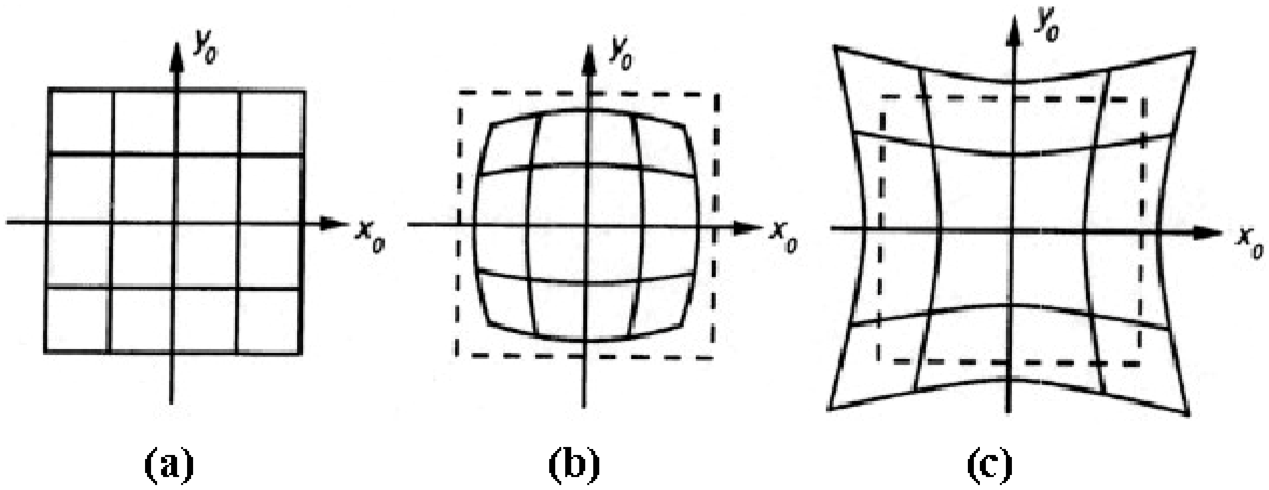
Assignment-1

Distortion and Correction

Radial Distortion

- Optic lens cause distortion in imaging system.
- Distortion can be irregular
- Radially symmetric distortions are the most common.



Radial distortion. (a) Ideal image with no distortion, (b) Barrel Distortion, (c) Pincushion Distortion.

Types of Radial Distortion

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- Distortion can be irregular
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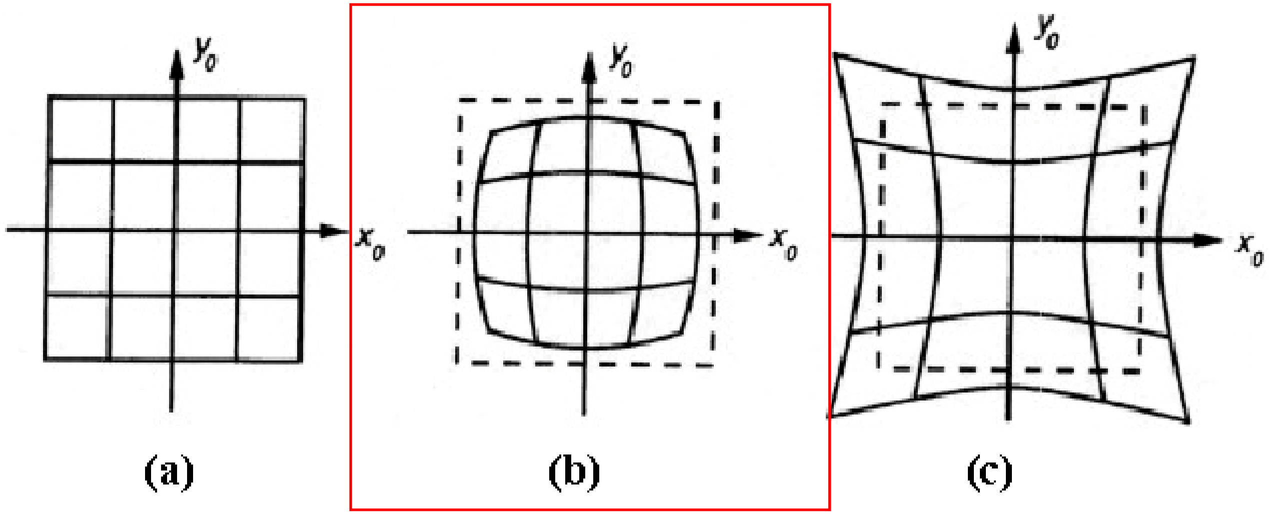
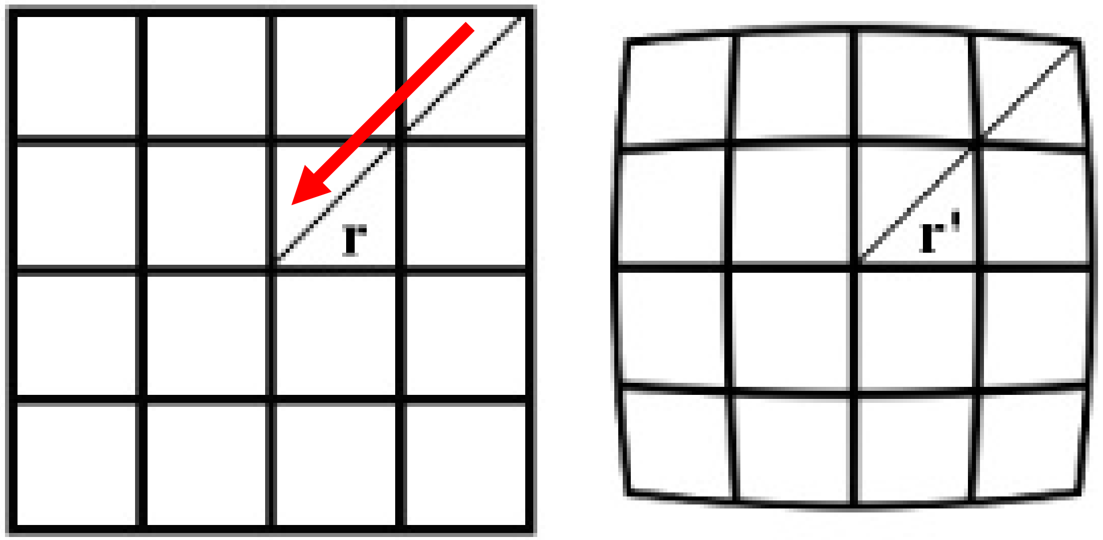


Radial distortion. (a) Ideal image with no distortion, (b) Barrel Distortion, (c) Pincushion Distortion.

