Digital Image Processing

COSC 6380/4393

Lecture – 18

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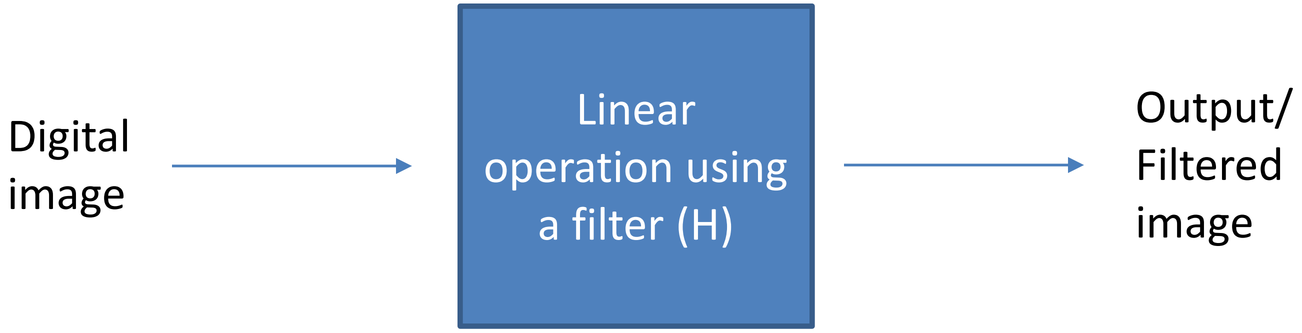
Slides from Dr. Shishir K Shah, and Frank Liu

Linear Image Filtering



Linear Image Filtering

- Correlation and Convolution are basic operations that we will perform to extract information from images
- Two operations
  - Correlation
    - Used as a tool to measure the similarity between two signals
  - Convolution
    - Used to modify one signal using another signal.
- The two operations in essence are the same with a minor difference.

