Shenxin Zhou 3036339565

1.

1. X= Professor Gao is ~~the~~ part of the commitee

①. $P(X) = \dfrac{\binom{16}{4}\binom{4}{1}}{\binom{16}{4}\binom{5}{2}} = \dfrac{2}{5}$

②. Y= Professor Adler is the chair.

$P(Y) = \dfrac{\binom{15}{3}\binom{5}{2}}{\binom{6}{1}\binom{16}{4}\binom{5}{2}} = \dfrac{1}{24}$

---

①. X= ~~at least 2~~ women contained in commitee.

$P(X \geq 2) = 1-(P(X=0)+P(X=1))$

$= 1 - \dfrac{\binom{17}{6}}{\binom{21}{6}} - \dfrac{\binom{4}{1}\binom{17}{5}}{\binom{21}{6}} \approx 0.315$

$\approx$

② Y= non-binary professors contained in commitee

$P(Y=0) = \dfrac{\binom{19}{6}}{\binom{21}{6}} = 0.5$

③. Z = no-binary is the chair.

$P(Z) = \dfrac{\binom{19}{5}\binom{2}{1}}{\binom{6}{1}\binom{21}{6}} + \dfrac{\binom{19}{4}\binom{2}{2}\binom{2}{1}}{\binom{6}{1}\binom{21}{6}}$

≈ 0.095

2.

2. ① C = answered a question rightly.

K = knew the answer.

~~$P(C) = 4$~~

$P(K) = 60\%$, $P(C|K) = 1$

$P(C|K^c) = \frac{1}{4}$

$$P(K|C) = \frac{P(C|K)\cdot P(K)}{P(C)}$$

$$= \frac{P(C|K)\cdot P(K)}{P(C|K)\cdot P(K) + P(C|K^c)\cdot P(K^c)}$$

$$= \frac{0.6\times 1}{1\times 0.6 + \frac{1}{4}\cdot 0.4} = \frac{6}{7}$$

② $\because P(C) = 0.7$

$\therefore C \sim Bin(10, 0.7)$

$$P(C \geq 6.5) = P(C=7) + P(C=8) + P(C=9) + P(C=10)$$

$$= \binom{10}{7}0.7^7\cdot 0.3^3 + \binom{10}{8}0.7^8\cdot 0.3^2 + \binom{10}{9}0.7^9\cdot 0.3^1 + \binom{10}{10}0.7$$

$= 0.65$

③ assume p is the probability that she pass, which we get at problem 2.

S = the student pass.

according to the question,

$S \sim Geo(n, p)$

$\therefore E(S) = \frac{1}{p}$

$= 1.54$

3.

'3.

①. since passengers behave independently,

$\therefore$ ~~Pa~~ X = passengers ~~show up~~ ~~booked~~ will show up

$X \sim Bin(6, 0.8)$

$\therefore E(X) = np = 4.8$

② according to the problem:

$$P(X \le 4) = 1 - P(X=5) - P(X=6)$$

$$= 1 - \binom{6}{5} 0.8^5 \, 0.2^1 - \binom{6}{6} 0.8^6$$

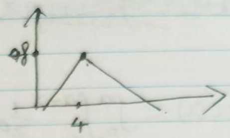
$$= 0.34$$

①.

n = number of people booked.

X = passengers shows up., $X \sim bin(n, 0.8)$

$\therefore$ Revenue = $\begin{cases} 80 - 8a, & a > 4 \\ 14a - 8, & a \leq 4 \end{cases}$



Since: $E[g(x)] = \sum g(x) P(X=k)$

$\therefore E[Revenue(a)] = \sum_{a=0}^{n} \binom{n}{a} \cdot 0.8^{a} \cdot 0.2^{n-a} \cdot$ Revenue.

Through the graph, we can found, when $a = 4$, Revenue is the max,

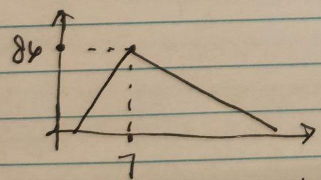
And. $E[X] = np$, so when $E[X] = 4$,

$n = 5$

②. according to ①.

Revenue = $\begin{cases} 140 - 8a & a > 7 \\ 14a - 14 & a \leq 7 \end{cases}$

and the same: $E[Revenue(a)] = \sum_{a=0}^{n} \binom{n}{a} 0.8^{a} \cdot 0.2^{n-a} \cdot$ Revenue.



according to graph, when $a = 7$, Revenue is the max,

So, $n = 8.75 \approx 9$,

4.

4.