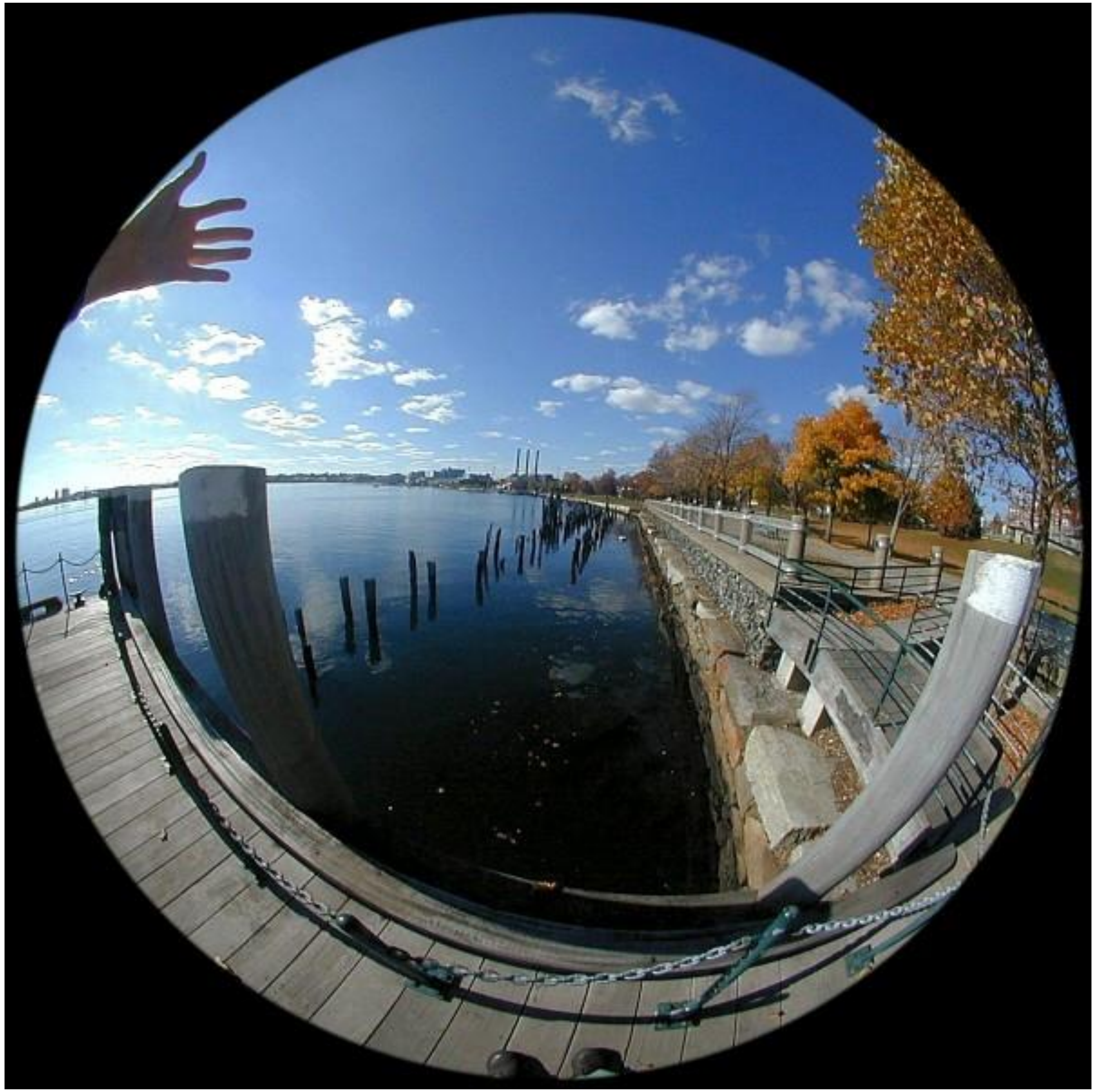
Barrel Distortion

- Geometric transformation where coordinates are mapped to points closer to the center (optic center, lens center)
- Points farther from the center undergo larger distortion, and are displaced more from their original position



Barrel Distortion

- Straight lines at image center, remain straight.
- Straight lines away from the center are bent.
- Circle concentric to the center remains a circle but reduces in radius.



Assignment Objectives

- Understand image distortion

1. Distortion
2. Naïve Image Correction
3. Correction with Interpolation
    1. Nearest neighbour interpolation
    2. Bilinear interpolation

Assignment Objectives

- Understand image distortion

1. Distortion

   Given a normal image, we apply distortion.

2. Naïve Image Correction:

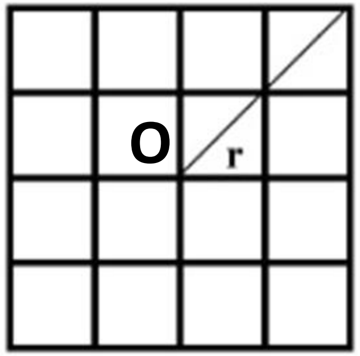
   Given a distorted image, we apply correction to remove distortion

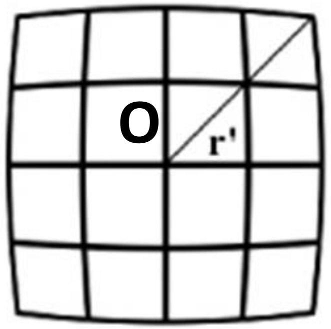
3. Correction with Interpolation

   Given a distorted image, we apply correction using interpolation (Nearest neighbour interpolation, Bilinear interpolation)

Simplified Distortion Functions

Assuming the origin ($\mathbf{O}$) for the image is at the center.





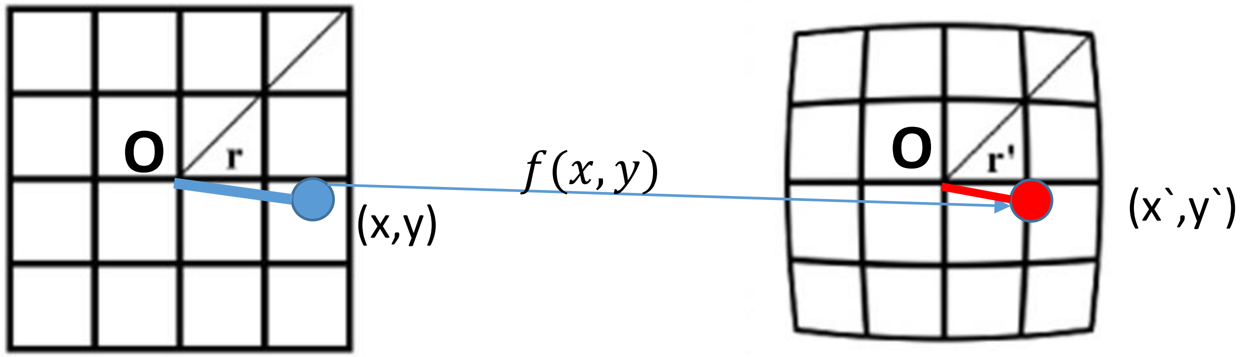
Simplified Distortion Functions

Assuming the origin (**O**) for the image is at the center.

A point (x, y) in the original image, when distorted is mapped to co-ordinate (x`, y`), using the function f

$$(x`, y`) = \ f(x, y) = (\frac{1}{1+kr}x, \frac{1}{1+kr}y)$$

Where, r is the distance of (x,y) from the image center, and k is distortion parameter



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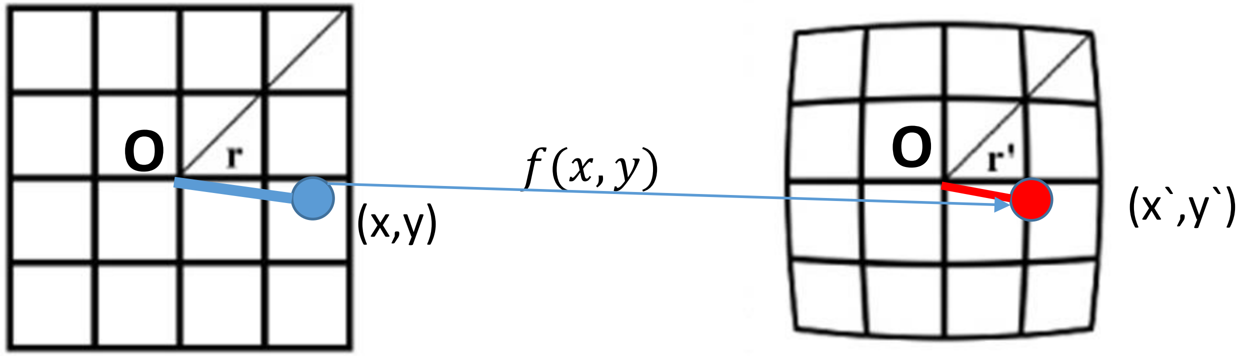
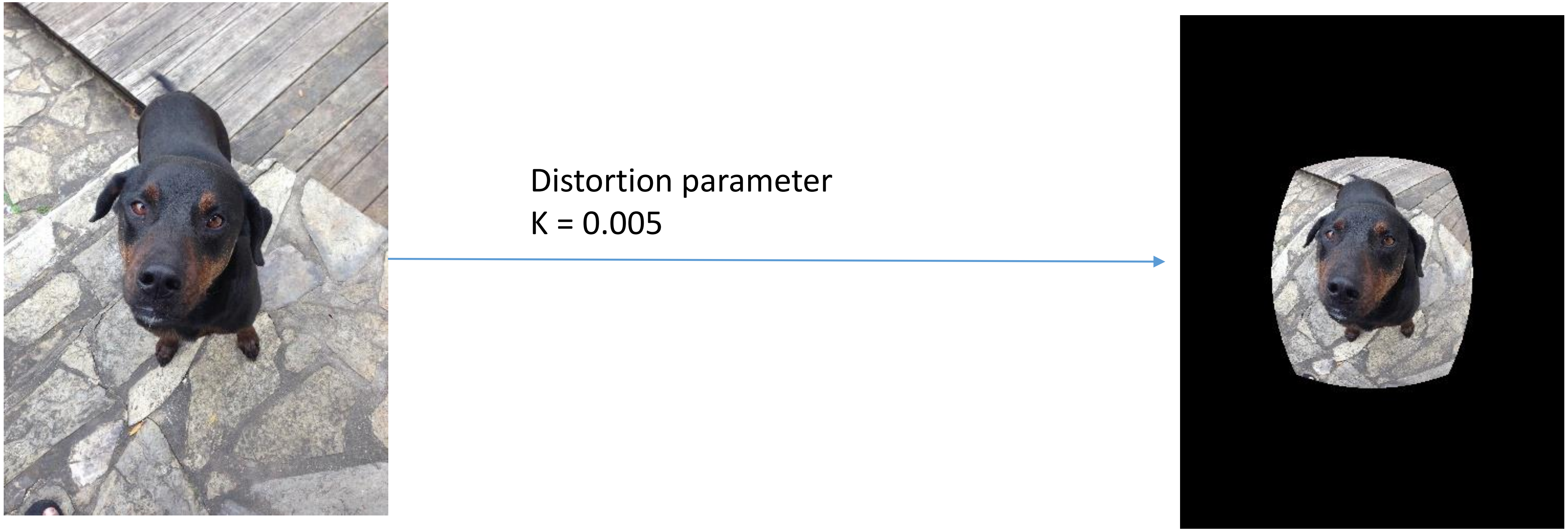


Note that the distorted point is at distance r` from the origin and not r anymore.

