

ELECENG 3EJ4

Lab. 5 Active Filter Circuits

Instructor: Dr. Chen

Jasmine Dosanjh – dosanj5 – 400531879

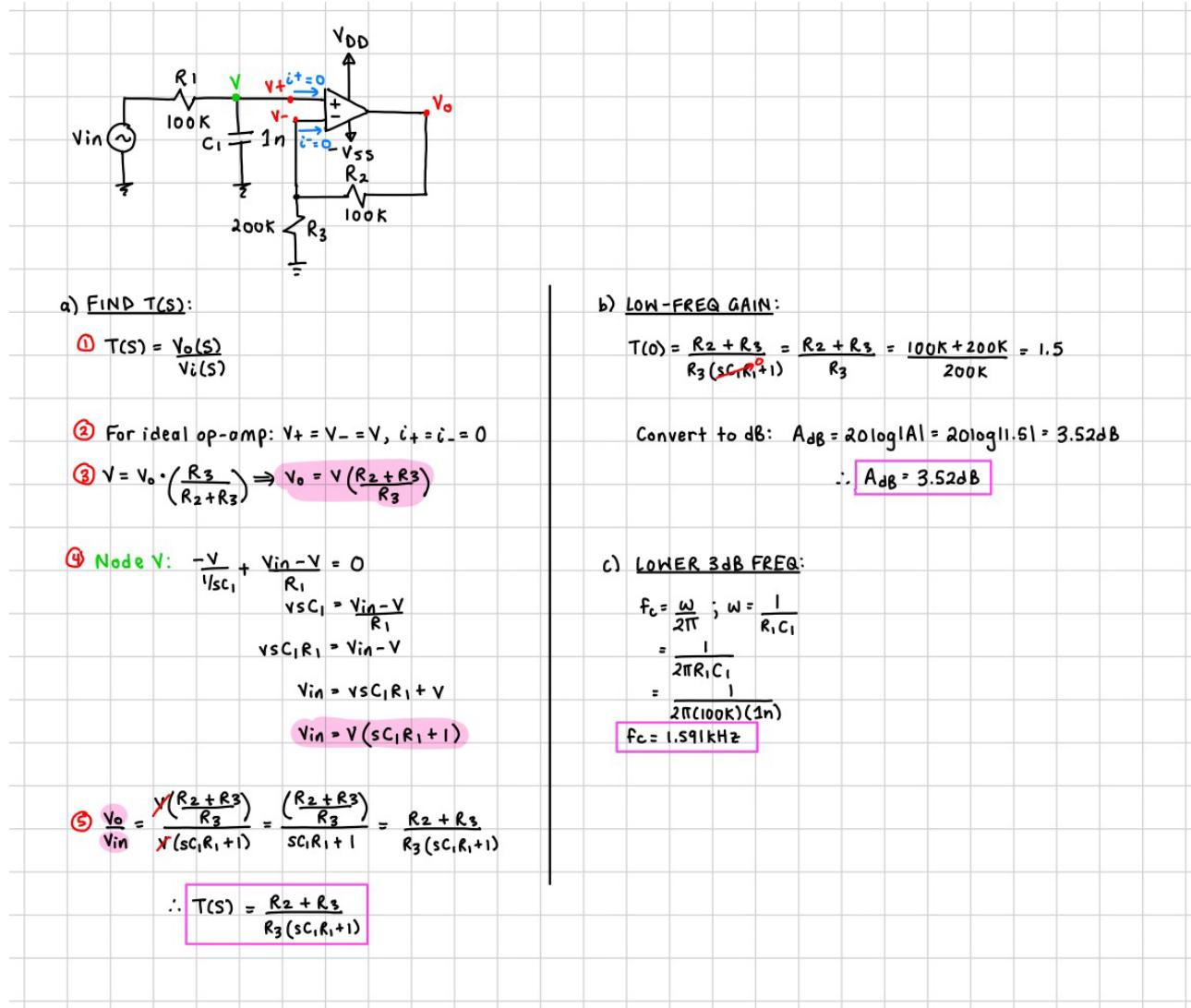
November 30, 2025

Part 1: First-Order Low-pass Filter

Q1. (20 Points)

(1) Find the transfer function of the first-order LPF, its low-frequency gain, and its -3dB frequency f_c . (2) Compare the calculated low-frequency gain and the -3dB frequency f_c with the simulated data from Step 1.3 and the measured data from Step 1.8, respectively. Justify/discuss the observation and comparison.

