delta\Theta$-valued martingale, $\{(s^t, \mu^t)\}_{t=1}^\infty$, on some filtered probability space such that, for all $t \in \mathbb{N}$, either $s^{t+1} = s^t$ almost surely or $\mu^{t+1} = \mu^t$ almost surely. We say the bimartingale has *initial value* (s, μ) if $(s^1, \mu^1) = (s, \mu)$ almost surely. Given a measurable subset Z of $\mathcal{R} \times \Delta\Theta$, we say the bimartingale has *terminal values in* Z if the almost-sure limit of the martingale is contained in Z almost surely.

Lemma 2. *If s_1 is an equilibrium $P1$-payoff, then some bimartingale exists with initial value (s_I, μ_0) and terminal values in* $\mathrm{gr}(F_1^*)$.

Our goal now is to show the bimartingale given by Lemma 2 gives an AMS-equilibrium $P1$-payoff. The lemma below, which follows from a definition of AMS equilibrium as pointed out by Hart (1985),[8] gives a sufficient condition for a payoff $s_1 \in \mathcal{R}$ to be an AMS-equilibrium $P1$-payoff. We call a distribution over posterior beliefs, $p \in \Delta\Delta\Theta$, that averages to μ_0 an *information policy* and let $\mathcal{I}(\mu_0) = \{p \in \Delta\Delta\Theta : \int_{\Delta\Theta} \mu \mathrm{d}p(\mu) = \mu_0\}$ denote the set of all information policies.

Lemma 3. *Let $s_1 \in \mathcal{R}$. Suppose some finite-support information policy, $p \in \mathcal{I}(\mu_0)$, exists such that*

$$p\left(\{\mu \in \Delta\Theta : s_1 \in F_1^*(\mu)\}\right) = 1. \tag{2.3}$$

Then, s_1 is an AMS-equilibrium $P1$-payoff.

The following lemma, essentially proved as part of Proposition 4 in Lipnowski and Ravid (2020), serves as a link between the previous two lemmata.

Lemma 4. *Suppose $V : \Delta\Theta \rightrightarrows \mathbb{R}$ is a Kakutani correspondence. If some bimartingale exists with initial value (s_1, μ_0) and terminal values in $\mathrm{gr}(V)$, then some $p \in \mathcal{I}(\mu_0)$ exists such that*

$$p\left(\{\mu \in \Delta\Theta : s_1 \in V(\mu)\}\right) = 1. \tag{2.4}$$

Proof. We prove the contrapositive statement. Consider $s_1 \in \mathcal{R}$ such that $p \in \mathcal{I}(\mu_0)$ exists for which (2.4). In the proof of Proposition 4, Lipnowski and Ravid

[8]See the second paragraph of section 6 in Hart (1985).

(2020) construct a continuous and biconvex function $B : \mathcal{R} \times \Delta\Theta \to \mathbb{R}$ such that $B|_{\mathrm{gr}(V)} = 0$ and $B(s_1, \mu_0) > 0$. The result then follows from Theorem 4.7 in Aumann and Hart (1986). ■

We are now ready to prove our main result.

Proof of Proposition 1. . Let $s_1 \in \mathcal{R}$ be an equilibrium $P1$-payoff. Lemma 2 delivers a bimartingale with initial value (s_1, μ_0) and terminal values in $\mathrm{gr}(F_1^*)$. By Lemma 4 and 1, some $p \in \mathcal{I}(\mu_0)$ exists that satisfies (2.3). Because Θ is finite, Carathéodory's theorem implies p may be chosen to ensure $\mathrm{supp}(p)$ is finite. Hence, s_1 is an AMS-equilibrium $P1$-payoff by Lemma 3. ■

