Circuits and Systems Lab Manual

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Electrical Engineering 2CJ4

2021

McMaster University

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Circuits and Systems

AD2 Experiments Manual for

Elec Eng 2CJ4

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2021

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Finally, I would to thank my wife and my beautiful children for their support during the preparation of this manual.

Dr. Mohamed Bakr

Hamilton, September 2020

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 1)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment

- Analog Discovery 2

Components

- 1 x 22kΩ resistor
- 1 x 100kΩ resistor
- 2 x 10kΩ resistor
- 2 x 47kΩ resistor
- Op-Amp: LM358P
  LM358 Dual operational amplifier (see Figure 1) consists of two independent and high-gain frequency-compensated operational amplifiers designed to operate from a single supply or split supply over a wide range of voltages.

Objective

- To become familiar with the basic properties of the operational amplifier circuits

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Background:

The circuit symbol of Op-Amp is shown in Figure 2. There are two input terminals, non-inverting (labelled as V_1 in Figure 2) and inverting (labelled as V_2). The corresponding input voltages are denoted by V_p and V_n, respectively. The output voltage V_o is equal to the difference of the two input voltages multiplied by the gain A_v (which is a big number)

$$V_0 = A \times (V_p - V_n)$$

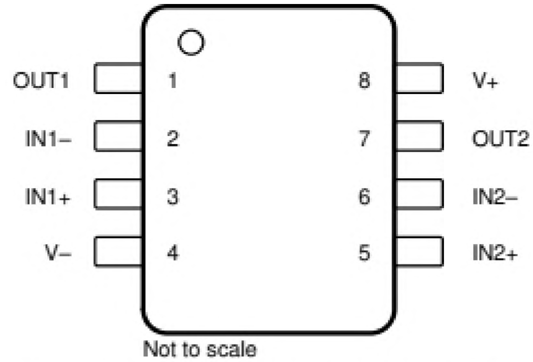

Figure 1: LM358P Dual Operational Amplifier

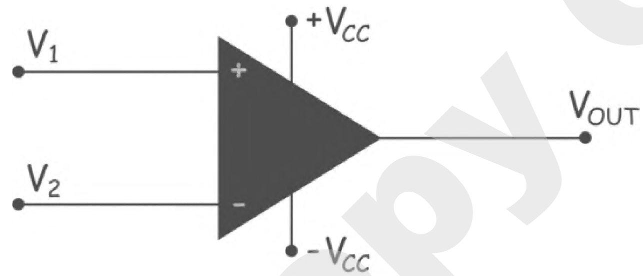

Figure 2: Circuit Symbol of Op-Amp

The maximum and minimum output voltages of Op-Amp are limited by the power supply sources ($+V_{cc}$ and $-V_{cc}$). This is true for both open-loop (see Figure 3), and, as you will find out in the lab, closed-loop configurations.

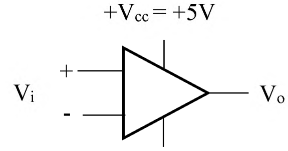

Figure 3: An open-loop configuration

Note that the input voltage range (for the linear active region) is given by

$$\frac{-V_{cc}}{A_v} < V_p - V_n < \frac{+V_{cc}}{A_v}$$

As a result of the large amplification A_v (say $A_v = 100000$), the input voltage difference ($V_p - V_n$) must be small (in order to operate in the linear active region)

Example:

i. Analyze and plot the relation between the output voltage V_o and the input voltage difference $V_p - V_n$, in which you should mark the linear active region and the saturation region.

Solution:

Assume that we have two power supplies of -15 volts and +15 volts (see Figure 4). The linear region is the part that is in orange. It is clear that the output voltage is linearly dependent on the input voltage for this region, hence the name linear active region. Once the input voltage achieves a certain Maximum or Minimum value, the output voltage becomes constant and saturates.

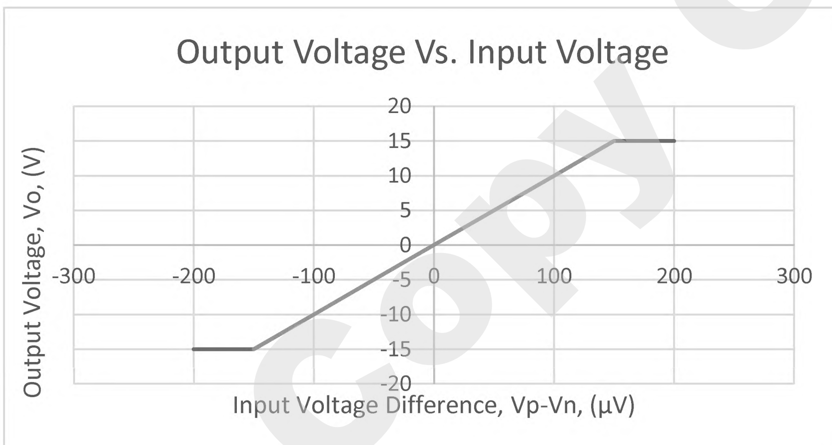

Figure 4: Output Voltage-Input Voltage graph

ii. In this experiment you will design closed-loop Op-Amp circuits to verify the linearity and non-linearity of the operational amplifier. Why not use the open-loop circuit directly? (Figure 3)

Solution:

The reason the open-loop circuit is not used is because given that we have a high gain, we want to amplify the signal within any desired value between the maximum and minimum power supplies. A feedback circuit is needed for this purpose

iii. Given the circuit in Figure 5, with R_1 = 22kΩ, R_2 = 100kΩ, $V_{cc} = 5\ V$, and $-V_{cc} = -5\ V$, express the gain $A = \frac{V_o}{V_i}$ (where V_o is the output signal and V_i is the input signal) as a function of R_1 and R_2. Determine the linear active region and saturation region.

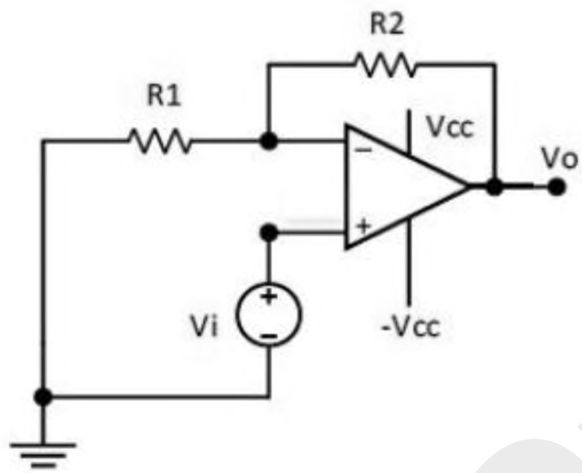

Figure 3

Solution:

There are two important facts which are useful for analyzing the Op-Amp circuits.

a. The input current is zero.

b. The voltage difference between the inverting and non-inverting inputs is zero due to the high gain (when the amplifier is used with negative feedback)

Looking at the circuit, $V_p = V_i$ and therefore $V_n = V_p = V_i$. Carrying out nodal analysis at node V_n gives us:

$$\frac{V_i}{R_1} = \frac{V_o - V_i}{R_2}$$

Multiplying by $R_2$and $\frac{1}{V_i}$ on both sides results in:

$$\frac{R_2}{R_1} = \frac{V_o - V_i}{V_i}$$

Therefore the gain of this circuit, $\frac{V_0}{V_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{100}{22} = \frac{61}{11} \approx 5.5454$. This is an example of a non-inverting Op-Amp circuit.

The input voltage range (for the linear active region) is given by:

$$\frac{-V_{cc}}{A_v} < V_p - V_n < \frac{+V_{cc}}{A_v}$$

Plugging in the values:

$$\frac{-5}{\frac{61}{11}} < V_i < \frac{+5}{\frac{61}{11}}$$

Therefore, $V_o = \frac{61}{11}V_i$ for this input voltage range:

$$\frac{-55}{61} < V_i < \frac{+55}{61}$$

If $V_i < \frac{-55}{61}$ then $V_o = -5\ V$ and if $V_i > \frac{+55}{61}$ then $V_o = +5\ V$ (saturation region).

iv. Build the circuit in Figure 5, with the values given from part iii and with V_i being a 1 kHz square wave with an amplitude of 500 mV and offset of 0V. Plot V_i and V_o using the oscilloscope tool on the Analog Discovery 2.

Solution:

Start by connecting the lead **1+** to the node V_i and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage waveform will be plotted on channel 2. Set the time base to 500 μs/div, and the range on channel 1 and 2 to 1 V/div. The resulting waveform is shown below:

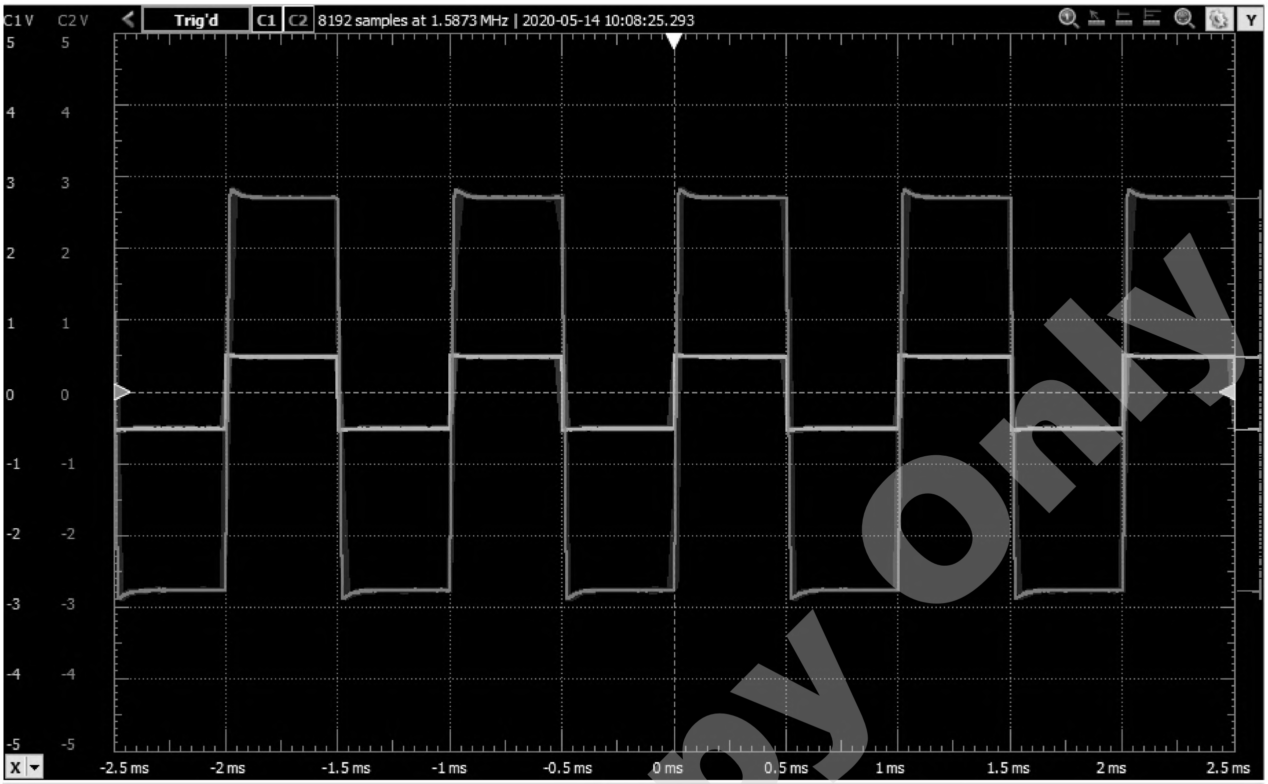

Figure 4

v. Using the circuit from part iv, estimate the gain using the Analog Discovery 2.

Solution:

Since the amplitude of the input signal is 500 mV, we are in the linear active region. The theoretical gain was calculated to be 5.5454. Measure the peak to peak voltage of the input and output signals using the "Measurements" tool. The peak to peak voltages were found to be 5.6188 V for the output signal and 1.0580 V for the input signal.

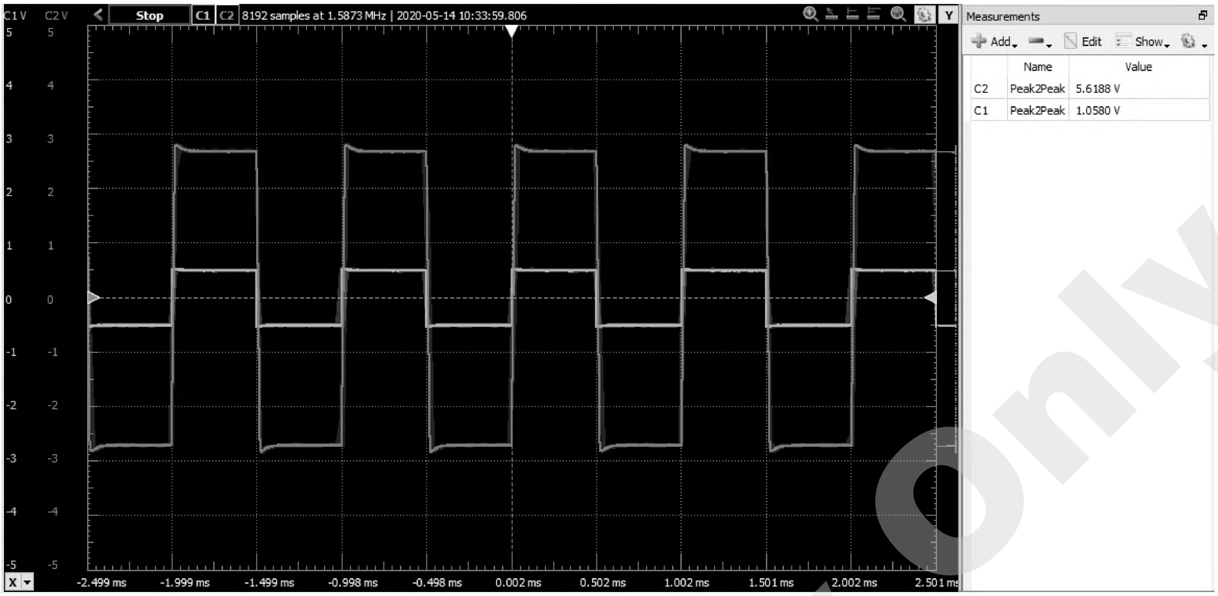

Figure 5

Dividing these two numbers gives us the actual gain 5.3108, which is close to the theoretical.

Note: If we were to change the input signal to have an amplitude of 2 or 5 volts and make it 100% symmetric (so that it becomes a straight line), the output signal will not increase and will be at a value close to 4.5 volts (which is close to the maximum power supply of 5 volts), therefore the Op-Amp will be in saturation.

Experiment:

Given the circuit in Figure 8 with $R_1 = 10\text{k}\Omega$, $R_2 = 47\text{k}\Omega$, $+V_{cc} = 5\ V$, and $-V_{cc} = -5\ V$, express the gain $A = \frac{V_o}{V_i}$ as a function of R_1 and R_2 and determine the linear active region and saturation region.

i. Build the circuit with the values given and with V_i being a 1 kHz square wave with amplitudes of 200 mV, 2 V, and 5 V and an offset of 0V, where you should only observe the peak to peak magnitude. Plot V_i and V_o using the oscilloscope tool on the Analog Discovery 2 in the linear active region and saturation region.

ii. Using the circuit from part i, estimate the gain using the Analog Discovery 2. Compare your analytical results with your experimental measures.

iii. Repeat parts i-ii for the following values: $+V_{cc} = 2.5\ V$, $-V_{cc} = -2.5\ V$. Does the gain change? Explain.

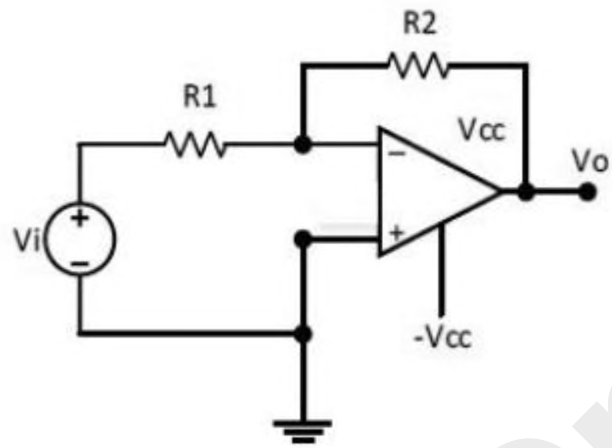

Figure 6

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Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 2)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment

- Analog Discovery 2

Components

- 2 x 10kΩ resistor
- 1 x 25kΩ resistor
- 1 x 22kΩ resistor
- 2 x 4.7kΩ resistor
- Op-Amp LM358P

Objective

- The objective of this lab is to help you investigate the characteristics of the Schmitt trigger circuit by using the operational amplifier (Op-Amp). You will also learn the input-output transition function of the Schmitt trigger.

Background:

The Schmitt trigger uses positive feedback to add hysteresis to the input-output transition threshold. It was invented by Otto Schmitt in 1934.

The main characteristics of the Schmitt trigger are summarized as follows:

- Consider the circuit as illustrated in Figure 1. Given the DC input $v_{in}(t)$, v_{out} would be either positively saturated or negatively saturated in practice (Can you explain why?). Denote the positive saturation output voltage and the negative saturation output voltage as V_{sat+} and V_{sat-} $(V_{sat+} > V_{sat-})$, respectively. If the power supply of Op-Amp is $\pm V_{cc}$, e.g., $\pm 5V$ what is the range of V_{sat+} and V_{sat-}? Please observe it carefully in the following experiment. Given the DC input, V_{th} (threshold voltage) also takes two possible values, which are denoted by V_{th_1} and V_{th_2} $\left(V_{th_1} > V_{th_2}\right)$.

- If $v_{in}(t) < \min(V_{th_1}, V_{th_2}) = V_{th_2}$, we can find that $v_{out} = V_{sat+}$ and hence using voltage division, $V_{th_1} = \frac{R_2}{R_1+R_2} V_{sat+}$. If $v_{in}(t) > \max(V_{th_1}, V_{th_2}) = V_{th_1}$, we can find that $v_{out} = V_{sat-}$ and hence $V_{th_2} = \frac{R_2}{R_1+R_2} V_{sat-}$.

- Now suppose that $V_{min} = min_t v_{in}(t) < V_{th_2}$ and $V_{max} = max_t v_{in}(t) > V_{th_1}$. Increase $v_{in}(t)$ gradually from V_{min}, which is the region that $v_{in}(t) < V_{th_2}$, to V_{max}, which is the region that $v_{in}(t) > V_{th_1}$.
  - When $v_{in}(t) < V_{th_2}$, we have $v_{out} = V_{sat+}$ and $V_{th} = V_{th_1}$. Now increase $v_{in}(t)$ such that $V_{th_2} < v_{in}(t) < V_{th_1}$; the output remains the same. If we further increase $v_{in}(t)$ such that $v_{in}(t) > V_{th_1}$, the output transition occurs and we have $v_{out} = V_{sat-}$ and $V_{th} = V_{th_2}$.
  - Likewise, when $v_{in}(t) > V_{th_1}$, we have $v_{out} = V_{sat-}$ and $V_{th} = V_{th_2}$. Now decrease $v_{in}(t)$ such that $V_{th_2} < v_{in}(t) < V_{th_1}$; again the output remains the same. If we further decrease $v_{in}(t)$ such that $v_{in}(t) < V_{th_2}$, the output transition occurs and we have $v_{out} = V_{sat+}$ and $V_{th} = V_{th_1}$.
  - The *hysteresis gap* is denoted by $V_{gap} = V_{th_1} - V_{th_2}$. Consider the case where $v_{in}(t)$ is corrupted by noise. For $v_{in}(t) < V_{th_2}$ or $v_{in}(t) > V_{th_1}$, if V_{gap} > *noise peak-peak amplitude*, then the output is unaffected. This characteristic is called noise immunity, which is very useful in practice, especially when you want to measure the frequency of a waveform or counting the number of pulses in a noisy environment. For the above circuit, we have $V_{gap} = V_{th_1} - V_{th_2} = \frac{R_2}{R_1+R_2}(V_{sat+} - V_{sat-}) \approx \frac{R_2}{R_1+R_2} V_{pp}$, where $V_{pp} = 2V_{cc}$ is the range of the DC power supply.

- We define the center of the threshold as $\overline{V}_{th} = \frac{1}{2}(V_{th_1} + V_{th_2})$. Now consider the circuit in Figure 2. In this experiment, you will prove that the hysteresis gap V_{gap} remains the same if we change the reference voltage, V_{ref}, from zero to some non-zero values such that $V_{sat-} < V_{ref} < V_{sat+}$.

- For an Op-Amp which works in the positive feedback mode, when the transition of the output occurs, the change of the output will be fed into the positive input pin which boosts the speed of the transition even if the input signal changes relatively slowly. This phenomenon is commonly referred to as *regenerative feedback.*

Example:

vi. Given the shown circuit, determine V_{gap}, V_{th_1}, and V_{th_2} for a DC power supply of $\pm 5V$.

