

Robust Predictions in Games with Rational Inattention^{*}

Tommaso Denti[†]
Cornell University

Doron Ravid[‡]
University of Chicago

June 16, 2023

Abstract

We derive robust predictions in games involving flexible information acquisition, also known as *rational inattention* (Sims, 2003). These predictions remain accurate regardless of the specific methods players employ to gather information. Compared to scenarios where information is predetermined, rational inattention reduces welfare and introduces additional constraints on behavior. We show these constraints generically do not bind; the two knowledge regimes are behaviorally indistinguishable in most environments. Yet, we demonstrate the welfare difference they generate is substantial: optimal policy depends on whether one assumes information is given or acquired. We provide the necessary tools for policy analysis in this context.

1. Introduction

The economics discipline has long recognized that information matters for incentives. Many studies have also noted the reverse, namely, that incentives shape information. The theory of rational inattention, initiated by Sims (2003), is a case in point. Motivated by people’s limited cognition, this theory postulates that agents pay attention as if they flexibly acquire information in an optimal fashion, trading off costs and benefits. This approach has been very successful, seeing a wide range of applications all across economics.¹ There is, however, a caveat: rational inattention models often require one to commit to difficult-to-test details of the information acquisition environment. This paper addresses this concern by developing

^{*}We thank Alex Bloedel, Benjamin Brooks, Modibo Camara, Marina Halac, Emir Kamenica, Ilia Krasikov, Elliot Lipnowski, Andrew McClellan, Stephen Morris, Luciano Pomatto, Daniel Rappoport, Phil Reny, and Joe Root for helpful discussions.

[†]tjd237@cornell.edu

[‡]dravid@uchicago.edu

¹See, e.g., Maćkowiak, Matějka, and Wiederholt (2023) for a recent review.

a framework for making predictions under rational inattention that are robust to the exact structure of agents’ learning technologies.

For concreteness, consider the following example.

Example 1. Two investors, Ann and Bob, are choosing whether or not to invest in a new project, and if so, which. There are two new projects: project A and project B . Each investor can either invest in one of these projects, or put their funds in the market. There are also two equally likely payoff states, θ_A and θ_B , that describe which project is better. If both investors fund project $t \in \{A, B\}$, they both get a payoff of 2 if t is the better project (i.e., if the state is θ_t), and payoff of 1 in the alternative state. If the two investors fund different projects, or one of the investors chooses to put their funds in the market, they both receive a payoff of 0 regardless of the state. In this scenario, the only impediment for Ann and Bob to outperform the market is miscoordination.

The above is an example of (what is commonly called) a *base game*. A base game describes an economic environment of interest: the state of fundamentals and its prior distribution, agents in the role of players, actions, and preferences. Given a base game, we aim to make predictions about behavior and welfare.

Standard rational inattention models obtain predictions by combining a base game with what we term an *information technology*, which specifies players’ learning capabilities regarding the true payoff state. In early studies, this technology assumes that information costs are proportional to entropy reduction (e.g., Matějka and McKay, 2015), and that players can flexibly gather information, provided that it is independent of the other agents’ information (e.g., Maćkowiak and Wiederholt, 2009; Yang, 2015). Recent papers consider more general information technologies, allowing for different cost functions and potentially correlated signals.²

Together, a base game and an information technology define an *information acquisition game*. This game begins with the players covertly choosing what costly information to acquire. Subsequently, each player privately uses the acquired knowledge to take an action. Predictions are derived by solving for Nash equilibrium.

We depart from the standard approach, and do not pair the base game with a fixed information technology. Instead, we simultaneously find all the predictions one can obtain with a technology that is consistent with rational inattention. Specifically, we consider the information technologies that satisfy three properties. First, no information comes at zero cost. Second, (strictly) more informative signals (in the sense of Blackwell 1951, 1953) cost (strictly) more. And third, information choice is flexible. These properties are satisfied

²See, e.g., Morris and Yang (2022); Ravid, Roesler, and Szentes (2022); Denti (forthcoming); Hebert and La’O (forthcoming)

by virtually all applications of rational inattention. We are agnostic whether signals are independent or correlated across players.

Theorem 1 characterizes all behavioral and welfare implications of rational inattention in a given base game. Behavior is summarized by the joint distribution of the actions taken by the players and the payoff state drawn by nature; we call such distribution the *outcome* of the game. Welfare is given by the *value* each player obtains from the game, that is, each player’s payoff, net any costs they pay for information.

We show rational inattention can generate any outcome that satisfies two constraints. To describe these constraints, consider the classical metaphor that views outcomes as being generated by a mediator who provides players with private but potentially correlated action recommendations after viewing the state. The first constraint rational inattention generates is *obedience*, which means no player can benefit by deviating from the mediator’s recommendations. This constraint represents optimal behavior given a fixed signal structure. We follow the literature (Bergemann and Morris, 2013, 2016), and use the term *Bayes correlated equilibrium* (BCE) to refer to obedient outcomes.³

The second constraint is *separation*: recommendations that result in different posterior beliefs about others’ behavior and the state cannot share an optimal reply. In other words, distinct beliefs have separate best responses. The separation constraint originates from Denti (2021), and stems from players’ information acquisition incentives. Intuitively, outcomes that violate separation involve the acquisition of non-instrumental information, a sub-optimal choice: by not learning this information, players can save on costs without impacting their gross payoffs from the base game. Thus, Theorem 1 says an outcome can be generated by rational inattention if and only if it is a separated BCE (sBCE).

Theorem 1 also provides tight bounds on the net payoff each player can get from a given equilibrium outcome. One bound is the utility the player expects from the outcome; that is, the gross payoff the player obtains by always following the mediator’s recommendation. We refer to this quantity as the outcome’s *gross value*. The second bound is the outcome’s *uninformed value*, which is given by the highest payoff the player can get by unilaterally deviating from all of the mediator’s recommendation to a fixed, deterministic action. Note that for obedient outcomes, this deviation must be unprofitable, meaning the uninformed value is lower than the gross value. Theorem 1 shows that, fixing a separated BCE, each player can attain any payoff that is above her uninformed value, and below her gross value.

Using Theorem 1 we study the impact information costs have on welfare, and whether rationally inattentive agents naturally seek special kinds of information. For this purpose, we compare the predictions made by rational inattention to those generated by models where information is an exogenous variable. Under exogenous information, players do not

³Forges (1993) refers to such outcomes as *universal Bayesian solutions*.

choose what they know; instead, they are endowed with a fixed signal structure. Bergemann and Morris (2013, 2016) show that an outcome can be induced by some signal structure if and only if the outcome is a BCE (i.e., it is obedient). Welfare is given by the outcome’s gross payoff, since no learning costs are paid when information is predetermined. Thus, Theorem 1 implies that, compared to exogenous information, rational inattention generates fewer outcomes, and lower payoffs per outcome.

In some cases, the separation constraint can dramatically shrink the BCE set. For a demonstration, consider Example 1.

Example 1 (Continued). Any outcome in which the two investors perfectly coordinate their decision is obedient: each investor is always happy to take the same action as the other investor. However, among these outcomes, the only ones that are consistent with rational inattention and satisfy the separation constraint are those that assign a probability of either zero or one to the event where both investors put their money in the market.

For an explanation, note that for perfectly-coordinated outcomes, a “market” recommendation and a “project t ” recommendation yield different posterior beliefs regarding the other agent’s choice: the former recommendation implies the other investor is not going to fund any project, whereas the latter recommendation means they will fund project t . Notice though, that funding t is a best reply for both beliefs, because funding some project weakly dominates investing in the market. Hence, the two recommendations share a best response, but lead to different beliefs. It follows that any perfect-coordination outcome where sometimes neither investor funds any project and sometimes both investors choose the same project violates separation, and so is inconsistent with rational inattention.

In fact, one can show a stronger result: the only sBCE in which some investor forgoes funding any project with positive probability is the BCE in which both investors always keep their money in the market. Thus, the sBCE set is *nowhere dense* in the BCE set.

Turns out, however, that situations like Example 1, where the separation constraint binds, are rare. Specifically, we show in Theorem 2 that, for generic preferences, one can approximate every obedient outcome with outcomes that are both obedient and separated. In other words, perturbing the payoffs of any base game with a large set of non-separated BCEs turns it into a game where essentially all BCEs are separated. Thus, unless one commits to a highly structured economic setting or makes specialized assumptions on agents’ information technologies, rational inattention is behaviorally indistinguishable from exogenous information.

Nevertheless, even when rational inattention and exogenous information yield the same behavioral predictions, they may have very different welfare implications. Intuitively, when information is exogenous, the value of information must go to the players. By contrast,

in the worst case under rational inattention, players must pay this value to purchase their information. Theorem 3 shows information has positive value in the worst-case outcome under rational inattention for a generic set of base games. Thus, there is a robust class of environments where players' worst-case welfare under rational inattention is significantly lower than their minimal payoffs under exogenous information.

For a heuristic treatment of our results on genericity, consider again Example 1.

Example 1 (Continued). Suppose we perturb the economy so that the market's returns are reduced by $\epsilon > 0$. Such a perturbation makes investing in the market strictly dominated. As a result, the game has no BCE in which the investors keep their money in the market. Thus, consistent with Theorem 2, the perturbation pushes all of the non-separated BCEs we described above out of the BCE set.⁴

Next, we explain that for all $\epsilon > 0$, the worst-case welfare for the investors is lower under rational inattention than under exogenous information. To find the worst case under rational inattention, one needs to find the sBCE that minimizes the uninformed value. The unique minimizer turns out to be the outcome where the two investors always coordinate on the inferior project. Whereas the gross payoff of this BCE is 1, its uninformed value is 0.5: the best an investor can do without seeing the mediator's recommendation is to blindly invest in one of the projects (e.g., project *A*), and so match the other investor's decision only when that project is the inferior one. By contrast, if the gross value were to determine players' payoffs, the worst BCE for the investors gives them a payoff of 0.6.⁵

The above-mentioned differences in welfare can impact optimal policy. We give a proof of concept in Section 5. We consider a utilitarian social planner who understands the economic environment (i.e., the base game), but does not know the source of players' information (i.e., the information technology). The planner takes a worst-case approach, as is common in the robust mechanism and contract design literature.⁶ We characterize the set of binary-action symmetric games in which this planner's welfare evaluations depend on whether she assumes players' information is given or acquired. We then apply this characterization to a regime change game, in which several investors choose whether or not to attack a distressed financial institution. The institution fails if the number of attackers surpasses a threshold, which represents the institution's fundamentals. We show that the distribution of the fundamentals impacts the worst-case welfare under exogenous information, but has

⁴The reader should not infer from Example 1 that the fragility of weakly dominated actions is the driving force behind Theorem 2; see Online Appendix F.

⁵The BCE that minimizes the gross value in the perturbed example is the one where, conditional on the state, both investors fund the inferior project with probability 0.6. In the complementary event, the two investors miscoordinate in a symmetric fashion: both the event in which Ann invests in project *A* and Bob invests in project *B*, and the event with the opposite investment pattern occur with probability 0.2.

⁶See, e.g., Carroll (2019c) for a recent survey of this literature.

no effect under rational inattention. Thus, whereas the planner may choose to bolster the institution’s fundamentals if she thinks information is exogenous, she never does so if she believes players are rationally inattentive.

We conclude the paper with two additional inquiries. The first is the study of the non-generic environments for which the separation constraint is binding. We obtain a tight characterization of these base games (Proposition 2), and show that separation is an all-or-nothing refinement of BCE: the sBCE set is either dense or nowhere dense in the set of BCEs (Theorem 4). The second line of inquiry characterizes the outcomes attainable under rational inattention as learning costs vanish. Such cheap-learning limits have been suggested in the literature as an equilibrium selection device for games with a commonly known state.⁷ We provide a general answer that ranges across all cheap learning environments: a full-information Nash equilibrium is attainable as a limit equilibrium outcome for some sequence of information technologies if and only if it is in the closure of the sBCE set.

In sum, our paper makes three main contributions. First, we show that in generic settings, the economic environment imposes no restrictions on the kinds of information that rationally inattentive agents may acquire. Consequently, studies that use rational inattention to investigate the nature of agents’ information must either take a stand on agents’ information technologies, or rely on non-generic aspects of the economic setting. Second, our work emphasizes the importance of recognizing the active acquisition of information when evaluating players’ welfare. In particular, erroneously assuming information is given in situations where it is actually acquired may result in overstating the benefits players get from their information, and so can result in misleading welfare conclusions. Finally, we make a methodological contribution by developing a framework for obtaining robust predictions in games with rational inattention.

Related literature. By opening the door to robust analysis under rational inattention, our paper contributes to the literature on robustness in game-theoretic predictions, mechanism design, information design, and contracting. Our paper is especially pertinent to the work that uses Bergemann and Morris’ (2013; 2016) BCE solution concept to obtain reliable predictions in games with incomplete information (e.g., Bergemann, Brooks, and Morris, 2017; Du, 2018; Brooks and Du, 2021).⁸ In particular, we show one can extend

⁷See, e.g., Yang (2015); Hoshino (2018); Denti (2021); Morris and Yang (2022); Denti (forthcoming).

⁸Other related studies regarding robustness that lie outside the BCE framework are Carroll (2019a) and Carroll (2019b). Carroll (2019a) studies the design of a socially efficient bilateral trade mechanism that is robust to agents’ abilities to influence the information structure at a cost. His analysis deviates from the rational inattention framework by assuming agents already know their own valuation, and by allowing for actions that give information to other players. Carroll (2019b) considers a principal who contracts with an expert who can acquire information via a Blackwell experiment. The principal is uncertain about the expert’s information technology, knowing only that it contains some fixed set of experiments. Carroll’s (2019b) analysis differs from ours in several ways, the most substantive of which being that he only derives

the robustness criterion of any such study to include endogenously determined information à la rational inattention, as long as the focus is only on the moments of the action-state distribution (e.g., the seller’s revenue in an auction), and the separation constraint is not binding. Even when separation binds, our genericity result means such constraint should be taken seriously only if the analyst is confident about non-generic features of the economic environment. For studies that focus on welfare (e.g., Bergemann, Brooks, and Morris, 2015), our framework provides a road-map for understanding whether their conclusions continue to hold once one accounts for the cost of information.⁹

We also contribute to the literature on rational inattention in games (e.g., Yang, 2015; Ravid, 2020; Denti, 2021; Morris and Yang, 2022; Denti, forthcoming; Hebert and La’O, forthcoming). Within this literature, the closely related work of Denti (2021) is the first to consider the question of robustness. Denti (2021) studies a two-player signaling game where the receiver needs to pay a cost to monitor the sender’s actions. He shows that, when costs are strictly monotone, off-path beliefs play no role in equilibrium. He then characterizes the set of behavioral predictions consistent with some strictly monotone cost function. As part of this characterization, he obtains what is essentially the single-player version of our Theorem 1: when the sender takes every action with positive probability, the receiver’s behavior is consistent with some strictly monotone cost function if and only if it satisfies obedience and separation. Theorem 1 expands on Denti’s (2021) result by allowing for multiple players, and by considering players’ welfare from a given outcome (Denti, 2021, does not discuss payoffs). In addition, our analysis of generic games and the comparison to exogenous information have no parallel in Denti (2021).

One can interpret some of our results as providing a test for rational inattention in a strategic, multi-agent setting: simply check whether the observed outcome satisfies obedience and separation. Moreover, our genericity result suggests that testing separation requires a highly structured and controlled environment. Thus, our paper can be seen as adding to the growing literature on the testable implications of rational inattention (e.g., Caplin and Dean, 2015; Caplin, Dean, and Leahy, 2022; Denti, 2022; Lipnowski and Ravid, 2022). Within this literature, the most closely related paper is Caplin and Dean (2015).¹⁰

the optimal contract under a worst-case objective, and does not span the set of attainable payoffs and outcomes from a fixed contract.

⁹The worst-case policy analysis of Section 5 is also related to the literature on robust Bayesian persuasion (e.g., Dworczak and Pavan, 2022; Kosterina, 2022; Lipnowski, Ravid, and Shishkin, 2023). Particularly relevant are studies who take a worst-case approach to study optimal information provision in a binary action games (Inostroza and Pavan, 2022; Halac, Lipnowski, and Rappoport, 2022; Morris, Oyama, and Takahashi, 2022). We do not consider information provision, whereas these studies do not consider information acquisition.

¹⁰Another related paper is de Clippel and Rozen (2022). They consider a two-staged game where the first stage player chooses how much to obfuscate the state, and the second stage player chooses what to learn about that state whenever it is obfuscated. In their setting, the second player essentially faces a single agent

They develop a test for whether a single agent’s choices in multiple menus are consistent with costly information acquisition, but do not require costs to be strictly monotone in information. Their characterization includes obedience, as well as another condition called *no improving attention cycles* (NIAC), which restricts the agent’s behavior across decision problems. Since we consider a fixed base game, the NIAC restriction does not apply in our setting. By contrast, Caplin and Dean’s (2015) characterization does not require separation, because they allow for costs that are not strictly monotone. Hence, their characterization differs from ours in that it only considers a single agent, does not require costs to be strictly monotone, and accounts for multiple decision problems.

Our work also expands on the uses of correlated equilibrium and its cousins for spanning the set of predictions attainable across various ways of extending a base game (e.g., Aumann, 1974, 1987; Myerson, 1982; Forges, 1993; Bergemann and Morris, 2016; Doval and Ely, 2020). An early and closely related paper within this literature is Lipman and Srivastava (1990). They consider base games without payoff uncertainty, and ask which obedient outcomes (i.e., correlated equilibria) can be attained by extending the game to allow players to acquire costly information about a common payoff irrelevant state space. They maintain two assumptions on players’ information technology: costs are (ordinally) symmetric across players, and players must use partitional information. They show an obedient outcome can be generated from an information technology satisfying their assumptions if and only if it satisfies a cyclical monotonicity condition across players that is similar to Caplin and Dean’s (2015) NIAC. Our work differs from theirs in that we allow for payoff uncertainty, asymmetric costs, and non-partitional information, and that we require costs to be strictly increasing in informativeness. In addition, Lipman and Srivastava (1990) have no analog of our payoff bounds, which play a central role in our paper.

2. Setup

A finite number of players have to engage in a game with uncertain payoffs; we call such a game the *base game*. Before playing the base game, the players can covertly acquire costly information to reduce the uncertainty they face. We call *information technology* the description of learning resources. Together, a base game and an information technology define an *information acquisition game*, our main object of study. We focus on two predictions: the *equilibrium outcome*, which details the distribution of the players’ actions in the base game, and the *equilibrium value*, which is the players’ expected payoffs net of information costs. We are interested in the outcomes and values that can be generated as we range

decision problem. Using this fact, de Clippel and Rozen (2022) apply Caplin and Dean’s (2015) results to obtain testable prediction that are valid across all cost functions. They then test these predictions in a lab experiment.

over all information technologies that represent *rational inattention*, i.e., when information choice is flexible and more information is more costly. In this section, we precisely define all these objects and provide the associated notation.

Base Game. Let I be a finite set of players, with typical element i . Each player i has to choose an action a_i from a finite set A_i . As usual, we define $A_{-i} = \prod_{j \neq i} A_j$ and $A = A_i \times A_{-i}$. Accordingly, we use $a_{-i} = (a_j)_{j \neq i}$ to denote the action profile of all players other than i , and $a = (a_i, a_{-i})$ to denote the entire action profile. Throughout the paper we adopt the same notational conventions for all Cartesian products indexed by $I \setminus \{i\}$ and I .

Players are expected utility maximizers who care about each other's actions as well as an exogenous variable θ , which is drawn from a finite set Θ according to a full-support probability measure $\pi \in \Delta(\Theta)$. We denote by $u_i : A \times \Theta \rightarrow \mathbb{R}$ player i 's von Neumann-Morgenstern utility function.

We call a tuple $\mathcal{G} = (I, \Theta, \pi, (A_i, u_i)_{i \in I})$ a **base game**.

Information Technologies. Before taking an action in the base game, each player has the opportunity to acquire information about θ as well as other exogenous quantities of potential interests (e.g., sunspots, noisy public information). We succinctly represent them by a single variable z taking values in a finite set Z ; a Markov kernel $\zeta : \Theta \rightarrow \Delta(Z)$ details the conditional distribution of z given θ .

Following Blackwell (1951), we model the acquisition of information using experiments. An experiment for player i is a Markov kernel $\xi_i : Z \times \Theta \rightarrow \Delta(X_i)$, where X_i is a finite space of signal realizations privately observable by player i . The functions ξ_i details how the distribution of i 's signal x_i depends on θ and z . To simplify the exposition, we assume the signal space is rich; specifically, we assume X_i has more elements than A_i and $Z \times \Theta$.

By construction, the players' signals are conditionally independent given θ and z . However, they may be correlated given θ only. Thus, our framework incorporates a form of correlated information acquisition, as in, among others, Hellwig and Veldkamp (2009), Myatt and Wallace (2012), Hebert and La'O (forthcoming), and Denti (forthcoming). Indeed, one can simply view z as a modeling device for situations in which players have access to information sources with correlated noise (e.g., newspapers with similar slants, consultants from the same firm).

The acquisition of information faces two kinds of frictions. First, each player is constrained in the kind of experiments she can use: player i can only choose experiments that lie in a given set \mathcal{E}_i . Second, experiments come at a cost, where $C_i : \mathcal{E}_i \rightarrow \mathbb{R}_+$ denotes i 's cost function. As a normalization, we assume the existence of an experiment that costs zero; that is, $C_i(\xi_i) = 0$ for some $\xi_i \in \mathcal{E}_i$.

We call a tuple $\mathcal{T} = (Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$ an **information technology**.

Information Acquisition Games. Together, a base game \mathcal{G} and an information technology \mathcal{T} define an **information acquisition game**. The game begins with the realization of the state of the world $\omega = (z, \theta)$. Without observing the state, the players simultaneously and covertly choose experiments, and pay their costs. Then, each player privately observes the outcome of their own experiment and takes an action. We use $\sigma_i : X_i \rightarrow \Delta(A_i)$ to denote player i 's action plan in this game, and let Σ_i be the set of i 's action plans.

