Digital Image Processing

COSC 6380/4393

Lecture – 9

Feb 14th, 2023

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Slides from Dr. Shishir K Shah and Frank (Qingzhong) Liu

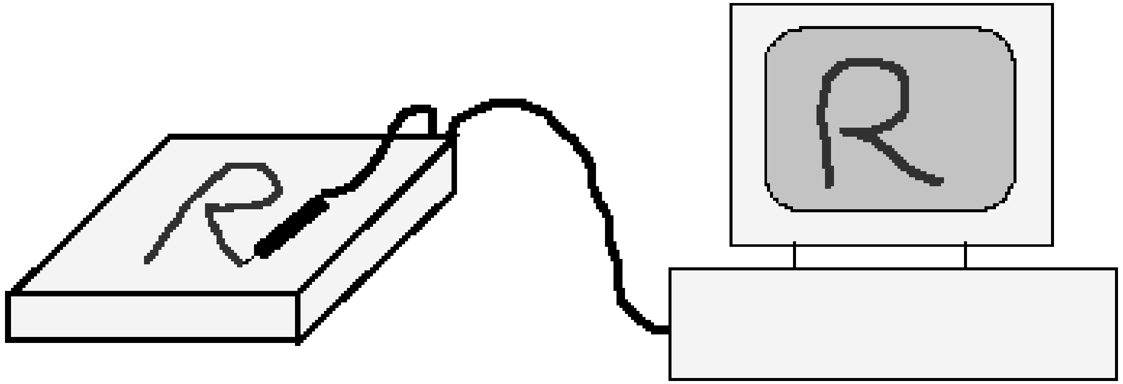
Review: BINARY IMAGES

- How do **binary images** arise?
- Since **binary = bi-valued,** the (logical) values ’0’ or ’1’ usually indicate the **absence** or **presence** of an **image** property in an associated gray-level image:
  - Points of high or low intensity (brightness)
  - Points where an object is present or absent
  - More abstract properties, such as smooth vs. nonsmooth, etc.
- **Convention** - We will make the associations
- ’1’ = BLACK
- ’0’ = WHITE

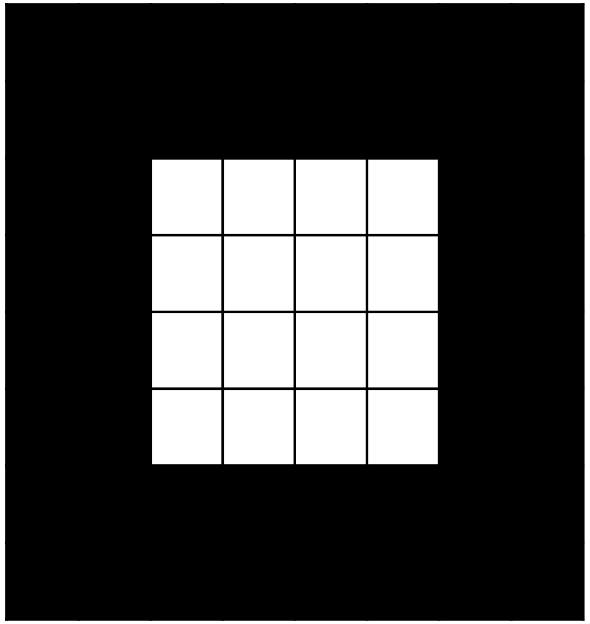
Review: BINARY IMAGE GENERATION

- **Tablet-Based Input:**
- Binary images can derive from **simple sensors** with binary output
- Simplest example: **tablet, resistive pad,** or **light pen**
- All pixels **initially** assigned value '0':

  I = [I(i, j)], I(i, j) = '0' for all (i, j) = (row column)
- When pressure or light is applied at (i_0 , j_0), the image is assigned the value '1': I(i_0 , j_0) = '1'
- This continues until the user completes the drawing



Review: Grey Level → Binary Image



8X8 image → white box
on black background

$Threshold$(T) →

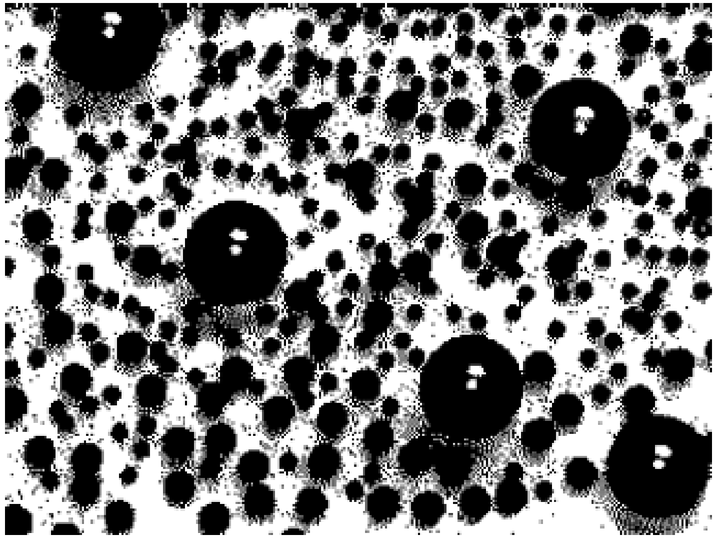
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

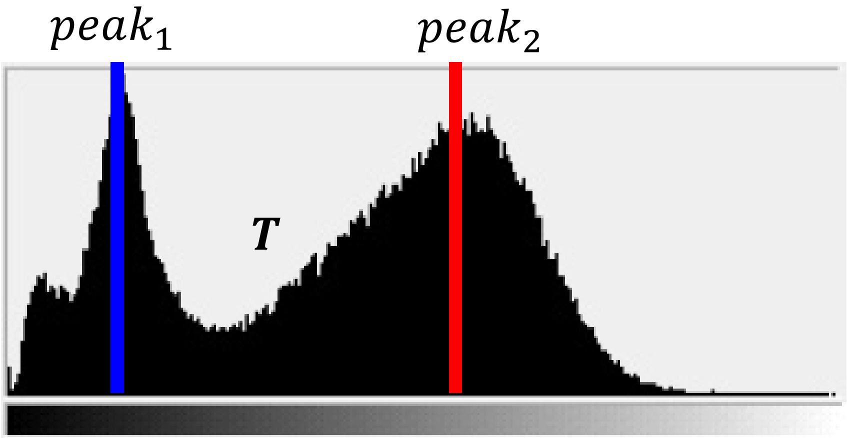
Binary image

What is good value of T?

Review: Example: How to find T

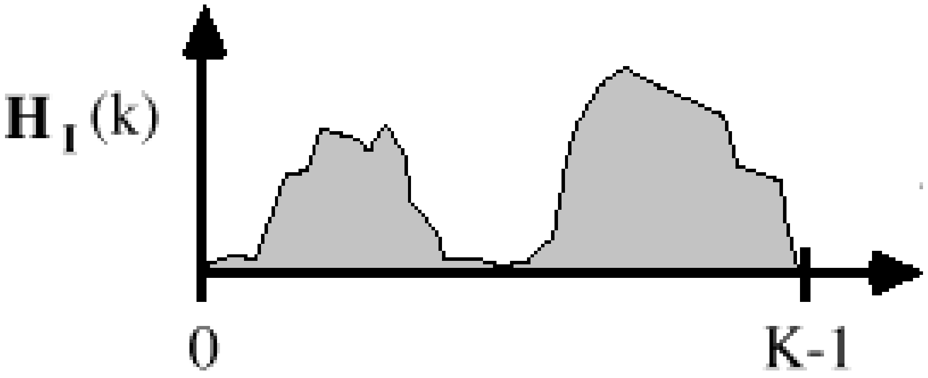
- Determine peaks
- Choose T between peaks(say average)





Review: Algorithm

$$Initialize\ T = K/2$$
$$Do$$
$$\quad Compute\ \mu_1 = E(X) \forall X < T$$
$$\quad Compute\ \mu_2 = E(X) \forall X \geq T$$
$$\quad Set\ T = \frac{\mu_1 + \mu_2}{2}$$
$$While\ \Delta\mu_1 != 0\ \& \Delta\mu_2 != 0$$



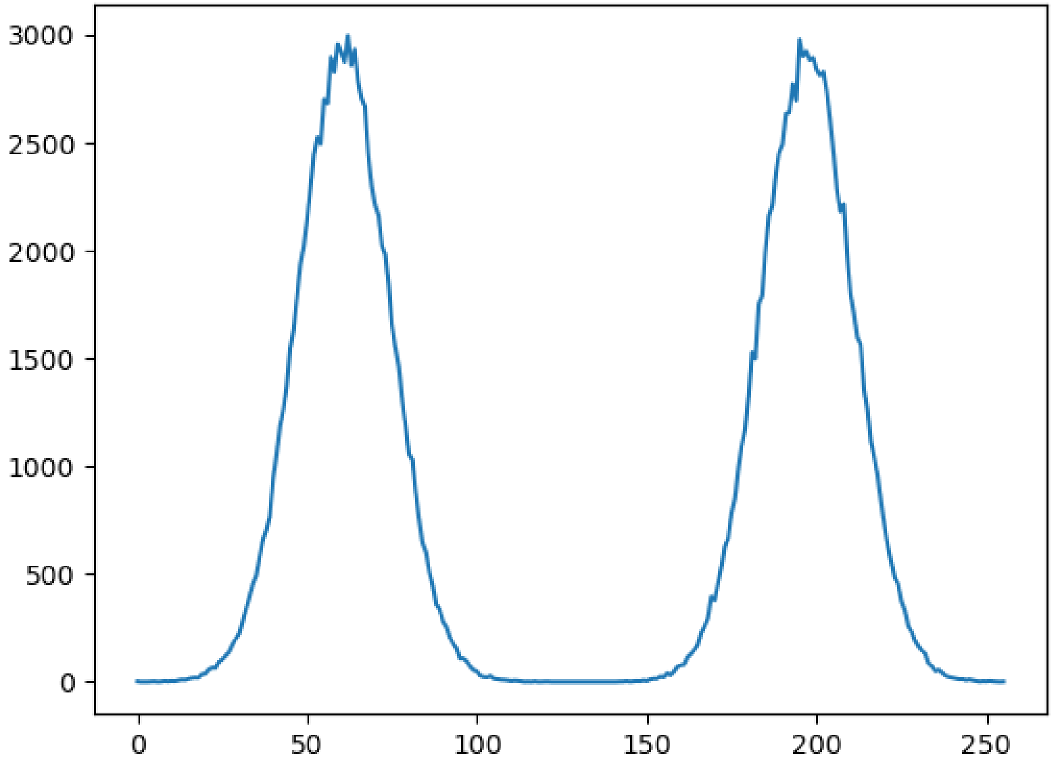
bimodal histogram
well separated peaks

AKA: Expectation Maximization (simple version)

