Optimal risk management

1. Introduction

INVESTOR I	INVESTOR II
- Profit 120%	- Profit 10%
- Loss 60%	- Loss 5%
- Every second trade earns a profit	- Every second trade earns a profit
- 10 trades	- 10 trades

Strategy effectiveness

INVESTOR I					INVESTOR II			
Period	Capital	Profit + 120%	Loss - 60%		Period	Capital	Profit + 10%	Loss - 5%
0	**10 000**				0	**10 000**		
1	22 000	12 000			1	11 000	1000	
2	8 800		-13 200		2	10 450		-550
3	19 360	10 560			3	11 495	1 045	
4	7 744		-11 616		4	10 920		-575
5	17 037	9 293			5	12 012	1 092	
6	6 815		-10 222		6	11 412		-600
7	14 992	8 178			7	12 553	1 141	
8	5 997		-8995		8	11 925		-628
9	13 193	7 196			9	13 118	1 193	
10	**5 277**		**-7 916**		10	**12 462**		**-656**

Figure 1: https://www.xtb.com/en/learn-to-trade/introduction-to-risk-management?is_mobile=1

While preparing myself for becoming an amateur CFD trader, I came across the following problem in one of the basic lessons posted on the website of a broker that I was currently using, illustrated in Figure 1:

Two investors are playing the same strategy, which has a 50% chance of winning each time. Both investors start with the same balance in their account. Both of them play 10 times. Both of them choose a risk to reward ratio of 1:2. However, while trading, they risk different *percentages of their capital. The first one aims for a 120% profit with a potential loss of -60%. The second one aims for a 10% profit, for a -5% loss. By looking at Figure 1, we can see that Investor I will lose 47% of his account, while the other investor will win 24.6% of his initial balance.*

As an engineer, I thought to myself: can these numbers be improved ? Is there an optimal amount to aim for take profit and stop loss for a given reward to risk ratio ? How does the model change when trading in the real world ?

The results will be categorized as:

1. **Ideal** results, which would give optimal results in an ideal world.
2. **Real** results, which take losses and other trading costs into account.
3. **Optimal** results, that combine ideal and real results in an optimal way.
4. **Helper** results, that express different formulas in a convenient/insightful way.

2. An initial model

Let's begin by denoting

Definition 1 *b is the initial balance before a sequence of trades*

Definition 2 *p is the take profit expressed as a percentage of the account balance,* $\frac{profit}{b}$*, thus the profit after a successful trade can be expressed as pb*

Definition 3 *l is the stop loss expressed as a percentage of the account balance,* $\frac{|loss|}{b}$*, thus the profit after an unsuccessful trade can be expressed as -lb*

Definition 4 *r is the reward to risk ratio (inverse of risk to reward ratio),* $r = \frac{p}{l},\ p = rl$

Definition 5 *n is the total number of trades,* n_w *is the number of winning trades,* n_l*is the number of losing trades.* $n = n_w + n_l$

Definition 6 w_r *is win ratio, expressed as*$\frac{n_w}{n}$ *.* w_r *can represent the frequentionist probability of a winning trade to occur.*

Definition 7 w_l : *loss ratio, expressed as* $\frac{n_l}{n}$*.* w_l *can represent the frequentionist probability of a losing trade to occur.*

Theorem 1 $w_r + w_l = \frac{n_w}{n} + \frac{n_l}{n} = \frac{n}{n} = 1$*. Thus,* $w_l = 1 - w_r$

Definition 8 *e is the equity, or the money left in the trader's account after a sequence of n trades*

Theorem 2 *The equity after a sequence of n trades, with an initial balance b, a take profit p and a stop loss l, with a win probability w_r is given by:*

$$e = b(1+p)^{n_w}(1-l)^{n_l}$$

Theorem 3 *The equity after a sequence of n trades, with an initial balance b, a win ratio w_r and a lose ratio w_l, a profit pb and a loss -lb. Consider the random variable of a trade that can result in either a take profit p or stop loss l, $X = \{x_p, x_l\}$*

$$e = E[X] = w_r pb - w_l lb = b(pw_r - lw_l) = b[rlw_r - l(1-w_r)]$$
$$e = bl[rw_r - 1 + w_r]$$

Let's first begin with the special case when $w_r = 50\%,\ thus\ n_w = n_l = \frac{n}{2}$

The equity of the trader, after n trades can be expressed using Theorem 2:

$$e = b[(1+p)(1-l)]^{\frac{n}{2}} \tag{1}$$

It is easy to plug in the numbers from Figure1:

The first trader will end up having

$$10000[(1+1.2)(1-0.6)]^5 = 5227.3 \qquad \text{(E1)}$$

The second trader will end up having

$$10000[(1+0.1)(1-0.05)]^5 = 12462 \qquad \text{(E2)}$$

The final equity of the traders depends on the term $(1+p)(1-l)$. By expressing $p = rl$ (Definition 4),

equation (1) becomes

$$e = b[(1+rl)(1-l)]^{\frac{n}{2}} = b\left[-l^2 r + (r-1)l + 1\right]^{\frac{n}{2}} \tag{2}$$

The quantity $-l^2 r + (r-1)l + 1$ depicts a concave parabola (its vertex is a maximum). The maximum can be found by setting the derivative with respect to l to 0:

$$\frac{d}{dl}\left[-l^2 r + (r-1)l + 1\right] = -2rl + (r-1) = 0 \tag{3}$$

$$l = \frac{r-1}{2r},$$
$$p = \frac{r-1}{2}$$

If the traders used $p = \frac{2-1}{2} = 0.5, l = \frac{2-1}{4} = 0.25$, their gains would have been:

$$10000[(1+0.5)(1-0.25)]^5 = \mathbf{18020} \qquad \text{(E3)}$$

The initial sum would have almost doubled !