1. Distortion



Distortion parameter
K = 0.005

Inputs:
Input image
Distortion parameter (k)

Image Co-ordinate system

- Note that, since we use a (row, col) convention.
- By default, the Image co-ordinates have x-axis pointing downwards, and y-axis to the right.

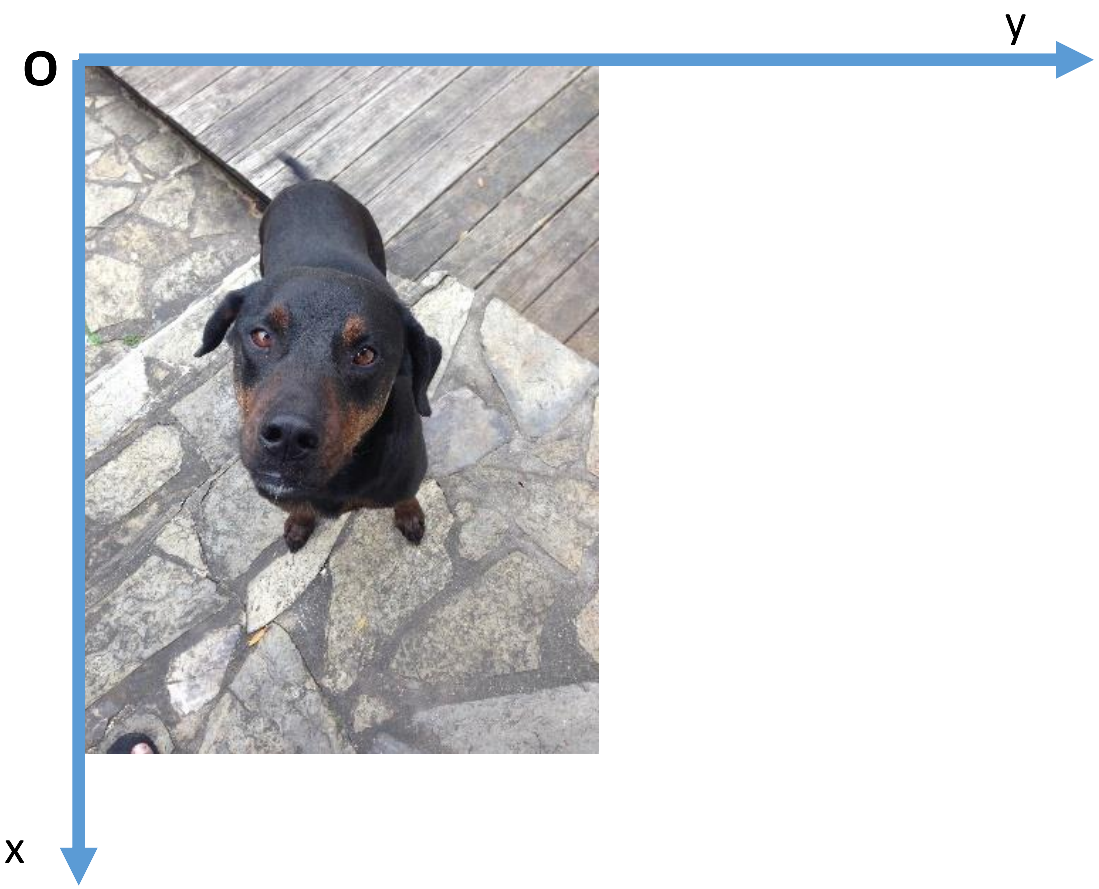
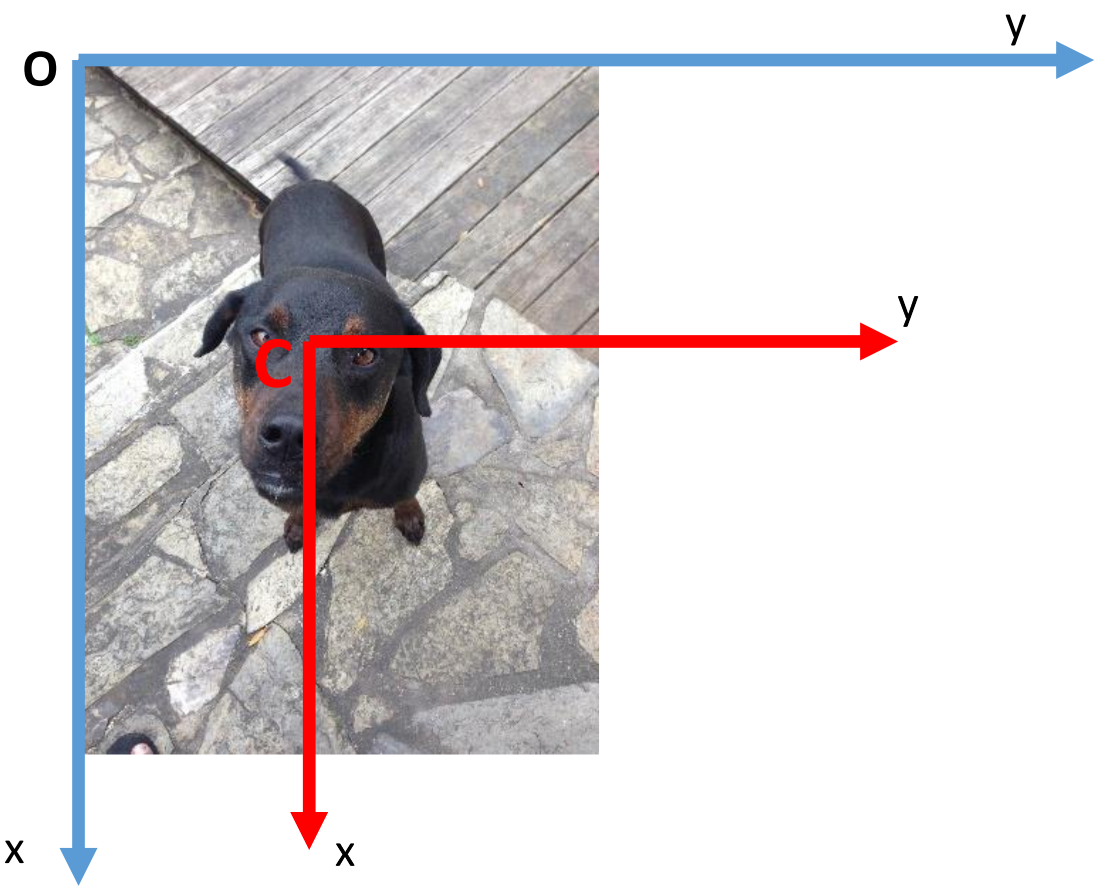


Image Co-ordinate system

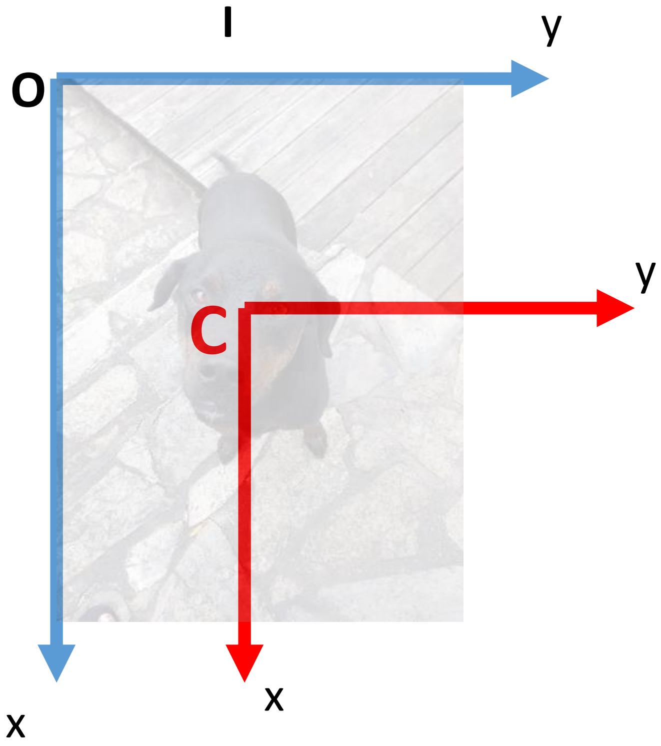
- Note that, since we use a (row, col) convention.
- By default, the Image co-ordinates have x-axis pointing downwards, and y-axis to the right.
- Distortion is applied over the center C of the image.
- Need to apply a change in coordinate system before applying the distortion function



Distortion Steps

Let I be the input image, and the C $\left(c_x, c_y\right)$ its center.

