MA 471
Project 2 report

Group 1

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In the first project, we have studied linear time series analysis.
Now we look into the non-linear time series models

There are several non-linear time series models.
In this project, we focus on the GARCH model which is a generalized ARCH model.

ARCH model

ARCH stands for Auto-Regressive Conditional Heteroskedasticity.
Heteroskedasticity is nothing but volatility.(deviation)
Conditional represents that the volatility of the series is dependent on the time where we are at currently.
Autoregressive here means that the volatility at a certain time point depends on the previous time points before it.

ARCH(1) model

In this basic ARCH model, the volatility follows an AR(1) model

$$Var(a_t)=\boldsymbol{\sigma}_t$$

$$\boldsymbol{\sigma}_t=\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1\boldsymbol{\sigma}_{t-1}$$

We model this using the formula

$$\varepsilon_t=w_t(\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1\varepsilon_{t-1}{}^2)^{1/2}$$

Where w_t is white noise.

ARCH(p) model

This model includes the values of p previous times into account for the volatility of the present time.

$$\varepsilon_t = w_t(\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \varepsilon_{t-1}^2 + \ldots\ldots + \boldsymbol{\alpha}_p \varepsilon_{t-p}^2)^{1/2} = w_t \boldsymbol{\sigma}_t$$

GARCH

GARCH is a generalized ARCH model where the present volatility is not only determined on the previous values of the time series but also the volatilities at previous time points.

GARCH(1,1)

This basic GARCH model is formulated using

$$\varepsilon_t = w_t(\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \varepsilon_{t-1}^2 + \boldsymbol{\beta}_1 \boldsymbol{\sigma}_{t-1})^{½} = \varepsilon_t \boldsymbol{\sigma}_t$$

Here volatility of present time is given a function of the value of time series at t-1 and also the volatility of the time series at time t-1.

GARCH(p,q)

This model takes into account p previous values of the time series and the q previous volatilities to calculate the volatility of the current time.
The formula given for this is

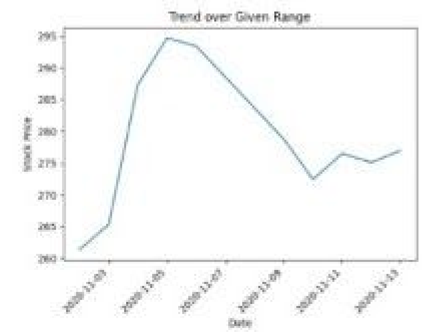
$$\varepsilon_t = w_t(\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \varepsilon_{t-1}^2 + \ldots\ldots + \boldsymbol{\alpha}_p \varepsilon_{t-p}^2 + \boldsymbol{\beta}_1 \boldsymbol{\sigma}_{t-1} + \ldots\ldots \boldsymbol{\beta}_q \boldsymbol{\sigma}_{t-q})^{1/2} = w_t \boldsymbol{\sigma}_t$$

We have also added Value at Risk(VaR) and Expected Shortfall(Es) metric for the stocks,

Value at Risk(VaR) Value at Risk (VaR) is a statistic that measures and quantifies the level of financial risk of a portfolio or position over a specific time frame. VaR modeling determines the potential for loss in the entity being assessed and the probability of occurrence for the defined loss. In other words, a 95% 1-day VaR of -0.05 implies we can be 95% certain that 1-day return of this portfolio/stock would be better than -0.05 or with 5% probability returns are worse than -0.05

Expected Shortfall(ES) Expected shortfall is also called Conditional Value at Risk(CVaR) which is an alternative to VaR to be used as a risk metric. The expected shortfall at 95% confidence interval implies the expected return of the stock in the worst 5% cases, .i.e expectation of the tail end of the distribution.

The following results are generated for **Facebook, Inc.** from 1-11-2020 to 15-11-2020



Historical Data plot

Return Data plot

The VaR is calculated at 95% and 99% confidence intervals.

VaR at 99% confidence interval is: -0.09263395067908509

VaR at 95% confidence interval is: -0.045009000783775344

The expected shortfall is calculated at 95 and 99% confidence intervals.