3. A more general initial model

But what if the traders used trading strategies that had an arbitrary w_r win ratio?
Let's denote:
$n_w : n_w = w_r n,\ number\ of\ winning\ trades$
$n_l : n_l = (1-w_r)n,\ number\ of\ losing\ trades$
$n = n_w + n_l$
Now, equation (1) becomes:

$$e = b(1+p)^{n_w}(1-l)^{n_l} \qquad (4)$$

Re-writing it again only in terms of r and l,

$$e = b(1+rl)^{n_w}(1-l)^{n_l} \qquad (5)$$

The term of interest to be maximized is now

$$(1+rl)^{n_w}(1-l)^{n_l} \qquad (6)$$

By setting the derivative with respect to l to 0, we get

$$(1-l)^{n_l}(rl+1)^{n_w+1}(rln - rn_w + n_l) = 0 \qquad (7)$$

Which yields:

$$l = \frac{rn_w - n_l}{rn_w + rn_l}, p = rl \qquad (8)$$

Note: Kelly criterion[reference]:

$$l = \frac{rn_w - n_l}{rn_w + rn_l} = \frac{rn_w - n_l}{rn} = \frac{r\frac{n_w}{n} - \frac{n_l}{n}}{r} \Longrightarrow \boldsymbol{l = \frac{rw_r - (1 - w_r)}{r}} \tag{9}$$

$$p = rl = rn_w - n_l$$

$$\boldsymbol{p = rw_r - (1 - w_r) = w_r(r+1) - 1}$$

$$\boldsymbol{l = \frac{w_r(r+1) - 1}{r}}$$

Theorem 4 *The maximum equity that can be achieved for a given win ratio w_r and a reward-to-risk-ratio r, given a number of trades n given an initial account balance b is given by:*

$$\boldsymbol{e = b[w_r(r+1)]^{nw_r}\left[1 - \frac{w_r(r+1) - 1}{r}\right]^{n(1-w_r)}}$$

$$Let's\ denote\ this\ optimal\ factor\ as\ O_f(w_r, r, n) = [w_r(r+1)]^{nw_r}\left[1 - \frac{w_r(r+1) + 1}{r}\right]^{1-nw_r}$$