The solution concept we adopt is Nash equilibrium. A strategy for player i consists of an experiment $\xi_i \in \mathcal{E}_i$ and an action plan $\sigma_i \in \Sigma$. A strategy profile $(\xi_i^*, \sigma_i^*)_{i \in I}$ is an equilibrium if for all players i , (ξ_i^*, σ_i^*) maximizes

$$\left[\sum_{a, x, z, \theta} u_i(a, \theta) \sigma_i(a_i | x_i) \xi_i(x_i | z, \theta) \prod_{j \neq i} \sigma_j^*(a_j | x_j) \xi_j^*(x_j | z, \theta) \zeta(z | \theta) \pi(\theta) \right] - C_i(\xi_i).$$

over all $\xi_i \in \mathcal{E}_i$ and $\sigma_i \in \Sigma_i$. The objective function consists of two terms: the value of information (in square brackets) and the cost of information. As common in applications, value and cost are additively separable.

Equilibrium Predictions. We summarize the equilibria of information acquisition games using two statistics of the players' behavior: the outcome and the value.

The **outcome** is the joint distribution $p \in \Delta(A \times \Theta)$ of the players' actions and the payoff-relevant state. Note that the marginal distribution of θ must be the prior π ; we denote by $\Delta_\pi(A \times \Theta)$ the set of probability measures over $A \times \Theta$ whose marginal on Θ is π .

The **value** is the vector $v = (v_i)_{i \in I} \in \mathbb{R}^I$ assigning each player the expected payoff in the game. Observe that this payoff includes players' information acquisition costs and so v_i may differ from the expectation of u_i under p .

Blackwell order. To give a precise definition of "more information," we build on the classic ranking of experiments due to Blackwell (1951, 1953). Given a pair of experiments ξ_i and ξ'_i , we say ξ_i **Blackwell dominates** ξ'_i (denoted $\xi_i \succsim \xi'_i$) if there exists a Markov kernel $g : X_i \rightarrow \Delta(X_i)$ such that for every $x_i \in X_i$, $\theta \in \Theta$, and $z \in Z$ with $\zeta(z | \theta) > 0$,

$$\xi'_i(x_i | z, \theta) = \sum_{x'_i \in X_i} g(x_i | x'_i) \xi(x'_i | z, \theta).$$

Intuitively, ξ_i Blackwell dominates ξ'_i if one can generate ξ'_i by "garbling" the output of ξ_i . As shown by Blackwell, $\xi_i \succsim \xi'_i$ if and only if player i is better off observing the output of

ξ_i rather than the output of ξ'_i (holding fixed other players' behavior). In this sense, ξ_i is more informative than ξ'_i . We write $\xi_i \succ \xi'_i$ whenever $\xi_i \succsim \xi'_i$ and $\xi'_i \not\succ \xi_i$.

Rational Inattention. Our aim is to study the outcomes and values that can be generated in equilibrium as we fix the base game and range over all information technologies that represent rational inattention. Consistent with the literature, we interpret rational inattention as information technologies where the set of feasible experiments is flexible, and where it is costly to acquire more information.

We say a set of feasible experiments \mathcal{E}_i is **flexible** if, whenever a given amount of information is feasible, a lower amount of information is feasible as well: whenever $\xi_i \in \mathcal{E}_i$ and $\xi_i \succsim \xi'_i$, then $\xi'_i \in \mathcal{E}_i$. We say a cost function C_i is **monotone** if less informative experiments are cheaper to acquire: for all $\xi_i, \xi'_i \in \mathcal{E}_i$ such that $\xi_i \succsim \xi'_i$ (resp., $\xi_i \succ \xi'_i$), we have $C_i(\xi_i) \geq C_i(\xi'_i)$ (resp., $C_i(\xi_i) > C_i(\xi'_i)$). We say a technology \mathcal{T} represents **rational inattention** if for every player, the set of feasible experiments is flexible and the cost function is monotone.

Many authors regard flexibility as the key difference between rational inattention and traditional information-acquisition models (see, e.g., Maćkowiak, Matějka, and Wiederholt, 2023, Section 2). Applications of rational inattention often assume all experiments are feasible: in the language of this paper, $\mathcal{E}_i = \Delta(X_i)^{Z \times \Theta}$. Our results are unchanged if we adopt this more extreme definition of flexibility. The reason is that there are no observable differences between experiments that are unfeasible and experiments that are excessively costly.

By pairing flexibility with monotonicity, we postulate that players can save on costs by only acquiring the information they actually use in making decisions. Matějka and McKay (2015) provide a standard example of monotone cost function: $C_i(\xi_i)$ equals the expected reduction in uncertainty about θ and z , as measured by Shannon entropy, from observing the output of ξ_i . More broadly, one could substitute Shannon's entropy with any other strictly concave measure of uncertainty (Caplin, Dean, and Leahy, 2022). One could also consider increasing transformations of these costs (Denti, 2022; Zhong, 2022), or any differentiable cost function whose derivative is strictly convex (Lipnowski and Ravid, 2022).¹¹

3. A Characterization of Rational Inattention

In this section we characterize the outcomes and values that can be generated via rational inattention. To provide a benchmark, we first review the case of exogenous information.

¹¹In each information acquisition game, prior beliefs are exogenously determined by π and ζ . Thus, the issue of experiment-based vs. posterior-based information costs that sometimes arise in applications of rational inattention (see, e.g., Ravid, 2020; Denti, Marinacci, and Rustichini, 2022) is irrelevant here.

The class of information acquisition games includes situations in which players' information is predetermined. One can obtain them by considering technologies in which each player has only *one* feasible experiment (whose cost is zero by our normalization). Such technologies are identified by a tuple $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ that we call **information structure**. Of course, an information acquisition game $(\mathcal{G}, \mathcal{S})$ is simply a conventional game of incomplete information à la Harsanyi.

Among other results, Bergemann and Morris (2016) characterize the equilibrium outcomes that can arise in a game of incomplete information as we fix the base game and range over all information structures: they call them Bayes correlated equilibria. A **Bayes correlated equilibrium** (BCE) of a base game \mathcal{G} is an outcome $p \in \Delta_\pi(A \times \Theta)$ that satisfies the following constraint: for all $i \in I$ and $a_i, b_i \in A_i$,

$$\sum_{a_{-i}, \theta} (u_i(a_i, a_{-i}, \theta) - u_i(b_i, a_{-i}, \theta)) p(a_i, a_{-i}, \theta) \geq 0. \quad (1)$$

Following standard terminology, we name (1) the **obedience constraint**. The standard way of viewing this constraint is through the lens of a mediator who generates p by observing the state and stochastically sending an action recommendation to each player. The players are willing to follow these recommendations if and only if the obedience constraint is satisfied.¹²

Calculating players' values under exogenous information is straightforward. If outcome p arises under exogenous information in base game \mathcal{G} , player i 's expected payoff is

$$\bar{v}_i(p) := \sum_{a, \theta} u_i(a, \theta) p(a, \theta).$$

We call $\bar{v}_i(p)$ the **gross value** for player i , since it ignores information costs. Let $\bar{v}(p) := (\bar{v}_i(p))_{i \in I}$ be the vector of gross values.

What changes when information is endogenous? Our analysis highlights two main differences between rational inattention and exogenous information.

The first difference is that outcomes generated by costly information acquisition must satisfy an additional “separation constraint.” To present this constraint, we require a few definitions. Given an outcome $p \in \Delta_\pi(A \times \Theta)$, a player $i \in I$, and an action $a_i \in A_i$, let $p(a_i) := \sum_{a_{-i}, \theta} p(a_i, a_{-i}, \theta)$ be the probability of player i taking action a_i under p , and let

$$\text{supp}_i(p) := \{a_i \in A_i : p(a_i) > 0\}$$

¹²Our definition of BCE corresponds to the specialization of Bergemann and Morris's (2016) definition to the case in which players' original type spaces are degenerate.

be the set of i 's actions that have positive probability. For each $a_i \in \text{supp}_i(p)$, let $p_{a_i} \in \Delta(A_{-i} \times \Theta)$ be the conditional distribution of the actions of the players other than i and the payoff-relevant state: for all $a_{-i} \in A_{-i}$ and $\theta \in \Theta$,

$$p_{a_i}(a_{-i}, \theta) := \frac{p(a_i, a_{-i}, \theta)}{p(a_i)}.$$

Finally, denote the set of player i 's best responses to p_{a_i} via

$$BR(p_{a_i}) := \operatorname{argmax}_{b_i \in A_i} \sum_{a_{-i}, \theta} u_i(b_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta).$$

Then an outcome $p \in \Delta_\pi(A \times \Theta)$ satisfies the **separation constraint** if for all $i \in I$ and $a_i, b_i \in \text{supp}_i(p)$,

$$p_{a_i} \neq p_{b_i} \quad \text{implies} \quad BR(p_{a_i}) \cap BR(p_{b_i}) = \emptyset. \quad (2)$$

In other terms, an outcome satisfies the separation constraint if distinct beliefs have separate best responses. We refer to a BCE that satisfies the separation constraint as a **separated BCE** (sBCE).

The second difference between rational inattention and exogenous information is in the set of payoffs players obtain from a given outcome. When information is given, each player i 's value is completely determined by the outcome p . By contrast, under rational inattention the same outcome can arise under multiple cost functions, and so is consistent with a *set* of values. As we demonstrate next, this set of values is convex, with an upper bound given by the gross value $\bar{v}_i(p)$. The lower bound is given by (what we call) the outcome's "uninformed value," which is i 's maximal value if she receives no information, and others' actions and θ are distributed according to that outcome. Formally, the **uninformed value** of an outcome p in a base game \mathcal{G} for player i is given by

$$\underline{v}_i(p) := \max_{b_i \in A_i} \sum_{a, \theta} u_i(b_i, a_{-i}, \theta) p(a, \theta).$$

Let $\underline{v}(p) := (\underline{v}_i(p))_{i \in I}$ be the vector of uninformed values.¹³

The next result summarizes our characterization of rational inattention in games:

¹³The uninformed value is related to the notion of *coarse correlated equilibrium* (see, e.g., Roughgarden, 2016, Definition 13.5). Coarse correlated equilibrium relaxes the obedience constraint, requiring instead that no player can benefit from deviating to a single action from all of the mediator's recommendation; that is, $\bar{v}_i(p) \geq \underline{v}_i(p)$ for every i .

Theorem 1. *Fix a base game \mathcal{G} . A rational-inattention technology \mathcal{T} exists that induces the outcome-value pair (p, v) in an equilibrium of $(\mathcal{G}, \mathcal{T})$ if and only if*

- (i) *the outcome p is a separated BCE, and*
- (ii) *for every $i \in I$, $v_i = \underline{v}_i(p) = \bar{v}_i(p)$ or $v_i \in [\underline{v}_i(p), \bar{v}_i(p))$.*

Thus, an outcome-value pair is consistent with rational inattention if and only if two conditions hold. First, the outcome satisfies both the obedience constraint and the separation constraint. And second, each player's value is weakly above her uninformed value, but strictly below her gross value.

We now explain why the theorem's conditions are necessary. That every outcome generated by rational inattention must be obedient follows from the necessity of the obedience constraint under exogenous information. The reason is that any outcome one can attain when players choose their information must also be attainable if players were exogenously endowed with their chosen experiment. To understand why the separation constraint is necessary, consider a player i who takes with positive probability a pair of actions a_i and b_i such that $p_{a_i} \neq p_{b_i}$. As in BCE, we can interpret a_i and b_i as signals. With rational inattention, informative signals are costly. To save on information costs, the player could substitute a_i and b_i with a *single* action recommendation c_i . For this not to be profitable, it must be that either $c_i \notin BR(p_{a_i})$ or $c_i \notin BR(p_{b_i})$. Since the choice of c_i is arbitrary, it must be that $BR(p_{a_i}) \cap BR(p_{b_i}) = \emptyset$.

To conclude our discussion of necessity, we now describe the origin of the theorem's payoff bounds. To see why $\underline{v}_i(p)$ is a lower bound on player i 's payoff, suppose we have an information technology and an equilibrium that induces the outcome p . By assumption, player i always has the option of remaining uninformed at no cost. Therefore, i 's optimal payoff must be higher than $\underline{v}_i(p)$. That $\bar{v}_i(p)$ is the maximal payoff player i can attain in an equilibrium that induces the outcome p follows from information acquisition costs being non-negative. Moreover, these costs must be strictly positive whenever $\underline{v}_i(p) \neq \bar{v}_i(p)$: to generate p in this case player i must acquire *some* information. Therefore, player i can actually attain her gross value from an outcome only if that value coincides with the outcome's uninformed value.

We now briefly review our proof that conditions (i) and (ii) of Theorem 1 are sufficient. The proof is constructive, and is based on a result by Denti (2021). Denti (2021) studies a signaling game where the receiver has to pay a cost to monitor the sender's action. Among other results, Denti (2021) shows that, when the sender takes every action with positive probability, any receiver behavior satisfying (what we call in this paper) the obedience constraint and the separation constraint can be justified via rational inattention.

To prove the “if” direction of Theorem 1, we first modify Denti’s (2021) single-agent construction so that it generates any payoff between the uninformed and the gross value. We emphasize Denti (2021) focuses on equilibrium outcomes and does not discuss achievable payoffs. Then, we use our modification of Denti’s (2021) single-agent result to construct a flexible and monotone technology that enables players to simultaneously acquire information about the same state space (in Denti, 2021, only *one* player endogenously acquires information).

Theorem 1 shows that any BCE that arises from rational inattention must satisfy the separation constraint. Next, we record that the separation constraint never eliminates all of a game’s Bayes correlated equilibria; that is, the set of separated BCEs is non-empty.

Corollary 1. *Every base game \mathcal{G} admits a separated BCE.*

The proof is straightforward (details omitted): A technology where all feasible experiments are free and uninformative is flexible and monotone. The corresponding game with information acquisition admits an equilibrium by standard arguments (information is de facto exogenous). It follows from Theorem 1 that the outcome of such an equilibrium is a separated BCE.¹⁴

We conclude the section with a brief discussion of what happens if we allow for non-flexible and non-monotone technologies. To accommodate such technologies, we need to adjust Theorem 1’s statement in two ways. First, the separation constraint is no longer necessary. And second, players can attain their gross value even when it differs from their uninformed value. We refer the reader to Online Appendix G for the precise details.

4. Generic Environments

Theorem 1 shows rational inattention differs from exogenous information in two ways. First, it reduces the set of equilibrium outcomes: an additional separation constraint must be satisfied. Second, it expands the set of achievable payoffs for any given equilibrium outcome. In this section, we prove *only* the second difference is meaningful in generic environments.

We adopt the following notion of genericity: We fix a finite set of players I , a finite set of payoff states Θ , a full-support prior $\pi \in \Delta(\Theta)$, and a finite set of actions A_i for each player $i \in I$. To obtain a base game, it remains to specify a profile of utility function $u = (u_i)_{i \in I}$. We identify u with an element of the Euclidean space $\mathbb{R}^{I \times A \times \Theta}$, and say a statement is true for **generic** u if the closure of the subset in $\mathbb{R}^{I \times A \times \Theta}$ for which it is false has Lebesgue measure zero. We denote by $BCE(u)$ and $sBCE(u)$ the sets of BCEs and separated BCEs for the base game corresponding to u .

¹⁴Another immediate consequence of this argument is that in the special case in which Θ is a singleton, all (pure or mixed) Nash equilibria of the base game are separated BCEs.

Theorem 2. *For generic u , the set $sBCE(u)$ is dense in the set $BCE(u)$.*

Thus, the environments in which rational inattention predict different outcomes than incomplete information are knife edge.¹⁵ An important caveat to this result is the notion of genericity we use: it is the most common for the static games we study in this paper, but also the most permissive. For example, according to this notion of genericity, many important economic applications—such as auctions or oligopolies—are non-generic. The notion of genericity is also not appropriate if the base game is not actually static but represents the strategic form of a primitive dynamic game. We discuss sBCE in non-generic environments in Section 6.1.

One might be tempted to think that Theorem 2 follows from indifferences being “fragile.” This intuition works, but only for the single agent case. When I has one element, the set of *strict* BCEs—i.e., the set of outcomes p such that $BR(p_{a_i}) = \{a_i\}$ for every player i and $a_i \in \text{supp}_i(p)$ —is generically dense in the BCE set. Theorem 2 then follows from noting that every strict BCE is separated.¹⁶

The situation radically changes when there are at least two players. The reason is that, with two or more players, indifferences emerge in equilibrium in generic fashion: when I has more than one element, there exists an open set of games where *no* BCE is strict. For example, consider the games in a neighborhood of Matching Pennies: such games have a unique correlated equilibrium (the fully-mixed Nash equilibrium) in which both players are indifferent between both actions. Hence, to prove Theorem 2 beyond the single-agent case, one cannot hope to show that indifferences are somewhat fragile.

Our proof of Theorem 2 combines two independent lemmas. The first lemma shows that for any BCE p and utility profile u , there exists a perturbation of u that makes p separated. In other words, every BCE of a given game is the limit of separated BCEs of nearby games.

The second lemma shows that for generic games, any BCE that is a limit of separated BCE in nearby games is also a separated BCE when the game is held fixed. Specifically, the second lemma shows that the correspondence $u \mapsto \text{cl}(sBCE(u))$, which takes utilities to the closure of the sBCE set, generically is upper hemicontinuous (in fact, continuous).

To prove this second lemma, we employ ideas from Blume and Zame (1994), who study the algebraic geometry of Nash, perfect, and sequential equilibria. In particular, we use the Tarsky-Sierberg Theorem to show that $u \mapsto \text{cl}(sBCE(u))$ has semi-algebraic graph, and so must be continuous for all utility profiles outside a closed low-dimensional manifold. Combined, these two lemmas imply that the set of games where the sBCE set is not dense in the BCE set is contained in a closed low-dimensional manifold. The theorem follows.

¹⁵It is easy to construct generic examples where the set of separated BCEs is not close. In particular, the statement “ $sBCE(u) = BCE(u)$ for generic u ” is false. It is also not true that “ $\text{cl}(sBCE(u)) = BCE(u)$ for all u .”

¹⁶See Online Appendix H for a detailed argument.

Theorem 2 implies that, in generic games, rational inattention and exogenous information are outcome equivalent. Next, we show this equivalence does not extend to players' welfare. In other words, even though the two knowledge regimes generically yield the same behavioral predictions, they can have very different welfare implications.

For $u \in \mathbb{R}^{I \times A \times \Theta}$, let $V_R(u)$ be the closure of set of attainable value vectors under rational inattention. By Theorem 1, these are given by the set of value vectors that lie between the uninformed value and the gross value of some limit of separated BCEs:¹⁷

$$V_R(u) = \{v \in [\underline{v}(p, u), \bar{v}(p, u)] : p \in \text{cl}(sBCE(u))\}, \quad (3)$$

where $\underline{v}(p, u)$ and $\bar{v}(p, u)$ make explicit the dependence of uninformed and gross values on the players' utility functions.

We also denote by $V_I(u)$ the set of attainable value vectors under exogenous information. Bergemann and Morris's (2016) analysis implies this is given by the set of gross value vectors attainable in some BCE,

$$V_I(u) = \{\bar{v}(p, u) : p \in BCE(u)\}. \quad (4)$$

For an arbitrary u , there is no simple relationship between $V_R(u)$ and $V_I(u)$: $sBCE(u)$ is a subset of $BCE(u)$, but $\bar{v}(p, u)$ is an element of $[\underline{v}(p, u), \bar{v}(p, u)]$, so one cannot easily conclude that $V_R(u)$ contains or is contained by $V_I(u)$, or neither.

In generic environments, the comparison is simpler:

Theorem 3. *For generic u , $V_I(u) \subseteq V_R(u)$. In addition, if $|I| \geq 2$, $|\Theta| \geq 2$, and $|A_i| \geq 2$ for all players i , then the set of u for which $V_I(u) \subset V_R(u)$ has non-empty interior.*

Thus, in generic environments, rational inattention expands the set of achievable payoffs, and it does so in a non-trivial way (i.e., with strict inclusion) for a class of environments of positive measure. The result suggests that rational inattention and exogenous information may have different welfare implications even when they are outcome equivalent. In the next section, we make this idea concrete in an application to robust policy analysis.

We conclude this section by sketching the proof of Theorem 3. The first part of the theorem immediately follows from Theorem 2, together with (3) and (4): for generic u , $\text{cl}(sBCE(u)) = BCE(u)$, and therefore

$$V_I(u) \subseteq \{v \in [\underline{v}(p, u), \bar{v}(p, u)] : p \in BCE(u)\} = V_R(u).$$

To show the strict inclusion, the key step involves finding a utility profile u^* and a BCE

¹⁷For two vector of real numbers $\lambda, \gamma \in \mathbb{R}^K$ such that $\lambda \leq \gamma$, we use the box, or ordered interval notation $[\lambda, \gamma] = \{\eta \in \mathbb{R}^K : \lambda \leq \eta \leq \gamma\}$ to denote the set of all vectors between λ and γ .

p^* of the corresponding base game with two properties. First, each player has exactly two positive probability actions, both of which are strict best responses; thus, in particular, p^* is separated. And second, for every player i , p^* is the unique minimizer of i 's uniformed value across all BCEs of u^* . The first property implies that $\underline{v}_i(p^*, u^*) < \bar{v}_i(p^*, u^*)$ for all players i —the equality $\underline{v}_i(p^*, u^*) = \bar{v}_i(p^*, u^*)$ holds if and only if i has a single action that is a best response to all action recommendations. With the aid of the second property, then we show that

$$\inf_{p \in sBCE(u^*)} \sum_i \underline{v}_i(p, u^*) = \sum_i \underline{v}_i(p^*, u^*) < \min_{p \in BCE(u^*)} \sum_i \bar{v}_i(p, u^*).$$

This allows us to deduce that $V_I(u^*) \subset V_R(u^*)$. Finally, we use a continuity argument to show that $V_I(u) \subset V_R(u)$ for all games u in a neighborhood of u^* .

5. Application: Robust Welfare Analysis

In this section we demonstrate how one can use our results to conduct robust welfare analysis in an economy where agents exhibit rational inattention.

We consider an economy that consists of a fixed set of agents, I , who play an information acquisition game, $(\mathcal{G}, \mathcal{T})$. The structure of the game depends on the policy enacted by a utilitarian social planner. The planner has a good understanding of the policy's material implications, i.e., she knows a given policy leads to a given \mathcal{G} . However, the planner is unsure about the accompanying information technology \mathcal{T} .

