a)

$$\frac{Pr[A_1(D)=O]}{Pr[A_1(D')=O]} \leq e^{\epsilon_1}$$

$$\frac{Pr[A_2(D)=O]}{Pr[A_2(D')=O]} \leq e^{\epsilon_2}$$

$$\frac{Pr[A_3(D)=O]}{Pr[A_3(D')=O]} \leq e^{\epsilon_3}$$

$\vdots$

$$\frac{Pr[A_n(D)=O]}{Pr[A_n(D')=O]} \leq e^{\epsilon_n}$$

$\Rightarrow$ so for each Algorithm A_i $\frac{Pr[A_i(D)=O]}{Pr[A_i(D')=O]} \leq e^{\epsilon_i}$ holds

by applying the definition of differential privacy to each A_i

$$\Longrightarrow \prod_{i=1}^{n} \frac{Pr[A_i(D)=O]}{Pr[A_i(D')=O]} \leq \prod_{i=1}^{n} e^{\epsilon_i}$$

$$= \prod_{i=1}^{n} \frac{Pr[A_i(D)=O]}{Pr[A_i(D')=O]} \leq e^{\sum_{i=1}^{n} \epsilon_i}$$

thus it satisfies $\left(\sum_{i=1}^{n} \epsilon_i\right)$-DP b)

```
def funct_of_part_b(dataset, count_query, epsilon)
        if count_query(dataset) ≥ e^epsilon
            return large
        else
            return small
```

$$\frac{Pr[A(D)=large]}{Pr[A(D')=large]} \leq e^{\epsilon} \Rightarrow \frac{Pr[count\_query(D) \geq e^{\epsilon}]}{Pr[count\_query(D') \geq e^{\epsilon}]} \leq e^{\epsilon} \Rightarrow \frac{\frac{1}{e^{\epsilon}}}{Pr[count\_query(D') \geq e^{\epsilon}]} \leq e^{\epsilon}$$

$\Rightarrow Pr[count\_query(D') \geq e^{\epsilon}] \geq \frac{1}{e^{2\epsilon}}$ so the length of neighbouring dataset D' must be bounded with $e^{2\epsilon}$

so if the length of D' between $[e^{\epsilon}, e^{2\epsilon}]$ it is DP for $A(D) = large$

$$\frac{Pr[A(D)=small]}{Pr[A(D')=small]} \leq e^{\epsilon} \Rightarrow \frac{Pr[count\_query(D) < e^{\epsilon}]}{Pr[count\_query(D) < e^{\epsilon}]} \leq e^{\epsilon} \Rightarrow \frac{\frac{1}{e^{\epsilon}}}{Pr[count\_query(D') \leq e^{\epsilon}]} < e^{\epsilon} \Rightarrow Pr[count\_query(D') < e^{\epsilon}] \geq \frac{1}{e^{2\epsilon}}$$

so if the length of D' between $[-e^{\epsilon}, e^{\epsilon}]$ it is DP for $A(D) = small$

c) We know that adding random noise drawn from a Laplace distribution with $\mu=0$ and scale $\geq \frac{S(q)}{\epsilon}$ satisfies ϵ-DP. In this case we are adding a Laplace distribution with mean $=0$ scale $=\epsilon$

$$\frac{Pr[A(D)=0]}{Pr[A(D')=0]} = \frac{Pr[q(D)+r=0]}{Pr[q(D')+r=0]} = \frac{Pr[r=0-q(D)]}{Pr[r=0-q(D')]} = \frac{\frac{1}{2\epsilon} * e^{\left(\frac{-|0-q(D)|}{\epsilon}\right)}}{\frac{1}{2\epsilon} * e^{\left(\frac{-|0-q(D')|}{\epsilon}\right)}}$$

$$= e^{\left(\frac{-|0-q(D)|+|0-q(D')|}{\epsilon}\right)}$$ *Apply Reverse Triangle Inequality then $e^{\left(\frac{|0-q(D')-0+q(D)|}{\epsilon}\right)}$

it becomes

$$e^{\frac{S(q)}{\epsilon}} \xrightarrow[SD]{} e^{\frac{S(q)}{\epsilon}} \leq e^{\epsilon}$$ if $\frac{S(q)}{\epsilon} \leq \epsilon$ if $S(q) \leq \epsilon^2$ then DP holds