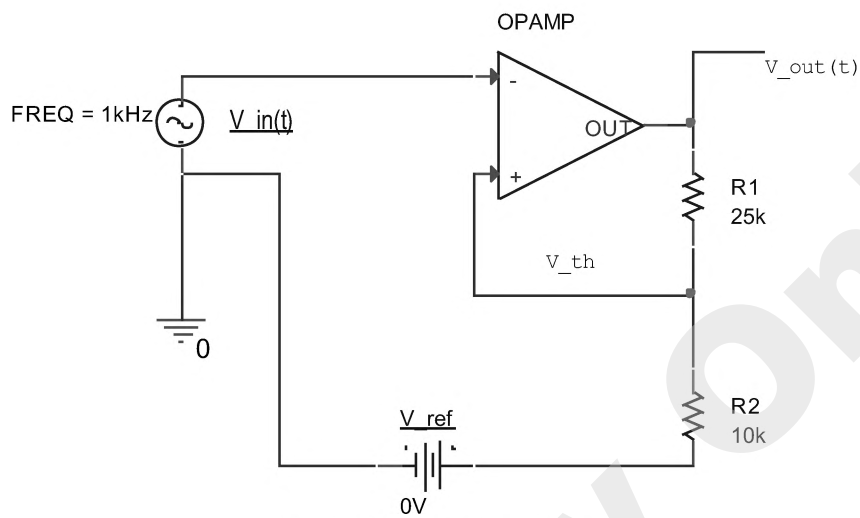

Figure 1: The Schmitt trigger

Solution:

We know from the background section that $V_{th_1} = \frac{R_2}{R_1+R_2} V_{sat+} = \frac{10k\Omega}{25k\Omega+10k\Omega} \times 5 = \frac{10}{7} V \approx 1.42857\ V$. Morevoer, $V_{th_2} = \frac{R_2}{R_1+R_2} V_{sat-} = \frac{10k\Omega}{25k\Omega+10k\Omega} \times -5V = -\frac{10}{7} V \approx -1.42857\ V$. Therefore, $V_{gap} = V_{th_1} - V_{th_2} = \frac{10}{7} V + \frac{10}{7} V = \frac{20}{7} V \approx 2.8571\ V$.

vii. Repeat the same calculations given $R_1 = 10k\Omega$ and all other values fixed.

Solution:

A similar calculation to part i will be done and therefore $V_{th_1} = \frac{10k\Omega}{10k\Omega+10k\Omega} \times 5V = 2.5V$, V_{th_2}would be -2.5V, and so then $V_{gap} = 5V$.

viii. Build the circuit in Figure 2 with $v_{in}(t)$ being a 1kHz sine wave with amplitude of 3V and an offset of 0V. Calculate the actual V_{th_1}, V_{th_2} and V_{gap} using the Analog Discovery 2 and the component values from part i.

Solution:

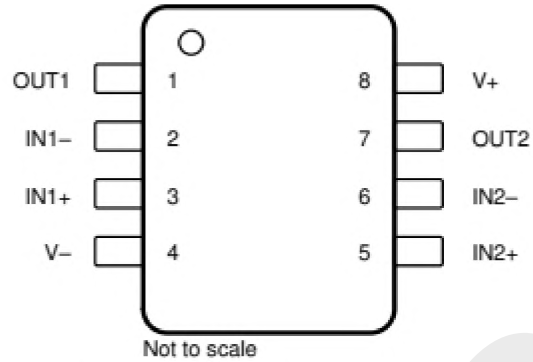

Figure 2: LM358P Datasheet

Following the datasheet of the Op-Amp, connect the lead **1+** to the node $v_{in}(t)$, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node v_{out}, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 100 µs/div and position to -1.5 ms, and the range on channels 1 and 2 to 1 V/div. The resulting waveform is shown below:

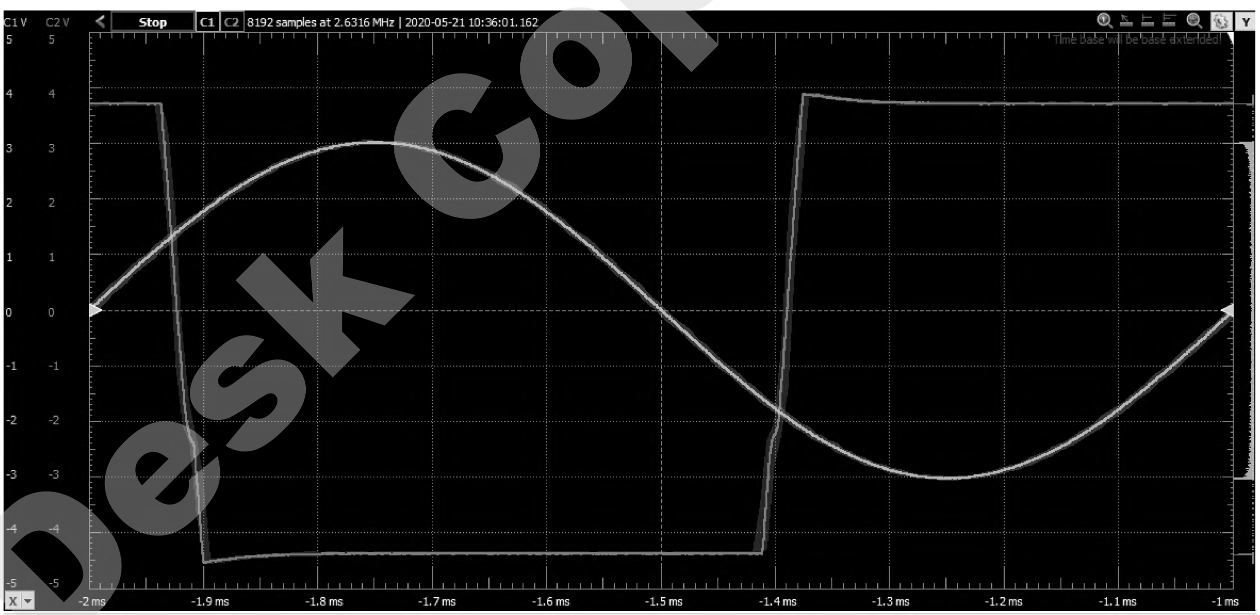

Figure 3: Resulting Waveform

To get V_{th_1} it is the transition from the output voltage graph to go from V_{sat+} to V_{sat-} and where channel 1 signal intersects channel 2 signal. We can use the pulse quick measure to get approximately 1.251 V. Furthermore, V_{th_2} is the transition from the output voltage graph to go from V_{sat-} to V_{sat+} and where channel 1 signal intersects channel 2 signal. It was calculated to be around -1.5892 V. Therefore, V_{gap} = 1.251 V + 1.5892 V = 2.8402 V. These values are close to the theoretical ones.

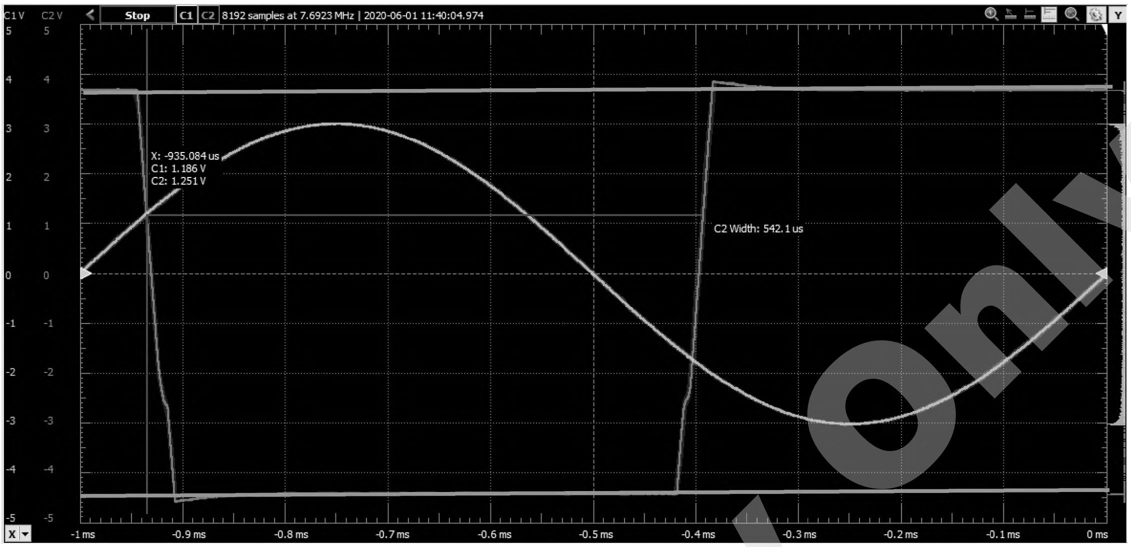

Figure 4: Calculating the Positive Threshold Voltage The top orange line is $V_{sat+}$and the bottom orange line is V_{sat-}.

Figure 5: Calculating the Negative Threshold Voltage. The top orange line is $V_{sat+}$and the bottom orange line is V_{sat-}.

Build the circuit from part ii and with the same properties of the input voltage source.

Solution:
Same as part iii but changing R_1 to be 10kΩ, V_{th_1} was found to be around 2.07 V and V_{th_2} was found to be around -2.4482 V. V_{gap} is calculated to be 4.5182 V. These values are close to the theoretical results.

Experiment:

i. From the background section, explain why when we increase or decrease $v_{in}(t)$ such that $V_{th_2} < v_{in}(t) < V_{th_1}$ the output remains the same.

ii. Given the circuit from Figure 2 in the example section, fill in the following table using $V_{ref} = 0V, 2V, R_1 = 4.7k\Omega, 22k\Omega$ and $R_2 = 4.7k\Omega$ (assuming V_{sat+} = 5V and V_{sat-} = -5V). Include one sample calculation for any row.

(V_{ref}, R_1, R_2)	V_{th_1} *(theoretical)*	V_{th_2} *(theoretical)*	V_{gap} *(theoretical)*
($0V$, 4.7$k\Omega$, 4.7$k\Omega$)			
($0V$, 22$k\Omega$, 4.7$k\Omega$)			
($2V$, 4.7$k\Omega$, 4.7$k\Omega$)			
($2V$, 22$k\Omega$, 4.7$k\Omega$)			

iii. Measure the actual V_{th_1}, V_{th_2}, and V_{gap} by building the circuits with $v_i(t)$ being a sine wave, square wave, or a traingular wave of amplitude 5V with a 0V offset and filling in the values in the following table. Include the resulting waveforms as well as circuits. (Hint: you will need to analyze the circuit if V_{ref} is a value that is not zero)

(V_{ref}, R_1, R_2)	V_{th_1} *(measured)*	V_{th_2} *(measured)*	V_{gap} *(measured)*
($0V$, 4.7$k\Omega$, 4.7$k\Omega$)			
($0V$, 22$k\Omega$, 4.7$k\Omega$)			
($2V$, 4.7$k\Omega$, 4.7$k\Omega$)			
($2V$, 22$k\Omega$, 4.7$k\Omega$)			

iv. What is the percentage difference between the calculated and measured voltages?

v. What do you notice about the hysteresis gap V_{gap} if we change V_{ref} from zero to some non-zero value?

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Term II, January – April 2021

Laboratory Experiments (Set 3)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2

Components
- 2 x 1kΩ resistor
- 2 x 10kΩ resistor
- 2 x 100kΩ resistor
- 1 x 2.2MΩ resistor
- 1 x 1nF (102) capacitor
- 1 x 100nF (104) capacitor
- Op-Amp LM358P

Objective
- To learn how to construct differentiator and integrator circuits using Op-Amps.

Background:

A simple differentiator based on the RC Op-Amp circuit is shown in Figure 1. The ideal input-output relationship for this differentiator is given by

$$v_o(t) = -RC\frac{dv_i(t)}{dt}$$

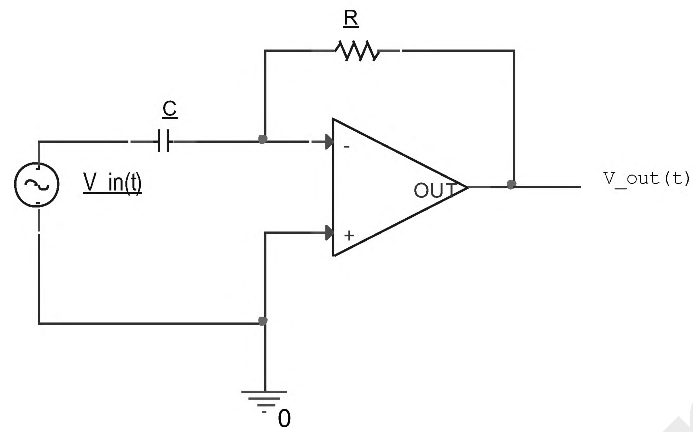


Figure 7: A simple differentiator based on the RC Op-Amp circuit

A simple integrator based on the RC Op-Amp circuit is shown in Figure 2. The ideal input-output relationship for this differentiator is given by

$$v_o(t) = -\frac{1}{RC}\int_0^t v_i(x)dx + v_o(0)$$

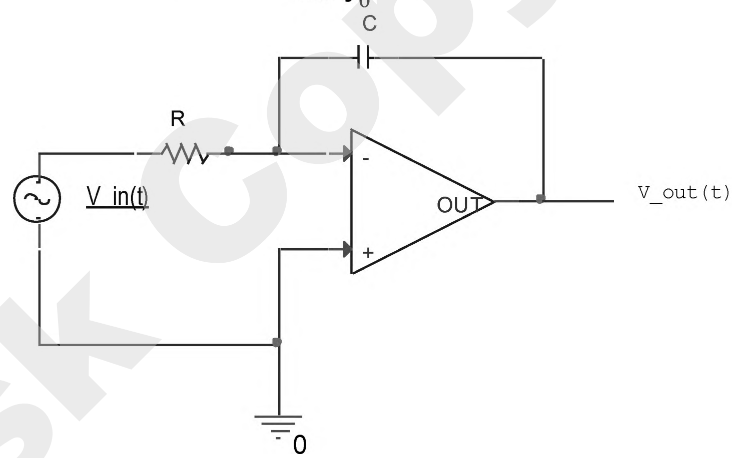


Figure 8: A simple integrator based on the RC Op-Amp circuit

Where v_o is the output voltage. In practice, the output of the differentiator in Figure 1 is quite sensitive to the noise perturbation, especially in the high frequency regime (why?). For this reason, we incorporate a series resistor at the input as shown in Figure 3 (what is the role of the resistor R_2?).

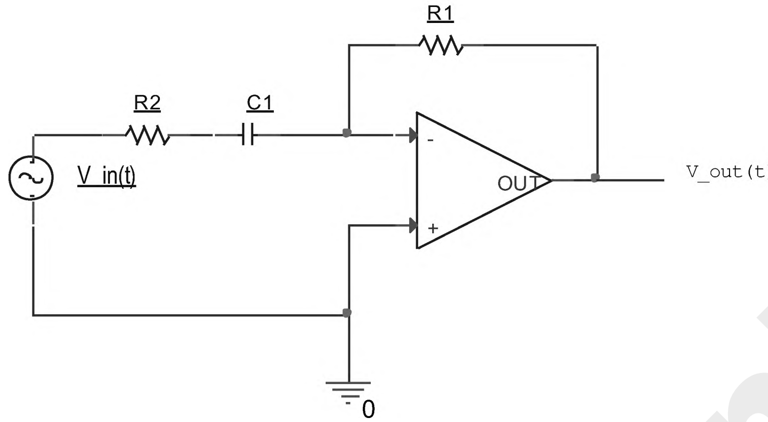


Figure 9: A practical differentiator

Similarly, instead of using the integrator in Figure 2, we use the one shown in Figure 4, where a resistor parallel to the feedback capacitor is added. The value of R_4 is chosen to be large enough so that the bias current going through this branch is negligible.

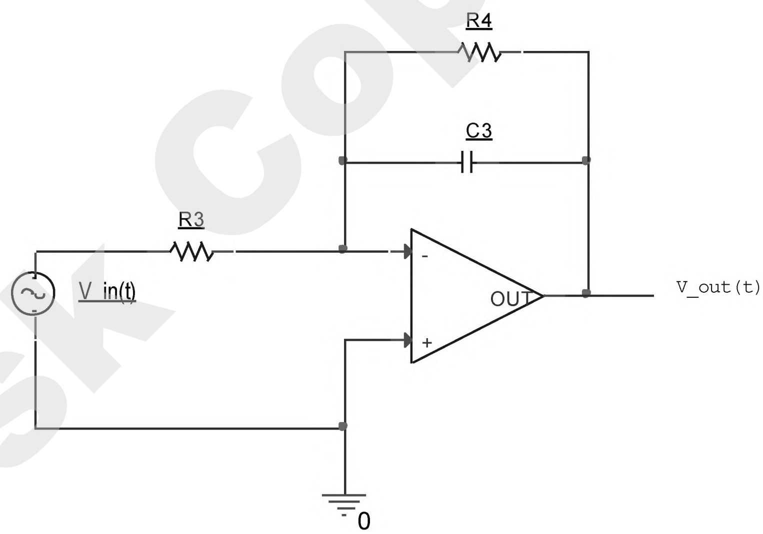


Figure 10: A practical integrator

Example:

ix. Given the circuit in Figure 3, assume $R_1 = 10k\Omega$, $R_2 = 1k\Omega$, $C_1 = 1nF$, $V_{cc}^{+} = +5V$, and $V_{cc}^{-} = -5V$. Consider two types of inputs: 1) the triangle wave, 2) the sine wave (both with frequency of 4KHz and peak-to-peak amplitude of 1V). Determine the output voltage.

Solution:

We know from the background section that the output voltage of a differentiator circuit is $v_o(t) = -RC\frac{dv_i(t)}{dt}$, where R is considered to be R_1, C is C_1, and $\frac{dv_i(t)}{dt}$ is the slope for each line in the triangular wave . When the input voltage is a triangular wave, the period is $\frac{1}{4kHz} = 0.25\ ms$, as seen in Figure 5. Therefore, if we plug in the known values we get:

$$v_o(t) = -(10k\Omega)(1nF)\left(\frac{0.5}{0.0625\ ms}\right) = -0.08\ V;\ 0 < t < 0.0625\ ms\ \&\ 0.1875 < t < 0.25\ ms$$

This means that $v_0(t) = 0.08V\ for\ 0.0625\ ms < t < 0.1875\ ms$. The ouput graph then will be a square wave osciallating between 0.08 V and -0.08 V.

Using the same logic, if the input were a sine wave then the output voltage would be:

$$v_o(t) = -(10k\Omega)(1nF)\left(\frac{d\,(0.5\sin 2\pi(4000)t)}{dt}\right) = -\frac{\pi}{25}\cos 8000\pi t$$

x. Build the circuit in Figure 3 and determine the output voltages using the analog discovery 2. Do the results match the theoretical results?

Solution:

Assuming the proper circuit connections have been made, connect the lead **1+** to the node V_i, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the inductor voltage will be plotted on channel 2. Set the time base to 100 µs/div, and the range on channel 1 and 2 to 500 mV/div. Set the wave generator to be a triangular wave and the resulting waveform is shown below:

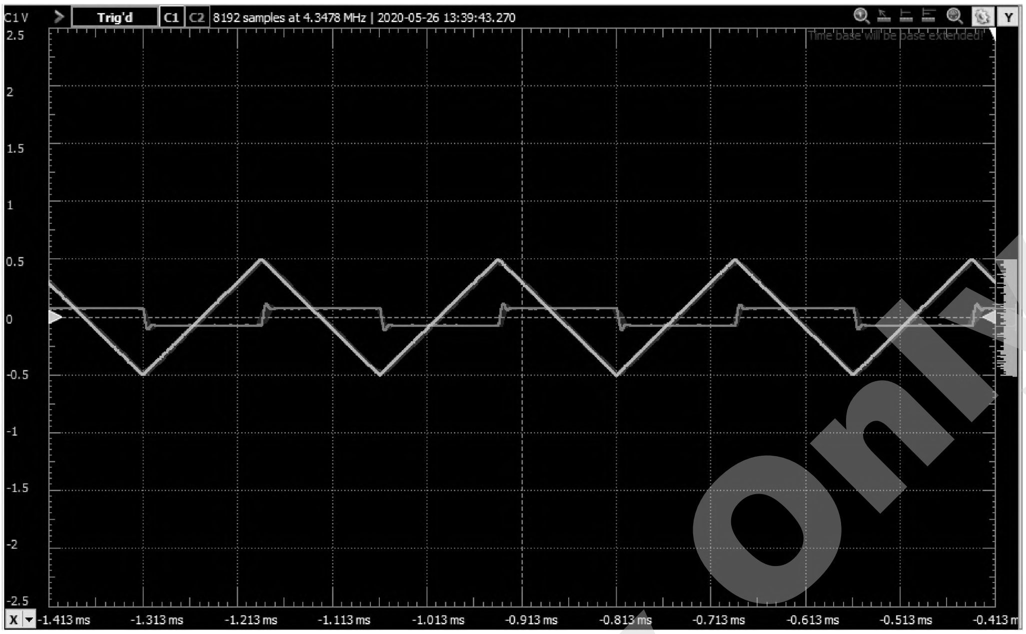


Figure 11: Output signal is in blue and input signal is in yellow

Using the "Y Cursors" tools, measure the maximum and minimum voltage of the output signal. The maximum was measured to be 0.085878 V and the minimum was -0.085878 V. Switch the wave generator type to be a sine function and using the same time base and channel ranges and the resulting waveform is shown below:

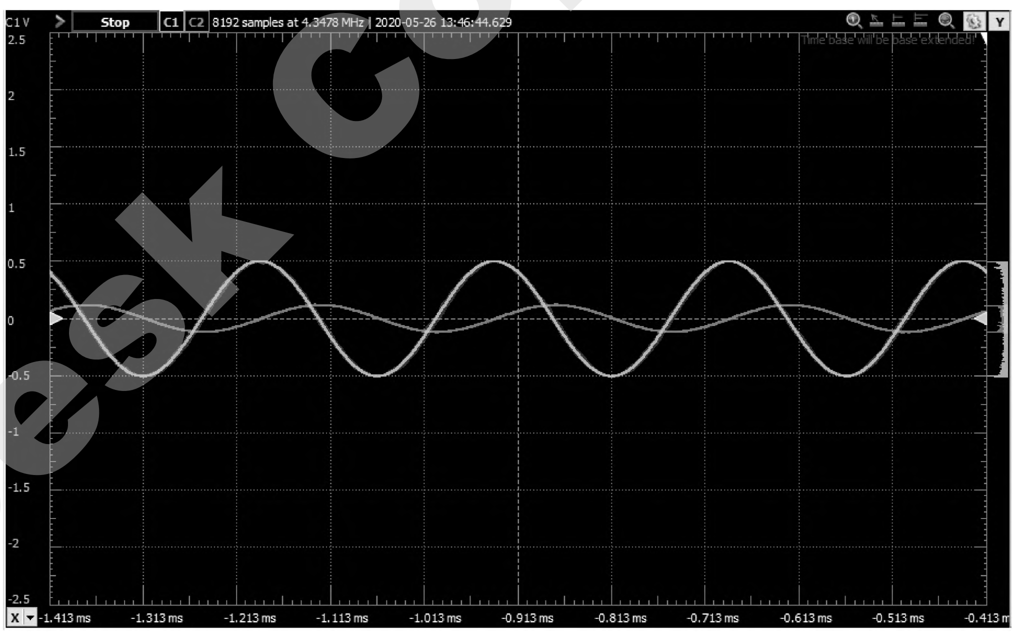


Figure 12: Input signal is in yellow and output signal is in blue

The maximum and minimum voltage values at the output were 0.1212 V and -0.1215 V, respectively. The measured results close to the theoretical results.

