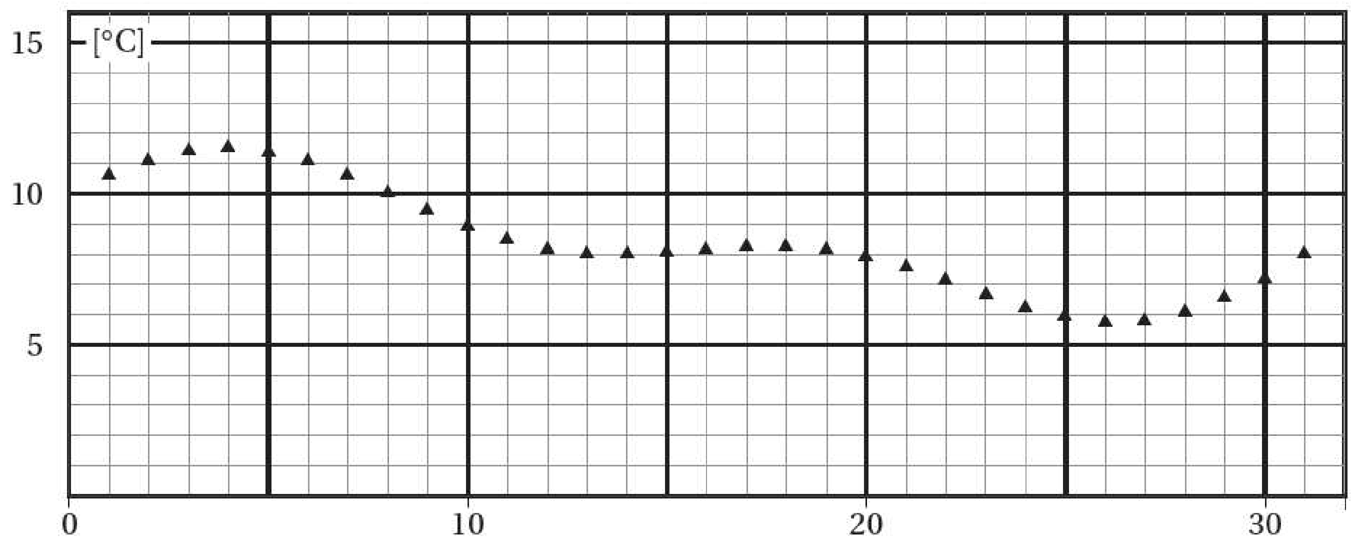
Digital Signal Processing

What Is Digital Signal Processing?

- A signal, technically yet generally speaking, is a a formal description of a phenomenon evolving over time or space
- By signal processing we denote any manual or “mechanical” operation which modifies, analyzes or otherwise manipulates the information contained in a signal

Consider the simple example of ambient temperature: once we have agreed upon a formal model for this physical variable – Celsius degrees, for instance – we can record the evolution of temperature over time in a variety of ways and the resulting data set represents a temperature “signal”.

Simple processing operations can then be carried out even just by hand: for example, we can plot the signal on graph paper as in the figure, or we can compute derived parameters such as the average temperature in a month.



Temperature measurements over a month.