$$e = bO_f(w_r, r, n)$$

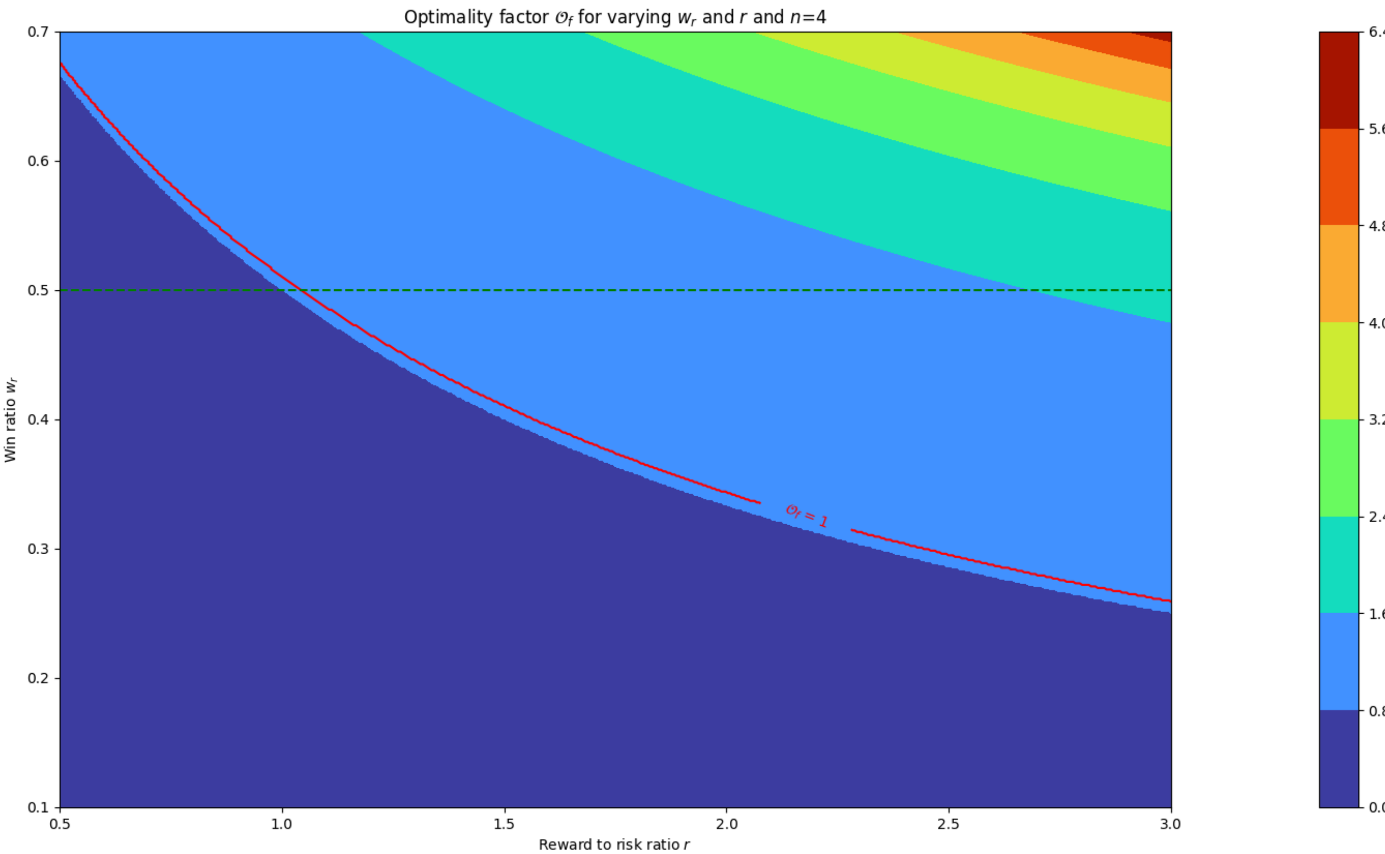


Figure 2: Optimality factor plotted for n = 4

Let's take a (not so fortunate) example of: $r = 2.75,\ n_w = 3, n_l = 7 \rightarrow l = 0.045, p = 0.125$
$e = 10000 * (1 + 0.125)^3 * (1 - 0.045)^7 = 10315$

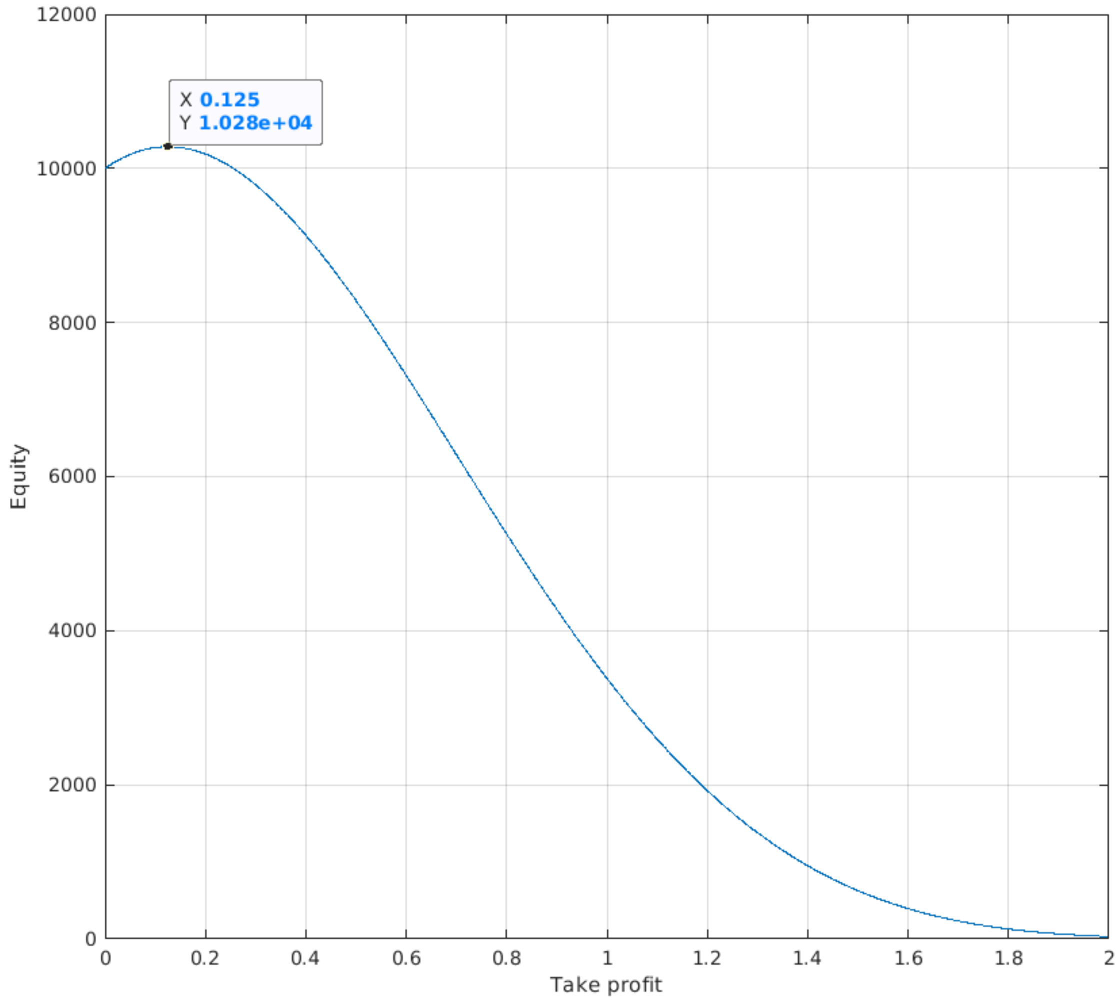


Figure 3: Equity vs take profit plot, showing that the maximum point was found correctly