Experiment:

vi. Given the circuit in Figure 4, assume $R_3 = 10k\Omega$, $R_4 = 2.2M\Omega$, $C_3 = 100nF(104), V_{cc}^{+} = +5V$, and $V_{cc}^{-} = -5V$. Consider two types of inputs: 1) a square wave, 2) a sine wave (both with frequency of 1 KHz and peak-to-peak amplitude of 2V). Determine the output voltage and plot the relationship between the input voltage and the output voltage.

vii. Build the circuit in Figure 4 using the analog discovery 2 and measure the corresponding outputs. Compare your theoretical analysis with your measured responses.

viii. Set the frequency to 10 Hz or lower. Check whether the integrator functions properly and explain your finding.

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 4)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment

- Analog Discovery 2

Components

- 2 x 10kΩ resistor
- 1 x 25kΩ resistor
- 1 x 100kΩ resistor
- 1 x 10nF (103) capacitor
- 1 x 100nF (104) capacitor
- 1 x 50kΩ resistor
- 1 x 22kΩ resistor
- 1 x 1kΩ resistor
- Op-Amp LM358P

Objective

- To learn how to analyze First-order circuits.

Background:

Natural and Forced Responses

Consider the following first-order differential equation with constant coefficients a_0 and a_1 such that $a_1 \neq 0$,

$$a_1 x'(t) + a_0 x(t) = f(t) \tag{1}$$

The solution to (1) is denoted by $x(t) = x_n(t) + x_f(t)$, where $x_n(t)$ and $x_f(t)$ are the natural response and the forced responses, respectively. In particular, $x_n(t)$ is the solution to the homogeneous equation

$$a_1 x'(t) + a_0 x(t) = 0. \tag{2}$$

It takes the form of $x_n(t) = Ke^{St}$, where K and S are constants. Substituting the expression of $x_n(t)$ back to (2), we have

$$(a_1 S + a_0)Ke^{St} = 0 \tag{3}$$

As $K \neq 0$ and $e^{St} > 0$ (otherwise the solution is trivial), we have

$$S = -\frac{a_0}{a_1} \tag{4}$$

The value of K can be determined by the initial condition on $x(0^+)$ and $f(0^+)$, which will be discussed later. Examples of forced solution and the corresponding trial solutions are listed here for your reference.

Table 1: Forms of forced solutions

$f(t)$	Trial Function
a	A
$at + b$	$At + B$
$at^n + bt^{n-1} + \cdots$	$At^n + Bt^{n-1} + \cdots$
$ae^{\sigma t}$	$Ae^{\sigma t}$
$a\cos wt + b\sin wt$	$A\cos wt + B\sin wt$

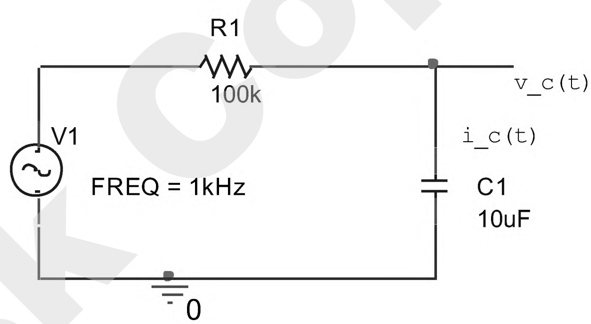


Figure 13: A first-order RC Circuit

Solving the first-Order RC Circuits with DC Source

Consider the first-order RC circuit depicted in Fig. 1. Using KVL and the properties of a capacitor, we have:

$$R_1 i_c(t) + v_c(t) = v_e(t), \tag{5a}$$
$$i_c(t) = C_1 v_c'(t), \tag{5b}$$

where $i_c(t)$ is the charging current of C_1, $v_c(t)$ is the end-to-end voltage of C, and $v_e(t)$ is the value of the voltage source. Hence the voltage $v_c(t)$ can be fully characterized by the first order differential equation:

$$R_1C_1v_c'(t)+v_c(t) = v_e(t). \tag{6}$$

If we know the expression of $v_e(t)$ and the initial conditions, then $v_c(t)$ can be solved in a closed form.

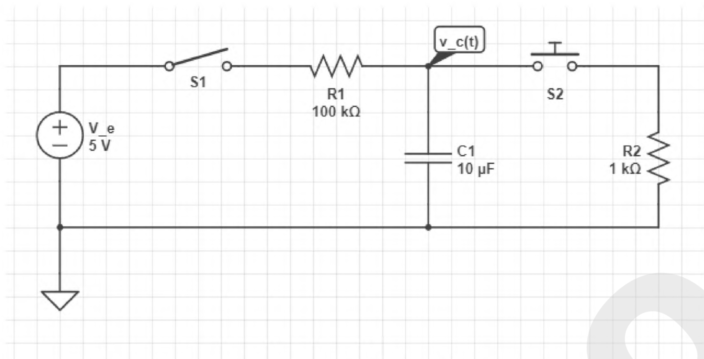


Figure 14: A first-order RC circuit with DC source

In what follows, we consider the first-order circuit with DC source (step response) in Fig. 2. First press S_2 for a while, and then close switch S_1 at time $t = t_0$. We assume that

$$v_c(t = t_0^+) = V_c,$$
$$v_e(t = t_0^+) = V_e,$$

where V_c is the initial voltage of C_1. Clearly, if we press S_2 for sufficiently long time with S_1 open circuited, then we have $V_c = 0$. From (6), for $t \geq t_0^+$, we have

$$R_1C_1v_c'(t)+v_c(t) = v_e(t), \qquad t \geq t_0^+, \tag{7}$$

where

$$v_e(t) = \begin{cases} 0, & t < t_0 \\ V_e, & t \geq t_0 \end{cases}$$

In view of (4), the natural response to $v_c(t)$ is given by

$$v_{c,n}(t) = Ke^{S(t-t_0)}, \tag{8}$$

where $S = -\frac{1}{R_1C_1}$, and K can be determined by the initial conditions. From table 1, we find that the forced response is a ocnstant number; as such we denote $v_{e,f}(t) = A, t > t_0^+$. Since

$$R_1C_1v_{c,f}'(t)+v_{c,f}(t) = V_e, \qquad t \geq t_0^+,$$

it follows that $v_{c,f}(t) = V_e, t \geq t_0^+$. Note that

$$v_c(t) = v_{c,n}(t) + v_{c,f}(t) = Ke^{-\frac{t-t_0}{R_1C_1}} + V_e. \tag{9}$$

For $t = t_0^+$, we have $v_c(t_0^+) = K + V_e = V_c$, which implies $K = V_c - V_e$. Therefore,

$$v_c(t) = (V_c - V_e)e^{-\frac{t-t_0}{R_1C_1}} + V_e,\ t > t_0^+, \tag{10}$$

which is illustrated in Fig. 3.

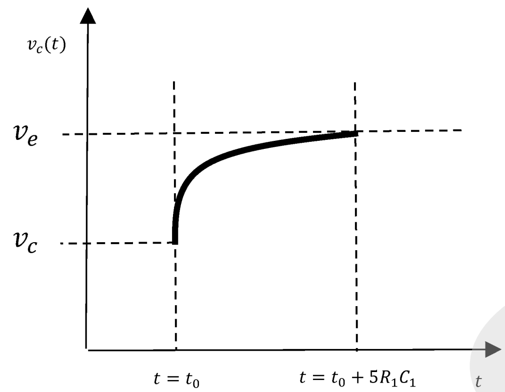


Figure 15: Charging Process for the first-order circuit with DC source

First-order RC Circuit in the Relaxation Oscillator

The relaxation oscillator is an important circuit that uses a Schmitt trigger to charge and discharge the capacitor. It is widely used in low frequency function signal generators. In this experiment, we will study the first-order RC circuit in the relaxation oscillator.

The Principle of the Relaxation Oscillator

Consider the relaxation oscillator depicted in Fig. 4, where the Schmitt trigger is used to charge and discharge the capacitor C through the resistor R_3 in an alternating way. Without loss of generality, we assume that in the beginning the output $v_{out} = V_{sat}^+$. As a consequence, $v_c(t) < V_{th_1} < V_{sat}^+$ (can you explain why?), and the capacitor is charging. When $v_c(t)$ reaches V_{th_1}, the output voltage changes to its opposite saturation limit V_{sat}^- and the trigger threshold changes to V_{th_2}. As the output is now negative and $v_c(t) > V_{th_2} > V_{sat}^-$, the capacitor discharges through R_3. Similarly, when $v_c(t)$ decreases to V_{th_2}, the

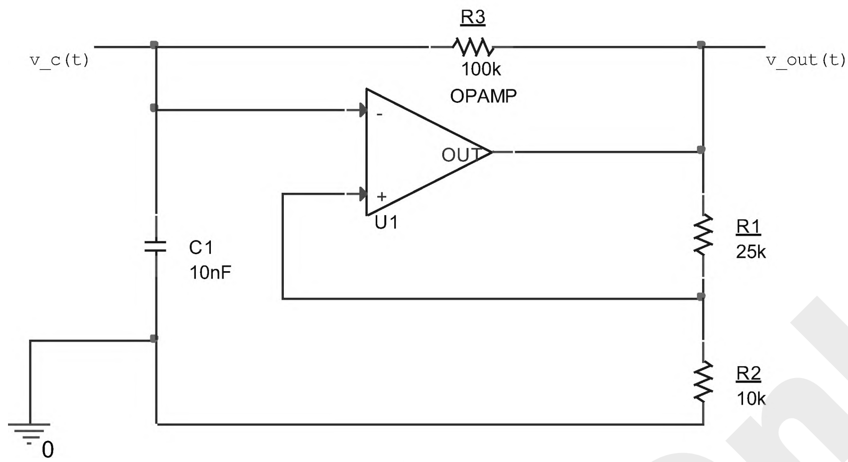


Figure 16: A simple relaxation oscillator

Output switches back to V_{sat}^{+} and the threshold moves to V_{th_1}. Consequently, the current through R_3 changes sign and the capacitor is charging again. In this way, $v_c(t)$ oscillates between V_{th_1} and V_{th_2}.

The Period (Frequency) of the Relaxation Oscillator

In what follows, we will calculate the period (frequency) of the relaxation oscillator. We shall ignore the current from and into the negative input pin of the Op-Amp. As a consequence, R_3 and C form a first-order circuit.

- Consider the charging process from time 0. Assuming the capacitor is charging but $v_c(t) < V_{th_1}$, we have:

$$R_3 C v_c'(t) + v_c(t) = v_{out}(t) \tag{11}$$

where $v_c(0^+) = V_{th_2}$ and $v_{out}(t)$ can be treated as a DC source with $v_{out}(0^+) = V_{sat}^{+}$. Now by (10), if $v_c(t) < V_{th_1}$, then

$$v_c(t) = \left(V_{th_2} - V_{sat}^{+}\right) e^{\frac{-t}{R_3 C}} + V_{sat}^{+}. \tag{12}$$

- Consider the discharging process from time 0. Assuming $v_c(t) > V_{th_2}$, we have

$$R_3 C v_c'(t) + v_c(t) = v_{out}(t) \tag{13}$$

where $v_c(0^+) = V_{th_1}$ and $V_{out}(0^+) = V_{sat}^{-}$. Hence, if $v_c(t) > V_{th_2}$, then

$$v_c(t) = \left(V_{th_1} - V_{sat}^{+}\right) e^{\frac{-t}{R_3 C}} + V_{sat}^{-} \tag{14}$$

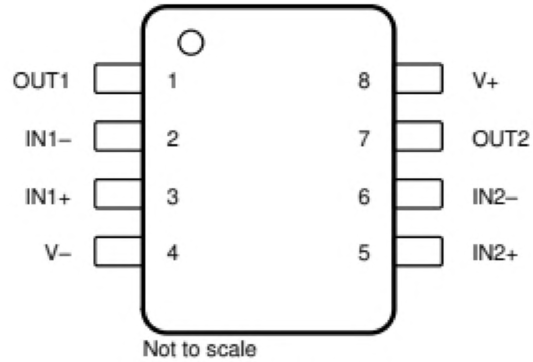


Figure 17: LM358P Datasheet

- Denote T_1 (T_2) as a time at which $v_c(t)$ changes from V_{th_2} (V_{th_1}) to V_{th_1} (V_{th_2}). We have:

$$(V_{th_2} - V_{sat}^{+})e^{\frac{-T_1}{R_3C}} + V_{sat}^{+} = V_{th_1}, \tag{15a}$$
$$(V_{th_1} - V_{sat}^{-})e^{\frac{-T_2}{R_3C}} + V_{sat}^{-} = V_{th_2}, \tag{15b}$$

which implies

$$T_1 = R_3C \ln \frac{V_{sat}^{+} - V_{th_2}}{V_{sat}^{+} - V_{th_1}}, \tag{16a}$$
$$T_2 = R_3C \ln \frac{V_{sat}^{-} - V_{th_1}}{V_{sat}^{-} - V_{th_2}}. \tag{16b}$$

As a result,

$$T = T_1 + T_2 = R_3C(\ln \frac{V_{sat}^{+} - V_{th_2}}{V_{sat}^{+} - V_{th_1}} + \ln \frac{V_{sat}^{-} - V_{th_1}}{V_{sat}^{-} - V_{th_2}}) \tag{17}$$

and $f = \frac{1}{T}$.

Example:

xi. Given the circuit in Fig. 4, calculate the period *T*, assuming that $V_{sat}^{+} = 5.0V$ and $V_{sat}^{-} = -5.0\,V$.

Solution:

Since the relaxation oscillator is a Schmitt trigger circuit, we need to first find V_{th_1} and V_{th_2}. Recall from Set 2 that $V_{th_1} = \frac{R_2}{R_1+R_2} V_{sat}^{+} = \frac{10k\Omega}{25k\Omega+10k\Omega} \times 5V = \frac{10}{7}V \approx 1.43V$. $V_{th_2} = \frac{R_2}{R_1+R_2}\, V_{sat}^{-} = \frac{10k\Omega}{25k\Omega+10k\Omega} \times -5V = -\frac{10}{7}V \approx -1.43V$.

Using equation (17) and plugging in the required values, we get the period T to be approximately 1.18 ms.

xii. Build the circuit in Figure 4 and compare your theoretical results to the actual results.

Solution:

Assuming the proper circuit connections have been made, connect the lead **1+** to the node V_c(t), and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the capacitor voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_{out}(t), and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 500 µs/div, and the range on channel 1 and 2 to 500 mV/div and the resulting waveform is shown below:

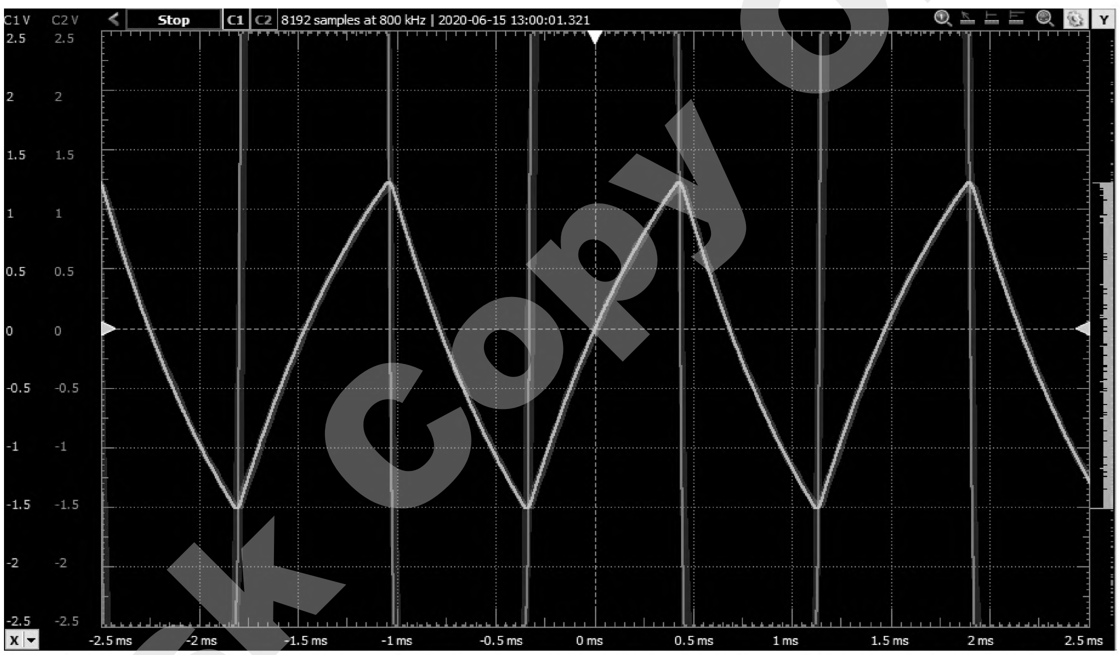


Figure 18: Resultant waveform. Channel 1 is the capacitor voltage and channel 2 is the output voltage

Using the "Y Cursors" tool, measure the maximum and minimum voltages of channel 1 (which are V_{th_1} and V_{th_2}). The maximum was measured to be 1.2309 V and the minimum was -1.479 V. Using the vertical measure to get the period T of channel 1's signal, it was measured to be 1.46 ms.

Comparing these values to the theoretical results, they are considered close to each other.

xiii. Given the circuit in Fig. 7, calculate the period T, assuming that $V_{sat}^{+} = 5.0V$ and $V_{sat}^{-} = -5.0\ V$. (D_1 and D_2 are ideal with built-in potential of approximately 0.7V)

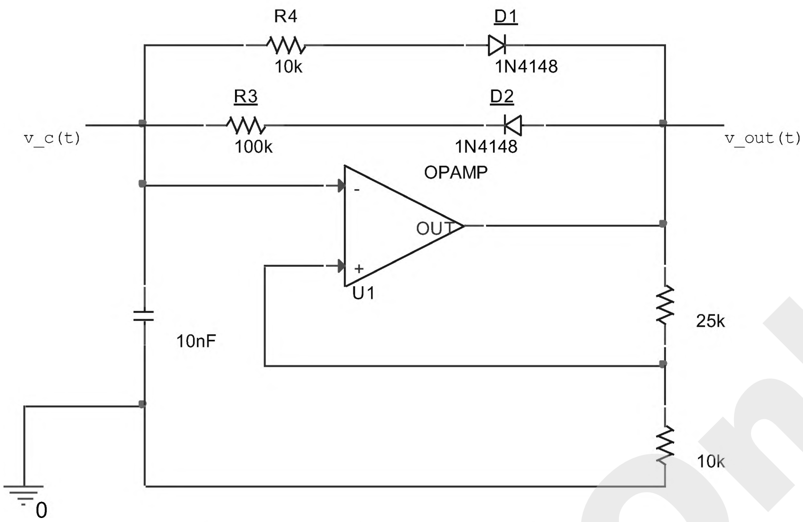


Figure 19: A modified relaxation oscillator

Solution:

If we were to reanalyze the Fig. 7 circuit to find the period T, we would get:

$$T = C(R_4 \times (\ln\frac{V_{sat}^{+}-0.7V-V_{th_2}}{V_{sat}^{+}-0.7V-V_{th_1}}) + R_3 \times (\ln\frac{V_{sat}^{-}+0.7V-V_{th_1}}{V_{sat}^{-}+0.7V-V_{th_2}}))$$

$$T = (10nF)(10k\Omega \times (\ln\frac{5V-0.7V-(-\frac{10}{7}V)}{5V-0.7V-(\frac{10}{7}V)}) + 100k\Omega \times (\ln\frac{-5V+0.7V-\frac{10}{7}V}{-5V+0.7V-(-\frac{10}{7}V)}))$$

This results in a T value of 0.760 ms.

xiv. Build the circuit in Fig. 7 and compare your theoretical results to the actual results.