3 Examples

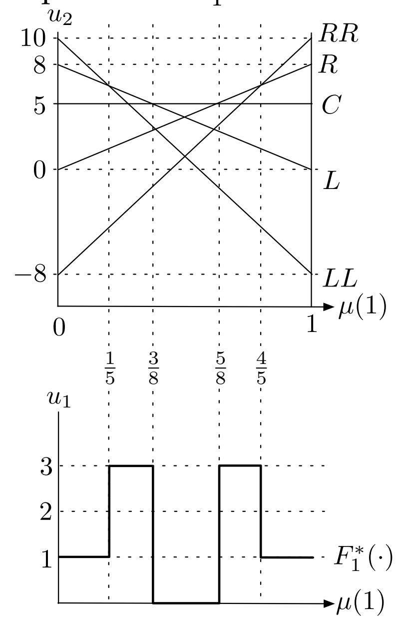
Proposition 1 does not extend to characterizing equilibrium payoff vectors. We demonstrate this fact by adapting two examples from the cheap-talk literature.[9] The first example, adapted from Aumann and Hart (2003), demonstrates some equilibrium payoff vector exists that is not an AMS-equilibrium payoff vector. Nevertheless, the set of equilibrium Pj-payoffs in this example coincides with the set of AMS-equilibrium Pj-payoffs for each player j. In Example 2, adapted from Lipnowski and Ravid (2020), we show an equilibrium $P2$-payoff exists that is not attainable in an AMS equilibrium. Taken together, the examples demonstrate that whereas assuming player 1's preferences are state independent simplifies the characterization of equilibrium $P1$-payoffs, the same simplification does not apply to the set of attainable equilibrium payoff vectors or the set of equilibrium $P2$-payoffs.

[9] In the cheap-talk version of the examples, the informed player has no "action" to take but can send a cheap-talk message to the uninformed player, whose action is payoff relevant. Because our environment has no explicit communication technology, we adapt the original examples by allowing the informed player to have least two payoff-irrelevant actions.

Example 1 (Example 2.6 in Aumann and Hart (2003))**.** Two possible states, $\Theta := \{0, 1\}$, are equally likely, and player 2 has five possible actions, $A_2 := \{LL, L, C, R, RR\}$. The figure below shows the payoffs associated with each action under each state, player 2's best response as a function of his belief that the state is B, $\mu(1)$, and player 1's value correspondence, F_1^*, as a function of $\mu(1)$.

Figure 3.1: Player 2's best response and F_1^*.

$u_1(\cdot\|\theta), u_2(\cdot\|\theta)$	LL	L	C	R	RR
$\theta = 0$	$1, 10$	$3, 8$	$0, 5$	$3, 0$	$1, -8$
$\theta = 1$	$1, -8$	$3, 0$	$0, 5$	$3, 8$	$1, 10$



As explained in Aumann and Hart (2003), $(2, 8)$ is an equilibrium payoff vector of this game. One can achieve this payoff, for example, by first performing a jointly controlled lottery with equal probabilities,[10] and depending on the outcome of the jointly controlled lottery, player 1 communicates the state either fully yielding a payoff of $(1, 10)$,[11] or partially so that player 2's posterior belief, $\mu(1)$, is either $\frac{1}{4}$ or $\frac{3}{4}$ yielding a payoff of $(3, 6)$.[12]

[10] For example, player 1 chooses first-stage action uniformly and player 2 chooses first-stage action uniformly among $\{LL, RR\}$. Then, player 1 fully reveals if and only if $a_1^1 = 1$ or $a_2^1 = LL$, and partially reveals (in the manner specified below) if $a_1^1 = 0$ and $a_2^1 = RR$.

[11] For example, player 1 chooses $a_1^2 = \theta$ if and only if the state is $\theta \in \Theta$.

[12] For example, player 1 chooses $a_1^2 = 1$ with probability $\frac{1}{4}$ if the state is 1 and with probability $\frac{3}{4}$ if the state is 0.

We now argue $(2, 8)$ cannot be an AMS-equilibrium payoff vector. First, observe that jointly controlled lotteries cannot be part of any AMS-equilibrium strategy, because players do not ignore player 2's action in responding to the lottery. To induce a payoff of 2 for player 1 in an AMS equilibrium, the posterior belief for player 2 must therefore always be one of $\frac{1}{5}$, $\frac{3}{8}$, $\frac{5}{8}$, or $\frac{4}{5}$. However, with such beliefs, player 2's expected payoffs are strictly below 8. Hence, it follows that $(2, 8)$ cannot be an AMS-equilibrium payoff vector.

