$$(1-\epsilon)\left\|x_i-x_j\right\|_2^2 \leq \left\|Jx_i-Jx_j\right\|_2^2 \leq (1+\epsilon)\left\|x_i-x_j\right\|_2^2 \quad \forall\, i,j \in [n]$$

The given parameters by the question are not true for some cases since when k is bigger than m, the dimension of the realizations is not reduced, they increase. So that, I tried to test the Lemma by my own parameters and the results are shown below. For the test I used
m = (10,100,1000),
k = (5,10,50,100) but only when k is smaller than m,
ε = (0.01,0.1)
n = 10 (for hardware easiness).

The used parameters for this run are as follows:
n: 10
m: 10
k: 5
ε: 0.01
Mean of 10 random runs: 0.010753968253968255

The used parameters for this run are as follows:
n: 10
m: 10
k: 5
ε: 0.1
Mean of 10 random runs: 0.24896913580246913

The used parameters for this run are as follows:
n: 10
m: 100
k: 5
ε: 0.01
Mean of 10 random runs: 0.03909347442680776

The used parameters for this run are as follows:
n: 10
m: 100
k: 5
ε: 0.1
Mean of 10 random runs: 0.2143809523809524

The used parameters for this run are as follows:
n: 10
m: 100
k: 10

ε: 0.01
Mean of 10 random runs: 0.02840652557319224

The used parameters for this run are as follows:
n: 10
m: 100
k: 10
ε: 0.1
Mean of 10 random runs: 0.3099841269841269

The used parameters for this run are as follows:
n: 10
m: 100
k: 50
ε: 0.01
Mean of 10 random runs: 0.08918871252204585

The used parameters for this run are as follows:
n: 10
m: 100
k: 50
ε: 0.1
Mean of 10 random runs: 0.6414426807760142

The used parameters for this run are as follows:
n: 10
m: 1000
k: 5
ε: 0.01
Mean of 10 random runs: 0.012087301587301588

The used parameters for this run are as follows:
n: 10
m: 1000
k: 5
ε: 0.1
Mean of 10 random runs: 0.23038712522045857

The used parameters for this run are as follows:
n: 10
m: 1000

k: 10
ε: 0.01
Mean of 10 random runs: 0.04327248677248677

The used parameters for this run are as follows:
n: 10
m: 1000
k: 10
ε: 0.1
Mean of 10 random runs: 0.40536155202821866

The used parameters for this run are as follows:
n: 10
m: 1000
k: 50
ε: 0.01
Mean of 10 random runs: 0.08802116402116403

The used parameters for this run are as follows:
n: 10
m: 1000
k: 50
ε: 0.1
Mean of 10 random runs: 0.6565414462081128

The used parameters for this run are as follows:
n: 10
m: 1000
k: 100
ε: 0.01
Mean of 10 random runs: 0.12161816578483246

The used parameters for this run are as follows:
n: 10
m: 1000
k: 100
ε: 0.1
Mean of 10 random runs: 0.8434435626102292

The above results are copied from python console. As we can see from the results the biggest significance is with the choice of ε since when the test is run by ε = 0.1 instead of

0.01, the results are increased from around %10 to %70 approximately. Also, increasing m and k increased the accuracy but not too much like the effect of ε. It can be said that the Lemma is also effective with small dimensions but the choice of ε is critical.