COSC 3337 : Data Science I

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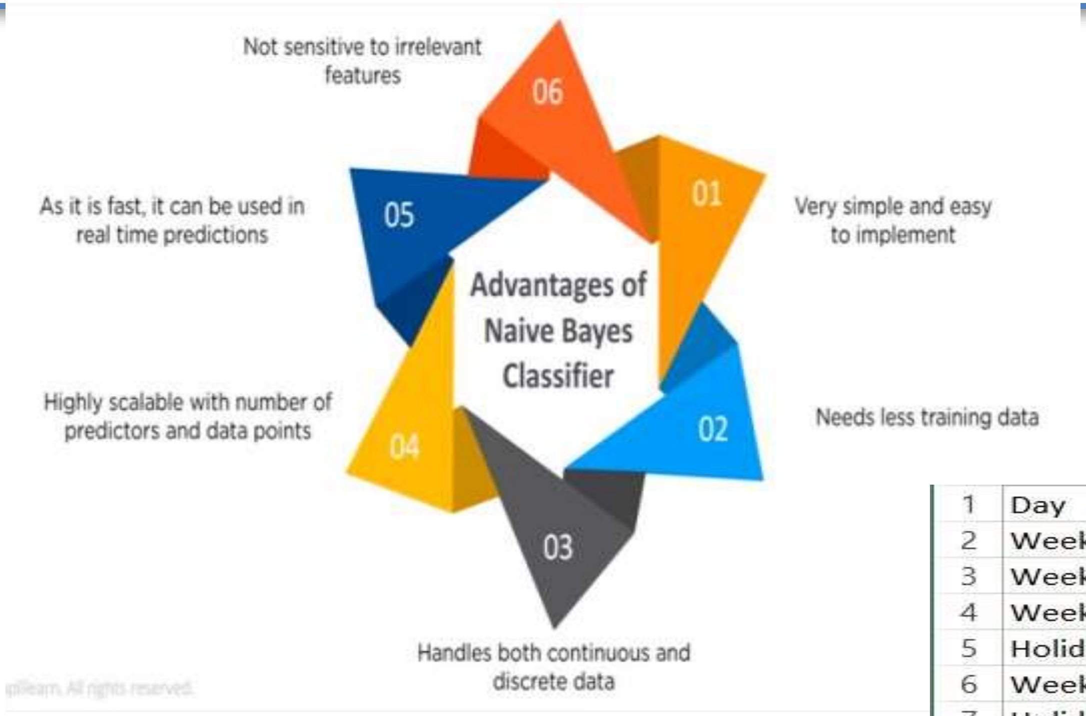
Naïve Bayes



1	Day	Discount	Free Delivery	Purchase
2	Weekday	Yes	yes	yes
3	Weekday	Yes	yes	yes
4	Weekday	No	No	No
5	Holiday	Yes	yes	yes
6	Weekday	Yes	yes	yes
7	Holiday	No	No	No
8	Weekend	Yes	No	yes
9	Weekday	Yes	yes	yes
10	Weekend	Yes	yes	yes
11	Holiday	Yes	yes	yes
12	Holiday	No	yes	yes
13	Holiday	No	No	No
14	Weekend	Yes	yes	yes
15	Holiday	Yes	yes	yes
16				

Independent variables

discount , free delivery, Day

Based on this dataset containing three input types of *Day*, *Discount* and *Free Delivery*, we will populate frequency tables for each attribute