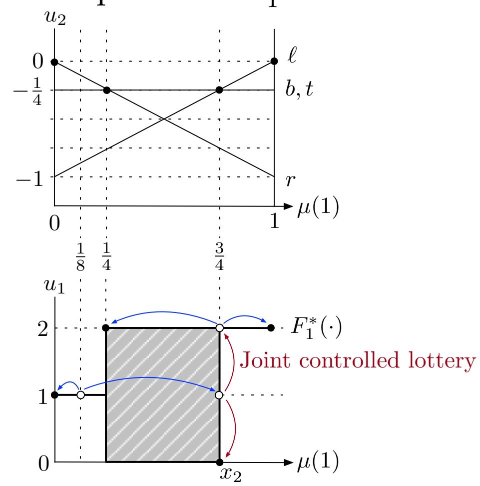
However, let us observe that the set of equilibrium $P2$-payoffs and the set of AMS-equilibrium $P2$-payoffs coincide. First, observe that in any equilibrium, player 2's payoff must lie in $[5, 10]$. Lemma 3 together with the fact that player 1's value correspondence is symmetric means any $s_1 \in [0, 3]$ is an AMS-equilibrium $P1$-payoff, and any such s_1 can be achieved by inducing a symmetric posterior belief around $\frac{1}{2}$. Because such distribution over posterior beliefs can induce any expected payoff for player 2 in $[5, 10]$, it follows that any equilibrium $P2$-payoff is an AMS-equilibrium $P2$-payoff.

We now demonstrate that the last observation, namely, that the set of equilibrium $P2$-payoffs and the set of AMS-equilibrium $P2$-payoffs coincide, does not hold generally.

Example 2 (Appendix C.3 in Lipnowski and Ravid (2020))**.** Two states, $\Theta := \{0, 1\}$, are possible, and the prior belief is that the state is 1 with probability $\frac{1}{8}$. Player 2 has four possible actions, $A_2 := \{\ell, b, t, r\}$. The figure below shows the payoffs associated with each action under each state, player 2's best response as a function of his belief that the state is 1, $\mu(1)$, and player 1's value correspondence, F_1^*, as a function of $\mu(1)$.

Figure 3.2: Player 2's best response and F_1^*.

$u_1(\cdot \mid \theta), u_2(\cdot \mid \theta)$	ℓ	b	t	r
$\theta = 0$	$1, -1$	$0, -\frac{1}{4}$	$2, -\frac{1}{4}$	$2, 0$
$\theta = 1$	$1, 0$	$0, -\frac{1}{4}$	$2, -\frac{1}{4}$	$2, -1$



By Lemma 3, because $F_1^*(\mu) = \{1\}$ for any $\mu(1) \leq \frac{1}{8}$, player 1's payoff must be 1 in any AMS equilibrium. Moreover, in any AMS equilibrium, player 2's maximum payoff is $-\frac{1}{24}$, corresponding to a distribution over beliefs $\mu(1) = 0$ with probability $\frac{5}{6}$ and $\mu(1) = \frac{3}{4}$ with probability $\frac{1}{6}$.[13] However, as explained in Lipnowski and Ravid (2020), players may perform a jointly controlled lottery (with equal probabilities) following realization of posterior belief $\frac{3}{4}$ (red arrows), and player 1 could further communicate (upper set of blue arrows) so that player 2's payoff will be supported by the solid dots as shown in the figure, which must yield a strictly higher payoff than $-\frac{1}{24}$. Such splits (called diconvexifications) are allowed under Hart's (1985) characterization, and it follows that the resulting player-2 payoff is an equilibrium $P2$-payoff. Thus, some equilibrium $P2$-payoff exists that is not an AMS-equilibrium $P2$-payoff.

In the case in which, instead, the *uninformed* player (player 2) has state-independent preferences, Shalev (1994) shows every equilibrium payoff vector is a fully revealing AMS-equilibrium payoff vector. In the case in which the informed player (player 1) has state-independent preference, however, fully revealing AMS-equilibrium

[13]For example, player 1 chooses $a_1^1 = 1$ with probability 1 if the state is 1 and with probability $\frac{1}{21}$ if the state is 0.

$P1$-payoffs do not characterize equilibrium $P1$-payoffs. For instance, in Example 1, attainable payoffs for player 1 (e.g., 5) exist that are unattainable with a fully revealing AMS equilibrium; and Example 2 has the feature that no fully revealing AMS equilibrium exists; hence, none of the nonempty set of attainable payoffs for player 1 can be attained in a fully revealing AMS equilibrium.

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