Blackwell Correlated Equilibrium

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Information Acquisition Games

Base Game

A **base game** is a tuple, $\mathcal{G} = (I, \Theta, \pi, A, (u_i)_{i \in I})$, consisting of

- a finite set I of players,
- a finite payoff-relevant state Θ ("payoff state"),
- a prior for the payoff state, $\pi \in \Delta(\Theta)$,
- a finite set of action profiles, $A = \prod_{i \in I} A_i$,
- a utility function $u_i : A \times \Theta \to \mathbb{R}$ for each player i.

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Information Technologies

An **information technology**, $\mathcal{T} = (Z, \zeta, (X_i, \mathcal{E}_i, C_i)_{i \in I})$, consists of

- a finite payoff-irrelevant state Z ("correlation state"),
- a conditional distribution for $z \in Z$, $\zeta : \Theta \to \Delta(Z)$,
- a finite set of signal realizations X_i for every i,
- a set of feasible experiments for player i, $\mathcal{E}_i \subseteq (\Delta(X_i))^{Z \times \Theta}$,
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Information Acquisition Game

Together, $(\mathcal{G}, \mathcal{T})$ induce an **information acquisition game**:

1. Simultaneously, each i chooses an experiment, $\xi_i \in \mathcal{E}_i$.
2. Nature determines (z, θ), draws signals, $(x_i)_{i \in I}$.
3. Each player i observes their signal x_i and takes an action a_i.
4. Each player i gets a payoff

$$u_i(a, \theta) - C_i(\xi_i).$$

Strategies and Equilibrium

A strategy for player i in this game consists of

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$(\xi_i^*, \sigma_i^*)_{i \in I}$ is **an equilibrium** if for all i, (ξ_i^*, σ_i^*) maximizes

$$\mathbb{E}_{\left(\xi_i, \sigma_i, (\xi_j^*, \sigma_j^*)_{j \neq i}\right)}\Big[u_i(a_i, a_{-i}, \theta)\Big] - C_i(\xi_i)$$

over all feasible experiments $\xi_i \in \mathcal{E}_i$ and action plans σ_i.

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- A vector $v = (v_i)_{i\in I}$ consisting of each player's payoff,

$$v_i = \sum_{a,\theta} u_i(a,\theta)p(a,\theta) - C_i(\xi_i^*).$$

  We refer to v as the equilibrium **value**.

Monotone Information Technologies

Main focus: technologies where learning less is easier.

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An information technology $\mathcal{T}$ is **(Blackwell) monotone** if

(i) If $\xi_i \in \mathcal{E}_i$ and $\xi_i \succsim \xi_i'$, then $\xi_i' \in \mathcal{E}_i$.

(ii) If $\xi_i, \xi_i' \in \mathcal{E}_i$ and $\xi_i \succsim \xi_i'$, then $C(\xi_i) \geq C(\xi_i')$.

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Related Literature

- **Rational inattention:** Sims (2003), . . . , Yang (2015), Hoshino (2018), Ravid (2020), Angeletos and Sastry (2021), Ravid, Roesler, and Szentes (2022), Hebert and La'O (2022), Morris and Yang (2022), Denti (forthcoming). . .

- **Espionage games:** Solan and Yariv (2004), de Clippel and Rozen (2021), Denti (2021). . .

- **Robust predictions:** Bergemann and Morris (2005, 2013), Chassang (2013) Bergemann, Brooks, and Morris (2015, 2017), Carroll (2017). . .

- **Correlated equilibrium:** Aumann (1974, 1987), Myerson (1986, 1997), Forges (1986, 1993, 2006), Lipman and Srivastava (1990), Bergemann and Morris (2016), Doval and Ely (2020). . .

Exogenous and General Information Technologies

Some Notation

For an outcome p and $a_i \in A_i$, let

$$p(a_i) := \sum_{a_{-i},\theta} p(a_i, a_{-i}, \theta).$$

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For $a_i \in \operatorname{supp}_i p$, let p_{a_i} be p's conditional distribution given a_i,

$$p_{a_i}(a_{-i}, \theta) := p(a_i, a_{-i}, \theta)/p(a_i).$$

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$$p_{a_i}(a_{-i}, \theta) := p(a_i, a_{-i}, \theta)/p(a_i).$$

Finally, define i's best response set given a_i,

$$BR_i(p_{a_i}) := \operatorname{argmax}_{b_i \in A_i} \sum_{a_{-i}, \theta} u_i(b_i, a_{-i}, \theta) p_{a_i}(a_{-i}, \theta).$$

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The **gross value** of an outcome p is $\bar{v}_i(p) = \sum_{a,\theta} u_i(a,\theta)p(a,\theta)$.