① $\because \int_0^1 cxe^x = 1$

$\therefore c(xe^x|_0^1 - \int_0^1 e^x dx) = 1$

$c = 1$

②. $Var(2+X) = Var(X)$

$E[X] = \int_0^1 x^2 e^x dx$

$= x^2e^x|_0^1 - (2\int_0^1 xe^x dx)$

$= e - 2$

$E[X^2] = \int_0^1 x^3 e^x dx$

$= x^3e^x|_0^1 - (3\int_0^1 x^2e^x dx)$

$= 6 - 2e$

$\therefore Var(X) = E[X^2] - E[X]^2$

$= -e^2 + 2e + 2$

5.

when $1<a<2$,

5. ①. $f(x) = \begin{cases} 1 & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ $\quad f(y) = \begin{cases} \frac{1}{2} & 0<y<2 \\ 0 & \text{otherwise} \end{cases}$

$f_{X+Y}(a) = \frac{1}{2}\int_0^2 f_X(a-y)\,dy$

when: $0 \le a \le 1$, $f_X(a-y) = 1$ when $0<y<a$, $f_X(a-y)=0$, when $y>a$

$\therefore f_{X+Y}(a) = \frac{1}{2}\int_0^a dy = \frac{1}{2}a$

when: $1<a<2$, $f_X(a-y)=1$ when $0<a-y<1 \Rightarrow a-1<y<a$

$f_{X+Y}(a) = \frac{1}{2}\int_{a-1}^{a} dy = \frac{1}{2}$

when $3>a>2$, $f_X(a-y)=1$ when $a-1<y<2$.

$f_{X+Y}(a) = \frac{1}{2}\int_{a-1}^{2} dy = \frac{3}{2} - \frac{1}{2}a$

$\therefore f_{X+Y}(a) = \begin{cases} \frac{1}{2}a & 0 \le a \le 1 \\ \frac{1}{2} & 1<a \le 2 \\ \frac{3}{2} - \frac{1}{2}a & 2<a<3 \\ 0 & \text{otherwise} \end{cases}$

②. according to ①,

$F_{X+Y}(a) = \int_0^a \int_0^{a-x} \frac{1}{2}\,dy\,dx$

when. $0<a \le 1$

$F_{X+Y}(a) = \frac{1}{4}a^2$

when $1<a \le 2$

$F_{X+Y}(a) = \int_0^1 \int_0^{a-x} \frac{1}{2}\,dy\,dx$

$= \frac{1}{2}a - \frac{1}{4}$

when $2<a \le 3$

$F_{X+Y}(a) = \int_2^a \left(\frac{3}{2} - \frac{a}{2}\right)da + \int_0^1 \frac{1}{2}a\,da + \int_1^2 \frac{1}{2}\,da$

$= -\frac{1}{4}a^2 + \frac{3}{2}a - \frac{4}{5}$

$\Rightarrow F_{X+Y}(a) = \begin{cases} 0 & a<0 \\ \frac{1}{4}a^2, & 0 \le a \le 1 \\ \frac{1}{2}a - \frac{1}{4} & 1<a \le 2 \\ -\frac{1}{4}a^2 + \frac{3a}{2} - \frac{5}{4}, & 2<a \le 3 \\ 1, & a>3 \end{cases}$

6.

6.

① $\because X \sim bin(n, p) \quad p = 0.5$

$\therefore E[X] = 0.5n$

$P(X \geq \frac{3}{4}n) \leq \dfrac{E[X]}{\frac{3}{4}n} = \dfrac{2}{3}$

② $Var(X) = np(1-p) = \frac{1}{4}n$

$\therefore P(X \geq 0.5n + \frac{1}{4}n) \leq \dfrac{\frac{1}{4}n}{\frac{1}{4}n + (\frac{1}{4}n)^2} = \dfrac{4}{n+4}$

7.

Inspiration: No inspiration, I have thought of this on my own.

question: The joint density function of X, Y is given by:

$$f(x, y) = 4e^{-2(x+y)} \quad , x > 0, \ y > 0$$

Find: $E[X|Y]$, $E[Y]$, $E[X]$, and $corr(X, Y)$

Solution:

First find the marginal density of Y:

$$f_Y(y) = \int_0^\infty 4e^{-2(x+y)} dx = 4e^{-2y} \int_0^\infty e^{-2x} dx$$
$$= 2e^{-2y}$$

$\therefore Y \sim Exp(2)$

So, $E[Y] = \frac{1}{2}$

$\because E[X|Y] = \int_0^\infty x f_{X|Y}(x, y) dx$

$f_{X|Y}(x, y) = \dfrac{f(x, y)}{f_Y(y)} = 2e^{-2x}$

$\therefore X|Y \sim EXP(2)$

$\therefore E[X|Y] = \frac{1}{2}$

$\therefore E[X] = E[E[X|Y]] = \frac{1}{2}$

$\therefore E[XY] = \int_0^\infty \int_0^\infty xy f(x, y) dx dy$

$= \frac{1}{4}$

$\therefore cov(X, Y) = E[XY] - E[X]E[Y]$

$= 0$

$corr(X, Y) = 0$