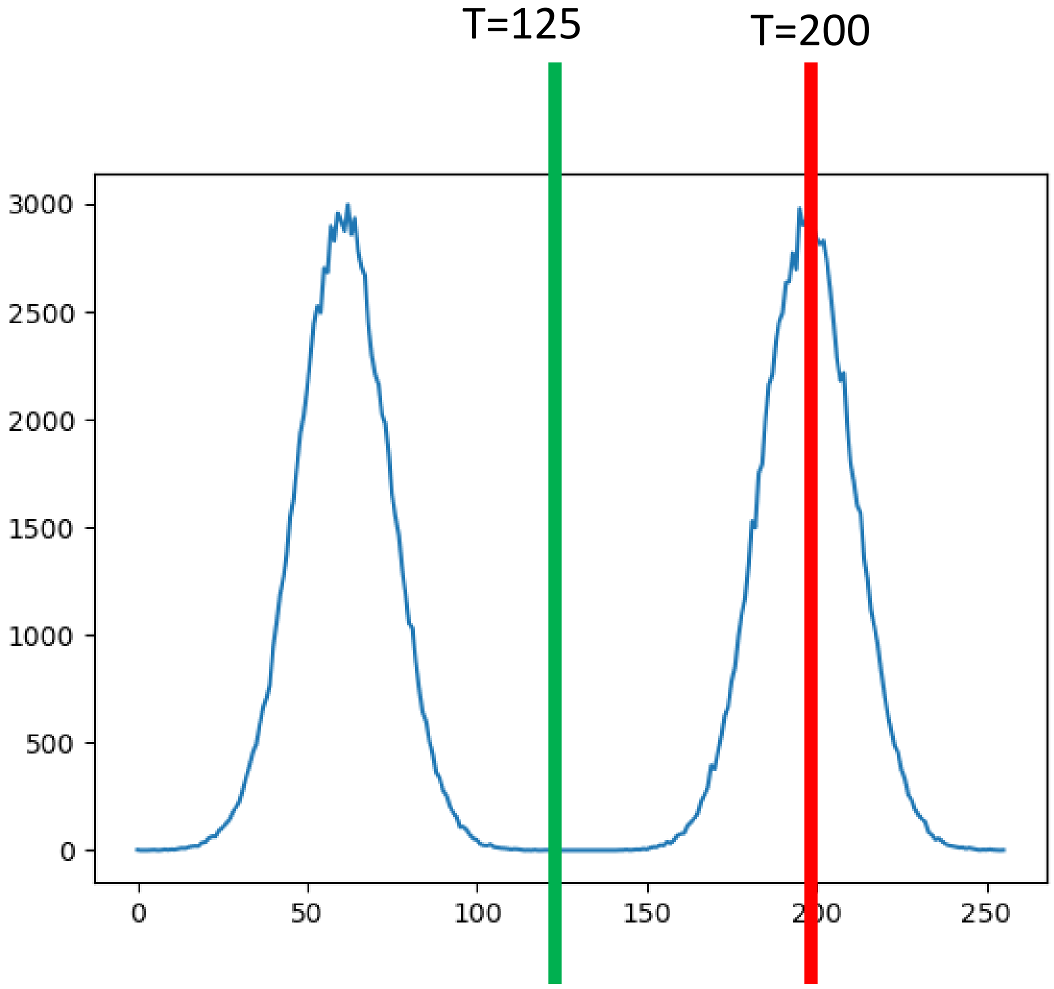
Otsu's Binarization

- Popular thresholding method
- It works on the histogram of the image
- It assumes that the histogram is bimodal