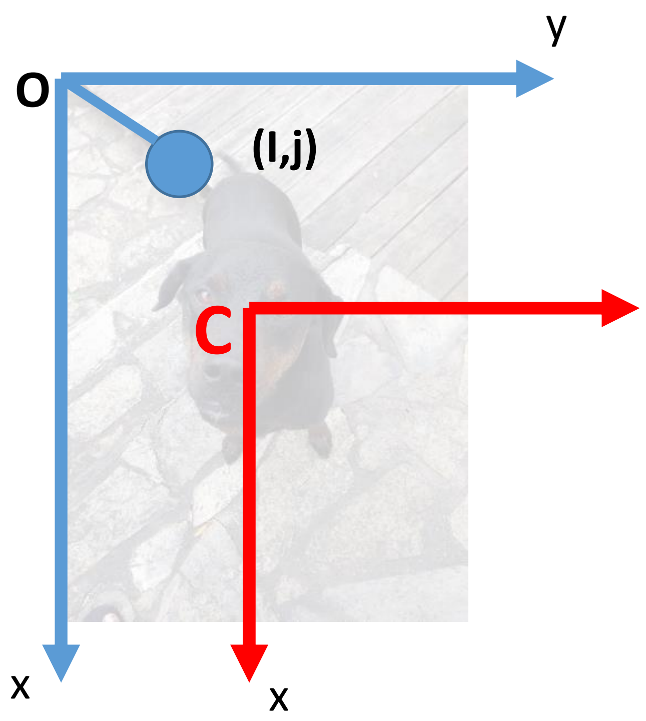
1. Initialize a zeros matrix for distorted image (I_D) with same dims as input image.

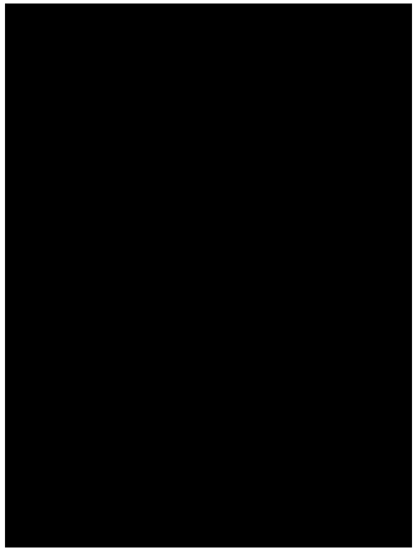


Distortion Steps

Let I be the input image, and the C $\left(c_x, c_y\right)$ its center.

1. Initialize a zeros matrix for distorted image (I_D) with same dims as input image.
2. For each (i, j) in image I:





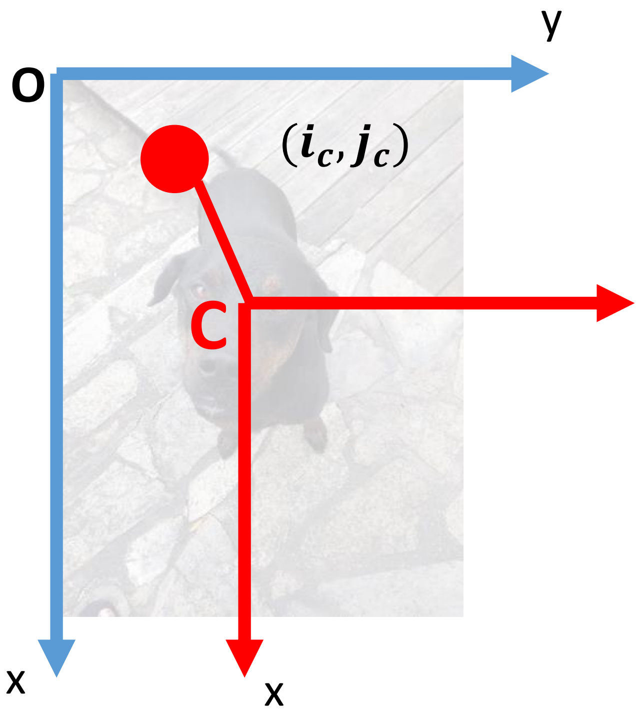
I_D

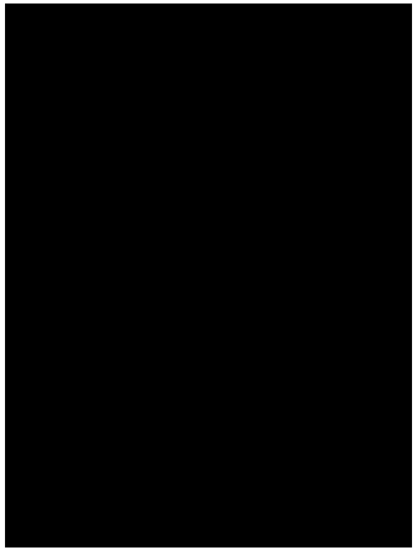
Distortion Steps

Let I be the input image, and the C $\left(c_x, c_y\right)$ its center.

1. Initialize a zeros matrix for distorted image (I_D) with same dims as input image.
2. For each (i, j) in image I:
    1. Apply change in co-ordinate system, and find the point (i_c, j_c) with respect to center C.

$$(i_c, j_c) = (i - c_x, j - c_y)$$





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1. Initialize a zeros matrix for distorted image (I_D) with same dims as input image.
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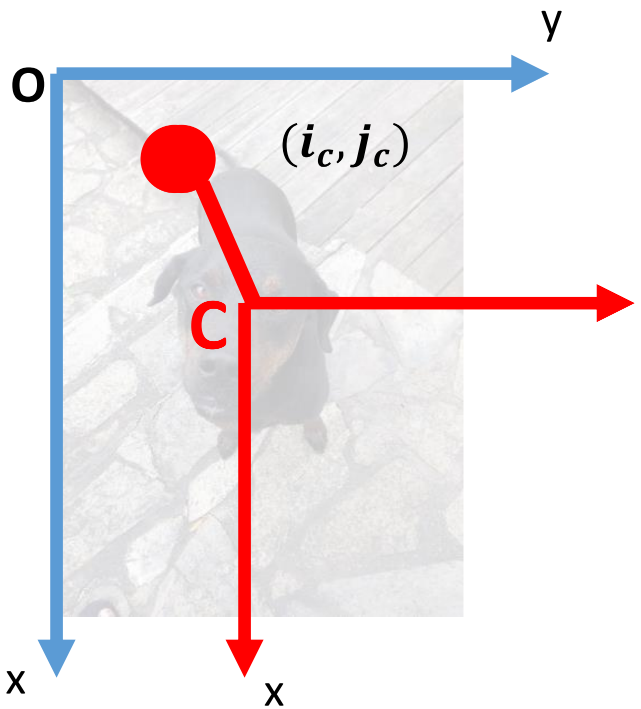
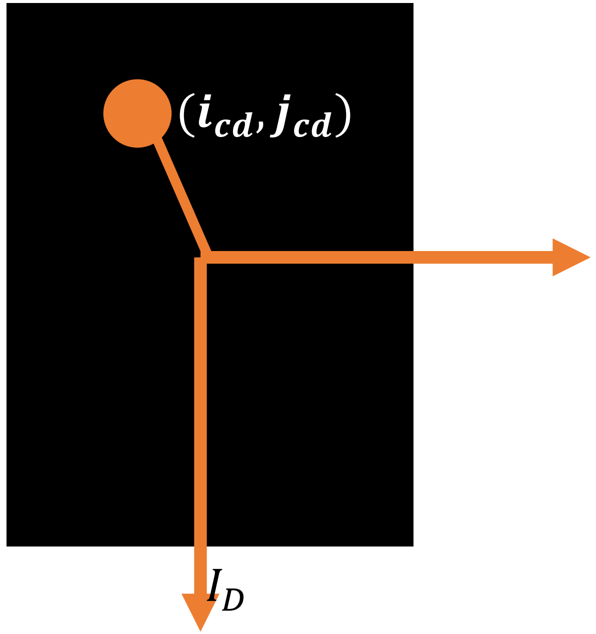
    1. Apply change in co-ordinate system, and find the point (i_c, j_c) with respect to center C.
    2. Apply the distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})

$$(i_{cd}, j_{cd}) = f(i_c, j_c)$$

Note that (i_{cd}, j_{cd}) is still with respect to center C.