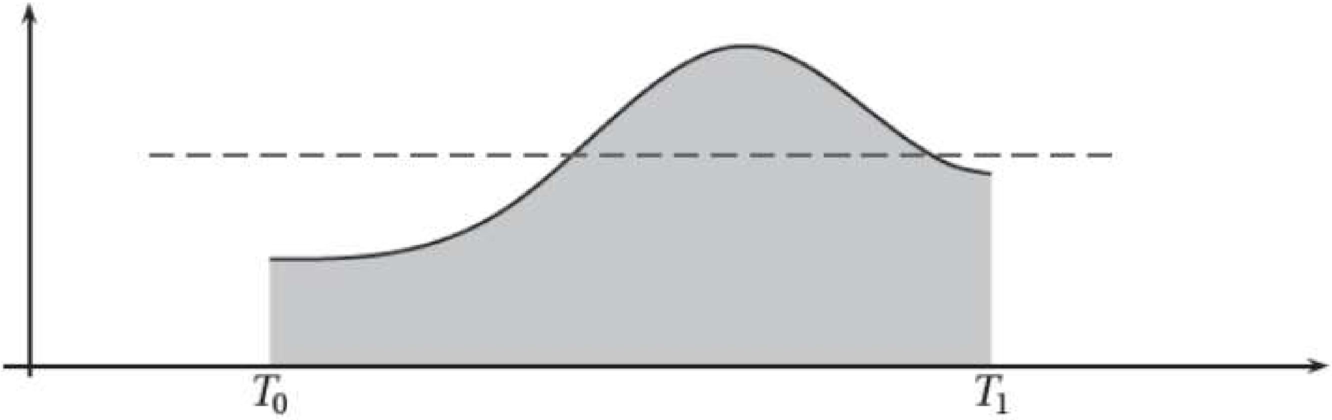
Discrete Time

Discrete Time - motivation

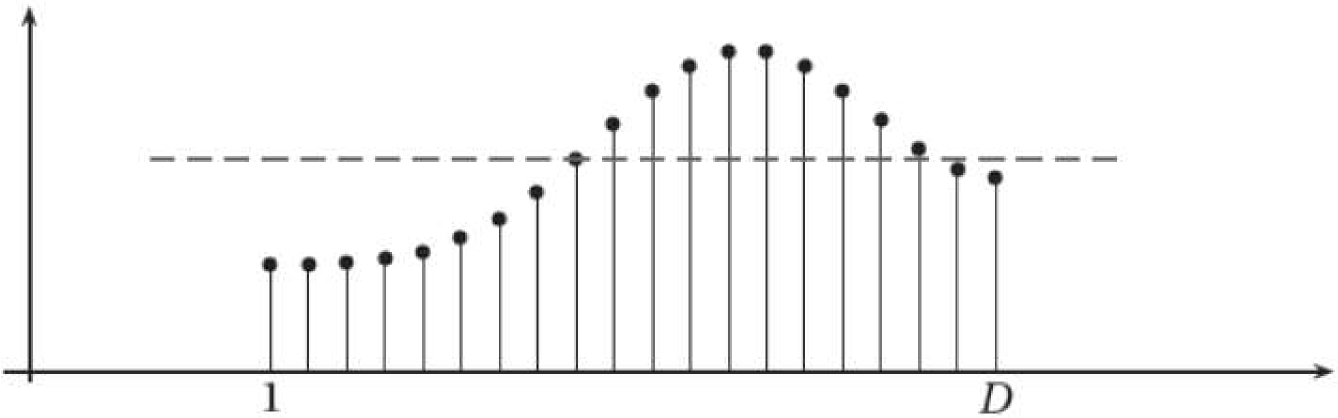
The problem with these analog recordings is that they are not abstract signals but a conversion of a physical phenomenon into another physical phenomenon: the temperature (which measured by **thermograph**), for instance, is converted into the amount of ink on paper while the sound pressure wave is converted into the physical depth of the groove (**phonograph**).

The advent of electronics did not change the concept: an audio tape, for instance, is obtained by converting a pressure wave into an electrical current and then into a magnetic deflection. The fundamental consequence is that, for analog signals, a different signal processing system needs to be designed explicitly for each specific formof recording.

Consider for instance the problem of computing the average temperature over a certain time interval.



$$\bar{C} = \frac{1}{T_1 - T_0} \int_{T_0}^{T_1} f(t)\,dt$$



$$\hat{C} = \frac{1}{D}\sum_{n=1}^{D} c_n$$

Discrete Time - definitions

- Discrete-time signals are often derived by sampling a continuous-time signal, such as speech, with an analogto- digital (AD) converter.
- For example, a continuous-time signal xa(t) that is sampled at a rate of fs = l/Ts samples per second (Ts sampling interval) produces the sampled signal x(n), which is related to xa(t) as follows:

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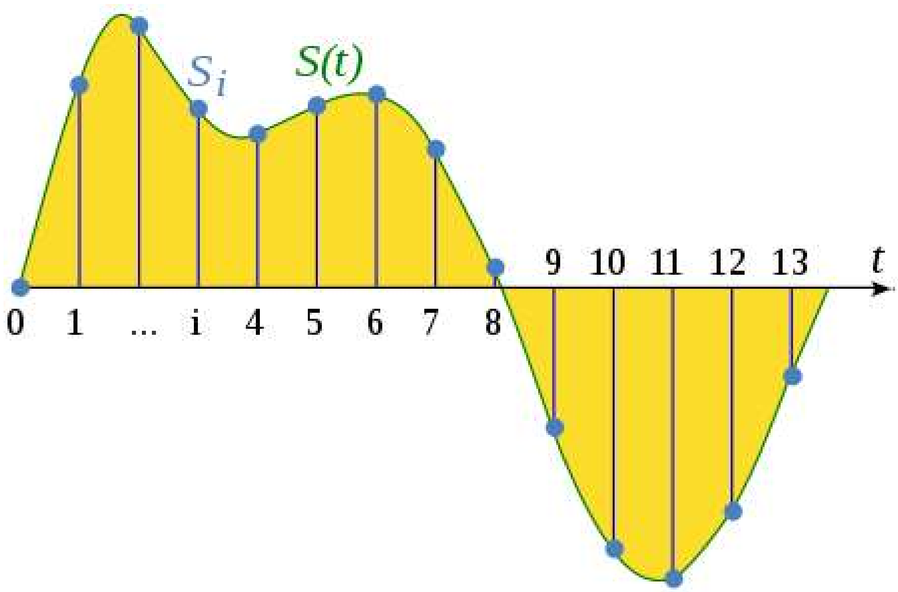