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An outcome p is a **Bayes correlated equilibrium** (BCE) if:

(i) the marginal of p over Θ is π,

(ii) the obedience constraint holds: for all $i \in I$ and $a_i \in \operatorname{supp}_i p$,

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Bergmann and Morris (2016):

an exogenous $\mathcal{T}$ exists that induces (p, v) if and only if

$$p \text{ is a BCE}, \quad \text{and} \quad v_i = \bar{v}_i(p).$$

An Example

	L	R
T	$0, 0$	$3, 1$
B	$2, 2$	$2, 2$

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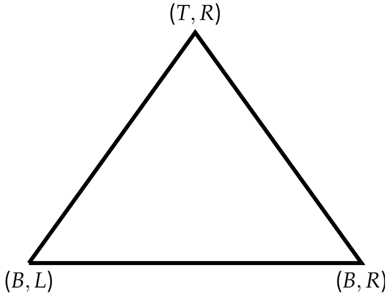

Note: R dominates L, and is strictly better if row player takes T with positive prob.

So, $p(T,L) = 0$ in every BCE, i.e.,

$$p \in \Delta(\{(B,L),(B,R),(T,R)\}).$$

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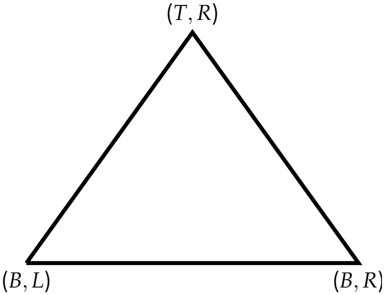

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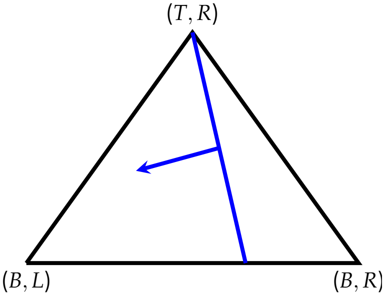

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Obedience for B yields:

$$p(B, L) \geq 0.5p(B, R).$$

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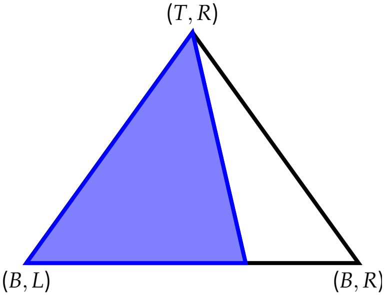
So p is a BCE iff

$p(T, L) = 0$ and $p(B, L) \geq 0.5p(B, R)$.



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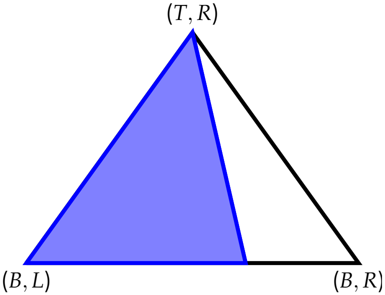
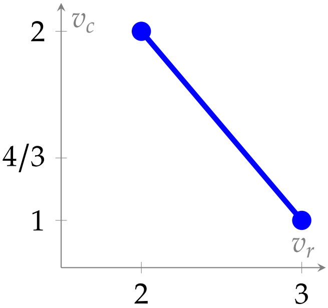
The set of feasible values is

$$v_r \in [2,3],\ v_c = 4 - v_r.$$





Define the **uninformed** value of p to be

$$\underline{v}_i(p) = \max_{b_i \in A_i} \sum_{a,\theta} u_i(b_i, a_{-i}, \theta) p(a, \theta).$$

General Information Technologies

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Proposition 1. A (p, v) is induced by some $\mathcal{T}$ if and only if

(i) p is a BCE,

(ii) for every i, $v_i \in [\underline{v}_i(p), \bar{v}_i(p)]$.