We focus on two cases regarding the source of the agents' information. The planner either postulates that information is exogenously given, or that it is generated by rational inattention. In both cases, the planner identifies a policy with the corresponding base game \mathcal{G} and employs a robust criterion that evaluates it according to the worst-case utilitarian welfare across all relevant information technologies \mathcal{T} and ensuing equilibria.

For the exogenous information case, Bergemann and Morris's (2016) results imply one can find the social value of a policy \mathcal{G} by minimizing the sum of the agents' gross payoffs across all BCEs. Specifically, let $\bar{w}(p)$ be the utilitarian welfare of an outcome p assuming the players' payoffs are given by their gross value,

$$\bar{w}(p) := \sum_i \bar{v}_i(p).$$

Then the planner's value of a base game \mathcal{G} under exogenous information is

$$\bar{w} := \min_{p \in BCE} \bar{w}(p),$$

where BCE is the set of Bayes correlated equilibria of \mathcal{G} . Since the BCE set is defined by linear inequalities, and $\bar{w}(p)$ is linear in p , one can compute \bar{w} via linear programming.

What about a planner who postulates that agents are rationally inattentive? Theorem 1 provides the answer: such a planner evaluates each base game according to the lowest sum of *uninformed* values that is attainable in some *separated* BCE. More precisely, for an outcome p , let $\underline{w}(p)$ be the utilitarian welfare implied by p in the base game \mathcal{G} if players' payoffs are given by their uninformed value,

$$\underline{w}(p) := \sum_i v_i(p).$$

Then Theorem 1 suggests that a planner who assumes information is endogenous would evaluate each game according to the minimum of $\underline{w}(p)$ across all separated BCEs p ,

$$\inf_{p \in sBCE} \underline{w}(p).$$

Recall, however, that Theorem 2 says that the separation constraint does not bind for generic games. As such, imposing the separation constraint is only appropriate if the planner is absolutely certain of the structure of the base game. Whereas such certainty might be justifiable in certain cases, here we take the perspective of a cautious planner who, in an economy where agents are rationally inattentive, evaluates \mathcal{G} according to the worst-case value of $\underline{w}(p)$ across all BCE,

$$\underline{w} := \min_{p \in BCE} \underline{w}(p).$$

Since the BCE set is defined by linear inequalities, and $\underline{w}(p)$ is convex in p , one can compute \underline{w} using convex programming.

Clearly, the planner's decisions under rational inattention and exogenous information can differ only if the two generate different values, that is, if $\underline{w} \neq \bar{w}$. Because each players' uninformed value is always lower than her gross value, we have $\underline{w} \leq \bar{w}$; that is, welfare under rational inattention is always lower than the welfare under exogenous information. Below we characterize when the inequality is strict in symmetric binary-action games.

A base game $\mathcal{G} = (I, \Theta, \pi, (A_i, u_i)_{i \in I})$ has **binary actions** if for every player i , A_i contains two elements. It is **symmetric** if $A_i = A_j$ for all $i, j \in I$, and if for every permutation $\phi : I \rightarrow I$, player i , action profile a , and payoff state θ ,

$$u_i(a_\phi, \theta) = u_{\phi(i)}(a, \theta),$$

where $a_\phi := (a_{\phi(j)})_{j \in I}$ is the action profile such that each player j takes action $a_{\phi(j)}$. Given a symmetric base game, an outcome $p \in \Delta(A \times \Theta)$ is **symmetric** if for every permutation

ϕ , action profile a , and payoff state θ ,

$$p(a, \theta) = p(a_\phi, \theta).$$

We denote the set of symmetric outcomes by $\Delta_\pi^{sy}(A \times \Theta)$. As one might hope, focusing on symmetric outcomes is without loss for welfare analysis in symmetric games:

Lemma 1. *If \mathcal{G} is symmetric, then both $\min_{p \in BCE} \underline{w}(p)$ and $\min_{p \in BCE} \bar{w}(p)$ admit symmetric optimal solutions.*

The next result characterizes the binary-action symmetric games for which rational inattention yields strictly lower worst-case welfare than exogenous information.

Proposition 1. *Let \mathcal{G} be a symmetric, binary-action base game. Then, $\underline{w} < \bar{w}$ if and only if all $p^* \in \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p)$ satisfy the following condition:*

$$a_i \in \operatorname{supp}_i(p^*) \text{ and } BR(p_{a_i}^*) = \{a_i\} \quad \text{for all } i \in I \text{ and } a_i \in A_i. \quad (5)$$

Moreover, in this case,

$$\operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p) \subseteq \operatorname{argmin}_{p \in BCE} \underline{w}(p).$$

Thus, one can check whether $\underline{w} < \bar{w}$ by examining the minimizers of $\underline{w}(p)$ among *all* symmetric outcomes p , ignoring the players' obedience constraints. In particular, one needs to check whether all these minimizers recommend both actions to every player, and only send recommendations that induce unique best responses. An immediate implication of the above proposition is that, in a symmetric binary-action game, if $\underline{w} < \bar{w}$, then

$$\underline{w} = \min_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p).$$

We now briefly explain the proposition's proof. The key observation is that in a binary action game, a BCE p satisfies (5) if and only if $\bar{v}_i(p) > \underline{v}_i(p)$ holds for every player i . To get intuition for the “if” direction, note that having *two* recommendations that lead to strict best responses means players get a strictly positive benefit from following them. This benefit creates a wedge between the gross value $\bar{v}_i(p)$, which accounts for the value of information, and the uninformed value $\underline{v}_i(p)$, which does not. For the converse direction, note that a violation of (5) means some player i has an action that is optimal across all of the mediator's recommendations. As such, player i loses nothing by ignoring the mediator's recommendations and taking that action. In other words, player i 's gross value equals her uninformed value.

Armed with the above observation, we prove the proposition in two steps. The first step shows $\underline{w} < \bar{w}$ holds if and only if all optimal solutions of $\min_{p \in BCE^{sy}} \underline{w}(p)$ satisfy (5). This

step follows from applying the above-mentioned observation to symmetric outcomes. The second step shows that all optimal solutions of $\min_{p \in BCE^{sy}} \underline{w}(p)$ satisfy (5) if and only if all optimal solutions of $\min_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p)$ satisfy (5). Loosely speaking, the reason is as follows: if the obedience constraint does not bind at the optimum—as (5) dictates—then it can be relaxed, and therefore minimizing over $p \in BCE^{sy}$ is the same as minimizing over $p \in \Delta_\pi^{sy}(A \times \Theta)$.

Proposition 1 enables us to find circumstances where the planner’s optimal policy depends on whether she believes players’ information is exogenously given, or generated by rational inattention. We demonstrate this fact below in a regime change game.

Example 2. We consider a regime change game whereby a status quo is abandoned if a sufficiently large number of players take an action against it. Such games are well-studied and have been used to model a variety of social phenomena, including currency crises, bank runs, debt crises, and political revolts.¹⁸

In our application, there are n identical investors, $i \in I = \{1, \dots, n\}$, each of which decides whether to speculate against (i.e., attack) a distressed financial institution ($a_i = 1$) or not ($a_i = 0$). Speculating costs $k \in (0, 1)$. If enough investors speculate, the institution fails (i.e., the attack succeeds), generating a profit of 1 to the speculators, and an externality of $-x$ to all passive investors, where $x \in (0, \infty)$. The payoff state $\theta \in \Theta \subseteq \{1, \dots, n\}$, determines the number of speculators required for the attack to succeed. We assume that $\min \Theta > 1$, and $\max \Theta < n - 1$, meaning no single investor can go against the will of all the others.¹⁹ We summarize these payoffs below:

	$\sum_j a_j \geq \theta$	$\sum_j a_j < \theta$
$u_i(1, a_{-i}, \theta)$	$1 - k$	$-k$
$u_i(0, a_{-i}, \theta)$	$-x$	0

We focus on finding conditions under which there is a difference between the worst-case welfare under rational inattention and exogenous information.²⁰ By Proposition 1, answering this question requires us to minimize the sum of the uninformed values across all symmetric outcomes, ignoring obedience constraints. The following claim characterizes the solutions to this problem.

¹⁸See, e.g., Obstfeld (1996); Morris and Shin (1998); Goldstein and Pauzner (2005); Morris and Shadmehr (forthcoming).

¹⁹A fortiori, $n > 3$ because $2 \leq \min \Theta \leq \max \Theta < n - 1$.

²⁰It is easy to verify that, in this example, rational inattention and exogenous information are outcome equivalent, that is, the sBCE set is dense in the BCE set. Indeed, the regime change game admits a strict BCE p where all players take both actions with positive probability (e.g., a convex combinations of the pure Nash equilibria in which everyone speculates or no-one speculates). Thus, any BCE q can be approximated by a separated BCE $(1 - \epsilon)q + \epsilon p$ as $\epsilon \rightarrow 0$.

Claim 1. In the regime change game, a symmetric p^* minimizes $\underline{w}(p)$ across all outcomes $p \in \Delta_\pi^{sy}(A \times \Theta)$ if and only if the following conditions hold: for all payoff state θ ,

$$p^* \left(\left\{ (a, \theta) : \theta - 1 \leq \sum_i a_i \leq \theta \right\} \right) = 0, \quad (6)$$

$$p^* \left(\left\{ (a, \theta) : \sum_i a_i > \theta \right\} \right) = \frac{k}{1+x}. \quad (7)$$

The two optimality conditions have simple interpretations. First, no investor is ever pivotal. And second, the attack succeeds with probability $k/(1+x)$. To obtain these conditions, we show that p^* minimizes $\underline{w}(p)$ across all $p \in \Delta_\pi^{sy}(A \times \Theta)$ only if each individual investor i is *indifferent* between never speculating and always speculating when other investors play according to p^* . Minimizing across all symmetric outcomes satisfying this condition then delivers the result.

Finding a symmetric p^* that satisfies the obedience constraints in addition to the claim's conditions is easy: have all investors attack together with probability $k/(1+x)$ regardless of the state, and no one speculates otherwise. Calculating the sum of the uninformed values from p^* immediately give the worst-case welfare under rational inattention,

$$\underline{w} = \underline{w}(p^*) = -\frac{nxk}{1+x}. \quad (8)$$

Hence, under rational inattention, the planner's value decreases in the size of the externality x and the cost of betting on the financial institution's demise k , but does not depend on π ; that is, the planner's value does not depend on the institution's fundamentals.

Next we argue a planner who views information as exogenous may adopt different policies than a planner who thinks investors are rationally inattentive. To do so, we characterize when these two knowledge environments attain the same worst-case welfare. The appendix provides a general characterization (see Claim 14); the claim below specializes this characterization to the case where Θ is binary.

Claim 2. Suppose $\Theta = \{\underline{\theta}, \bar{\theta}\}$, where $\bar{\theta} \geq \underline{\theta}$. Then $\underline{w} = \bar{w}$ if and only if $\bar{\theta} - \underline{\theta} \geq 3$ and

$$1 - \frac{1}{3} (\bar{\theta} - \underline{\theta}) \pi(\bar{\theta}) \leq \frac{k}{1+x} \leq \frac{1}{3} (\bar{\theta} - \underline{\theta}) (1 - \pi(\bar{\theta})). \quad (9)$$

In other terms, with binary θ , worst-case welfare under rational inattention is the same as under exogenous information if and only if the state is sufficiently uncertain in the sense that $\bar{\theta} - \underline{\theta} \geq 3$, and the probability of $\bar{\theta}$ is not too extreme compared to $k/(1+x)$ —e.g., (9) fails if $\pi(\bar{\theta})$ goes to zero or one. In particular, the worst-case welfare under rational inattention is always strictly below the welfare under exogenous information when there's

certainty about the institution’s fundamentals (i.e., when θ is deterministic).

The proof of Claim 2 is rather detailed; here we provide only a rough intuition. Combining Proposition 1 and Claim 1, one can show the worst-case welfare under exogenous information coincides with the worst-case welfare under rational inattention only if a symmetric outcome exists that satisfies two conditions. First, an investor who does not see the mediator’s recommendation is indifferent between attacking and not attacking. And second, for an investor who does see the mediator’s recommendation, either not speculating is a best response to a “speculate” recommendation, or vice versa. Appealing to Bayes rule, one can show that an outcome satisfies these conditions if and only if there is limited overlap between the event where many investors are attacking and the event in which the attack succeeds. Claim 2 follows from showing a sufficient disconnect between these two events is attainable if and only if there is enough uncertainty about θ . Intuitively, disconnecting the two events is easy when θ obtains both high and low values with large probability: one can have the number of speculators just come short of a successful attack when θ is high, and come just above the threshold when θ is low. The same cannot be done when θ is deterministic. In that case, successful attacks necessarily involve more speculating investors than failed ones.

A takeaway is that, unlike under rational inattention, a planner who views information as exogenous may adopt policies that change the institution’s fundamentals. For a concrete illustration, consider two policies \mathcal{G} and \mathcal{G}' that differ only in the institution’s fundamentals, that is, in the set of states Θ and their distribution $\pi \in \Delta(\Theta)$. Suppose \mathcal{G} satisfies the conditions of Claim 2, but \mathcal{G}' does not. Worst-case welfare under rational inattention is the same for \mathcal{G} and \mathcal{G}' : $\underline{w} = \underline{w}'$ by (8). By contrast, \mathcal{G} generates lower welfare under exogenous information: $\bar{w} < \bar{w}'$ by (8) and Claim 2. Consequently, a planner who views information as being exogenous would pay some amount to change the institution’s fundamentals.

6. Further Results

In this section, we discuss additional results regarding non-generic environments and equilibria with almost-free information. All related proofs are in the Online Appendix.

6.1. Non-generic Environments

Theorem 2 shows the separation constraint has no bite for generic games. However, many economic environments, such as auctions, are non-generic. In this section we characterize the (non-generic) environments where the separation constraint has substantive impact. We also show that the impact of separation has an all-or-nothing flavor: if the sBCE set is not dense in the BCE set, it is in fact *nowhere dense*.

Fix a base game \mathcal{G} . For the separation constraint to bite, players must have weak incentives to follow the mediator's action recommendations. A special case is one where, whenever the mediator recommends player i action b_i in a BCE of the game, the player would be equally happy to take action a_i . Myerson (1997) calls this scenario “jeopardization”: action a_i **jeopardizes** action b_i if, for every BCE p such that $b_i \in \text{supp}_i(p)$, $a_i \in BR(p_{b_i})$.²¹ We denote by $J(b_i)$ the set of actions that jeopardizes b_i .

Every action jeopardizes itself by the obedience constraint; hence, $J(b_i)$ is not empty. A sufficient condition for jeopardization is weak domination: if $u_i(a_i, a_{-i}, \theta) \geq u_i(b_i, a_{-i}, \theta)$ for all $a_{-i} \in A_{-i}$ and $\theta \in \Theta$, then a_i jeopardizes b_i . But the concept of jeopardization is broader than weak domination. For example, in Matching Pennies, Heads and Tails jeopardize each other, even if neither action is weakly dominant.

Weak incentives are necessary but not sufficient for the separation constraint to be binding: it must also be that different action recommendations induce distinct posterior beliefs. Next we introduce a class of BCE in which such separation is most pronounced. Say a BCE p is **maximally supported** if the support of every other BCE is contained by the support of p . A maximally-supported BCE p is **minimally mixed** if

$$q_{a_i} \neq q_{b_i} \quad \text{implies} \quad p_{a_i} \neq p_{b_i}$$

for every BCE q , $i \in I$, and $a_i, b_i \in \text{supp}_i(q)$.

For an interpretation of minimal mixing, take the perspective of a mediator who wants to implement a BCE p . When $p_{a_i} = p_{b_i}$, the mediator can replace the distinct recommendations of playing a_i and b_i with a single recommendation of mixing between the two actions with probabilities $p(a_i)/(p(a_i) + p(b_i))$ and $p(b_i)/(p(a_i) + p(b_i))$. A BCE p is minimally mixed if a mediator has the least amount of opportunities to implement p recommending mixed actions. Whereas minimally mixed BCEs seem esoteric at first, they are in fact, ubiquitous: the set of minimally mixed BCEs is open and dense in the BCE set (see Lemma 20 in the Online Appendix).

Our next result uses the concepts of jeopardization and minimally mixed BCEs to characterize when the BCE set and the closure of the sBCE set coincide.

Proposition 2. *The following statements are equivalent:*

- (i) *The sBCE set is dense in the BCE set.*
- (ii) *A minimally mixed sBCE exists.*

²¹Myerson defines jeopardization for games without payoff uncertainty; here we give the obvious extension to games where Θ is not a singleton.

(iii) For every BCE p , $i \in I$, $a_i, b_i \in \text{supp}_i(p)$,

$$p_{a_i} \neq p_{b_i} \quad \text{implies} \quad J(a_i) \cap J(b_i) = \emptyset.$$

The result shows how jeopardization and minimal mixing can be used in applications to study sBCE. To verify that the sBCE set is dense in the BCE set, it is enough to produce a minimally mixed sBCE. To verify that the sBCE set is not dense in the BCE set, it is enough to produce a BCE in which two actions induce distinct beliefs and share a common jeopardizing action. As shown by Myerson (1997), the jeopardizing actions can be easily computed from the dual of the system of linear inequalities that defines BCE. In Online Appendix I.2, we simplify the analysis of minimally mixed BCE by describing how to find the actions that induce different beliefs for some BCE.

Next, we build on Proposition 2 and obtain that sBCE is an all-or-nothing refinement of BCE.

Theorem 4. *The sBCE set is either dense or nowhere dense in the BCE set.*

For a rough explanation, consider first the case in which a minimally mixed sBCE exists. Then, by Proposition 2, the sBCE set is dense in the BCE set. Consider now the case in which a minimally mixed sBCE does not exist. By Proposition 2, the sBCE set is not dense in the BCE set. To reach the stronger conclusion that the sBCE set is nowhere dense in the BCE set, we use the fact that the set of minimally mixed BCE is open and dense in the BCE set.

Thus, whereas the separation constraint does not bind in most circumstances, whenever it does bind, it significantly restricts the set of attainable outcomes.

6.2. Vanishing Cost Equilibrium

The case in which the cost of information is very small can be interpreted as a perturbation of complete information. As such, it can serve as a device for selecting equilibria of games where the state of the environment is commonly known.²² Existing works study this limit scenario under various restrictions on the information technology; next we obtain a robust characterization by leveraging the connection between rational inattention and separated BCE.

Fix a base game \mathcal{G} . An outcome $p \in \Delta_\pi(A \times \Theta)$ is a **complete-information Nash**

²²See, e.g., Yang (2015); Hoshino (2018); Denti (2021, forthcoming); Morris and Yang (2022). Ravid et al. (2022) conduct a similar exercise in which only one player can acquire information.

equilibrium if for every $\theta \in \Theta$ there is a mixed action profile $\alpha_\theta = (\alpha_{\theta,i})_{i \in I}$ such that

$$\begin{aligned} \frac{p(a, \theta)}{\pi(\theta)} &= \prod_{i \in I} \alpha_{\theta,i}(a_i), & \text{for all } a \in A, \\ \alpha_{\theta,i} &\in \arg \max_{\beta_i \in \Delta(A_i)} \sum_a u_i(a, \theta) \beta_i(a_i) \prod_{j \neq i} \alpha_{\theta,j}(a_j), & \text{for all } i \in I. \end{aligned}$$

Notice that every complete-information Nash equilibrium is a BCE. Such BCE is generated by an information structure in which the payoff state θ is commonly known and the correlation state z is degenerate.

Under what conditions a complete-information Nash equilibrium is the result of almost-frictionless information acquisition? To answer this question, we first make precise what we mean by “almost frictionless” information acquisition.

A technology \mathcal{T} represents **unconstrained rational inattention** if for every player i , all experiments are feasible (i.e., $\mathcal{E}_i = \Delta(X_i)^{Z \times \Theta}$), and the cost function C_i is monotone. An outcome $p \in \Delta_\pi(A \times \Theta)$ is a **vanishing cost equilibrium** if for every $\epsilon > 0$, there exist an unconstrained rational-inattention technology \mathcal{T} and an equilibrium (ξ, σ) of the information-acquisition game $(\mathcal{G}, \mathcal{T})$ such that, denoting by q the outcome of (ξ, σ) ,

$$\max C_i(\mathcal{E}_i) \leq \epsilon \quad \text{and} \quad |p(a, \theta) - q(a, \theta)| \leq \epsilon$$

for all $i \in I$, $a \in A$, and $\theta \in \Theta$.

The next theorem characterizes what complete-information Nash equilibria are vanishing cost equilibria.

Theorem 5. *A complete-information Nash equilibrium p is a vanishing cost equilibrium if and only if it belongs to the closure of the sBCE set.*

The “only if” side of Theorem 5 is an immediate consequence of Theorem 1: since outcomes of rational inattention games are separated BCEs, any vanishing cost equilibrium p must be the limit of a sequence of separated BCEs. The “if” side of Theorem 5 requires more work. Theorem 1 only guarantees the existence of a sequence of rational-inattention games whose outcomes converge to p : it does not give that information costs go to zero along the sequence. To prove this additional property, we use the fact that p is a complete-information Nash equilibrium.

Recall that, by Theorem 2, the sBCE set generically is dense in the BCE set. Thus, a corollary of Theorem 5 is that, generically, all complete-information Nash equilibria are vanishing cost equilibria.

Corollary 2. *For generic u , every complete-information Nash equilibrium is a vanishing cost equilibrium.*

As a consequence, if one intends to use rational inattention to select equilibria of complete information games, one needs either to focus on specialized economic environments or take a stance on the nature of the information acquisition technology.

Finally, we remark that not all vanishing cost equilibria are complete-information Nash equilibria. We provide a complete characterization of vanishing cost equilibria in Online Appendix J.2. In particular, we show that *convex combinations* of complete-information Nash equilibria can also be the result of almost-frictionless information acquisition.

7. Conclusion

Since its introduction by Sims (2003), rational inattention has emerged as a powerful theory, with a wide range of applications throughout economics. This theory serves both as a portable model of limited cognition, and as a way of anchoring information to incentives. But for all its merits, the theory’s current formalization has a significant drawback: researchers must make hard-to-verify assumptions on players’ information acquisition capabilities. The current paper addresses this concern by developing a framework for spanning all predictions consistent with rational inattention in a given economic setting.