Spatial Correlation Operator

f

0	0	0	1	0	0	0	0

☆ w

1	2	3	2	8

f

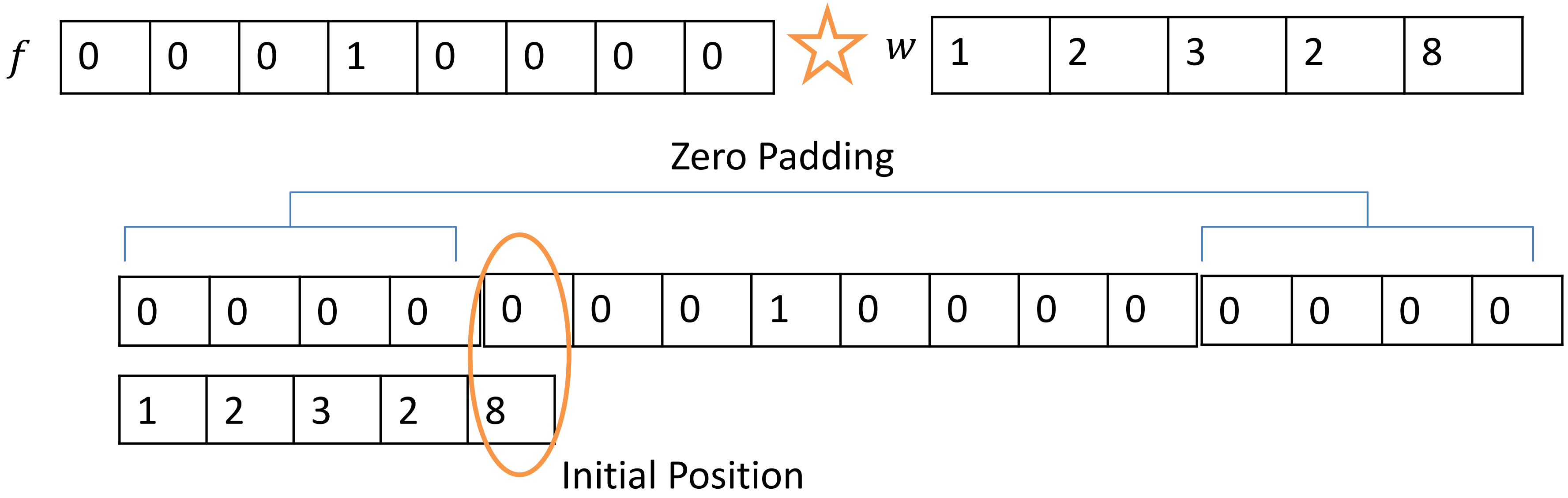
0	0	0	1	0	0	0	0

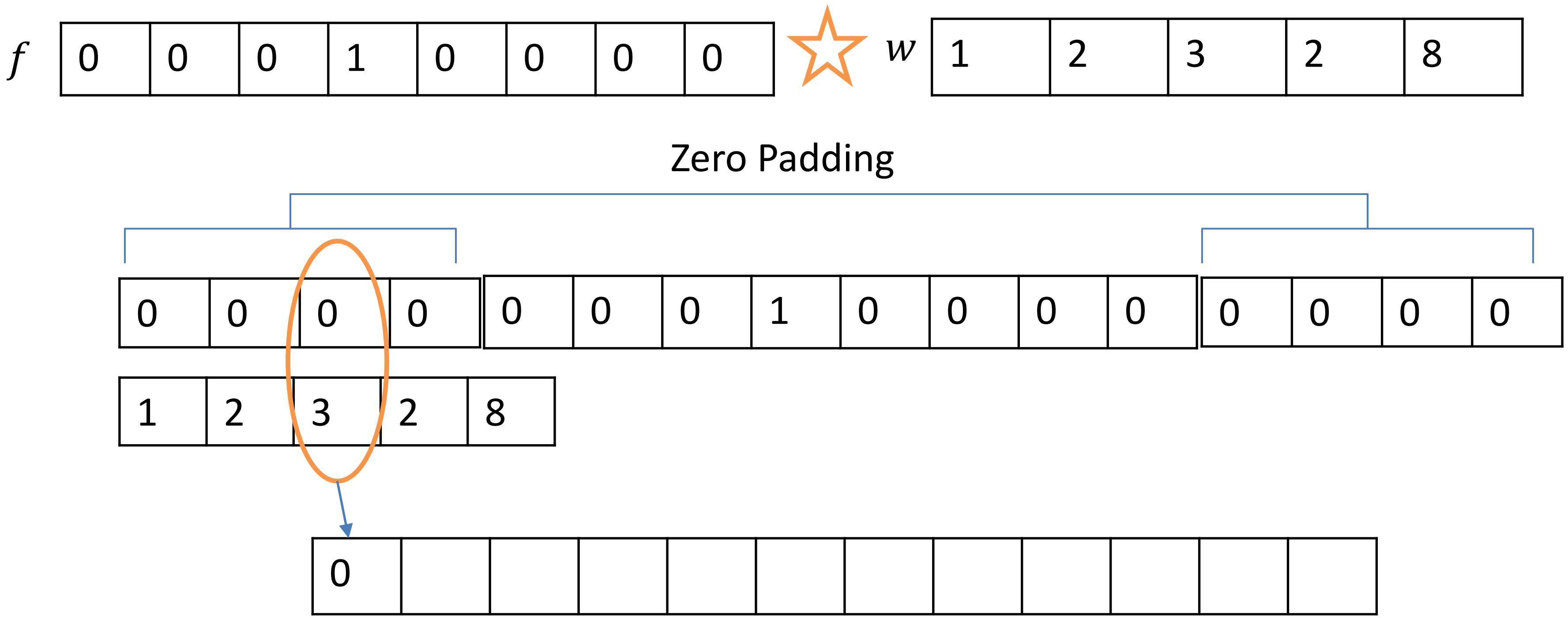
$\star$ w

1	2	3	2	8

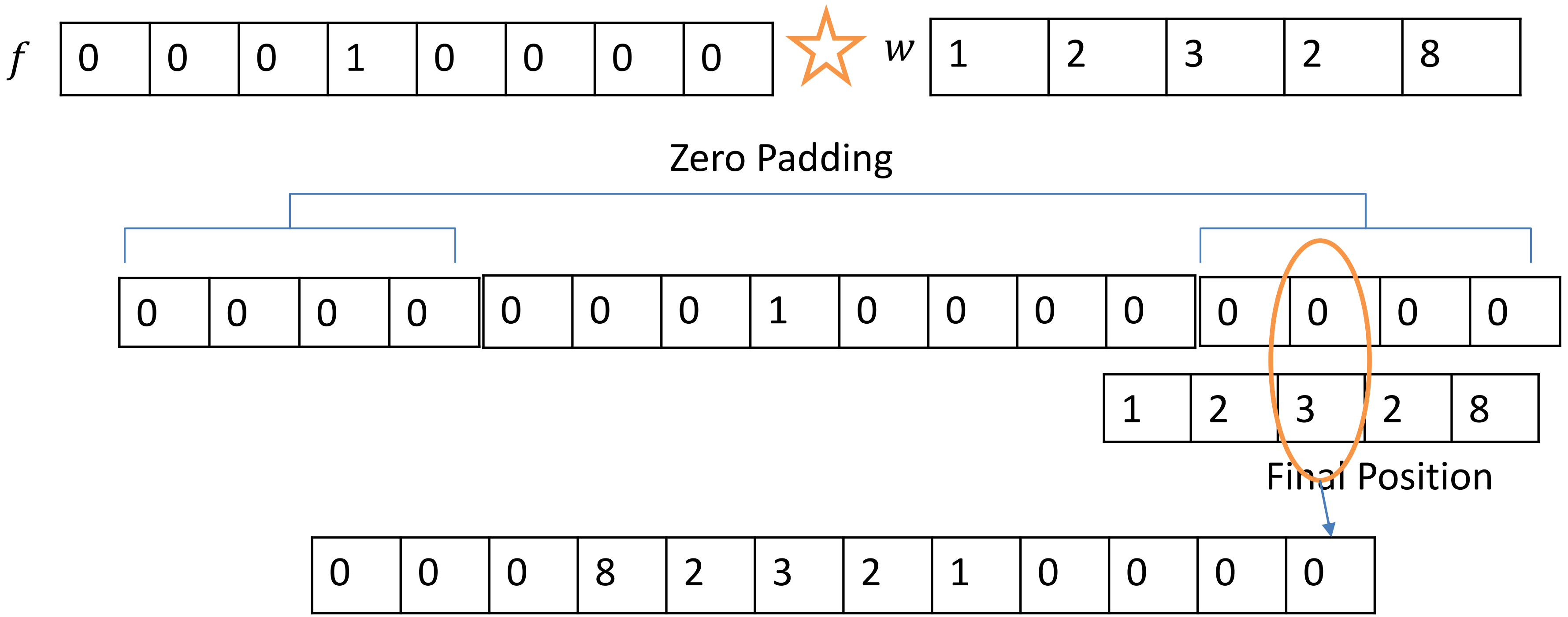
0	0	0	1	0	0	0	0

1	2	3	2	8


f
0 0 0 1 0 0 0 0
☆
w
1 2 3 2 8
Zero Padding
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Initial Position



f
0 0 0 1 0 0 0 0
w
1 2 3 2 8
Zero Padding
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
0


f
0 0 0 1 0 0 0 0
w
1 2 3 2 8
Zero Padding
0 0 0 0
0 0 0 1 0 0 0 0
0 0 0 0
1 2 3 2 8
Final Position
0 0 0 8 2 3 2 1 0 0 0 0

f

0	0	0	1	0	0	0	0

☆ w

1	2	3	2	8

Full Correlation result

0	0	0	8	2	3	2	1	0	0	0	0

f

0	0	0	1	0	0	0	0

$\star$ w

1	2	3	2	8

Cropped Correlation result

0	8	2	3	2	1	0	0

Spatial Correlation Operator

The correlation of a filter $w(x)$ of size m
with an signal $f(x)$, denoted as $w(x) \star f(x)$

