

# COSC 3337 : Data Science I



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# Density-based Approaches



- Why Density-Based Clustering methods?
  - Discover clusters of arbitrary shape.
  - Clusters – Dense regions of objects separated by regions of low density
- DBSCAN – the first density based clustering
- OPTICS – density based cluster-ordering
- DENCLUE – a general density-based description of cluster and clustering
- HDBSCAN Instead of taking an epsilon value as a cut level for the dendrogram, a different approach is taken
- AFFINITY PROPAGATION is a clustering algorithm based on the concept of "message passing" between data points

# DBSCAN: Density Based Spatial Clustering of Applications with Noise

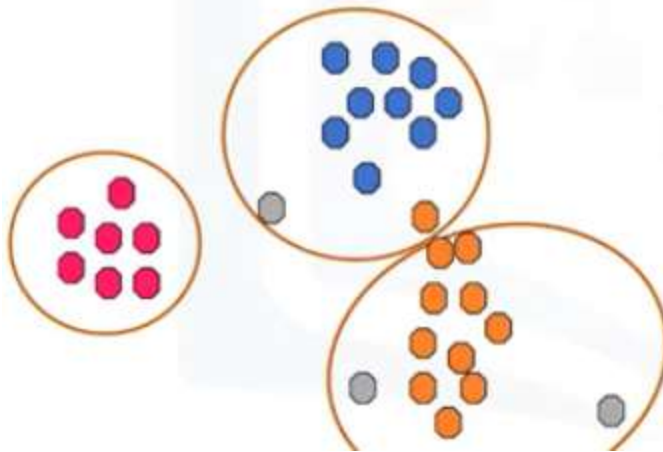


- Proposed by Ester, Kriegel, Sander, and Xu (KDD96)
- Relies on a density-based notion of cluster: A cluster is defined as a maximal set of density-connected points.
- Discovers clusters of arbitrary shape in spatial databases with noise

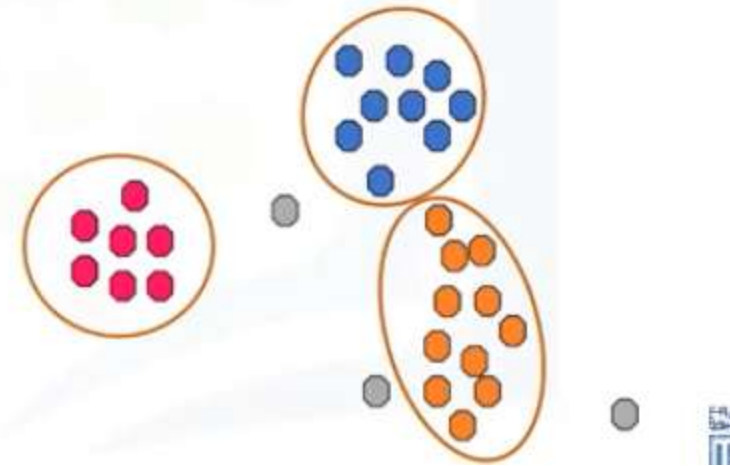
# Kmeans vs DBscan



- k-Means assigns all points to a cluster even if they do not belong in any



- Density-based Clustering locates regions of **high density**, and separates outliers

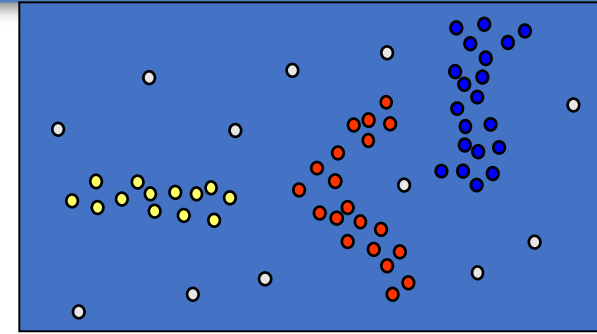


# Density-Based Clustering

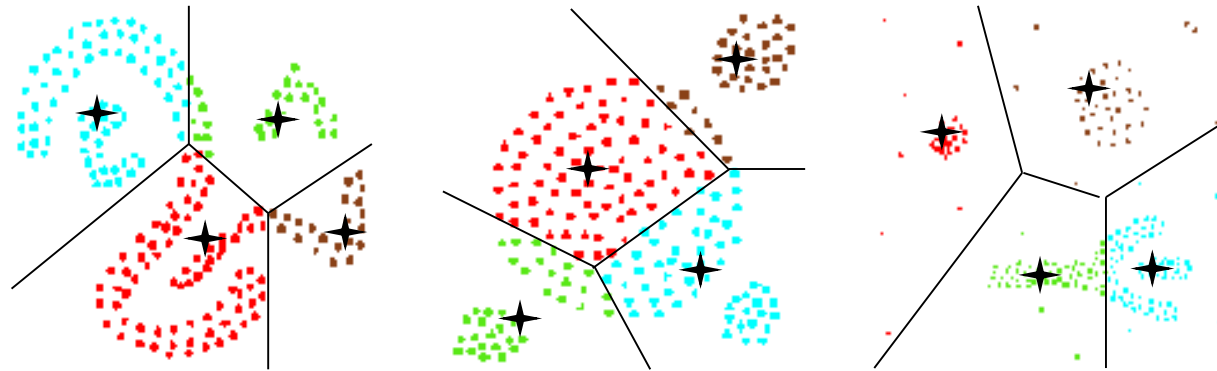


## ✦ *Basic Idea:*

Clusters are dense regions in the data space, separated by regions of lower object density



## • Why Density-Based Clustering?



Results of a  $k$ -medoid algorithm for  $k=4$

Different density-based approaches exist (see Textbook & Papers)  
Here we discuss the ideas underlying the DBSCAN algorithm

# Density Based Clustering: Basic Concept



- Intuition for the formalization of the basic idea
  - For any point in a cluster, the local point density around that point has to exceed some threshold
  - The set of points from one cluster is spatially connected
- Local point density at a point  $p$  defined by two parameters
  - $\varepsilon$  – radius for the neighborhood of point  $p$ :  
$$N_{\varepsilon}(p) := \{q \text{ in data set } D \mid \text{dist}(p, q) \leq \varepsilon\}$$
  - *MinPts* – minimum number of points in the given neighbourhood  $N(p)$

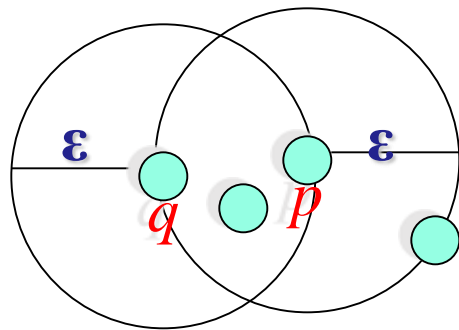
# $\varepsilon$ -Neighborhood



- $\varepsilon$ -Neighborhood – Objects within a radius of  $\varepsilon$  from an object.

$$N_{\varepsilon}(p) : \{q \mid d(p, q) \leq \varepsilon\}$$

- “High density” -  $\varepsilon$ -Neighborhood of an object contains at least *MinPts* of objects.



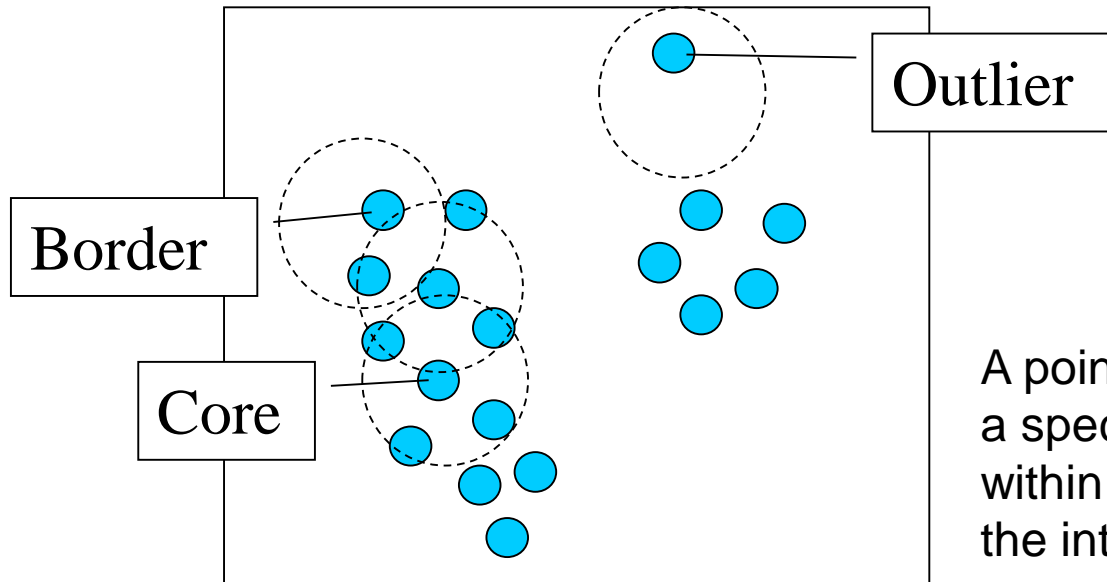
$\varepsilon$ -Neighborhood of  $p$

$\varepsilon$ -Neighborhood of  $q$

*Density of  $p$  is “high” (MinPts = 4)*

*Density of  $q$  is “low” (MinPts = 4)*

# Core, Border & Outlier



Given  $\epsilon$  and *MinPts*, categorize the objects into three exclusive groups.

