# ELECENG 2CJ4

# Lab 5 Report

## Butterworth Low-Pass Filter

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Jasmine Dosanjh – L03 – dosanj5 – 400531879

Akinniyi Chidiebube – L03 – akinniyc - 400501611

## 1. Introduction

Active filters are widely used in analog circuits, i.e. in power, communication, and control systems. In this experiment, we will study the second-order Butterworth Salley-key low-pass filter.

## 2. Operational Principle of the experiment

a) Deriving an expression for the transfer function of the filter using the Sallen-Key Transfer Function with $R_4 = 0,\ R_3 = \infty\ (K = 1)$

$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + s(R_1C_1 + R_2C_1) + 1}$$

Where $R_1 = R_2,\ C_1 = C_2$

Therefore,

$$H(s) = \frac{1}{R^2C^2s^2 + 2sRC + 1}$$

Let $s = j\omega = j(2\pi f)$

$$H(j \cdot 2\pi f) = \frac{1}{R^2C^2(j \cdot 2\pi f)^2 + 2(j \cdot 2\pi f)RC + 1}$$

$$H(j \cdot 2\pi f) = \frac{1}{[1 - (2RC\pi f)^2] + j(4\pi fRC)}$$

$$|H(j \cdot 2\pi f)| = \frac{1}{\sqrt{[1 - (2RC\pi f)^2]^2 + [4\pi fRC]^2}}$$

b) The cut-off frequency of the filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(10k\Omega)(100nF)} = 159.15\ Hz$$

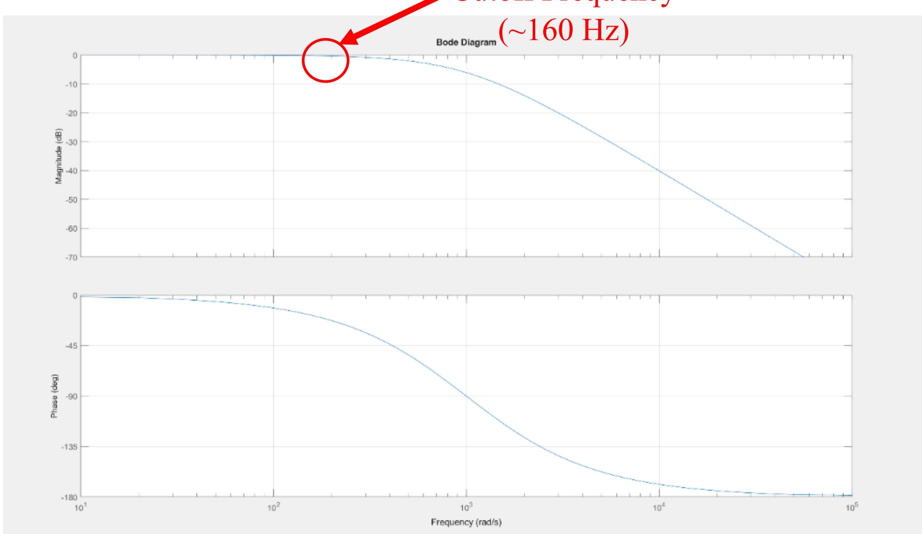
c) Evaluating the filter transfer function for the varying frequencies can be seen in the Table below.

Table 1. Theoretical Gain for Varying Frequencies

| Frequency | $Gain \left\lvert\frac{V_O}{V_I}\right\rvert$ (analytical) |
|---|---|
| 50 Hz | 0.91017 |
| 100 Hz | 0.71695 |
| 200 Hz | 0.3877 |
| 500 Hz | 0.092 |
| 1 kHz | 0.0247 |
| 1.1 kHz | 0.0205 |
| 1.2 kHz | 0.0173 |
| 1.3 kHz | 0.0147 |
| 1.4 kHz | 0.0128 |
| 1.5 kHz | 0.0111 |
| 1.6 kHz | 0.0098 |
| 1.7 kHz | 0.0087 |
| 1.8 kHz | 0.0078 |
| 1.9 kHz | 0.0070 |
| 2 kHz | 0.0063 |
| 5 kHz | 0.0010 |

The resulting waveform is shown below:

Cutoff Frequency
(~160 Hz)



## 3. Measurement results

The AD3 measurement results are included in the table below.