Example with General Info Tech

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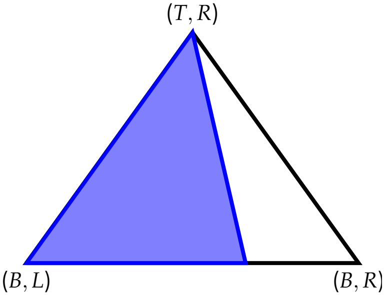
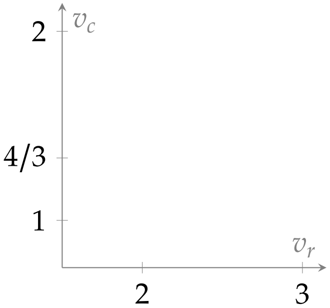
Allowing for general, non-exogenous $\mathcal{T}$ does not change the set of attainable outcomes.





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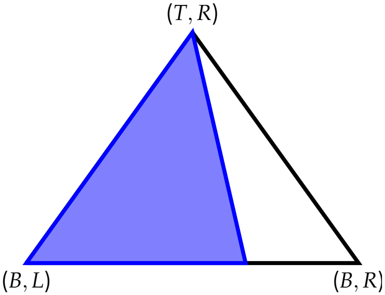

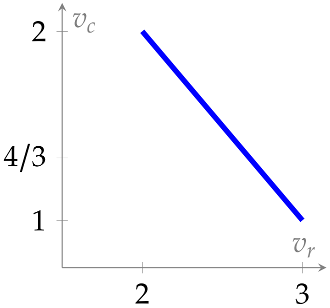

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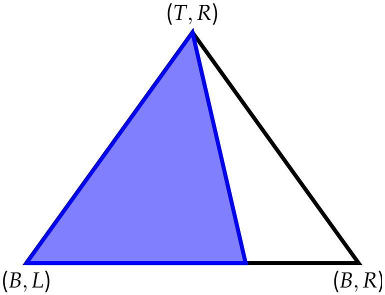

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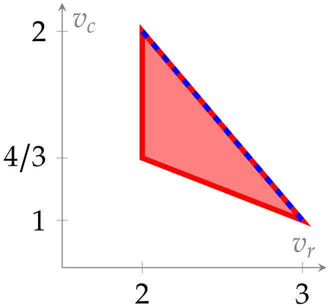
$$2 - \frac{1}{3} v_r \leq v_c \leq 4 - v_r, \; v_r \in [2, 3].$$



Monotone Information Technologies

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$$\text{for all } a_i, b_i \in \operatorname{supp}_i p, \text{if } p_{a_i} \neq p_{b_i}, \text{ then } BR_i(p_{a_i}) \cap BR_i(p_{b_i}) = \varnothing.$$

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Theorem 1. A monotone $\mathcal{T}$ exists that induces (p, v) if & only if

(i) p is a BKE,

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Sketch of "only if" proof:

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Theorem 1. A monotone $\mathcal{T}$ exists that induces (p, v) if & only if

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Sketch of "if" proof: extend Denti (2021).

Example with Monotone Information Tech

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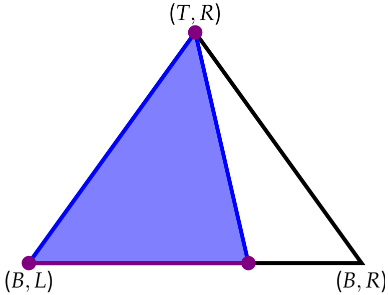
B	$2, 2$	$2, 2$

Claim. A BCE p is a BKE iff

$$p(T, R) \in \{0, 1\}.$$



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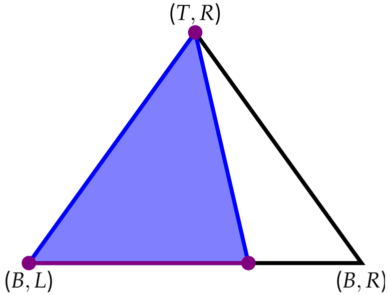