$$w(x) \star f(x) = \sum_{s=-a}^{a} w(s) f(x+s)$$

Spatial Convolution Operator

f	0	0	0	1	0	0	0	0

$\otimes$

w	1	2	3	2	8

f

0	0	0	1	0	0	0	0

$\otimes$ w

1	2	3	2	8

0	0	0	1	0	0	0	0

8	2	3	2	1

$w \; rotated \; by \; 180^0$

f

0	0	0	1	0	0	0	0

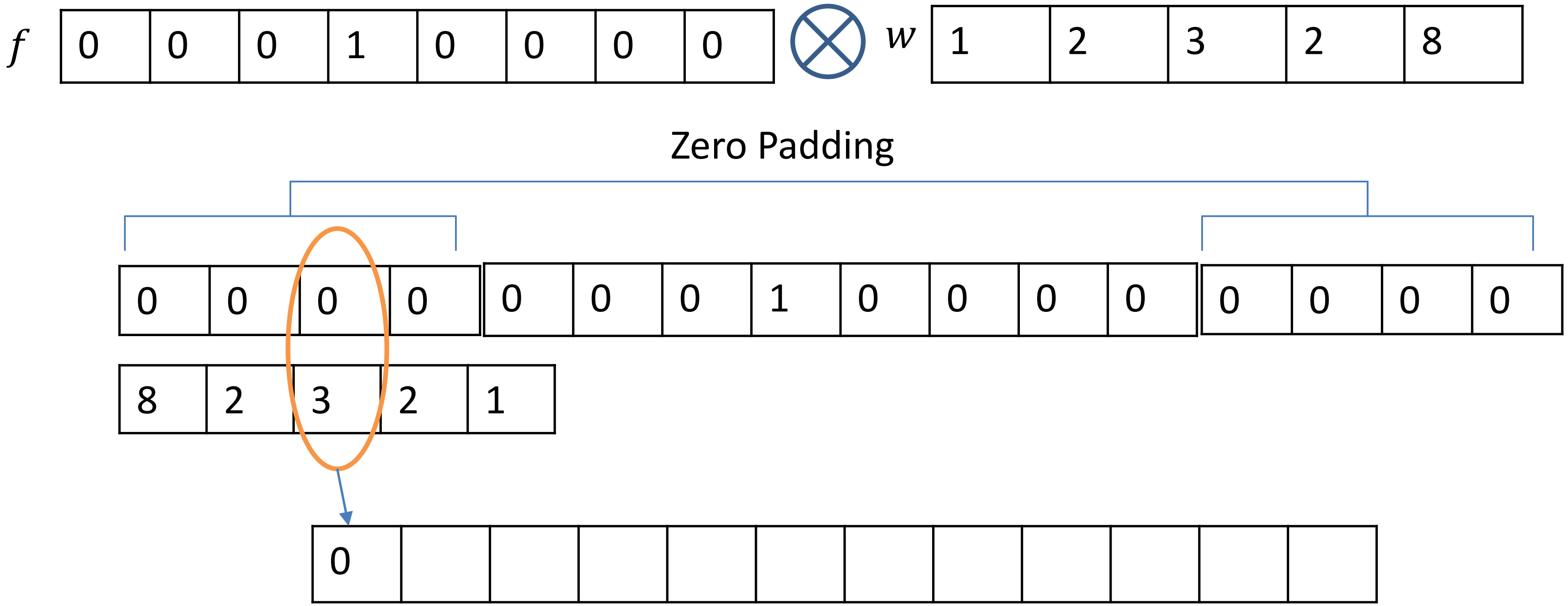
$\otimes$ w

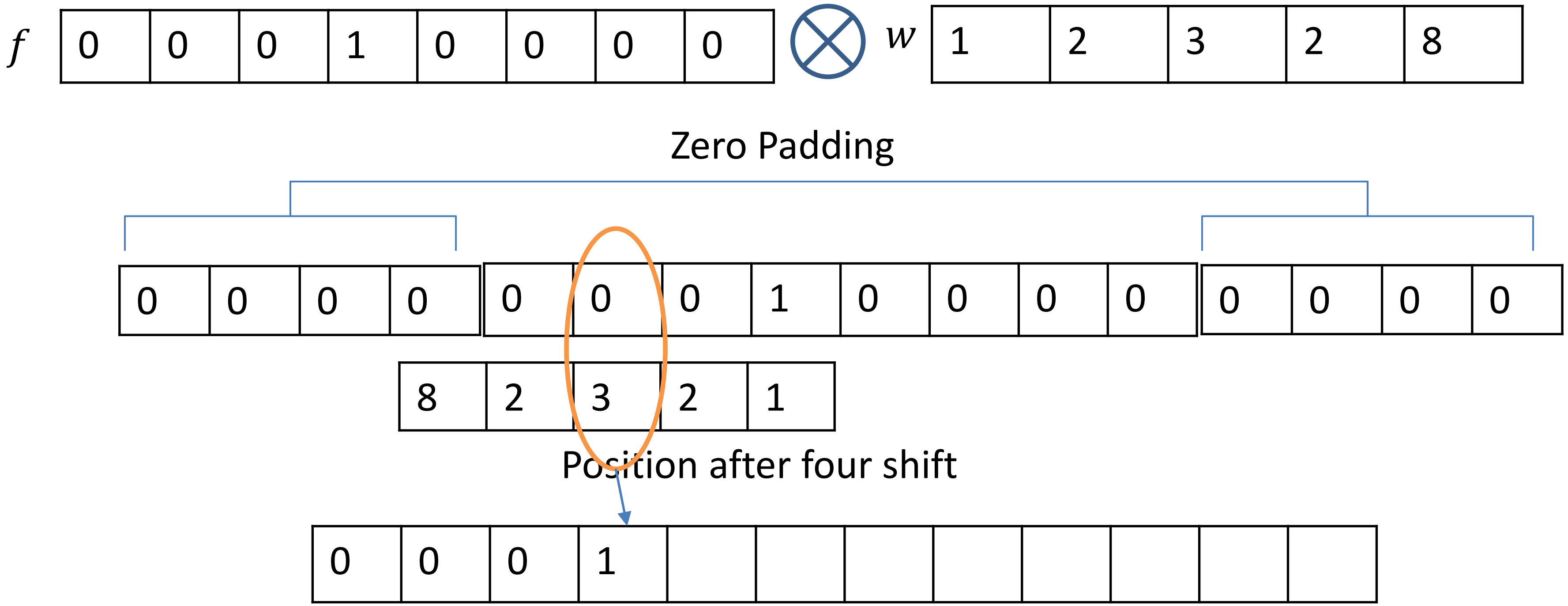
1	2	3	2	8

Zero Padding

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	2	3	2	1											

Initial Position


f
0 0 0 1 0 0 0 0
w
1 2 3 2 8
Zero Padding
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
8 2 3 2 1
0



f
0 0 0 1 0 0 0 0
w
1 2 3 2 8
Zero Padding
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
8 2 3 2 1
Position after four shift