$$\mathbf{x} = [x(0),\ x(1), \ldots, x(N-1)]^T$$

$$x(n) = x_a(nT_s)$$

Discrete Time - Type of signals

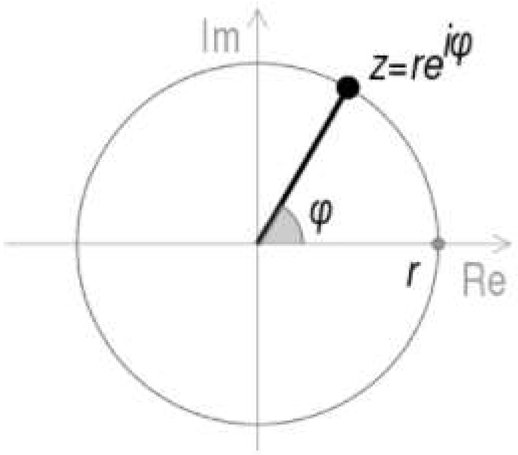
exponential sequence

$$x(n) = a^n$$

periodic sequence - Euler's formula

$$a = e^{j\omega_0}$$

$$e^{jn\omega_0} = \cos(n\omega_0) + j\sin(n\omega_0)$$



In general, a discrete-time signal may be complex-valued. In fact, in a number of important applications such as digital communications, complex signals arise naturally. A complex signal may be expressed either in terms of its real and imaginary parts,

$$z(n) = a(n) + jb(n) = \mathrm{Re}\{z(n)\} + j\mathrm{Im}\{z(n)\}$$

or in polar form in terms of its magnitude and phase,

$$z(n) = |z(n)| \exp[j \arg\{z(n)\}]$$

The magnitude may be derived from the real and imaginary parts as follows:

$$|z(n)|^2 = \mathrm{Re}^2\{z(n)\} + \mathrm{Im}^2\{z(n)\}$$

whereas the phase may be found using

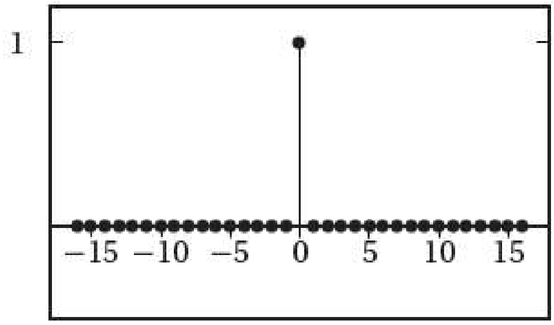
$$\arg\{z(n)\} = \tan^{-1}\frac{\mathrm{Im}\{z(n)\}}{\mathrm{Re}\{z(n)\}}$$

If $z(n)$ is a complex sequence, the *complex conjugate*, denoted by $z^*(n)$, is formed by changing the sign on the imaginary part of $z(n)$:

$$z^*(n) = \mathrm{Re}\{z(n)\} - j\mathrm{Im}\{z(n)\} = |z(n)| \exp[-j \arg\{z(n)\}]$$

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



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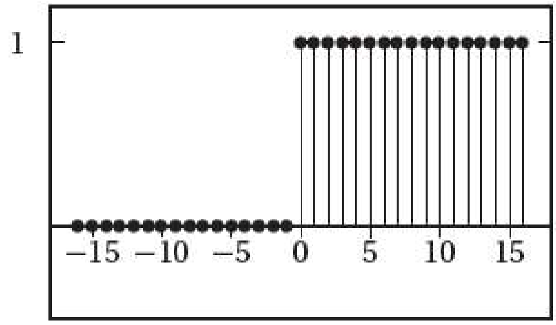

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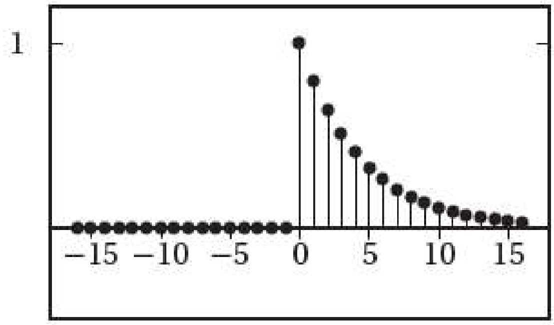

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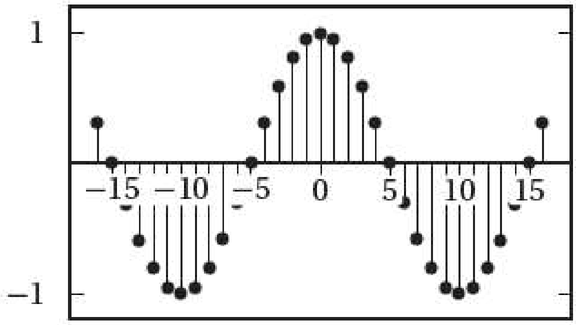

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