Therefore, in order to maximize profits, by following the simplistic model introduced in the first section of this article, the traders should set their take profit p and stop loss l times their balance b,
where p and l have been found in (9)

Kelly criterion appendix:

$Denote\ p = \frac{n_w}{n} is\ the\ probability\ of\ winning, q = \ 1 - p\ is\ the\ probability\ of\ losing, b = r, the\ net\ odds\ as\ defined\ by\ John\ Larry\ Kelly\ Jr,$

$$l = \frac{bp - q}{b} = \frac{p(b+1) - 1}{b} \tag{K1}$$

Which is also known by the name of **Kelly criterion.**

4.Maximizing profitability

Theorem 5 C *denotes the cost per trade for the given instrument*

From Theorem 4,

$$e = bO_f(w_r, r, n) - n\mathrm{C} \geqslant 0$$
$$\text{Thus, in order for the sequence of trades to be profitable,}$$
$$bO_f(w_r, r, n) - n\mathrm{C} \geqslant 0 \tag{10}$$
$$O_f(w_r, r, n) \geqslant n\mathrm{C}$$
$$\frac{O_f(w_r, r, n)}{n} \geqslant \mathrm{C}$$

Theorem 6 *In order for a given trading strategy for a particular instrument which inquires specific trading costs to be profitable, we need to ensure that, given the win ratio w_r, reward to risk ratio r, the number of trades to be made n, and the cost associated with one trade C that the following inequality holds*

$$\frac{O_f(w_r, r, n)}{n} \geqslant \mathrm{C}$$

Corollary 1 *Trading strategies for different instruments can be ranked by their optimality ranking factor*

$$\mathfrak{R}_f(w_r, r, n, \mathrm{C}) = \frac{O_f(w_r, r, n)}{\mathrm{C}n}$$
$$\text{with the minimal requirement that } \mathfrak{R}_f(w_r, r, n, \mathrm{C}) \geqslant 1$$

When trading, we are aiming to navigate price action "swings" in order to make profits. Let's derive a condition for our trades to be profitable, taking the transaction costs into account:

tp_swing : *price action swing that triggers a take profit*
sl_swing : *price action swing that triggers a stop loss*
$P(swing)$: *probability of the price action swing to happen*
$E(swing)$: *expected value of the price action swing*
$V(swing)$: *valuation function of a given price action swing, expressed in a given currency*
$\mathcal{P}_{net}$: *the net profit (profit after transaction costs)*
$\mathcal{P}_{gross}$: *the gross profit (profit before transaction costs)*
C : *transaction costs*

$$\mathcal{P}_{gross} = \mathcal{P}_{net} + \mathrm{C}$$

generally, using the definition of expected value [reference]:

$$E(swing) = P(swing)V(swing) \tag{11}$$

thus, within our established take profit / stop loss optimal trading framework,

$$\mathcal{P}_{gross} = E(tp\_swing) - E(sl\_swing) = \mathcal{P}_{net} + \mathrm{C} \tag{12}$$

$$P(tp\_swing)V(tp\_swing) - P(sl\_swing)V(sl\_swing) = \mathcal{P}_{net} + \mathrm{C} \tag{13}$$

recall that the probability of the take profit swing to happen is nothing but the win ratio w_r, and the probability of the stop loss swing to happen is the loss raio $w_l = 1 - w_r$
Also, $V(tp\_swing) = pb$ and $V(sl\_swing) = lb$, thus (13) becomes:

$$\begin{aligned}
w_r pb - w_l lb &= \mathcal{P}_{net} + \mathrm{C} \\
b(w_r p - w_l l) &= \mathcal{P}_{net} + \mathrm{C} \qquad (14) \\
b[w_r p - (1 - w_r)l] &= \mathcal{P}_{net} + \mathrm{C} \\
b[w_r(p + l) - l] &= \mathcal{P}_{net} + \mathrm{C} \\
b[w_r(rl + l) - l] &= \mathcal{P}_{net} + \mathrm{C}
\end{aligned}$$

$$\boldsymbol{bl[w_r(r+1) - 1] = P_{net} + C} \tag{15}$$

Recall that $\boldsymbol{l = \frac{rw_r - (1 - w_r)}{r}}$, therefore (15) becomes:

$$\begin{aligned}
b\frac{rw_r - (1 - w_r)}{r}[w_r(r + 1) - 1] &= \mathcal{P}_{net} + \mathrm{C} \\
b\frac{(rw_r + w_r - 1)^2}{r} &= \mathcal{P}_{net} + \mathrm{C} \qquad (16)
\end{aligned}$$