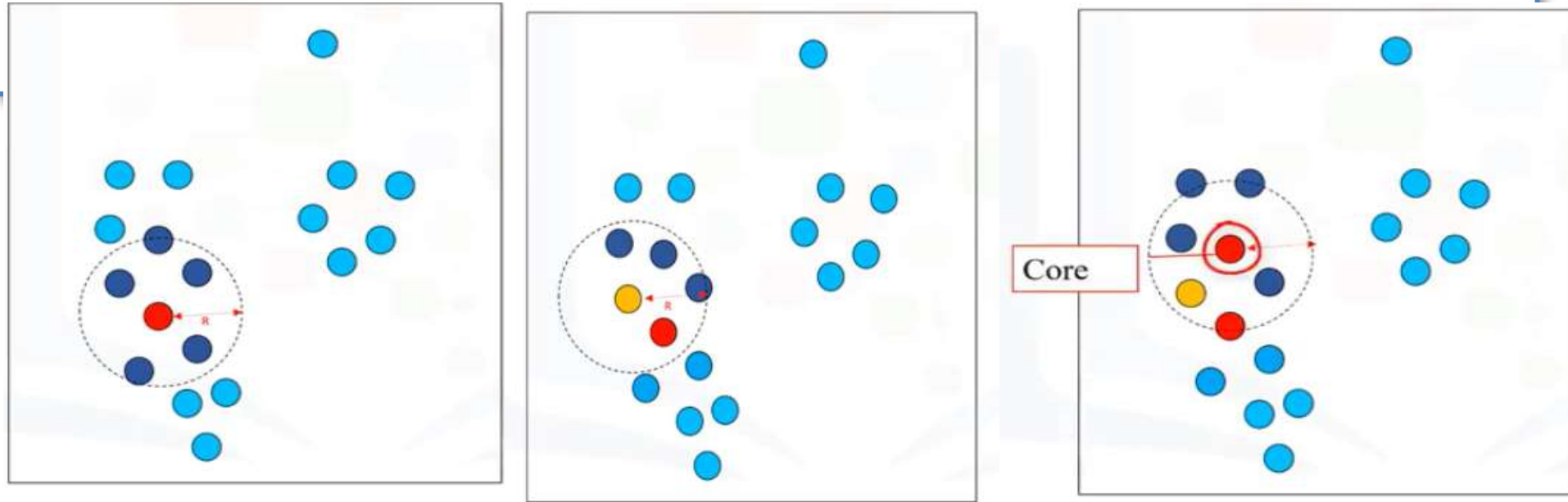
A point is a **core point** if it has more than a specified number of points (MinPts) within Eps. These are points that are at the interior of a cluster.

A **border point** has fewer than MinPts within Eps, but is in the neighborhood of a core point.

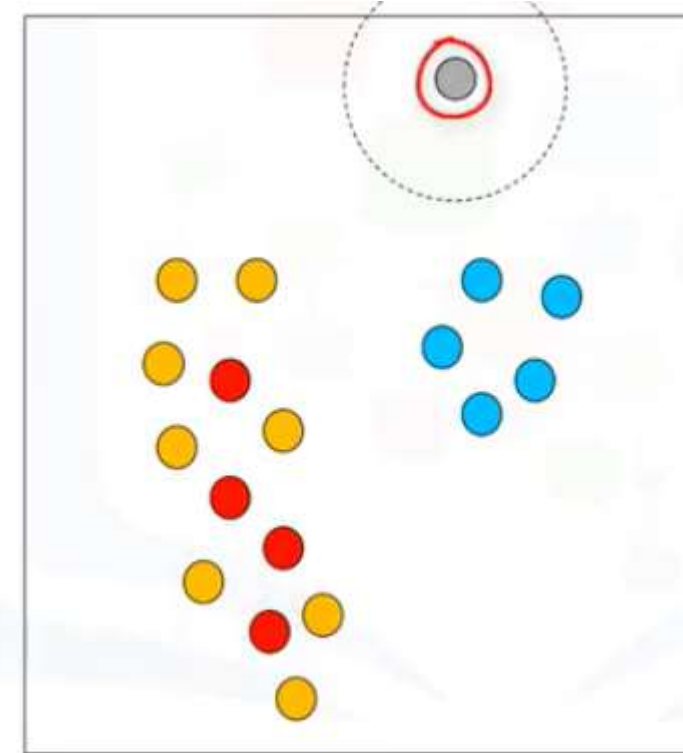
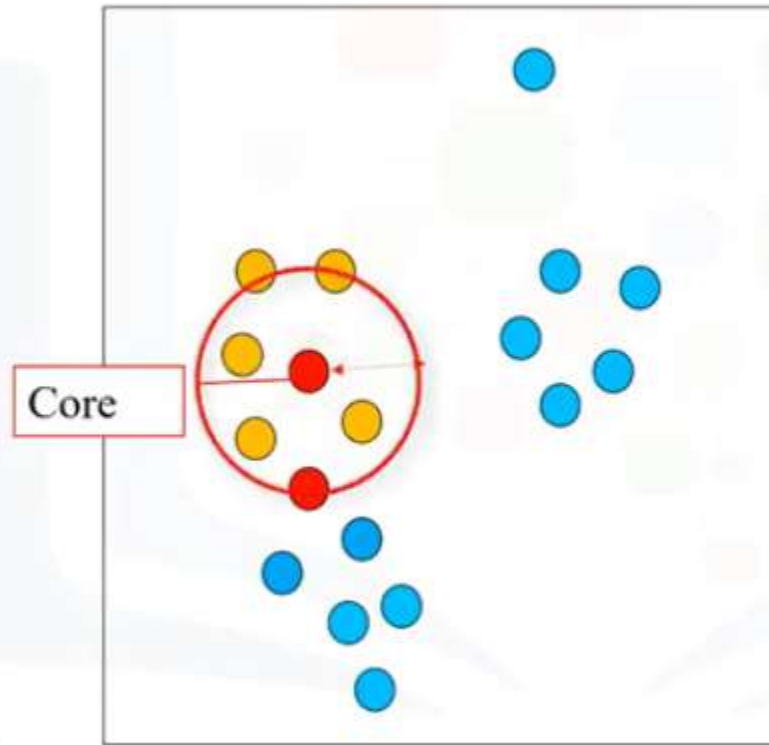
A **noise point** is any point that is not a core point nor a border point.

$\epsilon = 1\text{unit}$ ,  $\text{MinPts} = 5$

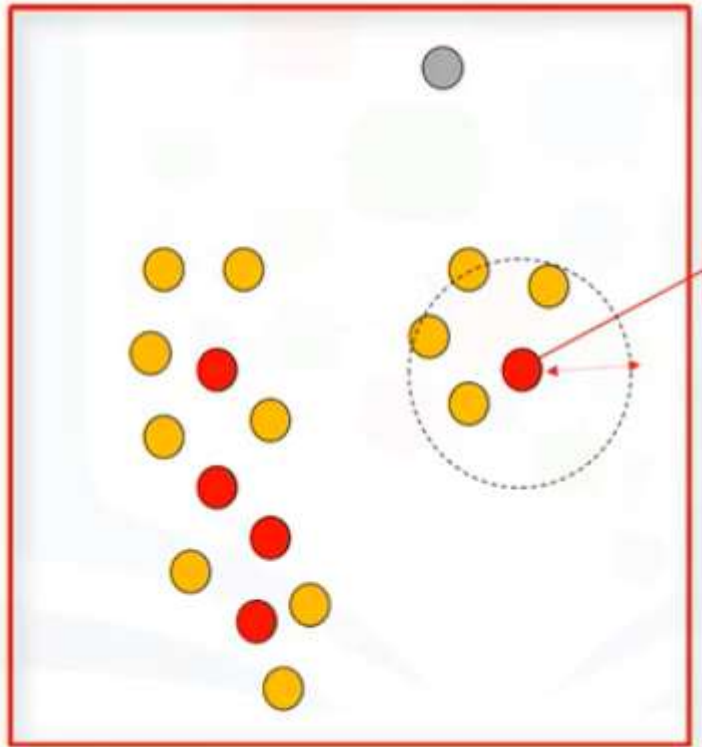




Border point if  
 1- less than Minpoints within radius  
 or  
 2- reachable from core point