We also use our framework to make two conceptual contributions. First, we show that in generic settings, the economic environment imposes no restrictions on the types of information that may emerge under rational inattention. Hence, studies employing rational inattention to understand the shape of agents’ information must either make specialized assumptions about the underlying information technology, or rely on highly-structured characteristics of the economic environment. Second, our work underscores the significance of considering the costs of information acquisition when assessing players’ welfare. Specifically, mistakenly assuming that information is given when it is actually acquired can lead one to overestimate the benefits players derive from their information, and so may result in misleading welfare conclusions.

Appendix

A. Preliminary results

The next sections introduce lemmas we will use to prove the results from the main text. The lemmas may be of independent interest, so we collect them here, in individual sections.

A.1. Single-agent lemmas on rational inattention

This section presents single-agent results on costly information acquisition. We take the perspective of an individual i who has to choose an action $a_i \in A_i$ whose utility $w_i(a_i, \omega)$ depends on an uncertain state of nature $\omega \in \Omega$. Both A_i and Ω are finite. Let $\rho \in \Delta(\Omega)$ be the prior distribution of the state; ρ may not have full support.

Before choosing an action, the decision maker can run an experiment $\xi_i : \Omega \rightarrow \Delta(X_i)$ at a cost $C_i(\xi_i) \in \mathbb{R}_+$. Let $\mathcal{E}_i \subseteq \Delta(X_i)^\Omega$ be the set of feasible experiments. The signal space X_i is finite and contains more elements than the sets Ω and A_i .

Overall, the decision maker faces the following information-acquisition problem:

$$\max_{\xi_i \in \mathcal{E}_i, \sigma_i \in \Sigma_i} \left[\sum_{\omega, x_i, a_i} w_i(a_i, \omega) \sigma_i(a_i | x_i) \xi_i(x_i | \omega) \rho(\omega) \right] - C_i(\xi_i) \quad (10)$$

where Σ_i is the set of all action plans $\sigma_i : X_i \rightarrow \Delta(A_i)$.

In accordance with the terminology used in the main text, we write $\xi_i \succsim \xi'_i$ if there is a Markov kernel $g : X_i \rightarrow \Delta(X_i)$ such that for every $x_i \in X_i$ and $\omega \in \Omega$ with $\rho(\omega) > 0$,

$$\xi'_i(x_i | \omega) = \sum_{x'_i \in X_i} g(x_i | x'_i) \xi(x'_i | \omega). \quad (11)$$

We say that \mathcal{E}_i is **flexible** if, whenever $\xi_i \succsim \xi'_i$ and $\xi_i \in \mathcal{E}_i$, then $\xi'_i \in \mathcal{E}_i$. We also say that $C_i : \mathcal{E}_i \rightarrow \mathbb{R}_+$ is **monotone** if, whenever $\xi_i, \xi'_i \in \mathcal{E}_i$ are such that $\xi_i \succsim \xi'_i$ (resp., $\xi_i \succ \xi'_i$), then $C_i(\xi_i) \geq C_i(\xi'_i)$ (resp., $C_i(\xi_i) > C_i(\xi'_i)$).

Next we characterize the pairs (ξ_i, σ_i) that are optimal solutions of (10) for some flexible \mathcal{E}_i and monotone C_i . We will use the following notation. Given a pair (ξ_i, σ_i) , we denote by $\mu_i \in \Delta(A_i \times X_i \times \Omega)$ the induced probability measure over actions, signals, and states:

$$\mu_i(a_i, x_i, \omega) = \sigma_i(a_i | x_i) \xi_i(x_i | \omega) \rho(\omega).$$

We also denote by $\mu_i(x_i)$ the unconditional probability of signal x_i :

$$\mu_i(x_i) = \sum_{a_i, \omega} \mu(a_i, \omega).$$

For all x_i such that $\mu_i(x_i) > 0$, let $\mu_{x_i} \in \Delta(\Omega)$ be the conditional distribution of the state:

$$\mu_{x_i}(\omega) = \frac{\sum_{a_i} \mu_i(a_i, x_i, \omega)}{\mu_i(x_i)}.$$

Let $BR(\mu_{x_i})$ be the corresponding set of best responses:

$$BR(\mu_{x_i}) = \operatorname{argmax}_{a_i \in A_i} \left[\sum_{\omega} u_i(a_i, \omega) \mu_{x_i}(\omega) \right].$$

Lemma 2. *A flexible $\mathcal{E}_i \subseteq \Delta(X_i)^\Omega$ and a monotone $C_i : \mathcal{E}_i \rightarrow \mathbb{R}_+$ exist such that (ξ_i, σ_i) is an optimal solution of (10) if and only if the following conditions hold:*

(i) *For all signals x_i such that $\mu_i(x_i) > 0$,*

$$\sigma_i(BR(\mu_{x_i}) | x_i) = 1.$$

(ii) *For all signals x_i and x'_i such that $\mu_i(x_i) > 0$ and $\mu_i(x'_i) > 0$,*

$$\mu_{x_i} \neq \mu_{x'_i} \quad \text{implies} \quad BR(\mu_{x_i}) \cap BR(\mu_{x'_i}) = \emptyset.$$

In addition, for every $\lambda_i \in (0, 1]$, one can choose C_i so that

$$C_i(\xi_i) = \lambda_i \left(\sum_{a_i, x_i, \omega} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) - \max_{a_i \in A_i} \sum_{\omega} w_i(a_i, \omega) \rho(\omega) \right).$$

Lemma 2 extends a result in Denti (2021). Denti (2021) uses a stronger version of Blackwell's informativeness: $\xi_i \succsim^* \xi'_i$ if there is a Markov kernel $g : X_i \rightarrow \Delta(X_i)$ such that for every $x_i \in X_i$ and $\omega \in \Omega$,

$$\xi'_i(x_i | \omega) = \sum_{x'_i \in X_i} g(x_i | x'_i) \xi(x'_i | \omega). \quad (12)$$

The difference between \succsim and \succsim^* is that (11) holds on the support of ρ while (12) on Ω .

We say that \mathcal{E}_i is **flexible*** if, whenever $\xi_i \succsim^* \xi'_i$ and $\xi_i \in \mathcal{E}_i$, then $\xi'_i \in \mathcal{E}_i$. We also say that $C_i : \mathcal{E}_i \rightarrow \mathbb{R}_+$ is **monotone*** if, whenever $\xi_i, \xi'_i \in \mathcal{E}_i$ are such that $\xi_i \succsim^* \xi'_i$ (resp., $\xi_i \succ^* \xi'_i$), then $C_i(\xi_i) \geq C_i(\xi'_i)$ (resp., $C_i(\xi_i) > C_i(\xi'_i)$).

Lemma 3 (Denti, 2021). *A flexible* $\mathcal{E}_i \subseteq \Delta(X_i)^\Omega$ and a monotone* $C_i : \mathcal{E}_i \rightarrow \mathbb{R}_+$ exist such that (ξ_i, σ_i) is an optimal solution of (10) if and only if the following conditions hold:*

(i) *For all signals x_i such that $\mu_i(x_i) > 0$,*

$$\sigma_i(BR(\mu_{x_i}) | x_i) = 1.$$

(ii) *For all signals x_i and x'_i such that $\mu_i(x_i) > 0$ and $\mu_i(x'_i) > 0$,*

$$\mu_{x_i} \neq \mu_{x'_i} \quad \text{implies} \quad BR(\mu_{x_i}) \cap BR(\mu_{x'_i}) = \emptyset.$$

(iii) *For all signals x_i and states ω , if $\xi_i(x_i | \omega) > 0$ then $\mu_i(x_i) > 0$.*

In addition, one can choose C_i so that

$$C_i(\xi_i) = \sum_{\omega, x_i, a_i} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) - \max_{a_i \in A_i} \sum_{\omega} w_i(a_i, \omega) \rho(\omega).$$

Proof of Lemma 2. To use Lemma 3, we define $\Omega^* = \text{supp}(\rho)$. Let ρ^* be the restriction of ρ to Ω , and let w_i^* be the restriction of w_i to $A_i \times X_i \times \Omega^*$. By construction, ρ^* has full support on Ω^* ; thus, for experiments defined on Ω^* , the rankings \succeq and \succeq^* coincide.

“If.” Suppose (ξ_i, σ_i) satisfy Lemma 2-(i) and Lemma 2-(ii). Let ξ_i^* be the restriction of ξ_i to Ω^* , and let μ_i^* be the restriction of μ_i to $A_i \times X_i \times \Omega^*$.

Lemma 2-(i) implies that for all signals x_i such that $\mu_i^*(x_i) > 0$,

$$\sigma_i(BR(\mu_{x_i}^*) | x_i) = 1.$$

Lemma 2-(ii) implies that for all signals x_i and x'_i such that $\mu_i^*(x_i) > 0$ and $\mu_i^*(x'_i) > 0$,

$$\mu_{x_i}^* \neq \mu_{x'_i}^* \quad \text{implies} \quad BR(\mu_{x_i}^*) \cap BR(\mu_{x'_i}^*) = \emptyset.$$

Since ρ^* has full support, if $\xi_i^*(x_i | \omega^*) > 0$ for $x_i \in X_i$ and $\omega^* \in \Omega^*$, then $\mu_i^*(x_i) > 0$.

Thus, by Lemma 3, there exist a flexible* $\mathcal{E}_i^* \subseteq \Delta(X_i)^{\Omega^*}$ and monotone* $C^* : \mathcal{E}^* \rightarrow \mathbb{R}_+$ such that (ξ_i^*, σ_i) is an optimal solution of

$$\max_{\xi_i^* \in \mathcal{E}_i^*, \sigma_i \in \Sigma_i} \left[\sum_{\omega^*, x_i, a_i} w_i^*(a_i, \omega^*) \sigma_i(a_i | x_i) \xi_i^*(x_i | \omega^*) \rho^*(\omega^*) \right] - C_i^*(\xi_i^*). \quad (13)$$

In addition, we can choose the cost function so that

$$C_i^*(\xi_i^*) = \sum_{\omega^*, x_i, a_i} w_i^*(a_i, \omega^*) \mu_i^*(a_i, x_i, \omega) - \max_{a_i \in A_i} \sum_{\omega^*} w_i^*(a_i, \omega^*) \rho^*(\omega^*).$$

Fix $\lambda_i \in (0, 1]$. Let \mathcal{E}_i be the set of all $\xi'_i : \Omega \rightarrow \Delta(X_i)$ such that $\xi \succeq \xi'$. For $\xi' \in \mathcal{E}_i$, define $C_i(\xi'_i) = \lambda_i C_i^*(\xi_i^*)$ where ξ_i^* is the restriction of ξ'_i to Ω^* . Note that C_i is well defined because, if $\xi_i \succeq \xi'_i$ and ξ_i^* is the restriction of ξ' to Ω^* , then $\xi_i^* \succeq^* \xi_i^*$, which in turn implies $\xi_i^* \in \mathcal{E}_i^*$ (given that \mathcal{E}_i^* is flexible*). Note that

$$C_i(\xi_i) = \lambda_i \left(\sum_{a_i, x_i, \omega} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) - \max_{a_i \in A_i} \sum_{\omega} w_i(a_i, \omega) \rho(\omega) \right).$$

Clearly, \mathcal{E}_i is flexible. Since C_i^* is monotone*, C_i is monotone. Moreover, since (ξ_i^*, σ_i) is an optimal solution of (13), then (ξ_i, σ_i) is an optimal solution of

$$\max_{\xi'_i \in \mathcal{E}_i, \sigma_i \in \Sigma_i} \left[\sum_{\omega, x_i, a_i} w_i(a_i, \omega) \sigma_i(a_i | x_i) \xi'_i(x_i | \omega) \rho(\omega) \right] - \frac{1}{\lambda_i} C_i(\xi'_i).$$

By construction of \mathcal{E}_i , (ξ_i, σ_i) is also optimal solution of

$$\max_{\xi'_i \in \mathcal{E}_i, \sigma_i \in \Sigma_i} \left[\sum_{\omega, x_i, a_i} w_i(a_i, \omega) \sigma_i(a_i | x_i) \xi'_i(x_i | \omega) \rho(\omega) \right].$$

Combining these two facts, we obtain that (ξ_i, σ_i) is an optimal solution of

$$\max_{\xi'_i \in \mathcal{E}_i, \sigma_i \in \Sigma_i} \left[\sum_{\omega, x_i, a_i} w_i(a_i, \omega) \sigma_i(a_i | x_i) \xi'_i(x_i | \omega) \rho(\omega) \right] - C_i(\xi'_i).$$

This concludes the proof of the “if” part of Lemma 2.

“Only if.” Let (ξ_i, σ_i) be an optimal solution of (10) for some flexible \mathcal{E}_i and some monotone C_i . Let ξ^* be the restrictions of ξ to Ω^* , and let μ_i^* be the restriction of μ_i to $A_i \times X_i \times \Omega^*$.

We denote by \mathcal{E}_i^* the set of all $\xi_i^* : \Omega^* \rightarrow \Delta(X_i)$ for which there exists $\xi'_i \in \mathcal{E}_i$ such that $\xi_i^*(\cdot | \omega^*) = \xi'_i(\cdot | \omega^*)$ for all $\omega^* \in \Omega^*$. For $\xi_i^* \in \mathcal{E}_i^*$, let $C_i^*(\xi_i^*)$ be the infimum of $C_i(\xi'_i)$ over all $\xi'_i \in \mathcal{E}_i$ such that ξ_i^* is the restriction of ξ'_i to Ω^* . Notice that, since C is monotone, $C_i^*(\xi_i^*) = C_i(\xi'_i)$ for all $\xi_i^* \in \mathcal{E}_i^*$ and $\xi'_i \in \mathcal{E}_i$ such that ξ_i^* is the restriction of ξ'_i to Ω^* .

Since \mathcal{E}_i is flexible, \mathcal{E}_i^* is flexible*. Since C is monotone, C^* is monotone*. Moreover, since (ξ_i, σ_i) is a optimal solution of (10) for \mathcal{E}_i and C_i , then (ξ_i^*, σ_i) is an optimal solution of

$$\max_{\xi_i^* \in \mathcal{E}_i^*, \sigma_i \in \Sigma_i} \left[\sum_{\omega^*, x_i, a_i} w_i^*(a_i, \omega^*) \sigma_i(a_i | x_i) \xi_i^*(x_i | \omega^*) \rho^*(\omega^*) \right] - C_i^*(\xi_i^*).$$

Then, Lemma 3-(i) implies Lemma 2-(i), and Lemma 3-(ii) implies Lemma 2-(ii). \square

Next we refine Lemma 2 by showing that one can put a bound on the cost of all exper-

iments. For short, we define

$$\begin{aligned}\underline{w}_i(\mu_i) &= \max_{b_i \in A_i} \sum_{a_i, x_i, \omega} w_i(b_i, \omega) \mu_i(a_i, x_i, \omega) = \max_{b_i \in A_i} \sum_{\omega} w_i(b_i, \omega) \rho(\omega), \\ \bar{w}_i(\mu_i) &= \sum_{a_i, x_i, \omega} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega), \\ \hat{w}_i(\mu_i) &= \sum_{\omega} \max_{b_i \in A_i} \sum_{a_i, x_i} w_i(b_i, \omega) \mu_i(a_i, x_i, \omega) = \sum_{\omega} \max_{b_i \in A_i} w_i(b_i, \omega) \rho(\omega).\end{aligned}$$

Lemma 4. *If (ξ_i, σ_i) satisfies Lemma 2-(i) and Lemma 2-(ii), then there exists a monotone $C_i : \Delta(X_i)^\Omega \rightarrow \mathbb{R}_+$ such that (ξ_i, σ_i) is an optimal solution of (10), with $\mathcal{E}_i = \Delta(X_i)^\Omega$. Moreover, for every $\lambda_i \in (0, 1]$, one can choose C_i so that*

$$\begin{aligned}C_i(\xi_i) &= \lambda_i (\bar{w}_i(\mu_i) - \underline{w}_i(\mu_i)), \\ \max_{\xi'_i \in \Delta(X_i)^\Omega} C_i(\xi'_i) &\leq \lambda_i + \hat{w}_i(\mu_i) - [(1 - \lambda_i) \bar{w}_i(\mu_i) + \lambda_i \underline{w}_i(\mu_i)].\end{aligned}$$

Proof. By Lemma 2, there are flexible $\mathcal{E}'_i \subseteq \Delta(X_i)^\Omega$ and monotone $C'_i : \mathcal{E}'_i \rightarrow \mathbb{R}_+$ such that (ξ_i, σ_i) is an optimal solution of (10). In addition, we can assume that

$$C'_i(\xi_i) = \lambda_i (\bar{w}_i(\mu_i) - \underline{w}_i(\mu_i)).$$

Note that the upper bound on information costs we wish to obtain can be re-written as

$$\max_{\xi'_i \in \Delta(X_i)^\Omega} C_i(\xi'_i) \leq \lambda_i + C'_i(\xi_i) + \hat{w}_i(\mu_i) - \bar{w}_i(\mu_i).$$

Let $C_i^* : \Delta(X_i)^\Omega \rightarrow \mathbb{R}_+$ be any finite monotone cost function defined over the set of all experiments (e.g., the entropy cost of Matějka and McKay, 2015). Let $K_i > 0$ be a positive constant such that for all $\xi'_i \in \Delta(X_i)^\Omega$,

$$K_i C_i^*(\xi'_i) \leq \lambda_i.$$

We define $C_i : \Delta(X_i)^\Omega \rightarrow \mathbb{R}_+$ as follows. For $\xi_i \succeq \xi'_i$, we set $C_i(\xi'_i) = C'_i(\xi'_i)$, and for $\xi_i \not\succeq \xi'_i$, we set

$$C_i(\xi'_i) = K_i C_i^*(\xi'_i) + C'_i(\xi_i) + \hat{w}_i(\mu_i) - \bar{w}_i(\mu_i).$$

Notice that for all $\xi'_i \in \Delta(X_i)^\Omega$,

$$C_i(\xi'_i) \leq \lambda_i + C'_i(\xi_i) + \hat{w}_i(\mu_i) - \bar{w}_i(\mu_i).$$

Thus, C_i has the desired bound. It remains to check that C_i is monotone, and that (ξ_i, σ_i)

is an optimal solution of (10) when all experiments are feasible and costs are given by C_i .

To verify that C_i is monotone, let ξ'_i and ξ''_i a pair of experiments. If $\xi'_i \sim \xi''_i$, then either both experiments are dominated by ξ_i , in which case

$$C_i(\xi'_i) = C'_i(\xi'_i) = C'_i(\xi''_i) = C_i(\xi''_i)$$

because C'_i is monotone, or neither experiment are dominated by ξ_i , in which case

$$C_i(\xi'_i) - C_i(\xi''_i) = K_i(C_i^*(\xi'_i) - C_i^*(\xi''_i)) = 0$$

because C_i^* is monotone. In any case, if $\xi'_i \sim \xi''_i$, then $C_i(\xi'_i) = C_i(\xi''_i)$.

Suppose now that $\xi'_i \succ \xi''_i$. If $\xi_i \succeq \xi'_i$, then $C_i(\xi'_i) > C_i(\xi''_i)$ because C'_i is monotone. If $\xi_i \not\succeq \xi'_i$, then $C_i(\xi'_i) > C_i(\xi''_i)$ because C_i^* is monotone. Finally, consider the case in which $\xi_i \not\succeq \xi'_i$ and $\xi_i \succeq \xi''_i$. We have that

$$\begin{aligned} C_i(\xi'_i) &= K_i C_i^*(\xi'_i) + C'_i(\xi_i) + \hat{w}_i(\mu_i) - \bar{w}_i(\mu_i) \\ &> C'_i(\xi_i) + \hat{w}_i(\mu_i) - \bar{w}_i(\mu_i) \\ &\geq C'_i(\xi''_i) = C_i(\xi''_i). \end{aligned}$$

where the strict inequality follows from C_i^* being monotone and $\xi_i \not\succeq \xi'_i$, and the weak inequality from C'_i being monotone and $\xi_i \succeq \xi''_i$. Overall, we conclude that C_i is monotone.

Next we check that (ξ_i, σ_i) is an optimal solution of (10) when all experiments are feasible and the cost of information is C_i . Let $\xi'_i \in \Delta(X_i)^\Omega$ and $\sigma'_i \in \Sigma_i$ be an alternative solution of (10). If $\xi_i \succeq \xi'_i$, then $\xi'_i \in \mathcal{E}'_i$ because $\xi_i \in \mathcal{E}'_i$ and \mathcal{E}'_i is flexible. Thus, since (ξ_i, σ_i) is an optimal solution of (10) when the set of feasible of experiments is \mathcal{E}'_i and the cost of information is C'_i , then (ξ'_i, σ'_i) cannot be strictly better than (ξ_i, σ_i) when the costs of ξ_i and ξ'_i are given by C_i , given that C_i coincide with C'_i on \mathcal{E}'_i .

Consider now the case in which $\xi_i \not\succeq \xi'_i$. The net payoff from (ξ'_i, σ'_i) is at most

$$\begin{aligned} \sum_{\omega} \max_{a_i} w_i(a_i, \omega) \rho(\omega) - C_i(\xi'_i) &= - \left(K_i C_i^*(\xi'_i) + C'_i(\xi_i) - \sum_{\omega, x_i, a_i} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) \right) \\ &\leq \sum_{\omega, x_i, a_i} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) - C'_i(\xi_i) \\ &= \sum_{\omega, x_i, a_i} w_i(a_i, \omega) \mu_i(a_i, x_i, \omega) - C_i(\xi_i). \end{aligned}$$

Overall, we conclude that (ξ_i, σ_i) is an optimal solution of (10) when all experiments are feasible and the cost of information is C_i . \square

A.2. Equilibrium information structures

In this section we characterize the information structures that can arise in an equilibrium of a rational inattention game for a fixed base game \mathcal{G} .