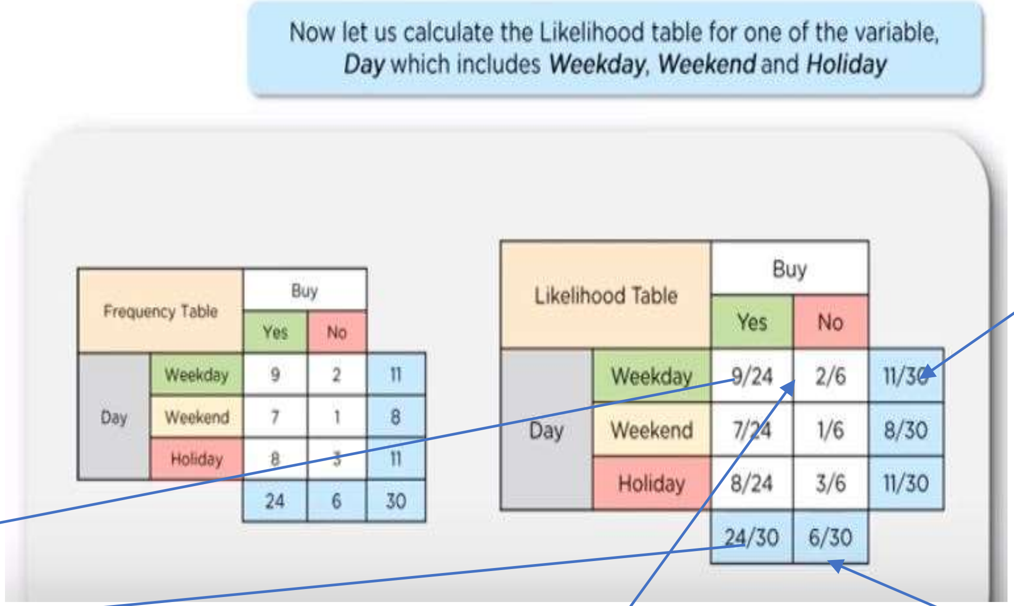
Frequency Table		Buy	
		Yes	No
Discount	Yes	19	1
	No	5	5

Frequency Table		Buy	
		Yes	No
Free Delivery	Yes	21	2
	No	3	4

Frequency Table		Buy	
		Yes	No
Day	Weekday	9	2
	Weekend	7	1
	Holiday	8	3

Likelihood

Probability
P(Buy No|Weekday)
(0.33*0.2)/0.37=
0.179


Now let us calculate the Likelihood table for one of the variable, *Day* which includes ***Weekday***, ***Weekend*** and ***Holiday***

Frequency Table		Buy: Yes	Buy: No	
Day	Weekday	9	2	11
	Weekend	7	1	8
	Holiday	8	3	11
		24	6	30

Likelihood Table		Buy: Yes	Buy: No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

P(Weekday)
11/30
0.37

Probability
P(Buy |Weekday)
(9/24*24/30)/0.37=
0.375*0.8/0.37=
0.817

P(Weekday |Buy No)
=2/6=0.33

P(not to buy)
6/30
0.2

The customer is more likely to buy during weekday

Likelihood Tables

Similarly, we can find the likelihood of occurrence of an event involving all three variables

Frequency Table		Buy	
		Yes	No
Discount	Yes	9	2
	No	5	14

Frequency Table		Buy	
		Yes	No
Free Delivery	Yes	6	3
	No	5	16

Frequency Table		Buy	
		Yes	No
Day	Weekday	9	2
	Weekend	7	1
	Holiday	8	3

Frequency Table		Buy	
		Yes	No
Day	Weekday	3	7
	Weekend	8	2
	Holiday	9	1

Likelihood Table		Buy		
		Yes	No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy	
		Yes	No
Discount	Yes	19	1
	No	5	5

Frequency Table		Buy		
		Yes	No	
Discount	Yes	19/24	1/6	20/30
	No	5/24	5/6	10/30
		24/30	6/30	

LET US USE THESE 3 LIKELIHOOD TABLES TO CALCULATE WHETHER A CUSTOMER WILL PURCHASE A PRODUCT ON A SPECIFIC COMBINATION OF DAY, DISCOUNT AND FREE DELIVERY OR NOT

HERE, LET US TAKE A COMBINATION OF THESE FACTORS:

- DAY = HOLIDAY
- DISCOUNT = YES
- FREE DELIVERY = YES

Likelihood Tables

Likelihood Tables

Likelihood Table		Buy: Yes	Buy: No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy: Yes	Buy: No	
Discount	Yes	19/24	1/6	20/30
	No	5/24	5/6	10/30
		24/30	6/30	

Frequency Table		Buy: Yes	Buy: No	
Free Delivery	Yes	21/24	2/6	23/30
	No	3/24	4/6	7/30
		24/30	6/30	

Calculating Conditional Probability of purchase on the following combination of day, discount and free delivery:

Where B equals:

- Day = Holiday
- Discount = Yes
- Free Delivery = Yes

Let A = No Buy

P (A|B) = P (No Buy | Discount = Yes, Free Delivery = Yes, Day = Holiday)

$$= \frac{P(\text{Discount = Yes} \mid \text{No}) * P(\text{Free Delivery = Yes} \mid \text{No}) * P(\text{Day = Holiday} \mid \text{No}) * P(\text{No Buy})}{P(\text{Discount=Yes}) * P(\text{Free Delivery=Yes}) * P(\text{Day=Holiday})}$$

$$= \frac{(1/6) * (2/6) * (3/6) * (6/30)}{(20/30) * (23/30) * (11/30)}$$

$$= 0.178$$

Probability Not To Buy

Normalize probabilities to get the likelihood of events

Likelihood Tables

Likelihood Table		Buy: Yes	Buy: No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

Frequency Table		Buy: Yes	Buy: No	
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		24/30	6/30	

Calculating Conditional Probability of purchase on the following combination of day, discount and free delivery:

Where B equals:

- Day = Holiday
- Discount = Yes
- Free Delivery = Yes

Let A = Buy

P (A|B) = P (Yes Buy | Discount = Yes, Free Delivery = Yes, Day = Holiday)

$$= \frac{P(\text{Discount}=\text{Yes} \mid \text{Yes}) * P(\text{Free Delivery}=\text{Yes} \mid \text{Yes}) * P(\text{Day}=\text{Holiday} \mid \text{Yes}) * P(\text{Yes Buy})}{P(\text{Discount}=\text{Yes}) * P(\text{Free Delivery}=\text{Yes}) * P(\text{Day}=\text{Holiday})}$$

$$= \frac{(19/24) * (21/24) * (8/24) * (24/30)}{(20/30) * (23/30) * (11/30)}$$

$$= 0.986$$

Probability To Buy

Conditional probability of purchase
Probability of Purchase =0.986
Probability of No purchase =0.178

Sum of probabilities = 0.986+0.178 =1.164

Likelihood of Purchase= 0.986/1.164 = 84.71 %
Likelihood of no purchase =0.178/1.164= 15.29 %

➔
An average customer will buy on a Holiday with discount and free delivery

Likelihood Tables

Likelihood Table		Buy: Yes	Buy: No	
Day	Weekday	9/24	2/6	11/30
	Weekend	7/24	1/6	8/30
	Holiday	8/24	3/6	11/30
		24/30	6/30	

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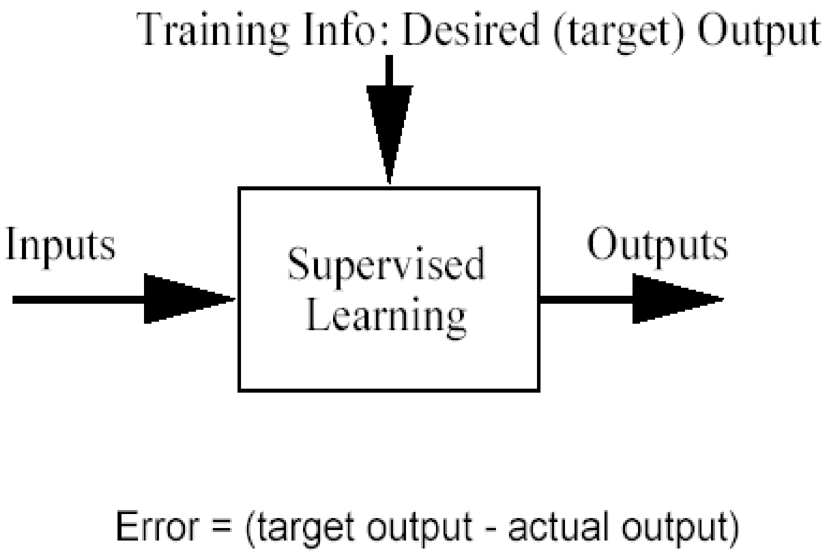
	No	3/24	4/6	7/30
		24/30	6/30	

Classification problem

- Training data: examples of the form (d,h(d))
  - where d are the data objects to classify (inputs)
  - and h(d) are the correct class info for d, $h(d) \in \{1, \ldots K\}$
- Goal: given d_{new}, provide $h(d_{new})$



Bayes Classifier

- A probabilistic framework for solving classification problems
- **A, C** random variables
- Joint probability: **Pr(A=a,C=c)**
- Conditional probability: **Pr(C=c | A=a)**
- Relationship between joint and conditional probability distributions

$$\Pr(C,A) = \Pr(C \mid A) \times \Pr(A) = \Pr(A \mid C) \times \Pr(C)$$

- **Bayes Theorem**:

$$P(C \mid A) = \frac{P(A \mid C)P(C)}{P(A)}$$

Example of Bayes Theorem

- Given:
  - A doctor knows that meningitis causes stiff neck 50% of the time
  - Prior probability of any patient having meningitis is 1/50,000
  - Prior probability of any patient having stiff neck is 1/20