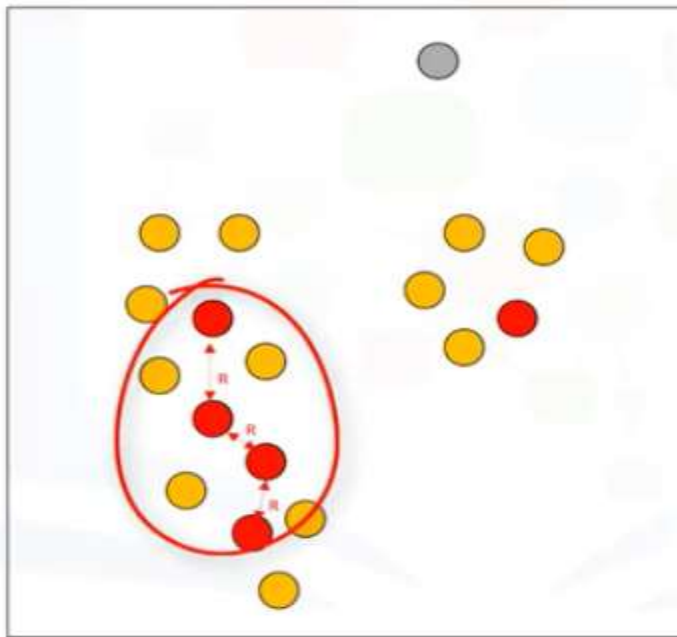


Outlier point if  
 1- it is not a core point  
 or  
 2- NOT reachable from core point