(1) Deriving the transfer function of the first-order LPF, its low-frequency gain, and its -3 dB frequency f_c :



(2) Next we must compare the calculated low-frequency gain and the -3 dB frequency f_c with the simulated data from Step 1.3 and measured data from Step 1.8.

Find Low-Frequency Gain:

Steps 1.3 and 1.8 gain is calculated by using the output voltage found below and the input voltage which is set to 100 mV:

	Frequency	M(V(Vout))	P(V(Vout))
	Hz	Volts	Degrees
Step 1.3	1	0.150000077	-0.036069295
Step 1.8	1	0.150369252	-0.037047176

Figure 1. Simulated and Measured Data for Gain

$$\text{Step 1.3: } A_v = T(0) = \frac{V_o}{V_{in}} = \frac{0.15 V}{0.1 V} = 1.5 \frac{V}{V} = 3.52 \text{ dB}$$

$$\text{Step 1.8: } A_v = T(0) = \frac{V_o}{V_{in}} = \frac{0.1504 V}{0.1 V} = 1.504 \frac{V}{V} = 3.54 \text{ dB}$$

Find -3 dB frequency f_c :

The -3 dB frequency f_c is the frequency at which the amplitude becomes $\frac{1}{\sqrt{2}} = 0.707$ of its low frequency value, or the phase changes 45° . This gives us:

- Step 1.3: $V_o = 0.707 \times 0.15 V = 0.1061 V \Rightarrow f_c = 1577.68 \text{ Hz}$
- Step 1.8: $V_o = 0.707 \times 0.1504 V = 0.1063 V \Rightarrow f_c = 1672.99 \text{ Hz}$

	Frequency	M(V(Vout))	P(V(Vout))
	Hz	Volts	Degrees
Step 1.3	1577.6832	0.106318853	-44.74587498
Step 1.8	1672.9903	0.106586544	-43.84864946

Figure 2. Simulated and Measured Data for f_c

By observing Table 1 below, the simulated and measured data are similar to the data obtained in Part 1. The simulated data is closer to the ideal values since it's also performed in ideal scenarios. It does not have external factors such as component tolerances, wire resistances, etc. Whereas, the physical measurement is subject to these errors, causing the measured values to have a higher deviation from the ideal values.

Table 2. Summary for Question 1 Part 2 Data

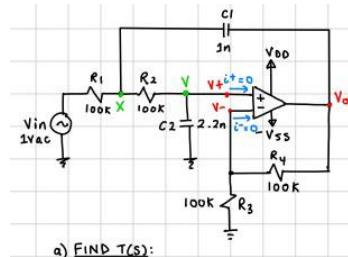
	A_{fL} (dB)	f_c (Hz)
Calculation	3.52	1591
Simulation	3.52	1577.68
Measurement	3.54	1672.99

Part 2: Second-Order Low-pass Filter

Q2. (20 Points)

(1) Derive the transfer function and calculate the low-frequency gain. (2) Verify the calculated gain using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

(1) Deriving the transfer function of the second-order LPF and its low-frequency gain:



a) FIND $T(s)$:

$$\textcircled{1} \quad T(s) = \frac{V_o(s)}{V_i(s)}$$

\textcircled{2} For ideal op-amp: $V_+ = V_- = V$, $C_+ = C_- = 0$

$$\textcircled{3} \quad V = V_o \cdot \left(\frac{R_3}{R_4 + R_3} \right)$$

$$\textcircled{4} \quad \text{Node } V: \quad -\frac{V}{sC_2} + \frac{V_x - V}{R_2} = 0$$

$$V s C_2 = \frac{V_x - V}{R_2}$$

$$V s C_2 R_2 = V_x - V$$

$$V_x = V(s C_2 R_2 + V)$$

$$V_x = V(s C_2 R_2 + 1)$$

$$V_x = V_o \cdot \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1)$$

$$\textcircled{5} \quad \text{Node } X: \quad \frac{V_{in} - V_x}{R_1} + \frac{V - V_x}{R_2} + \frac{V_o - V_x}{sC_1} = 0$$

$$V_{in} - V_x \cdot \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + V_o \cdot \left(\frac{R_3}{R_4 + R_3} \right) - V_o \cdot \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + V_o - V_o \cdot \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) = 0$$