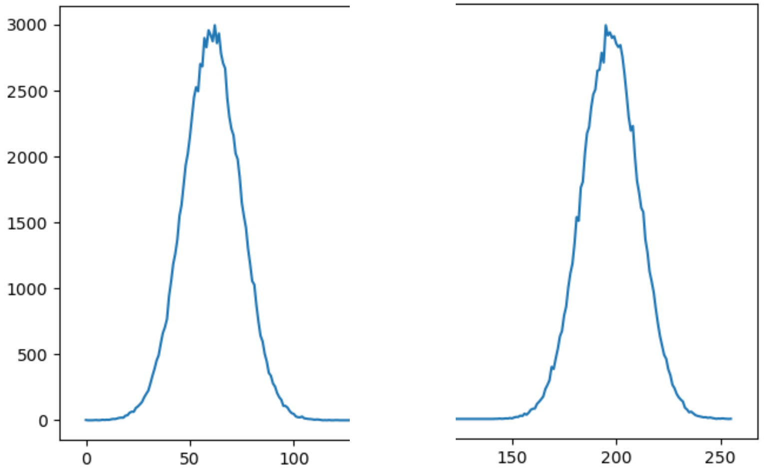
Bimodal Histogram



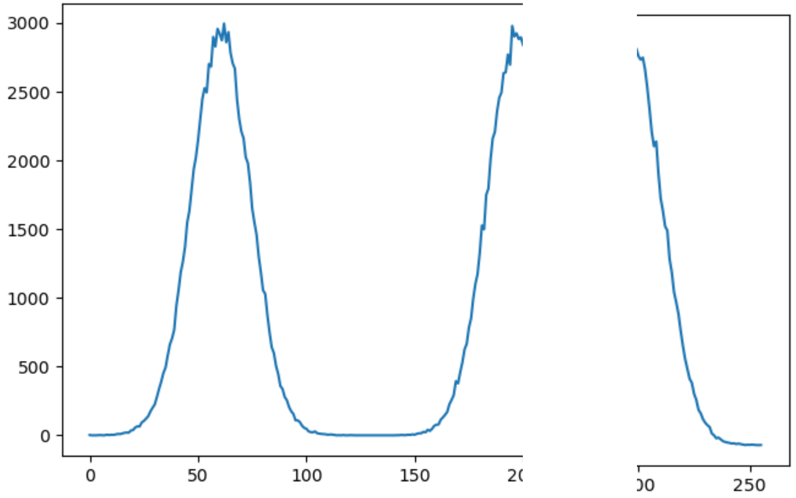
Bimodal Histogram



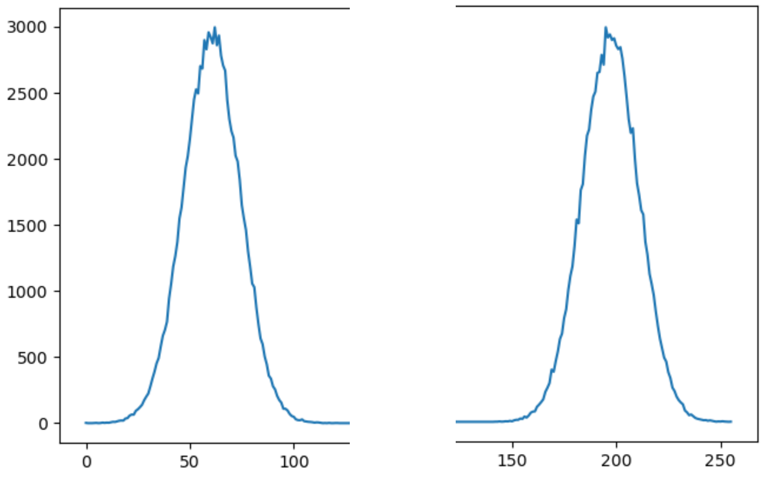
T=125


3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

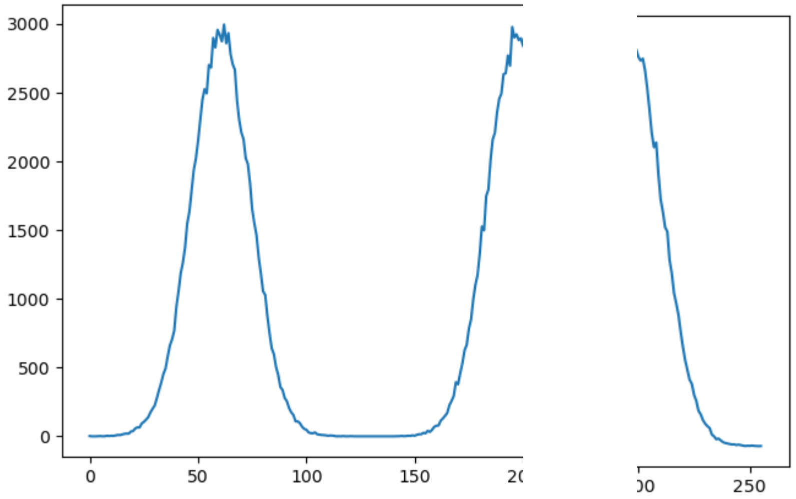
T=200


3000
2500
2000
1500
1000
500
0
0
50
100
150
250

T=125



T=200



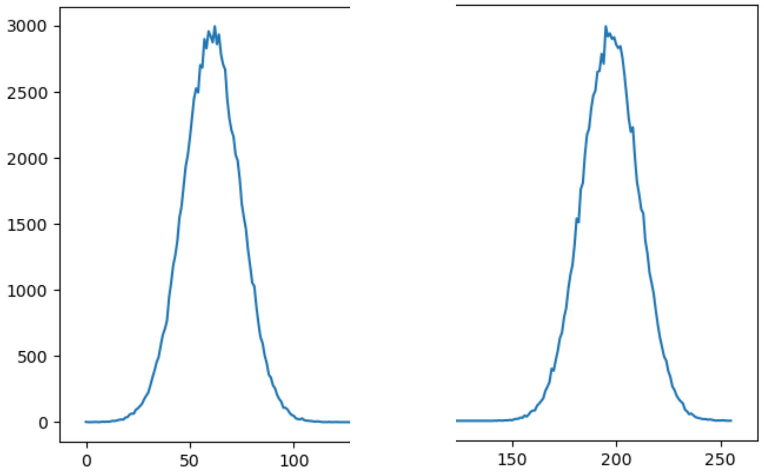
Compute a statistical value

Compute a statistical value

Compare

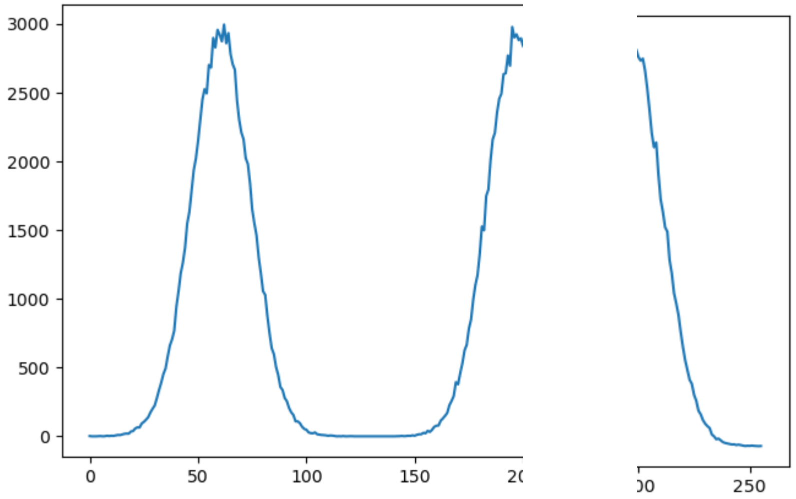
Pick Threshold

T=125



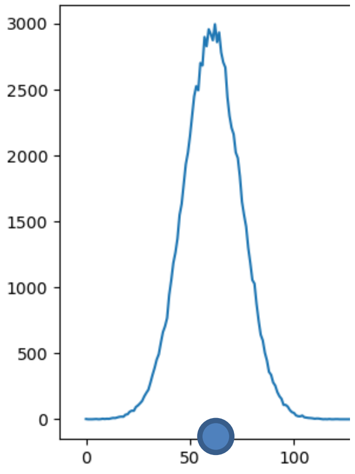
What metric to compute?

T=200

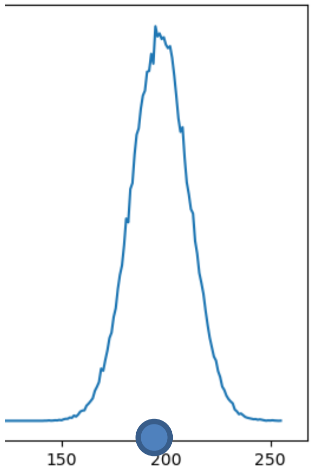


What metric to compute?