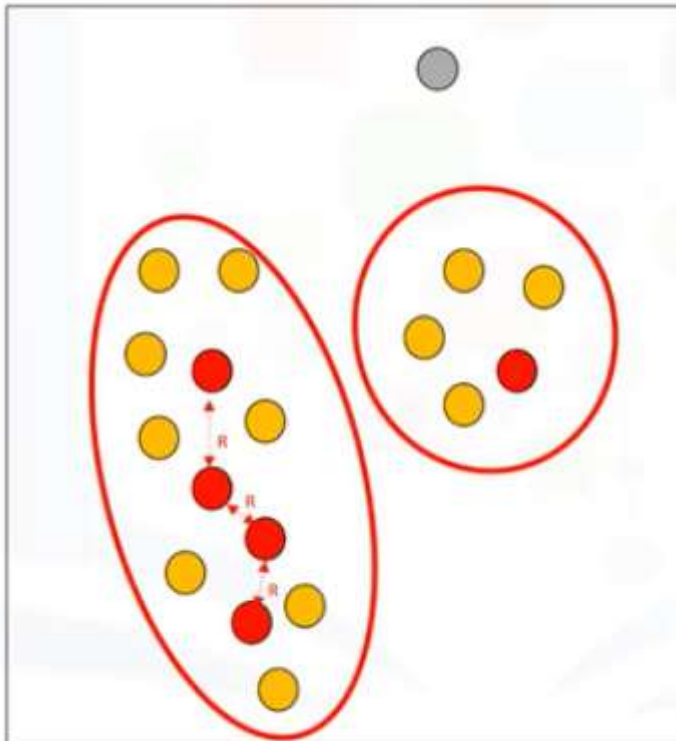


Visit all points and label them as  
Core  
Border  
Outlier/Noise

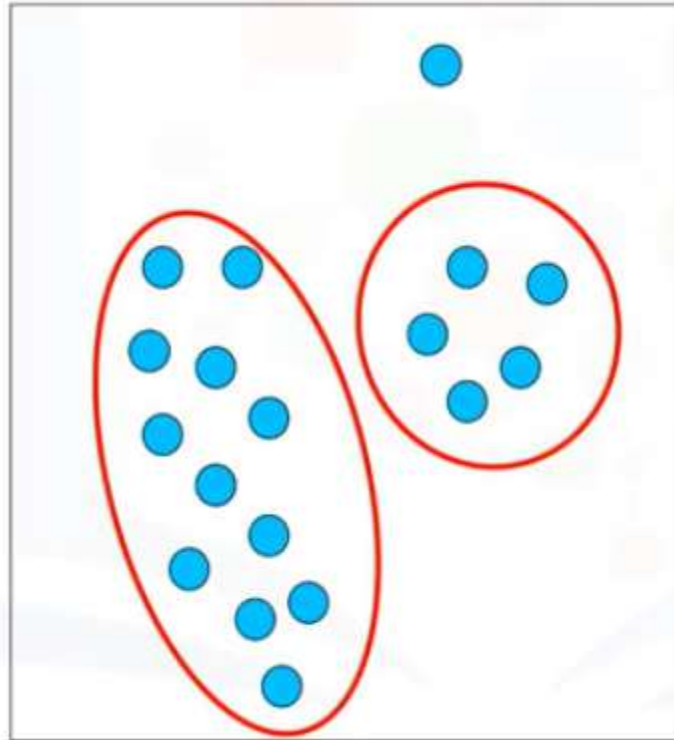
Connect all core points



And all their borders to form a cluster

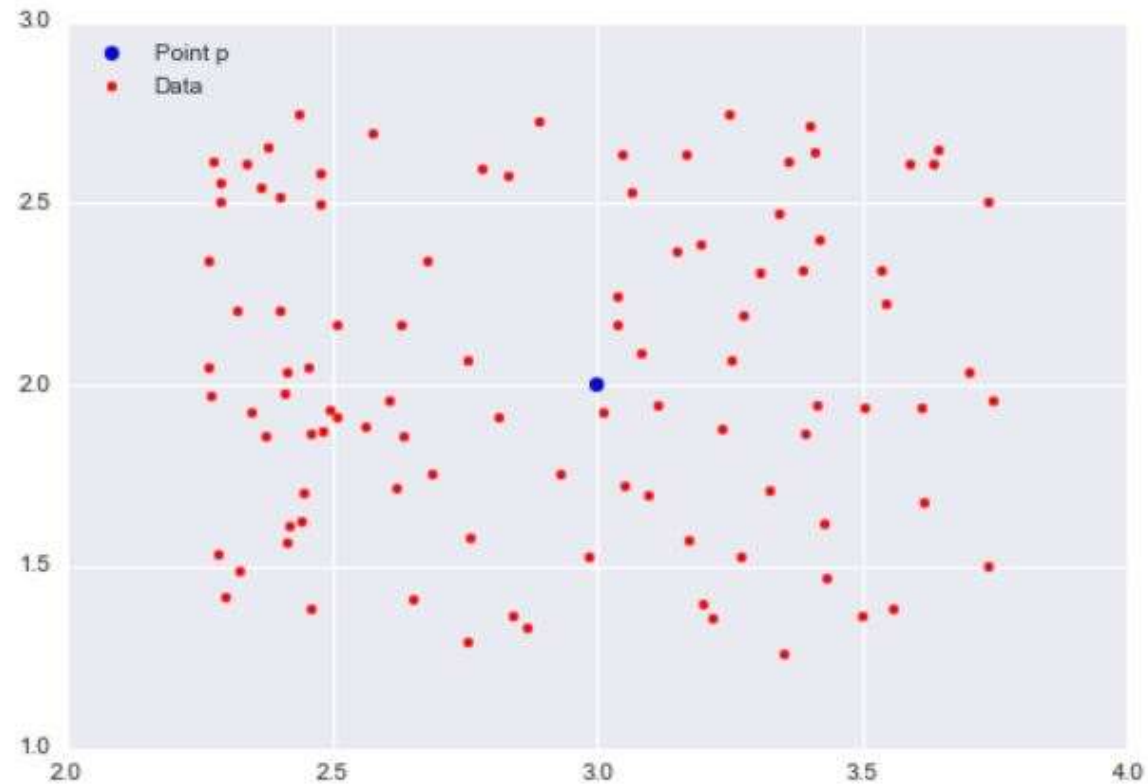


# Advantage of DBSCAN

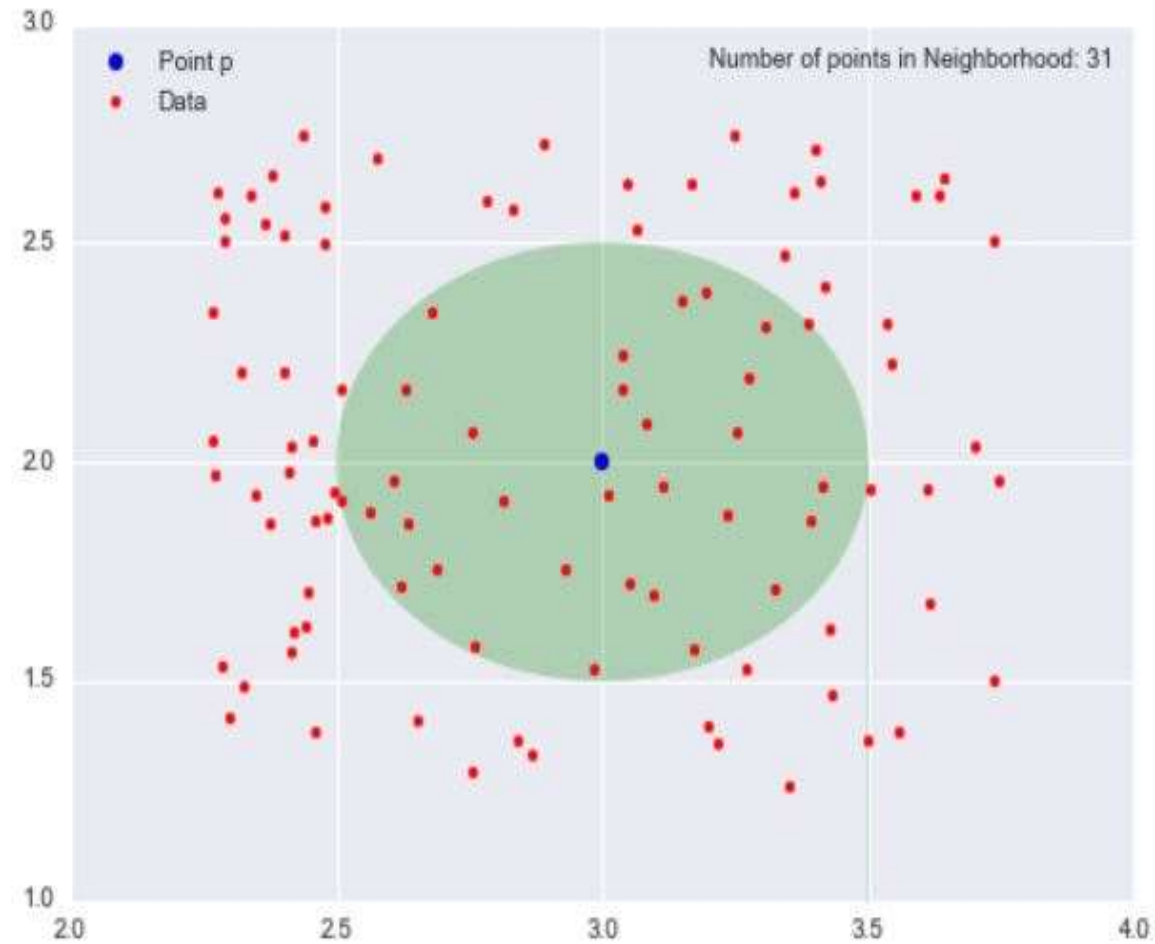


1. Arbitrarily shaped clusters
2. Robust to outliers
3. Does not require specification of the number of clusters

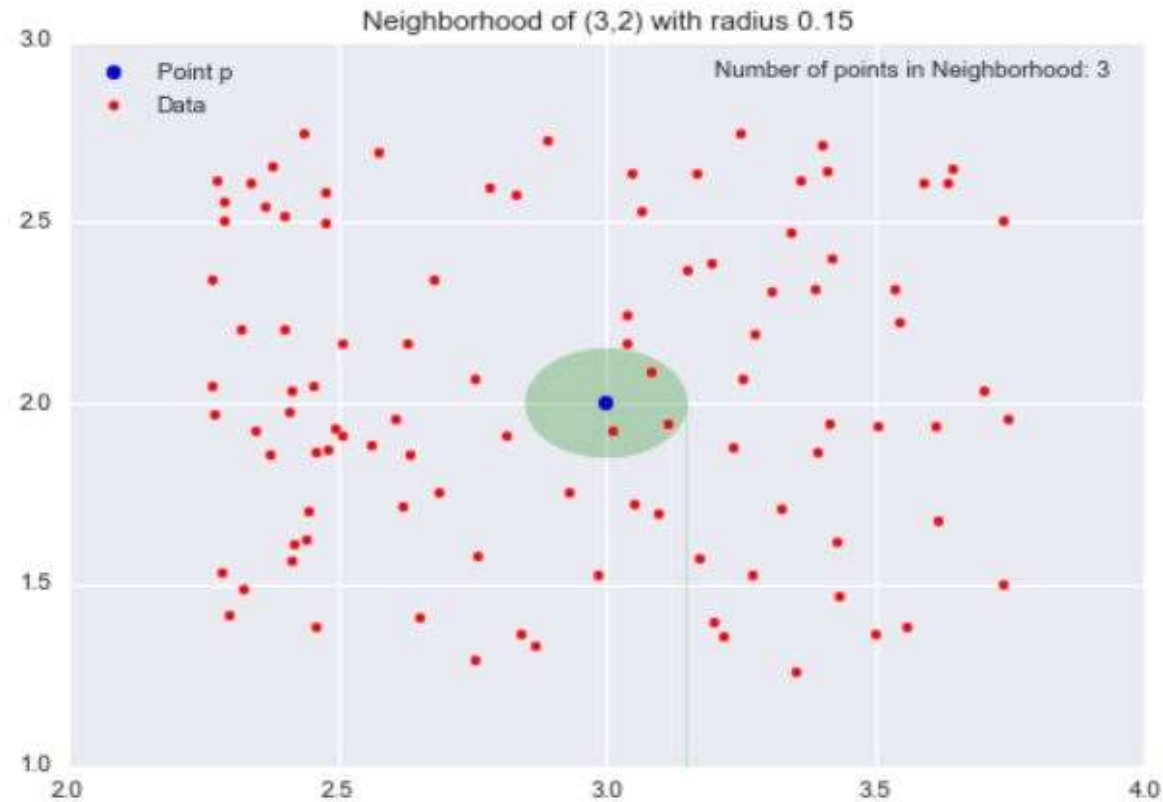
100 data points, select (3,2)



$\epsilon = 0.5 \rightarrow 31$  points



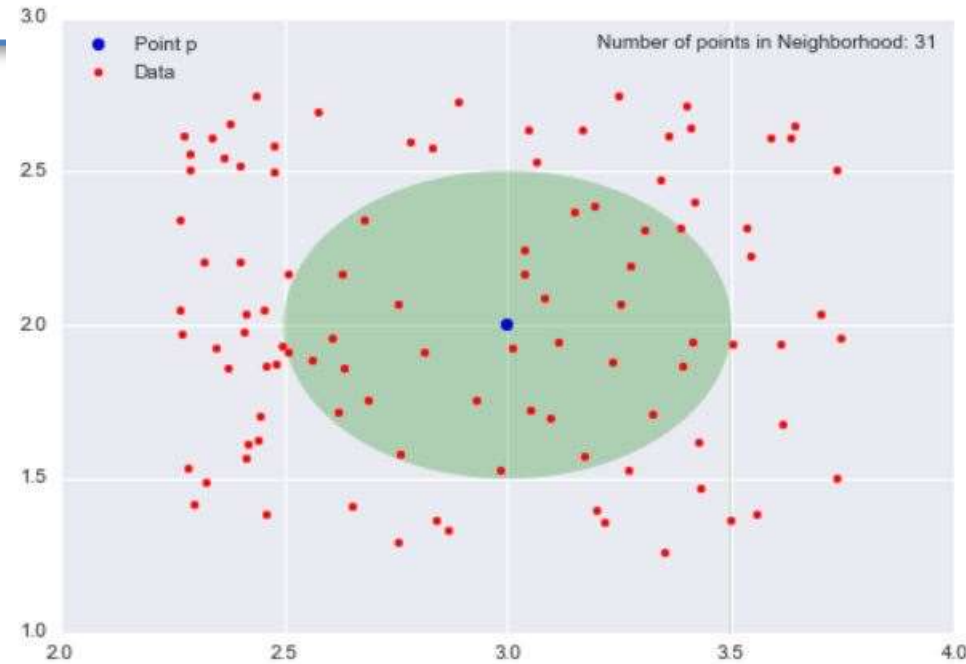
$\epsilon = 0.15 \rightarrow 3$  points



Decreasing  $\epsilon$  from 0.5 to 0.15 (a 70% reduction), the number of points is decreased in our neighborhood from 31 to 3 (a 90% reduction)