Table 1. Measured Gain for Varying Frequencies

| Frequency | $Gain \left\|\frac{V_O}{V_I}\right\|$ (measured) |
|---|---|
| 50 Hz | 0.9103 |
| 100 Hz | 0.72546 |
| 200 Hz | 0.403125 |
| 500 Hz | 0.098871 |
| 1 kHz | 0.027475 |
| 1.1 kHz | 0.0248446 |
| 1.2 kHz | 0.019552 |
| 1.3 kHz | 0.016714 |
| 1.4 kHz | 0.014499 |
| 1.5 kHz | 0.012661 |
| 1.6 kHz | 0.011122 |
| 1.7 kHz | 0.009964 |
| 1.8 kHz | 0.008905 |
| 1.9 kHz | 0.008006 |
| 2 kHz | 0.007211 |
| 5 kHz | 0.001385 |

Sample Measurement:

For f = 50Hz:

$$Gain = \left|\frac{V_O(amplitude)}{V_I\ (amplitude)}\right| = \frac{1.8149\ V}{1.9938\ V} = 0.9103$$

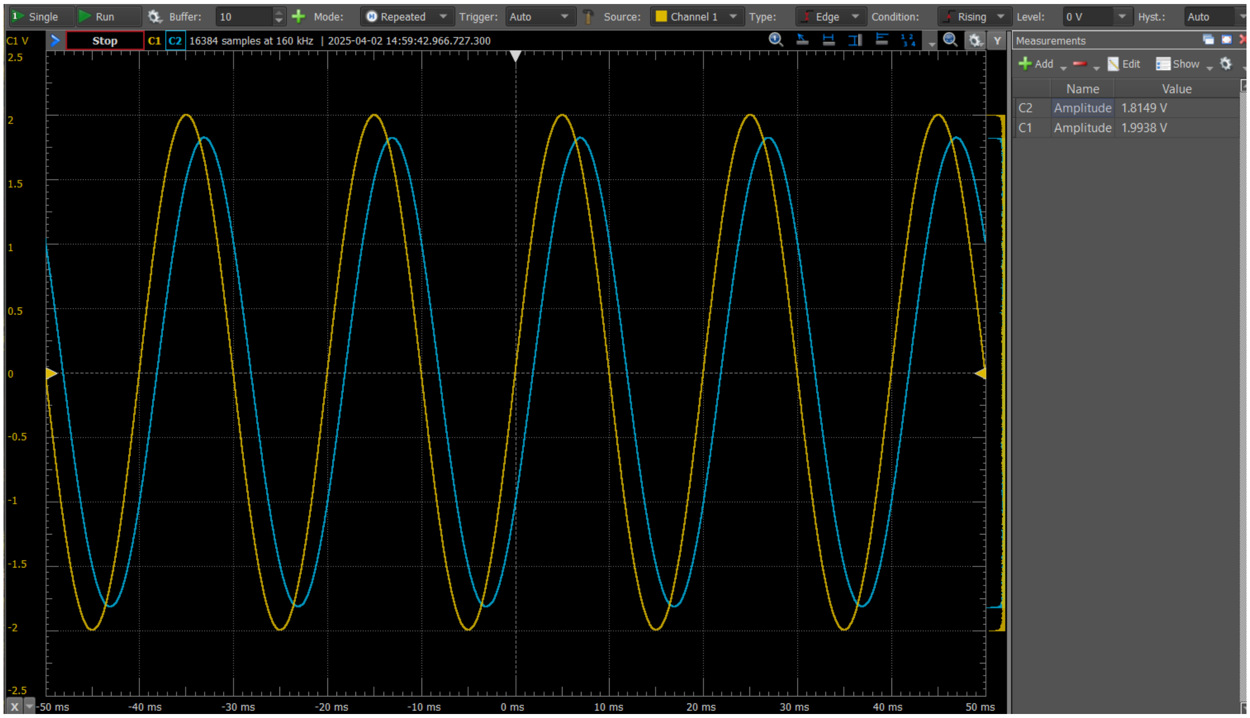



Figure 1. Resulting waveform for 50 Hz

## 4. Discussion

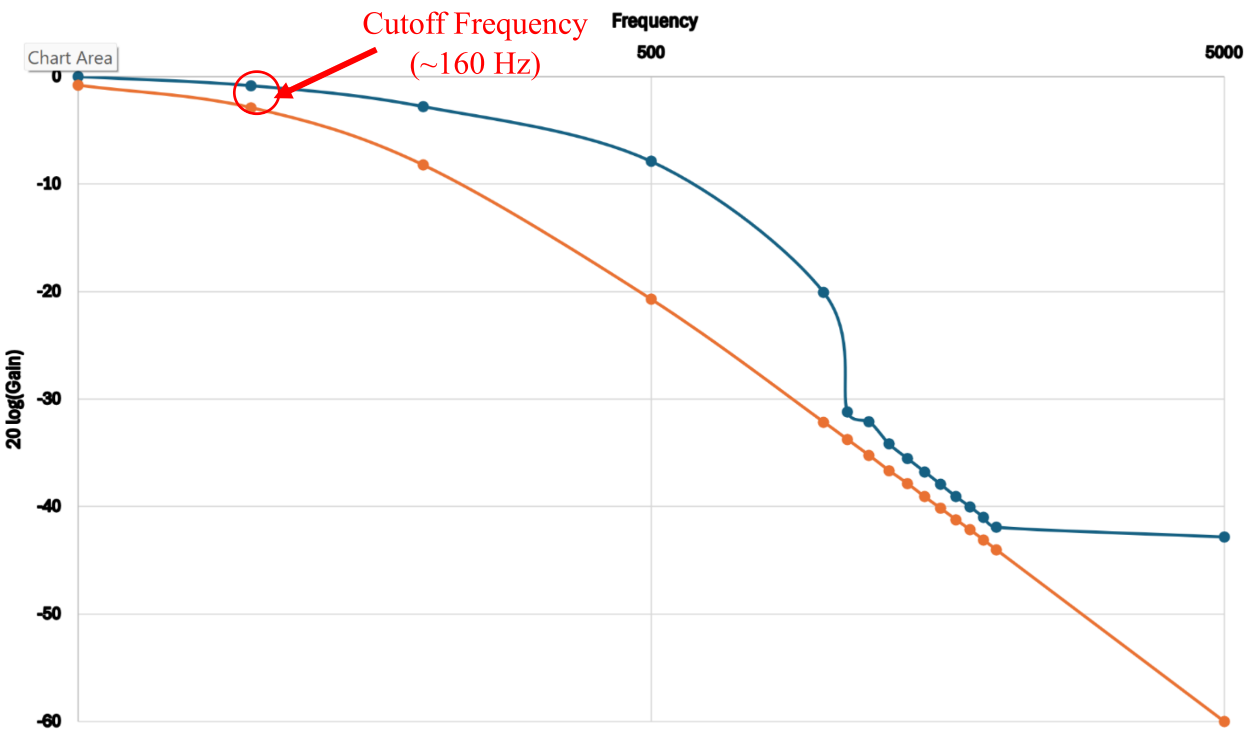



Figure 2 Graph of Measured Decibel Values against Frequency (On a logarithmic scale)

The theoretical vs measured results are very similar. This can be seen from the two graphs in the provided document. They both show that the cutoff is approx. 159.15 Hz but there is some error due to component tolerances.