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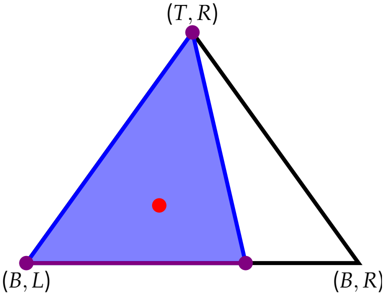

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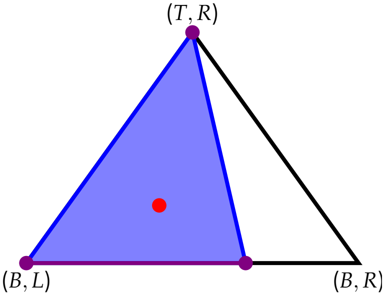

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But R is always a BR for column player.

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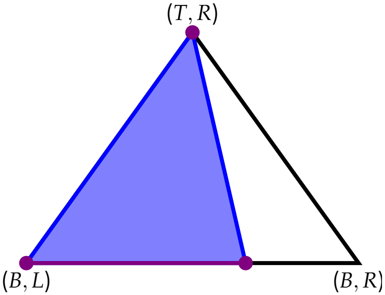

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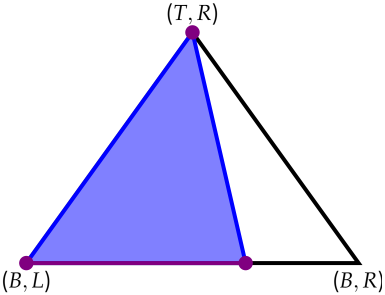

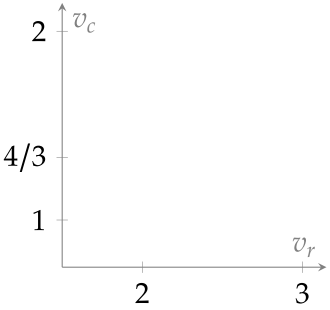

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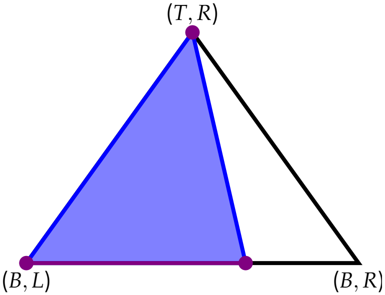

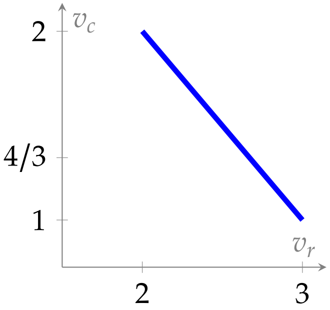

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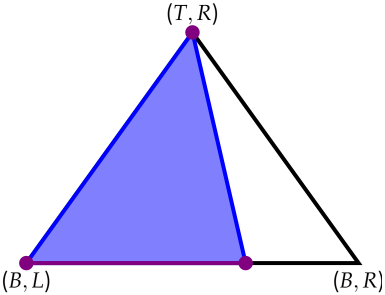

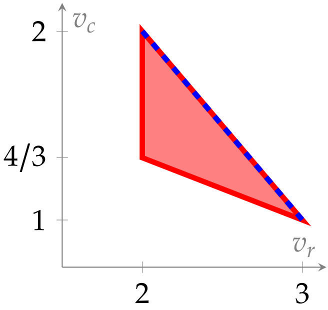

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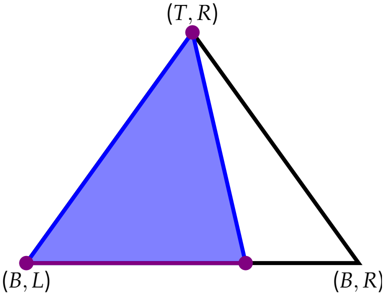

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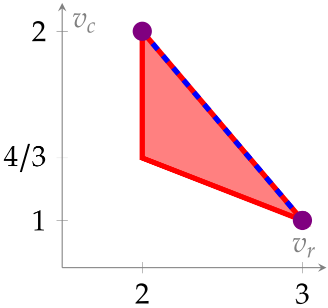

Bayes vs. Blackwell

Structural Differences between BCE and BKE

Structurally, the BCE set is a polytope, so closed & convex.