T=125

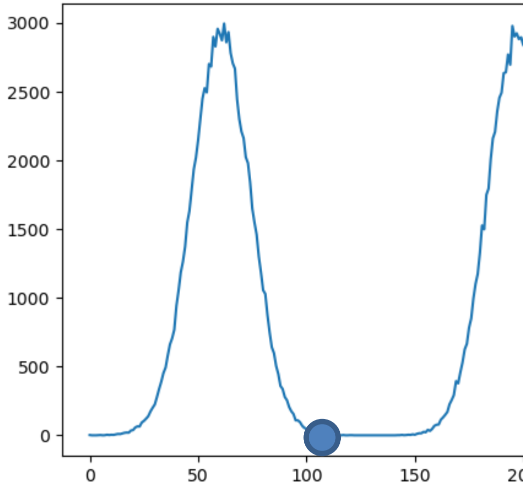


Mean = 62.03

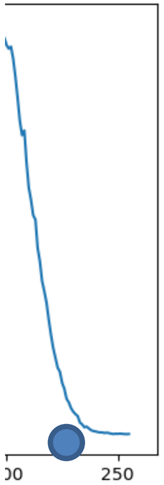


Mean = 195.8

T=200

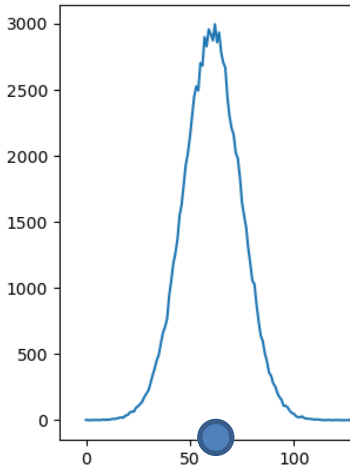


Mean = 107.54



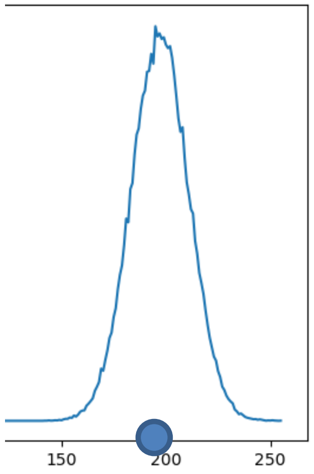
Mean = 225.4

T=125



Mean = 62.03

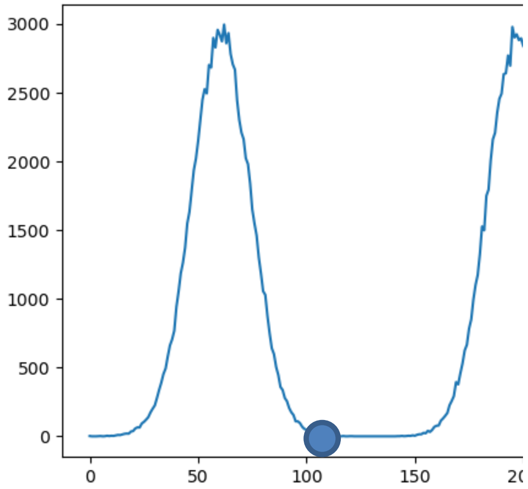
Var = 189.3



Mean = 195.8

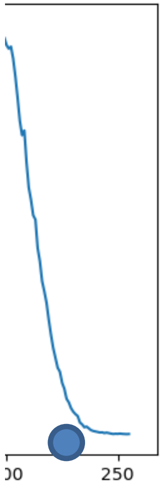
Var = 190.0

T=200



Mean = 107.54

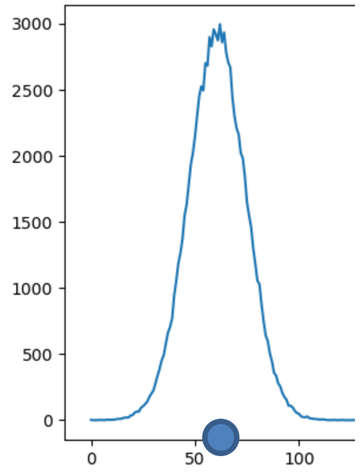
Var = 3709.54



Mean = 225.4

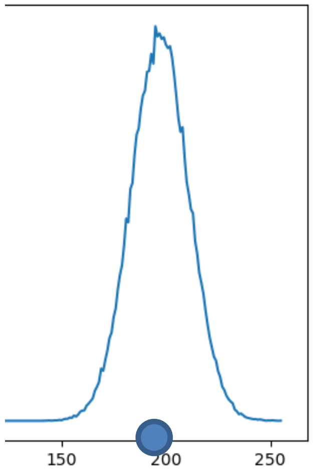
Var = 259.1

T=125



Mean = 62.03

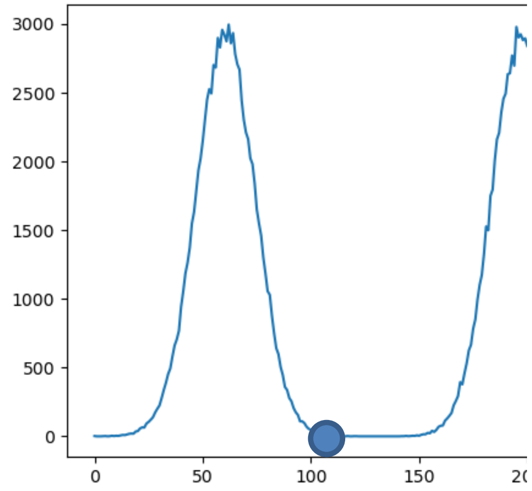
Var = 189.3



Mean = 195.8

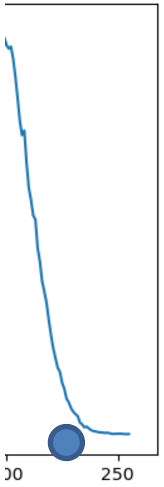
Var = 190.0

T=200



Mean = 107.54

Var = 3709.54



Mean = 225.4

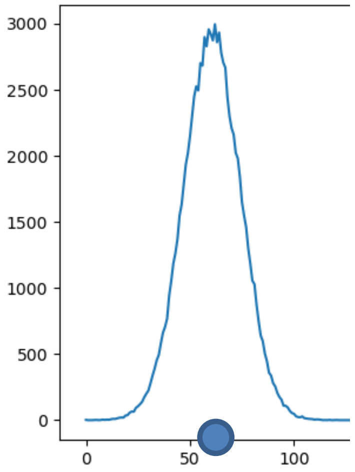
Var = 259.1

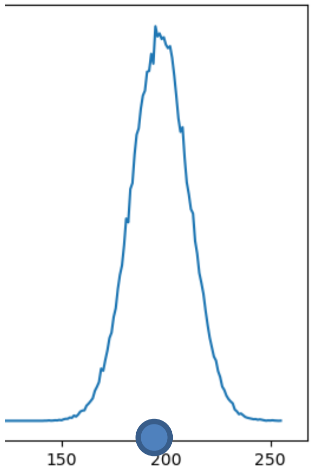
Intra-class Variance

Compare intra-class var

Pick Threshold that minimizes this value

T=125



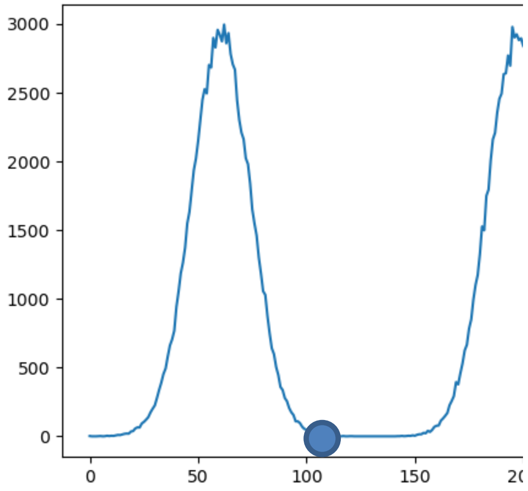


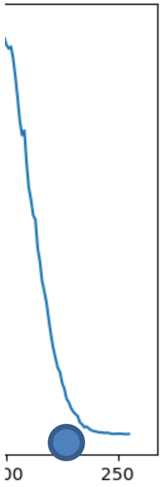
Mean = 62.03
Var = 189.3

+

Mean = 195.8
Var = 190.0

T=200





Mean = 107.54
Var = 3709.54

+

Mean = 225.4
Var = 259.1

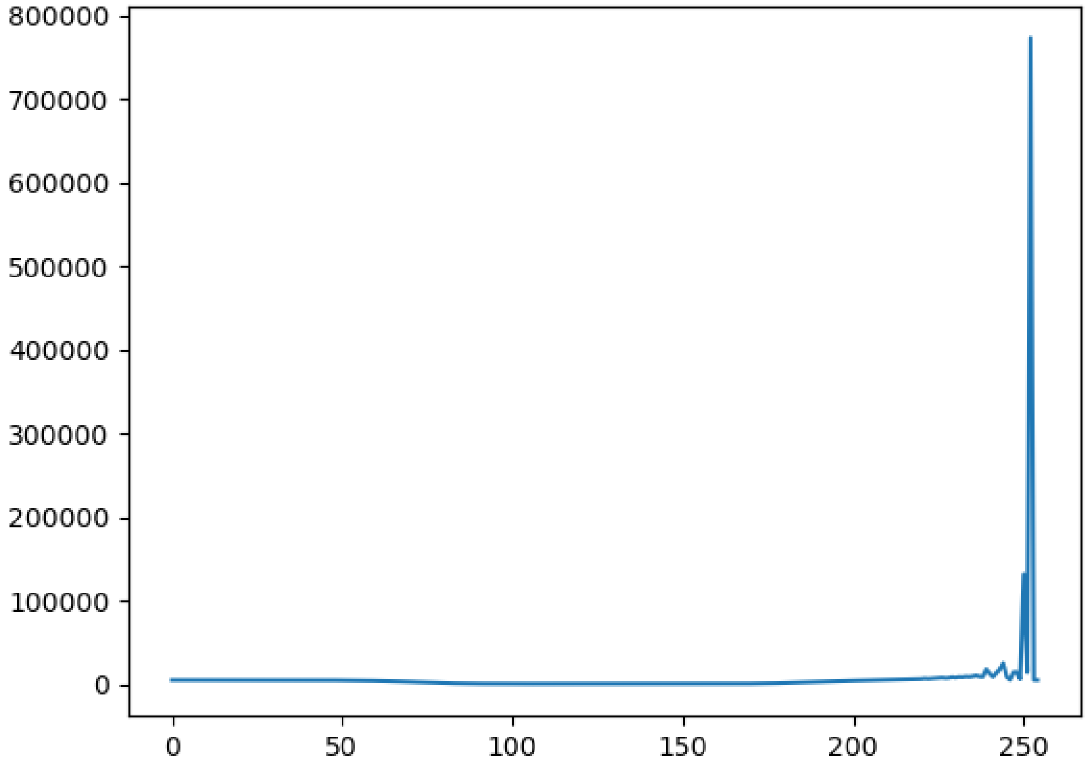
364 $\underbrace{\hspace{6em}}_{}$ Intra-class Variance 4370

Compare intra-class var

Pick Threshold that minimizes this value

Fn = Var1 + var 2

fn



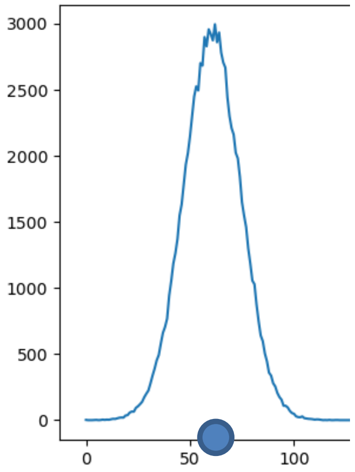
Threshold

Extreme values

Low values on either side of the valley

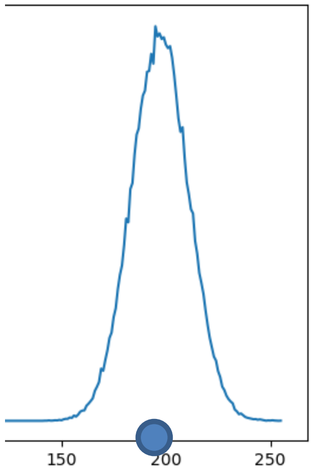
Weighted Sum

T=125



Mean = 62.03

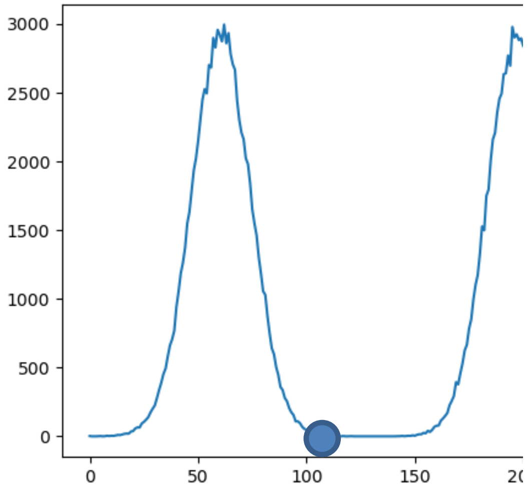
Var = 189.3



Mean = 195.8

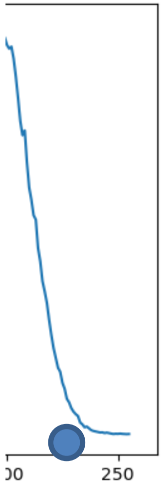
Var = 190.0

T=200



Mean = 107.54

Var = 3709.54



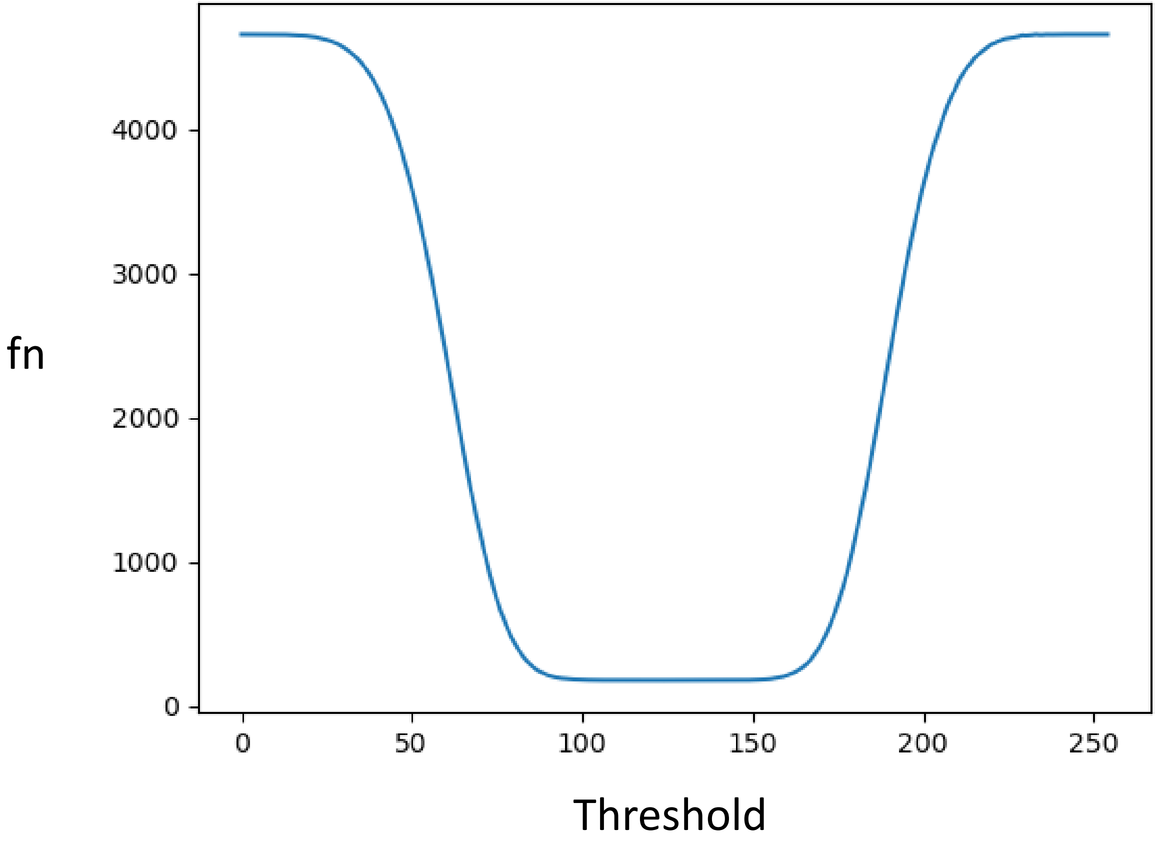
Mean = 225.4

Var = 259.1

$$fn = w_1\ var1 + \ w_2 var2$$

$$w_1 = \sum_{i=0}^{t} p(i),\ w_2 = \sum_{i=t+1}^{255} p(i),$$

$$fn = w_1\, var1 + w_2 var2$$



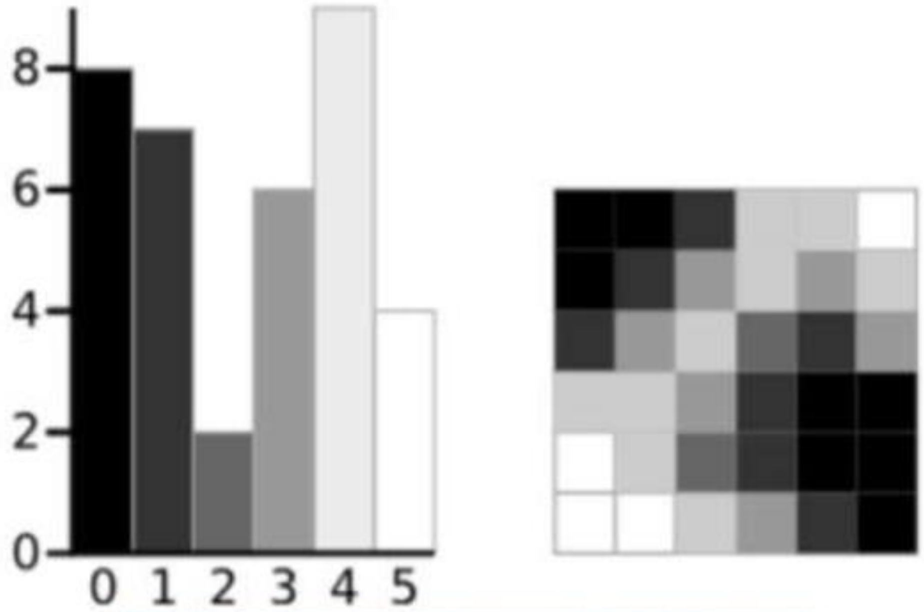
Extreme values

Low values on either side of the valley

Algorithm

1. Compute Histogram
2. Compute probabilities
3. Iterate through all possible threshold values (t=0 to t= 255)
   1. Compute weights (q_1, q_2)
   2. Compute mean (μ_1, μ_2)
   3. Compute intra-class variance $(\boldsymbol{\sigma_1^2}, \boldsymbol{\sigma_2^2})$
   4. Compute weighted sum of intra-class variance $(\boldsymbol{q_1\sigma_1^2} + \boldsymbol{q_2\sigma_2^2})$
4. Pick threshold that minimizes the weighted sum of intraclass variance