Solution:

Assuming the proper circuit connections have been made, connect the lead **1+** to the node V_c(t), and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the capacitor voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_{out}(t), and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 200 µs/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

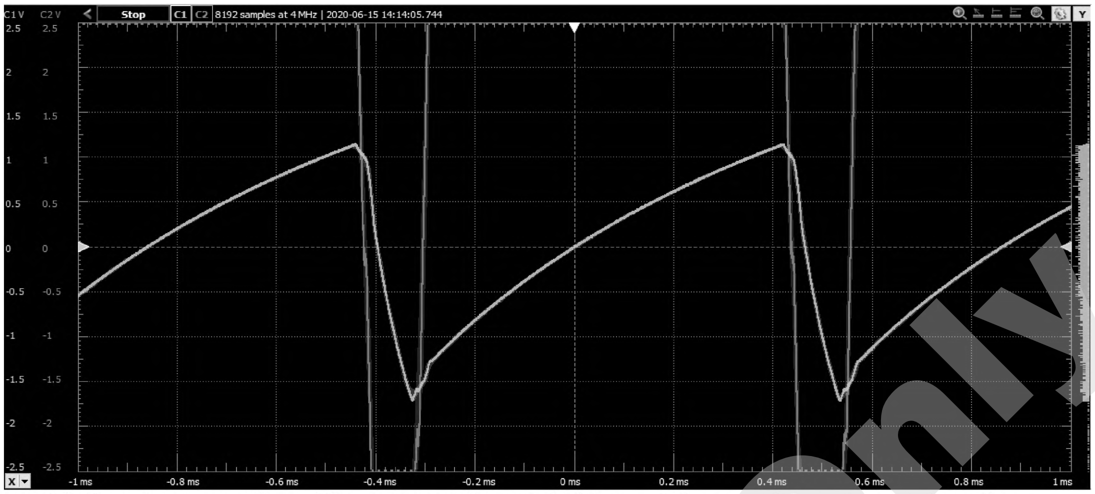


Figure 20: Resultant waveform. The output voltage is in blue and the capacitor voltage is in yellow

The experimental time period was found to be 859.5 μs, which is close to the theoretical.

Experiment:

i. Given the circuit in Fig. 9, calculate the period T and frequency f of the oscillator.

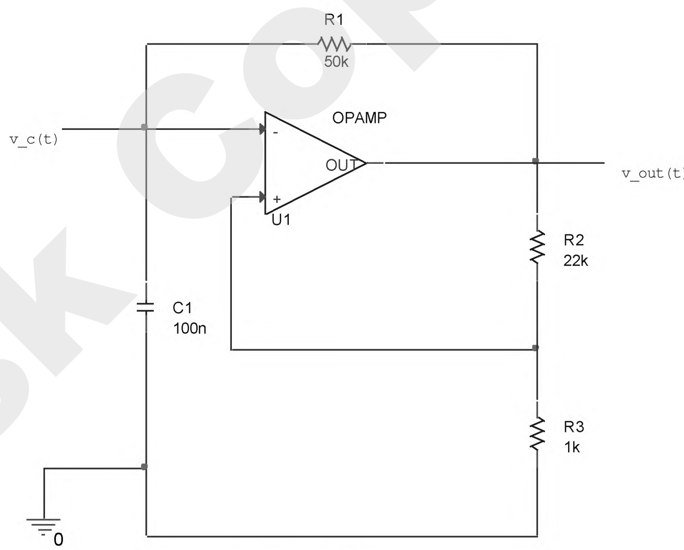


Figure 21

ii. Build the circuit in Fig. 9 and plot the voltage of the capacitor and the output voltage with respect to time (assuming $V_{sat} = \pm 5V$). Measure the time period T using the Analog Discovery 2 and compare it to the theoretical result.

iii. Can you build a circuit by using another Op-Amp LM358P to generate a triangular output? Explain.

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 5)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2
- Matlab

Components
- 1 x 10Ω resistor
- 3 x 10kΩ resistor
- 1 x 25kΩ resistor
- 1 x 100kΩ resistor
- 2 x 10nF (103) capacitor
- Op-Amp LM358P

Objective

Active filters are widely used in analog circuits, e.g., in power, communication, and control systems. In this experiment, we will study the second-order Butterworth Sallen-key low-pass filter.

Background:

The Sallen-Key topology was introduced by R. P. Sallen and E. L. Key of MIT Lincoln Laboratory in 1955.

The Transfer Function

Consider the circuit in Fig. 1. To simplify our analysis, we assume that the Op-Amp is ideal and it works in the linear region. Hence,

$$V_p = V_n. \tag{1}$$

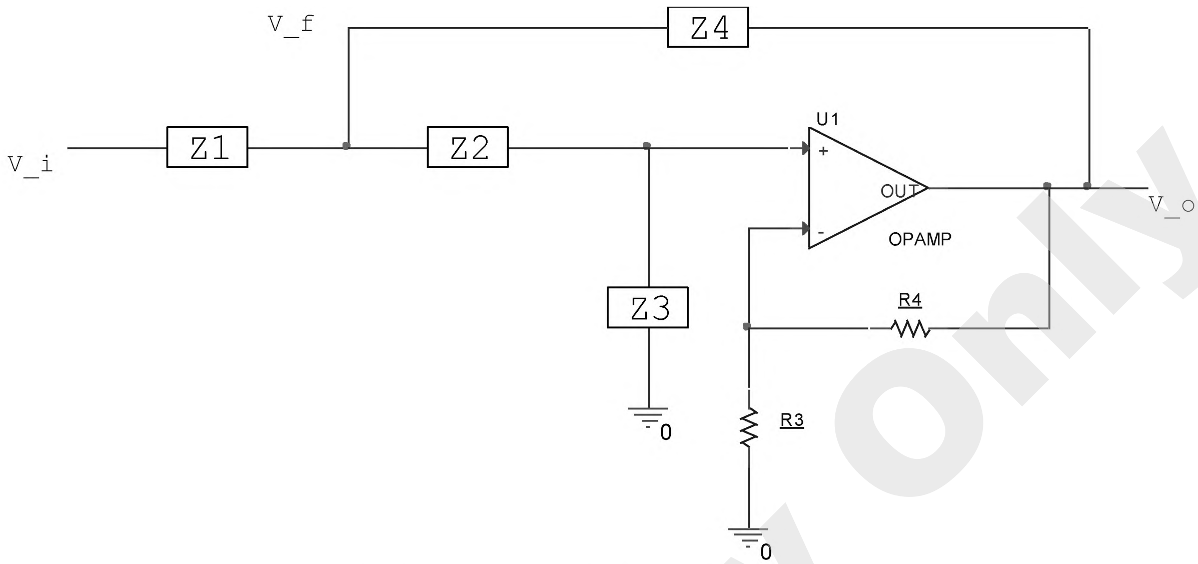


Figure 22: A generic Sallen-Key filter topology

Where V_f is the voltage at node f, V_i is the input voltage, and V_o is the output voltage. Also note that

$$V_p = V_f \frac{Z_3}{Z_2 + Z_3}, \tag{2a}$$

$$V_n = V_o \frac{R_3}{R_3 + R_4}. \tag{2b}$$

It follows by KCL that

$$\frac{V_i - V_f}{Z_1} = \frac{V_f - V_o}{Z_4} + \frac{V_f}{Z_2 + Z_3} \tag{3}$$

Combining the above equations yields

$$\frac{V_o}{V_i} = \frac{K}{\frac{Z_1}{Z_3}\frac{Z_2}{Z_4} + \frac{Z_1}{Z_3} + \frac{Z_2}{Z_3} + \frac{Z_1(1-K)}{Z_4} + 1} \tag{4a}$$

$$\frac{V_o}{V_i} = \frac{KZ_3Z_4}{Z_1Z_2 + Z_1Z_4 + Z_2Z_4 + Z_1Z_3(1-K) + Z_3Z_4}, \tag{4b}$$

Where $K = 1 + \frac{R_4}{R_3}$.

Low-Pass Sallen-Key Circuit

Consider the filter in Figure 2. We let $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = \frac{1}{sC_1}$ and $Z_4 = \frac{1}{sC_2}$. The transfer function for this low-pass filter is given by

$$H(s) = \frac{K}{R_1R_2C_1C_2s^2 + s\big(R_1C_1 + R_2C_1 + R_1C_2(1-K)\big) + 1}. \tag{5}$$

It can be verified that the cutoff frequency, $f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$ and quality factor, $Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1+R_2C_1+R_1C_2(1-K)}$. Setting $R_1 = R_2 = R$, $C_1 = C_2 = C$, and $K \approx 1$ gives $f_c = \frac{1}{2\pi RC}$ and $Q = \frac{1}{2}$.

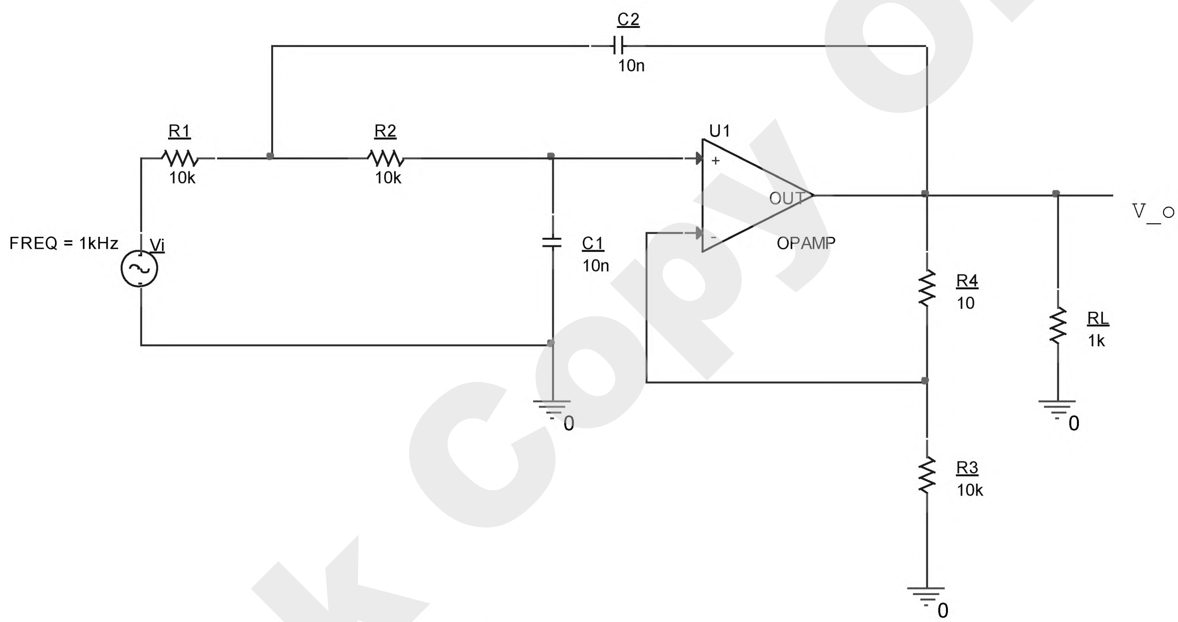


Figure 23: Low-pass Sallen-Key circuit

Example:

xv. Given the circuit in Fig. 2, calculate the cutoff frequency of the filter.

Solution:

Using the formula from the background section, $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(10^4\Omega)(10\times10^{-9}F)} = 1591.55\ Hz.$

xvi. Plot the theoretical transfer function using Matlab given the component values in Figure 2.

Solution:

Since we are given the values of the components and we know the generic transfer function from the background section and all the parameters, we can use MATLAB to obtain the plot using the following code:

```
%component values
R1 = 10000;
R2 = 10000;
R3 = 10000;
R4 = 10;
C1 = 10e-9;
C2 = 10e-9;

%generic transfer function parameters and equation
K = 1+(R4/R3);
numerator = [K];
denominator = [R1*R2*C1*C2 R1*C1+R2*C1+R2*C2*(1-K) 1]

%plotting
H = tf(numerator,denominator)
bode(H)
grid on

%cutoff frequency
Fc = 1/(2*pi*sqrt(R1*R2*C1*C2))
```

Figure 24: Matlab Code

The resulting waveform is shown below:

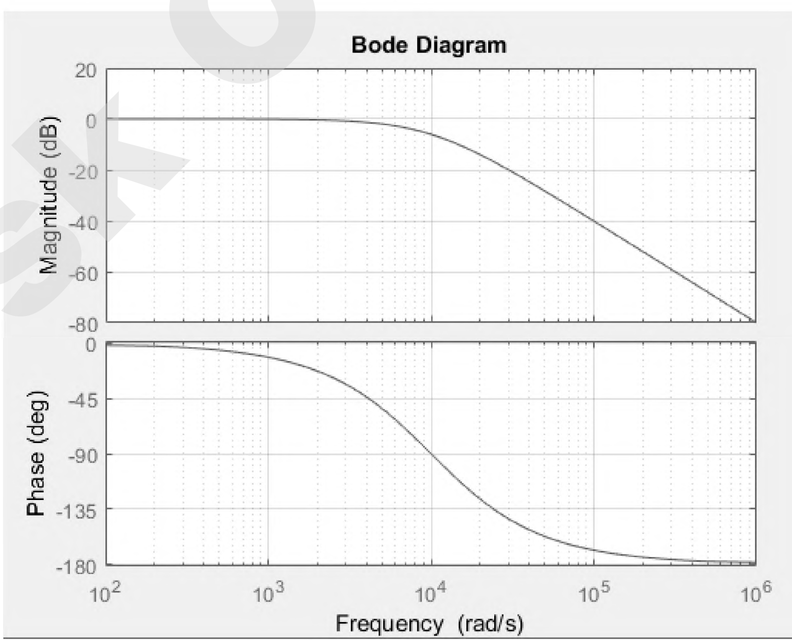


Figure 25: Bode Diagram

xvii. Build the circuit in Fig. 2 with V_i being a 1 kHz sine wave with an amplitude of 2V and an offset of 0V and the other components the same ($V_{cc} = \pm 5V$). Plot V_i and V_o using the oscilloscope tool on the Analog Discovery 2.

Solution:

Following the datasheet of the Op-Amp, connect the lead **1+** to the node $V_i(t)$, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 500 us/div and position to 0s, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

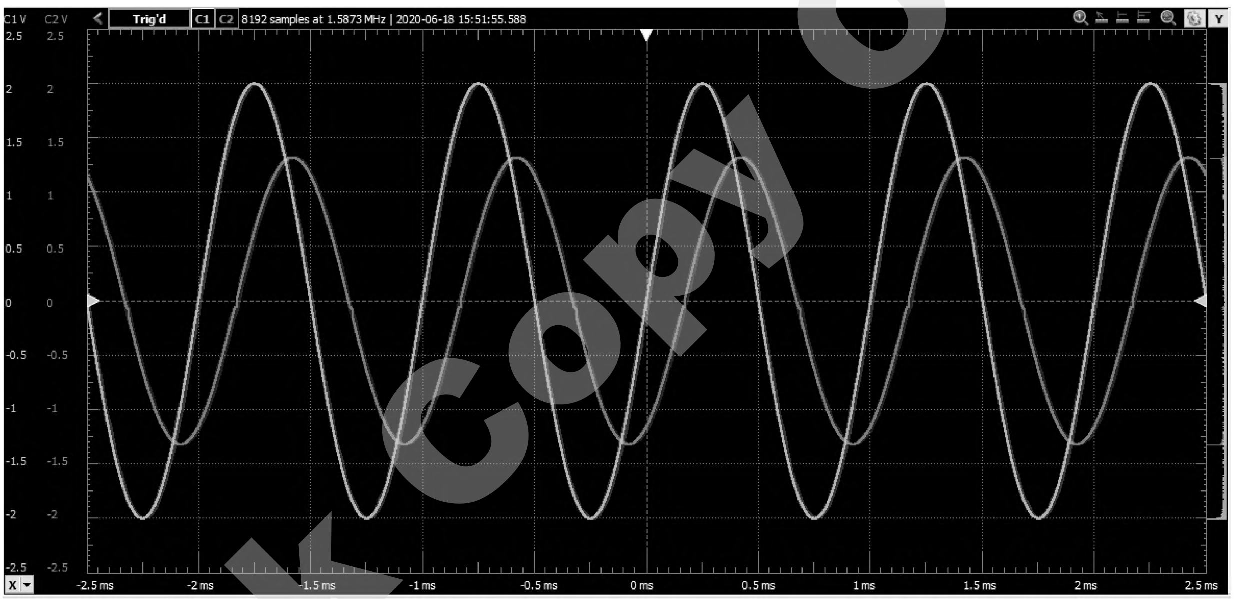


Figure 26: Resultant waveform

xviii. Sweep the input signal frequency and observe how the output voltage changes. Compare it to the theoretical result.

Solution:

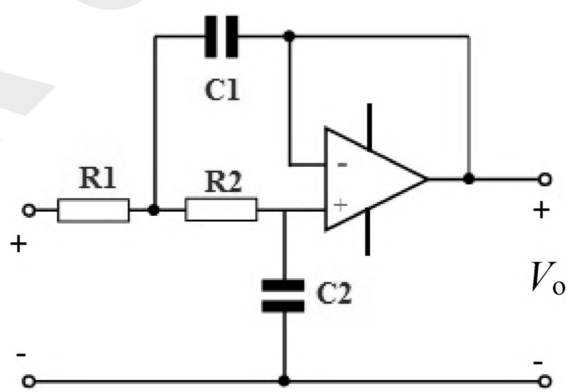
We will start with an input signal with a frequency range between 50 *Hz* up and 5 *kHz*. We will measure the output amplitude and calculate the gain. We will go by the step from the Analog Discovery 2 and once we get to the 1 *kHz* marks we will go up by 0.1 *kHz* until 2 *kHz*. The cutoff frequency, which is around 1.5 *kHz*, can be thus observed.

Table 2

Frequency	V_o (V)	V_i (V)	Gain (V_o/ V_i)
50 *Hz*	1.48145	1.816	0.81578
100 *Hz*	1.42035	1.685	0.84294
200 *Hz*	1.5721	1.989	0.79040
500 *Hz*	1.3825	1.98935	0.69495
1 *kHz*	0.96995	1.9733	0.49154
1.1 *kHz*	0.8999	1.9762	0.45537
1.2 *kHz*	1.0093	1.97485	0.51108
1.3 *kHz*	0.95545	1.97315	0.48423
1.4 *kHz*	0.908	1.9706	0.46077
1.5 *kHz*	0.86295	1.9691	0.43825
1.6 *kHz*	0.8096	1.95895	0.41328
1.7 *kHz*	0.7608	1.96	0.38816
1.8 *kHz*	0.71525	1.9583	0.36524
1.9 *kHz*	0.6717	1.95745	0.34315
2 *kHz*	0.6352	1.9588	0.32428
5 *kHz*	0.154455	1.9507	0.07918

The theoretical and measured values are very similar.

Experiment:



Consider the shown Butterworth Low-Pass Filter. Take R_1=R_2= 10 KΩ and C_1=C_2=100 nF.
a) Derive an expression for the transfer function of the filter.
b) Evaluate the filter transfer function abs(Vo/V_i) using the transfer function derived in part (a) for the frequencies shown in the table

c) Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude of 2V and offset of 0V ($V_{cc} = \pm 5V$).
d) What is the cut-off frequency of this filter?
d) How do the theoretical and measured results compare? Comment on your results.

Frequency	abs (V_o/ V_i) (analytical)	abs (V_o/ V_i) (measured)
50 Hz		
100 Hz		
200 Hz		
500 Hz		
1 kHz		
1.1 kHz		
1.2 kHz		
1.3 kHz		
1.4 kHz		
1.5 kHz		
1.6 kHz		
1.7 kHz		
1.8 kHz		
1.9 kHz		
2 kHz		
5 kHz		

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 6)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2
- MATLAB

Components
- 1 x 100nF capacitor
- 1 x 50nF capacitor
- 2 x 10kΩ resistor
- 1 x 3kΩ resistor
- 2 x 1kΩ resistor
- 1 x 6 kΩ resistor
- Op-Amp LM358P

Objective
- Derive the transfer function of a Sallen-key band pass filter
- Plotting voltage waveforms for a Sallen-key band pass filter
- Determining the modulus of the transfer function
- Measuring the amplitude of the input and output voltages and comparing it to the modulus of the transfer function

---

Example:

xix. Find the transfer function, $\frac{V_o}{V_{in}}$, of the following circuit (Assuming the Op-amp is ideal).