By contrast, the BKE set need not be closed nor convex:

- For non-convexity, see the example.
- For non-closedness, can consider a single agent problem.

Next: Ask when BCE equals the closure of BKE.

Jeopardization

Definition (Myerson 1997). An action a_i **jeopardizes** b_i if for every BCE p with $p(b_i) > 0$,

$$a_i \in BR_i(p_{b_i}).$$

Let $J(b_i)$ be the set of actions jeopardizing b_i.

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Remarks:

- Every action jeopardizes itself.
- Weak domination $\implies$ jeopardization.
- But the converse is false, e.g. matching pennies.

Minimal Mixing

A BCE p has **maximal support** if for every other BCE q,

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Lemma 1.

The minimally mixed BCE set is open & dense in the BCE set.

When does BKE refine BCE: Characterization

Proposition 2. The following are equivalent:

(i) The BKE set is dense in the BCE set.

(ii) A minimally mixed BKE exists.

(iii) For any BCE p, $i \in I$, $a_i, b_i \in \mathrm{supp}_i(p)$,

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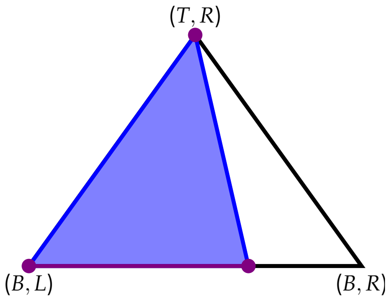

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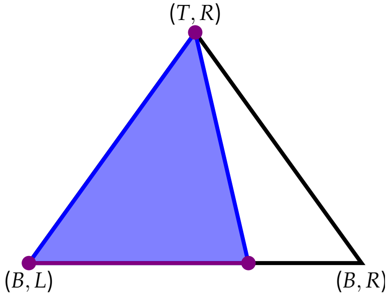

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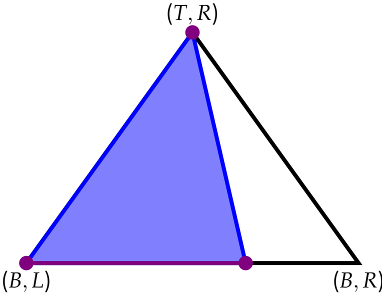

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(ii) requires a minimally mixing BKE to exist.

Since minimally mixing requires maximal support, and

$$\text{BKE} = \left\{ p \in \text{BCE} : p(T, R) \in \{0, 1\} \right\},$$

no minimally mixing BKE exists.

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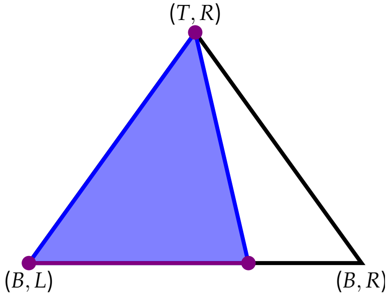

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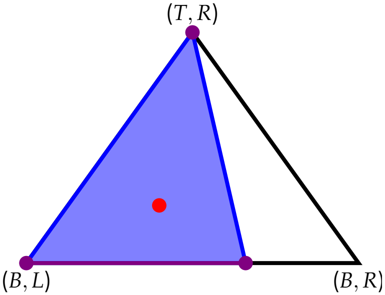

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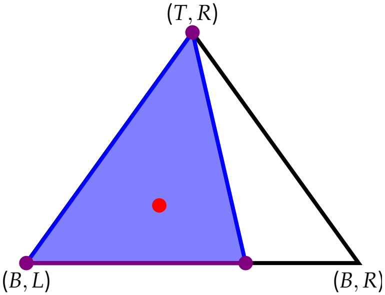

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Corollary 1.

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Thus, $\exists$a minimally mixed BKE $\implies$ BKE is dense in BCE.

GENERICALLY, BKE DOES NOT REFINE BCE

Theorem 2. Fix I, A, Θ, and full support π. Then the set

$$\left\{u := (u_i)_{i \in I} \in \mathbb{R}^{I \times A \times \Theta} : \mathrm{BCE}(u) \neq \mathrm{cl}\left(\mathrm{BKE}(u)\right)\right\}$$

is contained in a closed low-dimensional manifold of $\mathbb{R}^{I \times A \times \Theta}$.