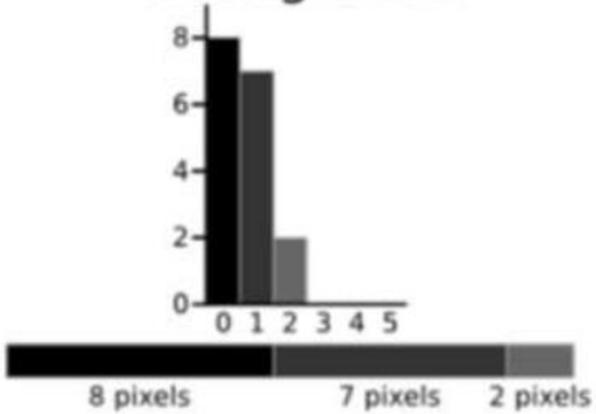
Example



A 6-level greyscale image and its histogram

- The calculations for finding the foreground and background variances (the measure of spread) for a single threshold are now shown in next slide.

Background

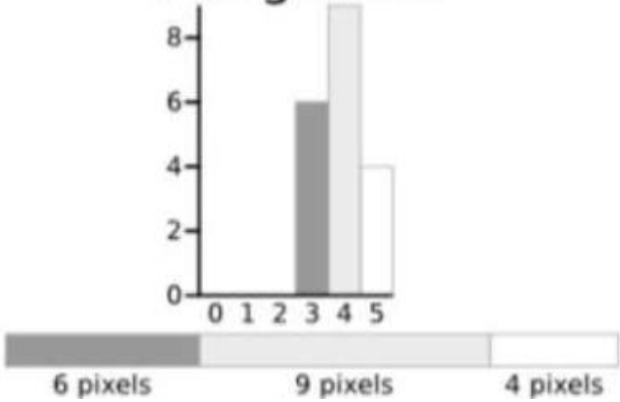


$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

$$\text{Mean} \quad \mu_b = \frac{(0 \times 8) + (1 \times 7) + (2 \times 2)}{17} = 0.6471$$

$$\begin{aligned}\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2 \times 8) + ((1-0.6471)^2 \times 7) + ((2-0.6471)^2 \times 2)}{17} \\ &= \frac{(0.4187 \times 8) + (0.1246 \times 7) + (1.8304 \times 2)}{17} \\ &= 0.4637\end{aligned}$$

Foreground



$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

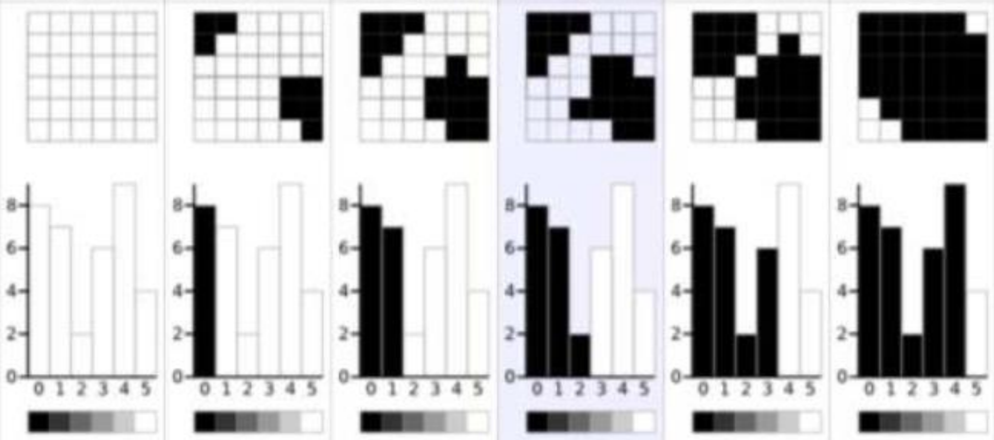
$$\text{Mean} \quad \mu_f = \frac{(3 \times 6) + (4 \times 9) + (5 \times 4)}{19} = 3.8947$$

$$\begin{aligned}\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2 \times 6) + ((4-3.8947)^2 \times 9) + ((5-3.8947)^2 \times 4)}{19} \\ &= \frac{(4.8033 \times 6) + (0.0997 \times 9) + (4.8864 \times 4)}{19} \\ &= 0.5152\end{aligned}$$

- The next step is to calculate the 'Within-Class Variance'. This is simply the sum of the two variances multiplied by their associated weights

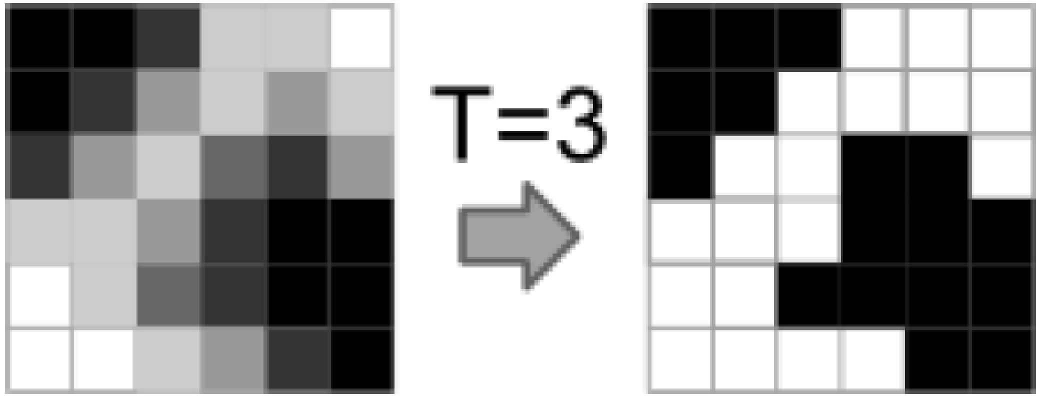
$$\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 = 0.4722 * 0.4637 + 0.5278 * 0.5152$$
$$= 0.4909$$

- This final value is the 'sum of weighted variances' for the threshold value 3. This same calculation needs to be performed for all the possible threshold values 0 to 5. The table below shows the results for these calculations. The highlighted column shows the values for the threshold calculated above



Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\sigma^2_W = 0.4909$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

- It can be seen that for the threshold equal to 3, as well as being used for the example, also has the lowest sum of weighted variances. Therefore, this is the final selected threshold. All pixels with a level less than 3 are background, all those with a level equal to or greater than 3 are foreground. As the images in the table show, this threshold works well.



Algorithm

1. Compute Histogram $\left(H(i)\right)$, with N different intensity values.
2. Compute probabilities $\boldsymbol{P(i) = \frac{H(i)}{\sum_i H(i)}}$

3. Iterate through all possible threshold values (t=0 to t= 255)

3.1 Calculate Weights

$$\boldsymbol{q_1(t) = \sum_{i=0}^{t} P(i) \qquad q_2(t) = \sum_{i=t+1}^{N} P(i)}$$

3.2 Compute mean

$$\mu_1(t) = \sum_{i=0}^{i=t} \frac{iP(i)}{q_1(t)}, \mu_2(t) = \sum_{i=t+1}^{i=N} \frac{iP(i)}{q_2(t)}$$

3.3 Compute intra-class variance

$$\boldsymbol{\sigma_1^2(t) = \sum_{i=0}^{t} \frac{\left(i - \mu_1(t)\right)^2 P(i)}{q_1(t)}} \qquad \boldsymbol{\sigma_2^2(t) = \sum_{i=t+1}^{N} \frac{\left(i - \mu_2(t)\right)^2 P(i)}{q_2(t)}}$$

3.4 Compute weighted sum of intra-class variance

$$\sigma_w{}^2(t) = q_1(t) \quad \sigma_1{}^2(t) + q_2(t) \quad \sigma_2{}^2(t)$$

4. Pick threshold that minimizes the weighted sum of intra-class variance

DISCUSSION OF HISTOGRAM TYPES

- We'll return to the histogram later in the context of **quantitative gray-level properties**
- Some general qualitative observations are worth making now
  - **Bimodal histograms** often imply **objects** and **background** of significantly different average brightnesses
  - **Bimodal histograms** are the easiest to threshold
  - The result of thresholding a **bimodal histogram** is (ideally) a simple binary image showing **object/background separation**
- Examples. Images of
  - Printed type
  - Blood cells in solution
  - Machine parts on an assembly line

HISTOGRAM TYPES

- **Multi-modal histograms** often occur when the image contains different objects of different average brightnesses on a uniform background
- **Flat or level histograms** usually imply more complex images, containing detail, non-uniform background, etc.
- Thresholding rarely gives perfect results
- Usually, some kind of **region correction** must be applied

THE BASIC LOGICAL OPERATIONS

- We will use only a **few simple** logical operations
- Suppose that X_1 ,..., X_n are **binary variables**
- For example, pixels from one or more binary images
- Here is the notation we will use:
- **Logical Complement**: $NOT(X_1)$ = complement of X_1
- **Logical AND**: $AND(X_1, X_2) = X_1 \wedge X_2$