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2. For each (i, j) in image:
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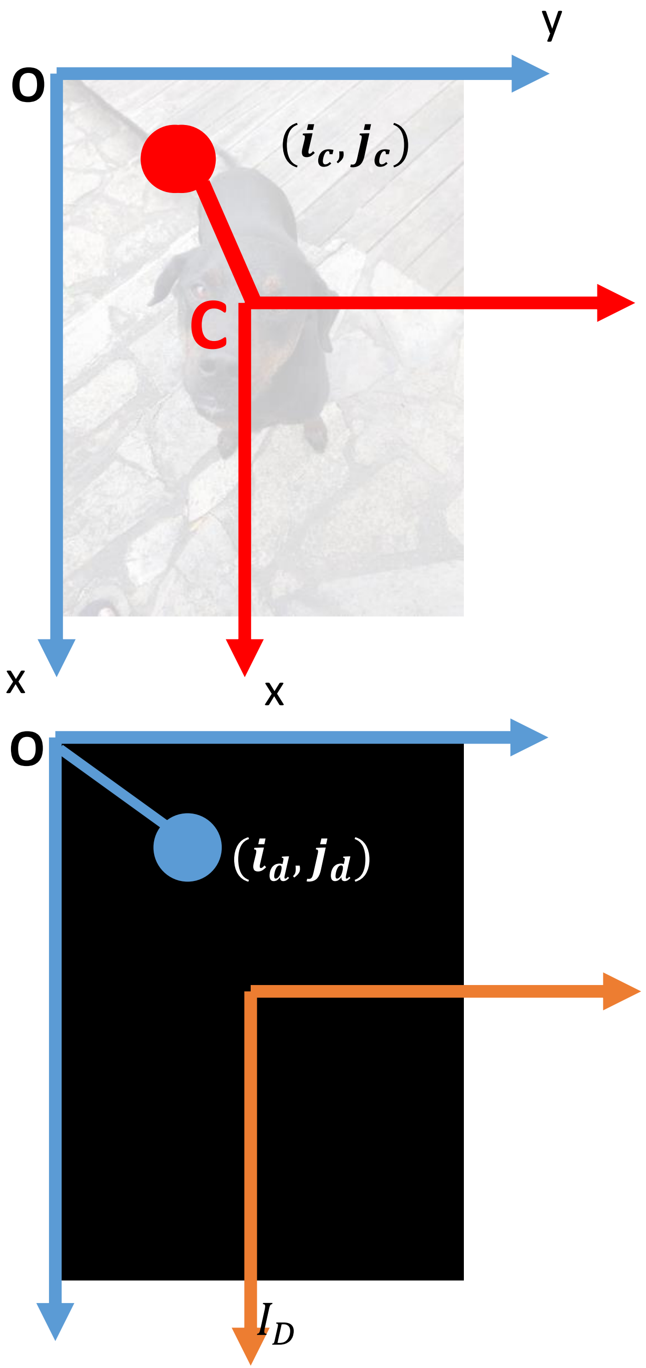
    2. Apply the distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})
    3. Apply change in co-ordinate system again to get (i_d, j_d)

$$(i_d, j_d) = (i_{cd} + c_x, j_{cd} + c_y)$$



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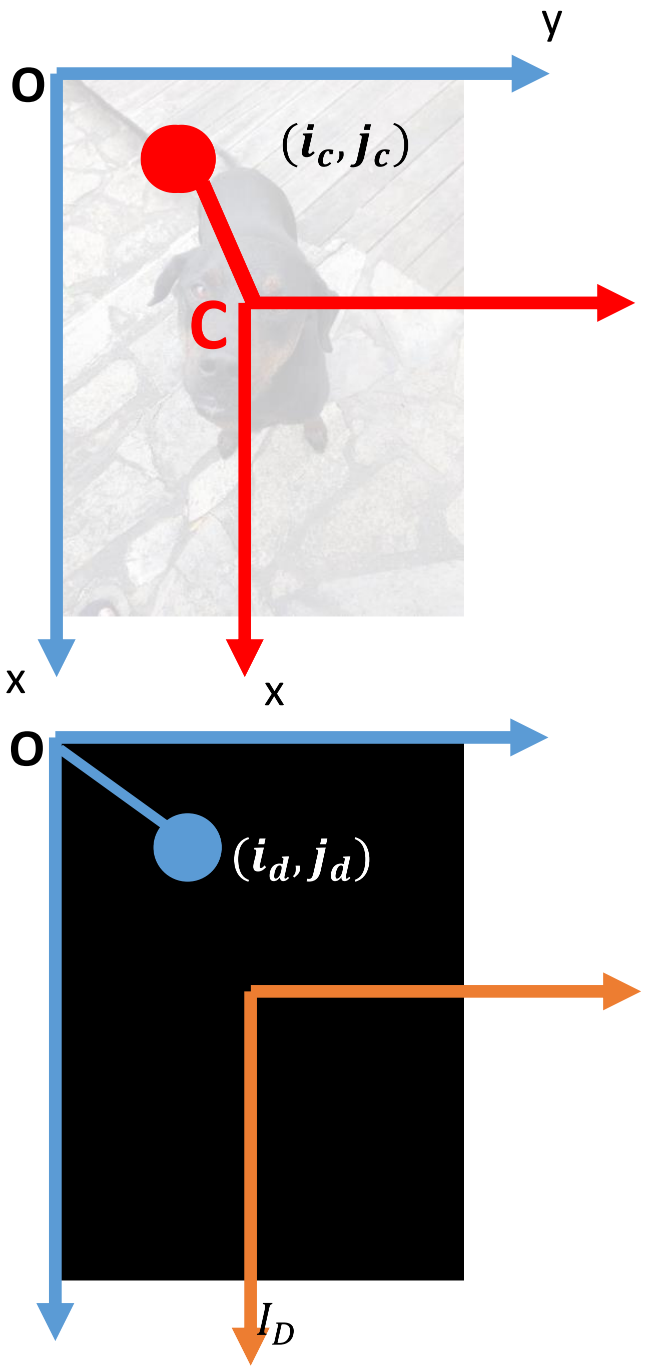
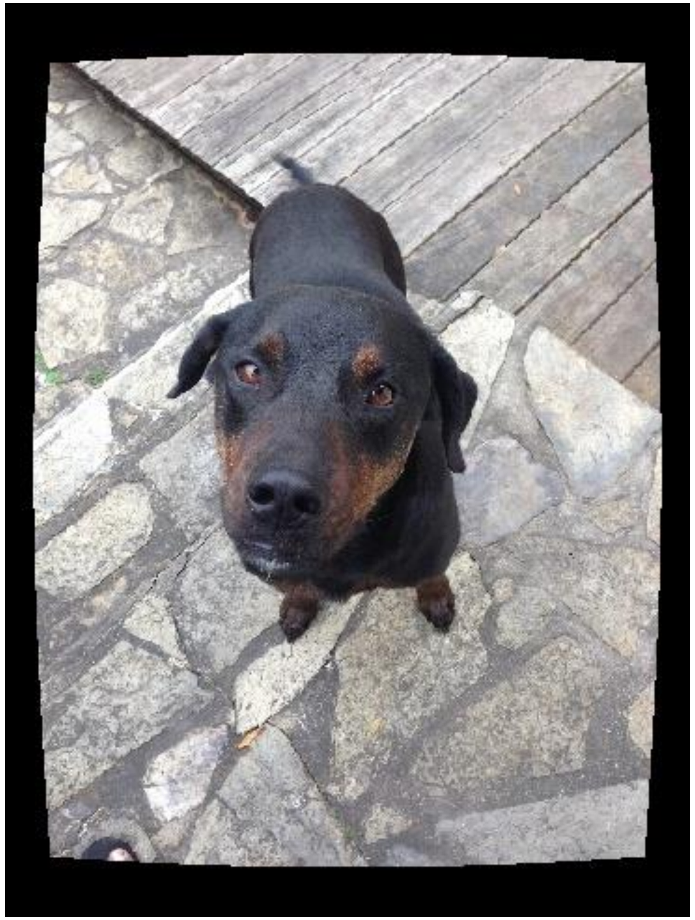
    3. Apply change in co-ordinate system again to get (i_d, j_d)
    4. Copy the pixel values from (i, j) in the original image at (i_d, j_d) in the distorted image.