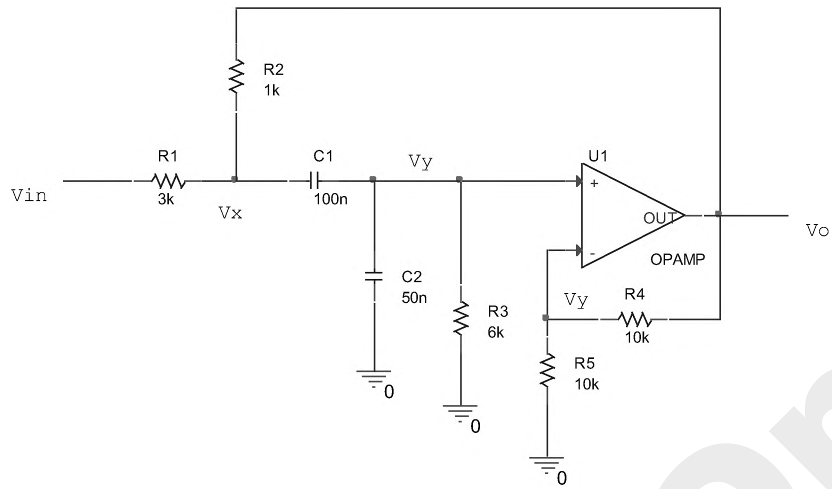


Figure 27: Sallen-key band pass filter

Solution:

We begin by carrying out KCL at V_x:

$$\frac{V_x - V_{in}}{R_1} + \frac{V_x - V_o}{R_2} + \frac{V_x - V_y}{Z_{C_1}} = 0. \tag{1}$$

where $Z_{C_1} = \frac{1}{SC_1}$. We can then find what V_y is through voltage division:

$$V_y = V_x \left(\frac{R_3SC_1}{R_3SC_1 + 1 + R_3SC_2}\right). \tag{2}$$

Next, we find V_y in terms of V_o:

$$V_y = V_o \left(\frac{R_5}{R_5 + R_4}\right). \tag{3}$$

Combining equations (2) and (3) and solving for V_x, we get:

$$V_x = V_o \left(\frac{R_5}{R_5 + R_4}\right)\left(\frac{R_3SC_1 + 1 + R_3SC_2}{R_3SC_1}\right). \tag{4}$$

We now have V_x and V_y in terms of V_o. We will substitute equations (3) and (4) into equation (1):

$$\frac{1}{R_1}V_o\left(\frac{R_5}{R_5+R_4}\right)\left(\frac{R_3SC_1+1+R_3SC_2}{R_3SC_1}\right)-\frac{V_{in}}{R_1}+\frac{1}{R_2}V_o\left(\frac{R_5}{R_5+R_4}\right)\left(\frac{R_3SC_1+1+R_3SC_2}{R_3SC_1}\right)-\frac{V_o}{R_2}$$
$$+SC_1V_o\left(\frac{R_5}{R_5+R_4}\right)\left(\frac{R_3SC_1+1+R_3SC_2}{R_3SC_1}\right)-SC_1V_o\left(\frac{R_5}{R_5+R_4}\right)=0.$$

Moving $\frac{V_{in}}{R_1}$ to the right side and factoring out $V_o\left(\frac{R_5}{R_5+R_4}\right)$ on the left hand side gives us:

$$V_o\left(\frac{R_5}{R_5+R_4}\right)\left[\frac{R_3SC_1+1+R_3SC_2}{R_1R_3SC_1}+\frac{R_3SC_1+1+R_3SC_2}{R_2R_3SC_1}-\frac{R_5+R_4}{R_2R_5}+\frac{R_3SC_1+1+R_3SC_2}{R_3}-SC_1\right]=\frac{V_{in}}{R_1}.$$

Therefore, the ratio of $\frac{V_o}{V_{in}}$ is,

$$\frac{V_o}{V_{in}}=\frac{1}{R_1}\frac{\frac{R_5}{R_5+R_4}}{\frac{R_3SC_1+1+R_3SC_2}{R_1R_3SC_1}+\frac{R_3SC_1+1+R_3SC_2}{R_2R_3SC_1}-\frac{R_5+R_4}{R_2R_5}+\frac{R_3SC_1+1+R_3SC_2}{R_3}-SC_1} \tag{5}$$

Let $K=\frac{R_5}{R_5+R_4}$ and multiply R_1 in the denominator to get:

$$\frac{V_o}{V_{in}}=K\frac{1}{\frac{R_3SC_1+1+R_3SC_2}{R_3SC_1}+\frac{R_1R_3SC_1+R_1+R_1R_3SC_2}{R_2R_3SC_1}-\frac{KR_1}{R_2}+\frac{R_1\,R_3SC_1+R_1\;+R_1\,R_3SC_2}{R_3}-R_1SC_1}.$$

Rearranging the denominator by multiplying $R_2R_3SC_1$ (common denominator) and then bringing it to the numerator gives us,

$$\frac{V_o}{V_{in}}=K\frac{R_2R_3SC_1}{R_2R_3SC_1+R_2+R_2R_3SC_2+R_1R_3SC_1+R_1+R_1R_3SC_2-KR_1R_3SC_1+R_1R_2SC_1+R_1R_2R_3C_1C_2S^2}.$$

Grouping like terms together and then dividing by $R_1R_2R_3C_1C_2$ in the denominator and moving it to the numerator gives us:

$$H(S)=\frac{V_o}{V_{in}}=\frac{K\left(\frac{1}{R_1C_2}\right)S}{S^2+S\left(\frac{\frac{C_1+C_2}{R_1}+\frac{C_2}{R_2}+\frac{C_1}{R_3}+\frac{C_1}{R_2}(1-K)}{C_1C_2}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}. \tag{6}$$

Plugging in the numbers for the components, we have:

$$H(S) = \frac{V_o}{V_{in}} = \frac{\frac{40000}{3}S}{S^2 + S\left(\frac{10000}{3}\right) + 44444444.44}. \tag{7}$$

xx. Plot the theoretical transfer function using MATLAB given the components in Figure 1.

Solution:

Since we are given the values of the components and we know the generic transfer function from the background section and all the parameters, we can use MATLAB to plot the transfer function:

```
clc;
clear;

%generic transfer function parameters and equation
K = 2;
numerator = [K*(20000/3) 0];
denominator = [1 10000/3 44444444.44]

%plotting
H = tf(numerator,denominator)
bode(H)
grid on
```

Figure 28: MATLAB code

the resulting waveform is shown below:

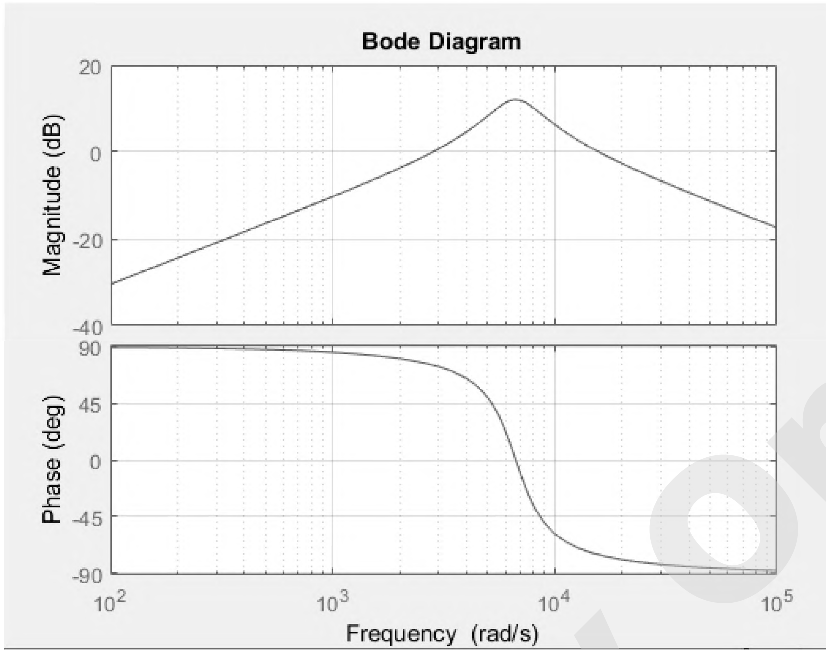


Figure 29: Bode Diagram

xxi. Determine the modulus of the transfer function you found from part i.

Solution:

If we take equation 7 and replace S with $j\omega$, where ω is the angular velocity of a signal (equal to $2\pi f$, f being the frequency), we get:

$$H(j\omega) = \frac{\frac{40000}{3} j\omega}{(j\omega)^2 + (j\omega)\left(\frac{10000}{3}\right) + 44444444.44}.$$

Simplifying the denominator:

$$H(j\omega) = \frac{\frac{40000}{3}\omega j}{44444444.44 - \omega^2 + \left(\frac{10000}{3}\omega\right) j}.$$

Therefore, the modulus of the transfer function for any ω would be the modulus of the numerator divided by the denominator,

$$|H(\omega)| = \frac{\frac{40000}{3}\omega}{\sqrt{(44444444.44 - \omega^2)^2 + \left(\frac{10000}{3}\omega\right)^2}}. \quad (8)$$

xxii. Build the circuit from Figure 1, with V_{in} being a 1 kHz sinusoidal wave with an amplitude of 0.5V and offset of 0V. Plot V_o and V_{in} using the oscilloscope tool on the Analog Discovery 2.

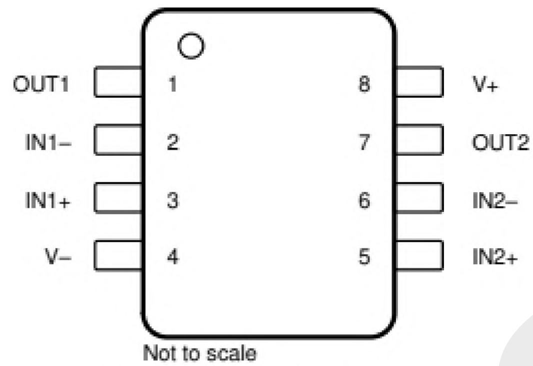


Figure 30: LM358P Datasheet

Solution:

After the circuit is properly assembled, connect the lead **1+** to the node V_{in}, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 1 ms/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

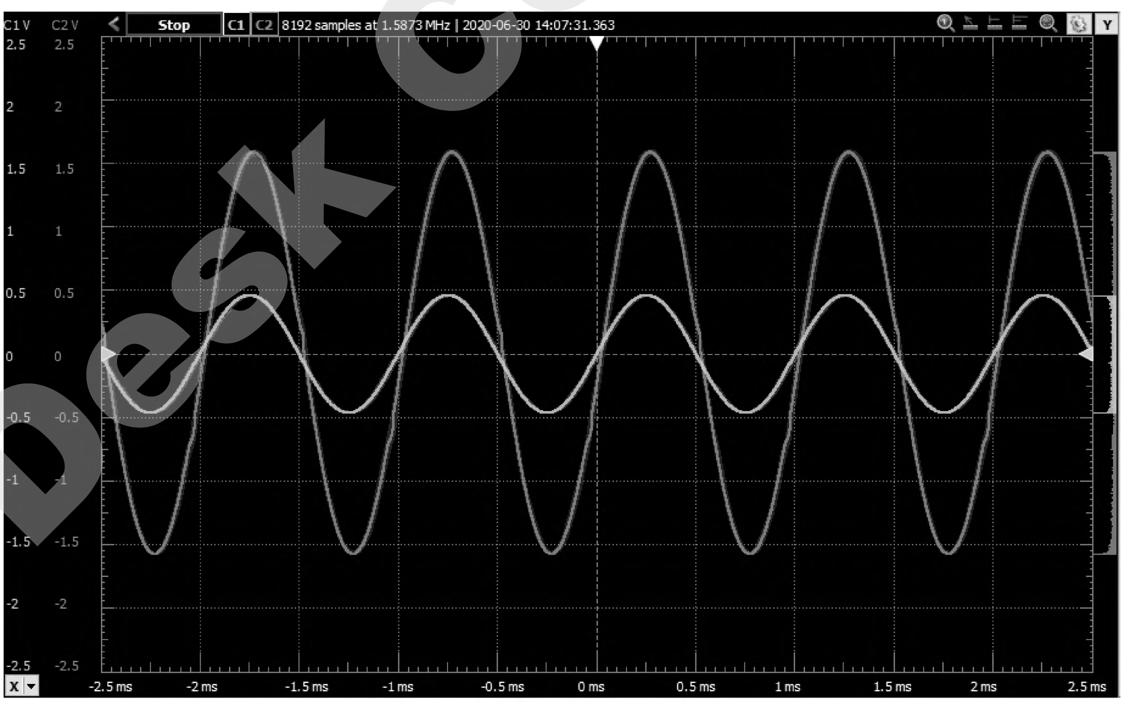


Figure 31: Resultant waveform

xxiii. Estimate the ratio of amplitudes of the output voltage and the input voltage using the Analog Discovery 2. Compare it to the theoretical result.

Solution:

If the ratio of amplitudes of $\frac{V_o}{V_{in}}$ on the Analog Discovery 2 is equal to the modulus of the transfer function at 1 kHz, then our results are accurate. To find the amplitude ratio we can use the "Measurements" tool on our Analog Discovery 2 and then using the "Maximum" tool and we get the ratio to be 3.428 (rounded to the nearest 3 decimal places). Since our f is 1 kHz, $\omega = 2\pi f$. Once we know ω we can plug it into equation 8 that we found from part iii and we get the modulus of the transfer function to be 3.892. therefore, the theoretical is roughly the same as the measured.

Experiment:

i. Find the transfer function $H(S)$ of the following circuit. (Assuming the Op-amp is ideal)

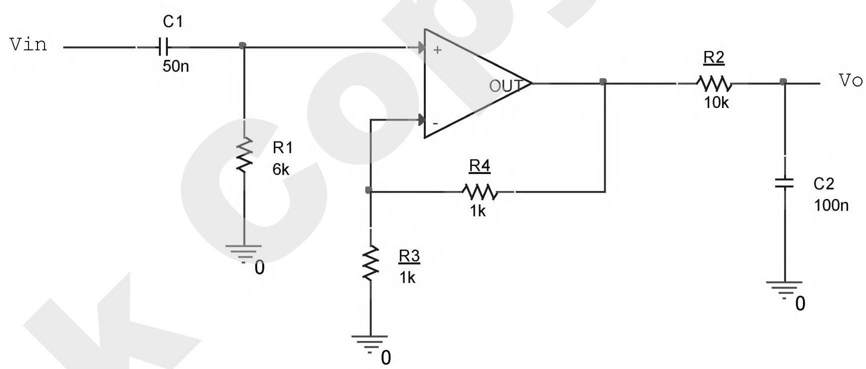


Figure 32

ii. Plot the theoretical transfer function using MATLAB given the components in Figure 6.
iii. Determine the modulus of the transfer function you found from part i.
iv. Build the circuit from Figure 6, with V_{in} being a 500 Hz sinusoidal wave with an amplitude of 0.5V and offset of 0V. Plot V_o and V_{in} using the oscilloscope tool on the Analog Discovery 2.
v. Estimate the ratio of amplitudes of the output voltage and the input voltage using the Analog Discovery 2. Compare it to the theoretical result.

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 7)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2
- MATLAB

Components
- 2 x 100nF capacitor
- 1 x 200nF capacitor
- 1 x 10kΩ resistor
- 1 x 5kΩ resistor
- 2 x 100kΩ resistor
- 1 x 50kΩ resistor
- Op-Amp LM358P

Objective
- Derive the transfer function of a notch filter
- Plotting voltage waveforms for a notch filter
- Determining the modulus of the transfer function
- Measuring the amplitude of the input and output voltages and comparing it to the modulus of the transfer function

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Example:

xxiv. Find the transfer function, $\frac{V_o}{V_{in}}$, of the following circuit (Assuming the Op-amp is ideal).

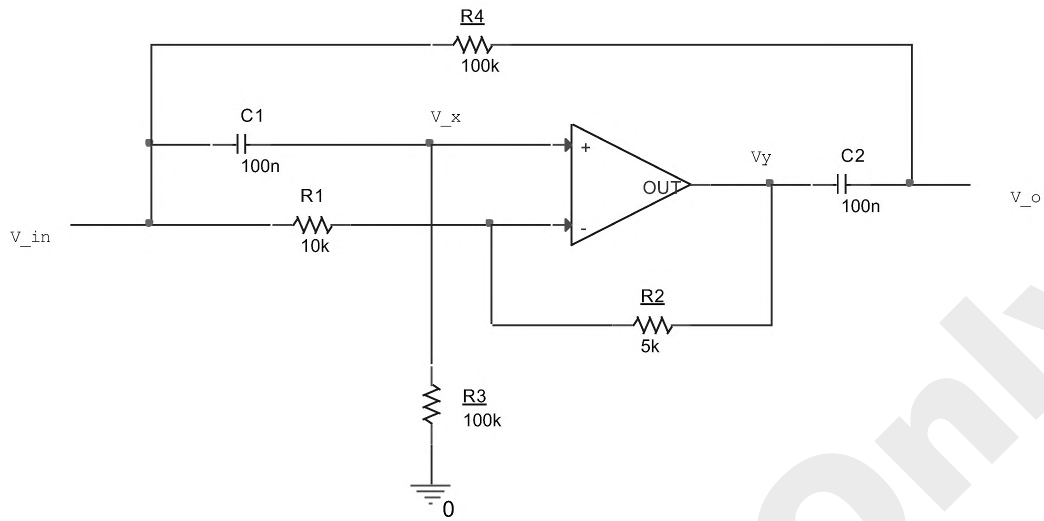

Figure 33: notch filter.

Solution:

We begin by using voltage division at V_x:

$$V_x = V_{in}\frac{R_3}{R_3 + Z_{C_1}} = V_{in}\frac{R_3SC_1}{R_3SC_1 + 1}, \tag{1}$$

where $Z_{C_1} = \frac{1}{SC_1}$. We can then find what V_y is by doing KCL at the inverting node,

$$\frac{V_{in} - V_x}{R_1} = \frac{V_x - V_y}{R_2}.$$

Isolating V_y and combining equation (1) for V_x, we get:

$$V_y = \frac{V_{in}\left(\frac{R_3SC_1}{R_3SC_1 + 1}\right)(R_1 + R_2) - V_{in}R_2}{R_1}. \tag{2}$$

We then use KCL at node V_o:

$$\frac{V_o - V_y}{Z_{C_2}} + \frac{V_o - V_{in}}{R_4} = 0. \tag{3}$$

We substitute equations (2) into equation (3) to have everything in terms of V_{in} and V_o,

$$SC_2\left(V_o-\frac{V_{in}\left(\frac{R_3SC_1}{R_3SC_1+1}\right)(R_1+R_2)}{R_1}+\frac{V_{in}R_2}{R_1}\right)+\frac{V_o}{R_4}-\frac{V_{in}}{R_4}=0.$$

Grouping the terms of V_o on the left side and terms with V_{in} on the right side gives us:

$$V_o\left(SC_2+\frac{1}{R_4}\right)=V_{in}\left(\frac{SC_2\left(\frac{R_3SC_1}{R_3SC_1+1}\right)(R_1+R_2)}{R_1}-\frac{R_2SC_2}{R_1}+\frac{1}{R_4}\right).$$

Therefore, the ratio of $\frac{V_o}{V_{in}}$ is,

$$\frac{V_o}{V_{in}}=\frac{\left(\frac{SC_2\left(\frac{R_3SC_1}{R_3SC_1+1}\right)(R_1+R_2)}{R_1}-\frac{R_2SC_2}{R_1}+\frac{1}{R_4}\right)}{\frac{R_4SC_2+1}{R_4}}. \tag{4}$$

Simplifying the numerator gives us,

$$\frac{V_o}{V_{in}}=\frac{\frac{R_1R_3R_4S^2C_1C_2-R_2R_4SC_2+R_1R_3SC_1+R_1}{R_1R_4(R_3SC_1+1)}}{\frac{R_4SC_2+1}{R_4}}.$$

If we simplify things even further, we notice that R_4 cancels out from the bottom part of the numerator and the denominator. We can then expand the denominator of the numerator $\left(R_1(R_3SC_1+1)\right)$ and the numerator of the denominator(R_4SC_2+1). This gives us,

$$\frac{V_o}{V_{in}}=\frac{R_1R_3R_4S^2C_1C_2-R_2R_4SC_2+R_1R_3SC_1+R_1}{R_1R_3R_4S^2C_1C_2+R_1R_3SC_1+R_1R_4SC_2+R_1}.$$

Grouping like terms together and then factoring out $R_1R_3R_4C_1C_2$ in the numerator and denominator gives us:

$$H(S)=\frac{V_o}{V_{in}}=\frac{S^2+S\left(\frac{1}{R_4C_2}-\frac{R_2}{R_1R_3C_1}\right)+\frac{1}{R_3R_4C_1C_2}}{S^2+S\left(\frac{1}{R_4C_2}+\frac{1}{R_3C_1}\right)+\frac{1}{R_3R_4C_1C_2}}. \tag{5}$$

Plugging in the numbers for the components gives us,

$$H(S) = \frac{V_o}{V_{in}} = \frac{S^2 + 50S + 10000}{S^2 + 200S + 10000}. \tag{6}$$

xxv. Plot the theoretical transfer function using MATLAB given the components in Figure 1.