We adopt the following notation. Given an information structure $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and a profile of action plans $\sigma = (\sigma_i)_{i \in I}$, we denote by $\nu \in \Delta(A \times X \times Z \times \Theta)$ the induced probability measure over actions, signals, and states:

$$\nu(a, x, z, \theta) = \left[\prod_{i \in I} \sigma_i(a_i | x_i) \xi_i(x_i | z, \theta) \right] \zeta(z | \theta) \pi(\theta). \quad (14)$$

Let $\nu(x_i)$ be the probability that player i observes signal x_i :

$$\nu(x_i) = \sum_{a, x_{-i}, z, \theta} \nu(a, x_i, x_{-i}, z, \theta).$$

For all x_i such that $\nu(x_i) > 0$, we denote by $\nu_{x_i} \in \Delta(A_{-i} \times X_{-i} \times Z \times \Theta)$ the conditional distribution of the others' actions, the others' signals, and the state:

$$\nu_{x_i}(a_{-i}, x_{-i}, z, \theta) = \frac{\sum_{a_i} \nu(a_i, a_{-i}, x_i, x_{-i}, z, \theta)}{\nu(x_i)}.$$

Let $BR(\nu_{x_i})$ be the corresponding set of best responses:

$$BR(\nu_{x_i}) = \operatorname{argmax}_{a_i \in A_i} \left[\sum_{a_{-i}, x_{-i}, z, \theta} u_i(a_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right].$$

With a slight abuse of notation, we also define

$$\begin{aligned} \underline{v}_i(\nu) &= \max_{b_i \in A_i} \sum_{a, x, z, \theta} u_i(b_i, a_{-i}, \theta) \nu(a, x, z, \theta), \\ \bar{v}_i(\nu) &= \sum_{a, x, z, \theta} u_i(a, \theta) \nu(a, x, z, \theta), \\ \hat{v}_i(\nu) &= \sum_{z, \theta} \max_{b_i} \sum_{a, x} u_i(b_i, a_{-i}, \theta) \nu(a, x, z, \theta). \end{aligned}$$

Lemma 5. *Let \mathcal{G} be a base game, $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ an information structure, and $\sigma = (\sigma_i)_{i \in I}$ a profile of action plans. For every player i , there exist a flexible \mathcal{E}_i and a monotone C_i such that (ξ, σ) is an equilibrium of $(\mathcal{G}, \mathcal{T})$, with $\mathcal{T} = (Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$, if and only if for every player i , the following conditions hold:*

(i) For all signals x_i such that $\nu(x_i) > 0$,

$$\sigma_i(BR(\nu_{x_i}) | x_i) = 1.$$

(ii) For all signals x_i and x'_i such that $\nu(x_i) > 0$ and $\nu(x'_i) > 0$,

$$\nu_{x_i} \neq \nu_{x'_i} \quad \text{implies} \quad BR(\nu_{x_i}) \cap BR(\nu_{x'_i}) = \emptyset.$$

In addition, for every player i and scalar $\lambda_i \in (0, 1]$, one can choose $\mathcal{E}_i = \Delta(X_i)^{Z \times \Theta}$ and $C_i : \Delta(X_i)^{Z \times \Theta} \rightarrow \mathbb{R}_+$ such that

$$\begin{aligned} C_i(\xi_i) &= \lambda_i (\bar{v}_i(\nu) - \underline{v}_i(\nu)), \\ \max_{\xi'_i \in \Delta(X_i)^{Z \times \Theta}} C_i(\xi'_i) &\leq \lambda_i + \hat{v}_i(\nu) - [(1 - \lambda_i)\bar{v}_i(\nu) + \lambda_i \underline{v}_i(\nu)]. \end{aligned}$$

Proof. The proof of Lemma 5 builds on Lemmas 2 and 4. Paralleling the notation in Section A.1, for every player i we define an auxiliary decision problem. Given $\Omega := Z \times \Theta$, we take $\rho \in \Delta(\Omega)$ and $w_i : A_i \times \Omega \rightarrow \mathbb{R}$ as follows:

$$\rho(z, \theta) = \zeta(z|\theta)\pi(\theta), \tag{15}$$

$$w_i(a_i, z, \theta) = \sum_{a_{-i}, x_{-i}} u_i(a, \theta) \left[\prod_{j \neq i} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right]. \tag{16}$$

In the single-agent problem, Ω is the set of possible states, ρ is the prior distribution of the state, and w_i is the utility function (A_i remains the set of feasible actions). Like in Section A.1, given a signal x_i generated by ξ_i with positive probability, we denote by $\mu_{x_i} \in \Delta(\Omega)$ the conditional distribution of ω , and we write $BR(\mu_{x_i})$ for the best response set.

We will use the following claims, which relate the single-agent problem to the primitive many-player environment.

Claim 3. For all signals x_i such that $\nu(x_i) > 0$,

$$BR(\mu_{x_i}) = BR(\nu_{x_i}).$$

Proof of the claim. For all actions a_i , we have

$$\begin{aligned} \sum_{\omega} w_i(a_i, \omega) \mu_{x_i}(\omega) &= \sum_{a_{-i}, x_{-i}, z, \theta} u_i(a, \theta) \left[\prod_{j \neq i} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right] \frac{\xi_i(x_i | z, \theta) \zeta(z|\theta) \pi(\theta)}{\nu(x_i)} \\ &= \sum_{a_{-i}, x_{-i}, z, \theta} u_i(a, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta). \end{aligned}$$

We conclude that $BR(\mu_{x_i}) = BR(\nu_{x_i})$. \square

Claim 4. For all signals x_i and x'_i such that $\nu(x_i) > 0$ and $\nu(x'_i) > 0$,

$$\nu_{x_i} = \nu_{x'_i} \text{ if and only if } \mu_{x_i} = \mu_{x'_i}.$$

Proof of the claim. Notice first that for all ω ,

$$\begin{aligned} \mu_{x_i}(\omega) &= \sum_{a_{-i}, x_{-i}} \nu_{x_i}(a_{-i}, x_{-i}, \omega), \\ \mu_{x'_i}(\omega) &= \sum_{a_{-i}, x_{-i}} \nu_{x'_i}(a_{-i}, x_{-i}, \omega). \end{aligned}$$

Thus, $\nu_{x_i} = \nu_{x'_i}$ implies $\mu_{x_i} = \mu_{x'_i}$. For the converse direction, suppose $\mu_{x_i} = \mu_{x'_i}$. Then one obtains the following equality chain for all a_{-i} , x_{-i} , z , and θ :

$$\begin{aligned} \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) &= \frac{\xi_i(x_i|z, \theta) \zeta(z|\theta) \pi(\theta) \prod_{j \neq i} \sigma_j(a_j|x_j) \xi_j(x_j|z, \theta)}{\phi(x_i)} \\ &= \frac{\xi_i(x_i|z, \theta) \rho(z, \theta)}{\nu(x_i)} \prod_{j \neq i} \sigma_j(a_j|x_j) \xi_j(x_j|z, \theta) \\ &= \mu_{x_i}(z, \theta) \prod_{j \neq i} \sigma_j(a_j|x_j) \xi_j(x_j|z, \theta) \\ &= \mu_{x'_i}(z, \theta) \prod_{j \neq i} \sigma_j(a_j|x_j) \xi_j(x_j|z, \theta) = \nu_{x'_i}(a_{-i}, x_{-i}, z, \theta). \end{aligned}$$

We deduce that $\nu_{x_i} = \nu_{x'_i}$. \square

We are now ready to complete the proof of Lemma 5. For the “only if” part, notice that if (ξ, σ) is an equilibrium of $(\mathcal{G}, \mathcal{T})$, then for every player i , (ξ_i, σ_i) solve (10) for the setting defined above. Lemma 2 then delivers two facts. First, for all signals x_i such that $\nu(x_i) > 0$,

$$\sigma_i(BR(\nu_{x_i})|x_i) = \sigma_i(BR(\mu_{x_i})|x_i) = 1,$$

where the first equality follows from Claim 3, and the second from Lemma 2-(i). We conclude Lemma 5-(i) holds. Second, for all signals x_i and x'_i such that $\nu(x_i) > 0$ and $\nu(x'_i) > 0$, if $\nu_{x_i} \neq \nu_{x'_i}$ then $\mu_{x_i} \neq \mu_{x'_i}$ (by Claim 4), which implies

$$\emptyset = BR(\mu_{x_i}) \cap BR(\mu_{x'_i}) = BR(\nu_{x_i}) \cap BR(\nu_{x'_i}),$$

where the first equality follows from Lemma 2-(ii), and the second from Claim 3. Thus, Lemma 5-(ii) also holds.

For the “if” part, suppose that Lemma 5-(i) and Lemma 5-(ii) hold. Reasoning as above,

one can use Claims 3 and 4 to obtain that Lemma 2-(i) and Lemma 2-(ii) are satisfied. It follows that for every player i , we can find a flexible \mathcal{E}_i and a monotone C_i such that (ξ_i, σ_i) solves (10) for the setting defined above. In addition, using Lemma 4, we can assume that $\mathcal{E}_i = \Delta(X_i)^\Omega$ and, given $\lambda_i \in (0, 1]$, that $C_i : \Delta(X_i)^\Omega \rightarrow \mathbb{R}_+$ is such that

$$C_i(\xi_i) = \lambda_i (\bar{w}_i(\mu_i) - \underline{w}_i(\mu_i)) = \lambda_i (\bar{v}_i(\nu) - \underline{v}_i(\nu)),$$

and for all $\xi'_i \in \Delta(X_i)^\Omega$,

$$\begin{aligned} C_i(\xi'_i) &\leq \lambda_i + \hat{w}_i(\mu_i) - [(1 - \lambda_i)\bar{w}_i(\mu_i) + \lambda_i \underline{w}_i(\mu_i)] \\ &= \lambda_i + \hat{v}_i(\nu) - [(1 - \lambda_i)\bar{v}_i(\nu) + \lambda_i \underline{v}_i(\nu)]. \end{aligned}$$

Going back to the many-player environment, we construct the desired technology \mathcal{T} by combining the information structure \mathcal{S} with $(\mathcal{E}_i, C_i)_{i \in I}$: $\mathcal{T} := (Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$. \square

A.3. Representing outcomes with information structures

Fix a base game \mathcal{G} . An information structure $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and a profile of action plans $\sigma = (\sigma_i)_{i \in I}$ **represent** an outcome $p \in \Delta(A \times \Theta)$ if for all $a \in A$ and $\theta \in \Theta$,

$$p(a, \theta) = \sum_{z, x} \left[\prod_{i \in I} \sigma_i(a_i | x_i) \xi_i(x_i | z, \theta) \right] \zeta(z | \theta) \pi(\theta).$$

Each outcome admits multiple representations. For example, as is well known, one can represent p as follows. Set $Z = A$, and for every $i \in I$, take X_i so that $X_i \supseteq A_i$. For every $z \in A$ and $\theta \in \Theta$, set $\zeta(z | \theta) = p(z, \theta) / \pi(\theta)$. For every $i \in I$, $x_i \in X_i$, $z \in A$, and $\theta \in \Theta$, take

$$\xi_i(x_i | z, \theta) = \begin{cases} 1 & \text{if } x_i = z_i, \\ 0 & \text{otherwise.} \end{cases}$$

And finally, for every $i \in I$ and $x_i \in A_i$,

$$\sigma_i(a_i | x_i) = \begin{cases} 1 & \text{if } a_i = x_i, \\ 0 & \text{otherwise.} \end{cases}$$

This outcome representation has been used, among others, by Aumann (1987) and Bergemann and Morris (2016) to study games of incomplete information. Next, we introduce outcome representations that are useful to analyze information acquisition games.

Let \mathcal{A}_i^p be the partition of A_i with the following properties: for $a_i \in \text{supp}_i(p)$,

$$\mathcal{A}_i^p(a_i) = \{b_i \in \text{supp}_i(p) : p_{a_i} = p_{b_i}\},$$

and for $a_i \notin \text{supp}_i(p)$,

$$\mathcal{A}_i^p(a_i) = A_i \setminus \text{supp}_i(p).$$

In the formulas above, $\mathcal{A}_i^p(a_i)$ is the cell of the partition that contains a_i .

A representation (\mathcal{S}, σ) of an outcome p is **canonical** if the following conditions hold:

- $Z \subseteq \prod_{i \in I} \Delta(\mathcal{A}_i^p)$;
- for every $i \in I$,

$$\mathcal{A}_i^p \subseteq X_i; \tag{17}$$

- for every $i \in I$, $x_i \in X_i$, $z \in Z$, and $\theta \in \Theta$,

$$\xi_i(x_i|z, \theta) = \begin{cases} z_i(x_i) & \text{if } x_i \in \mathcal{A}_i^p, \\ 0 & \text{otherwise;} \end{cases} \tag{18}$$

- for every $i \in I$, $x_i \in \mathcal{A}_i^p$, and $a_i \in A_i$,

$$\sigma_i(a_i|x_i) = \begin{cases} \frac{p(a_i)}{\sum_{b_i \in x_i} p(b_i)} & \text{if } a_i \in x_i \text{ and } a_i \in \text{supp}_i(p), \\ \frac{1}{|A_i| - |\text{supp}_i(p)|} & \text{if } a_i \in x_i \text{ and } a_i \notin \text{supp}_i(p), \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Observe that canonical representations of the same outcome (essentially) differ only in the specification of Z and ζ , the correlation state.

Lemma 6. *Every outcome $p \in \Delta_\pi(A \times \Theta)$ admits a canonical representation.*

To prove the existence of a canonical representation, we use the following decomposition of p in terms of the partitions \mathcal{A}_i^p , $i \in I$:

Lemma 7. *Fix an outcome $p \in \Delta_\pi(A \times \Theta)$. For all $a \in A$ and $\theta \in \Theta$,*

$$p(a, \theta) = \left[\prod_{i \in I} \sigma_i(a_i | \mathcal{A}_i^p(a_i)) \right] \sum_{b \in \mathcal{A}^p(a)} p(b, \theta), \tag{20}$$

where $\sigma_i(a_i | \mathcal{A}_i^p(a_i))$ defined by (19), and $\mathcal{A}^p(a) = \prod_{i \in I} \mathcal{A}_i^p(a_i)$.

Proof. It suffices to show that for all $a \in A$, $\theta \in \Theta$, and $i \in I$,

$$p(a, \theta) = \sigma_i(a_i | \mathcal{A}_i^p(a_i)) \sum_{b_i \in \mathcal{A}_i^p(a_i)} p(b_i, a_{-i}, \theta). \quad (21)$$

The desired result then follows from reasoning by induction on the number of players.

If $p(a_i) = 0$, then

$$p(a, \theta) = 0 = \sum_{b_i \in \mathcal{A}_i^p(a_i)} p(b_i, a_{-i}, \theta) = \sigma_i(a_i | \mathcal{A}_i^p(a_i)) \sum_{b_i \in \mathcal{A}_i^p(a_i)} p(b_i, a_{-i}, \theta).$$

Thus, if $p(a_i) = 0$, then (21) holds.

Suppose now $p(a_i) > 0$. By Bayes rule,

$$p(a, \theta) = p(a_i) p_{a_i}(a_{-i}, \theta).$$

Recall that $b_i \in \mathcal{A}_i^p(a_i)$ if and only if $p_{b_i} = p_{a_i}$; in addition, $\sum_{b_i \in \mathcal{A}_i^p(a_i)} \sigma_i(b_i | \mathcal{A}_i^p(a_i)) = 1$. Therefore,

$$\begin{aligned} p(a_i, b_{-i}, \theta) &= p(a_i) p_{a_i}(b_{-i}, \theta) \\ &= p(a_i) \sum_{b_i \in \mathcal{A}_i^p(a_i)} \sigma_i(b_i | \mathcal{A}_i^p(a_i)) p_{a_i}(a_{-i}, \theta) \\ &= p(a_i) \sum_{b_i \in \mathcal{A}_i^p(a_i)} \sigma_i(b_i | \mathcal{A}_i^p(a_i)) p_{b_i}(a_{-i}, \theta). \end{aligned}$$

Substituting in the definition of σ_i , one obtains (21). \square

Proof of Lemma 6. Take

$$Z = \prod_{i \in I} \{\delta_{B_i} : B_i \in \mathcal{A}_i^p\}, \quad (22)$$

where $\delta_{B_i} \in \Delta(\mathcal{A}_i^p)$ is the Dirac measure concentrated on B_i . For $z = (\delta_{B_i})_{i \in I}$ and $B = \prod_{i \in I} B_i$, define

$$\zeta(z | \theta) = \sum_{a \in B} \frac{p(a, \theta)}{\pi(\theta)}. \quad (23)$$

For $i \in I$, define X_i , ξ_i , and σ_i as in (17)-(19). Let $\mathcal{S} := (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and $\sigma := (\sigma_i)_{i \in I}$.

We need to show that for all $a \in A$ and $\theta \in \Theta$,

$$p(a, \theta) = \sum_{z, x} \left[\prod_{i \in I} \sigma_i(a_i | x_i) \xi_i(x_i | z, \theta) \right] \zeta(z | \theta) \pi(\theta). \quad (24)$$

Using Lemma 7 we get that

$$\begin{aligned}
p(a, \theta) &= \left[\prod_{i \in I} \sigma_i(a_i | \mathcal{A}_i^p(a_i)) \right] \sum_z \left[\prod_{i \in I} z_i(\mathcal{A}_i^p(a_i)) \right] \zeta(z | \theta) \pi(\theta) \\
&= \sum_z \left[\prod_{i \in I} \sigma_i(a_i | \mathcal{A}_i^p(a_i)) \xi_i(\mathcal{A}_i^p(a_i) | z, \theta) \right] \zeta(z | \theta) \pi(\theta) \\
&= \sum_{z, x} \left[\prod_{i \in I} \sigma_i(a_i | x_i) \xi_i(x_i | z, \theta) \right] \zeta(z | \theta) \pi(\theta),
\end{aligned}$$

where the first equality combines (20) with (22) and (23), the second equality follows from (18), and the last equality from (19). We conclude (24) holds. \square

The next lemma concerns canonical representations of separated BCE. With a slight abuse of notation, we define

$$\hat{v}_i(\mathcal{S}, \sigma) = \sum_{z, \theta} \zeta(z | \theta) \pi(\theta) \left[\max_{a_i} \sum_{x_{-i}, a_{-i}} u_i(a_i, a_{-i}, \theta) \prod_{j \neq i} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right].$$

Lemma 8. *Let $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and σ be a canonical representation of an outcome p . If p is a separated BCE, then for every player i there exist a flexible \mathcal{E}_i and a monotone C_i such that (ξ, σ) is an equilibrium of $(\mathcal{G}, \mathcal{T})$ with $\mathcal{T} = (Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$. Moreover, for every player i , one can set $\mathcal{E}_i = \Delta(X_i)^{Z \times \Theta}$ and, given $\lambda_i \in (0, 1]$, choose $C_i : \Delta(X_i)^{Z \times \Theta} \rightarrow \mathbb{R}_+$ such that*

$$\begin{aligned}
C_i(\xi_i) &= \lambda_i(\bar{v}_i(p) - \underline{v}_i(p)), \\
\max_{\xi'_i \in \Delta(X_i)^{Z \times \Theta}} C_i(\xi'_i) &\leq \lambda_i + \hat{v}_i(\mathcal{S}, \sigma) - [(1 - \lambda_i)\bar{v}_i(p) + \lambda_i \underline{v}_i(p)].
\end{aligned}$$

Proof. We need to show that the conditions of Lemma 5 hold; like in Section A.2, we denote by $\nu \in \Delta(A \times X \times Z \times \Theta)$ the probability measure over actions, signals, and states induced by $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and σ .

We will use the following claim:

Claim 5. For all $i \in I$, $x_i \in X_i$ such that $\nu(x_i) > 0$, and $a_i \in x_i$, $BR(\nu_{x_i}) = BR(p_{a_i})$.

Proof of the claim. Fix $i \in I$, $x_i \in X_i$ such that $\nu(x_i) > 0$, and $a_i \in x_i$. Observe first that for all $b_i \in x_i$,

$$p(b_i) = \sum_{x'_i, z, \theta} \sigma_i(b_i | x'_i) \xi_i(x'_i | z, \theta) \zeta(z | \theta) \pi(\theta) = \sum_{z, \theta} \sigma_i(b_i | x_i) \xi_i(x_i | z, \theta) \zeta(z | \theta) \pi(\theta),$$

where the first equality holds because (\mathcal{S}, σ) represents p , and the second equality because $\sigma_i(b_i|x'_i) > 0$ if and only if $b_i \in x'_i$. Thus,

$$\sum_{b_i \in x_i} p(b_i) = \nu(x_i). \quad (25)$$

Reasoning as above, we have that for all $a_{-i} \in A_{-i}$ and $\theta \in \Theta$,

$$p(a, \theta) = \sum_{x'_i, x_{-i}, z} \nu(a, x'_i, x_{-i}, z, \theta) = \sum_{x_{-i}, z} \nu(a, x_i, x_{-i}, z, \theta).$$

Then, we obtain that

$$\begin{aligned} p(a, \theta) &= \sigma_i(a_i|x_i) \sum_{b_i, x_{-i}, z} \nu(b_i, a_{-i}, x, z, \theta) \\ &= \frac{p(a_i)}{\sum_{b_i \in x_i} p(b_i)} \sum_{b_i, x_{-i}, z} \nu(b_i, a_{-i}, x, z, \theta) = p(a_i) \sum_{x_{-i}, z} \nu_{x_i}(a_{-i}, x_{-i}, z, \theta), \end{aligned}$$

where the first equality follows from a_i being conditionally independent of the other variable given x_i , and the second equality from (25). In sum, we deduce that

$$p_{a_i}(a_{-i}, \theta) = \sum_{x_{-i}, z} \nu_{x_i}(a_{-i}, x_{-i}, z, \theta).$$

It follows that $BR(\nu_{x_i}) = BR(p_{a_i})$. \square

We now complete the proof by verifying the conditions of Lemma 5. For Lemma 5-(i), take $x_i \in X_i$ and $a_i \in A_i$ such that $\nu(x_i) > 0$ and $\sigma_i(a_i|x_i) > 0$. By the construction of the action plan, $a_i \in x_i$. By the obedience constraint for p , $a_i \in BR(p_{a_i})$. From Claim 5, $BR(p_{a_i}) = BR(\nu_{x_i})$. Therefore, $a_i \in BR(\nu_{x_i})$. We deduce that Lemma 5-(i) holds.