$$D(i_d, j_d) = I(i, j)$$

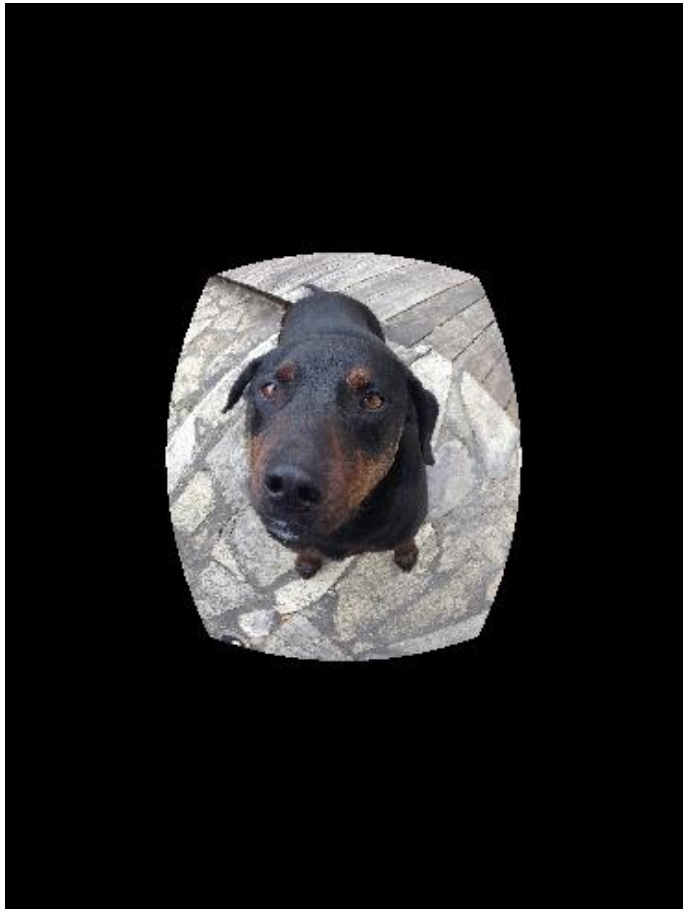


y
O
$(\boldsymbol{i_c}, \boldsymbol{j_c})$
C
x
x
O
$(\boldsymbol{i_d}, \boldsymbol{j_d})$
I_D

Results

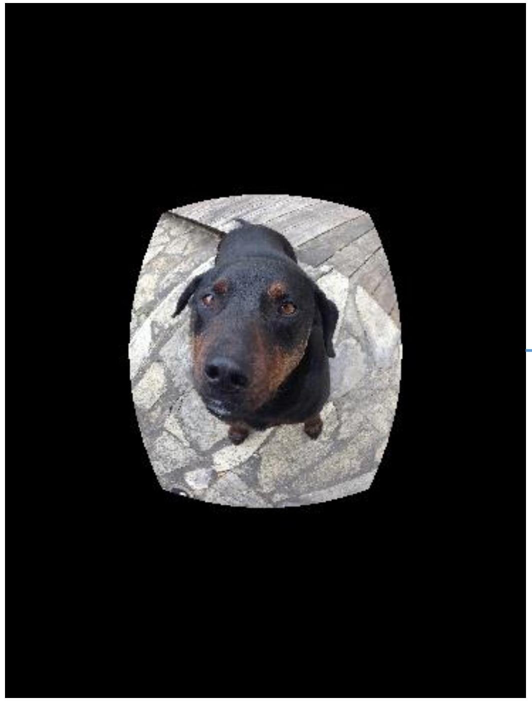


K = 0.0005

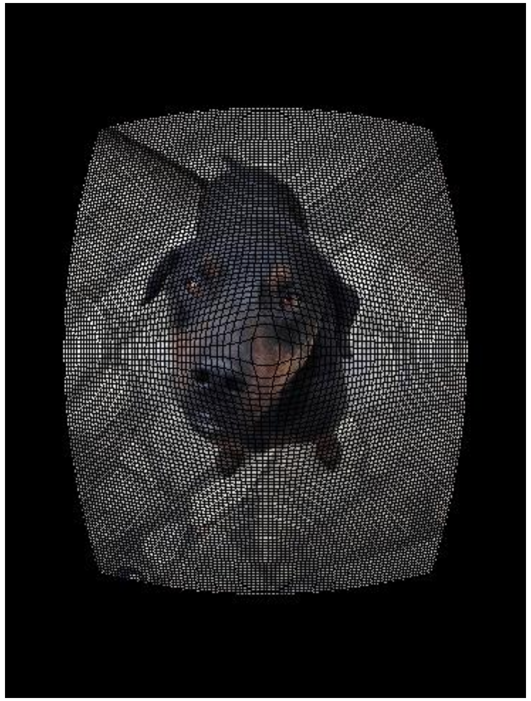


K = 0.005

2. Naïve Correction



Apply inverse distortion
K = 0.005



Corrected image

Inputs:
Distorted image
Distortion parameter (k)

Inverse distortion function

- We originally applied distortion using the function

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Mathematically the inverse distortion function is

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$$x` = \frac{1}{1 + kr}x \Rightarrow x = (1 + kr)x`$$

So, the inverse distortion function is

$$(x, y) = \left((1 + kr)x`, (1 + kr)y\right)$$

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However, r here is the distance of (x,y) from center. Since (x,y) is not known, we cannot compute r.

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So, the inverse distortion function is

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However, r here is the distance of (x,y) from center. Since (x,y) is not known, we cannot compute r

So, it is not possible to apply an exact inverse function.

Inverse distortion function

- We originally applied distortion using the function

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Mathematically the inverse distortion function is

$$x` = \frac{1}{1+kr}x \Rightarrow x = (1+kr)x`$$

So, the inverse distortion function is

$$(x,y) = f^{-1}(x`,y`) = \big((1+kr)x`,(1+kr)y`\big)$$

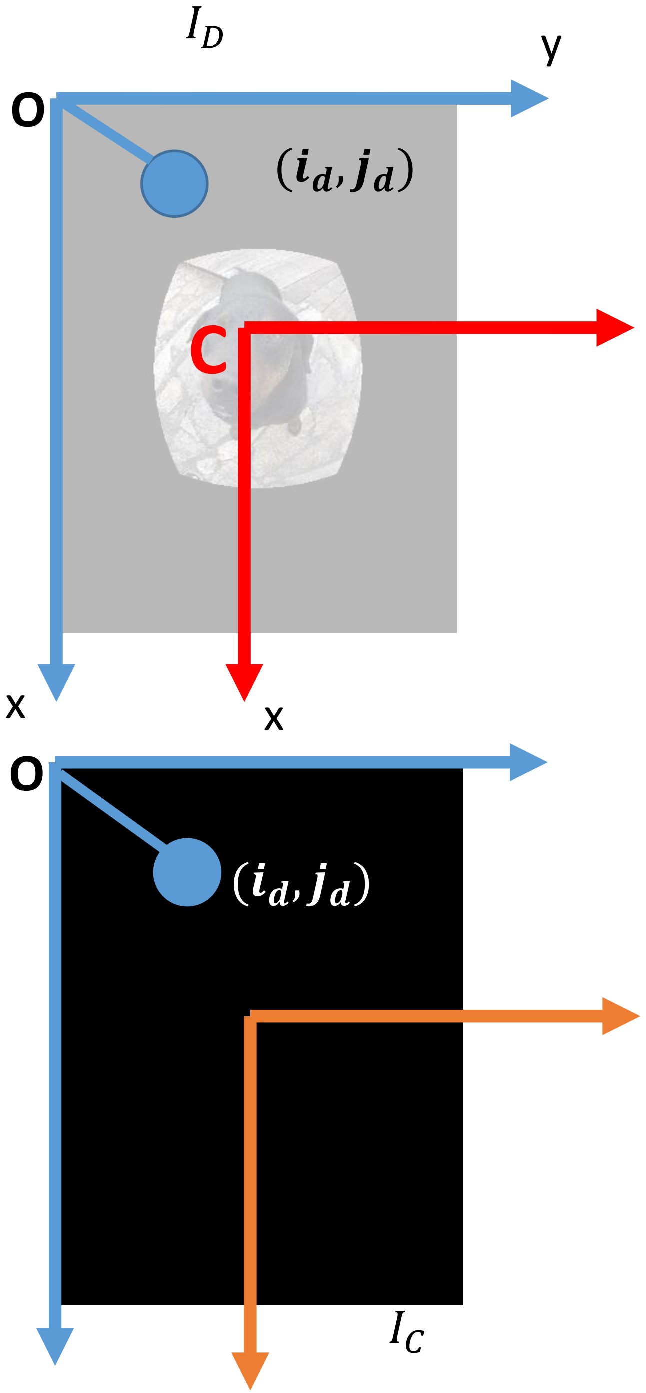
Alternatively, **we assume r to be the distance of the point $(\boldsymbol{x}`,\boldsymbol{y}`)$** from the center C.

(x`,y`) is known

Correction Naïve Steps

Let I_D be the distorted image, and the C $\left(c_x, c_y\right)$ its center.