Solution:

Since we are given the values of the components and we know the generic transfer function from the background section and all the parameters, we use MATLAB to obtain the plot using the shown M-file.

```
clc;
clear;

%generic transfer function parameters and equation

numerator = [1 50 10000];
denominator = [1 200 10000];

%plotting
H = tf(numerator,denominator)
bode(H)
grid on
```

Figure 34: MATLAB code

the resulting waveform is shown below:

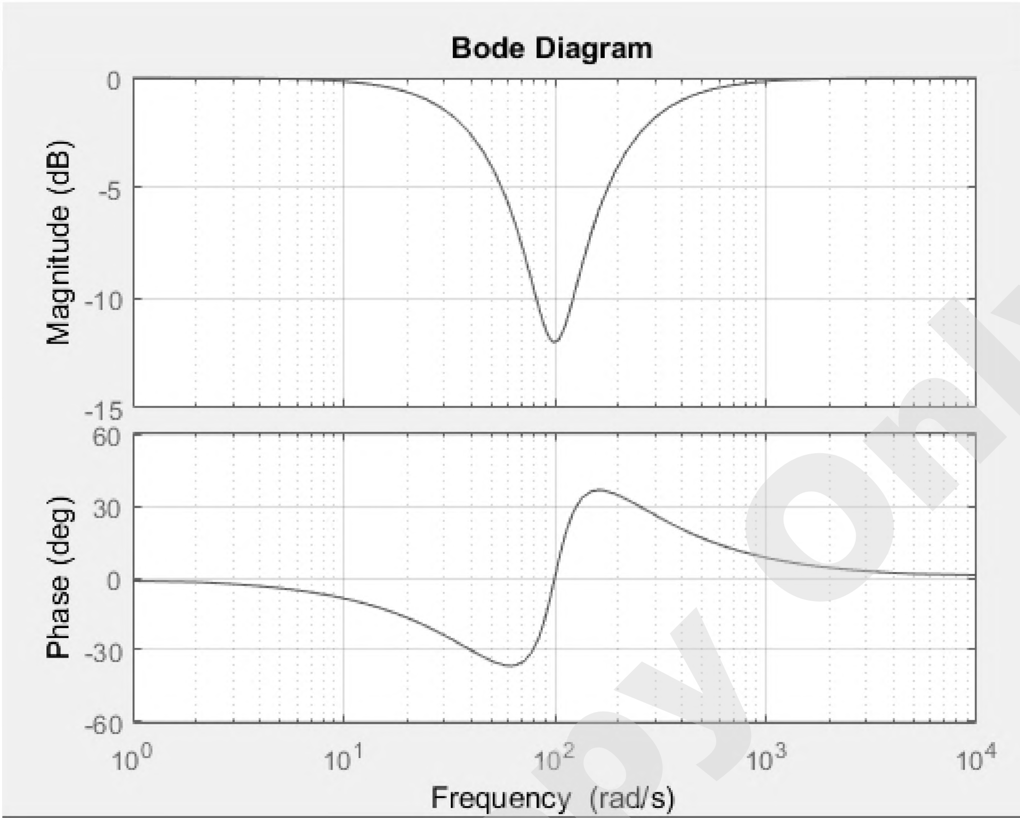

Figure 35: Bode Diagram

By looking at the bode diagram, we notice that the resonance frequency is approximately 16 Hz $\left(\frac{10^2}{2\pi}\right)$.

xxvi. Determine the modulus of the transfer function found in part i.

Solution:

If we take equation 6 and replace S with $j\omega$, where ω is the angular velocity of a signal (equal to $2\pi f$, f being the frequency), we get:

$$H(j\omega) = \frac{(j\omega)^2 + 50\omega j + 10000}{(j\omega)^2 + 200\omega j + 10000}.$$

Simplifying the denominator and numerator, we get:

$$H(j\omega) = \frac{10000 - \omega^2 + (50\omega)j}{10000 - \omega^2 + (200\omega)j}.$$

Therefore, the modulus of the transfer function for any ω would be the modulus of the numerator divided by the denominator,

$$|H(\omega)| = \frac{\sqrt{(10000 - \omega^2)^2 + (50\omega)^2}}{\sqrt{(10000 - \omega^2)^2 + (200\omega)^2}}. \qquad (7)$$

xxvii. Build the circuit from Figure 1, with V_{in} being a sinusoidal wave with an amplitude of 1V and offset of 0V. Plot V_o and V_{in} using the oscilloscope tool on the Analog Discovery 2 with an input frequency before the resonant frequency, at the resonant frequency, and after the resonant frequency.

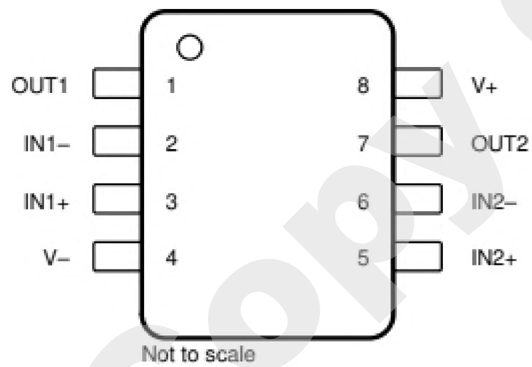

Figure 36: LM358P Datasheet

Solution:
Assemble the circuit components. Connect the lead **1+** to the node V_{in}, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the input voltage waveform will be plotted on channel 1. Then connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage will be plotted on channel 2. Set the time base to 50 ms/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

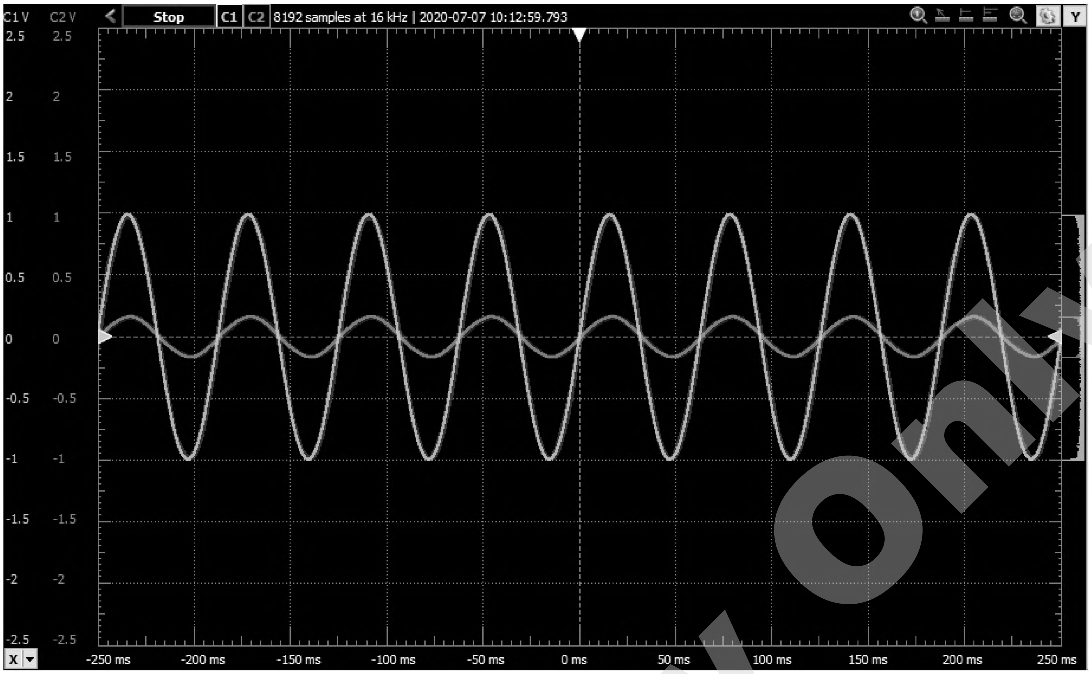

Figure 37: Resultant waveform when the input frequency is at the resonant frequency (16 Hz)

Set the time base to 1 ms/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

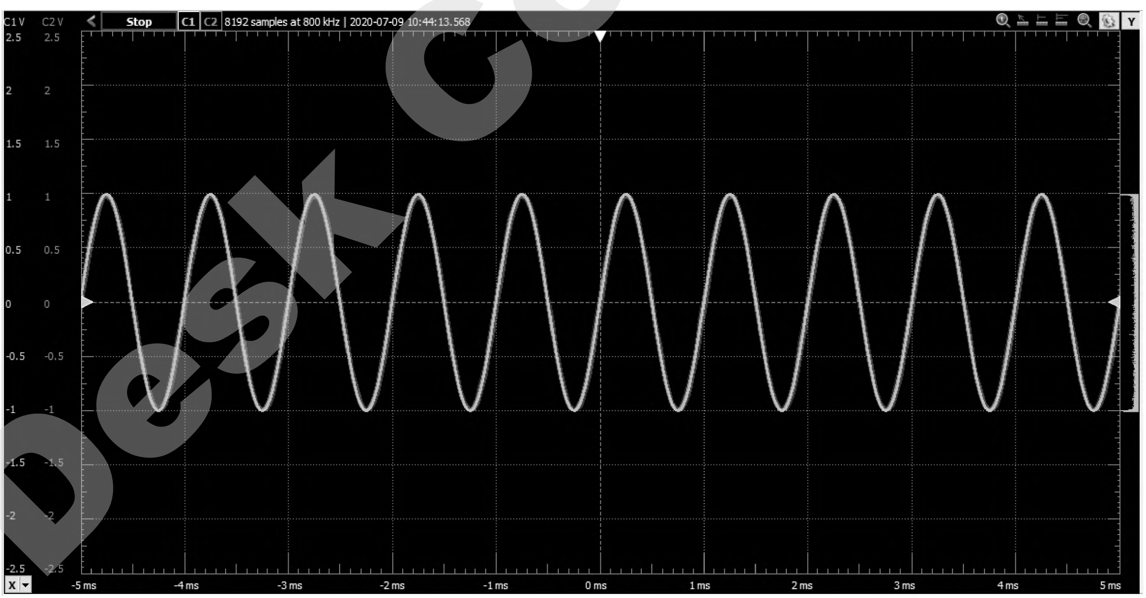

Figure 38: Resultant waveform when the input frequency is at 1kHz (after the resonant frequency)

Set the time base to 500 ms/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

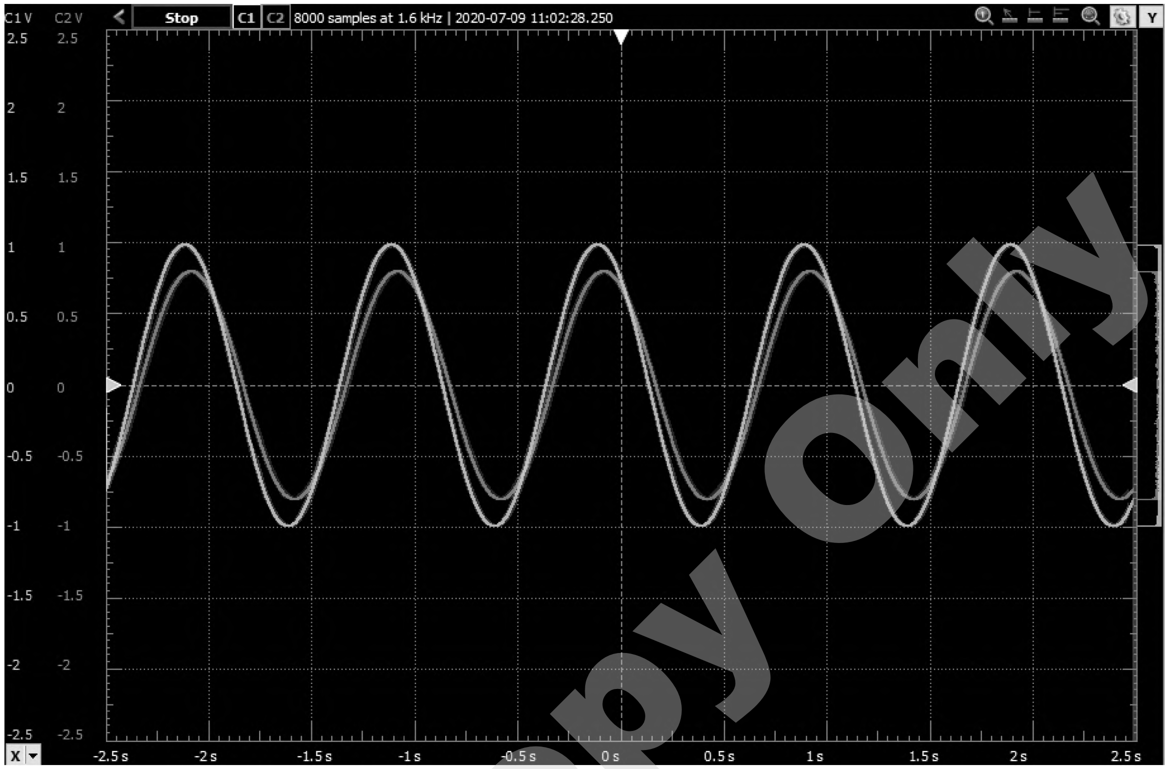

Figure 39: Resultant waveform when the input frequency is at 1 Hz (before the resonant frequency)

xxviii. Estimate the ratio of amplitudes of the output voltage and the input voltage using the Analog Discovery 2. Compare it to the theoretical result.

Solution:

If the ratio of amplitudes of $\frac{V_o}{V_{in}}$ on the Analog Discovery 2 is equal to the modulus of the transfer function at the different input frequencies, then our results are accurate. To find the amplitude ratio we can use the "Measurements" tool on our Analog Discovery 2 and then using the "Maximum" tool. When the input frequency is the resonant frequency (16 Hz) we get the ratio to be 0.166 (rounded to the nearest 3 decimal places). Since $\omega = 2\pi(15)$, we can plug it into equation 7 that we found from part iii and we get the modulus of the transfer function to be 0.250.

When the input frequency is bigger than the resonant frequency (1 kHz), we get the ratio of the amplitudes to be approximately 1 on the Analog Discovery 2. Theoretically, we get 0.9995 when we plug $\omega = 2\pi(1000)$ in equation 7.

When the input frequency is less than the resonant frequency (1 Hz) we get the ratio of the amplitudes to be approximately 0.81 on the Analog Discovery 2. Theoretically, we get 0.99 when we plug $\omega = 2\pi(1)$ in equation 7.

A notch removes a specific frequency and lets other frequencies pass through. Therefore, the theoretical is roughly the same as the measured.

Experiment:

1. Find the transfer function, $H(S)$ of the following circuit (Assuming the Op-amp is ideal).

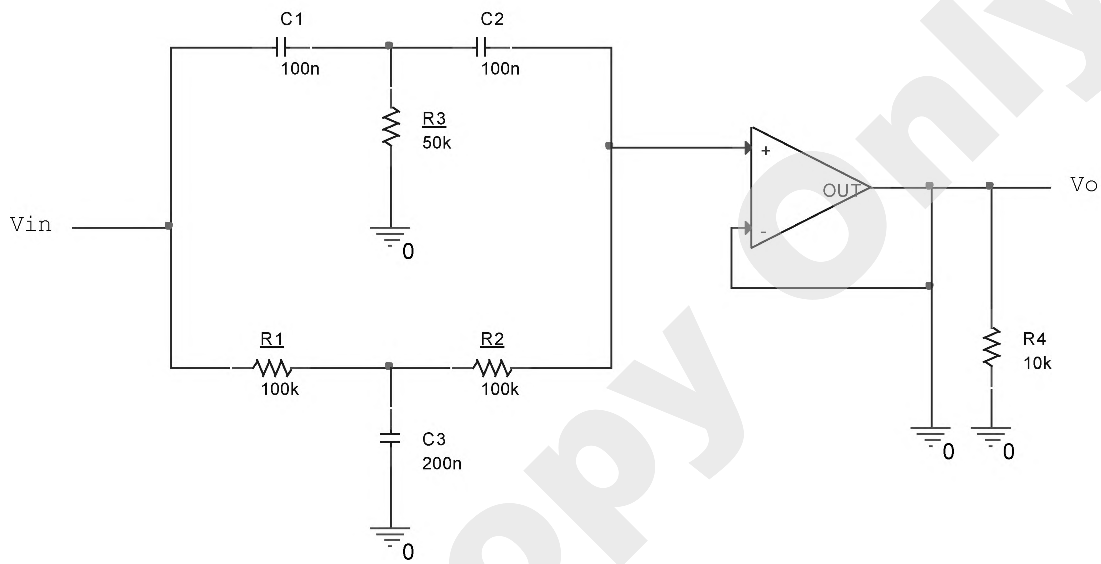

Figure 40

2. Plot the theoretical transfer function using MATLAB given the components.
3. Determine the modulus of the transfer function you found from part 1. Determine the notch frequency (the resonant frequency).
4. Build the circuit from Figure 1, with V_{in} being a sinusoidal wave with an amplitude of 1V and offset of 0V. Plot V_o and V_{in} using the oscilloscope tool on the Analog Discovery 2 with an input frequency before the resonant frequency, at the resonant frequency, and after the resonant frequency.
5. Estimate the ratio of amplitudes of the output voltage and the input voltage using the Analog Discovery 2. Compare it to the theoretical result.

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 8)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2

Components
- 2 x 4.7kΩ resistor
- 2 x 1kΩ resistor
- 1 x 10Ω resistor
- Op-Amp: LM358P

Objective
- Calculate the current through a component by applying principles of analyzing Op-amp circuits
- Plotting voltage waveforms
- Estimating voltage across a component
- Measuring the current through a component

Example:

Given the circuit in Figure 1, determine the current i_L.

Solution:

There are two things to remember when analyzing Op-amp circuits:

- The input current is zero
- The voltage difference between the inverting and non-inverting inputs is zero.

By performing voltage division at the inverting node, we get:

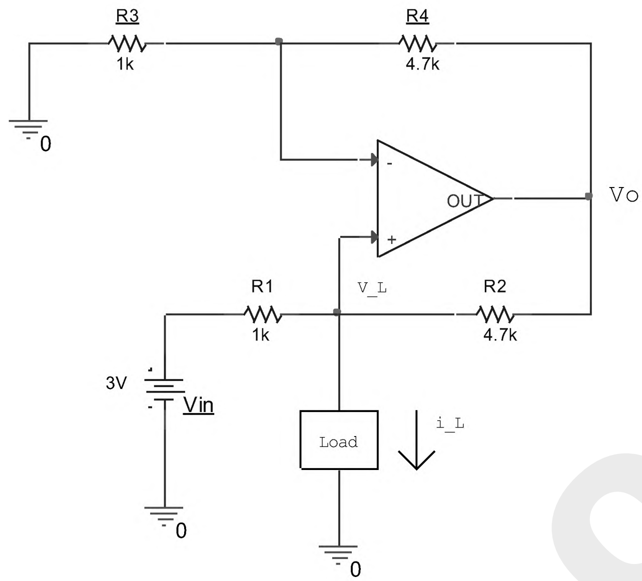

Figure 41

$$V^- = V_o\left(\frac{R_3}{R_3+R_4}\right). \tag{1}$$

Solving for V_o gives us:

$$V_o = V^-\left(1+\frac{R_4}{R_3}\right). \tag{2}$$

Carrying out nodal analysis at the node V_L gives us:

$$\frac{V_L - V_{in}}{R_1} + i_L + \frac{V_L - V_o}{R_2} = 0. \tag{3}$$

Plugging in equation (2) into equation (3) and knowing that $V^- = V_L$ we get,

$$\frac{V_L}{R_1} - \frac{V_{in}}{R_1} + i_L + \frac{V_L}{R_2} - \frac{\left(V_L\left(1+\frac{R_4}{R_3}\right)\right)}{R_2} = 0. \tag{4}$$

Expanding the brackets in the last term and knowing that $R_4 = R_2$ and $R_3 = R_1$, we get:

$$\frac{V_L}{R_1} - \frac{V_{in}}{R_1} + i_L + \frac{V_L}{R_2} - \frac{V_L}{R_2} - \frac{V_L R_2}{R_1 R_2} = 0$$

Cancelling out the necessary terms and isolating for i_L gives us,

$$i_L = \frac{V_{in}}{R_1} = \frac{3V}{1k\Omega} = 3mA. \tag{5}$$

This shows us that regardless of the load, the current is dependent only on the input voltage and R_1.

xxix. Build the circuit in Figure 1, with the load being a 10Ω resistor and $V_{cc} = \pm 5V$. Plot V_L (the voltage across the load) using the oscilloscope tool on the Analog Discovery 2.

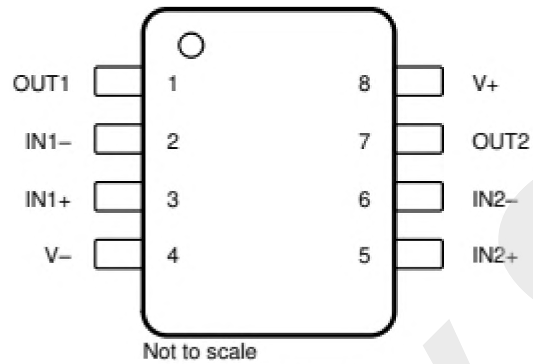

Figure 42: LM358P.

