Simulating Blood Flow in the Pulmonary Artery using Physics-Informed Neural Networks

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What is Pulmonary Arterial Hypertension (PAH)?



- Rare, progressive, mPAP > 20 mmHg at rest
- Late diagnosis: 2 to 3 years, 4 to 6 physicians
- Structural alterations of the vascular wall
- Highly invasive evaluation

Left pulmonary artery Left lung Healthy artery To right lung Severely narrowed artery Small arteries in lungs become narrowed or blocked

Subject-specific computational models:

- Non-invasive
- In silico experimentation
- Gain prediction capabilities

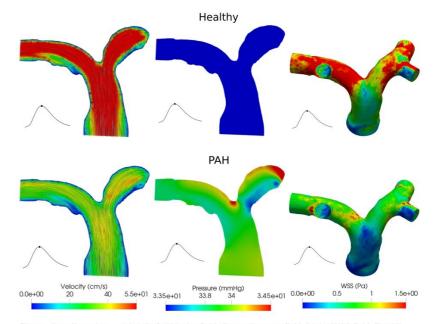


Fig. 10 Systolic peak (t = 1.9 s). Left: Velocity field. Center: Pressure field. Right: WSS field. Test VI

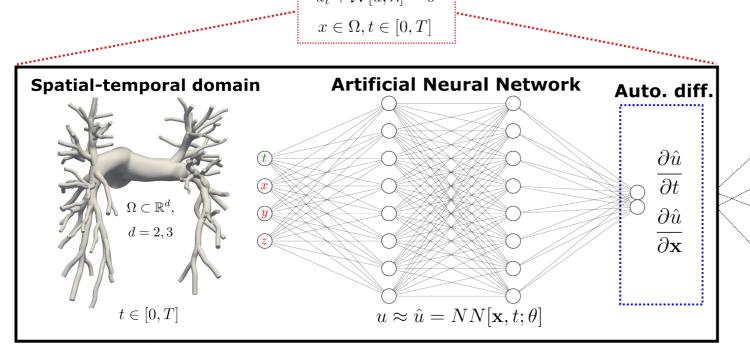
Physics-Informed neural networks (PINNs) for computational

modeling



$$u_t + \mathcal{N}[u, \lambda] = 0$$
$$x \in \Omega, t \in [0, T]$$





PDE based loss

$$\mathcal{L}_{PDE} = MSE(f(\hat{u}, \partial_t \hat{u}, \partial_x \hat{u}, ..., \lambda))$$

$$\mathcal{L}_{Data} = MSE(\hat{u}|_{\Omega} - u|_{Data})$$

$$\mathcal{L}_{IC} = MSE(\hat{u}|_{\Omega, t_0} - u|_{\Omega, t_0})$$

$$\mathcal{L}_{BC} = MSE(\partial_n \hat{u}|_{\partial\Omega} - \partial_n g|_{\partial\Omega})$$

- Universal approximation theorem
- Meshless
- Mixed (coupled) formulations
- Forward and inverse problems

 $<\epsilon$

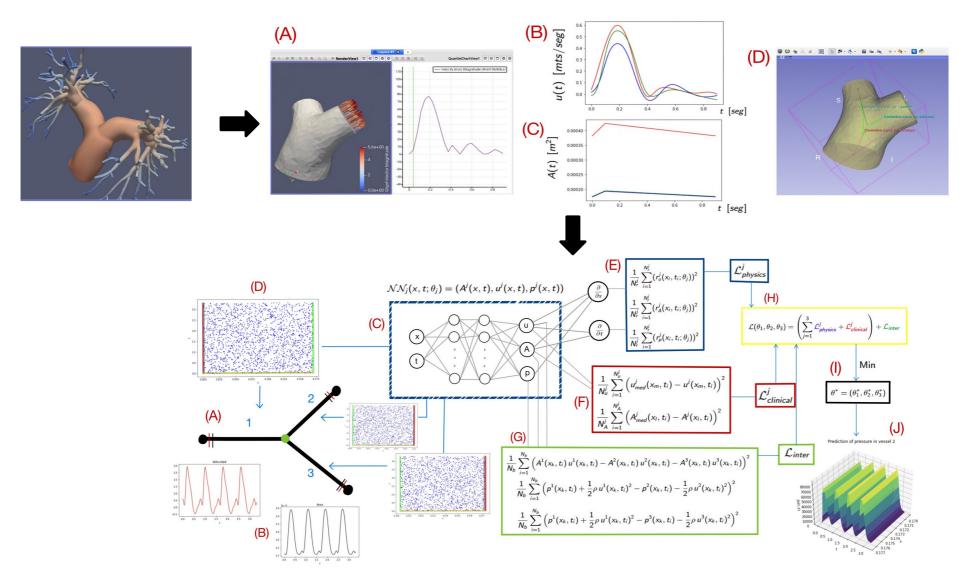
END

 $-\frac{\partial \mathcal{L}}{\partial \theta}, \frac{\partial \mathcal{L}}{\partial \lambda} \longleftarrow \mathcal{L} = w_1 \mathcal{L}_{PDE} + w_2 \mathcal{L}_{Data} + w_3 \mathcal{L}_{IC} + w_4 \mathcal{L}_{BC}$