X_1	$NOT(X_1)$
0	1
1	0

TRUTH TABLE

X_1	X_2	$X_1 \wedge X_2$
0	0	0
0	1	0
1	0	0
1	1	1

TRUTH TABLE

LOGICAL OPERATIONS

- **Multi-Variable Logical AND:**

$$\mathrm{AND}(X_1, X_2, \ldots, X_n) = X_1 \wedge X_2 \wedge \cdots\cdots \wedge X_{n-1} \wedge X_n$$

$$= \bigwedge_{i=1}^{n} X_i$$

$$= 1 \qquad \text{if } X_1 = X_2 = X_3 = \cdots\cdots = X_{n-1} = X_n = 1 \text{ (all 1's)}$$

$$= 0 \qquad \text{otherwise}$$

- **Logical OR**: OR(X1, X2) = X1 ∨ X2

X_1	X_2	$X_1 \vee X_2$
0	0	0
0	1	1
1	0	1
1	1	1

TRUTH TABLE

LOGICAL OPERATIONS

- **Multi-Variable Logical OR:**

$$OR(X_1, X_2, \ldots, X_n) = X_1 \vee X_2 \vee \cdots\cdots \vee X_{n-1} \vee X_n$$

$$= \bigvee_{i=1}^{n} X_i$$

$$= 0 \qquad \text{if } X_1 = X_2 = X_3 = \cdots\cdots = X_{n-1} = X_n = 0 \text{ (all 0's)}$$

$$= 1 \qquad \text{otherwise}$$

SIMPLE BOOLEAN ALGEBRA PROPERTIES

1. NOT [NOT(X)] = X
2. $X1 \wedge X2 \wedge X3 = (X1 \wedge X2) \wedge X3$
   $= X1 \wedge (X2 \wedge X3)$
3. $X1 \vee X2 \vee X3 = (X1 \vee X2) \vee X3$
   $= X1 \vee (X2 \vee X3)$
4. $X1 \wedge X2 = X2 \wedge X1$
5. $X1 \vee X2 = X2 \vee X1$
6. $(X1 \wedge X2) \vee X3 = (X1 \vee X3) \wedge (X2 \vee X3)$
7. $(X1 \vee X2) \wedge X3 = (X1 \wedge X3) \vee (X2 \wedge X3)$
8. $\mathrm{NOT}(X1 \wedge X2) = \mathrm{NOT}(X1) \vee \mathrm{NOT}(X2)$
9. $\mathrm{NOT}(X1 \vee X2) = \mathrm{NOT}(X1) \wedge \mathrm{NOT}(X2)$

SIMPLE BOOLEAN ALGEBRA PROPERTIES

1. NOT [NOT(X)] = X
2. $X1 \wedge X2 \wedge X3 = (X1 \wedge X2) \wedge X3$
   $= X1 \wedge (X2 \wedge X3)$ (Associative Law)
3. $X1 \vee X2 \vee X3 = (X1 \vee X2) \vee X3$
   $= X1 \vee (X2 \vee X3)$ (Associative Law)
4. $X1 \wedge X2 = X2 \wedge X1$ (Commutative Law)
5. $X1 \vee X2 = X2 \vee X1$ (Commutative Law)
6. $(X1 \wedge X2) \vee X3 = (X1 \vee X3) \wedge (X2 \vee X3)$ (Distributive Law)
7. $(X1 \vee X2) \wedge X3 = (X1 \wedge X3) \vee (X2 \wedge X3)$ (Distributive Law)
8. $\text{NOT}(X1 \wedge X2) = \text{NOT}(X1) \vee \text{NOT}(X2)$ (DeMorgan's Law)
9. $\text{NOT}(X1 \vee X2) = \text{NOT}(X1) \wedge \text{NOT}(X2)$ (DeMorgan's Law)

BOOLEAN ALGEBRA

- **Binary Majority** (odd # of variables only)

X_1	X_2	X_3	$MAJ(X_1, X_2, X_3)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

TRUTH TABLE

BOOLEAN ALGEBRA

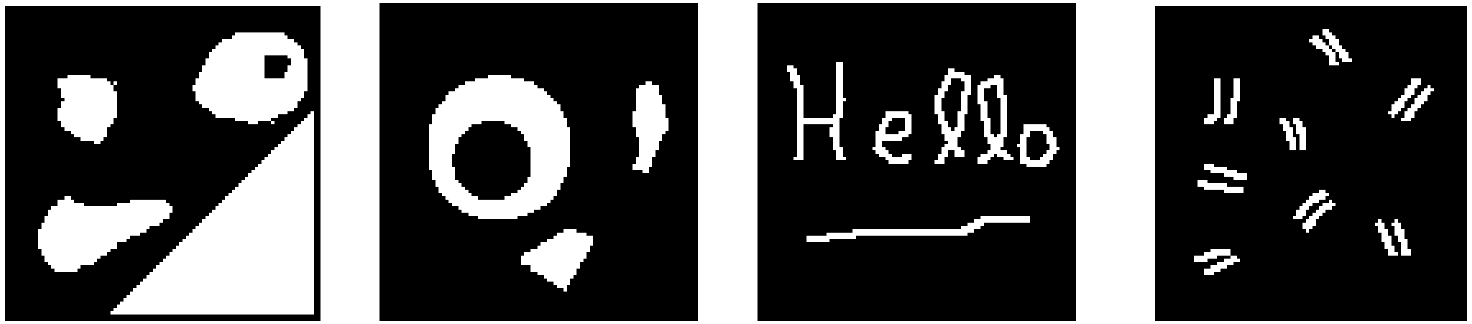
- **Multi-Variable Binary Majority:**
- $MAJ(X_1, X_2, \ldots, X_n) = 1$ if more 1's than 0's = 0 if more 0's than 1's
- **Comments**
  - **Any** binary operation can be created from 'NOT', 'AND', 'OR' - Boolean
  - Algebra is an entire math discipline built on these
  - However, we will restrict ourselves to using 'NOT', 'AND', 'OR', and 'MAJ' in a few simple applications

LOGICAL OPERATIONS ON IMAGES

- Let $\mathbf{I}_1$, $\mathbf{I}_2$, ..., $\mathbf{I}_n$ be binary images
- We define logical operations on images on a point-wise basis

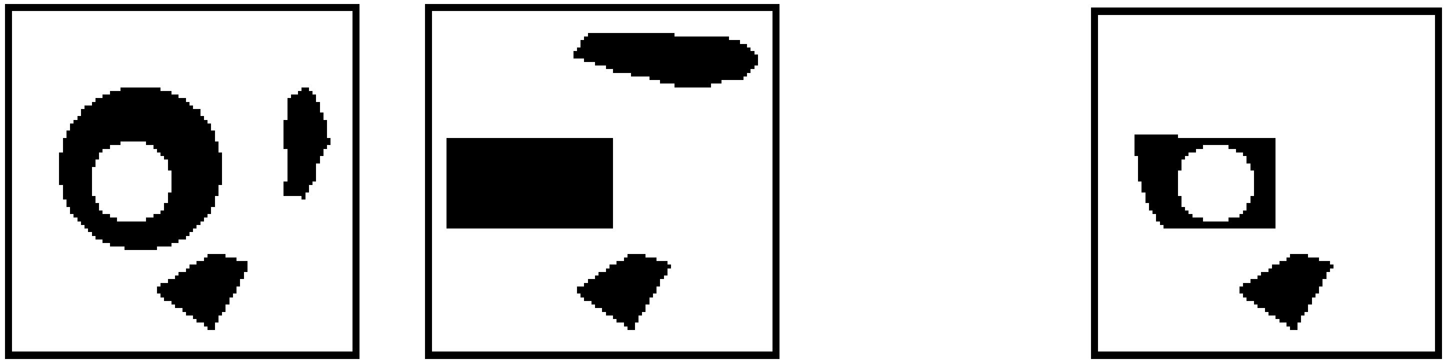
The complement of an image:

- $\mathbf{J}_1$ = NOT($\mathbf{I}_1$) if J_1 (i, j) = NOT[I_1 (i, j)] for all (i, j)
- This reverses the **contrast** - it creates a **binary negative**:



BINARY AND

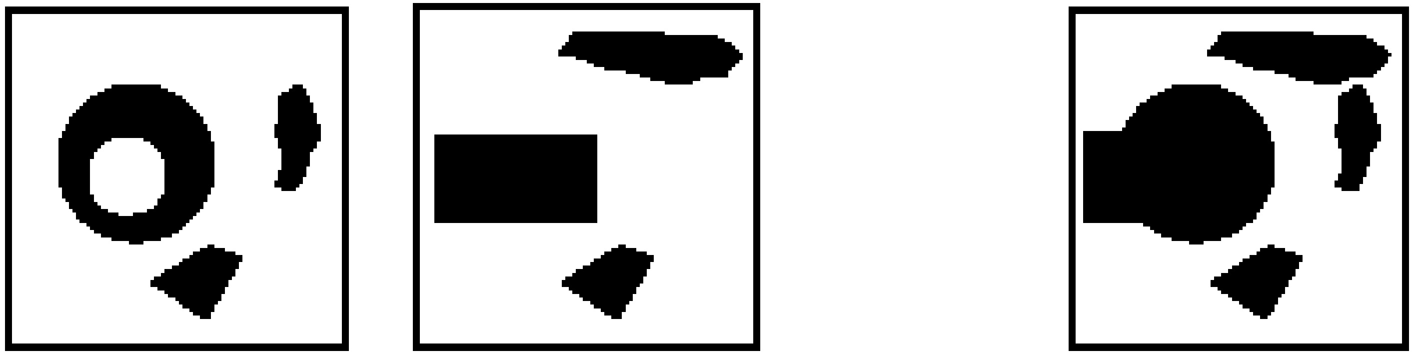
- The AND or intersection of two images:
- $\mathbf{J}_2 = \text{AND}(\mathbf{I}_1 , \mathbf{I}_2) = \mathbf{I}_1 \wedge \mathbf{I}_2$ if $J_2 (i, j) = \text{AND}[I_1 (i, j), I_2 (i, j)]$ for all (i, j)
- Shows the overlap of BLACK regions in $\mathbf{I}_1$ and $\mathbf{I}_2$



$\mathbf{I}_1$ $\mathbf{I}_2$ $\mathbf{J}_2 = \mathbf{I}_1 \wedge \mathbf{I}_2$

BINARY OR

- The OR or union of two images:
- $\mathbf{J}_3 = \mathrm{OR}(\mathbf{I}_1 , \mathbf{I}_2) = \mathbf{I}_1 \vee \mathbf{I}_2$
  if $J_3 (i, j) = \mathrm{OR}[\, I_1 (i, j), I_2 (i, j)\,]$ for all (i, j)
- Shows the **overlap** of the WHITE regions in $\mathbf{I}_1$ and $\mathbf{I}_2$