Solution:

Assemble the circuit components. Connect the lead **2+** to the node V_L, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the load voltage will be plotted on channel 2. Set the time base to 1 ms/div, and the range on channel 2 to 500 mV/div. The resulting waveform is shown below:

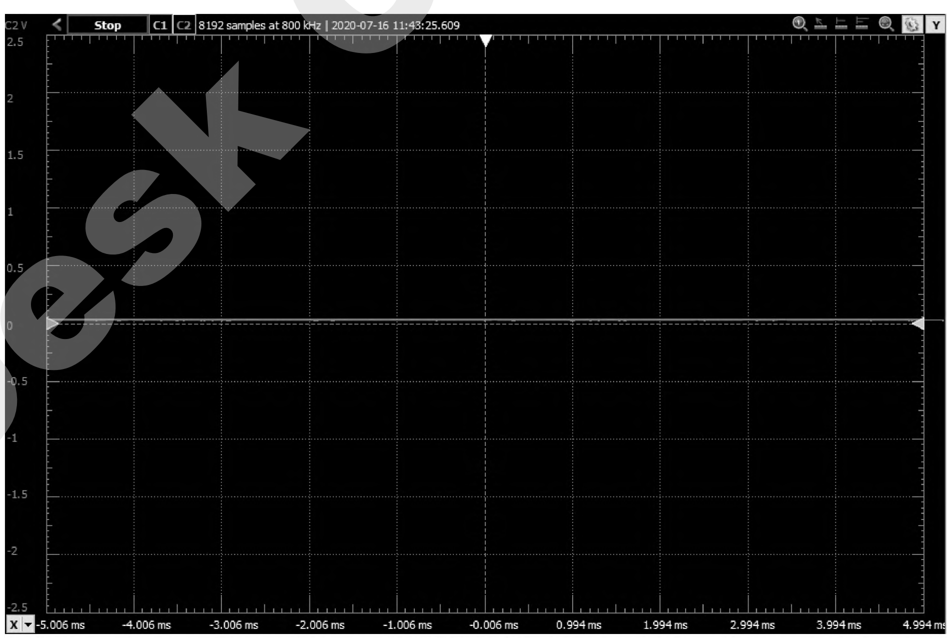

Figure 43

Using the circuit from part i, estimate the current going through the load using the Analog Discovery 2. Compare the theoretical result to the actual.

Solution:

Using the maximum tool on the Analog Discovery 2, we measure the voltage to be around 0.033V. By using Ohm's law to isolate for the current, $i_L = V_L/R = 0.033V/10\Omega = 3.3mA$. The theoretical was $3mA$, which is close to the measured.

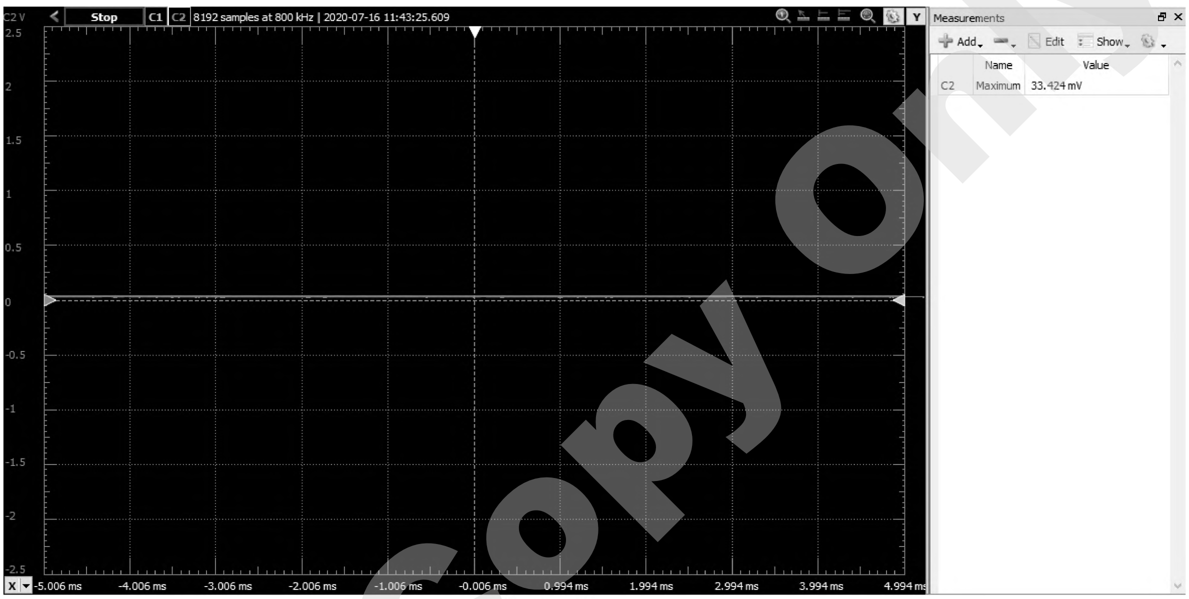

Figure 44

Experiment:

Consider the shown circuit.

a) Show that the current flowing through the load resistance R_L is ideally independent from the value of that resistor.

b) Evaluate analytically the current in R_L for the R_L values shown in the table for the values R_1 = 1KΩ and Vcc=5V.

b) Construct the shown circuit and measure the current through the resistance R_L using the same values in part (b).

c) Compare your analytical versus measured results. Comment on your results.

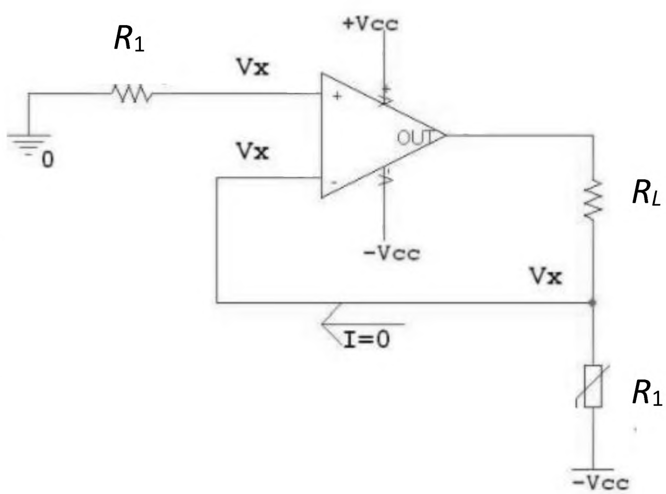

Table 3

R_L	I_L(analytical)	I_L(measured)
1KΩ		
3KΩ		
6KΩ		
10KΩ		
50 KΩ		

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 9)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment
- Analog Discovery 2

Components
- 1 x 290kΩ resistor
- 3 x 100nF capacitor
- 4 x 1nF capacitor
- 3 x 10kΩ resistor
- 4 x 1kΩ resistor
- Op-Amp LM358P

Objective
- Calculate the oscillation frequency given an oscillator circuit
- Plotting voltage waveforms for an oscillator circuit
- Estimating the period of an oscillator circuit
- Measuring the period of an oscillator circuit

Background:

There are different kinds of oscillator circuits in the real world. In his experiment, you will look at a phase shift oscillator circuit. A phase shift oscillator circuit shifts an input signal by 180 degrees. It does that through stages such as the RC network shown in Figure 1. Since each RC sage shifts an input signal by 60 degrees, a total of 3 sections are needed to shift the signal to 180 degrees.

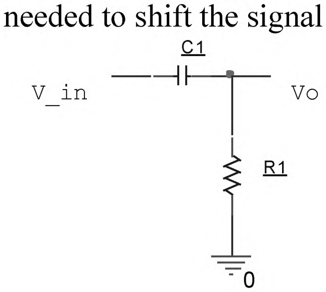

Figure 45

Example:

Find the oscillation frequency and period of the following phase shift oscillator.

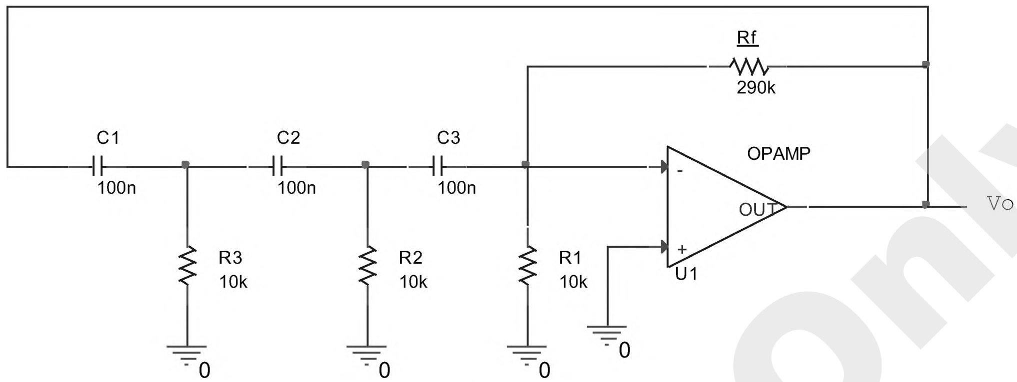

Figure 46

Solution:

Let us take the RC network and analyze it as shown in Figure 2,

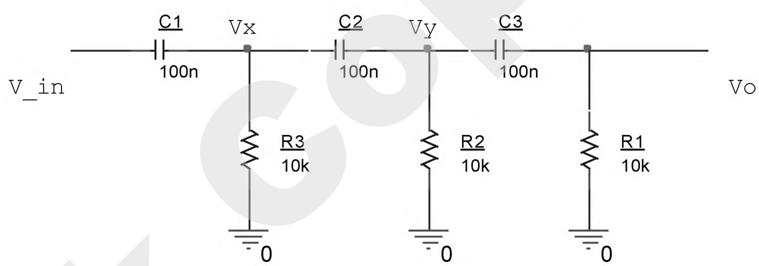

Figure 47

We will find the equation for $\frac{V_o}{V_{in}}$ by first using KCL at node V_o. Looking at the circuit, $C = C_1 = C_2 = C_3$ and $R = R_1 = R_2 = R_3$,

$$\frac{V_o}{R} + (V_o - V_y)SC = 0. \tag{1}$$

Isolating V_y gives us:

$$V_y = \frac{\left(\frac{1}{R} + SC\right)}{SC} V_o. \tag{2}$$

Next, we will use KCL at node V_y:

$$(V_y - V_x)SC + \frac{V_y}{R} + (V_y - V_o)SC = 0. \tag{3}$$

Plugging equation (2) into equation (3) and collecting and simplifying terms gives us:

$$V_x = \frac{\left(\frac{2}{R} + SC + \frac{\frac{1}{R} + SC}{SRC}\right)}{SC} V_o. \tag{4}$$

Finally, we will use KCL at node V_x and plugging in equations 2 and 4 and simplifying terms gives us,

$$\frac{V_o}{V_{in}} = \frac{SC}{\frac{2}{R} + SC + \frac{\frac{1}{R} + SC}{SRC} + \frac{\frac{2}{R} + SC + \frac{\frac{1}{R} + SC}{SRC}}{SRC} + \frac{1}{R} + \frac{\frac{1}{R} + SC}{SRC}}.$$

Having a common denominator and further simplifying the equation gives us:

$$\frac{V_o}{V_{in}} = \frac{1}{\frac{1}{S^3R^3C^3} + \frac{5}{S^2C^2R^2} + \frac{6}{SCR} + 1}. \tag{5}$$

We will then let $S = j\omega$ and isolate for V_{in} which gives us:

$$V_{in} = V_o\left(-\frac{1}{j\omega^3R^3C^3} - \frac{5}{\omega^2R^2C^2} + \frac{6}{j\omega RC} + 1\right).$$

Multiplying by j in the numerator and denominator of only the imaginary terms gives us:

$$V_{in} = V_o\left(\frac{j}{\omega^3R^3C^3} - \frac{5}{\omega^2R^2C^2} - \frac{6j}{\omega RC} + 1\right).$$

Setting the imaginary terms to 0, we get:

$$\frac{1}{\omega^3R^3C^3} - \frac{6}{\omega RC} = 0.$$

Solving for ω and the frequency f gives us,

$$\omega = \frac{1}{\sqrt{6}RC},$$

$$f = \frac{1}{2\pi RC\sqrt{6}}. \tag{6}$$

Plugging in the values for R and C gives us 64.97 Hz and since the period is $1/f$ this gives us 15.4 ms.

Build the circuit in Figure 1 with $V_{cc} = \pm 2V$. Measure the period of the output voltage and compare the theoretical to the actual.

Assuming the proper circuity has been implemented, connect the lead **2+** to the node V_o, and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the inductor voltage will be plotted on channel 2. Set the time base to 5 ms/div, and the range on channel 1 and 2 to 500 mV/div. The resulting waveform is shown below:

Figure 48

Using the quick measure tool on the Analog Discovery 2 to measure the period or one cycle of the output signal as shown below,

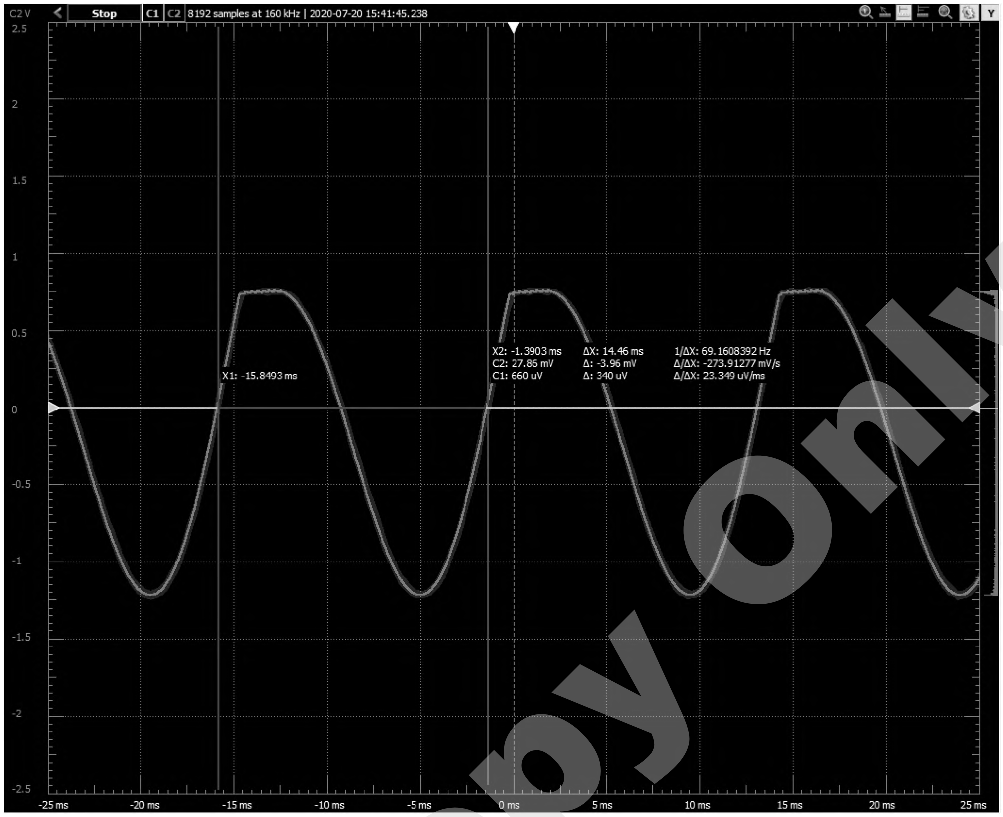

Figure 49

We notice that the period is around 14.46 ms and the frequency is 69.16 Hz, which is close to the theoretical result from part i.

Experiment:

ix. Find the oscillation frequency and the period in the following phase shift oscillator circuit.

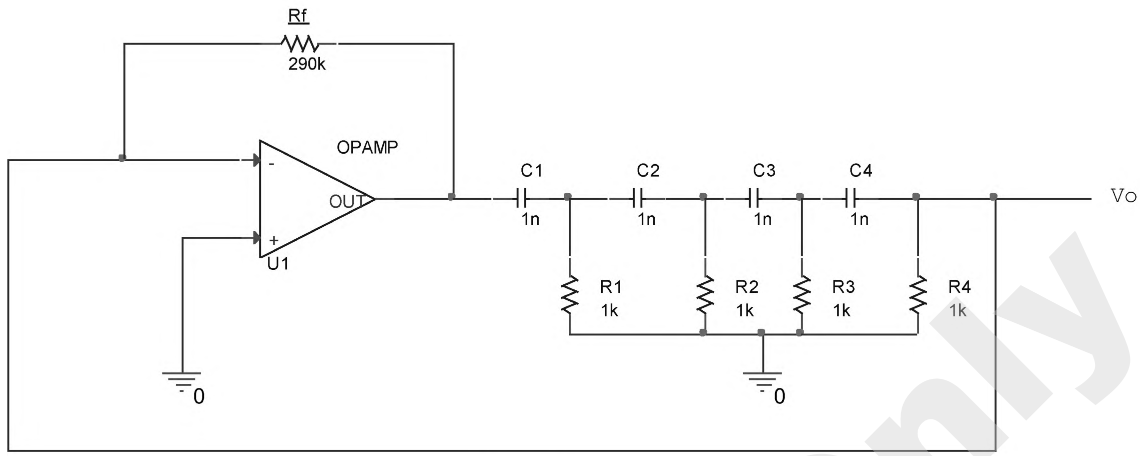

Figure 50

x. Build the circuit in Figure 5 with $V_{cc} = \pm 2V$. Measure the period of the output voltage and compare the theoretical to the actual.

xi. What happens if you change the values of R_1, R_2, R_3, and R_4 to $10k\Omega$? What output voltage do you get? Measure the period of the new output voltage.

xii. What is the purpose of R_f?

ELEC ENG 2CJ4
Circuits and Systems

Term II, January – April 2021

Laboratory Experiments (Set 10)

Prepared by: Dr. M. H. Bakr and Y. Asham

Test Equipment

- Analog Discovery 2

Components

- 1 x 50kΩ resistor
- 1 x 35kΩ resistor
- 1 x 30kΩ resistor
- 2 x 0.01uF capacitor
- 1 x 1MΩ resistor
- 1 x 680kΩ resistor
- 2 x 10 kΩ resistor
- 1 x 100kΩ resistor
- 1 x 200kΩ resistor
- Op-Amp LM358P

Objective

- To learn how to analyze First-order circuits.
- To learn how different circuits can be integrated into another circuit

---

Background:

Natural and Forced Responses

Consider the following first-order differential equation with constant coefficients a_0 and a_1 such that $a_1 \neq 0$,

$$a_1 x'(t) + a_0 x(t) = f(t) \tag{1}$$

The solution to (1) is denoted by $x(t) = x_n(t) + x_f(t)$, where $x_n(t)$ and $x_f(t)$ are called the natural response and the forced response, respectively. In particular, $x_n(t)$ is the solution to the following homogeneous equation

$$a_1 x'(t) + a_0 x(t) = 0 \tag{2}$$

and it takes the form of $x_n(t) = Ke^{St}$, where K and S are constants. Substituting the expression of $x_n(t)$ back to (2), we have

$$(a_1 S + a_0)Ke^{St} = 0 \tag{3}$$

As $K \neq 0$ and $e^{St} > 0$ (otherwise the solution is trivial), we have

$$S = -\frac{a_0}{a_1} \tag{4}$$

The value of K can be determined by the initial condition on $x(0^+)$ and $f(0^+)$, which will be discussed later. Some functional forms of the forced solution are listed here for your reference.

Table 4: Forms of forced solutions

$f(t)$	Trial Function
a	A
$at + b$	$At + B$
$at^n + bt^{n-1} + \cdots$	$At^n + Bt^{n-1} + \cdots$
$ae^{\sigma t}$	$Ae^{\sigma t}$
$a\cos wt + b\sin wt$	$A\cos wt + B\sin wt$

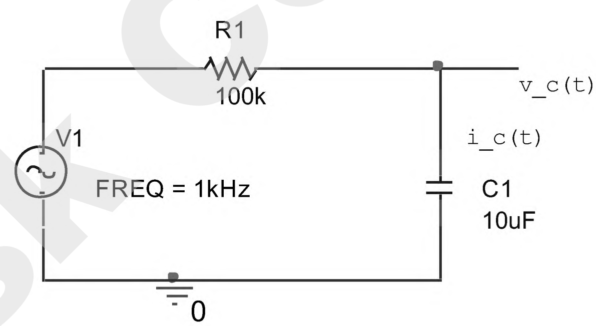

Figure 51: A first-order RC Circuit

Solving the first-Order RC Circuits with DC Source

Consider the first-order RC circuit depicted in Fig. 1. It follows by the KVL and the property of capacitor that

$$R_1 i_c(t) + v_c(t) = v_e(t), \tag{5a}$$
$$i_c(t) = C_1 v_c'(t), \tag{5b}$$

where $i_c(t)$ is the charging current of C_1, $v_c(t)$ is the end-to-end voltage of C, and $v_e(t)$ is the value of the voltage source. Hence the voltage $v_c(t)$ can be fully characterized by the first order differential equation

$$R_1C_1v_c'(t)+v_c(t) = v_e(t). \tag{6}$$

If we know the expression of $v_e(t)$ and the initial conditions, then $v_c(t)$ can be solved in closed-form.

