Math 3339

Homework 4 (Jointly distributed variables, Chapter 5)

Name:_____ PeopleSoft ID:_____

Instructions:

- Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
- Print out this file use or software and complete the problems.
- Write in black ink or dark pencil or type your solutions in the space provided. You must show all work for full credit.
- Submit this assignment at http://www.casa.uh.edu under "Assignments" and choose **hw4**.
- Total possible points: **15**
- You can use RStudio for any of these problems unless otherwise indicated.

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1. Suppose we have a frequency function for two variables X and Y, $f(x,y) = \frac{x+y}{30}$, for $x = 0, 1, 2$ and $y = 0, 1, 2, 3$.
   a. Determine the marginal distributions of X and Y.
   b. Determine E(X) and E(Y).
   c. Determine E($X + Y$).
   d. If $Z = 2X + 10$, determine E(Z).
   e. Determine E(XY).
   f. Determine cov(X,Y).
   g. Are X and Y independent? Justify your answer.

2. Section 5.2.3
    a. Problem 1
    b. Problem 2
    c. Find the cumulative distribution for the previous density function.

3. Section 5.2.3
    a. Problem 3
    b. Problem 4
    c. Problem 5

4. Let X denote the amount of time for which a book on 2-hour reserve at a college library is checked out by a randomly selected student and suppose that X has cumulative distribution function, CDF

$$F(X) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Use this to compute the following:

a. P($X \le 1$)
b. P($0.5 \le X \le 1.5$)
c. P($1.5 < X$)
d. Determine the median checkout duration. That is find x such that $F(x) = 0.5$.
e. Compute $F'(x)$ to obtain the density function $f(x)$.
f. Determine E(X) and Var(X).

5. Section 5.3.2; problem 5

6. Section 5.5.4; problem 1

7. Section 5.5.4; problem 2

8. Section 5.5.4
    a. Problem 3
    b. Problem 4

9. In each case, determine the value of the constant c that makes the probability statement correct.
   a. $\Phi(c) = 0.9838$
   b. $P(0 \leq Z \leq c) = 0.291$
   c. $P(c \leq Z) = 0.121$
   d. $P(-c \leq Z \leq c) = 0.668$
   e. $P(c \leq |Z|) = 0.016$

10. Suppose the force acting on a column that helps to support a building is a normally distributed random variable X with mean value 15.0 kips and standard deviation 1.25 kips. Compute the following probabilities.
    a. $P(X \leq 15)$
    b. $P(X \leq 17.5)$
    c. $P(X \geq 10)$
    d. $P(14 \leq X \leq 18)$
    e. $P(|X - 15| \leq 3)$

11. A test was done to look at the maximum speed of mopeds. They found that the maximum speed had a normal distribution with a mean of 29 mph and standard deviation of 1.4 mph. Consider randomly selecting a moped.
    a. What is the probability that the maximum speed is at most 31 mph?
    b. What is the probability that the maximum speed is at least 29.8 mph?
    c. What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

12. Let X = the time between two successive arrivals at the drive-up window of a local bank. If X has an exponential distribution with $\lambda = 1$, compute the following:
    a. The expected time between two successive arrivals.
    b. The standard deviation of the time between successive arrivals.
    c. $P(X \leq 4)$
    d. $P(2 \leq X \leq 5)$

13. Suppose that the time spent online to do homework by a randomly selected student has a gamma distribution with mean 20 minutes and variance 80 minutes^2.
    a. What are the values of α and β?
    b. What is the probability that a student spends online at most 24 minutes?
    c. What is the probability that a student spends between 20 and 40 minutes online?