- If a patient has stiff neck, what's the probability he/she has meningitis?

$$P(M \mid S) = \frac{P(S \mid M)P(M)}{P(S)} = \frac{0.5 \times 1/50000}{1/20} = 0.0002$$

Bayes' Rule & Diagnosis

- Useful for assessing diagnostic probability from causal probability:
  - $P(Cause|Effect) = \dfrac{P(Effect|Cause) * P(Cause)}{P(Effect)}$

Likelihood **Prior**

$$P(a|b) = \frac{P(b|a) * P(a)}{P(b)}$$

Posterior

Normalization

Bayesian Classifiers

- Consider each attribute and class label as random variables

- Given a record with attributes (A_1, A_2,…,A_n)
  - Goal is to predict class C
  - Specifically, we want to find the value of C that maximizes $P(C| A_1, A_2,…,A_n)$

- Can we estimate $P(C| A_1, A_2,…,A_n)$ directly from data?

Bayesian Classifiers

- Approach:
  - compute the posterior probability $P(C \mid A_1, A_2, \ldots, A_n)$ for all values of C using the Bayes theorem

$$\boxed{P(C \mid A_1 A_2 \ldots A_n) = \frac{P(A_1 A_2 \ldots A_n \mid C) P(C)}{P(A_1 A_2 \ldots A_n)}}$$

  - Choose value of C that maximizes
    $P(C \mid A_1, A_2, \ldots, A_n)$
  - Equivalent to choosing value of C that maximizes
    $P(A_1, A_2, \ldots, A_n \mid C)\, P(C)$
- How to estimate $P(A_1, A_2, \ldots, A_n \mid C)$?

Naïve Bayes Classifier

Assume independence among attributes A_i when class is given:

$$P(A_1, A_2, \ldots, A_n | C_j) = P(A_1|C_j)\, P(A_2|C_j) \cdots P(A_n|C_j)$$

We can estimate $P(A_i| C_j)$ for all A_i and C_j.

New point X is classified to C_j if

$$P(C_j|X) = P(C_j) \prod_i P(A_i|C_j)$$

is maximal.

How to Estimate Probabilities from Data?

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

- Class: $P(C) = N_c/N$
  - e.g., $P(No) = 7/10$,
    $P(Yes) = 3/10$
- For discrete attributes:

  $$P(A_i \mid C_k) = |A_{ik}| / N_{c\ k}$$

  - where $|A_{ik}|$ is number of instances having attribute A_i and belongs to class C_k
  - Examples:

    P(Status=Married|No) = 4/7
    P(Refund=Yes|Yes)=0

How to Estimate Probabilities from Data?

- For continuous attributes:
  - Discretize the range into bins
    - one ordinal attribute per bin
    - violates independence assumption
  - Two-way split: (A < v) or (A > v)
    - choose only one of the two splits as new attribute
  - Probability density estimation:
    - Assume attribute follows a normal distribution
    - Use data to estimate parameters of distribution (e.g., mean and standard deviation)
    - Once probability distribution is known, can use it to estimate the conditional probability $P(A_i|c)$

How to Estimate Probabilities from Data?

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6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

- Normal distribution:

$$P(A_i \mid c_j) = \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} e^{-\frac{(A_i-\mu_{ij})^2}{2\sigma_{ij}^2}}$$

  - One for each (A_i, c_i) pair

- For (Income, Class=No):
  - If Class=No
    - sample mean = 110
    - sample variance = 2975

$$P(Income=120 \mid No) = \frac{1}{\sqrt{2\pi}(54.54)} e^{-\frac{(120-110)^2}{2(2975)}} = 0.0072$$

Example of Naïve Bayes Classifier

Given a Test Record:

$$X = (\text{Refund} = \text{No}, \text{Married}, \text{Income} = 120\text{K})$$

naive Bayes Classifier:

P(Refund=Yes|No) = 3/7
P(Refund=No|No) = 4/7
P(Refund=Yes|Yes) = 0
P(Refund=No|Yes) = 1
P(Marital Status=Single|No) = 2/7
P(Marital Status=Divorced|No)=1/7
P(Marital Status=Married|No) = 4/7
P(Marital Status=Single|Yes) = 2/7
P(Marital Status=Divorced|Yes)=1/7
P(Marital Status=Married|Yes) = 0

For taxable income:
If class=No: sample mean=110
sample variance=2975
If class=Yes: sample mean=90
sample variance=25

- P(X|Class=No) = P(Refund=No|Class=No) $\times$ P(Married| Class=No) $\times$ P(Income=120K| Class=No) = 4/7 $\times$ 4/7 $\times$ 0.0072 = 0.0024

- P(X|Class=Yes) = P(Refund=No| Class=Yes) $\times$ P(Married| Class=Yes) $\times$ P(Income=120K| Class=Yes) = 1 $\times$ 0 $\times$ 1.2 $\times$ 10^{-9} = 0

Since P(X|No)P(No) > P(X|Yes)P(Yes)

Therefore P(No|X) > P(Yes|X)
=> Class = No

Naïve Bayes Classifier

- If one of the conditional probability is zero, then the entire expression becomes zero
- Probability estimation:

$$\text{Original} : P(A_i \mid C) = \frac{N_{ic}}{N_c}$$

$$\text{Laplace} : P(A_i \mid C) = \frac{N_{ic} + 1}{N_c + N_i}$$

$$\text{m-estimate} : P(A_i \mid C) = \frac{N_{ic} + mp}{N_c + m}$$

N_i: number of attribute values for attribute A_i

p: prior probability

m: parameter

Example of Naïve Bayes Classifier

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

A: attributes

M: mammals

N: non-mammals

$$P(A|M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A|N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A|M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A|N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

P(A|M)P(M) > P(A|N)P(N)

=> Mammals

Implementation details

- Computing the conditional probabilities involves multiplication of many very small numbers
    - ➢ Numbers get very close to zero, and there is a danger of numeric instability
- We can deal with this by computing the logarithm of the conditional probability

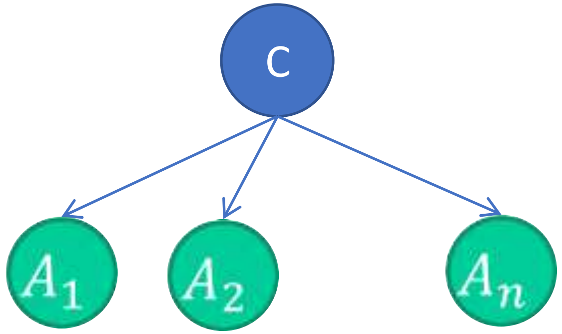
$$\log P(C|A) \sim \log P(A|C) + \log P(A)$$
$$= \sum_i \log P(A_i|C) + \log P(A)$$

Naïve Bayes

- Robust to isolated noise points
- Handle missing values by ignoring the instance during probability estimate calculations
- Robust to irrelevant attributes
- Independence assumption may not hold for some attributes
  - Use other techniques such as Bayesian Belief Networks (BBN)
- Naïve Bayes can produce a probability estimate, but it is usually a very biased one
  - Logistic Regression is better for obtaining probabilities.

Generative vs Discriminative models

- Naïve Bayes is a type of a generative model
  - Generative process:
    - First pick the category of the record
    - Then given the category, generate the attribute values from the distribution of the category
    - Conditional independence given C



- We use the training data to learn the distribution of the values in a class

Generative vs Discriminative models

- Logistic Regression and SVM are discriminative models
    - The goal is to find the boundary that discriminates between the two classes from the training data
- In order to classify the language of a document, you can
    - Either learn the two languages and find which is more likely to have generated the words you see
    - Or learn what differentiates the two languages.