$$V_o \left[-\left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + \frac{(R_3)}{R_1} - \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + \frac{1 - \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1)}{\frac{1}{sC_1}} \right] = -\frac{V_{in}}{R_1}$$

$$-\left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + R_1 \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + R_1 s C_1 \left(1 - \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) \right) = -\frac{V_{in}}{V_o}$$

$$-\left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + R_1 \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + R_1 s C_1 \left(1 - \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) \right) = -\frac{V_{in}}{V_o}$$

$$-\left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) + R_1 \left(\frac{R_3}{R_4 + R_3} \right) \left[1 - (s C_2 R_2 + 1) \right] + R_1 s C_1 \left(1 - \left(\frac{R_3}{R_4 + R_3} \right) (s C_2 R_2 + 1) \right) = -\frac{V_{in}}{V_o}$$

$$-\frac{R_3}{R_4 + R_3} (s C_2 R_2 + 1) - R_1 R_3 \frac{s C_2 R_2}{R_4 + R_3} + R_1 s C_1 \left(\frac{R_4 + R_3 - s R_2 R_3 C_2 - R_3}{R_4 + R_3} \right) = -\frac{V_{in}}{V_o}$$

$$-\frac{R_3 (s C_2 R_2 + 1)}{R_4 + R_3} - \frac{s R_1 R_3 C_2}{R_4 + R_3} + \frac{R_1 s C_1 (R_4 - s R_2 R_3 C_2)}{R_4 + R_3} = -\frac{V_{in}}{V_o}$$

$$-\frac{R_3 (s C_2 R_2 + 1) - s R_1 R_3 C_2 + R_1 s C_1 (R_4 - s R_2 R_3 C_2)}{R_4 + R_3} = -\frac{V_{in}}{V_o}$$

$$-\frac{R_3 s C_2 R_2 - R_3 - s R_1 R_3 C_2 + R_1 s C_1 R_4 - s^2 R_1 R_2 R_3 C_1 C_2}{R_4 + R_3} = -\frac{V_{in}}{V_o}$$

$$-\frac{s^2 R_1 R_2 R_3 C_1 C_2 - s(R_1 R_3 C_2 - C_1 R_1 R_4 + R_2 R_3 C_2) - R_3}{R_4 + R_3} = -\frac{V_{in}}{V_o}$$

$$\frac{R_4 + R_3}{s(R_1 R_2 R_3 C_1 C_2) + s(R_1 R_3 C_2 - C_1 R_1 R_4 + R_2 R_3 C_2) + R_3} = \frac{V_o}{V_{in}}$$

Substituting the known quantities:

$$T(s) = \frac{R_4 + R_3}{s^2(R_1 R_2 R_3 C_1 C_2) + s(R_1 R_3 C_2 - C_1 R_1 R_4 + C_2 R_2 R_3) + R_3}$$

$$T(s) = \frac{2 \times 10^5}{s^2(2.2 \times 10^{-3}) + s(34) + 10^5} = \frac{9.09 \times 10^7}{s^2 + s(1.55 \times 10^4) + 4.55 \times 10^7}$$

Find Low-Frequency Gain:

Using the General Form of a 2nd-order Filter:

$$T(s) = \frac{a_o}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

The gain can be found by: $A_v = T(0) = \frac{a_o}{\omega_o^2} = \frac{9.09 \times 10^7}{4.55 \times 10^7} = 1.998 \frac{V}{V} = 6.02 \text{ dB}$

(2) Next we must compare the calculated low-frequency gain with the simulated data from Step 2.2 and measured data from Step 2.6.

	Frequency	DB(V(Vout))	P(V(Vout))
	Hz	dBV	Degrees
Step 2.2	1	6.020586918	-0.122644171
Step 2.6	1	6.021863237	-0.12935246

Figure 3. Simulated and Measured Gain

All three methods provide a gain of approximately 6.02 dB, implying that the simulated and measured data align closely with the calculated gain in Part 1.