density = mass/volume

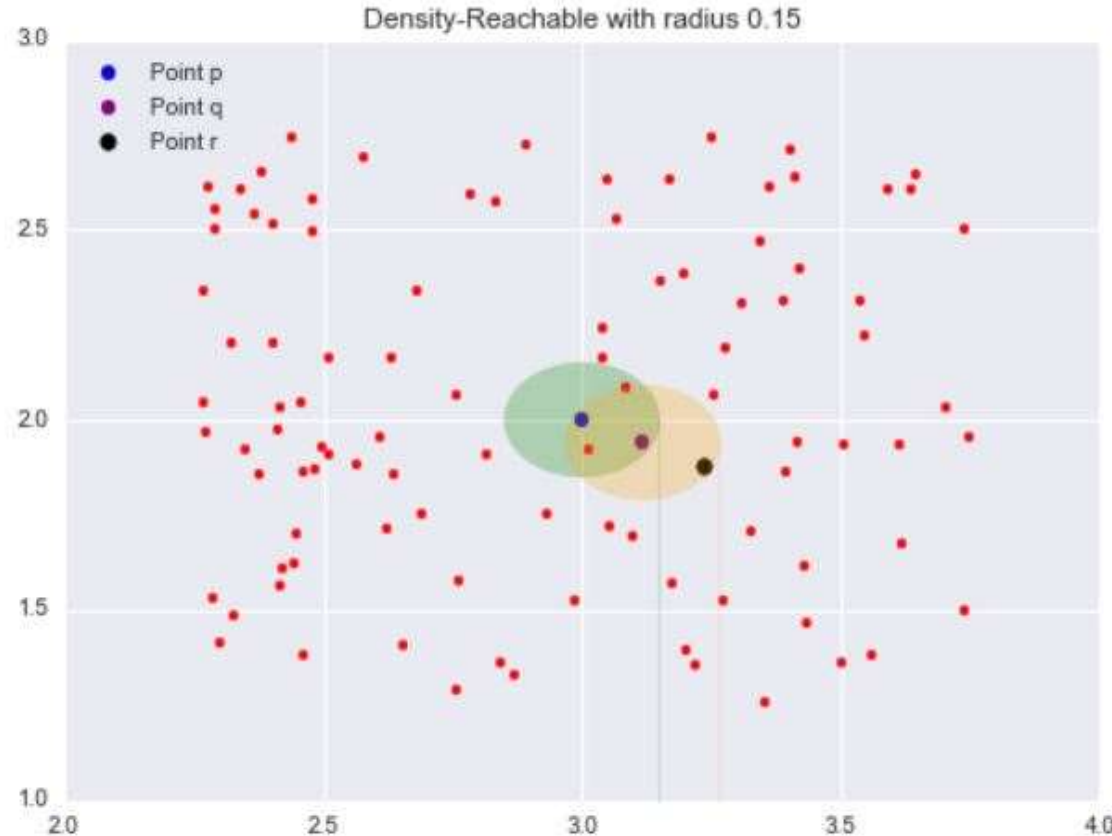


density = number of data points /  $\pi 0.5^2$

local density approximation at \* p = (3,2) is calculated as  
density = mass/volume =  $31 / (\pi/4) = 124/\pi \approx 39.5$

→ Cluster points who have similar local density approximations

- Starting at a point **p**, then the point **r** is **density-reachable** (friends of friend) from the point **p**.
- (directly-reachable** of a core point **p** are its “friends”)
- “friends of a friend of a friend ... of a friend” are also **density-reachable**



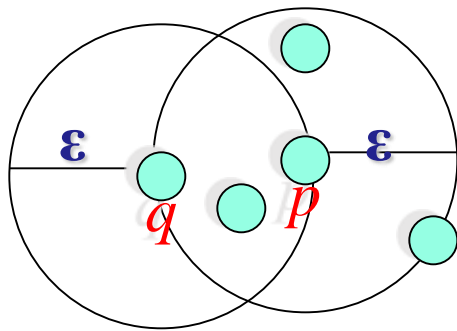
By picking **larger values of  $\epsilon$** , more points become density-reachable, and by choosing **smaller values of  $\epsilon$** , **fewer** points become density-reachable.

# Density-Reachability



## ■ Directly density-reachable

□ An object  $q$  is directly density-reachable from object  $p$  if  $p$  is a core object and  $q$  is in  $p$ 's  $\epsilon$ -neighborhood.



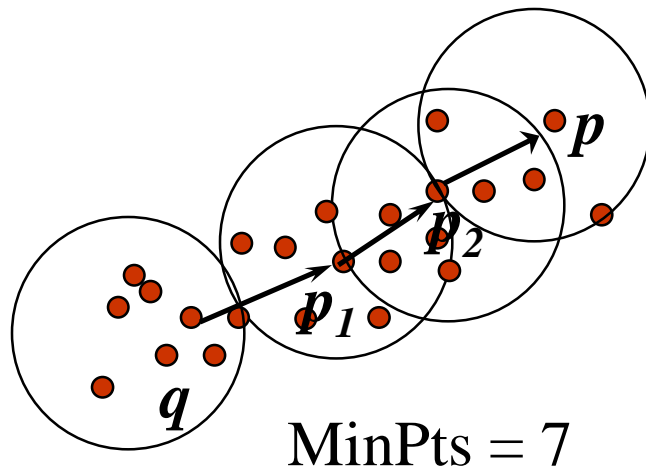
MinPts = 4

- $q$  is directly density-reachable from  $p$
- $p$  is not directly density-reachable from  $q$ ?
- Density-reachability is asymmetric.

# Density-reachability



- Density-Reachable (directly and indirectly):
  - A point  $p$  is directly density-reachable from  $p_2$ ;
  - $p_2$  is directly density-reachable from  $p_1$ ;
  - $p_1$  is directly density-reachable from  $q$ ;
  - $p \leftarrow p_2 \leftarrow p_1 \leftarrow q$  form a chain.



■  $p$  is (indirectly) density-reachable from  $q$

■  $q$  is not density-reachable from  $p$ ?

# Density-Connectivity

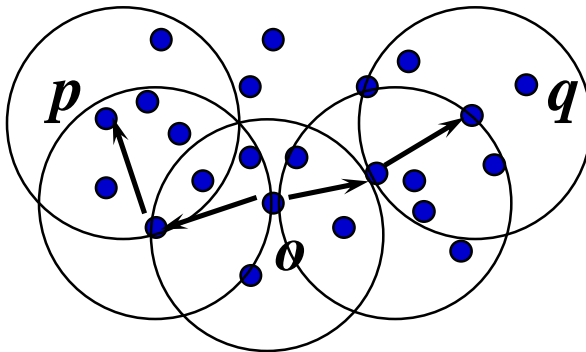


## ■ Density-reachable is not symmetric

□ not good enough to describe clusters

## ■ Density-Connected

□ A pair of points  $p$  and  $q$  are density-connected if they are commonly density-reachable from a point  $o$ .



■ Density-connectivity is symmetric

# Formal Description of Cluster

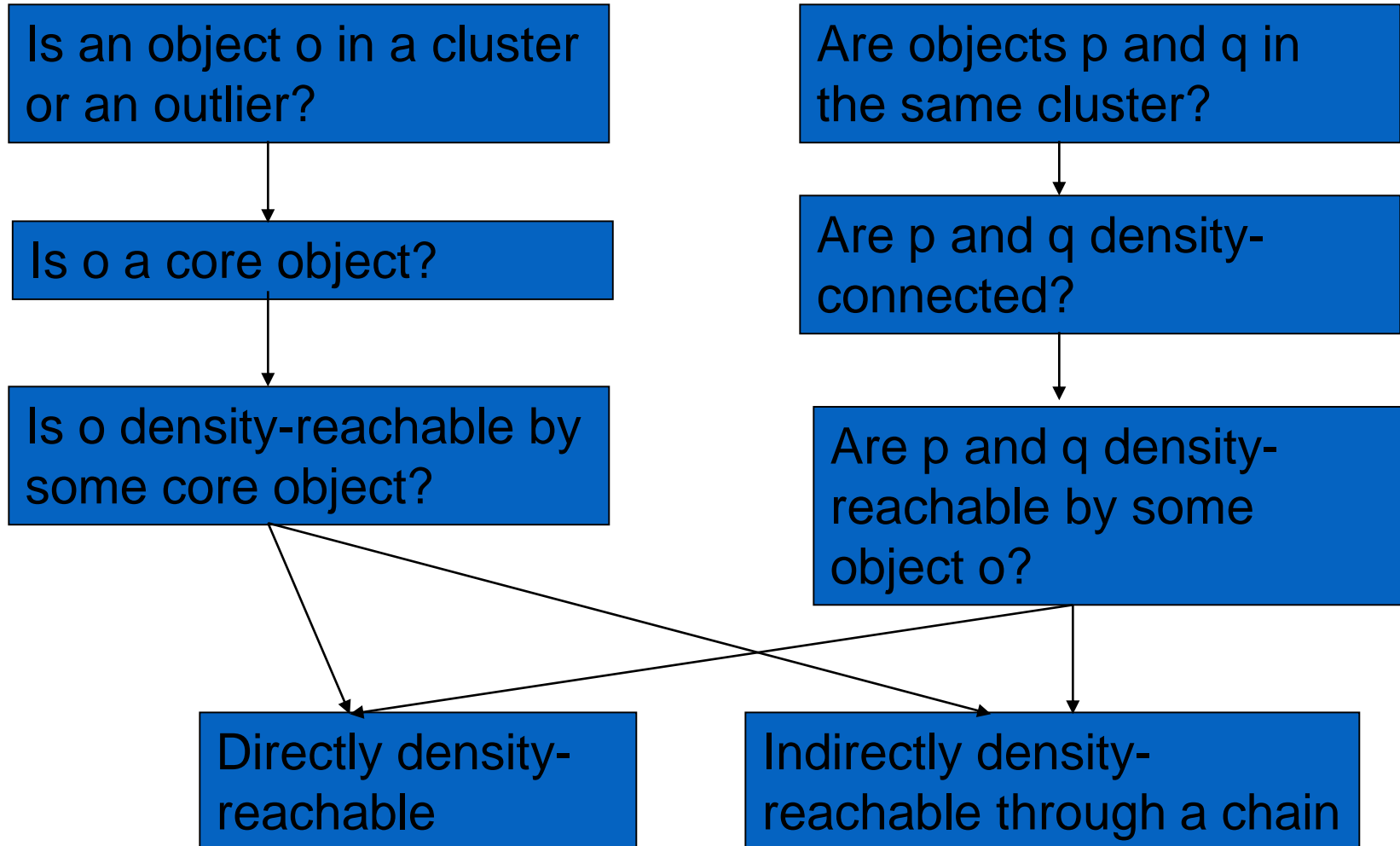


- Given a data set  $D$ , parameter  $\varepsilon$  and threshold  $\text{MinPts}$ .
- A cluster  $C$  is a subset of objects satisfying two criteria:
  - *Connected*:  $\forall p, q \in C$ :  $p$  and  $q$  are density-connected.
  - *Maximal*:  $\forall p, q$ : if  $p \in C$  and  $q$  is density-reachable from  $p$ , then  $q \in C$ . (avoid redundancy)



$P$  is a core object.

