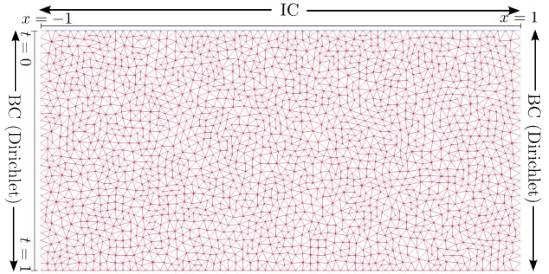
Physical model

$$u_t + \mathcal{N}[u, \lambda] = 0$$

$$x \in \Omega, t \in [0, T]$$

1

Spatial-temporal domain



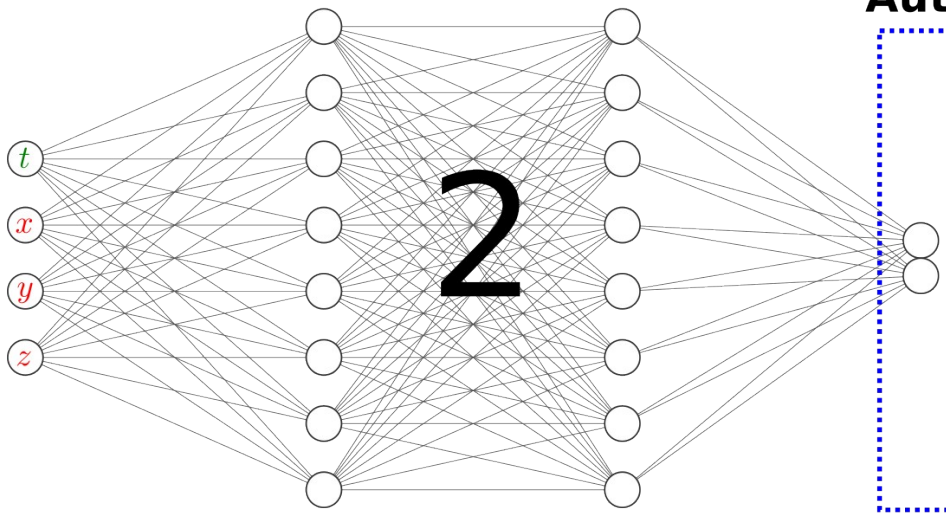
$$\Omega \subset \mathbb{R}^d,$$

$$d = 2, 3$$

$$t \in [0, T]$$

6

Artificial Neural Network



2

$$u \approx \hat{u} = NN[\mathbf{x}, t; \theta]$$

Auto. diff.

3

$$\frac{\partial \hat{u}}{\partial t}$$

$$\frac{\partial \hat{u}}{\partial \mathbf{x}}$$

PDE based loss

$$\mathcal{L}_{PDE} = MSE(f(\hat{u}, \partial_t \hat{u}, \partial_x \hat{u}, ..., \lambda))$$

$$\mathcal{L}_{Data} = MSE(\hat{u}|_\Omega - u|_{Data})$$

$$\mathcal{L}_{IC} = MSE(\hat{u}|_{\Omega,t_0} - u|_{\Omega,t_0})$$

$$\mathcal{L}_{BC} = MSE(\partial_n \hat{u}|_{\partial\Omega} - \partial_n g|_{\partial\Omega})$$

4

$$\mathcal{L} = w_1\mathcal{L}_{PDE} + w_2\mathcal{L}_{Data} + w_3\mathcal{L}_{IC} + w_4\mathcal{L}_{BC}$$

$$\frac{\partial \mathcal{L}}{\partial \theta}, \frac{\partial \mathcal{L}}{\partial \lambda}$$

5

$< \epsilon$

N

Y

END