Q3. (20 Points)

Calculate (1) the pole frequency f_o , (2) the cut-off frequency (or -3dB frequency) f_c , (3) the pole quality factor Q , (4) the peak value of the magnitude of the transfer function, $|T(s)|_{max}$, and (5) the frequency f_{max} where the peak value of the magnitude of the transfer function happens. Verify the calculated f_c using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

(1) The pole frequency f_o is found below:

$$\textcircled{1} \quad f_o = \frac{\omega_o}{2\pi} = \frac{\sqrt{4.55 \times 10^7}}{2\pi} = 1074 \text{ Hz}$$

(2) The cutoff (-3 dB frequency) f_c is the frequency at which the amplitude becomes $\frac{1}{\sqrt{2}} = 0.707$ of its low frequency value. This gives us:

$$② |T(j\omega_c)| = \frac{1}{\sqrt{2}} |T(0)| = \frac{A_0}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$T(j\omega_c) = \frac{9.09 \times 10^7}{-\omega^2 + (j\omega_c)(1.55 \times 10^4) + 4.55 \times 10^7}$$

Real Imag Real

$$|T(j\omega_c)| = \left| \frac{9.09 \times 10^7}{\sqrt{(4.55 \times 10^7 - \omega_c^2)^2 + (1.55 \times 10^4 \omega_c)^2}} \right| = \frac{2}{\sqrt{2}}$$

Imag Real

$$\frac{(4.55 \times 10^7 - \omega_c^2)^2 + (1.55 \times 10^4 \omega_c)^2}{(9.09 \times 10^7)^2} = \frac{1}{2}$$

$$2.07 \times 10^{15} - 9.1 \times 10^7 \omega_c^2 + \omega_c^4 + 2.4 \times 10^8 \omega_c^2 = 4.13 \times 10^{15}$$

$$\omega_c^4 + (1.49 \times 10^8) \omega_c^2 - 2.06 \times 10^{15} = 0$$

* * *

$$a = \omega_c^2$$

$$a^2 + (1.49 \times 10^8) a - 2.06 \times 10^{15} = 0$$

$$a = 1.27 \times 10^7$$

$$\omega_c = \sqrt{a} = \pm 3563.7 \text{ rad/s}$$

$$f_c = \frac{\omega_c}{2\pi} = 567.2 \text{ Hz}$$

Verifying f_c using data obtained in Step 2.2 and Step 2.6:

The calculated cutoff frequency $f_c = 567.2 \text{ Hz}$ occurs at a cutoff gain of:

$$A_c = 20 \log \frac{2}{\sqrt{2}} = 3.01 \text{ dB}$$

Figure 4 shows that all three methods corresponding to this cutoff frequency occur a gain of approximately 3.01 dB, implying that the simulated and measured data align closely with the calculations above.

	Frequency	DB(V(Vout))	P(V(Vout))
	Hz	dBV	Degrees
Step 2.2	565.5555	3.036726579	-59.17249618
Step 2.6	567.0174	2.961067521	-57.67976626

Figure 4. Simulated and Measured Cutoff Frequency

(3) The pole quality factor Q is found below:

$$③ \frac{\omega_0}{Q} = 1.55 \times 10^4$$

$$Q = \frac{\omega_0}{\omega_0/Q} = \frac{\sqrt{4.55 \times 10^7}}{1.55 \times 10^4} = 0.435$$

(4) The peak value of the magnitude of the transfer function $|T(s)|_{max}$ is found below. Since $Q < \frac{1}{\sqrt{2}}$, the system has no bump at the pole frequency, and thus the maximum is at DC where $\omega = 0$:

$$\textcircled{4} \quad |T(s)|_{max} = |T(j0)| = |T(0)| = 6.02 \text{ dB}$$

(Same as Q2 Part 1)

(5) The frequency f_{max} where the peak value of the magnitude of the transfer function happens occurs at 0 Hz ($\omega = 0$). This value then drops at the pole frequency $f_o = 1074 \text{ Hz}$.

This can be verified by observing the simulated data from Step 2.2 (Figure 4) and measured data from Step 2.6 (Figure 5). The peak of the Transfer function occurs at 0 Hz, with a gain of approximately 6.02 dB. This gain stays constant until the pole frequency is reached around 1074 Hz.