1. Initialize a zeros matrix for corrected image (I_C) with same dims as input image.
2. For each (i_d, j_d) in distorted image:



I_D
y
o
$(\boldsymbol{i_d}, \boldsymbol{j_d})$
C
x
x
o
$(\boldsymbol{i_d}, \boldsymbol{j_d})$
I_C

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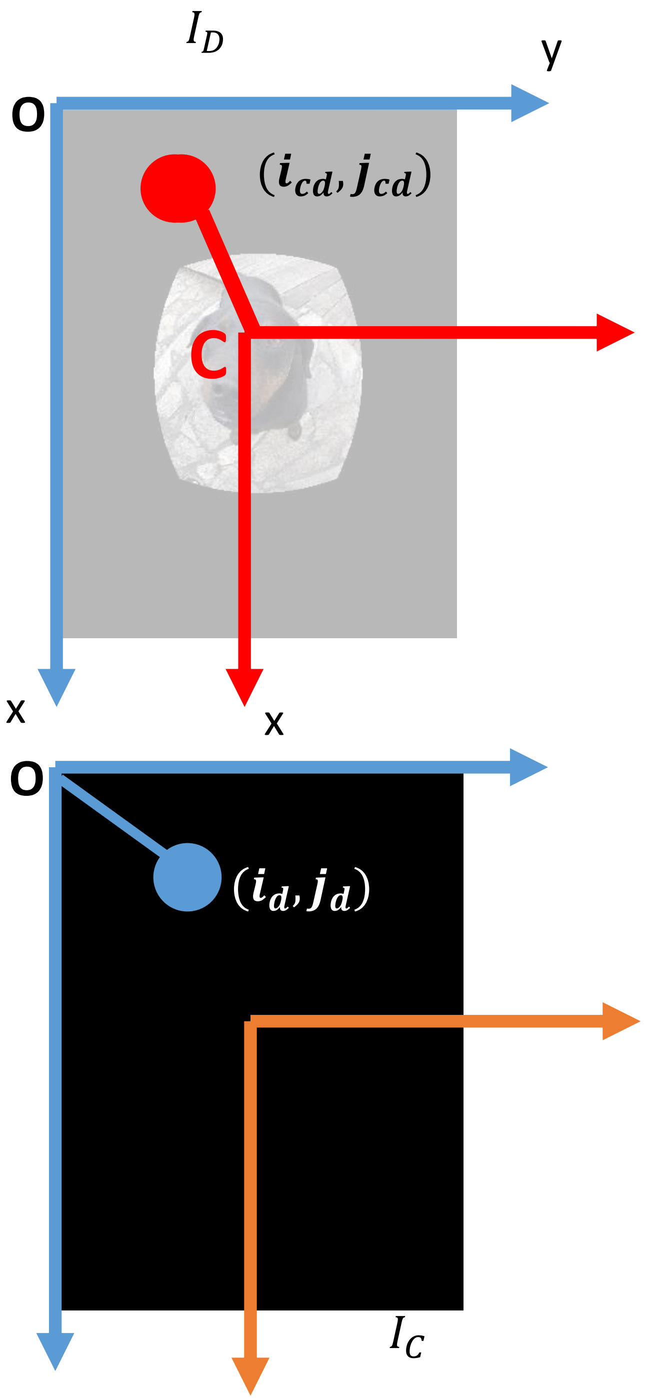

I_D
y
O
$(\boldsymbol{i_{cd}}, \boldsymbol{j_{cd}})$
C
x
x
O
$(\boldsymbol{i_d}, \boldsymbol{j_d})$
I_C

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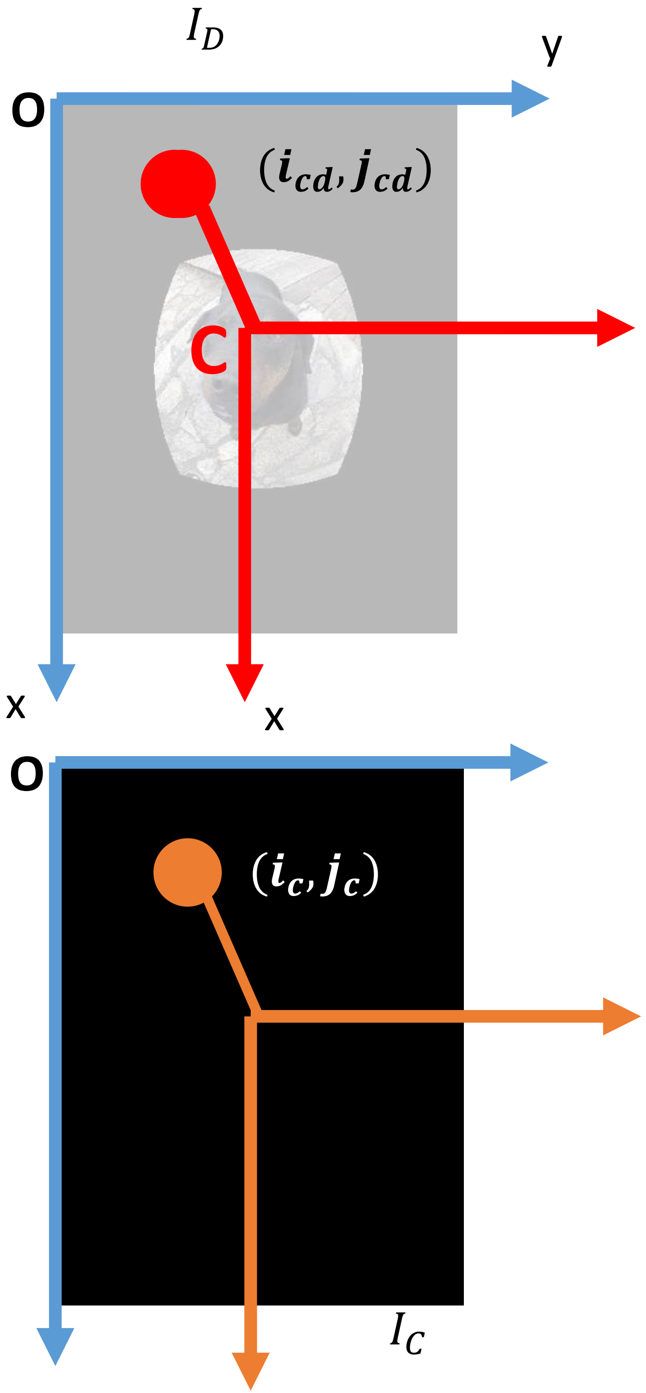
    2. Apply the inverse distortion function on (i_{cd}, j_{cd}) to get (i_c, j_c)

$$(i_c, j_c) = f^{-1}(i_{cd}, j_{cd})$$



I_D
y
O
$(\boldsymbol{i_{cd}}, \boldsymbol{j_{cd}})$
C
x
x
O
$(\boldsymbol{i_c}, \boldsymbol{j_c})$
I_C

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2. For each (i, j) in distorted image:
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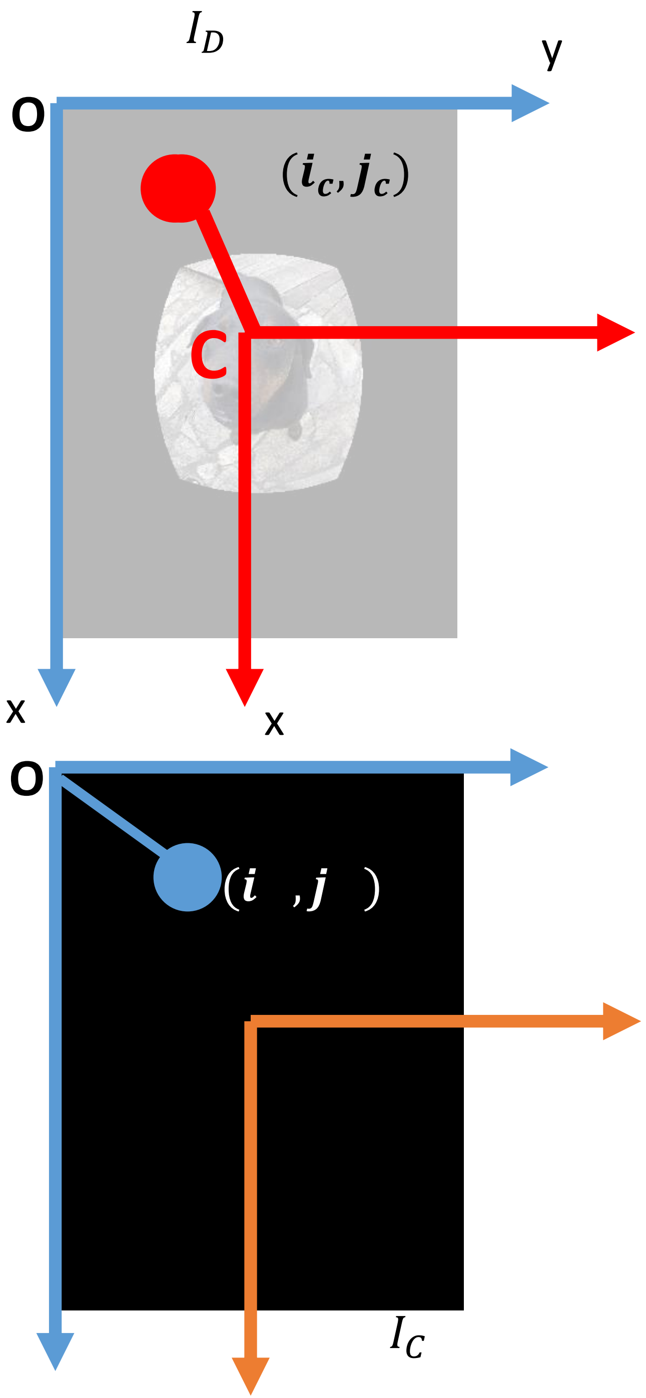
    2. Apply the inverse distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})

