

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

This function represents a rational number

```
def numer(x):
    return x('n')
```

Constructor is a higher-order function

```
def denom(x):
    return x('d')
```

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
```

```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```

Executing a for statement:  
for <name> in <expression>:  
    <suite>

1. Evaluate the header <expression>, which must yield an iterable value (a list, tuple, iterator, etc.)
2. For each element in that sequence, in order:
  - A. Bind <name> to that element in the current frame
  - B. Execute the <suite>

Unpacking in a for statement:

```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
```

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1
>>> same_count
2
```

A name for each element in a fixed-length sequence

..., -3, -2, -1, 0, 1, 2, 3, 4, ...

range(-2, 2)

Length: ending value – starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Identity:

```
<exp0> is <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to the same object
```

Equality:

```
<exp0> == <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to equal values
```

Identical objects are always equal values

iter(iterable):

Return an iterator over the elements of an iterable value

next(iterator):

Return the next element

```
>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4
>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

A generator function is a function that yields values instead of returning.

```
>>> def plus_minus(x):
...     yield x
...     yield -x
>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3
>>> def a_then_b(a, b):
...     yield from a
...     yield from b
>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

List comprehensions:

[&lt;map exp&gt; for &lt;name&gt; in &lt;iter exp&gt; if &lt;filter exp&gt;]

Short version: [&lt;map exp&gt; for &lt;name&gt; in &lt;iter exp&gt;]

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent
2. Create an empty result list that is the value of the expression
3. For each element in the iterable value of <iter exp>:
  - A. Bind <name> to that element in the new frame from step 1
  - B. If <filter exp> evaluates to a true value, then add the value of <map exp> to the result list

The result of calling repr on a value is what Python prints in an interactive session

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

The result of calling str on a value is what Python prints using the print function

```
>>> today = datetime.date(2019, 10, 13)
>>> print(today)
2019-10-13
```

str and repr are both polymorphic; they apply to any object repr invokes a zero-argument method \_\_repr\_\_ on its argument

```
>>> today.__repr__()
'datetime.date(2019, 10, 13)'
>>> today.__str__()
'2019-10-13'
```

Type dispatching: Look up a cross-type implementation of an operation based on the types of its arguments

Type coercion: Look up a function for converting one type to another, then apply a type-specific implementation.

Functions that aggregate iterable arguments

- sum(iterable[, start]) → value sum of all values
- max(iterable[, key=func]) → value largest value
- max(a, b, c, ..., [, key=func]) → value
- min(iterable[, key=func]) → value smallest value
- min(a, b, c, ..., [, key=func]) → value
- all(iterable) → bool whether all are true
- any(iterable) → bool whether any is true

Many built-in Python sequence operations return iterators that compute results lazily

map(func, iterable):

Iterate over func(x) for x in iterable

filter(func, iterable):

Iterate over x in iterable if func(x)

zip(first\_iter, second\_iter):

Iterate over co-indexed (x, y) pairs

reversed(sequence):

Iterate over x in a sequence in reverse order

list(iterable):

Create a list containing all x in iterable

tuple(iterable):

Create a tuple containing all x in iterable

sorted(iterable):

Create a sorted list containing x in iterable

def cascade(n):

if n &lt; 10:

print(n)

else:

print(n)

cascade(n//10)

print(n)

&gt;&gt;&gt; cascade(123)

123

12

1

12

123

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21,

def fib(n):

if n == 0:

return 0

elif n == 1:

return 1

else:

return fib(n-2) + fib(n-1)



Exponential growth. E.g., recursive fib

 $\Theta(b^n)$   $O(b^n)$ 

Incrementing n multiplies time by a constant

Quadratic growth. E.g., overlap

 $\Theta(n^2)$   $O(n^2)$ 

Incrementing n increases time by n times a constant

Linear growth. E.g., slow exp

 $\Theta(n)$   $O(n)$ 

Incrementing n increases time by a constant

Logarithmic growth. E.g., exp\_fast

 $\Theta(\log n)$   $O(\log n)$ 

Doubling n only increments time by a constant

Constant growth. Increasing n doesn't affect time

 $\Theta(1)$   $O(1)$ 

List &amp; dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
>>> a == b
False
```

```
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

```
>>> sum([1, 2])
3
>>> sum([1, 2], 3)
6
>>> sum()
0
>>> all([False, True])
False
>>> all()
True
>>> any([False, True])
True
>>> any()
False
>>> max(1, 2)
2
>>> max([1, 2])
2
>>> max([1, -2], key=abs)
-2
```

You can copy a list by calling the list constructor or slicing the list from the beginning to the end.

```
>>> suits = ['coin', 'string', 'myriad']
>>> suits.pop()
'string'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
>>> suits
['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Remove and return the last element

Remove a value

Add all values

Replace a slice with values

Add an element at an index

False values:

```
>>> bool(0)
False
>>> bool(1)
True
>>> bool('')
False
>>> bool('0')
True
>>> bool(())
False
>>> bool({})
True
>>> bool({})
False
>>> bool(0)
False
>>> bool(lambda x: 0)
True
```



Global frame

make\_withdraw\_list(balance)

withdraw

func make\_withdraw\_list(balance) [parent=Global]

```
list
0
75
```

It changes the contents of the b list

f1: make\_withdraw\_list [parent=Global]

balance

withdraw

Return value

amount

25

Return value

75

f2: withdraw [parent=f1]

amount

25

Return value

75

Name bound outside of withdraw def

Element assignment changes a list

```
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

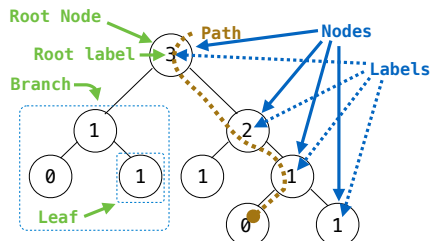
withdraw = make_withdraw_list(100)
withdraw(25)
```