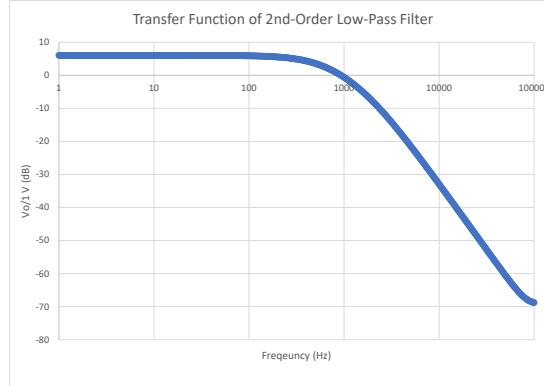


Figure 4. Step 2.2 Simulated Transfer Function

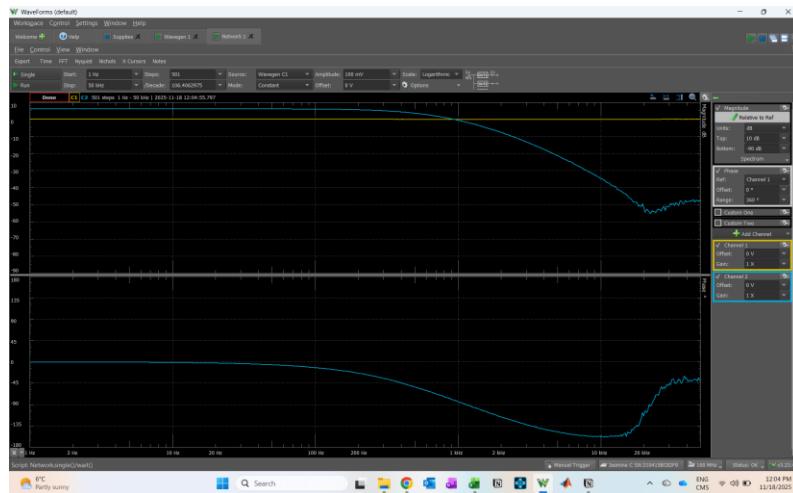


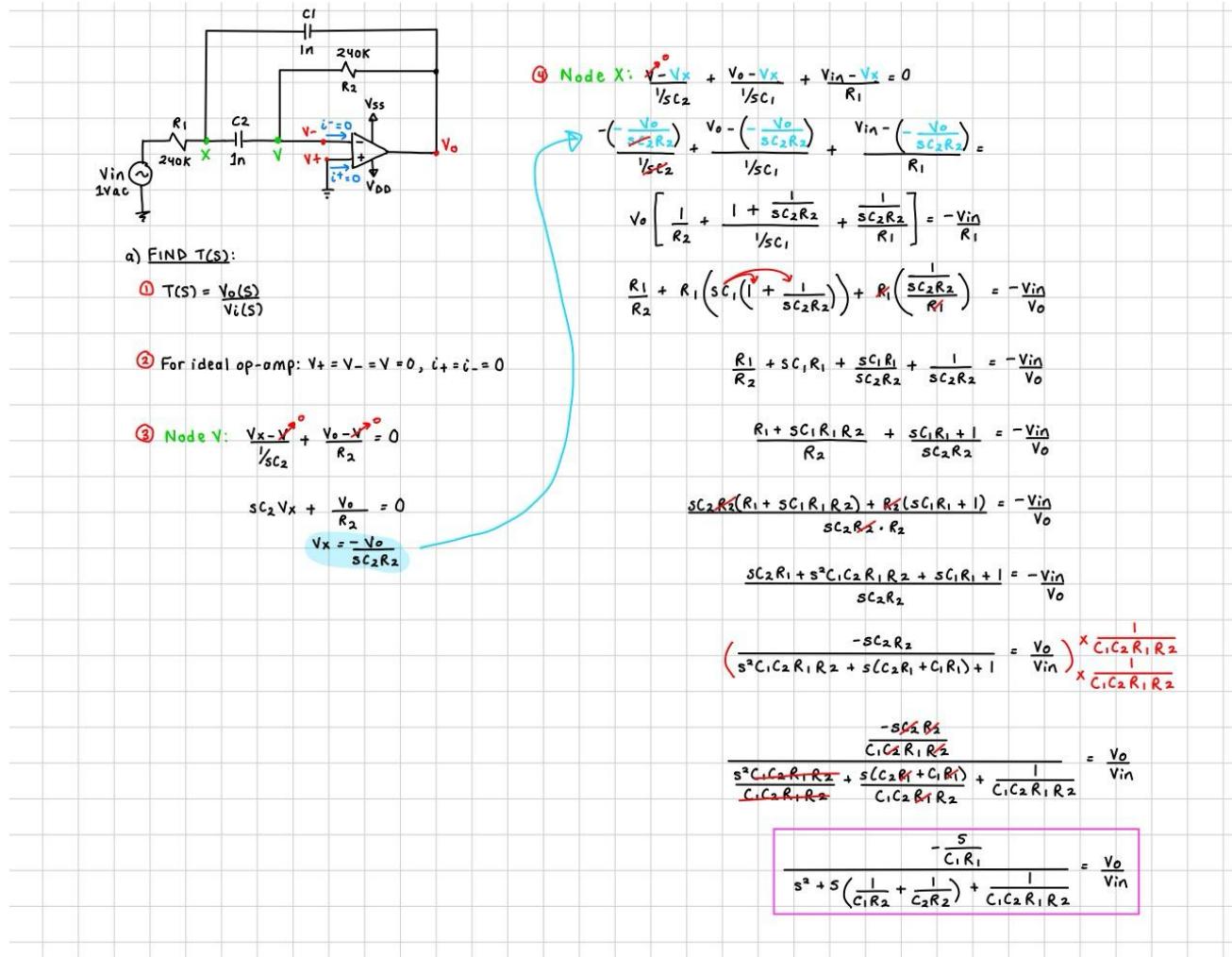
Figure 5. Step 2.6 Physical Implementation of Transfer Function