PINNs used in blood flow modeling



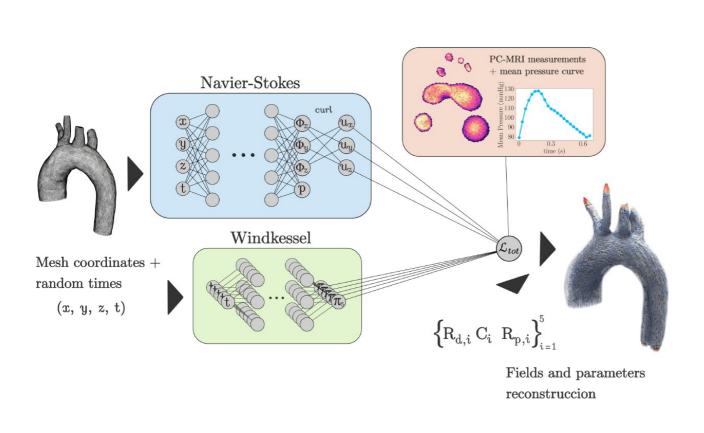
Jara 2023. Pulmonary artery blood pressure estimation using PINNs

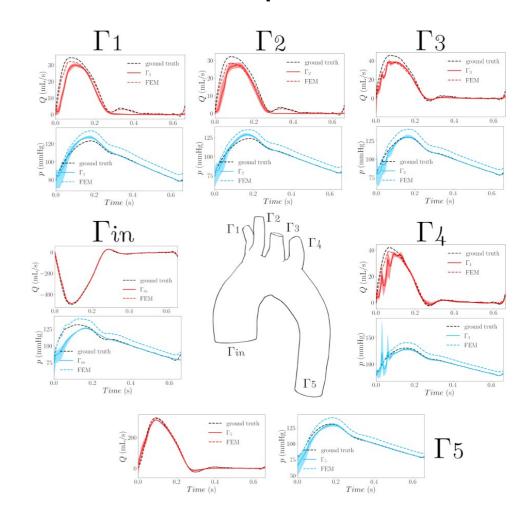


PINNs used in blood flow modeling



Garay 2023. Physics-informed neural networks for blood flow inverse problems





Our approach

Ν

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END



Adimensional Steady Navier-Stokes model

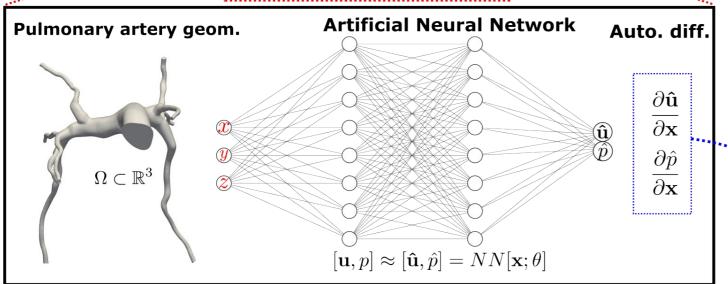
$$\mathbf{u}^* \cdot \nabla \mathbf{u}^* - \frac{1}{Re} \Delta \mathbf{u}^* + \nabla p^* = \mathbf{0}$$

$$\nabla \cdot \mathbf{u}^* = 0$$

$$BC: \quad \mathbf{u}^* = \mathbf{0}, \quad \frac{1}{Re} \frac{\partial \mathbf{u}^*}{\partial \mathbf{n}} - \mathbf{n}p^* = \mathbf{0}$$

Parameters:

- Density:
- Viscosity:
- Velocity:
- Length:
- Reynolds: 2500



Reference solution (Data):