## Recursive description:

- A tree has a root **label** and a list of **branches**
- Each branch is a **tree**
- A tree with zero branches is called a **leaf**

## Relative description:

- Each location is a **node**
- Each **node** has a **label**
- One node can be the **parent/child** of another



```
def tree(label, branches=[]):
```

```
    for branch in branches:
```

```
        assert is_tree(branch)
```

```
    return [label] + list(branches)
```

```
def label(tree):
```

```
    return tree[0]
```

```
def branches(tree):
```

```
    return tree[1:]
```

```
def is_tree(tree):
```

```
    if type(tree) != list or len(tree) < 1:
```

```
        return False
```

```
    for branch in branches(tree):
```

```
        if not is_tree(branch):
```

```
            return False
```

```
    return True
```

```
def is_leaf(tree):
```

```
    return not branches(tree)
```

```
def leaves(t):
```

```
    """The leaf values in t.
```

```
    >>> leaves(fib_tree(5))
```

```
    [1, 0, 1, 0, 1, 1, 0, 1]
```

```
    """
```

```
    if is_leaf(t):
```

```
        return [label(t)]
```

```
    else:
```

```
        return sum([leaves(b) for b in branches(t)], [])
```

```
class Tree:
```

```
    def __init__(self, label, branches=[]):
```

```
        self.label = label
```

```
        for branch in branches:
```

```
            assert isinstance(branch, Tree)
```

```
        self.branches = list(branches)
```

```
    def is_leaf(self):
```

```
        return not self.branches
```

```
    def leaves(tree):
```

```
        """The leaf values in a tree."""
```

```
        if tree.is_leaf():
```

```
            return [tree.label]
```

```
        else:
```

```
            return sum([leaves(b) for b in tree.branches], [])
```

```
class Link:
```

```
    empty = ()
```

```
    def __init__(self, first, rest=empty):
```

```
        assert rest is Link.empty or isinstance(rest, Link)
```

```
        self.first = first
```

```
        self.rest = rest
```

```
    def __repr__(self):
```

```
        if self.rest:
```

```
            rest = ' + repr(self.rest)
```

```
        else:
```

```
            rest = ""
```

```
        return 'Link(' + repr(self.first) + rest + ')'
```

```
    def __str__(self):
```

```
        string = '<'
```

```
        while self.rest is not Link.empty:
```

```
            string += str(self.first) + ' '
```

```
            self = self.rest
```

```
        return string + str(self.first) + '>'
```

## Anatomy of a recursive function:

- The **def statement header** is like any function
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
```

```
    """Sum the digits of positive integer n."""
```

```
    if n < 10:
```

```
        return n
```

```
    else:
```

```
        all_but_last, last = n // 10, n % 10
```

```
        return sum_digits(all_but_last) + last
```

```
def count_partitions(n, m):
```

```
    """Number of ways to partition n using m parts.
```

```
    E.g., count_partitions(6, 4)
```

```
    Explore two possibilities:
```

```
    • Use at least one 4
```

```
    • Don't use any 4
```

```
    Solve two simpler problems:
```

```
    • count_partitions(2, 4)
```

```
    • count_partitions(6, 3)
```

```
    Tree recursion often involves
```

```
    exploring different choices.
```

```
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

## Python object system:

**Idea:** All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

A new instance is created by calling a class

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

```
balance: 0 holder: 'Jim'
```

When a class is called:

1. A new instance of that class is created:

2. The **\_\_init\_\_** method of the class is called with the new object as its first argument (named **self**), along with any additional arguments provided in the call expression.

**\_\_init\_\_** is called a constructor

```
class Account:
    def __init__(self, account_holder):
```

```
        self.balance = 0
        self.holder = account_holder
```

```
    def deposit(self, amount):
```

```
        self.balance = self.balance + amount
```

```
        return self.balance
```

```
    def withdraw(self, amount):
```

```
        if amount > self.balance:
```

```
            return 'Insufficient funds'
```

```
        self.balance = self.balance - amount
```

```
        return self.balance
```

Function call: all arguments within parentheses

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

Method invocation: One object before the dot and other arguments within parentheses

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Call expression

Dot expression

&lt;expression&gt; . &lt;name&gt;

The **<expression>** can be any valid Python expression.

The **<name>** must be a simple name.

Evaluates to the value of the attribute looked up by **<name>** in the object that is the value of the **<expression>**.

To evaluate a dot expression:

1. Evaluate the **<expression>** to the left of the dot, which yields the object of the dot expression
2. **<name>** is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
3. If not, **<name>** is looked up in the class, which yields a class attribute value
4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

Account class attributes

```
interest: 0.02 0.04 0.05
(withdraw, deposit, __init__)
```

Instance attributes of jim\_account

```
balance: 0
holder: 'Jim'
interest: 0.08
```

Instance attributes of tom\_account

```
balance: 0
holder: 'Tom'
```

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
```

```
    """A bank account that charges for withdrawals."""
```

```
    withdraw_fee = 1
```

```
    interest = 0.01
```

```
    def withdraw(self, amount):
```

```
        return Account.withdraw(self, amount + self.withdraw_fee)
```

```
        or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom') # Calls Account.__init__
>>> ch.interest # Found in CheckingAccount
0.01
>>> ch.deposit(20) # Found in Account
20
>>> ch.withdraw(5) # Found in CheckingAccount
14
```