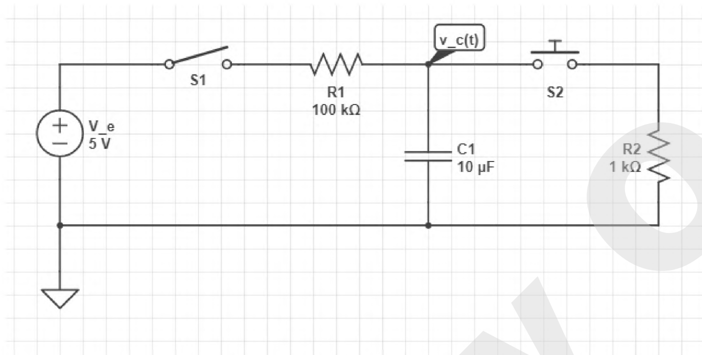

Figure 52: A first-order RC circuit with DC source

In what follows, we consider the first-order circuit with DC source (step response) in Fig. 2. First press S_2 for a while, and then close switch S_1 at time $t = t_0$. We assume that

$$v_c(t = t_0^+) = V_c,$$
$$v_e(t = t_0^+) = V_e,$$

where V_c is the initial voltage of C_1. Clearly, if we press S_2 for sufficiently long time with S_1 open-loop, then $V_c = 0$. From (6), for $t \geq t_0^+$, we have

$$R_1C_1v_c'(t)+v_c(t) = v_e(t), \qquad t \geq t_0^+, \tag{7}$$

where

$$v_e(t) = \begin{cases} 0, & t < t_0 \\ V_e, & t \geq t_0 \end{cases}$$

In view of (4), the natural response to $v_c(t)$ is given by

$$v_{c,n}(t) = Ke^{S(t-t_0)}, \tag{8}$$

where $S = -\frac{1}{R_1C_1}$, and K can be determined by the initial conditions. From table 1, we find that the forced response is a ocnstant number; as such we denote $v_{e,f}(t) = A, t > t_0^+$. Since

$$R_1C_1v_{c,f}'(t)+v_{c,f}(t) = V_e, \qquad t \geq t_0^+,$$

it follows that $v_{c,f}(t) = V_e, t \geq t_0^+$. Note that

$$v_c(t) = v_{c,n}(t) + v_{c,f}(t) = Ke^{-\frac{t-t_0}{R_1C_1}} + V_e. \tag{9}$$

For $t = t_0^+$, we have $v_c(t_0^+) = K + V_e = V_c$, which implies $K = V_c - V_e$. Therefore,

$$v_c(t) = (V_c - V_e)e^{-\frac{t-t_0}{R_1C_1}} + V_e,\ t > t_0^+, \tag{10}$$

which is illustrated in Fig. 3.

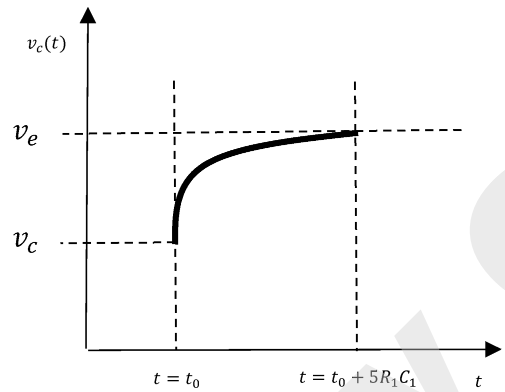

Figure 53: Charging Process for the first-order circuit with DC source

First-order RC Circuit in the Relaxation Oscillator

The relaxation oscillator is an important circuit that uses a Schmitt trigger to alternately charge and discharge the capacitor. And it is widely used in low frequency function signal generators. In this experiment, we will study the first-order RC circuit in the relaxation oscillator.

The Principle of the Relaxation Oscillator

Consider the relaxation oscillator depicted in Fig. 4, where the Schmitt trigger is used to alternately charge and discharge the capacitor C through the resistor R_3. Without loss of generality, we suppose that, initially, the output $v_{out} = V_{sat}^+$; as a consequence, $v_c(t) < V_{th_1} < V_{sat}^+$ (can you explain why?), and the capacitor is charged. When $v_c(t)$ reaches V_{th_1}, the output voltage reverses to its opposite saturation limit V_{sat}^- and the trigger threshold changes to V_{th_2}. As the output is negative now and $v_c(t) > V_{th_2} > V_{sat}^-$, the capacitor will be discahrged by R_3. Similarly, when $v_c(t)$ decreases to V_{th_2}, the output would

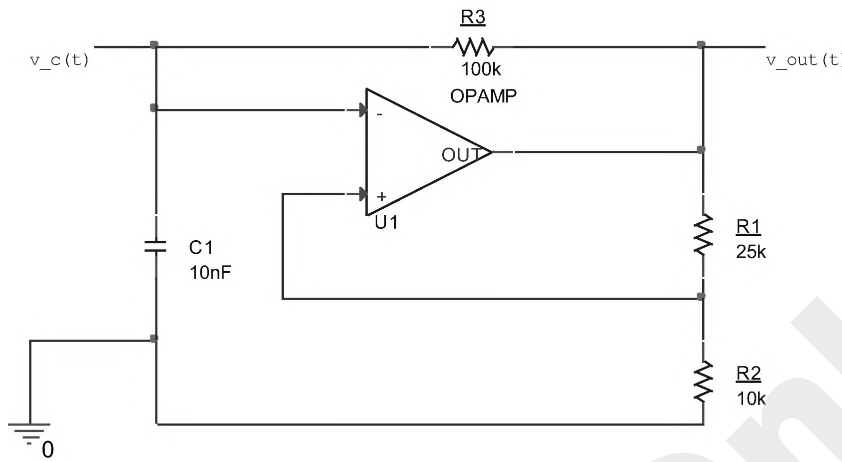

Figure 54: A simple relaxation oscillator

be triggered back to V_{sat}^{+} and the threshold moves to V_{th_1}. Consequently, the current through R_3 changes sign and the capacitor is charged again. In this way, $v_c(t)$ oscillates between V_{th_1} and V_{th_2}.

The Period (Frequency) of the Relaxation Oscillator

In what follows, we will calculate the period (frequency) of the relaxation oscillator. We shall ignore the current from and into the negative input pin of the Op-Amp. As a consequence, $R_3$and C form a first-order circuit.

- Consider the charging process from time 0. Assuming the capacitor is charged but $v_c(t) < V_{th_1}$, we have

$$R_3 C v_c'(t) + v_c(t) = v_{out}(t) \tag{11}$$

where $v_c(0^+) = V_{th_2}$ and $v_{out}(t)$ can be treated as a DC source with $v_{out}(0^+) = V_{sat}^{+}$. Now by (10), if $v_c(t) < V_{th_1}$, then

$$v_c(t) = \left(V_{th_2} - V_{sat}^{+}\right) e^{\frac{-t}{R_3 C}} + V_{sat}^{+}. \tag{12}$$

- Consider the discharging process from time 0. Assuming $v_c(t) > V_{th_2}$, we have

$$R_3 C v_c'(t) + v_c(t) = v_{out}(t) \tag{13}$$

where $v_c(0^+) = V_{th_1}$ and $V_{out}(0^+) = V_{sat}^{-}$. Hence, if $v_c(t) > V_{th_2}$, then

$$v_c(t) = \left(V_{th_1} - V_{sat}^{+}\right) e^{\frac{-t}{R_3 C}} + V_{sat}^{-} \tag{14}$$

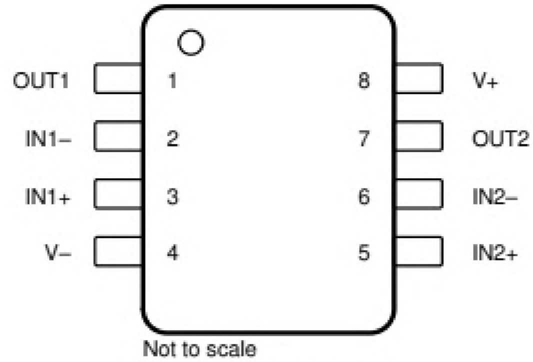

Figure 55: LM358P Datasheet

- Denote T_1 (T_2) as a time at which $v_c(t)$ changes from V_{th_2} (V_{th_1}) to V_{th_1} (V_{th_2}). We have

$$(V_{th_2} - V_{sat}^{+})e^{\frac{-T_1}{R_3C}} + V_{sat}^{+} = V_{th_1}, \quad (15a)$$

$$(V_{th_1} - V_{sat}^{-})e^{\frac{-T_2}{R_3C}} + V_{sat}^{-} = V_{th_2}, \quad (15b)$$

which implies

$$T_1 = R_3C\ln\frac{V_{sat}^{+} - V_{th_2}}{V_{sat}^{+} - V_{th_1}}, \quad (16a)$$

$$T_2 = R_3C\ln\frac{V_{sat}^{-} - V_{th_1}}{V_{sat}^{-} - V_{th_2}}. \quad (16b)$$

As a result,

$$T = T_1 + T_2 = R_3C(\ln\frac{V_{sat}^{+} - V_{th_2}}{V_{sat}^{+} - V_{th_1}} + \ln\frac{V_{sat}^{-} - V_{th_1}}{V_{sat}^{-} - V_{th_2}}) \quad (17)$$

and $f = \frac{1}{T}$.

Example:

Given the circuit in Fig. 6, calculate the period *T*, assuming that $V_{sat}^{+} = 5.0V$ and $V_{sat}^{-} = -5.0\ V$.

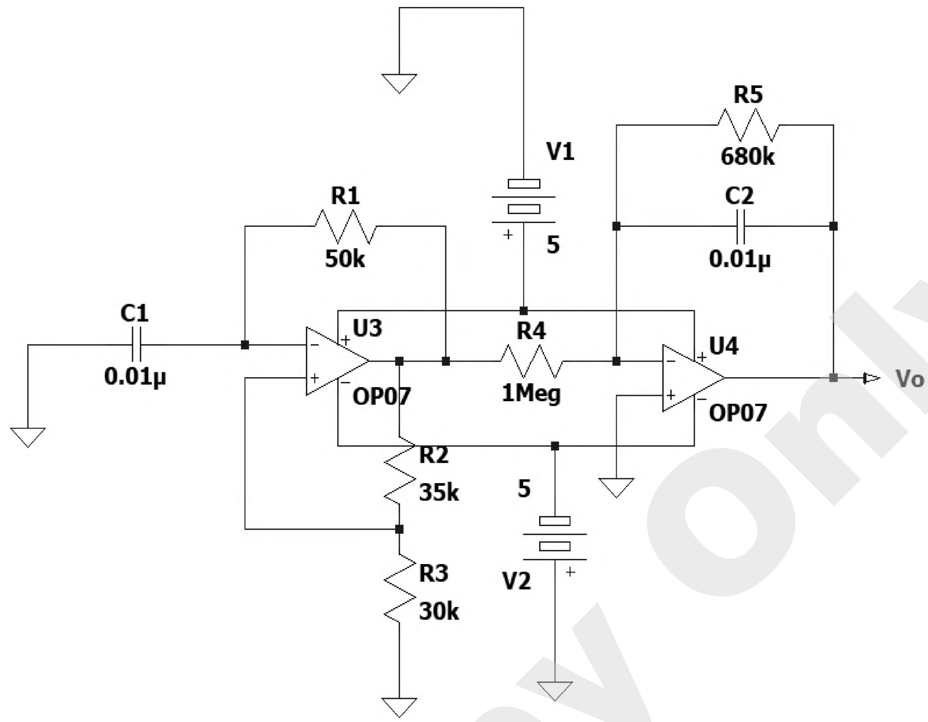

Figure 56

Solution:

The circuit in Figure 6 is actually two circuits combined into one. The first part is a relaxation oscillator circuit and the second circuit is an integrator circuit (see Figure 7).

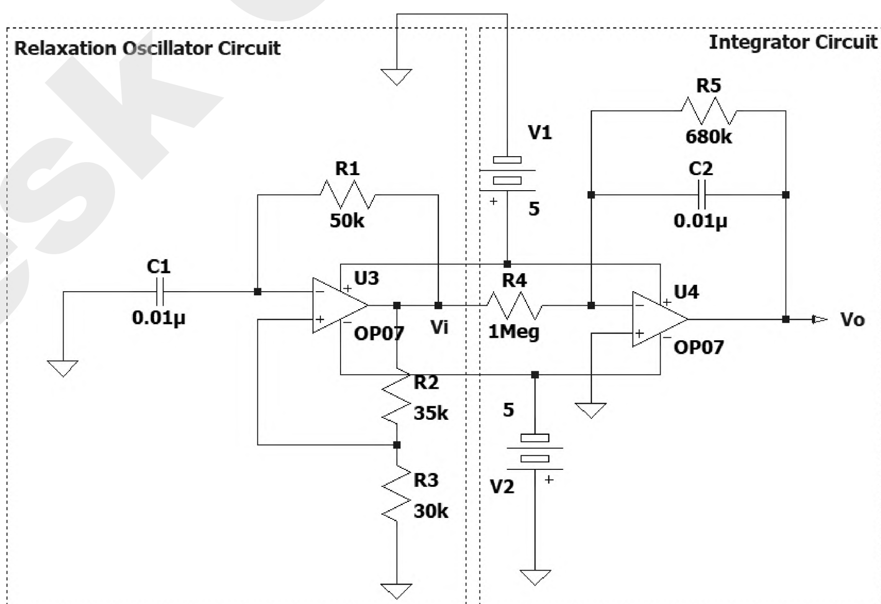

Figure 57

Using the formula from equation 17, we first need to find the values of V_{th_1} and V_{th_2}. The formula for V_{th_1} is $V_{th_1} = \frac{R_3}{R_3+R_2}V_{sat}^{+} = \frac{30k\Omega}{30k\Omega+35k\Omega} \times 5V \approx 2.31\ V$. Similarly, $V_{th_2} = -V_{th_1} = -2.31V$. therefore, the period of the oscillator circuit is $(50k\Omega)(0.01\mu F)(\ln\frac{5V-(-2.31V)}{5V-2.31V} + \ln\frac{-5V-2.31V}{-5V-(-2.31V)}) \approx 1ms$.

Recall the formula of an integrator circuit is: $v_o(t) = -\frac{1}{RC}\int_0^t v_i(x)dx + v_o(0)$, where R is the value of R_4 and C is $0.01\mu F$. Since the output of the relaxation oscillator circuit is a square wave, the output of the integrator circuit will be a triangular wave with the same period and frequency but inverted (so where the square wave is positive the triangular wave will have a negative slope and vice versa).

Build the circuit in Figure 6 and compare your theoretical results to the actual results.

Solution:

Assuming the proper circuit connections have been made, connect the lead **1+** to the node V_i, and the lead **1-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage from the relaxation oscillator will be plotted on channel 1. Then connect the lead **2+** to the node V_o(t), and the lead **2-** to any of the black wires on the device (labelled on the device as ↓). This connection means that the output voltage from the integrator circuit will be plotted on channel 2. Set the time base to 500 μs/div, and the range on channel 1 to 1 V/div and channel 2 to 500 mV/div and the resulting waveform is shown below:

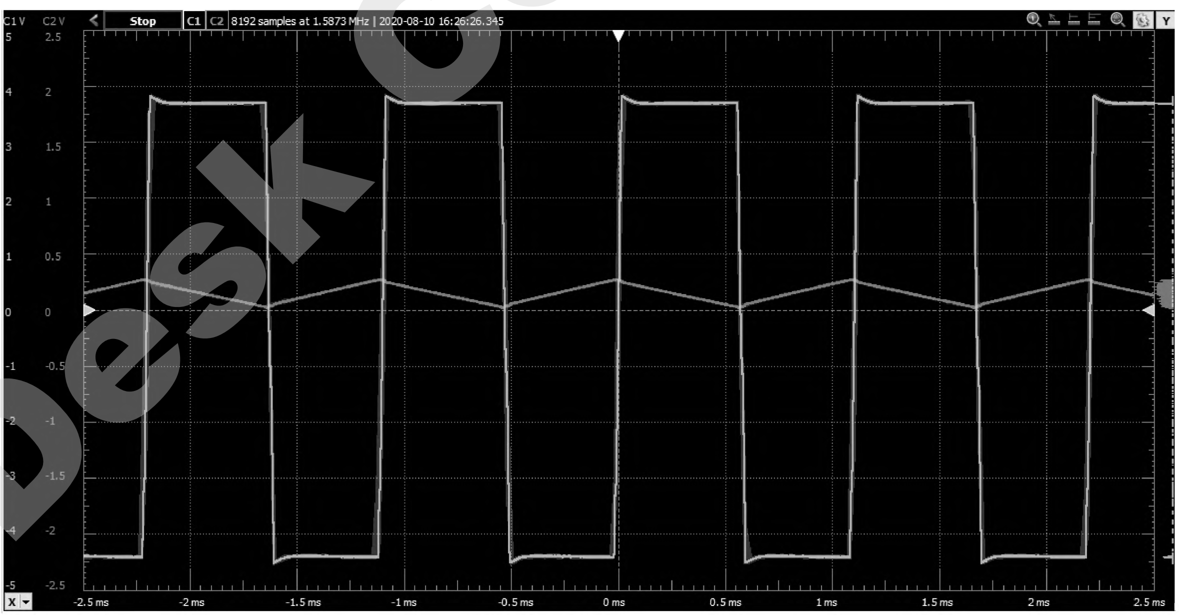

Figure 58

Using the quick measure vertical tool, measure the period of channel 1's signal and it was measured to be 1.098 ms. Comparing this value to the theoretical, they are considered close to each other.

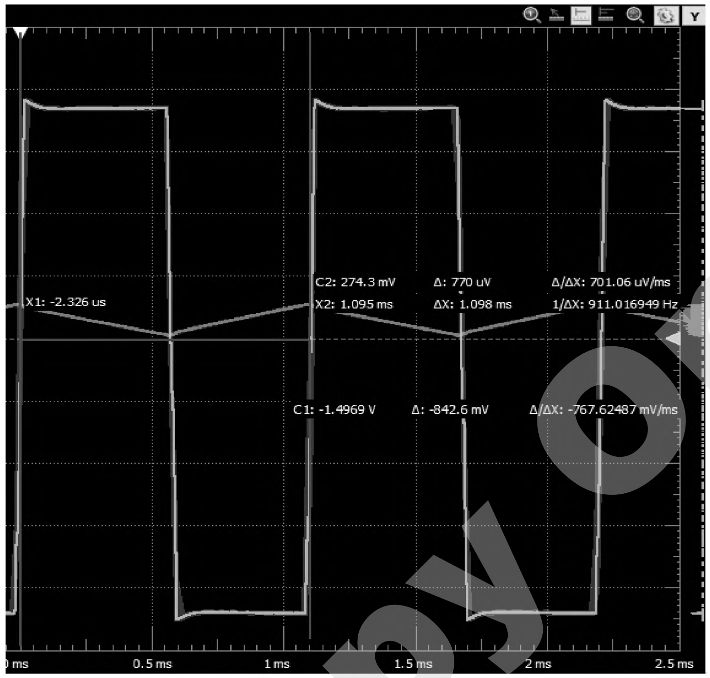

Figure 59

Experiment:

iv. Given the circuit in Figure 10, calculate the period T and frequency f.

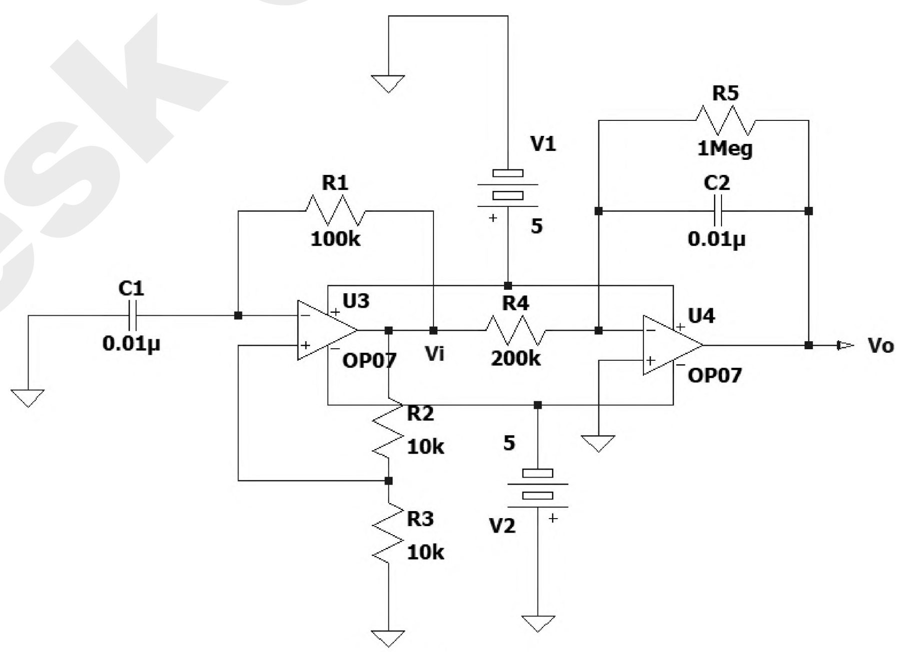

Figure 10

v. Build the circuit in Figure 10 and plot the voltage of V_i and the output voltage, V_o, with respect to time (assuming $V_{sat} = \pm 5V$). Measure the time period T using the analog discovery 2 and compare it to the theoretical result.

vi. Can you build a circuit by using another Op-Amp LM358P to generate a sinusoidal output? Explain.

vii. Can you build a circuit by using another Op-Amp LM358P to generate a square wave output? Explain.

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