For finding out the condition for profitability, set $\mathcal{P}_{net} \geqslant 0$, thus:

$$\boldsymbol{b\frac{[w_r(r + 1) - 1]^2}{r} \geqslant C} \tag{17}$$

Let's denote

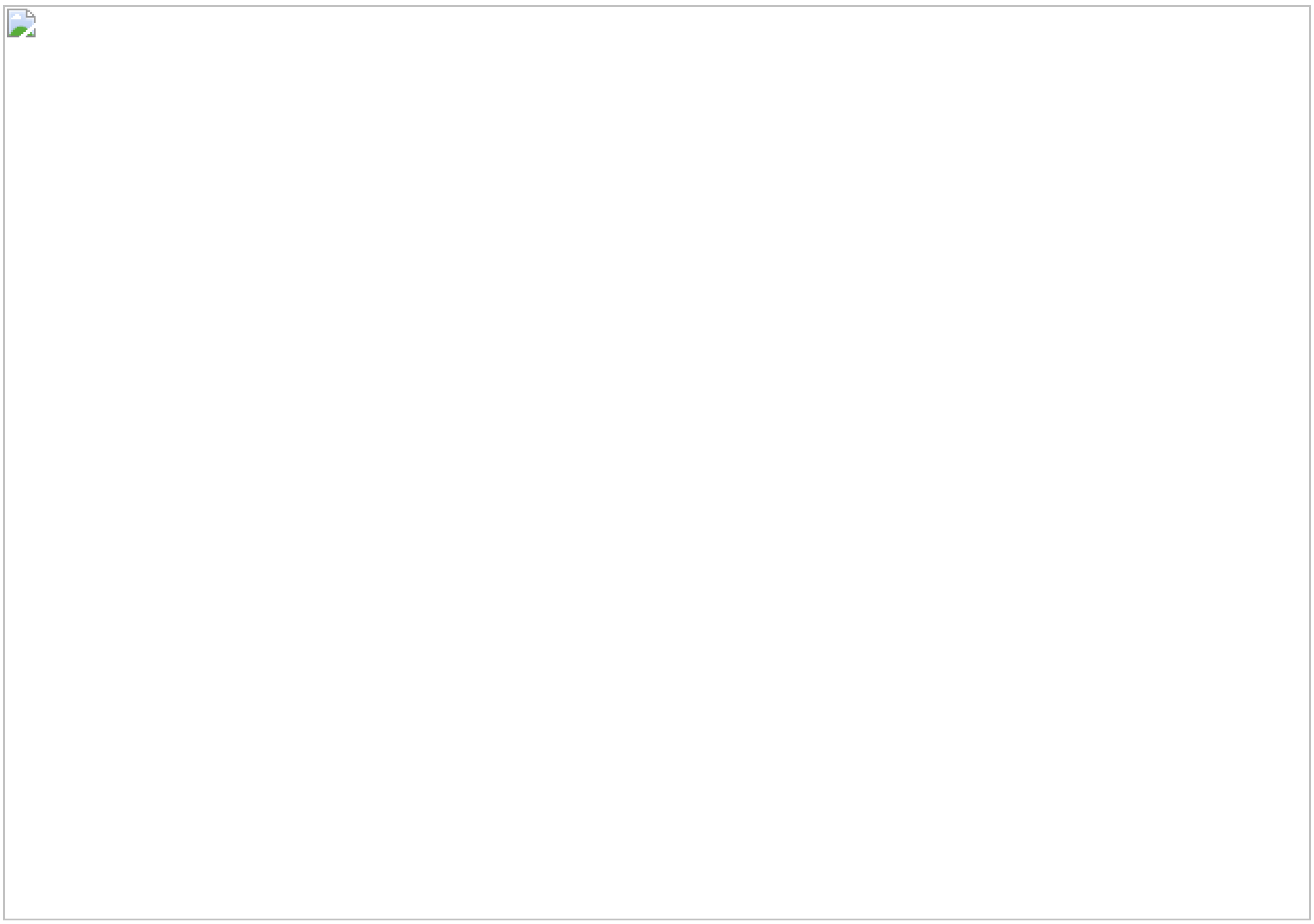
$$\mathcal{T}_{wr} = \frac{[w_r(r + 1) - 1]^2}{r} \tag{18}$$

$$b\mathcal{T}_{wr} \geqslant \mathrm{C} \tag{19}$$

$$\boldsymbol{b \geqslant \frac{\mathrm{C}}{\mathcal{T}_{wr}}} \tag{20}$$

The result can be represented as representing the trade-off between the win ratio and the reward to risk ratio, r, and how these two factors impact the profitability of the trades, considering transaction costs. It's a measure of the combined effect of the winning probability and the reward-to-risk on the total profitability.

- When the win ratio $w_r$is high, and the reward to risk ratio r is favorably high, the value of this expression is liikely to be high. This means that the trader can afford higher transaction costs
while maintaining profitability.
- When either the win ratio or the reward to risk ratio is low, the value of the expression is likely to be low, meaning that even low transaction costs can eat into the profitability.



Therefore, we can compare how different instruments and their transaction costs would perform under a given trading strategy

3.xx Finding out the real w_r and r

Until now, we have assumed that w_r and r are given. However, we need to find a way of approximating/predicting these values before trading.

Let's consider inspecting a set of $n_c$candlesticks, each defined by:

$i: \ the\ candlestick\ number\ in\ the\ sequence\ (1 \leqslant i \leqslant s$

$O_i : the\ open\ price\ of\ the\ i^{th} candlestick$

$H_i : the\ high\ price\ of\ the\ i^{th} candlestick$

$L_i : the\ low\ price\ of\ the\ i^{th} candlestick$

$C_i : the\ close\ price\ of\ the\ i^{th} candlestick$

We have to define the following metrics:

$D_{H_i} = |H_i - O_i|$

$D_{L_i} = |O_i - L_i|$

$A_H, A_L the\ averages\ for\ all\ D_{H_i}\ and\ D_{L_i}$

$M_H, M_L\ the\ median\ values\ for\ all\ D_{H_i}\ and\ D_{L_i}$

If $M_H > M_L$our strategy will be to buy, otherwise it will be identical but to sell.