0 0 0 1

f

0	0	0	1	0	0	0	0

$\otimes$ w

1	2	3	2	8

Full Convolution result

0	0	0	1	2	3	2	8	0	0	0	0

f

0	0	0	1	0	0	0	0

$\otimes$ w

1	2	3	2	8

Cropped Convolution result

0	1	2	3	2	8	0	0

Spatial Correlation Operator

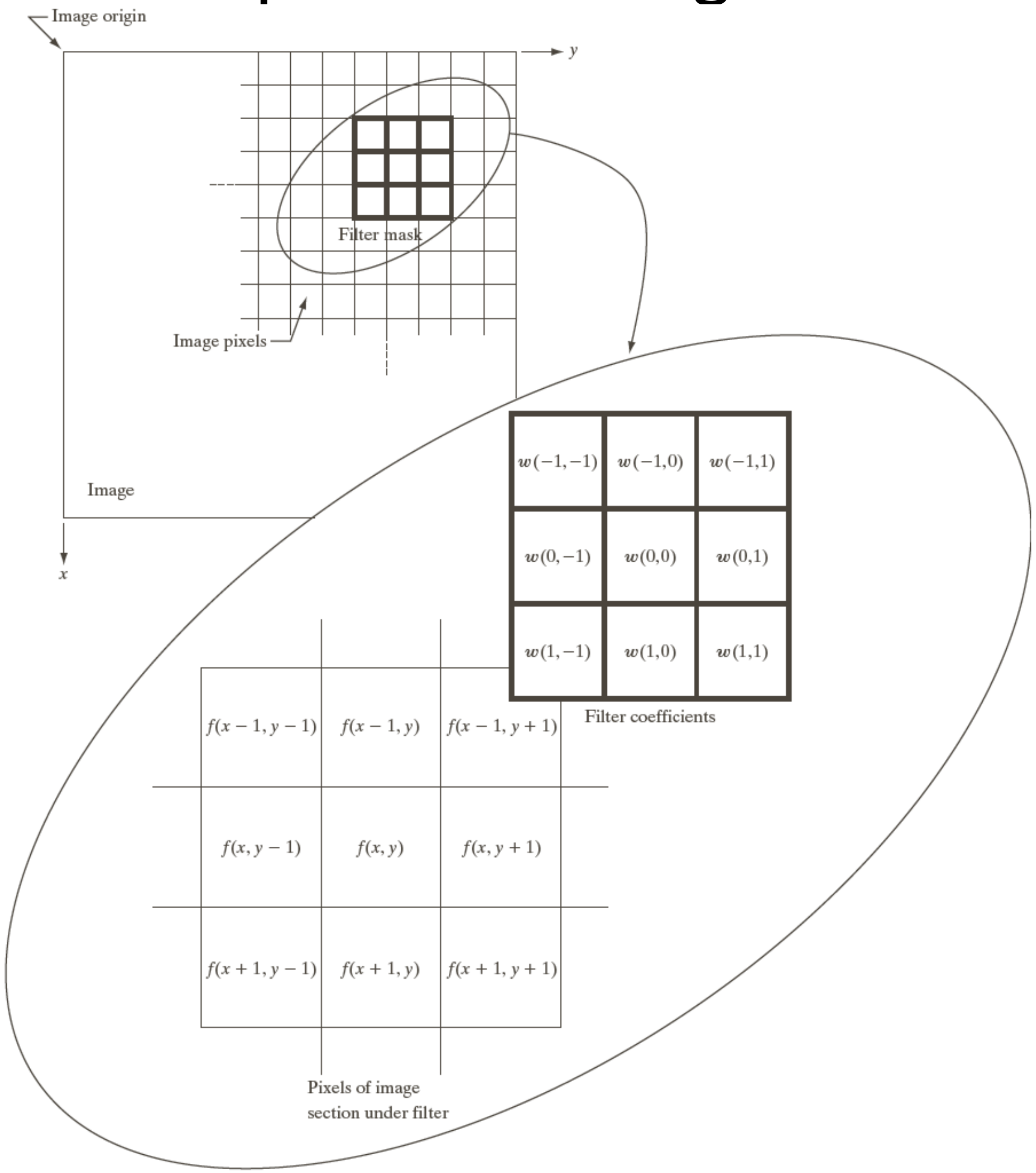
The correlation of a filter $w(x)$ of size m with an signal $f(x)$, denoted as $w(x) \otimes f(x)$

$$w(x) \otimes f(x) = \sum_{s=-a}^{a} w(s) f(x-s)$$

Spatial Filtering

Linear spatial filtering of an image of size MXN with a filter of size mXn is given by

Spatial Filtering



Spatial Correlation Operator

The correlation of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$, denoted as $w(x, y) \star f(x, y)$