# Review of Concepts



# DBSCAN Algorithm



Input: The data set  $D$

Parameter:  $\epsilon$ , MinPts

For each object  $p$  in  $D$

    if  $p$  is a core object and not processed then

$C =$  retrieve all objects density-reachable from  $p$

        mark all objects in  $C$  as processed

        report  $C$  as a cluster

    else mark  $p$  as outlier

    end if

End For

DBScan Algorithm



# DBSCAN: The Algorithm

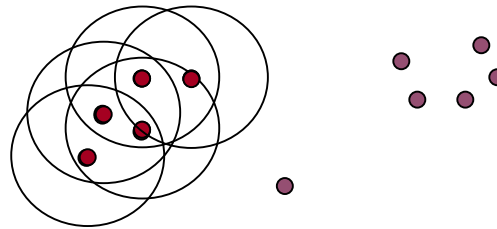


- Arbitrary select a point  $p$
- Retrieve all points density-reachable from  $p$  wrt  $Eps$  and  $MinPts$ .
- If  $p$  is a core point, a cluster is formed.
- If  $p$  is a border point, no points are density-reachable from  $p$  and DBSCAN visits the next point of the database.
- Continue the process until all of the points have been processed.

# DBSCAN Algorithm: Example



- Parameter
  - $\varepsilon = 2 \text{ cm}$
  - $\text{MinPts} = 3$

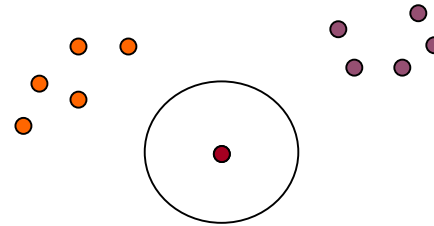


```
for each  $o \in D$  do  
  if  $o$  is not yet classified then  
    if  $o$  is a core-object then  
      collect all objects density-reachable from  $o$   
      and assign them to a new cluster.  
    else  
      assign  $o$  to NOISE
```

# DBSCAN Algorithm: Example



- Parameter
  - $\varepsilon = 2$  cm
  - $MinPts = 3$

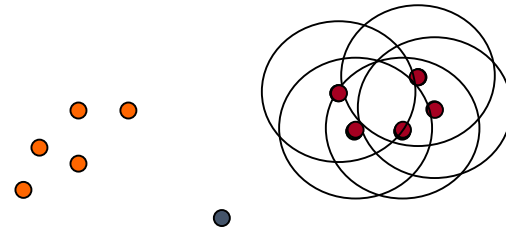


```
for each  $o \in D$  do  
  if  $o$  is not yet classified then  
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# DBSCAN Algorithm: Example



- Parameter
  - $\varepsilon = 2$  cm
  - $MinPts = 3$



```
for each  $o \in D$  do  
    if  $o$  is not yet classified then  
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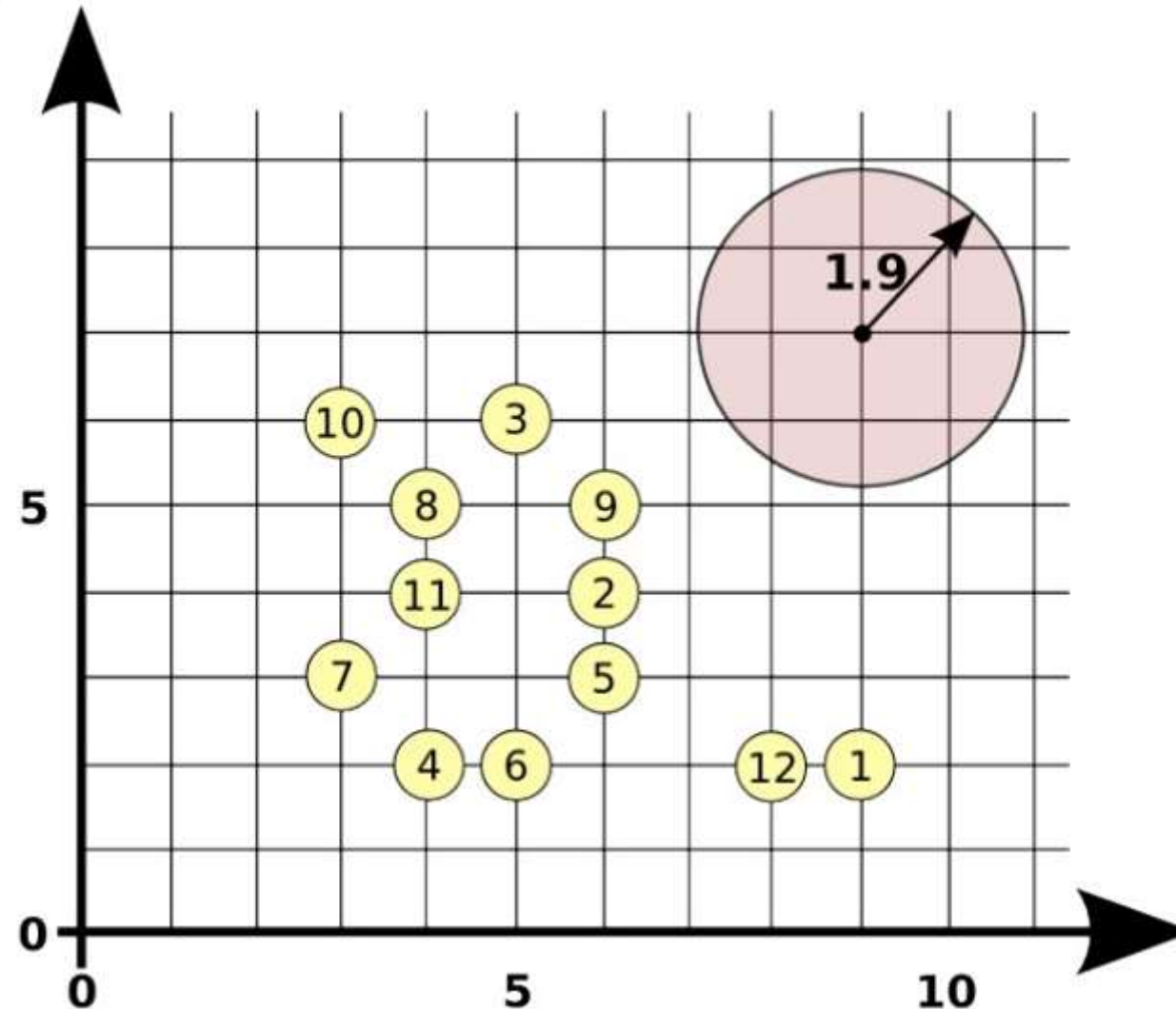
# DBSCAN $\text{eps}=2$ $\text{Minpts}=2$



If Epsilon is 2 and minpoint is 2, what are the clusters that DBScan would discover with the following 8 examples:  $A1=(2,10)$ ,  $A2=(2,5)$ ,  $A3=(8,4)$ ,  $A4=(5,8)$ ,  $A5=(7,5)$ ,  $A6=(6,4)$ ,  $A7=(1,2)$ ,  $A8=(4,9)$ .