Buy at the current open price. Take profit set to M_H. Stop loss set to C_i

The win ratio is simply $w_r = 0.5$ by the definiton of the median value.

The reward to risk ratio will be -

$$\overline{h_p} = \frac{1}{n}\sum_{i=1}^{n}\frac{p_h(i) - p_o(i)}{p_o}$$

$$\overline{l_p} = \frac{1}{n}\sum_{i=1}^{n}\frac{p_o(i) - p_l(i)}{p_o}$$

as well as their standard deviation

$$\sigma_{h_p} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{p_h(i) - p_o(i)}{p_o} - \overline{h_p}\right]^2$$

4. Spreading out

In real life, whenever one is trading CFDs, s/he will quickly notice that there is a difference between the **buy** and **sell** prices. That is, if an asset were to be bought at the price of 2.000 and sold immediately, the broker would set a lower selilng price eg. 1.997, with a settled difference, called spread. By keeping a difference between the buy and sell prices, the broker ensures that they will always win whenever the clients will make a transaction. [reference]

pip, Contract value, Leverage, Volume, Margin

The spread, or difference between Buy and Sell price, is typically measured in pip, or "price interest points". A pip can represent a fraction of the asset's price.

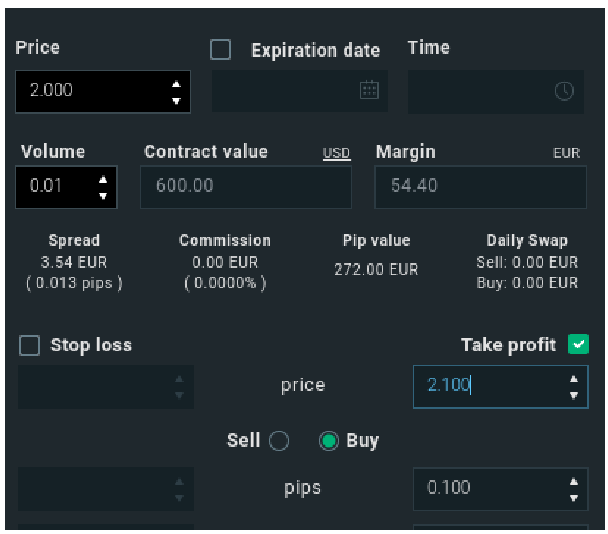


Figure 4: Typical trading order

A typical trading order, seen in Figure 3, includes the following elements:

- Initial price, the price at which the asset is traded
- Volume, the quantity of the asset
- Contract value, the cost of the total quantity of the asset
- Margin, a fraction of the contract value, which is to be withheld from the trader's balance, until the trade has finished.
- $Leverage = \frac{Margin}{Contract\ value}$. But more about leverage, in the later sections.

Typically, if one wanted to trade one unit of an asset priced at 2.000 $price\ units$, with a contract value of 600 *price units*, one would need to own that amount of money to begin with. However, due to the concept of leverage, only a fraction of the real cost of the asset will be needed by the trader in order to execute a transaction.

Let us denote:
$p_i : initial\ price$
$t_p : final\ price\ (take\ profit\ price)$
$v : volume$
$P : scaled\ pip\ value\ for\ v = 1\ (one\ unit) - if\ one\ pip = 10^{-4},\ P = pip\ value * 10^4$
$C : contract\ value\ =\ Pvp_i\ for\ one\ unit$
$s : spread,\ expressed\ in\ units\ (not\ in\ pips)$

Without spread, the total profit made during a transaction would be:

$$\boldsymbol{profit(t_p) = Ci = C\left(\frac{t_p}{p_i} - 1\right) = Pv(t_p - p_i)} \tag{21}$$

It is at this point when it becomes convenient to denote the "price increment" quantity
$i : price\ increment\ i = \dfrac{t_p}{p_i} - 1$
But when trading in an optimal way, the targeted profit is related to p as such:

$$bp = (t_p - p_i)Pv \tag{22}$$

$$\boldsymbol{t_p(v) = \frac{bp}{Pv} + p_i} \tag{23}$$

For example: We start with a balance of 10000, $p = 0.5$, $P = 300$, $p_i = 2.0$

$$t_p(v) = \frac{10000*0.5}{300v} + 2.0 = \frac{16.67}{v} + 2.0 = 3.67,\ for\ v = 10 \tag{E4}$$

Thus, because of the variable v that is in the trader's control, s/he can control the price swing increment needed for a given desired profit.

$$\boldsymbol{i(v) = |\frac{t_p}{p_i} - 1| = \frac{\frac{bp}{Pv} + p_i}{p_i} - 1 \Longrightarrow i(v) = \frac{bp}{Pvp_i}} \tag{24}$$

Where $i(v)$ denotes the price increment needed to reach the take profit value. Note that i could be kept constant by varying v, as long as b varies.