Expected shortfall at 99% confidence interval is: -0.14252997678205148

Expected shortfall at 95% confidence interval is: -0.0717497001908978

```
==============================================================
Dep. Variable:                  Close   No. (
Model:                  ARIMA(7, 0, 3)   Log
Date:                 Thu, 26 Nov 2020   AIC
Time:                         20:41:38   BIC
Sample:                     11-02-2020   HQIC
                          - 11-13-2020
Covariance Type:                   opg
==============================================================
                 coef    std err          z
--------------------------------------------------------------
const        279.9871      0.848    330.179
ar.L1          1.2729      0.891      1.429
ar.L2         -0.5919      0.749     -0.791
ar.L3         -0.7185      0.583     -1.233
ar.L4          0.7137      0.580      1.231
ar.L5          0.5918      0.757      0.782
ar.L6         -1.2754      0.763     -1.671
ar.L7          0.9964      0.358      2.785
ma.L1          0.8793      0.012     70.854
ma.L2         -0.9574      0.008   -119.951
ma.L3         -0.9206      0.011    -85.590
sigma2         0.0002      0.002      0.104
==============================================================
Ljung-Box (L1) (Q):                    2.04
Prob(Q):                               0.15
Heteroskedasticity (H):                0.02
Prob(H) (two-sided):                   0.01
==============================================================

Warnings:
[1] Covariance matrix calculated using the o
[2] Covariance matrix is singular or near-si
```

```
==============================================================

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[1] Covariance matrix calculated using the o
[2] Covariance matrix is singular or near-si
```
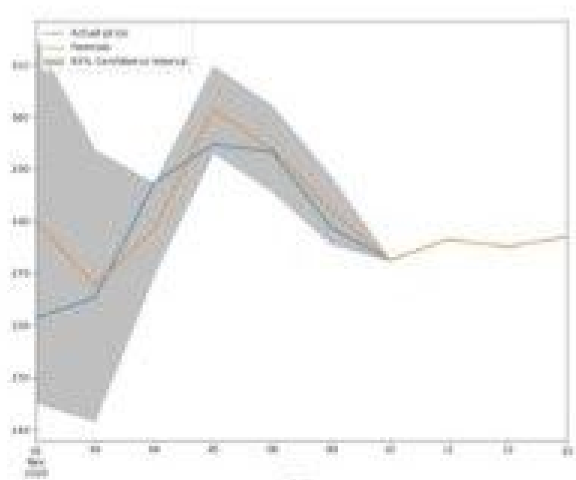

```
==============================================================
Dep. Variable:                    Close   R-sq
Mean Model:               Constant Mean   Adj.
Vol Model:                        GARCH   Log-
Distribution:                    Normal   AIC:
Method:              Maximum Likelihood   BIC:
                                          No. (
Date:                  Thu, Nov 26 2020   Df R
Time:                          20:41:38   Df M
                              Mean Model
==============================================================
                 coef    std err          t
--------------------------------------------------------------
mu             0.3082      1.156      0.267
                          Volatility Model
==============================================================
                 coef    std err          t
--------------------------------------------------------------
omega      8.7765e-08      4.627  1.897e-08
alpha[1]   1.9288e-10      0.599  3.219e-10
beta[1]        0.8811      0.516      1.708
==============================================================

Covariance estimator: robust
```



ARMA Model Forecast vs Actual Price over given Time Range

If we look at the summary of the GARCH model
We have the parameters of the GARCH equation

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  Close   R-squared:                      -0.013
Mean Model:             Constant Mean   Adj. R-squared:                 -0.013
Vol Model:                      GARCH   Log-Likelihood:               -23.3233
Distribution:                  Normal   AIC:                           54.6466
Method:            Maximum Likelihood   BIC:                           55.4355
                                        No. Observations:                    9
Date:                Thu, Nov 26 2020   Df Residuals:                        5
Time:                        22:50:56   Df Model:                            4
                                 Mean Model
   ========================================================================
                  coef    std err          t      P>|t|   95.0% Conf. Int.
   ------------------------------------------------------------------------
   mu           0.3082      1.156      0.267      0.790 [ -1.957,  2.573]
                              Volatility Model
   ========================================================================
                  coef    std err          t      P>|t|   95.0% Conf. Int.
   ------------------------------------------------------------------------
   omega    8.7765e-08      4.627  1.897e-08      1.000 [ -9.068,  9.068]
   alpha[1] 1.9288e-10      0.599  3.219e-10      1.000 [ -1.174,  1.174]
   beta[1]      0.8811      0.516      1.708  8.754e-02 [ -0.130,  1.892]
   ========================================================================

                        Covariance estimator: robust
```

The equations for GARCH for the above implementation is as follows

$$r_t = \mu + \epsilon_t$$

$$e_t = \sigma_t e_t$$

$$\sigma^2_t = \omega + \alpha \epsilon^2_{t-1} + \beta \sigma^2_{t-1}$$

We have α=1.9288e-10

β=0.8811

ω=9.7765e-08

μ=0.3082

So the mathematical equation for the GARCH model is

$$r_t = 0.3082 + \epsilon_t$$

$$\sigma^2_t = 9.7765\text{e-}08 + (1.9288\text{e-}10)\epsilon^2_{t-1} + 0.8811\sigma^2_{t-1}$$

References

Data retrieval API: https://github.com/ranaroussi/yfinance
Stat models: https://www.statsmodels.org/stable/index.html
Arch model: https://arch.readthedocs.io/en/latest/#
Analysis of Financial Time-series book bu Ruey S. Tsay