To verify Lemma 5-(ii), take $x_i, x'_i \in X_i$ such that $\nu(x_i) > 0$, $\nu(x'_i) > 0$, and $\nu_{x_i} \neq \nu_{x'_i}$. Let $a_i \in x_i$ and $b_i \in x'_i$. Since $\nu_{x_i} \neq \nu_{x'_i}$, we must have $x_i \neq x'_i$ and, therefore, $p_{a_i} \neq p_{b_i}$. By the separation constraint for p , $BR(p_{a_i}) \cap BR(p_{b_i}) = \emptyset$. From Claim 5, $BR(p_{a_i}) = BR(\nu_{x_i})$ and $BR(p_{b_i}) = BR(\nu_{x'_i})$. We deduce $BR(\nu_{x_i}) = BR(\nu_{x'_i}) = \emptyset$: Lemma 5-(ii) holds. \square

B. Proof of Theorem 1

The “if” statement of Theorem 1 follows immediately from Lemma 8—to apply the lemma, one needs a canonical representation for p , whose existence is guaranteed by Lemma 6.

Next we prove the “only if” statement of Theorem 1. Let (p, v) be the outcome-value pair induced by an equilibrium (ξ, σ) of a rational inattention game $(\mathcal{G}, \mathcal{T})$, with $\mathcal{T} =$

$(Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$. It follows the conditions of Lemma 5 must hold for the information structure $\mathcal{S} = (Z, \zeta, (X_i, \xi_i)_{i \in I})$ and the profile of action plans σ . We now use this fact to prove a connection between the best responses in the information acquisition game and optimal behavior given the mediator's recommendation. Like in Section A.2, we denote by $\nu \in \Delta(A \times X \times Z \times \Theta)$ the measure over actions, signals, and states induced by (\mathcal{S}, σ) . For $i \in I$ and $a_i \in \text{supp}_i(p)$, we denote by X_{a_i} the set of positive-probability signals that makes player i take action a_i :

$$X_{a_i} := \{x_i : \nu(x_i) > 0 \text{ and } \sigma_i(a_i|x_i) > 0\}.$$

Lemma 9. *For every $i \in I$ and $a_i \in \text{supp}_i(p)$,*

$$BR(p_{a_i}) = \bigcap_{x_i \in X_{a_i}} BR(\nu_{x_i}).$$

Proof. First we show that

$$BR(p_{a_i}) \subseteq \bigcap_{x_i \in X_{a_i}} BR(\nu_{x_i}).$$

Take $b_i \in BR(p_{a_i})$. By Lemma 5-(i), $a_i \in BR(\nu_{x_i})$ for all $x_i \in X_{a_i}$, which implies

$$\sum_{a_{-i}, x_{-i}, z, \theta} u_i(a_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \geq \sum_{a_{-i}, x_{-i}, z, \theta} u_i(b_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta). \quad (26)$$

Letting

$$\nu(x_i|a_i) = \frac{\sigma_i(a_i|x_i)\nu(x_i)}{p(a_i)},$$

simple algebra shows that

$$p_{a_i}(a_{-i}, \theta) = \sum_{x_i: \nu(x_i) > 0} \nu(x_i|a_i) \left[\sum_{x_{-i}, z} \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right]. \quad (27)$$

Therefore,

$$\begin{aligned} \sum_{x_i: \nu(x_i) > 0} \nu(x_i|a_i) \left(\sum_{a_{-i}, x_{-i}, z, \theta} u_i(a_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right) &= \sum_{a_{-i}, \theta} u_i(a_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta), \\ \sum_{x_i: \nu(x_i) > 0} \nu(x_i|a_i) \left(\sum_{a_{-i}, x_{-i}, z, \theta} u_i(b_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right) &= \sum_{a_{-i}, \theta} u_i(b_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta). \end{aligned}$$

As a consequence, since $b_i \in BR(p_{a_i})$, we have that

$$\begin{aligned} & \sum_{x_i: \nu(x_i) > 0} \nu(x_i | a_i) \left(\sum_{a_{-i}, x_{-i}, z, \theta} u_i(a_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right) \\ & \leq \sum_{x_i: \nu(x_i) > 0} \nu(x_i | a_i) \left(\sum_{a_{-i}, x_{-i}, z, \theta} u_i(b_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \right). \end{aligned}$$

It follows (26) holds with equality for all $x_i \in X_{a_i}$. Since $a_i \in BR(\nu_{x_i})$, $b_i \in BR(\nu_{x_i})$.

Now we show that

$$BR(p_{a_i}) \supseteq \bigcap_{x_i \in X_{a_i}} BR(\nu_{x_i}).$$

Let $b_i \in A_i$ such that $b_i \in BR(\nu_{x_i})$ for all $x_i \in X_{a_i}$, that is,

$$\sum_{a_{-i}, x_{-i}, z, \theta} u_i(c_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \leq \sum_{a_{-i}, x_{-i}, z, \theta} u_i(b_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta).$$

for all $c_i \in A_i$. Averaging across inequalities, we obtain that

$$\begin{aligned} & \sum_{x_i \in X_{a_i}} \nu(x_i | a_i) \sum_{a_{-i}, x_{-i}, z, \theta} u_i(c_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta) \\ & \leq \sum_{x_i \in X_{a_i}} \nu(x_i | a_i) \sum_{a_{-i}, x_{-i}, z, \theta} u_i(b_i, a_{-i}, \theta) \nu_{x_i}(a_{-i}, x_{-i}, z, \theta). \end{aligned}$$

Using (27), we obtain that

$$\sum_{a_{-i}, \theta} u_i(c_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) \leq \sum_{a_{-i}, \theta} u_i(b_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta).$$

We deduce that $b_i \in BR(p_{a_i})$. □

Next, we use the above to show the equilibrium outcome must satisfy the obedience constraint and the separation constraint.

Lemma 10. *The equilibrium outcome p is a sBCE.*

Proof. To check the obedience constraint, take $i \in I$ and $a_i \in \text{supp}_i(p)$. By Lemma 5-(i), $a_i \in BR(\nu_{x_i})$ for all $x_i \in X_{a_i}$. By Lemma 9, $a_i \in BR(p_{a_i})$.

To verify the separation constraint, let $i \in I$ and $a_i, b_i \in \text{supp}_i(p)$ such that $p_{a_i} \neq p_{b_i}$. Since $p_{a_i} \neq p_{b_i}$, there must be a pair of signals $x_{a_i} \in X_{a_i}$ and $x_{b_i} \in X_{b_i}$ such that $\mu_{x_{a_i}} \neq \mu_{x_{b_i}}$. By Lemma 5-(ii),

$$BR(\mu_{x_{a_i}}) \cap BR(\mu_{x_{b_i}}) = \emptyset.$$

By Lemma 9, $BR(p_{a_i}) \cap BR(p_{b_i}) = \emptyset$. \square

The next lemma concludes the proof of the “only if” statement of Theorem 1.

Lemma 11. *For every player i ,*

$$v_i = \underline{v}_i(p) = \bar{v}_i(p) \quad \text{or} \quad v_i \in [\underline{v}_i(p), \bar{v}_i(p)).$$

Proof. For every player i , $C_i(\xi_i) \geq 0$, which implies that $v_i \leq \bar{v}_i(p)$. In addition, uninformative experiments have zero cost by hypothesis. Thus, since (ξ_i, σ_i) is a best response to (ξ_{-i}, σ_{-i}) , we have that for any uninformative experiment ξ'_i ,

$$v_i \geq \max_{\sigma'_i} \sum_{a, x, z, \theta} u_i(a, \theta) \left[\sigma'_i(a_i | x_i) \xi_i(x_i | z, \theta) \prod_{j \neq i} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right] \zeta(z | \theta) \pi(\theta) = \underline{v}_i(p).$$

Overall, we conclude that $v_i \in [\underline{v}_i(p), \bar{v}_i(p)]$

If $\underline{v}_i(p) = \bar{v}_i(p)$, then $v_i \in [\underline{v}_i(p), \bar{v}_i(p)]$ implies $v_i = \underline{v}_i(p) = \bar{v}_i(p)$. Suppose instead that $\underline{v}_i(p) < \bar{v}_i(p)$. If

$$\sum_{a, x, z, \theta} u_i(a, \theta) \left[\prod_{j \in I} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right] \zeta(z | \theta) \pi(\theta) < \bar{v}_i(p),$$

then obviously $v_i < \bar{v}_i(p)$ because $C_i(\xi_i) \geq 0$. If, on the other hand,

$$\sum_{a, x, z, \theta} u_i(a, \theta) \left[\prod_{j \in I} \sigma_j(a_j | x_j) \xi_j(x_j | z, \theta) \right] \zeta(z | \theta) \pi(\theta) = \bar{v}_i(p),$$

then ξ_i cannot be uninformative because $\underline{v}_i(p) < \bar{v}_i(p)$. By monotonicity, $C_i(\xi_i) > 0$, which implies that $v_i < \bar{v}_i(p)$. In sum, if $\underline{v}_i(p) < \bar{v}_i(p)$, then $v_i \in [\underline{v}_i(p), \bar{v}_i(p))$. \square

C. Proof of Theorem 2

Throughout this section, we fix the set of players I , the set of payoff states Θ , the prior π (with full support), and an action set A_i for every player i . Given these, specifying the players' utilities is all that is left for defining a base game. As such, we use the profile of utility functions $u = (u_i)_{i \in I}$ as a shorthand for the base game it defines, writing for example $BCE(u)$ for the set of BCEs. We also denote by $sBCE(u)$ the set of sBCEs, by $\text{cl}(sBCE(u))$ the closure of $sBCE(u)$, and by $\|u\|$ the Euclidean norm.

Lemma 12. *For every $u \in \mathbb{R}^{I \times A \times \Theta}$, $p \in BCE(u)$, and $\epsilon > 0$, there exists $u' \in \mathbb{R}^{I \times A \times \Theta}$ such that $\|u - u'\| \leq \epsilon$ and $p \in sBCE(u')$.*

Proof. For each player $i \in I$, we consider a set $P_i \subseteq \Delta(A_{-i} \times \Theta)$ given by

$$P_i = \{p_{a_i} : a_i \in \text{supp}_i(p)\}.$$

Let n_i be the cardinality of P_i (of course, it could be that n_i is smaller than the cardinality of $\text{supp}_i(p)$). Reasoning inductively, we can find an enumeration p_1, \dots, p_{n_i} of the elements of P_i such that, for every $m_i \in \{1, \dots, n_i\}$, p_{m_i} is an extreme point of the convex hull of $\{p_1, \dots, p_{m_i}\}$.

By an hyperplane separation theorem (e.g., Rockafellar, 1970, Corollary 11.4.2) for every $m_i \in \{2, \dots, n_i\}$ we can find a function $f_{m_i} : A_{-i} \times \Theta \rightarrow \mathbb{R}$ such that

$$\sum_{a_{-i}, \theta} f_{m_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta) > 0 \geq \max_{l_i \in \{1, \dots, m_i-1\}} \sum_{a_{-i}, \theta} f_{m_i}(a_{-i}, \theta) p_{l_i}(a_{-i}, \theta). \quad (28)$$

For $m_i = 1$, we define $f_1(a_{-i}, \theta) = 1$ for all $a_{-i} \in A_{-i}$ and $\theta \in \Theta$.

For every $l_i \in \{1, \dots, n_i - 1\}$, we choose $t_{l_i} \in (0, 1]$ such that for every $m_i \in \{l_i + 1, \dots, n_i\}$,

$$\sum_{a_{-i}, \theta} f_{m_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta) > t_{l_i} \sum_{a_{-i}} f_{l_i}(a_{-i}) p_{m_i}(a_{-i}). \quad (29)$$

We can choose such a t_{l_i} because the left-hand side of (29) is positive—see (28). For $l_i = n_i$, we simply define $t_{n_i} = 1$.

For every $l_i \in \{1, \dots, n_i\}$, we define

$$s_{l_i} = \prod_{m_i=l_i}^{n_i} t_{m_i}.$$

Using (29), simple algebra shows that for every $l_i \in \{1, \dots, n_i - 1\}$ and $m_i \in \{l_i + 1, \dots, n_i\}$,

$$s_{m_i} \sum_{a_{-i}, \theta} f_{m_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta) > s_{l_i} \sum_{a_{-i}, \theta} f_{l_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta). \quad (30)$$

For $a_i \in \text{supp}_i(p)$, we define the function $g_{a_i} : A_{-i} \times \Theta \rightarrow \mathbb{R}$ by

$$g_{a_i}(a_{-i}, \theta) = s_{m_i} \cdot f_{m_i}(a_{-i}, \theta)$$

where m_i is such that $p_{a_i} = p_{m_i}$. For $a_i \notin \text{supp}_i(p)$, we define $g_{a_i} = 0$.

We claim that for all $a_i \in \text{supp}_i(p)$ and $b_i \notin \{c_i \in \text{supp}_i(p) : p_{c_i} = p_{a_i}\}$,

$$\sum_{a_{-i}, \theta} g_{a_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) > \sum_{a_{-i}, \theta} g_{b_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta). \quad (31)$$

To verify the claim, pick $m_i \in \{1, \dots, n_i\}$ such that $p_{a_i} = p_{m_i}$. From the left-hand side of (28) and the fact that $s_{m_i} > 0$, we obtain that

$$\sum_{a_{-i}, \theta} g_{a_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) = s_{m_i} \sum_{a_{-i}, \theta} f_{m_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta) > 0. \quad (32)$$

Hence, for $b_i \notin \text{supp}_i(p)$, we have

$$\sum_{a_{-i}, \theta} g_{a_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) > 0 = \sum_{a_{-i}, \theta} g_{b_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta)$$

where the equality follows from $g_{b_i} = 0$.

Assume now that $b_i \in \text{supp}_i(p)$. Choose l_i such that $p_{b_i} = p_{l_i}$. Since $p_{a_i} \neq p_{b_i}$, $m_i \neq l_i$. Suppose that $l_i > m_i$. It follows from the right-hand side of (28)—in (28) the roles of l_i and m_i are inverted—that

$$0 \geq \sum_{a_{-i}, \theta} f_{l_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta).$$

Thus, given that $s_{l_i} > 0$, we deduce that

$$0 \geq \sum_{a_{-i}, \theta} g_{b_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) = s_{l_i} \sum_{a_{-i}, \theta} f_{l_i}(a_{-i}, \theta) p_{m_i}(a_{-i}, \theta).$$

We obtain that

$$\sum_{a_{-i}, \theta} g_{a_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) > 0 \geq \sum_{a_{-i}, \theta} g_{b_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta)$$

where we use again (32). For the case $l_i < m_i$, the condition

$$\sum_{a_{-i}, \theta} g_{a_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta) > \sum_{a_{-i}, \theta} g_{b_i}(a_{-i}, \theta) p_{a_i}(a_{-i}, \theta)$$

is equivalent to (30). We conclude that (31) holds.

To complete the proof of the lemma, for every $\delta > 0$ we define $u' = (u'_i)_{i \in I}$ by

$$u'_i(a, \theta) = u_i(a, \theta) + \delta g_{a_i}(a_{-i}, \theta).$$

By choosing δ sufficiently small, we can make sure that $\|u - u'\| \leq \epsilon$. Since $p \in BCE(u)$, it follows from (31) that for all $i \in I$ and $a_i \in \text{supp}_i(p)$,

$$a_i \in BR'(p_{a_i}) \subseteq \{b_i \in \text{supp}_i(p) : p_{a_i} = p_{b_i}\}$$

where $BR'(p_{a_i})$ is the set of i 's best response to a belief p_{a_i} given utility function u'_i . Thus, $p \in sBCE(u')$. \square

A subset of a Euclidean space is **semi-algebraic** if it is defined by finite systems of polynomial inequalities. A correspondence between Euclidean spaces is semi-algebraic if its graph is semi-algebraic. The background knowledge on semi-algebraic sets that we use in this proof can be gathered from Blume and Zame (1994, Section 2).

Lemma 13. *The correspondences $u \mapsto BCE(u)$, $u \mapsto sBCE(u)$, and $u \mapsto \text{cl}(sBCE(u))$ are semi-algebraic.*

Proof. The BCE correspondence is semi-algebraic: for all $u \in \mathbb{R}^{I \times A \times \Theta}$ and $p \in \mathbb{R}^{A \times \Theta}$, $p \in BCE(u)$ if and only if the pair (u, p) is a solution to the following finite system of polynomial inequalities:

$$\begin{aligned} p(a, \theta) &\geq 0 && \text{for all } a \in A \text{ and } \theta \in \Theta, \\ \sum_a p(a, \theta) &= \pi(\theta) && \text{for all } \theta \in \Theta, \\ \sum_{a_{-i}, \theta} (u(a, \theta) - u(b_i, a_{-i}, \theta))p(a, \theta) &\geq 0 && \text{for all } i \in I \text{ and } a_i, b_i \in A_i. \end{aligned}$$

The sBCE correspondence is also semi-algebraic. To prove it, for every $u \in \mathbb{R}^{I \times A \times \Theta}$, $p \in \mathbb{R}^{A \times \Theta}$, $i \in I$, and $a_i, b_i, c_i \in A_i$, we denote by $F(u, p, a_i, b_i, c_i)$ the quantity

$$\sum_{a_{-i}, \theta} (u(a_i, a_{-i}, \theta) - u(c_i, a_{-i}, \theta))p(a_i, a_{-i}, \theta) + (u(b_i, a_{-i}, \theta) - u(c_i, a_{-i}, \theta))p(b_i, a_{-i}, \theta).$$

We observe that $p \in sBCE(u)$ if and only if $p \in BCE(u)$ and for every $i \in I$ there is $T_i \subseteq A_i \times A_i$ such that the pair (u, p) is a solution of the following finite system of polynomial inequalities:

$$\begin{aligned} \sum_{a_{-i}, \theta} (p(a_i, a_{-i}, \theta)p(b_i) - p(b_i, a_{-i}, \theta)p(a_i))^2 &= 0 && \text{for all } (a_i, b_i) \in T_i, \\ F(u, p, a_i, b_i, c_i) &> 0 && \text{for all } (a_i, b_i) \notin T_i \text{ and } c_i \in A_i. \end{aligned}$$

Thus, $p \in sBCE(u)$ if and only if it is the solution of one of finitely many systems of polynomial inequalities; we conclude that the sBCE correspondence is semi-algebraic.

The correspondence $u \mapsto \text{cl}(sBCE(u))$ is also semi-algebraic. Indeed, $p \in \text{cl}(sBCE(u))$ if and only if for every $\epsilon > 0$ there exists $q \in \mathbb{R}^{A \times \Theta}$ such that $\|p - q\| \leq \epsilon$ and $q \in sBCE(u)$. Thus, since the sBCE correspondence is semi-algebraic, the graph of the correspondence

$u \mapsto \text{cl}(sBCE(u))$ is defined by a first-order formula and therefore semi-algebraic by the Tarski-Seidenberg theorem (Blume and Zame, 1994, page 787). \square

We are ready to complete the proof of the theorem. By Lemma 13, the correspondence $u \mapsto \text{cl}(sBCE(u))$ is semi-algebraic. Hence, there is an open subsets U of $\mathbb{R}^{I \times A \times \Theta}$ such that the complement of U has Lebesgue measure zero, and $u \mapsto \text{cl}(sBCE(u))$ is continuous on U (Blume and Zame, 1994, page 786).

We claim that for all $u \in U$, $BCE(u) = \text{cl}(sBCE(u))$. To prove the claim, take any $u \in U$. Since $BCE(u)$ is closed and $sBCE(u) \subseteq BCE(u)$, $\text{cl}(sBCE(u)) \subseteq BCE(u)$. To verify the other inclusion, we use Lemma 12 to find a sequence of games $(u^n)_{n=1}^\infty$ such that $u^n \rightarrow u$ and, for every n , $p \in sBCE(u^n) \subseteq \text{cl}(sBCE(u^n))$. Since $\text{cl}(sBCE(u))$ is continuous at u , we have $p \in \text{cl}(sBCE(u))$; see Aliprantis and Border (2006, Theorem 17.16). Hence, $BCE(u) \subseteq \text{cl}(sBCE(u))$. We deduce that $BCE(u) = \text{cl}(sBCE(u))$, as desired.

We conclude that for generic u , the sBCE set is dense in the BCE set.

D. Proof of Theorem 3

Throughout this section, we fix the set of players I , the set of payoff states Θ , the prior π (with full support), and an action set A_i for every player i . Given these, specifying the players' utilities is all that is left for defining a base game. As such, we use the profile of utility functions $u = (u_i)_{i \in I}$ as a shorthand for the base game it defines, writing for example $BCE(u)$ for the set of BCEs.

The first part of the theorem immediately follows from Theorem 2, together with (3) and (4). Next, we assume that $|I| \geq 2$, $|\Theta| \geq 2$, and $|A_i| \geq 2$ for all players i , and show there is an open set $U \subseteq \mathbb{R}^{I \times A \times \Theta}$ such that $V_I(u) \subset V_R(u)$ for all $u \in U$.

To prove $V_I(u) \subset V_R(u)$, we will determine that

$$\min_{v \in V_R(u)} \sum_i v_i < \min_{v \in V_I(u)} \sum_i v_i. \quad (33)$$

Consistently with the notation of Section 5, we write $\underline{w}(p, u) = \sum_i \underline{v}_i(p, u)$ and $\bar{w}(p, u) = \sum_i \bar{v}_i(p, u)$. We also denote by $\underline{w}(u)$ the minimum of $\underline{w}(p, u)$ over all $p \in BCE(u)$, and by $\bar{w}(u)$ the minimum of $\bar{w}(p, u)$ over all $p \in BCE(u)$. Note that

$$\min_{v \in V_R(u)} \sum_i v_i = \min_{p \in \text{cl}(sBCE(u))} \underline{w}(p, u) \geq \underline{w}(u), \quad \text{and} \quad \min_{v \in V_I(u)} \sum_i v_i = \bar{w}(u).$$

The next lemma gives a sufficient condition for the existence of an open set $U \subseteq \mathbb{R}^{I \times A \times \Theta}$ such that (33) holds for all $u \in U$.