1. Create the distance matrix using Euclidean
2. Find Neighborhood of each points
3. Define core,border,outliers
4. Define clusters

# DBSCAN eps=1.9 Minpts=4



```
# Let's import all your dependencies first

from sklearn.cluster import DBSCAN
from sklearn import preprocessing
from sklearn.preprocessing import StandardScaler
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

```
# Import .csv file and convert it to a DataFrame object
df = pd.read_csv("customers.csv");

print(df.head())
```

	Channel	Region	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicassen
0	2	3	12669	9656	7561	214	2674	1338
1	2	3	7057	9810	9568	1762	3293	1776
2	2	3	6353	8808	7684	2405	3516	7844
3	1	3	13265	1196	4221	6404	507	1788
4	2	3	22615	5410	7198	3915	1777	5185

```
#FRESH: annual spending (m.u.) on fresh products (Continuous);
#MILK: annual spending (m.u.) on milk products (Continuous);
#GROCERY: annual spending (m.u.) on grocery products (Continuous);
#FROZEN: annual spending (m.u.) on frozen products (Continuous)
#DETERGENTS_PAPER: annual spending (m.u.) on detergents and paper products (Continuous)
#DELICATESSEN: annual spending (m.u.) on and delicatessen products (Continuous);
#CHANNEL: customers' Channel - Horeca (Hotel/Restaurant/Café) or Retail channel (Nominal) REGION
```

```
print(df.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 440 entries, 0 to 439
Data columns (total 8 columns):
Channel          440 non-null int64
```

```
# there is no missing value in the dataset and all the data is integer in type
```

```
print(df.describe())
```

	Channel	Region	Fresh	Milk	Grocery \
count	440.000000	440.000000	440.000000	440.000000	440.000000
mean	1.322727	2.543182	12000.297727	5796.265909	7951.277273
std	0.468052	0.774272	12647.328865	7380.377175	9503.162829
min	1.000000	1.000000	3.000000	55.000000	3.000000
25%	1.000000	2.000000	3127.750000	1533.000000	2153.000000
50%	1.000000	3.000000	8504.000000	3627.000000	4755.500000
75%	2.000000	3.000000	16933.750000	7190.250000	10655.750000
max	2.000000	3.000000	112151.000000	73498.000000	92780.000000

	Frozen	Detergents_Paper	Delicassen
count	440.000000	440.000000	440.000000
mean	3071.931818	2881.493182	1524.870455
std	4854.673333	4767.854448	2820.105937
min	25.000000	3.000000	3.000000
25%	742.250000	256.750000	408.250000
50%	1526.000000	816.500000	965.500000
75%	3554.250000	3922.000000	1820.250000
max	60869.000000	40827.000000	47943.000000

```
#most of the data in this dataset is continuous in nature except for two features: Channel and Region so we can drop them
df.drop(["Channel", "Region"], axis = 1, inplace = True)
```

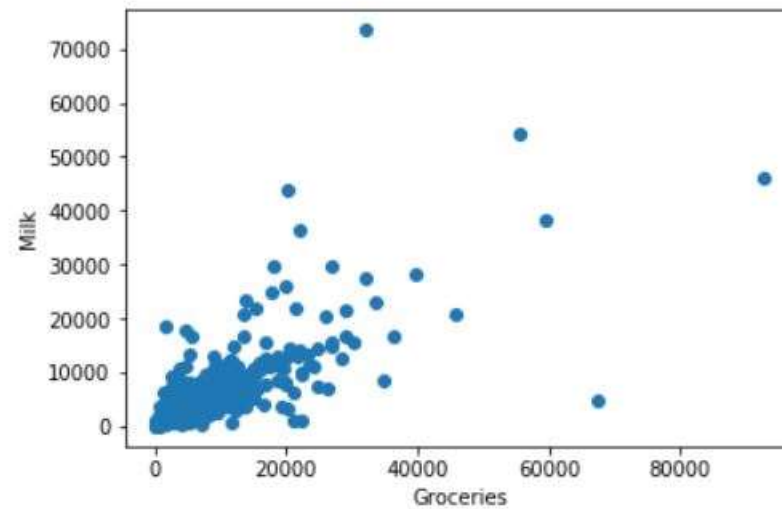
```
# Let's get a view of the data after the drop
```

```
print(df.head())
```

	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicassen
0	12669	9656	7561	214	2674	1338
1	7057	9810	9568	1762	3293	1776
2	6353	8808	7684	2405	3516	7844



```
# Let's plot the data now
x = df['Grocery']
y = df['Milk']
plt.scatter(x,y)
plt.xlabel("Groceries")
plt.ylabel("Milk")
plt.show()
```



```
#normalize each attribute by scaling it to 0 mean and unit variance.
```

```
df = df[["Grocery", "Milk"]]
df = df.as_matrix().astype("float32", copy = False)
```

```
stscaler = StandardScaler().fit(df)
df = stscaler.transform(df)
```

```
dbsc = DBSCAN(eps = .5, min_samples = 15).fit(df)
```

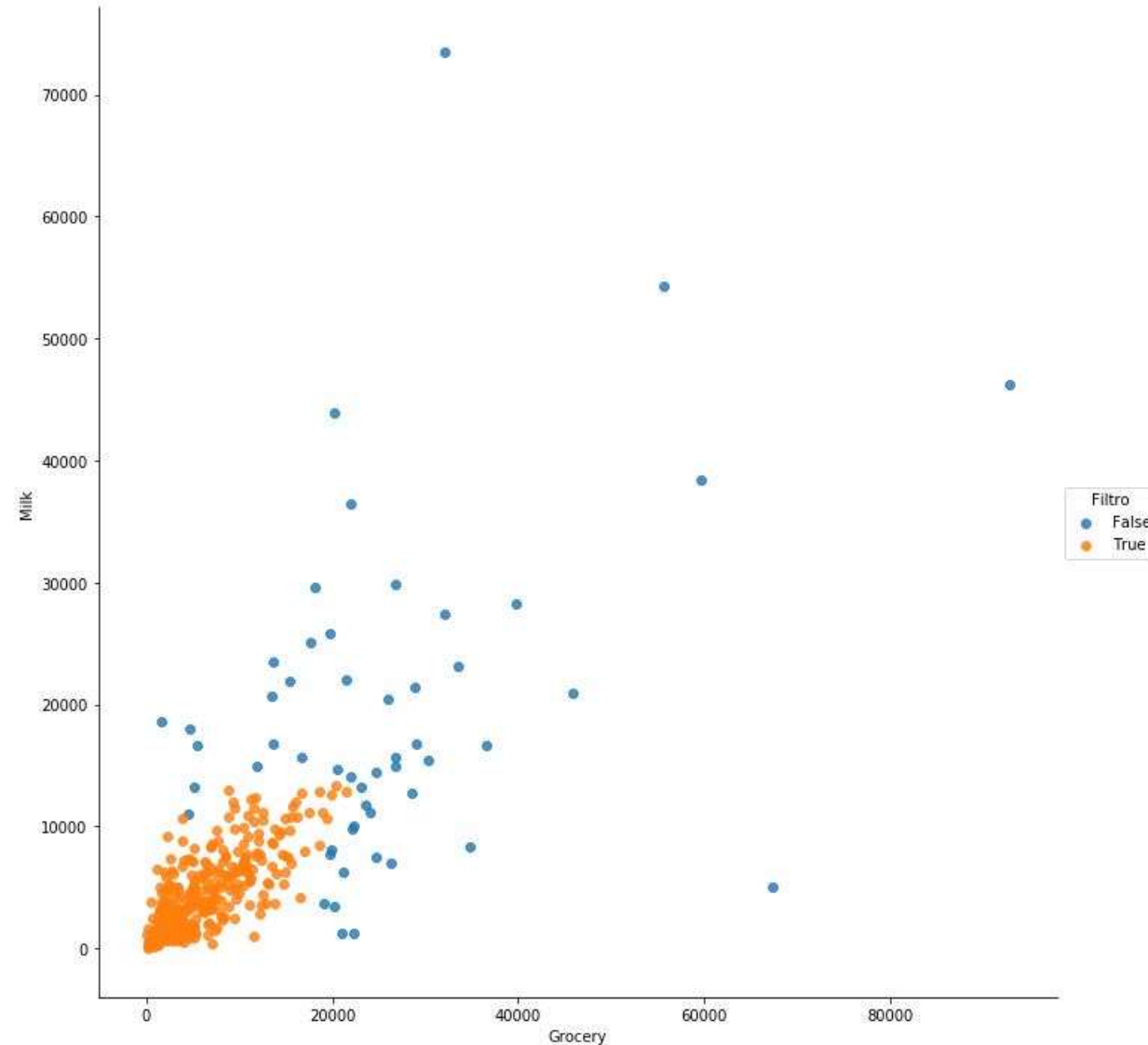
```
labels = dbsc.labels_
core_samples = np.zeros_like(labels, dtype = bool)
core_samples[dbsc.core_sample_indices_] = True
```