Consequently, for generic games one has

$$\mathrm{BCE} = \mathrm{cl}(\mathrm{BKE}).$$

Perturbing Example Gives BKE = cl(BCE)

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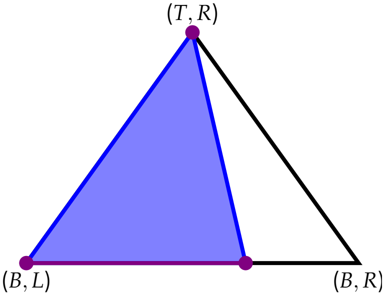

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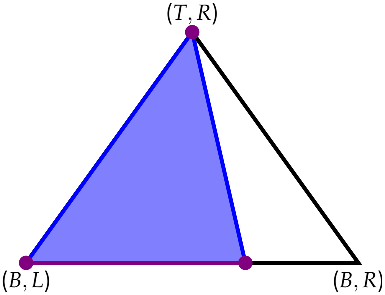

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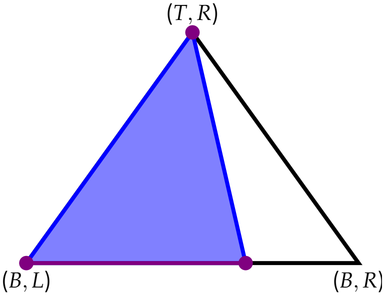

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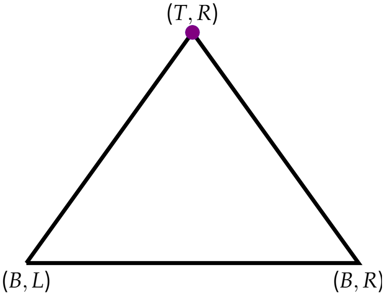

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Then,

$$\text{BCE} = \{p : p(T, R) = 1\} = \text{BKE}.$$

Jeopardization is Generic

Natural conjecture: jeopardization is non-generic.

However, consider matching-pennies:

- has a unique BCE, which equals the (fully-mixed) NE.
- for all i and a_i, $J(a_i) = A_i$.
- The same holds for all games around matching-pennies.

Theorem 2: Proof Sketch

Lemma 2. For any $u \in \mathbb{R}^{I \times A \times \Theta}$, $p \in \mathrm{BCE}(u)$, and $\epsilon > 0$, a $\tilde{u} \in B_\epsilon(u)$ exists such that $p \in \mathrm{BKE}(\tilde{u})$.

Lemma 3. The set $\{u : \mathrm{BCE}(u) = \mathrm{cl}\,(\mathrm{BKE}(u))\}$ is dense in $\mathbb{R}^{I \times A \times \Theta}$.

Lemma 4. The correspondences $\mathrm{BCE}(\cdot)$ and $\mathrm{cl}(\mathrm{BKE}(\cdot))$ are semi-algebraic.

Lemma (Blume and Zame, 1994). If $F : \mathbb{R}^N \rightrightarrows \mathbb{R}^M$ is a semi-algebraic correspondence with compact values, then F is continuous at every point outside a closed set with $\dim < N$.

Proof of Theorem 2. By Lemma 3, $\mathrm{BCE}(u) = \mathrm{cl}(\mathrm{BKE}(u))$ at any u at which both correspondences are continuous.

Bertrand Competition

Bertrand Competition with Stochastic MC

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Letting $\underline{\theta}_1 := \min \Theta_1$ and $\bar{\theta}_1 := \max \Theta_1$, we assume

$$\underline{\theta}_1 < \bar{\theta}_1 < \theta_2.$$

Ties are broken in favor of low cost firm (i.e., firm 1).

Max CS in BKE is lower CS than in BCE

Given a BCE p, define the expected consumer surplus,

$$\mathrm{CS}(p) = \mathbb{E}_p\left[\int_{\min\{a_1,a_2\}}^{1} D(t)\mathrm{d}t\right].$$

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Proposition 3.