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x+s, y+t)$$

Spatial Convolution Operator

The convolution of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$, denoted as $w(x, y) \otimes f(x, y)$

$$w(x, y) \otimes f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Origin $f(x, y)$

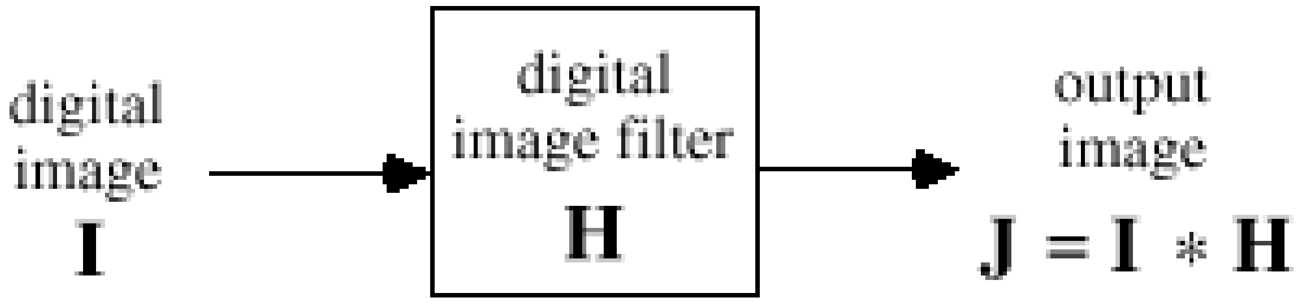
$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \qquad \begin{array}{ccc} & w(x, y) & \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}$$

(a)

FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

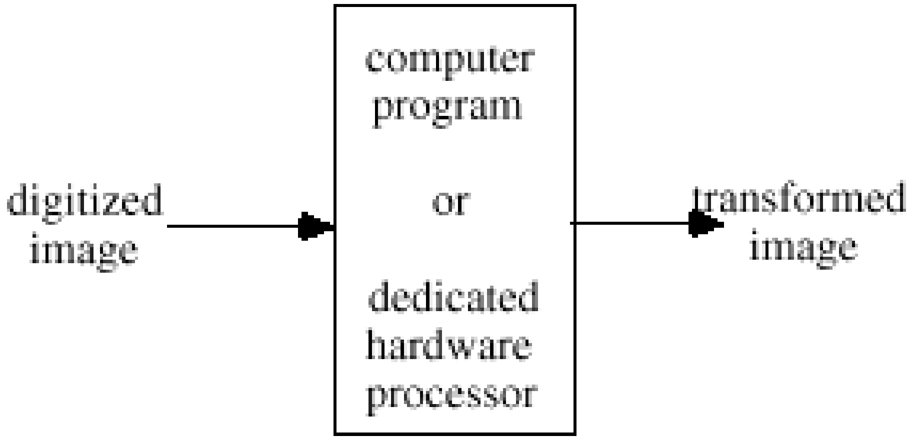
Linear Systems And Linear Image Filtering

- A process that accepts a signal or image **I** as input and transforms it by an act of linear convolution is a type of **linear system**
- **Example**



Goals of Linear Image Filtering

- Process sampled, quantized images to **transform** them into
  - images of **better quality** (by some criteria)
  - images with certain features **enhanced**
  - images with certain features **de-emphasized** or **eradicated**



Some Specific Goals

- **smoothing** - remove noise from bit errors, transmission, etc
- **deblurring** - increase **sharpness** of blurred images
- **sharpening** - emphasize significant features, such as **edges**
- **combinations** of these

Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction

Blurring is used in removal of small details and bridging of small gaps in lines or curves

Smoothing spatial filters include linear filters and nonlinear filters.

Spatial Smoothing Linear Filters

The general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ is given

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

where $m = 2a+1, \quad n = 2b+1.$

Two Smoothing Averaging Filter Masks

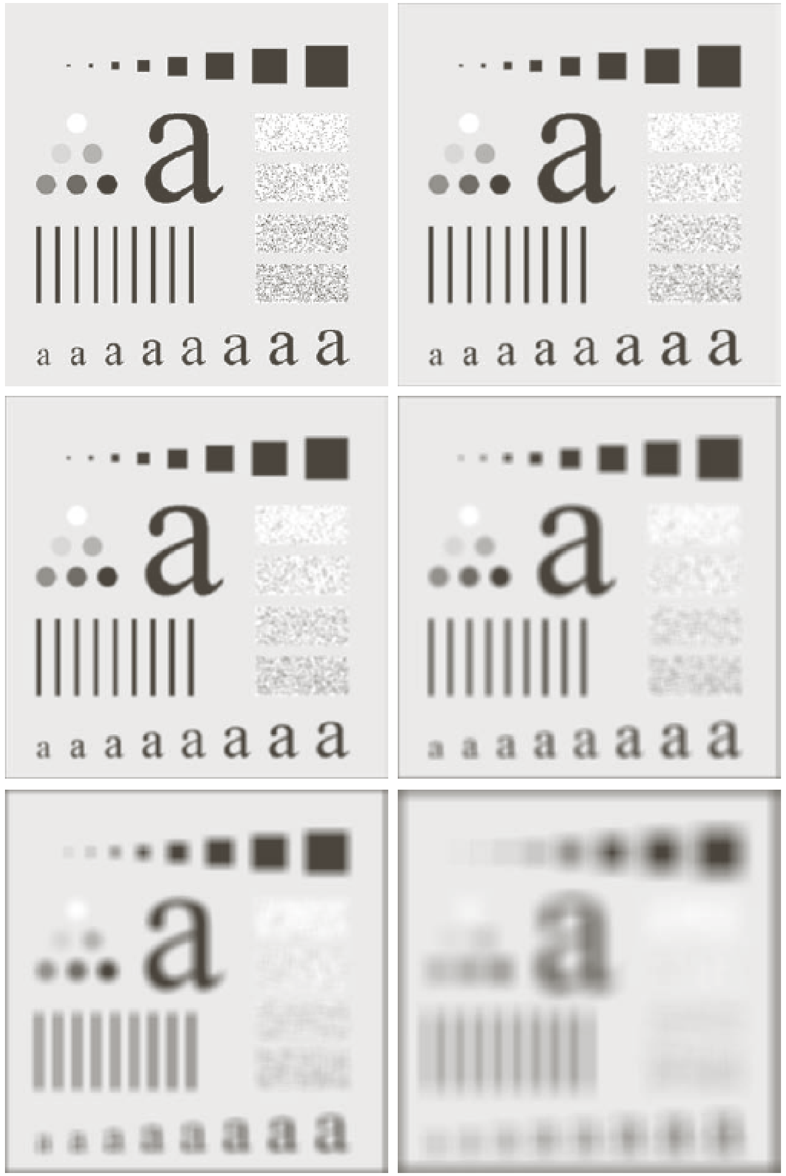
$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f