Lemma 14. Suppose $u^* \in \mathbb{R}^{I \times A \times \Theta}$ and $p^* \in \Delta_\pi(A \times \Theta)$ satisfy the following properties:

- (i) Each player i takes at least two actions at p^* : $|\text{supp}_i(p^*)| \geq 2$.
- (ii) p^* is a strict BCE: $BR(p_{a_i}) = \{a_i\}$ for all $i \in I$ and $a_i \in \text{supp}_i(p)$.
- (iii) p^* is the unique minimizer of $\underline{w}(p, u^*)$ over all $p \in BCE(u^*)$.

Then there is a neighborhood U of u^* such that, for all $u \in U$,

$$\min_{p \in \text{cl}(sBCE(u))} \underline{w}(p, u) < \bar{w}(u).$$

Proof. Take u^* and p^* that satisfy (i)-(iii). First, we verify that

$$\underline{w}(p^*, u^*) < \bar{w}(u^*). \quad (34)$$

Take $p \in BCE(u^*)$ such that $\bar{w}(u^*) = \bar{w}(p, u^*)$. If $p \neq p^*$, then $\bar{w}(p, u^*) \geq \underline{w}(p, u^*) > \underline{w}(p^*, u^*)$, where the strict inequality holds by (iii); thus, $\bar{w}(u^*) > \underline{w}(p^*, u^*)$. If instead $p = p^*$, then $\bar{w}(p^*, u^*) > \underline{w}(p^*, u^*)$ by (i) and (ii); thus $\bar{w}(u^*) > \underline{w}(p^*, u^*)$. Overall, we conclude that (34) holds, as desired.

The rest of the proof proceed by contradiction. To attain this contradiction, suppose there is a sequence $(u^n)_{n=1}^\infty$ converging to u^* such that

$$\min_{p \in \text{cl}(sBCE(u^n))} \underline{w}(p, u^n) = \bar{w}(u^n) \quad \text{for all } n. \quad (35)$$

By (ii), $p^* \in sBCE(u^n)$ for all n sufficiently large. Thus,

$$\underline{w}(p^*, u^n) \geq \min_{p \in \text{cl}(sBCE(u^n))} \underline{w}(p, u^n) \quad \text{for all } n \text{ large enough.} \quad (36)$$

Combining (35) and (36), we obtain that

$$\underline{w}(p^*, u^n) \geq \bar{w}(u^n) \quad \text{for all } n \text{ large enough.} \quad (37)$$

By standard arguments, $\underline{w}(p^*, u)$ is continuous in u . In addition, since the correspondence $u \mapsto BCE(u)$ is upper hemicontinuous, $\bar{w}(u) = \min_{p \in BCE(u)} \bar{w}(p, u)$ is lower semicontinuous in u (e.g., Aliprantis and Border, 2006, Lemma 17.3). It follows from (37) that

$$\underline{w}(p^*, u^*) = \liminf_{n \rightarrow \infty} \underline{w}(p^*, u^n) \geq \liminf_{n \rightarrow \infty} \bar{w}(u^n) \geq \bar{w}(u^*).$$

Hence, $\underline{w}(p^*, u^*) \geq \bar{w}(u^*)$, which contradicts (34). \square

To complete the proof of the theorem, we construct a utility profile u^* and an outcome p^* that satisfy the conditions of Lemma 14. This lemma then delivers a neighborhood U of u^* such that (33) holds for all $u \in U$, which in turn means that the set of u such that $V_I(u) \subset V_R(u)$ has non-empty interior.

We now construct u^* and p^* . Let n be the cardinality of I ; by hypothesis, $n \geq 2$. For every player i , we order the set of actions from 0 to m_i (where $m_i + 1$ is the cardinality of A_i): $A_i = \{0, \dots, m_i\}$. By hypothesis, A_i contains at least two distinct elements, thus $m_i \geq 1$. We also consider a partition $\Theta = \Theta_l \cup \Theta_h$ of the set of payoff states such that both Θ_l and Θ_h are nonempty; this is feasible because, by hypothesis, Θ contains at least two elements.

For player i , we define u_i^* as follows:

$$u_i^*(a, \theta) = \begin{cases} 0 & \text{if } a_i = 0, \\ \frac{1}{\pi(\Theta_l)} \left(-1 + \frac{1}{n-1} \sum_{j \neq i} a_j \right) & \text{if } a_i = 1 \text{ and } \theta \in \Theta_l, \\ \frac{1}{\pi(\Theta_h)} \left(2 - \frac{1}{n-1} \sum_{j \neq i} a_j \right) & \text{if } a_i = 1 \text{ and } \theta \in \Theta_h, \\ -1 & \text{if } a_i > 1. \end{cases}$$

Thus, action 0 is a safe action. Action 1 has a payoff that depends both on the state and on the average action of the opponents. For states in Θ_l , action 1 generates a *negative* baseline payoff of -1 , but there is also a *positive* externality from the actions of others; these payoffs are scaled by $1/\pi(\Theta_l)$. For states in Θ_h , action 1 generates a *positive* baseline payoff of 2, but there is also a *negative* externality from the actions of others; these payoffs are scaled by $1/\pi(\Theta_h)$. Any action outside $\{0, 1\}$ is strictly dominated by 0.

Let p^* be the outcome such that all players take action 0 when $\theta \in \Theta_l$, and all players take action 1 when $\theta \in \Theta_h$. It is clear that conditions (i) and (ii) of Lemma 14 are satisfied.

All that remains is to verify (iii). To do so, note first that p^* is the unique minimizer of $\sum_i \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p(a, \theta)$ over all $p \in \Delta_\pi(\{0, 1\}^I \times \Theta)$. Since any action outside $\{0, 1\}$ is strictly dominated, we deduce that p^* is the unique minimizer of $\sum_i \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p(a, \theta)$ over all $p \in BCE(u^*)$. Moreover, simple algebra shows that for all players i ,

$$\max_{b_i} \sum_{a, \theta} u_i^*(b_i, a_{-i}, \theta) p^*(a, \theta) = \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p^*(a, \theta).$$

In turn, this implies that

$$\underline{w}(p^*, u^*) = \sum_i \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p^*(a, \theta).$$

Therefore, every $p \in BCE(u^*) \setminus \{p^*\}$ has

$$\begin{aligned}\underline{w}(p, u^*) &= \sum_i \max_{b_i} \sum_{a, \theta} u_i^*(b_i, a_{-i}, \theta) p(a, \theta) \\ &\geq \sum_i \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p(a, \theta) \\ &> \sum_i \sum_{a, \theta} u_i^*(1, a_{-i}, \theta) p^*(a, \theta) = \underline{w}(p^*, u^*).\end{aligned}$$

we conclude that $\underline{w}(u^*) = \underline{w}(p^*, u^*)$ if and only if $p = p^*$, that is, (iii) of Lemma 14 holds. The proof is now complete.

E. Proofs of the results in Section 5

E.1. Proof of Lemma 1

We prove a slightly stronger result:

Claim 6. For every BCE p , there is a symmetric BCE q such that

$$\bar{w}(q) = \bar{w}(p) \quad \text{and} \quad \underline{w}(q) \leq \underline{w}(p).$$

Proof. Fix a BCE p . Let Φ be the set of permutations of I . For every permutation $\phi \in \Phi$, we define the outcome p_ϕ by

$$p_\phi(a, \theta) = p(a_\phi, \theta).$$

Note that player i in p_ϕ behaves as player $j = \phi^{-1}(i)$ in p . One can verify that p_ϕ because p is a BCE and the game is symmetric.

We define the outcome q by

$$q = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} p_\phi,$$

where $|\Phi|$ is the cardinality of Φ . As noted above, each p_ϕ is a BCE. Since the BCE set is convex, q is a BCE.

The outcome q is symmetric. Indeed, for every permutation $\psi \in \Phi$,

$$\Phi = \left\{ \psi^{-1} \circ \phi : \phi \in \Phi \right\}.$$

Thus, we deduce that

$$\begin{aligned} q(a_\psi, \theta) &= \frac{1}{|\Phi|} \sum_{\phi \in \Phi} p_\phi(a_\psi, \theta) = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} p_{(\psi^{-1} \circ \phi)}(a_\psi, \theta) \\ &= \frac{1}{|\Phi|} \sum_{\phi \in \Phi} p(a_\phi, \theta) = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} p_\phi(a, \theta) = q(a, \theta). \end{aligned}$$

We conclude q is symmetric.

To conclude the proof, we observe that

$$\bar{w}(q) = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} \bar{w}(p_\phi) = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} \bar{w}(p) = \bar{w}(p),$$

where the first equality holds because $\bar{w}(p_\phi)$ is affine in p_ϕ , and the second equality because the game is symmetric. Finally, we note that

$$\underline{w}(q) \leq \frac{1}{|\Phi|} \sum_{\phi \in \Phi} \underline{w}(p_\phi) = \frac{1}{|\Phi|} \sum_{\phi \in \Phi} \underline{w}(p) = \underline{w}(p),$$

where the first inequality holds because $\underline{w}(p_\phi)$ is convex in p_ϕ , and the second equality because the game is symmetric. \square

E.2. Proof of Proposition 1

By Lemma 1, \bar{w} is the value of the optimization problem

$$\min_{p \in BCE^{sy}} \bar{w}(p), \tag{38}$$

and \underline{w} is the value of the following optimization

$$\min_{p \in BCE^{sy}} \underline{w}(p). \tag{39}$$

We consider also the following optimization problem:

$$\min_{p \in \Delta_{\pi^{sy}}(A \times \Theta)} \underline{w}(p), \tag{40}$$

We proceed by successive claims.

Claim 7. The following conditions are equivalent:

- (i) $\underline{w} < \bar{w}$
- (ii) $\underline{v}_i(p) < \bar{v}_i(p)$ for all players i and optimal solutions p of (39).

Proof. We first prove that (i) implies (ii). Suppose $\underline{w} < \bar{w}$ and let p be an optimal solution of (39). Then, $\underline{w}(p) = \underline{w} < \bar{w} \leq \bar{w}(p)$. The inequality $\underline{w}(p) < \bar{w}(p)$ implies that $\underline{v}_i(p) < \bar{v}_i(p)$ for some player i . Since the game is symmetric and p is symmetric, $\underline{v}_i(p) < \bar{v}_i(p)$ for all players i .

We now prove that (ii) implies (i). Suppose $\underline{v}_i(p) < \bar{v}_i(p)$ for all players i and optimal solutions p of (39). Let $p \in BCE$ be an optimal solution of (38). If p is also an optimal solution of (39), then $\underline{w}(p) = \sum_i \underline{v}_i(p) < \sum_i \bar{v}_i(p) = \bar{w}(p)$ by hypothesis; thus, $\underline{w} < \bar{w}$. If instead p is not an optimal solution of (39), then $\underline{w} < \underline{w}(p) \leq \bar{w}(p) = \bar{w}$; thus, $\underline{w} < \bar{w}$. \square

Claim 8. For every $p \in BCE$ and $i \in I$, the following conditions are equivalent:

- (i) $\underline{v}_i(p) < \bar{v}_i(p)$.
- (ii) $a_i \in \text{supp}_i(p)$ and $BR(p_{a_i}) = \{a_i\}$ for all $a_i \in A_i$.

Proof. Condition (i) holds if and only if player i is strictly better by following the action recommendation of the mediator rather than best responding ex ante. In other terms, player i has no action a_i such that for all $b_i \in \text{supp}_i(p)$, $a_i \in BR(p_{b_i})$. Given that A_i has two elements, this is equivalent to condition (ii). \square

Claim 9. The following conditions are equivalent:

- (i) All optimal solutions of (39) satisfy (5).
- (ii) All optimal solutions of (40) satisfy (5).

In either case,

$$\underset{p \in BCE^{sy}}{\text{argmin}} \underline{w}(p) = \underset{p \in \Delta_{\pi}^{sy}(A \times \Theta)}{\text{argmin}} \underline{w}(p).$$

Proof. First we show that (i) implies (ii). Let p be an optimal solution of (39) and let q be an optimal solution of (40). For every $t \in [0, 1]$, define $p^t = (1 - t)p + tq$. Furthermore, set

$$s = \max\{t : p^t \in BCE^{sy}\}.$$

Note that s is well defined: the set BCE^{sy} is closed and $p^0 = p \in BCE^{sy}$.

We observe that p^s is an optimal solution of (39): since $\underline{w}(p^t)$ is convex in t ,

$$\underline{w}(p^s) \leq (1 - s)\underline{w}(p) + s\underline{w}(q) \leq \underline{w}(p) = \underline{w}.$$

Thus, p^s must satisfy (5). But this implies that $s = 1$; otherwise, one could find $\epsilon > 0$ sufficiently small so that $p^{s+\epsilon} \in BCE^{sy}$, contradicting the definition of p^s . This implies that $q = p^s$ satisfies (5).

Now we show that (ii) implies (i). Let p be an optimal solution of (39) and let q be an optimal solution of (40). Since q satisfies (5), q is a BCE. Thus, q is an optimal solution of (39). This implies that p is an optimal solution of (40), and therefore satisfies (5). \square

By combining the three claims above, we obtain Proposition 1.

E.3. Proof of Claim 1

We begin with a result that establishes a necessary condition for an outcome to solve the relaxed program from Proposition 1. To state the result, let $U_i(a_i, p)$ be player i 's payoff if she always takes action a_i while (a_{-i}, θ) is distributed according to p :

$$U_i(a_i, p) := \sum_{b_i, a_{-i}, \theta} u_i(a_i, a_{-i}, \theta) p(b_i, a_{-i}, \theta).$$

Note that for all $p \in \Delta_\pi^{sy}(A \times \Theta)$ and $i \in I$,

$$\underline{w}(p) = n \max\{U_i(0, p), U_i(1, p)\}.$$

Thus,

$$\operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p) = \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \max\{U_i(0, p), U_i(1, p)\}.$$

Claim 10. Every $p^* \in \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p)$ has $U_i(0, p^*) = U_i(1, p^*)$ for all players i .

Proof. We prove the contrapositive: if $p^* \in \Delta_\pi^{sy}(A \times \Theta)$ has $U_i(0, p^*) \neq U_i(1, p^*)$, then

$$p^* \notin \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p).$$

We first consider the case in which $U_i(0, p^*) > U_i(1, p^*)$. Let q be the outcome where all investors attack with probability one. Observe that $q \in \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} U_i(0, p)$, and that every $r \in \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} U_i(0, p)$ has the speculative attack succeeding with probability one. Hence, every such r has $U_i(0, r) < U_i(1, r)$, which implies that $p^* \notin \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} U_i(0, p)$. We deduce that $U_i(0, q) < U_i(0, p^*)$.

For every $\epsilon \in (0, 1)$, we define $q^\epsilon := \epsilon q + (1 - \epsilon)p^* \in \Delta_\pi^{sy}(A \times \Theta)$. Using the inequality $U_i(0, q) < U_i(0, p^*)$, we obtain that for all $\epsilon > 0$ small enough,

$$\underline{w}(q^\epsilon) = nU_i(0, p^\epsilon) = n(\epsilon U_i(0, q) + (1 - \epsilon)U_i(0, p^*)) < nU_i(0, p^*) = \underline{w}(p^*).$$

We conclude that $p^* \notin \operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p)$.

The argument for the case $U_i(0, p^*) < U_i(1, p^*)$ is similar, but with q being replaced by the outcome where no one ever speculates. \square

Thanks to Claim 10, to determine $\operatorname{argmin}_{p \in \Delta_\pi^{sy}(A \times \Theta)} \underline{w}(p)$, we can study the following “simpler” optimization problem:

$$\min_{p \in \Delta_\pi^{sy}(A \times \Theta)} U_i(0, p) \quad \text{s.t.} \quad U_i(0, p) = U_i(1, p). \quad (41)$$

Claim 11. An outcome $p \in \Delta_\pi^{sy}(A \times \Theta)$ is an optimal solution of (41) if and only if

$$p \left(\sum_{j \neq i} a_j = \theta - 1 \right) = 0 \quad \text{and} \quad p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k}{1+x}.$$

Proof. Writing the constraint $U_i(1, p) = U_i(0, p)$ in terms of p gives

$$p \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) - k = -xp \left(\sum_{j \neq i} a_j \geq \theta \right),$$

which is equivalent to

$$p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k - p \left(\sum_{j \neq i} a_j = \theta - 1 \right)}{1+x}.$$

It follows that every p that satisfies $U_i(1, p) = U_i(0, p)$ must yield

$$U_i(0, p) = -xp \left(\sum_{j \neq i} a_j \geq \theta \right) = -x \left[\frac{k - p \left(\sum_{j \neq i} a_j = \theta - 1 \right)}{1+x} \right].$$

Hence, we get that (41) is the same as

$$\begin{aligned} & \min_{p \in \Delta_\pi^{sy}(A \times \Theta)} p \left(\sum_{j \neq i} a_j = \theta - 1 \right) \\ & \text{s.t.} \quad p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k - p \left(\sum_{j \neq i} a_j = \theta - 1 \right)}{1+x}. \end{aligned}$$

Hence, to complete the proof, we only need to be sure that there is $p \in \Delta_\pi^{sy}(A \times \Theta)$ such that

$$p \left(\sum_{j \neq i} a_j = \theta - 1 \right) = 0 \quad \text{and} \quad p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k}{1+x}.$$

Such an outcome is easy to construct: with probability $k/(1+x)$, all players attack; with the remaining probability, no player attacks. \square

The following result connects what we have just found with the conditions in the statement of Claim 1.

Claim 12. For an outcome $p \in \Delta_\pi(A \times \Theta)$, the following conditions are equivalent:

(i) For all players i and payoff states θ ,

$$p \left(\sum_{j \neq i} a_j = \theta - 1 \right) = 0, \quad (42)$$

$$p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k}{1+x}. \quad (43)$$

(ii) For all payoff states θ ,

$$p \left(\theta - 1 \leq \sum_i a_i \leq \theta \right) = 0, \quad (44)$$

$$p \left(\sum_i a_i > \theta \right) = \frac{k}{1+x}. \quad (45)$$

Proof. First we show that (i) implies (ii). Since $\max \Theta < n$,

$$p \left(\sum_i a_i = \theta - 1 \right) = p \left(\sum_i a_i = \theta - 1, \text{ and } a_i = 0 \text{ for some } i \right).$$

Thus,

$$p \left(\sum_i a_i = \theta - 1 \right) \leq \sum_i p \left(\sum_{j \neq i} a_j = \theta - 1, \text{ and } a_i = 0 \right) = 0,$$

where the last equality follows from (42). Moreover, since $\min \Theta > 0$,

$$p \left(\sum_i a_i = \theta \right) = p \left(\sum_i a_i = \theta, \text{ and } a_i = 1 \text{ for some } i \right).$$

Thus,

$$p \left(\sum_i a_i = \theta \right) \leq \sum_i p \left(\sum_{j \neq i} a_j = \theta - 1, \text{ and } a_i = 1 \right) = 0,$$

where the last equality follows from (42). We conclude that (44) holds.

To prove (45), notice that

$$p \left(\sum_i a_i > \theta \right) = p \left(\sum_i a_i \geq \theta \right),$$

because we have just verified that $p(\sum_i a_i = \theta) = 0$. Then, fixing some player i^* ,

$$\begin{aligned} p\left(\sum_i a_i \geq \theta\right) &= p\left(\sum_{i \neq i^*} a_i \geq \theta, \text{ and } a_{i^*} = 0\right) + p\left(\sum_{i \neq i^*} a_i \geq \theta - 1, \text{ and } a_{i^*} = 1\right) \\ &= p\left(\sum_{i \neq i^*} a_i \geq \theta, \text{ and } a_{i^*} = 0\right) + p\left(\sum_{i \neq i^*} a_i \geq \theta, \text{ and } a_{i^*} = 1\right) \\ &= p\left(\sum_{i \neq i^*} a_i \geq \theta\right) = \frac{k}{1+x}, \end{aligned}$$

where the second equality holds by (42), and the last equality by (43). We deduce (45). This completes the proof that (i) implies (ii).

Now we show that (ii) implies (i). Observe that

$$p\left(\sum_{j \neq i} a_j = \theta - 1\right) = p\left(\sum_j a_j = \theta - 1, \text{ and } a_i = 0\right) + p\left(\sum_j a_j = \theta, \text{ and } a_i = 1\right).$$

By (44), the right-hand side is equal to zero: we deduce (42). We obtain (43) from the following chain of equalities:

$$\begin{aligned} p\left(\sum_{j \neq i} a_j \geq \theta\right) &= p\left(\sum_j a_j \geq \theta, \text{ and } a_i = 0\right) + p\left(\sum_j a_j > \theta, \text{ and } a_i = 1\right) \\ &= p\left(\sum_j a_j > \theta, \text{ and } a_i = 0\right) + p\left(\sum_j a_j > \theta, \text{ and } a_i = 1\right) \\ &= p\left(\sum_j a_j > \theta\right) = \frac{k}{1+x}, \end{aligned}$$

where the second equality follows from (44), and the last equality from (45). This completes the proof that (ii) implies (i). \square

Combining the three results above, we obtain Claim 1.

E.4. Proof of Claim 2

First, we obtain necessary and sufficient conditions for $\underline{w} < \bar{w}$ in the regime change game for an arbitrary number of states.

Claim 13. The inequality $\underline{w} < \bar{w}$ holds if and only if all symmetric outcomes p that satisfy

(6) and (7), also satisfy

$$p_{a_i=1} \left(\sum_j a_j \geq \theta \right) > \frac{k}{1+x}, \quad (46)$$

where $p_{a_i=1}$ is the conditional probability of (a_{-i}, θ) given $a_i = 1$.

Proof. By Proposition 1, $\underline{w} < \bar{w}$ if and only if all optimal solutions of $\min_{p \in \Delta_{\pi}^{sy}(A \times \Theta)} \underline{w}(p)$ satisfy (5). By Claim 1, the latter condition is equivalent to the following statement: all symmetric outcomes p that satisfy (6) and (7), also satisfy (5). Next we verify that, for all symmetric outcomes p that satisfy (6) and (7), the conditions (5) and (46) are equivalent.

Let p be a symmetric outcome that satisfy (6) and (7). First, note (7) implies the attack succeeds with a probability strictly between 0 and 1, and so players must both attack and not attack with positive probability due to symmetry. Hence $\text{supp}_i(p) = \{0, 1\} = A_i$.