About the Bayesian framework

- Allows us to combine <u>observed data</u> and <u>prior knowledge</u>
- Provides <u>practical learning algorithms</u>
- It is a <u>generative</u> (model based) approach, which offers a useful conceptual framework
  - This means that any kind of objects (e.g. time series, trees, etc.) can be classified, based on a probabilistic model specification

Example. 'Play Tennis' data

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
Day1	Sunny	Hot	High	Weak	*No*
Day2	Sunny	Hot	High	Strong	*No*
Day3	Overcast	Hot	High	Weak	Yes
Day4	Rain	Mild	High	Weak	Yes
Day5	Rain	Cool	Normal	Weak	Yes
Day6	Rain	Cool	Normal	Strong	*No*
Day7	Overcast	Cool	Normal	Strong	Yes
Day8	Sunny	Mild	High	Weak	*No*
Day9	Sunny	Cool	Normal	Weak	Yes
Day10	Rain	Mild	Normal	Weak	Yes
Day11	Sunny	Mild	Normal	Strong	Yes
Day12	Overcast	Mild	High	Strong	Yes
Day13	Overcast	Hot	Normal	Weak	Yes
Day14	Rain	Mild	High	Strong	*No*

Example

- Learning Phase
-

Outlook	Play=*Yes*	Play=*No*
Sunny	2/9	3/5
Overcast	4/9	0/5
Rain	3/9	2/5

Temperature	Play=*Yes*	Play=*No*
Hot	2/9	2/5
Mild	4/9	2/5
Cool	3/9	1/5

Humidity	Play=*Yes*	Play=*No*
High	3/9	4/5
Normal	6/9	1/5

Wind	Play=*Yes*	Play=*No*
Strong	3/9	3/5
Weak	6/9	2/5

$P(\text{Play}=Yes) = 9/14$ $P(\text{Play}=No) = 5/14$

Example :Test Phase

– **Given a new instance, predict its label**

$\mathbf{x}'$=(Outlook=*Sunny,* Temperature=*Cool,* Humidity=*High,* Wind=*Strong*)

– Look up tables achieved in the learning phrase

- P(Outlook=*Sunny*|Play=*Yes*) = 2/9
  P(Temperature=*Cool*|Play=*Yes*) = 3/9
  P(Huminity=*High*|Play=*Yes*) = 3/9
  P(Wind=*Strong*|Play=*Yes*) = 3/9
  P(Play=*Yes*) = 9/14

  P(Outlook=*Sunny*|Play=*No*) = 3/5
  P(Temperature=*Cool*|Play==*No*) = 1/5
  P(Huminity=*High*|Play=*No*) = 4/5
  P(Wind=*Strong*|Play=*No*) = 3/5
  P(Play=*No*) = 5/14

– Decision making with the MAP rule

P(*Yes*|$\mathbf{x}'$) ≈ [P(*Sunny*|Yes)P(*Cool*|*Yes*)P(*High*|Yes)P(*Strong*|*Yes*)]P(Play=*Yes*) = 0.0053

P(*No*|$\mathbf{x}'$) ≈ [P(*Sunny*|No) P(*Cool*|No)P(*High*|*No*)P(*Strong*|*No*)]P(Play=*No*) = 0.0206

Given the fact P(*Yes*|$\mathbf{x}'$) < P(*No*|$\mathbf{x}'$), we label $\mathbf{x}'$ to be "*No*".

Problem: Players will play if weather is sunny. Is this statement is correct?

- We can solve it using above discussed method of posterior probability.
- P(Yes | Sunny) = P(Sunny | Yes) * P(Yes) / P (Sunny)
- Here we have P (Sunny |Yes) = 3/9 = 0.33, P(Sunny) = 5/14 = 0.36, P(Yes)= 9/14 = 0.64
- Now, P (Yes | Sunny) = 0.33 * 0.64 / 0.36 = 0.60, which has higher probability.
- Naive Bayes uses a similar method to predict the probability of different class based on various attributes. This algorithm is mostly used in text classification and with problems having multiple classes.

How Naive Bayes algorithm works?

- Step 1: Convert the data set into a frequency table
- Step 2: Create Likelihood table by finding the probabilities like Overcast probability = 0.29 and probability of playing is 0.64.
- Step 3: Now, use Naive Bayesian equation to calculate the posterior probability for each class. The class with the highest posterior probability is the outcome of prediction.