```
filtro=list(core_samples)
```

```
filtro
True,
False,
True,
True,
True,
True,
True,
True,
True,
True,
True,
False,
True,
True,
True,
True,
True,
True,
False,
False,
```



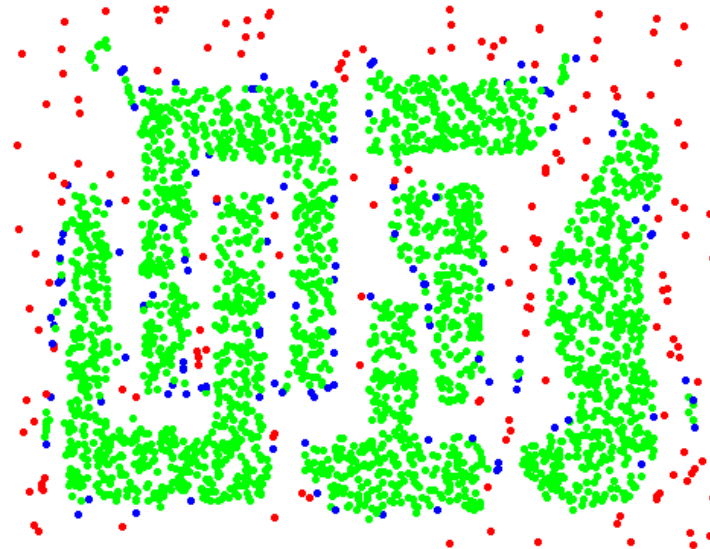
```
df0 = pd.read_csv("customers.csv");  
df0["Filtro"] = filtro  
  
sns.lmplot("Grocery", "Milk", data=df0, fit_reg=False, hue="Filtro", size=10)  
  
<seaborn.axisgrid.FacetGrid at 0x248ce343828>
```



# Example



Original Points



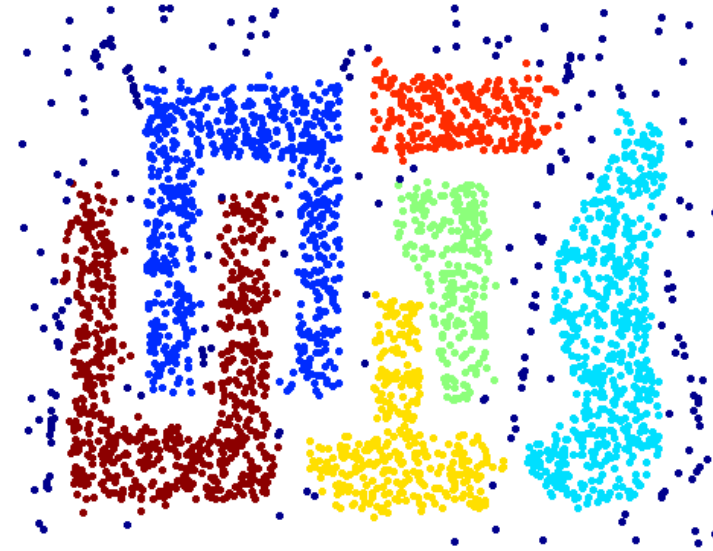
Point types: **core**,  
**border** and **outliers**

$\epsilon = 10$ , MinPts = 4

# When DBSCAN Works Well



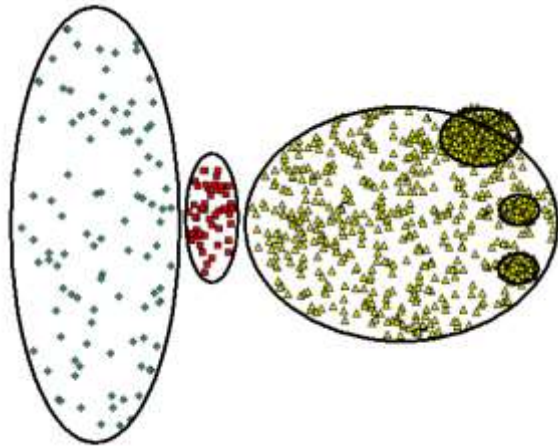
Original Points



Clusters

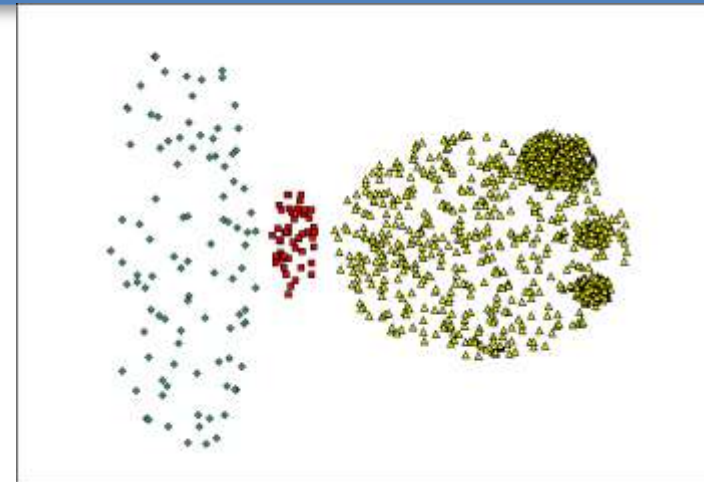
- Resistant to Noise
- Can handle clusters of different shapes and sizes

# When DBSCAN Does NOT Work Well

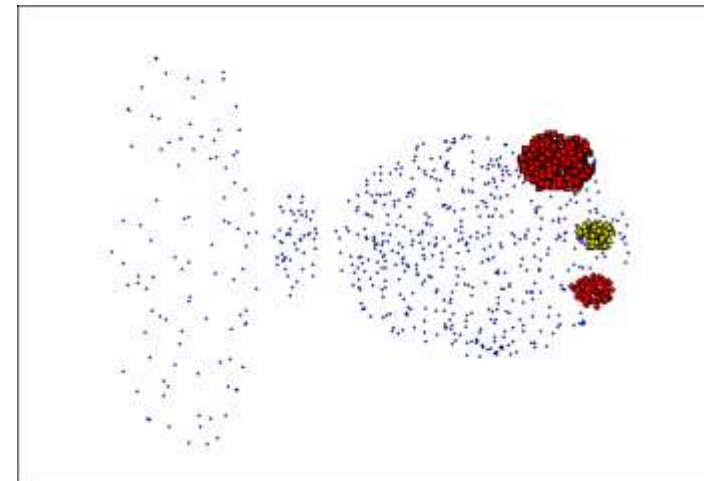


Original Points

- Cannot handle Varying densities
- sensitive to parameters



(MinPts=4, Eps=9.92).



(MinPts=4, Eps=9.75)



# DBSCAN: Sensitive to Parameters



Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

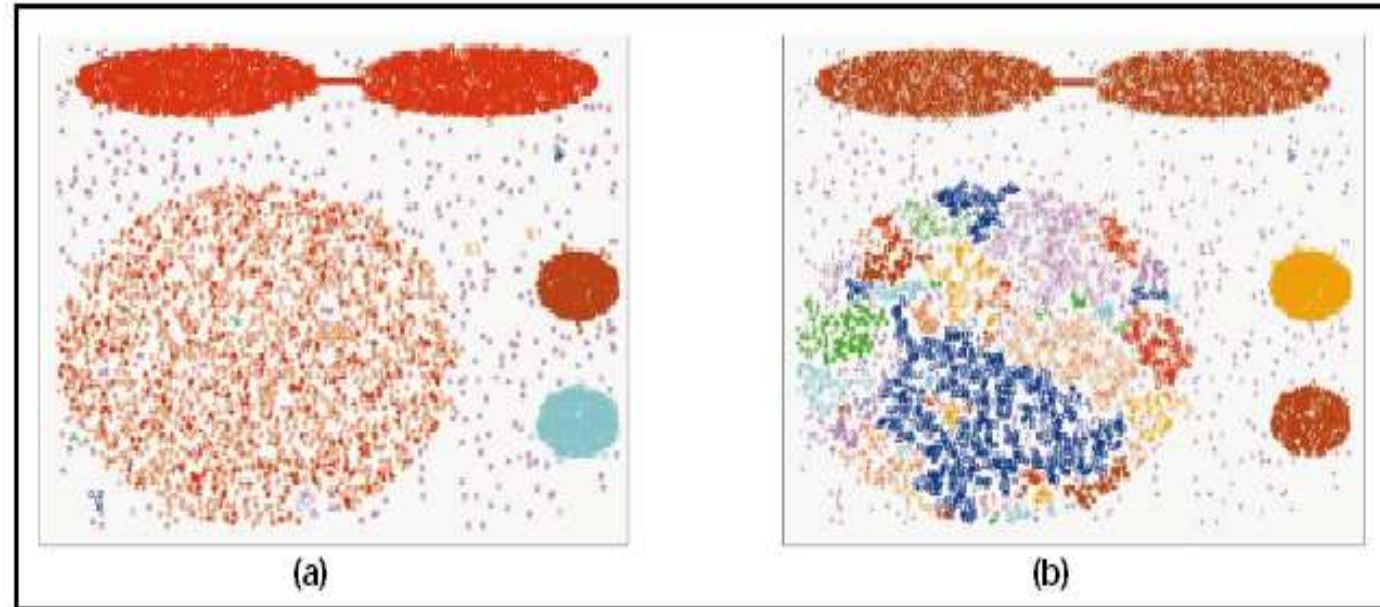


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.