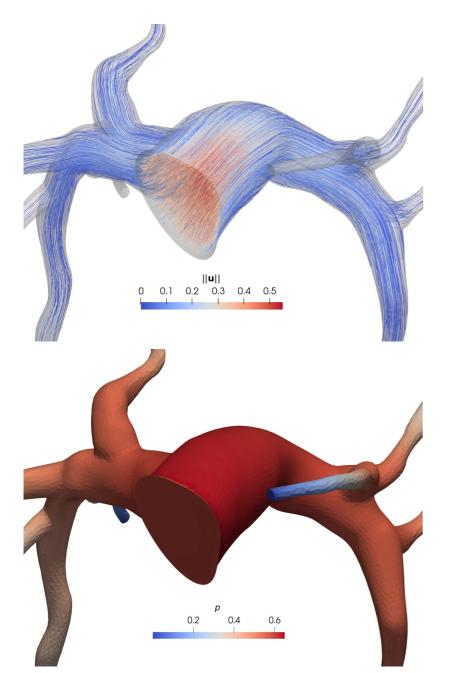
PDE based loss

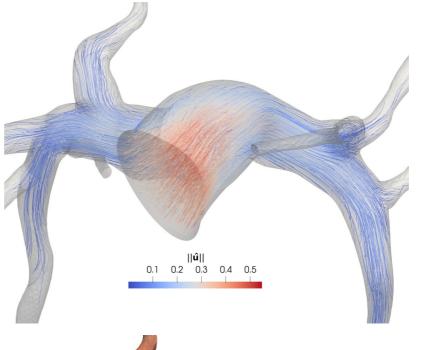
$$\mathcal{L} \leftarrow \frac{\partial \mathcal{L}}{\partial \theta}, \frac{\partial \mathcal{L}}{\partial \lambda} \leftarrow \mathcal{L} = w_1 \mathcal{L}_{PDE} + w_2 \mathcal{L}_{Data} + w_3 \mathcal{L}_{BC} \qquad \mathcal{L}_{Data} = MSE(\hat{f}(\hat{u}, \partial_t \hat{u}, \partial_x \hat{u}, ..., \lambda)) \\ \mathcal{L}_{Data} = MSE(\hat{u}|_{\Omega} - u|_{Data})$$

$$\mathcal{L}_{BC} = MSE(\partial_n \hat{u}|_{\partial\Omega} - \partial_n g|_{\partial\Omega})$$

Numerical reference

PINN solution

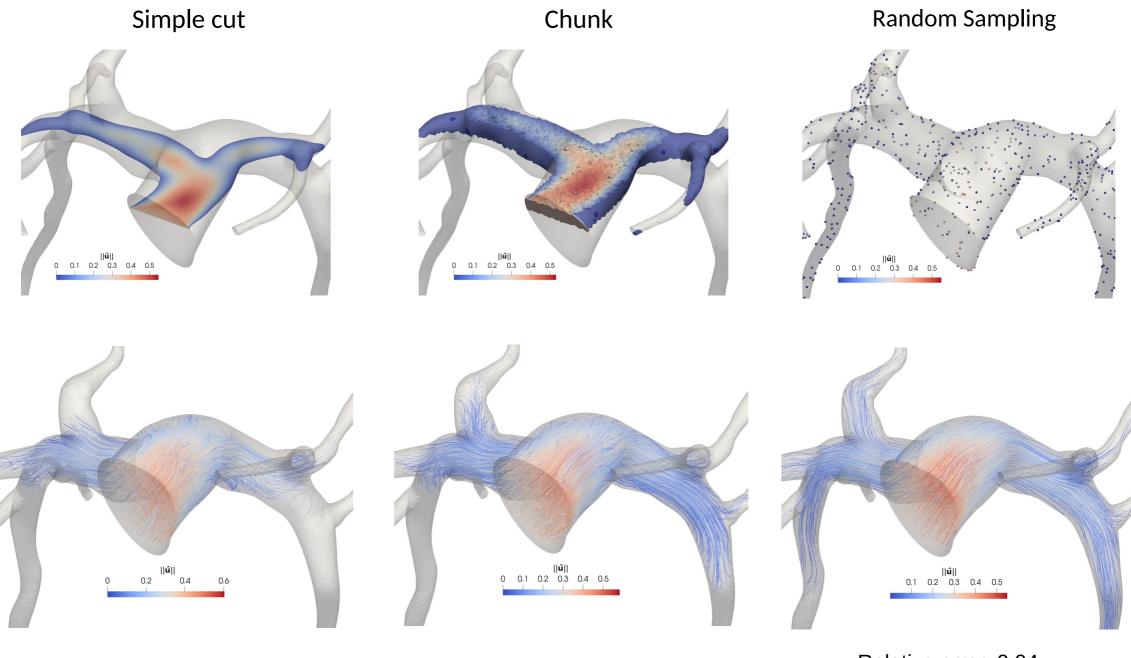




D2 0.4 0.6

- ModifiedMLP (Wang etal. 2020)
- 10 Hidden layers
- 100 Neuron
- Fourier Features
- 100 Epochs

Relative error: 0.04



Relative error: 0.11 Relative error: 0.09 Relative error: 0.04



Conclusions:

- Accurate estimation of the blood pressure in the pulmonary artery
- Accurate in high Reynolds numbers
- Accuracy depends on the spatial distribution of the input data

What is next?

- Transient Navier-Stokes
- Use 4D-flow (simulated) data
- Add Windkessel model









Thank you for your attention!

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