However,if the spread was $s = 0.013$, in E4 the trader would actually lose an additional Pvs:

Thus, the **real** profit that we would make in the market due to spreads is:

$$realprofit = profit - Pvs = Pv[(t_p - p_i) - s] \tag{25}$$

We can now express the quotient

$$q(i, p_i) = \frac{realprofit}{profit} = \frac{profit - Pvs}{profit} = 1 - \frac{s}{t_p - p_i} = 1 - \frac{s}{ip_i} \tag{26}$$

We can now plot the q quotient for $p_i = 2.0, P = 300, v = 1, s = 0.011$

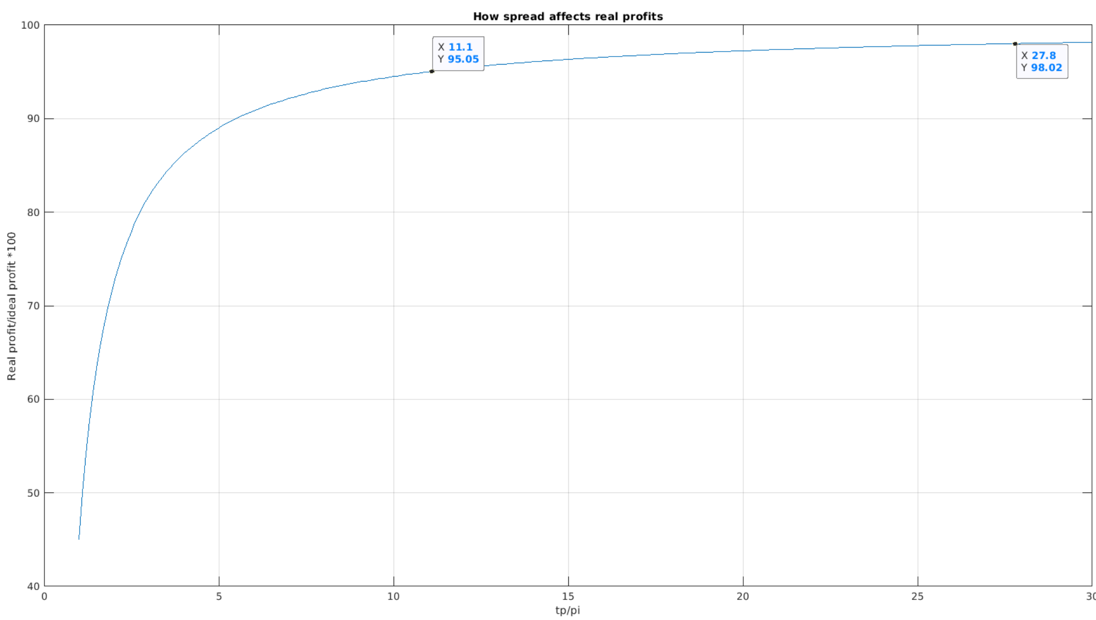


Figure 5: Plotting q(i,2.0)

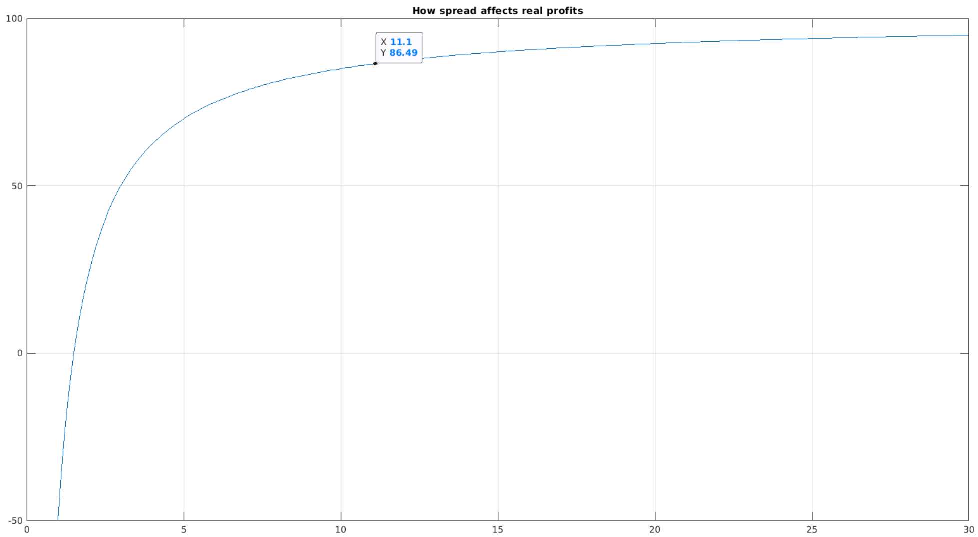


Figure 6: Plotting q(i,1.0)

By looking at Figure 4, one can note that the price would have to go 10% in the direction of the trade, in order for the trader to mark 95% instead of 100% of the theoretical profits. The spread affects the losses in the same way, thus a 100% theoretical loss is turned into a 105% loss.