Given $\text{supp}_i(p) = \{0, 1\}$, i 's obedience constraints are strict when

$$p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) - k > -x p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta \right), \quad (47)$$

$$p_{a_i=0} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) - k < -x p_{a_i=0} \left(\sum_{j \neq i} a_j \geq \theta \right). \quad (48)$$

By (6)—see also Claim 12—

$$p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) = p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta \right) \quad \text{and} \quad p_{a_i=0} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) = p_{a_i=0} \left(\sum_{j \neq i} a_j \geq \theta \right).$$

Thus, (47) and (48) hold if and only if

$$p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta \right) > \frac{k}{1+x} > p_{a_i=0} \left(\sum_{j \neq i} a_j \geq \theta \right).$$

By (6) and (7)—see also Claim 12—

$$p \left(\sum_{j \neq i} a_j \geq \theta \right) = \frac{k}{1+x}.$$

Thus, by the law of total probability, (47) and (48) hold if and only if

$$p_{a_i=1} \left(\sum_j a_j \geq \theta \right) = p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) = p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta \right) > \frac{k}{1+x}.$$

Overall, we conclude that, for all symmetric outcomes p that satisfy (6) and (7), the conditions (5) and (46) are equivalent. \square

Next we refine the characterization $\underline{w} < \bar{w}$ obtained in Claim 13. To state this refinement, denote the CDF of θ by $F(\theta) := \sum_{\theta' \leq \theta} \pi(\theta')$. Define also the cutoff state θ^* by

$$\theta^* := \min \left\{ \theta \in \Theta : F(\theta) \geq \frac{k}{1+x} \right\}$$

Claim 14. The inequality $\bar{w} > \underline{w}$ holds if and only if

$$F(\theta^*) (\theta^* - \mathbb{E}[\theta | \theta \leq \theta^*]) < \frac{k}{1+x} \left(3 - \frac{3k}{1+x} + \theta^* - \mathbb{E}[\theta] \right). \quad (49)$$

Proof. By Claim 13, $\underline{w} < \bar{w}$ is equivalent to

$$\begin{aligned} \frac{k}{1+x} &< \min_{p \in \Delta_{\pi}^{sy}(A \times \Theta)} p_{a_i=1} \left(\sum_{j \neq i} a_j \geq \theta - 1 \right) \\ \text{s.t. } & \text{(6) and (7)}. \end{aligned} \quad (50)$$

Hence, showing (49) and (50) are equivalent is sufficient. To show this equivalence, we first characterize the unique solution to the program on the right hand side of (50). This solution gives the value of the program, which we then compare to $k/(1+x)$.

We begin with an alternative way of representing symmetric outcomes. This representation is based on the observation that an outcome $p \in \Delta_{\pi}(A \times \Theta)$ is symmetric if and only if, conditional on the state, all action profiles with the same number of attackers have the same probability. Consequently, $p \in \Delta_{\pi}^{sy}(A \times \Theta)$ if and only if there is $Q : \Theta \rightarrow \Delta(\{0, \dots, n\})$ such that

$$p(a, \theta) = \binom{n}{\sum_j a_j} Q \left(\sum_j a_j \middle| \theta \right) \pi(\theta),$$

where $\binom{n}{\sum_j a_j}$ is the binomial coefficient. Thus, one can write

$$p(a_i = 1) = \sum_{\theta} \pi(\theta) \sum_{m=1}^n \frac{m}{n} Q(m|\theta).$$

Moreover, condition (6) is equivalent to $Q(\theta - 1|\theta) = Q(\theta|\theta) = 0$. Therefore,

$$p \left(\sum_{j \neq i} a_j \geq \theta - 1 \text{ and } a_i = 1 \right) = \sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} \frac{m}{n} Q(m|\theta),$$

and

$$p\left(\sum_j a_j \geq \theta\right) = \sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} Q(m|\theta).$$

Hence, letting

$$f(Q) = \frac{\sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} m Q(m|\theta)}{\sum_{\theta} \pi(\theta) \sum_{m=1}^n m Q(m|\theta)},$$

we can write the program on the right hand side of (49) as

$$\begin{aligned} \min_{Q: \Theta \rightarrow \Delta(\{0, \dots, n\})} & f(Q) \\ \text{s.t.} \quad & \sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} Q(m|\theta) = \frac{k}{1+x}, \\ & Q(\theta-1|\theta) = Q(\theta|\theta) = 0 \text{ for all } \theta. \end{aligned} \tag{51}$$

Since the constraint set is compact and the objective continuous, the above program admits a solution, Q^* . We now use perturbation-based arguments to show Q^* must satisfy a few properties:

1. $Q^*(m|\theta) = 0$ whenever $m \notin \{\theta-2, \theta+1\}$: if $Q^*(m|\theta) > 0$ for $m > \theta+1$ (resp., $m < \theta-2$), one can reduce the objective without violating the constraints by moving $\epsilon > 0$ mass from $Q^*(m|\theta)$ to $Q^*(\theta+1|\theta)$ (resp., $Q^*(\theta-2|\theta)$).
2. If $Q^*(\theta+1|\theta) > 0$, then $Q^*(\theta'+1|\theta') = 1$ for all $\theta' < \theta$: For a contradiction, suppose $Q^*(\theta+1|\theta) > 0$, but $Q^*(\theta'+1|\theta') < 1$ for some $\theta' < \theta$. For every $\epsilon > 0$, define the following perturbation Q^ϵ of Q :

$$Q^\epsilon(m|\hat{\theta}) = \begin{cases} Q^*(\theta+1|\theta) - \epsilon & \text{if } m = \theta+1, \hat{\theta} = \theta, \\ Q^*(\theta-2|\theta) + \epsilon & \text{if } m = \theta-2, \hat{\theta} = \theta, \\ Q^*(\theta'+1|\theta') + \epsilon \frac{\pi(\theta)}{\pi(\theta')} & \text{if } m = \theta'+1, \hat{\theta} = \theta', \\ Q^*(\theta'-2|\theta') - \epsilon \frac{\pi(\theta)}{\pi(\theta')} & \text{if } m = \theta'-2, \hat{\theta} = \theta', \\ Q^*(m|\hat{\theta}) & \text{otherwise.} \end{cases}$$

The contradiction assumption means Q^ϵ is feasible for all sufficiently small $\epsilon > 0$. Direct computation shows

$$\lim_{\epsilon \searrow 0} \frac{1}{\epsilon} (f(Q^\epsilon) - f(Q^*)) = \frac{\pi(\theta)(\theta' - \theta)}{\sum_{\hat{\theta}} \pi(\hat{\theta}) \sum_{m=1}^n m Q^*(m|\hat{\theta})} < 0,$$

contradicting the optimality of Q .

3. If $Q^*(\theta - 2|\theta) > 0$, then $Q^*(\theta' - 2|\theta') = 1$ for all $\theta' > \theta$: For a contradiction, suppose $Q^*(\theta - 2|\theta) > 0$, but $Q^*(\theta' - 2|\theta') < 1$ for some $\theta' > \theta$. For every $\epsilon > 0$, define the following perturbation Q^ϵ of Q :

$$Q^\epsilon(m|\hat{\theta}) = \begin{cases} Q^*(\theta - 2|\theta) - \epsilon & \text{if } m = \theta - 2, \hat{\theta} = \theta, \\ Q^*(\theta + 1|\theta) + \epsilon & \text{if } m = \theta + 1, \hat{\theta} = \theta, \\ Q^*(\theta' - 2|\theta') + \epsilon \frac{\pi(\theta)}{\pi(\theta')} & \text{if } m = \theta' - 2, \hat{\theta} = \theta', \\ Q^*(\theta' + 1|\theta') - \epsilon \frac{\pi(\theta)}{\pi(\theta')} & \text{if } m = \theta' + 1, \hat{\theta} = \theta', \\ Q^*(m|\hat{\theta}) & \text{otherwise.} \end{cases}$$

The contradiction assumption means Q^ϵ is feasible for all sufficiently small $\epsilon > 0$. Direct computation shows

$$\lim_{\epsilon \searrow 0} \frac{1}{\epsilon} (f(Q^\epsilon) - f(Q^*)) = \frac{\pi(\theta)(\theta - \theta')}{\sum_{\hat{\theta}} \pi(\hat{\theta}) \sum_{m=1}^n m Q(m|\hat{\theta})} < 0,$$

contradicting the optimality of Q^* .

The above conditions imply the optimal Q^* admits a cutoff $\tilde{\theta}$ such that $Q^*(\theta + 1|\theta) = 1$ for all $\theta < \tilde{\theta}$, $Q^*(\theta - 2|\theta) = 1$ for all $\theta > \tilde{\theta}$, and $Q^*(\{\tilde{\theta} + 1, \tilde{\theta} - 2\}|\tilde{\theta}) = 1$. Then, the constraint

$$\sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} Q(m|\theta) = \frac{k}{1+x}$$

pins down the optimum: we must have $\tilde{\theta} = \theta^*$, and

$$Q^*(\theta^* + 1|\theta^*) = \frac{1}{\pi(\theta^*)} \left(\frac{k}{1+x} - F(\theta^* - 1) \right).$$

Therefore, the inequality (50) becomes

$$\begin{aligned} \frac{k}{1+x} < f(Q^*) &= \frac{\sum_{\theta} \pi(\theta) \sum_{m \geq \theta+1} m Q^*(m|\theta)}{\sum_{\theta} \pi(\theta) \sum_{m=1}^n m Q^*(m|\theta)} \\ &= \frac{F(\theta^*) \mathbb{E}[\theta + 1|\theta \leq \theta^*] - \left(F(\theta^*) - \frac{k}{1+x} \right) (\theta^* + 1)}{\mathbb{E}[\theta] + \frac{k}{1+x} - 2 \left(1 - \frac{k}{1+x} \right)}. \end{aligned}$$

Rearranging the above equation gives (49). \square

Finally, we prove Claim 2 by specializing Claim 14 to two states.

Proof of Claim 2. We begin the proof by explicitly stating the implication of (49) for the

binary case. In particular, we show $\underline{w} < \bar{w}$ if and only if one of the following two conditions hold:

- (i) $\frac{k}{1+x} > \pi(\underline{\theta})$ and $\frac{k}{1+x} > \frac{1}{3}\pi(\underline{\theta})(\bar{\theta} - \underline{\theta})$.
- (ii) $\frac{k}{1+x} \leq \pi(\underline{\theta})$ and $\frac{k}{1+x} < 1 - \frac{1}{3}\pi(\bar{\theta})(\bar{\theta} - \underline{\theta})$.

To prove the above, we consider two cases, depending on the value of θ^* :

- *Case 1:* $k/(1+x) > \pi(\underline{\theta})$. Then $\theta^* = \bar{\theta}$, and the inequality (49) specializes to

$$\bar{\theta} - \mathbb{E}[\theta] < \frac{k}{1+x} \left(3 - \frac{3k}{1+x} + \bar{\theta} - \mathbb{E}[\theta] \right).$$

Substituting $\bar{\theta} - \mathbb{E}[\theta] = \pi(\underline{\theta})(\bar{\theta} - \underline{\theta})$ and rearranging gives

$$\left(1 - \frac{k}{1+x} \right) \pi(\underline{\theta})(\bar{\theta} - \underline{\theta}) < 3 \frac{k}{1+x} \left(1 - \frac{k}{1+x} \right),$$

which is equivalent to

$$\frac{1}{3}\pi(\underline{\theta})(\bar{\theta} - \underline{\theta}) < \frac{k}{1+x}.$$

Thus, we have established (i) is sufficient for $\underline{w} < \bar{w}$, and necessary if $\pi(\underline{\theta}) \geq \frac{k}{1+x}$.

- *Case 2:* Suppose now $k/(1+x) \leq \pi(\underline{\theta})$. Then $\theta^* = \underline{\theta}$. Thus, the inequality (49) is now

$$0 < \frac{k}{1+x} \left(3 - \frac{3k}{1+x} + \underline{\theta} - \mathbb{E}[\theta] \right).$$

Note $\underline{\theta} - \mathbb{E}[\theta] = -\pi(\bar{\theta})(\bar{\theta} - \underline{\theta})$. Therefore, the above inequality is equivalent to

$$\frac{k}{1+x} < 1 - \frac{1}{3}\pi(\bar{\theta})(\bar{\theta} - \underline{\theta}).$$

Hence, (ii) is sufficient for $\underline{w} < \bar{w}$, and necessary if $\pi(\underline{\theta}) \leq \frac{k}{1+x}$.

Next, we argue that a violation of one of the claim's conditions implies that either (i) or (ii) above hold. Suppose first $\bar{\theta} - \underline{\theta} < 3$. In this case, $\frac{1}{3}\pi(\underline{\theta})(\bar{\theta} - \underline{\theta}) < \pi(\underline{\theta})$, and so (i) holds whenever $\frac{k}{1+x} > \pi(\underline{\theta})$. If $\frac{k}{1+x} \leq \pi(\underline{\theta})$, then (ii) holds, because

$$1 - \frac{1}{3}\pi(\bar{\theta})(\bar{\theta} - \underline{\theta}) > 1 - \pi(\bar{\theta}) = \pi(\underline{\theta}) \geq \frac{k}{1+x}.$$

Suppose now $\bar{\theta} - \underline{\theta} \geq 3$, but (9) fails. Then one of the following inequality chains must hold: either

$$\frac{k}{1+x} > \frac{1}{3}(\bar{\theta} - \underline{\theta})(1 - \pi(\bar{\theta})) = \frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\underline{\theta}) \geq \pi(\underline{\theta}),$$

or

$$\frac{k}{1+x} < 1 - \frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\bar{\theta}) \leq 1 - \pi(\bar{\theta}) = \pi(\underline{\theta}).$$

Either way, $\underline{w} < \bar{w}$ holds: the first inequality chain implies (i), whereas the second inequality chain implies (ii).

To conclude the proof, we show that the claim's condition must hold if neither (i) nor (ii) hold. Suppose first that $\frac{k}{1+x} > \pi(\underline{\theta})$, but (i) fails. Then

$$\frac{1}{3}(\bar{\theta} - \underline{\theta})(1 - \pi(\bar{\theta})) = \frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\underline{\theta}) \geq \frac{k}{1+x} > \pi(\underline{\theta}),$$

meaning $\bar{\theta} - \underline{\theta} \geq 3$, and the right inequality in (9) holds. For the left inequality, note that

$$1 - \frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\bar{\theta}) \leq 1 - \pi(\bar{\theta}) = \pi(\underline{\theta}) < \frac{k}{1+x}.$$

Suppose now $\frac{k}{1+x} \leq \pi(\underline{\theta})$, but (ii) fails. Then,

$$1 - \pi(\bar{\theta}) \geq \frac{k}{1+x} \geq 1 - \frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\bar{\theta}).$$

The right inequality above delivers the left inequality in (9). Moreover, the implied inequality between the left most expression and the right most expression implies

$$\frac{1}{3}(\bar{\theta} - \underline{\theta})\pi(\bar{\theta}) \geq \pi(\bar{\theta}),$$

and so $\bar{\theta} - \underline{\theta} \geq 3$. Finally, to get the right inequality in (9), notice that

$$\frac{k}{1+x} \leq 1 - \pi(\bar{\theta}) \leq \frac{1}{3}(\bar{\theta} - \underline{\theta})(1 - \pi(\bar{\theta})),$$

where the last inequality holds because $\bar{\theta} - \underline{\theta} \geq 3$. □

References

- C. D. Aliprantis and K. C. Border. *Infinite Dimensional Analysis*. Springer, third edition, 2006.
- R. J. Aumann. Subjectivity and correlation in randomized strategies. *Journal of Mathematical Economics*, 1:67–96, 1974.
- R. J. Aumann. Correlated equilibrium as an expression of Bayesian rationality. *Econometrica*, 55:1–18, 1987.

- D. Bergemann and S. Morris. Robust predictions in games with incomplete information. *Econometrica*, 81:1251–1308, 2013.
- D. Bergemann and S. Morris. Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics*, 11:487–522, 2016.
- D. Bergemann, B. Brooks, and S. Morris. The limits of price discrimination. *American Economic Review*, 105:921–957, 2015.
- D. Bergemann, B. Brooks, and S. Morris. First-price auctions with general information structures: Implications for bidding and revenue. *Econometrica*, 85:107–143, 2017.
- D. Blackwell. Comparison of experiments. In J. Neyman, editor, *Proceedings of the 2nd Berkeley Symposium on Mathematical Statistics and Probability*, pages 93–102. University of California Press, 1951.
- D. Blackwell. Equivalent comparisons of experiments. *Annals of Mathematical Statistics*, 24:265–272, 1953.
- L. E. Blume and W. R. Zame. The algebraic geometry of perfect and sequential equilibrium. *Econometrica*, 62:783–794, 1994.
- B. Brooks and S. Du. Optimal auction design with common values: An informationally robust approach. *Econometrica*, 89:1313–1360, 2021.
- A. Caplin and M. Dean. Revealed preference, rational inattention, and costly information acquisition. *American Economic Review*, 105:2183–2203, 2015.
- A. Caplin, M. Dean, and J. Leahy. Rationally inattentive behavior: Characterizing and generalizing Shannon entropy. *Journal of Political Economy*, 130:1676–1715, 2022.
- G. Carroll. Information games and robust trading mechanisms. 2019a. URL <http://individual.utoronto.ca/carroll>.
- G. Carroll. Robust incentives for information acquisition. *Journal of Economic Theory*, 181:382–420, 2019b.
- G. Carroll. Robustness in mechanism design and contracting. *Annual Review of Economics*, 11:139–166, 2019c.
- G. de Clippel and K. Rozen. Communication, perception, and strategic obfuscation. 2022. URL <https://sites.google.com/brown.edu/kareenrozen>.

- T. Denti. Costly monitoring in signaling games. 2021. URL <https://sites.google.com/site/tommasojdenti>.
- T. Denti. Posterior separable cost of information. *American Economic Review*, 112:3215–59, 2022.
- T. Denti. Unrestricted information acquisition. *Theoretical Economics*, forthcoming.
- T. Denti, M. Marinacci, and A. Rustichini. Experimental cost of information. *American Economic Review*, 112:3106–23, 2022.
- L. Doval and J. C. Ely. Sequential information design. *Econometrica*, 88:2575–2608, 2020.
- S. Du. Robust mechanisms under common valuation. *Econometrica*, 86:1569–1588, 2018.
- P. Dworczak and A. Pavan. Preparing for the worst but hoping for the best: Robust (Bayesian) persuasion. *Econometrica*, 90:2017–2051, 2022.
- F. Forges. Five legitimate definitions of correlated equilibrium in games with incomplete information. *Theory and Decision*, 35:277–310, 1993.
- I. Goldstein and A. Pauzner. Demand–deposit contracts and the probability of bank runs. *Journal of Finance*, 60:1293–1327, 2005.
- M. Halac, E. Lipnowski, and D. Rappoport. Addressing strategic uncertainty with incentives and information. *AEA Papers and Proceedings*, 112:431–437, 2022.
- B. Hebert and J. La’O. Information acquisition, efficiency, and non-fundamental volatility. *Journal of Political Economy*, forthcoming.
- C. Hellwig and L. Veldkamp. Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies*, 76:223–251, 2009.
- T. Hoshino. Rational inattention in games. 2018. URL <https://sites.google.com/view/tetsuyahoshino>.
- N. Inostroza and A. Pavan. Adversarial coordination and public information design. 2022. URL <https://sites.google.com/view/nicolas-inostroza>.
- S. Kosterina. Persuasion with unknown beliefs. *Theoretical Economics*, 17:1075–1107, 2022.
- B. L. Lipman and S. Srivastava. Computation as a correlation device. *Games and Economic Behavior*, 2:154–172, 1990.

- E. Lipnowski and D. Ravid. Predicting choice from information costs. 2022. URL <https://arxiv.org/abs/2205.10434>.
- E. Lipnowski, D. Ravid, and D. Shishkin. Perfect Bayesian persuasion. 2023. URL <https://denisshishkin.com>.
- B. Maćkowiak and M. Wiederholt. Optimal sticky prices under rational inattention. *American Economic Review*, 99:769–803, 2009.
- B. Maćkowiak, F. Matějka, and M. Wiederholt. Rational inattention: A review. *Journal of Economic Literature*, 61:226–273, 2023.
- F. Matějka and A. McKay. Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, 105:272–298, 2015.
- S. Morris and M. Shadmehr. Inspiring regime change. *Journal of the European Economic Association*, forthcoming.
- S. Morris and H. S. Shin. Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review*, 88:587–597, 1998.
- S. Morris and M. Yang. Coordination and continuous stochastic choice. *The Review of Economic Studies*, 89:2687–2722, 2022.
- S. Morris, D. Oyama, and S. Takahashi. Implementation via information design in binary-action supermodular games. *Available at SSRN 3697335*, 2022.
- D. P. Myatt and C. Wallace. Endogenous information acquisition in coordination games. *Review of Economic Studies*, 79:340–374, 2012.
- R. B. Myerson. Optimal coordination mechanisms in generalized principal–agent problems. *Journal of Mathematical Economics*, 10:67–81, 1982.
- R. B. Myerson. Dual reduction and elementary games. *Games and Economic Behavior*, 21:183–202, 1997.
- R. F. Nau and K. F. McCardle. Coherent behavior in noncooperative games. *Journal of Economic Theory*, 50:424–444, 1990.
- M. Obstfeld. Models of currency crises with self-fulfilling features. *European Economic Review*, 40:1037–1047, 1996.
- R. R. Phelps. *Lectures on Choquet’s theorem*. Springer, 2001.

- D. Ravid. Ultimatum Bargaining with Rational Inattention. *American Economic Review*, 110:2948–63, 2020.
- D. Ravid, A.-K. Roesler, and B. Szentes. Learning before trading: on the inefficiency of ignoring free information. *Journal of Political Economy*, 130:346–387, 2022.
- R. T. Rockafellar. *Convex analysis*. Princeton University Press, 1970.
- T. Roughgarden. *Twenty lectures on algorithmic game theory*. Cambridge University Press, 2016.
- C. A. Sims. Implications of rational inattention. *Journal of Monetary Economics*, 50: 665–690, 2003.
- M. Yang. Coordination with flexible information acquisition. *Journal of Economic Theory*, 158:721–738, 2015.
- W. Zhong. Optimal dynamic information acquisition. *Econometrica*, 90:1537–1582, 2022.