Frequency Table, Likelihood

Weather	Play
Sunny	No
Overcast	Yes
Rainy	Yes
Sunny	Yes
Sunny	Yes
Overcast	Yes
Rainy	No
Rainy	No
Sunny	Yes
Rainy	Yes
Sunny	No
Overcast	Yes
Overcast	Yes
Rainy	No

Frequency Table		
Weather	No	Yes
Overcast		4
Rainy	3	2
Sunny	2	3
Grand Total	5	9

Likelihood table				
Weather	No	Yes		
Overcast		4	=4/14	0.29
Rainy	3	2	=5/14	0.36
Sunny	2	3	=5/14	0.36
All	5	9		
	=5/14	=9/14		
	0.36	0.64		

```
#Import Library of Gaussian Naive Bayes model
from sklearn.naive_bayes import GaussianNB
import numpy as np

#assigning predictor and target variables
x= np.array([[-3,7],[1,5], [1,2], [-2,0], [2,3], [-4,0], [-1,1], [1,1], [-
2,2], [2,7], [-4,1], [-2,7]])
Y = np.array([3, 3, 3, 3, 4, 3, 3, 4, 3, 4, 4, 4])
#Create a Gaussian Classifier
model = GaussianNB()

# Train the model using the training sets
model.fit(x, y)

#Predict Output
predicted= model.predict([[1,2],[3,4]])
print predicted
```

Output: ([3,4])

Review :Bayesian Classification: Why?

- A statistical classifier: performs *probabilistic prediction, i.e.,* predicts class membership probabilities
- Foundation: Based on Bayes' Theorem.
- Performance: A simple Bayesian classifier, *naïve Bayesian classifier*, has comparable performance with decision tree and selected neural network classifiers
- Incremental: Each training example can incrementally increase/decrease the probability that a hypothesis is correct — prior knowledge can be combined with observed data
- Standard: Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

Bayesian Theorem: Basics

- Let **X** be a data sample ("*evidence*"): class label is unknown
- Let H be a *hypothesis* that X belongs to class C
- Classification is to determine P(H|**X**), the probability that the hypothesis holds given the observed data sample **X**
- P(H) (*prior probability*), the initial probability
  - E.g., **X** will buy computer, regardless of age, income, …
- P(**X**): probability that sample data is observed
- P(**X**|H) (*posteriori probability*), the probability of observing the sample **X**, given that the hypothesis holds
  - E.g., Given that **X** will buy computer, the prob. that X is 31..40, medium income

Bayesian Theorem

- Given training data **X**, *posteriori probability of a hypothesis* H, P(H|**X**), follows the Bayes theorem

$$P(H|\mathbf{X}) = \frac{P(\mathbf{X}|H)P(H)}{P(\mathbf{X})}$$

- Informally, this can be written as

  posteriori = likelihood x prior/evidence

- Predicts **X** belongs to C_2 iff the probability $P(C_i|\mathbf{X})$ is the highest among all the $P(C_k|X)$ for all the *k* classes
- Practical difficulty: require initial knowledge of many probabilities, significant computational cost

Towards Naïve Bayesian Classifier

- Let D be a training set of tuples and their associated class labels, and each tuple is represented by an n-D attribute vector **X** = $(x_1, x_2, \ldots, x_n)$
- Suppose there are *m* classes $C_1, C_2, \ldots, C_m$.
- Classification is to derive the maximum posteriori, i.e., the maximal $P(C_i|\mathbf{X})$
- This can be derived from Bayes' theorem

$$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$

- Since P(X) is constant for all classes, only

$$P(C_i|\mathbf{X}) = P(\mathbf{X}|C_i)P(C_i)$$

needs to be maximized

Avoiding the 0-Probability Problem

- Naïve Bayesian prediction requires each conditional prob. be non-zero. Otherwise, the predicted prob. will be zero

- Ex. Suppose a dataset with 1000 tuples, income=low (0), income= medium (990), and income = high (10),
- Use Laplacian correction (or Laplacian estimator)
  - Adding 1 to each case

    Prob(income = low) = 1/1003

    Prob(income = medium) = 991/1003

    Prob(income = high) = 11/1003
  - The "corrected" prob. estimates are close to their "uncorrected" counterparts

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Naïve Bayesian Classifier: Comments

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence, therefore loss of accuracy
  - Practically, dependencies exist among variables
    - E.g., hospitals: patients: Profile: age, family history, etc.

    Symptoms: fever, cough etc., Disease: lung cancer, diabetes, etc.
    - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
- How to deal with these dependencies?
  - Bayesian Belief Networks

Naïve Bayesian Classifier: Training Dataset

Class:
C1:buys_computer = 'yes'
C2:buys_computer = 'no'

Data sample
X = (age <=30,
Income = medium,
Student = yes
Credit_rating = Fair)

age	income	student	credit_rating	_comp
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Naïve Bayesian Classifier: An Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Therefore, X belongs to class ("buys_computer = yes")