Example: Gross Representation of Objects

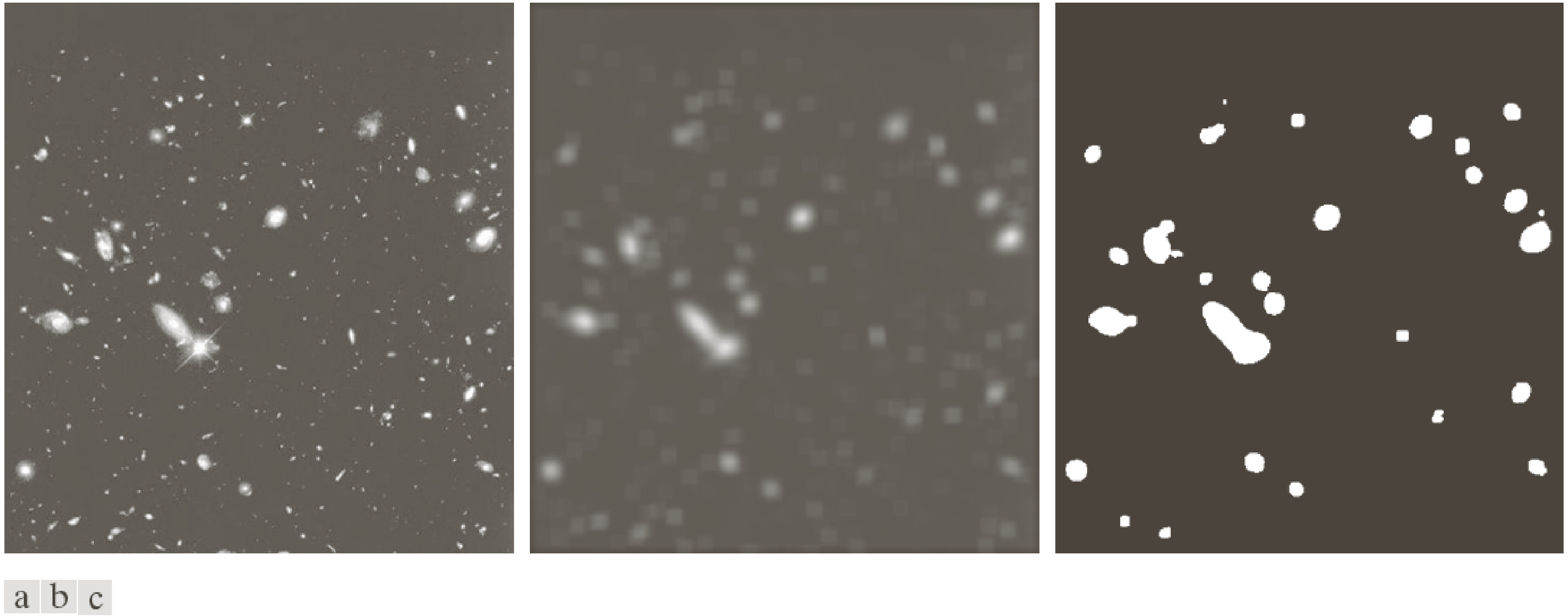
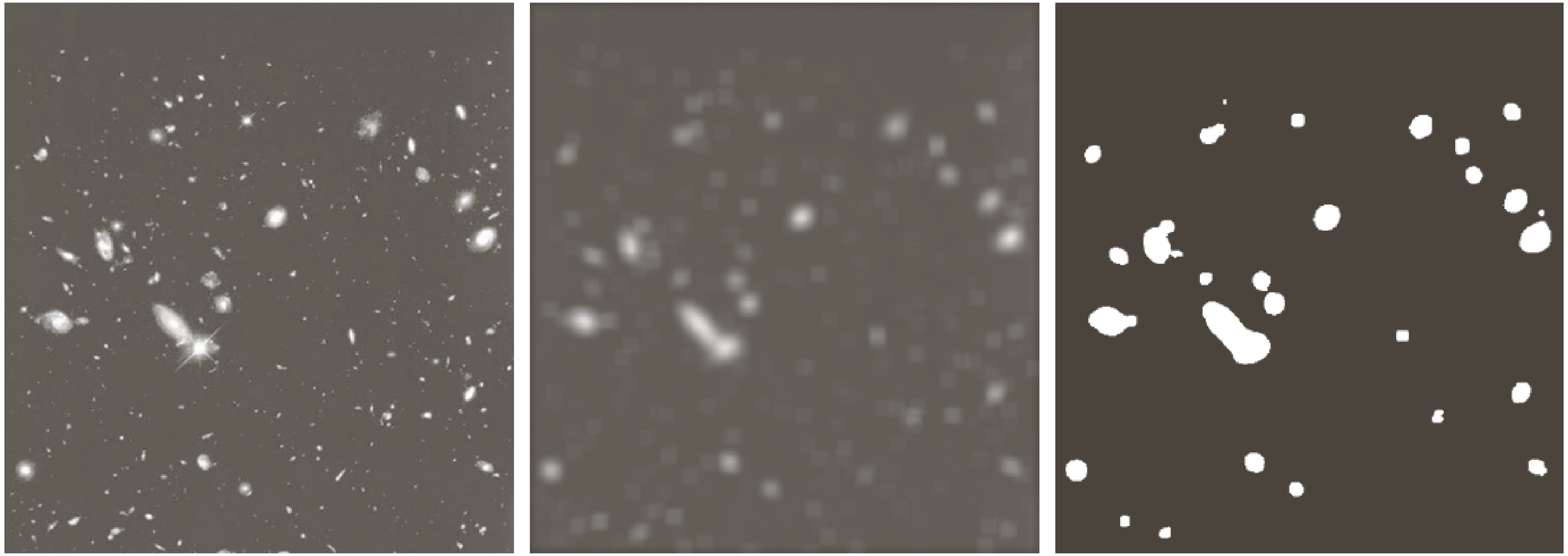


FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Blur an image for getting a gross representation.
Small objects get blended with background

Example: Gross Representation of Objects



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Blur an image for getting a gross representation.
Small objects get blended with background

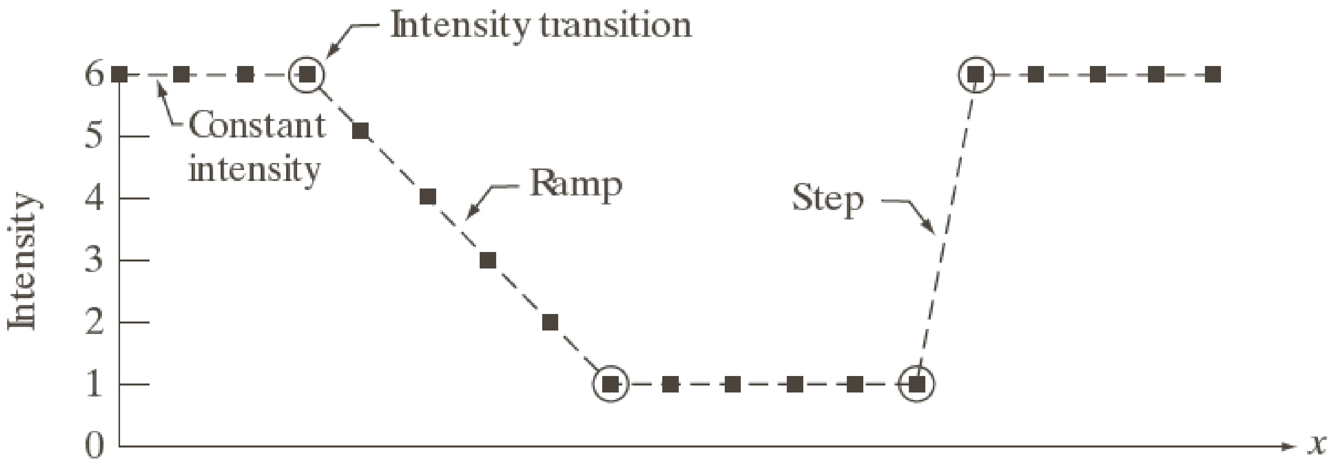
 Demo stars

Image Sharpening

- Objective is to highlight transition in intensity.
- Applications range from
  - Electronic printing
  - Medical imaging
  - Industrial inspection, etc…
- Image smoothing is accomplished by averaging

Image Sharpening

- Objective is to highlight transition in intensity.
- Applications range from
  - Electronic printing
  - Medical imaging
  - Industrial inspection, etc…
- Image smoothing is accomplished by averaging (averaging is analogous to integration)
- Sharpening can be accomplished by differentiation (first order and second order)



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Laplace Operator: Foundation

- The first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative of f(x) as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Laplace Operator

The second-order isotropic derivative operator is the Laplacian for a function (image) f(x,y)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)$$
$$-4f(x, y)$$

Laplace Operator

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Demo Filters

Sharpening Spatial Filters: Laplace Operator

Image sharpening in the way of using the Laplacian:

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

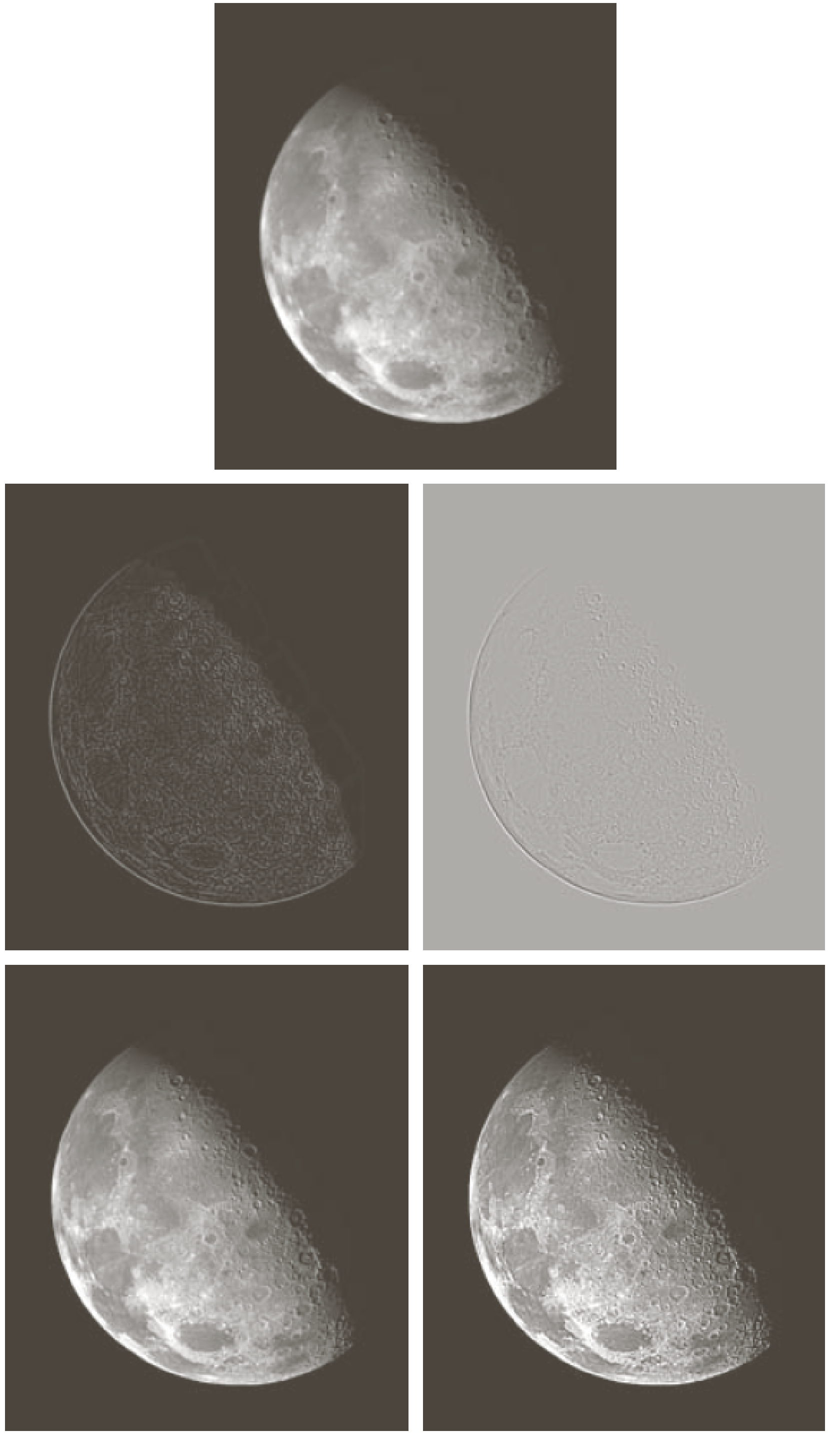
where,

$f(x,y)$ is input image,

$g(x,y)$ is sharpenend images,

$c =$ -1 if $\nabla^2 f(x,y)$ corresponding to Fig. 3.37(a) or (b)

and $c = 1$ if either of the other two filters is used.



a
b c
d e

FIGURE 3.38
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Unsharp Masking and Highboost Filtering

- Unsharp masking

  Sharpen images consists of subtracting an unsharp (smoothed) version of an image from the original image

  e.g., printing and publishing industry

- Steps

  1. Blur the original image
  2. Subtract the blurred image from the original
  3. Add the mask to the original

Unsharp Masking and Highboost Filtering

Let $\overline{f}(x, y)$ denote the blurred image, unsharp masking is

$$g_{mask}(x, y) = f(x, y) - \overline{f}(x, y)$$

Then add a weighted portion of the mask back to the original

$$g(x, y) = f(x, y) + k * g_{mask}(x, y) \qquad k \geq 0$$

when $k > 1$, the process is referred to as highboost filtering.

Unsharp Masking: Demo

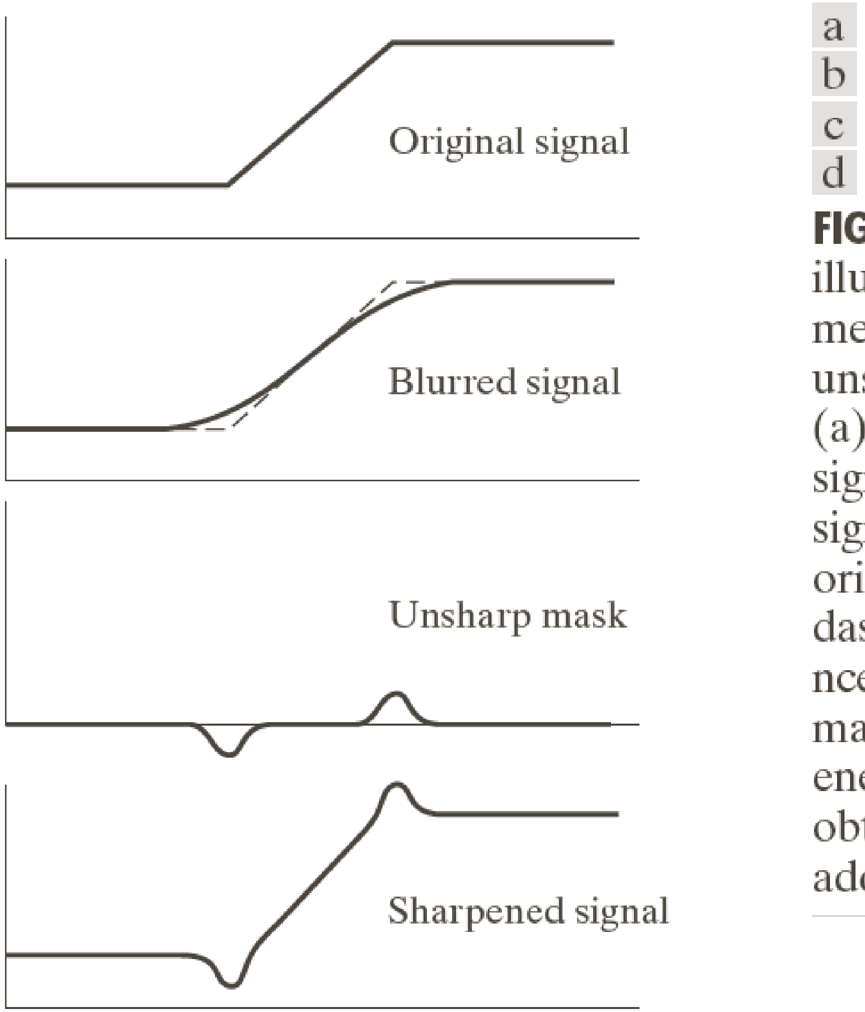


FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Unsharp Masking and Highboost Filtering: Example

a
b
c
d
e

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Demo Sharpen