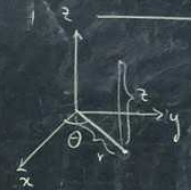
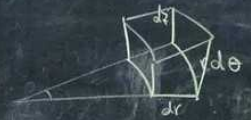
مختصات استوانه‌ای



$$(x,y,z) \approx (r,\theta,z)$$

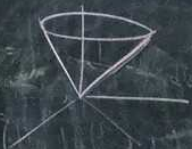


$$dV = r\,dr\,d\theta\,dz$$

مثال

رویه $z=r$ را در مختصات استوانه‌ای رسم کنید.

$$r=\sqrt{x^2+y^2}$$



مثال

رویه $r=2$ را رسم کنید.

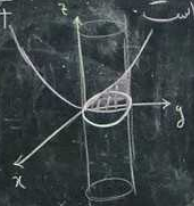
مثال

$\iiint_T x^2y^2\,dV$ را بیابید که در آن T ناحیه محدود به استوانه‌ی $x^2+y^2=y$

$$\Downarrow$$

$$x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$$

و بهمی‌گون $z=x^2+y^2$ و صفحه‌ی $z=0$ است.



$\int$

$$\int_0^{r^2} r^5 \sin^2\theta\cos^2\theta \, dz = r^7 \sin^2\theta\cos^2\theta$$

$$\int_0^{\sin\theta} r^7 \sin^2\theta\cos^2\theta \, dr = \sin^2\theta\cos^2\theta \, r^8/8 \Big|_0^{\sin\theta}$$

$$= \frac{\sin^{10}\theta}{8}\cos^2\theta$$

$$\int_0^{\pi}\int_0^{\sin\theta}\int_0^{r^2} r^2\cos^2\theta \, r^2\sin^2\theta \, r \, dz\,dr\,d\theta$$

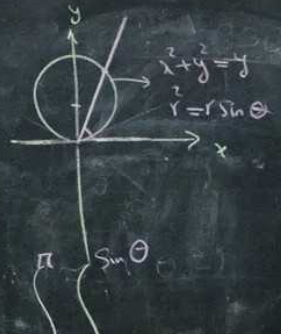
$$\theta \quad r \quad z$$

ناحیه‌ی انتگرال‌گیری

$$T = \left\{ (r,\theta,z) \;\middle|\; 0 \le \theta \le \pi,\ 0 \le r \le \sin\theta,\ 0 \le z \le r^2 \right\}$$

$$dv = r\,dr\,d\theta\,dz$$

$$x = r\cos\theta \quad / \quad y = r\sin\theta$$



$$\int_0^{\pi}\int_0^{\sin\theta}\int_r^2 \left(r^3\cos^3\theta + r^3\sin^3\theta\right) r\,dz\,dr\,d\theta$$

$$\int_0^2 r^4\left(\sin^3\theta+\cos^3\theta\right)dr = r^6\left(\sin^3\theta+\cos^3\theta\right)$$

$$\iiint_T (x^3+y^3)\,dV$$

$$z=0,\quad z=x^2+y^2,\quad x^2+y^2=4$$

$$\int \sin^{10} d\theta = \int_{10} \sin^8\theta\,d\theta + \frac{\cos\theta\sin^9\theta}{10}\,\frac{1}{8}\int \sin^8\theta\cos^2\theta\,d\theta =$$

$$\sin^{10} = \sin^8\left(\sin^2\right) = \sin^8(1-\cos^2\theta)$$

$$\frac{1}{8}\int \sin^{10}\theta\left(1-\sin^2\theta\right)d\theta =$$

$$= \frac{1}{8}\int_0^{\pi} \sin^{10}\theta - \sin^{12}\theta\,d\theta =$$

$$\int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{1+z^2}} r\,dr\,dz\,d\theta$$

حجم ناحیه‌ی 2

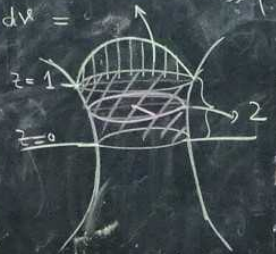
$$x^2+y^2+z^2=3$$

$$x^2+y^2-z^2=1 \Rightarrow$$

$$2z^2=2 \Rightarrow z=1$$

$$\iiint_T dv =$$

حجم مورد نظر؟



مثال

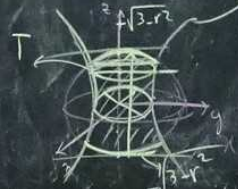
حجم ناحیه‌ی شامل مبدأ مختصات و محدود به هذلولی‌گون

$x^2+y^2+z^2=3$ را بیابید

و کره $x^2+y^2-z^2=1$

و بالای محور مختصات

$$r^2=1+z^2$$

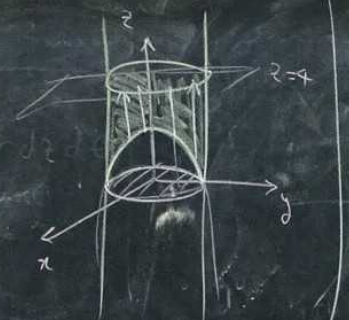


$$\int_0^{\sin\theta} r^6\left(\sin^3\theta+\cos^3\theta\right)dr$$

$$=\frac{\sin^7\theta}{7}\left(\underbrace{\sin^3\theta+\cos^3\theta}_{\sin(1-\cos^2\theta)}\right)$$

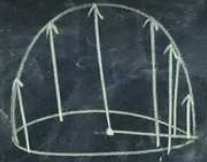
$$\int_0^{2\pi}\int_0^{1}\int_{1-r^2}^{4} r\,dz\,dr\,d\theta$$

θ r z



$x^2+y^2=1$

مثال حجم داخل استوانه $x^2+y^2=1$ و زیر صفحه $z=4$ و بالای کره‌ی $z=1-x^2-y^2$ را بیابید.



$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_{1}^{\sqrt{3-r^2}} r\,dz\,dr\,d\theta$$

حجم برابر

θ r z

$$\int_{1-r^2}^{4} r\,dz = (3+r^2)\,r = 3r + r^3$$

$$\int_{0}^{1} (3r + r^3)\,dr = A$$

$$\int_{0}^{2\pi} A\,d\theta = A \times 2\pi$$