Recall that $p = \dfrac{profit}{balance}, l = \dfrac{p}{r}$. But $realprofit \ = q{*}profit \rightarrow$

$$p_{real} = pq(i, p_i), l_{real} = lq\left(\frac{i}{r}, p_i\right) \tag{27}$$

If i could be kept fixed to 0.1 as the account balance varies, and we presume that the initial price does not vary much during a session of 10 trades from E3, let's assume $q(i) = q(0.1, 2.0) = 0.925$ for a win, and $q\left(\frac{i}{r}\right) = q(0.05, 2.0) = 0.85$ for a loss (taking a spread of 0.015)

$$10000\left[(1 + 0.5{*}0.925)\left(1 - 0.25{*}\frac{1}{0.85}\right)\right]^5 = 11725$$

However, if we keep $i = 0.6,\ q(0.2) = 0.985$ and $q(0.3) = 0.975$, we could instead get

16906, at the expense of having to predict price increments of 60%.

$$e = b(1 + p_{real})^{n_w}(1 - l_{real})^{n_l} \Longrightarrow \boldsymbol{e = b[1 + pq(i(v), p_i)]^{n_w}\left[1 - \frac{l}{q\left(\frac{i(v)}{r}, p_i\right)}\right]^{n_l}} \quad (28)$$

5. Margin calls

It is at this point of the article, when the definition of margin needs to be taken into account, if the aspiring trader wishes to keep his account for more than just a couple of misfortunate trades.

$$\boldsymbol{margin = LvPp_i}$$

$$m_l = \frac{equity}{margin} = \frac{equity}{leverage{*}volume{*}pip\ value{*}p_i} = \frac{balance + profit}{LPvp_i} = \frac{balance + Pvip_i}{LPvp_i} =$$

$$= \frac{\frac{balance}{Pv} + ip_i}{Lp_i} \rightarrow \boldsymbol{m_l = \frac{\frac{b}{Pv} + ip_i}{Lp_i}} - Constraint: Always\ trade\ at\ margin\ level > ml.$$

$L: leverage,\ a\ constant\ provided\ by\ the\ broker$
$P: pip\ value$
$v: volume$
$m_l: margin\ level$
$p_i: initial\ price$
$b: balance$

$$\boldsymbol{i = m_l L - \frac{b}{Pvp_i}} - price\ inc(dec)rement\ needed\ to\ get\ to\ a\ margin\ level.\ If\ we\ are\ to\ trade\ above\ m_l\ at$$

$$all\ costs,\ then\ i = m_l L - \frac{b}{Pvp_i} \leq -1\ (a\ price\ decrement\ of\ less\ than\ 100\%\ is\ not\ possible)$$

$b, m_l, L, P, p_i\ are\ given\ constants.$

$$\boldsymbol{m_l L + 1 \leq \frac{b}{Pvp_i}} \rightarrow \boldsymbol{v \leq \frac{b}{Pp_i[m_l L + 1]}}$$

$$\boldsymbol{v = \frac{b}{Pp_i[m_l L + 1]}} \quad (29)$$

$If\ we\ want\ to\ be\ above\ the\ margin\ level\ after\ suffering\ n\ losses,$

$$b(1 - l)^n \geq m_l LvPp_i$$

$$b(1-l)^n \geq m_l L P p_i \frac{b}{P p_i [m_l L + 1]}$$
$$(1-l)^n \geq \frac{m_l L}{m_l L + 1}$$
$$1 - l \geq \sqrt[n]{\frac{m_l L}{m_l L + 1}}$$

$$\boldsymbol{l \leq 1 - \sqrt[n]{\frac{m_l L}{m_l L + 1}}} = l_b \tag{30}$$

To summarize, after each trade, the optimal volume, take profit and stop loss have to be readjusted

6. Yes...probably

It was assumed until now that the trading strategy will give positive results with a w_r rate. Once a trade is entered, the framework for a Bernoulli is established: the result of the trade has two outcomes, either win or loss, with a win probability of w_r and a loss probability of $1 - w_r$.

If the trader was allowed to lose, for example, 10 times in a row, and then win 10 times in a row, if playing optimally, s/he would most surely get rich. However, even in the simplest case where $l = 0.25$, $(1-0.25)^{10} = 0.056$, meaning that if the trader started with a balance $b = 1000$, now his equity would be $e = 56.3$. (Picture that if the number of losses was 20, he would end up only with $e = 3.7$). The trading could not possibly continue in order to score 10 more wins, because, in order to trade an asset, the trader needs at least the $margin = leverage\ times\ contract\ value$

A natural question arises: what is the minimum number of consecutive losses needed for the trader's ruin?
Let's denote the equity at which the trader cannot use his account anymore with $e_t - terminal\ equity$

$$b(1-l)^k = e_t \rightarrow k = log_{1-l}\left(\frac{e_t}{b}\right) \tag{31}$$

Example: $b = 10000,\ l = 0.3, e_t = 100 \rightarrow k = log_{0.7}(0.01) = 12.9 \rightarrow k = 13$
$b = 10000,\ l = 0.4, e_t = 100 \rightarrow k = log_{0.6}(0.01) = 9 \rightarrow k = 9$

Therefore, (spread loss not taken into account)

$$base = 1 - \frac{rw + w - 1}{r}$$

Next: There is still a chance to touch the margin limit eventually. So, split balance into multiple "trading units". Eg: Balance is 5k: split into 5 1k trades that each follow optimal risk management.

NOTE: Find an algorithm with r, w as big as possible. Make sure that l stays between lower bound.s