The maximal CS under BCE is strictly larger than under BKE,

$$\max_{p\in\mathrm{BCE}} \mathrm{CS}(p) = \mathbb{E}\left[\int_{\theta_1}^{1} D(t)\mathrm{d}t\right] > \int_{\mathbb{E}[\theta_1]}^{1} D(t)\mathrm{d}t = \max_{p\in\mathrm{BKE}} \mathrm{CS}(p).$$

Moreover, every BKE corresponds to a no-information Nash.

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Proposition 2's condition (iii) implies cl(BKE) $\neq$ BCE.

Almost Free Information

Almost-Free Information

Several papers use almost-free flexible learning as an eqlbm selection device.

- **Coordination games:** Yang (2015), Denti (2021, forthcoming), Morris and Yang (forthcoming).
- **Monopoly pricing:** Ravid, Roesler, and Szentes (2022).
- **Perturbing the game:** Hoshino (2018).

Next: study all almost-free learning outcomes (holding game fixed).

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(ii) The equilibrium's outcome q is within ϵ of p,

$$\|q - p\| < \epsilon.$$

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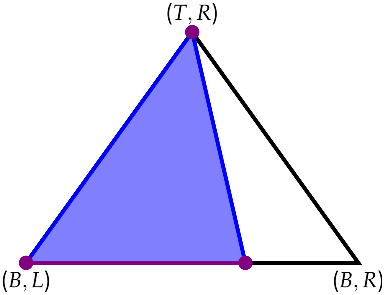
Theorem 3. p is an almost-free info outcome if and only if it is a full-info outcome and $p \in \mathrm{cl}(\mathrm{BKE})$.

Example: Full Info ≠ Almost-Free Info

	L	R
T	$0, 0$	$3, 1$
B	$2, 2$	$2, 2$



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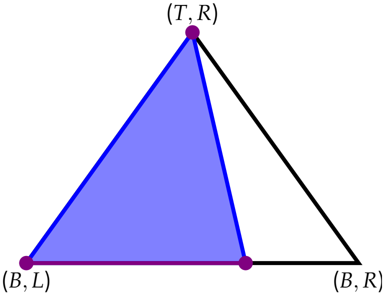

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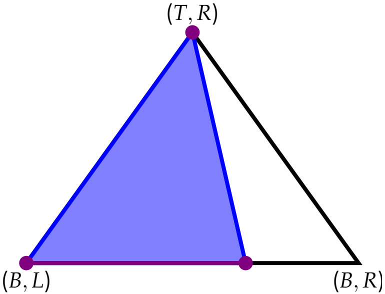

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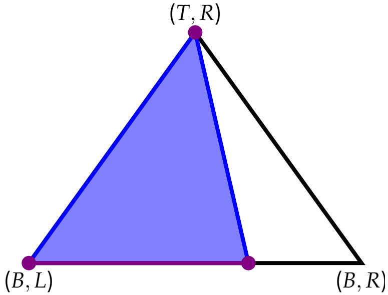

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Therefore, we get that

$$\text{almost-free info} \neq \text{full info}.$$

Related Literature

- **Rational inattention:** Sims (2003), . . . , Yang (2015), Hoshino (2018), Ravid (2020), Angeletos and Sastry (2021), Ravid, Roesler, and Szentes (2022), Hebert and La'O (2022), Morris and Yang (2022), Denti (forthcoming). . .

- **Espionage games:** Solan and Yariv (2004), de Clippel and Rozen (2021), Denti (2021). . .

- **Robust predictions:** Bergemann and Morris (2005, 2013), Chassang (2013) Bergemann, Brooks, and Morris (2015, 2017), Carroll (2017). . .

- **Correlated equilibrium:** Aumann (1974, 1987), Myerson (1986, 1997), Forges (1986, 1993, 2006), Lipman and Srivastava (1990), Bergemann and Morris (2016), Doval and Ely (2020). . .

Conclusion

1. BKE gives the outcomes that can arise across all Blackwell-monotone $\mathcal{T}$.

2. BKE differs from BCE when there is shared jeopardization and/or there is no minimal-mixing BKE, which is rare.

3. BKE is either dense or nowhere dense in the BCE set.

We also show:

- BKE significantly refines BCE in Bertrand competition.

- Almost-free learning-outcomes given by limit of BKEs that are convex hull of full-info Nash.