$\mathbf{I}_1$ $\mathbf{I}_2$ $\mathbf{J}_3 = \mathbf{I}_1 \vee \mathbf{I}_2$

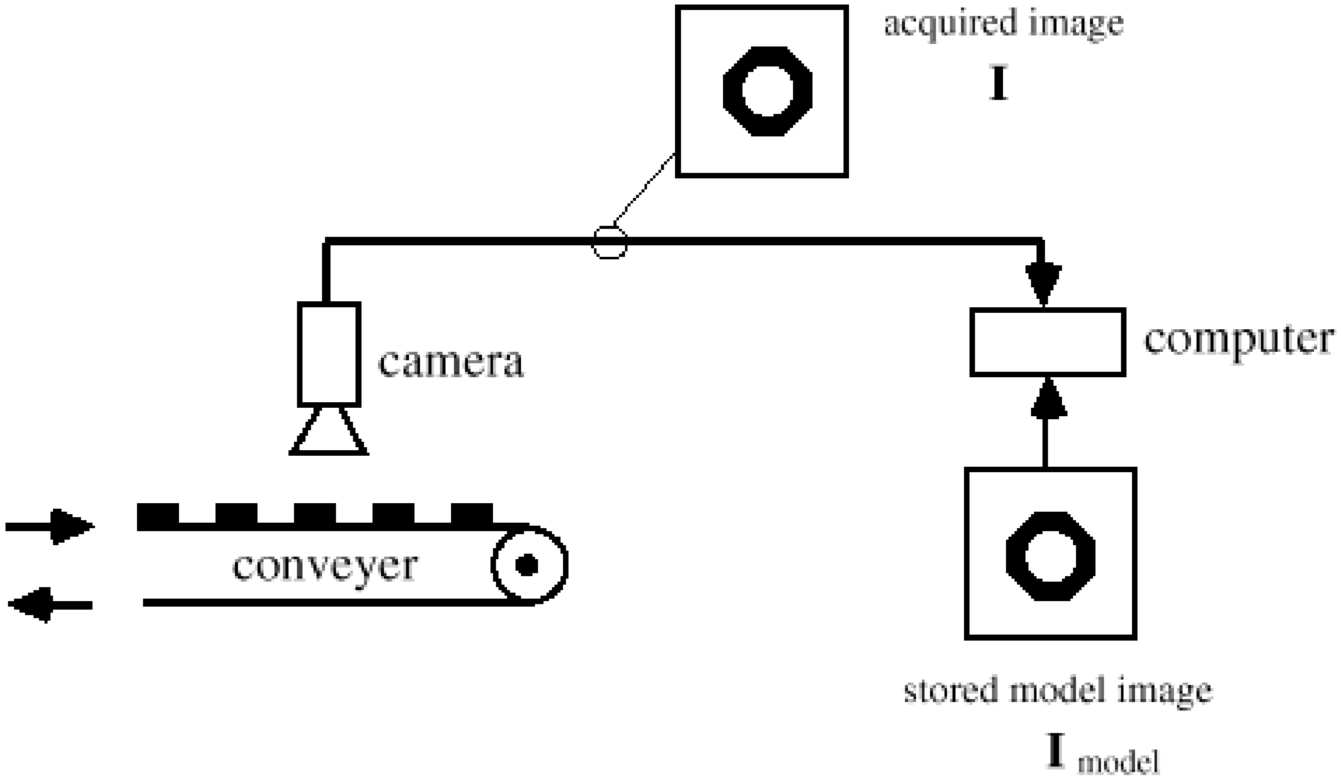
BINARY OPERATIONS

Comments

- The usefulness of **globally** applying AND, OR and MAJ to images is very limited.
- Later, we will find that AND, OR, and MAJ are very useful when applied to **small, local image regions**
- There are exceptions ...

EXAMPLE

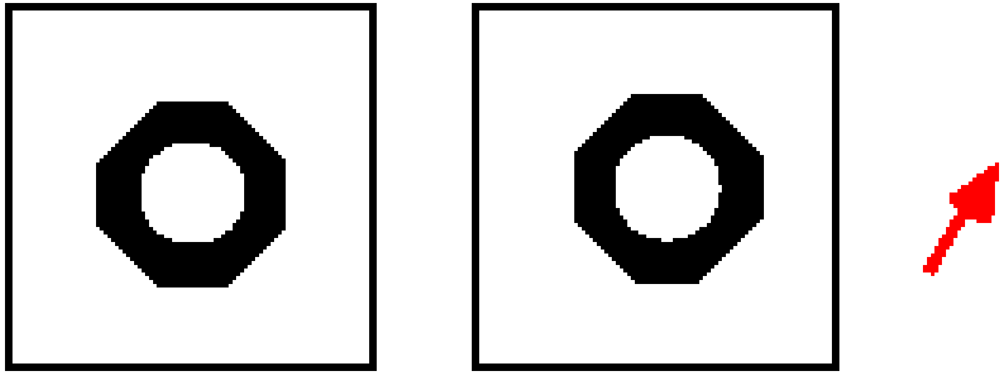
- An assembly-line image inspection system. Similar to many marketed by industry:



- **Objective**: Numerically compare the stored image $\mathbf{I}_{model}$ and the acquired image $\mathbf{I}$

EXAMPLE

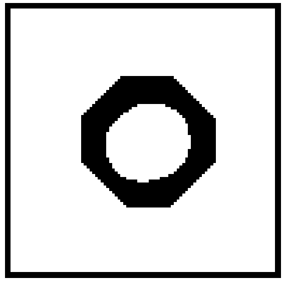
- Observe that the object in **I** has been shifted very slightly



$\mathbf{I}_{\text{model}}$ $\mathbf{I}$

Logical AND

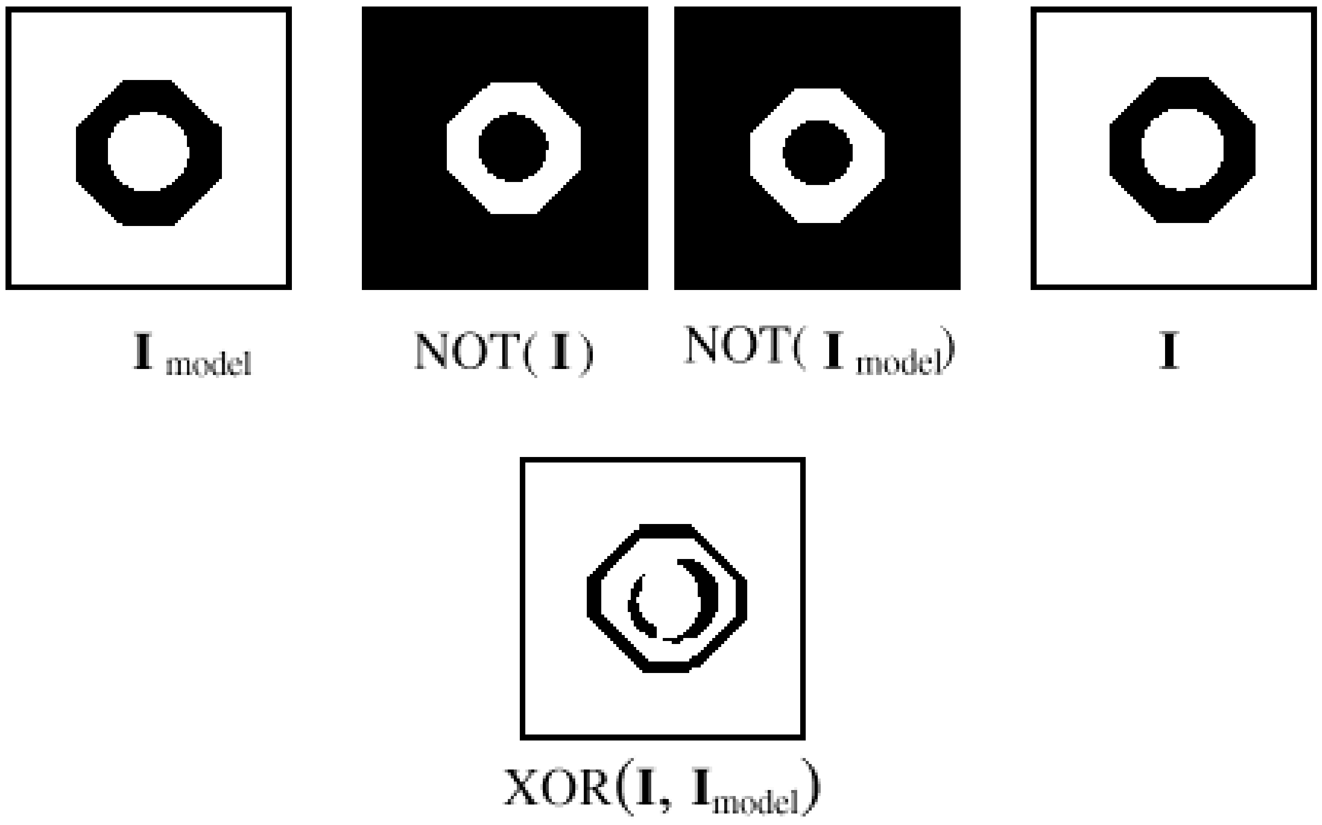
- The logical AND conveys the **overlap**



$\mathbf{I}_{\text{model}} \wedge \mathbf{I}$

- A measurement of the **displacement** is given by:
- $\text{XOR}(\mathbf{I}, \mathbf{I}_{\text{model}}) = \text{OR}\{\ \text{AND}[\mathbf{I}_{\text{model}}, \text{NOT}(\mathbf{I})], \text{AND}[\text{NOT}(\mathbf{I}_{\text{model}}), \mathbf{I}\ ]\}$

DISPLACEMENT



EXAMPLE

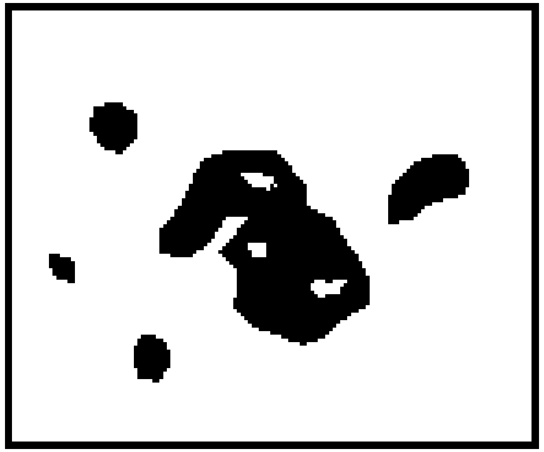
- XOR shows where the displacement errors occur
- To decide if there is a problem or flaw, the ratio or percentage