Part 3: Second-Order Bandpass Filter

Q4. (20 Points)

(1) Derive the transfer function and calculate the center frequency gain. (2) Verify the calculated gain using the simulated data obtained in Step 3.2 and the measured data obtained in Step 3.6, respectively.

(1) Deriving the transfer function of the second-order BPF, and center-frequency gain:



Substituting the known quantities:

$$T(s) = \frac{-\frac{s}{C_1 R_1}}{s^2 + s \left(\frac{1}{C_1 R_2} + \frac{1}{C_2 R_2} \right) + \frac{1}{C_1 C_2 R_1 R_2}}$$

$$T(s) = \frac{-s(4.167 \times 10^3)}{s^2 + s(8.3 \times 10^3) + 1.74 \times 10^7}$$

Find Center-Frequency Gain:

$$\omega_o = \sqrt{1.74 \times 10^7} = 4.171 \times 10^3 \text{ rad/s}$$

$$f_o = \frac{\omega_o}{2\pi} = 663.89 \text{ Hz}$$

The gain can be found by:

$$\begin{aligned} T(j\omega_o) &= \frac{-j\omega_o(4.167 \times 10^3)}{-\omega^2 + (j\omega_o)^2 + (j\omega_o)(8.3 \times 10^3) + 1.74 \times 10^7} \\ |T(j\omega_o)| &= \left| \frac{\omega_o(4.167 \times 10^3)}{\sqrt{(8.3 \times 10^3 \omega_o)^2 + (1.74 \times 10^7 - \omega_o^2)^2}} \right| \\ |T(j\omega_o)| &= \left| \frac{(4.167 \times 10^3)^2}{\sqrt{(8.3 \times 10^3)^2 (4.171 \times 10^3)^2 + ((1.74 \times 10^7 - (4.171 \times 10^3)^2)^2}}} \right| \\ |T(j\omega_o)| &= \frac{1.74 \times 10^7}{\sqrt{1.2 \times 10^{15} + 7.6 \times 10^6}} \\ |T(j\omega_o)| &= 0.5 \frac{V}{V} \\ A_o &= 20 \log 0.5 = -6.02 \text{ dB} \end{aligned}$$

(2) Verifying gain using the Step 3.2 and in Step 3.6 data:

Next we must verify the calculated low-frequency gain with the simulated data from Step 3.2 and measured data from Step 3.6.

	Frequency	DB(V(Vout))	P(V(Vout))
	Hz	dBV	Degrees
Step 3.2	663.4102514	-6.020589659	-180.0370809
Step 3.6	660.693448	-6.403270923	-179.1844497

Figure 6. Simulated and Measured Gain

All three methods provide a gain of approximately -6.02 dB at $f_o \approx 663 \text{ Hz}$, implying that the simulated and measured data align closely with the calculated gain in Part 1.