PERCENT = [# black pixels in XOR($\mathbf{I}$, $\mathbf{I}_{model}$)] / [# black pixels in $\mathbf{I}_{model}$]

- may be compared to a pre-determined tolerance percentage P
- If PERCENT > P, then the part may be **flawed or incorrectly placed**

BLOB COLORING

- A simple technique for **region classification** and **correction**
- **Motivation**: Gray-level image thresholding **usually** produces an imperfect binary image:
  - Extraneous blobs or holes due to noise
  - Extraneous blobs from thresholded objects of little interest
  - Nonuniform object/background surface reflectances

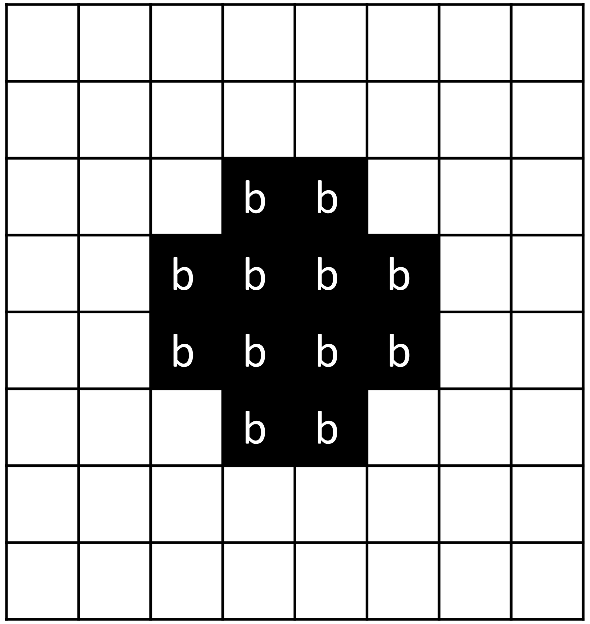


typical thresholded image result

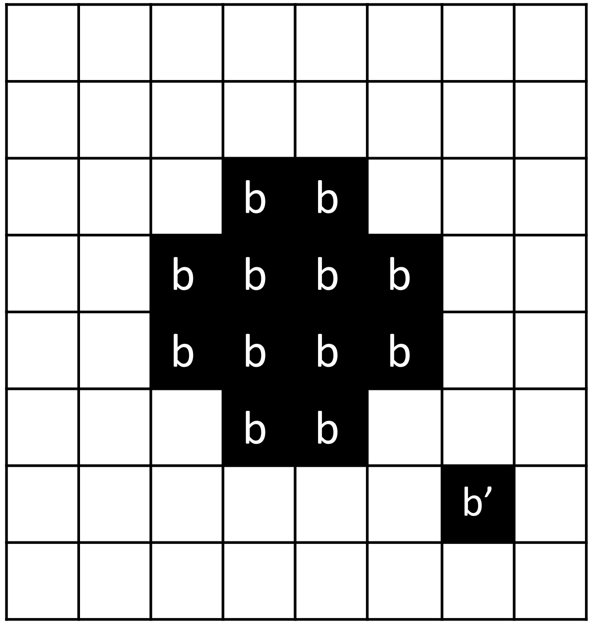
BLOB COLORING

- It is usually desired to **extract** a **small number of objects** or even a **single object** by thresholding
- Blob coloring is a very simple technique for **listing** all of the blobs or objects in a binary image

BLOB COLORING

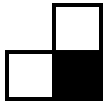


BLOB COLORING

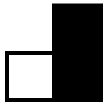


BLOB COLORING ALGORITHM

- For binary image **I**, define a "region color" array **R**:
- R(i, j) = **region number** of pixel I(i, j)
- Set **R** = **0** (all zeros) and k = 1 (k = region number counter)
- While scanning the image left-to-right and top-to-bottom **do**
  - if I(i, j) = 1 and I(i, j-1) = 0 and I(i-1, j) = 0 then
    - set R(i, j) = k and k = k + 1;

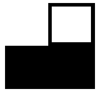


  - if I(i, j) = 1 and I(i, j-1) = 0 and I(i-1, j) = 1 then
    - set R(i, j) = R(i-1, j);

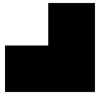


BLOB COLORING ALGORITHM (contd.)

- if I(i, j) = 1 and I(i, j-1) = 1 and I(i-1, j) = 0 then
  - set R(i, j) = R(i, j-1);



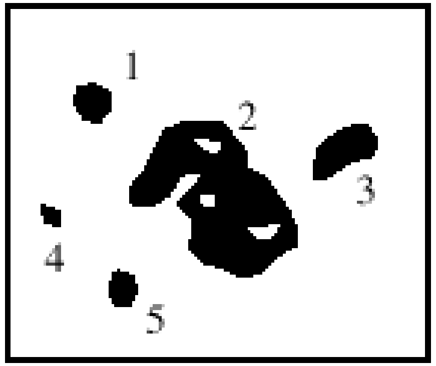
- if I(i, j) = 1 and I(i, j-1) = 1 and I(i-1, j) = 1 then
  - set R(i, j) = R(i-1, j);
  - if R(i, j-1) =/= R(i-1, j) then
  - record R(i, j-1) and R(i-1, j) as equivalent (same color)



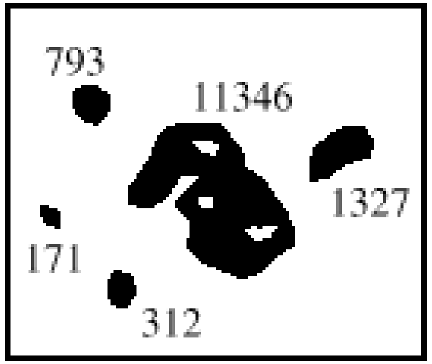
- Distinct integers or "colors" *k* are assigned to each blob
- Counting the pixels in each blob (by color) is then simple

EXAMPLE

- Using blob coloring



blob coloring result

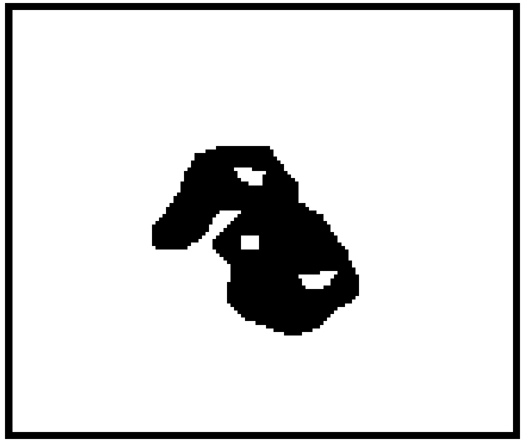


blob counting result

- "Color" of largest blob: **2**

REMOVING MINOR REGIONS

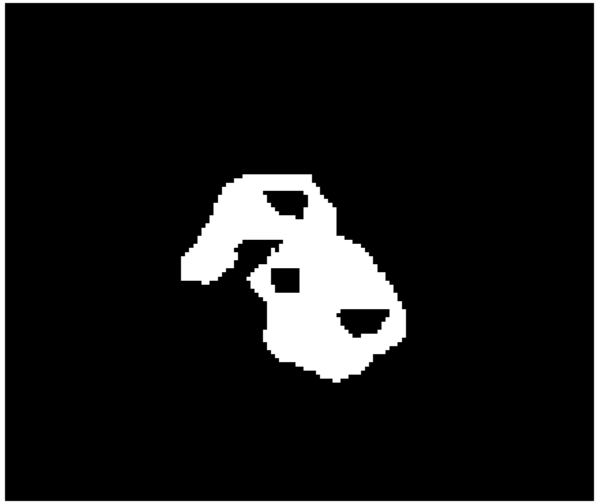
- Let m = "color" of largest region
- While scanning the image left-to-right and top-to-bottom **do**
- if I(i, j) = 1 and R(i, j) != m then
- set I(i, j) = 0;



minor region
removal

EXAMPLE

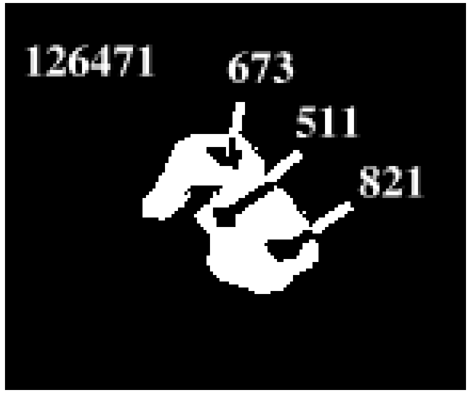
- The process is not complete! To obtain a cohesive, connected object, repeat the procedure on the WHITE pixels
- **Complement** the last result:



complement

EXAMPLE

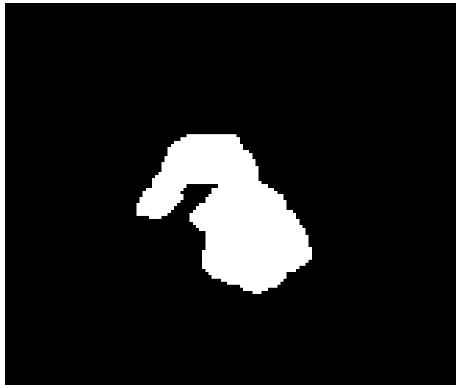
- Then apply all the same steps:



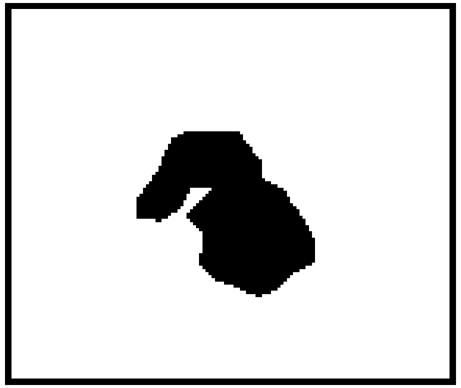
blob counting

- "Color" of largest blob: 1

EXAMPLE



minor region removal



complement

- Simple and effective, but doesn't "cure" everything