Q5. (20 Points)

Calculate (1) the center frequency ω_0 , (2) the pole quality factor Q , (3) the two -3dB frequencies ω_1 and ω_2 , and (4) the 3-dB bandwidth $BW = \omega_2 - \omega_1$. Verify the calculated results

(1) The center frequency ω_0 is found below:

$$\textcircled{1} \quad \omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} = \sqrt{\frac{1}{(240 \times 10^3)^2 (1 \times 10^{-9})^2}} = 4.167 \times 10^3 \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = 663.15 \text{ Hz}$$

(2) The pole quality factor Q is found below:

$$\textcircled{2} \quad T(s) = \frac{a_1 s}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

$$\frac{\omega_0}{Q} = 8.3 \times 10^3$$

$$Q = \frac{\omega_0}{\omega_0/Q} = \frac{4.167 \times 10^3}{8.3 \times 10^3} = 0.502$$

(3) The two -3dB (cutoff) frequencies ω_1 and ω_2 are the frequencies at which the amplitude becomes $\frac{1}{\sqrt{2}} = 0.707$ of its low frequency value. This gives us:

$$\textcircled{3} \quad |T(j\omega_{3dB})| = \frac{1}{\sqrt{2}} |T(0)| = \frac{A_U}{\sqrt{2}} = \frac{0.5}{\sqrt{2}}$$

$$|T(j\omega_{3dB})| = \left| \frac{\omega_{3dB} (4.167 \times 10^3)}{\sqrt{(8.3 \times 10^3 \omega_{3dB})^2 + (1.74 \times 10^7 - \omega_{3dB}^2)^2}} \right| = \frac{0.5}{\sqrt{2}}$$

$$\frac{(0.5)^2}{2} = \frac{(\omega_{3dB} (4.167 \times 10^3))^2}{(8.3 \times 10^3 \omega_{3dB})^2 + (1.74 \times 10^7 - \omega_{3dB}^2)^2}$$

$$\frac{1}{8} = \frac{\omega_{3dB}^2 (1.74 \times 10^7)}{(6.89 \times 10^7) \omega_{3dB}^2 + 3.02 \times 10^{14} - 3.48 \times 10^7 \omega_{3dB}^2 + \omega_{3dB}^4}$$

$$(6.89 \times 10^7) \omega_{3dB}^2 + 3.02 \times 10^{14} - 3.48 \times 10^7 \omega_{3dB}^2 + \omega_{3dB}^4 = (1.39 \times 10^8) \omega_{3dB}^2$$

$$\omega_{3dB}^4 - (1.05 \times 10^8) \omega_{3dB}^2 + 3.02 \times 10^{14} = 0$$

$$\alpha^2 - (1.05 \times 10^8) \alpha - 3.02 \times 10^{14} = 0$$

$$\alpha = \omega_{3dB}^2$$

$$\alpha_1 = 1.1 \times 10^8$$

$$\alpha_2 = 2.8 \times 10^6$$

$$\omega_{3dB1} = \sqrt{\alpha_1} = \pm 1.005 \times 10^4 \text{ rad/s} \Rightarrow f_{3dB1} = \frac{\omega_{3dB1}}{2\pi} = 1655 \text{ Hz}$$

$$\omega_{3dB2} = \sqrt{\alpha_2} = \pm 1.7 \times 10^3 \text{ rad/s} \Rightarrow f_{3dB2} = \frac{\omega_{3dB2}}{2\pi} = 270.6 \text{ Hz}$$

(4) The 3-dB bandwidth is found below:

$$BW = \omega_1 - \omega_2 = 1.005 \times 10^4 - 1.7 \times 10^3 = 8.3 \times 10^3 \text{ rad/s}$$

Verifying the calculated results

The calculated cutoff frequencies occur at a cutoff gain of:

$$A_c = 20 \log \frac{0.5}{\sqrt{2}} = -9.03 \text{ dB}$$

Figure 7 shows that all three methods corresponding to the cutoff frequencies, occur at gain of approximately -9.03 dB, implying that the simulated and measured data align closely with the calculations above. The simulated data is closer since it's performed in ideal scenarios. It does not have external factors such as component tolerances, wire resistances, etc. Whereas, the physical measurement is subject to these errors, causing the measured values to have a higher deviation from the ideal values.

	Frequency	DB(V(Vout))	P(V(Vout))
	Hz	dBV	Degrees
Step 3.2	272.6738966	-9.075976901	-134.708069
	1651.284035	-9.223828688	-226.2692261
Step 3.6	275.4228703	-9.407746129	-132.7913069
	1659.586907	-9.760595119	133.9582572

Figure 7. Simulated and Measured Values