    3. Apply change in co-ordinate system again to get (i, j)



I_D
y
O
$(\boldsymbol{i_c}, \boldsymbol{j_c})$
C
x
x
O
$(\boldsymbol{i} \ \ , \boldsymbol{j} \ \ )$
I_C

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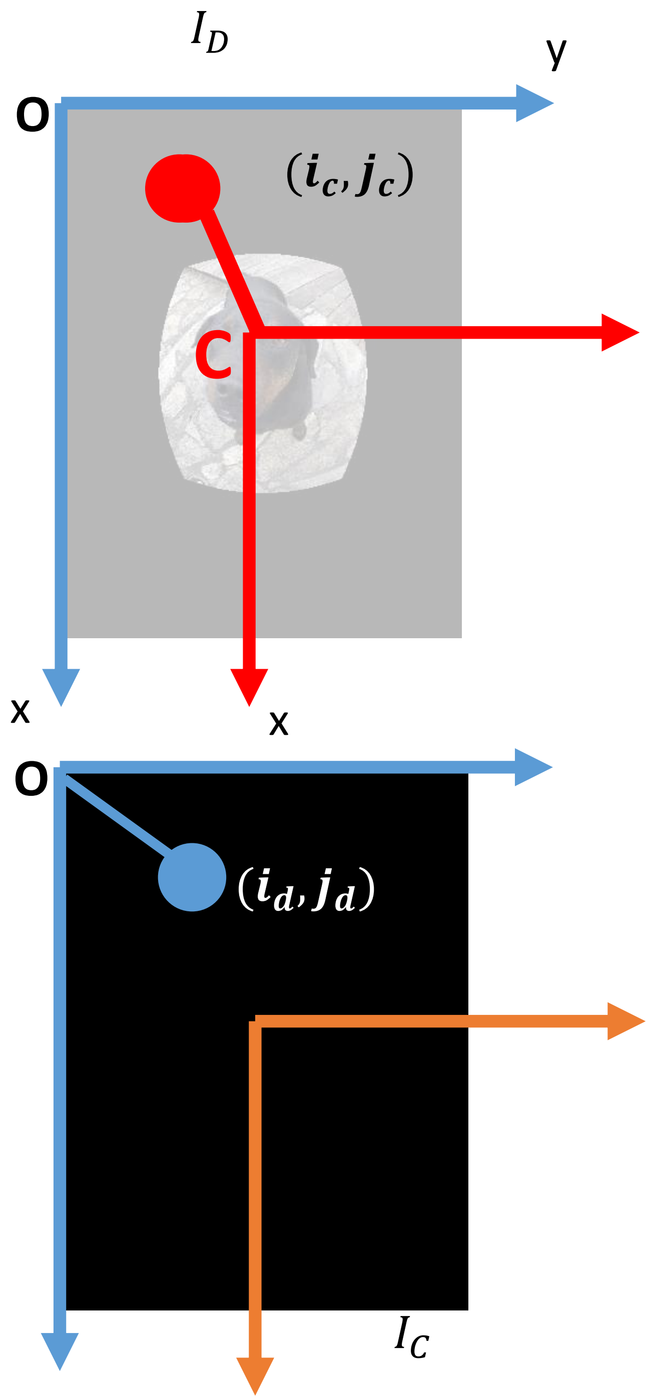
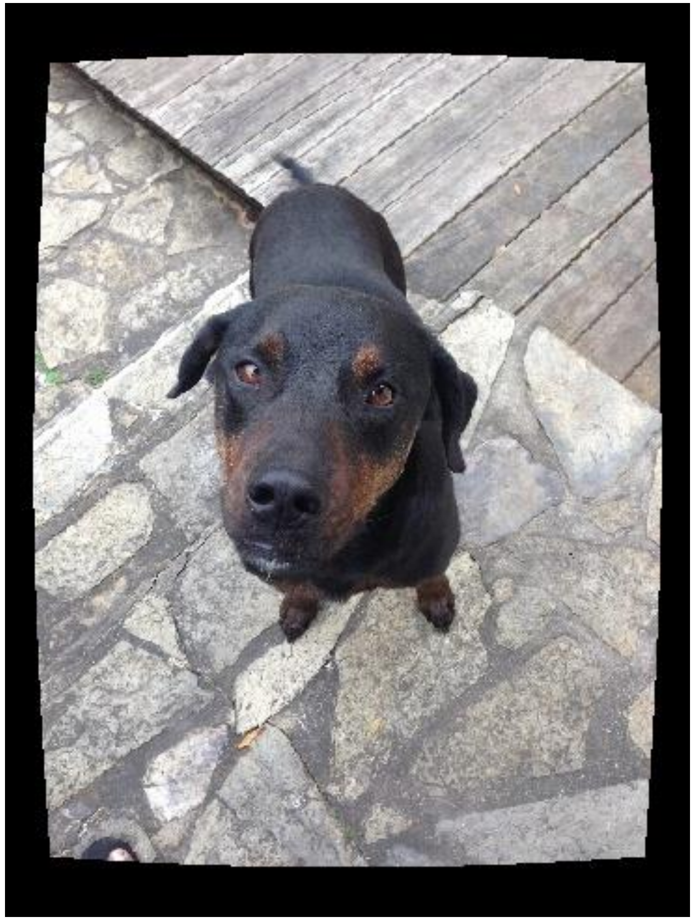
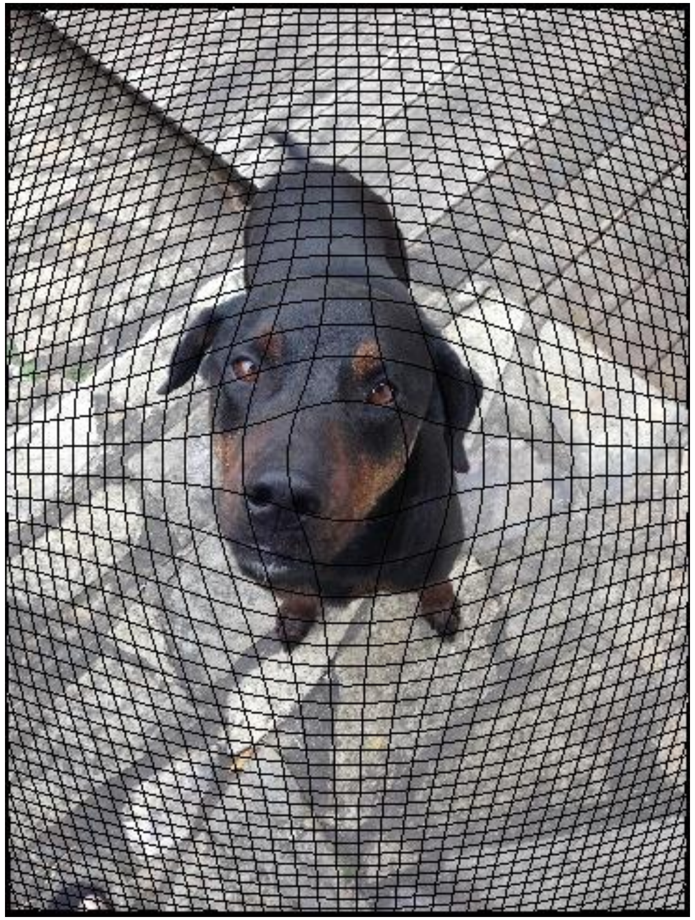
    2. Apply the inverse distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})
    3. Apply change in co-ordinate system again to get (i, j)
    4. Copy the pixel values from at (i_d, j_d) in the distorted image at (i, j) in the corrected image.

$$I_C(i, j) = I_D(i_d, j_d)$$


I_D
y
O
$(\boldsymbol{i_c}, \boldsymbol{j_c})$
C
x
x
O
$(\boldsymbol{i_d}, \boldsymbol{j_d})$
I_C

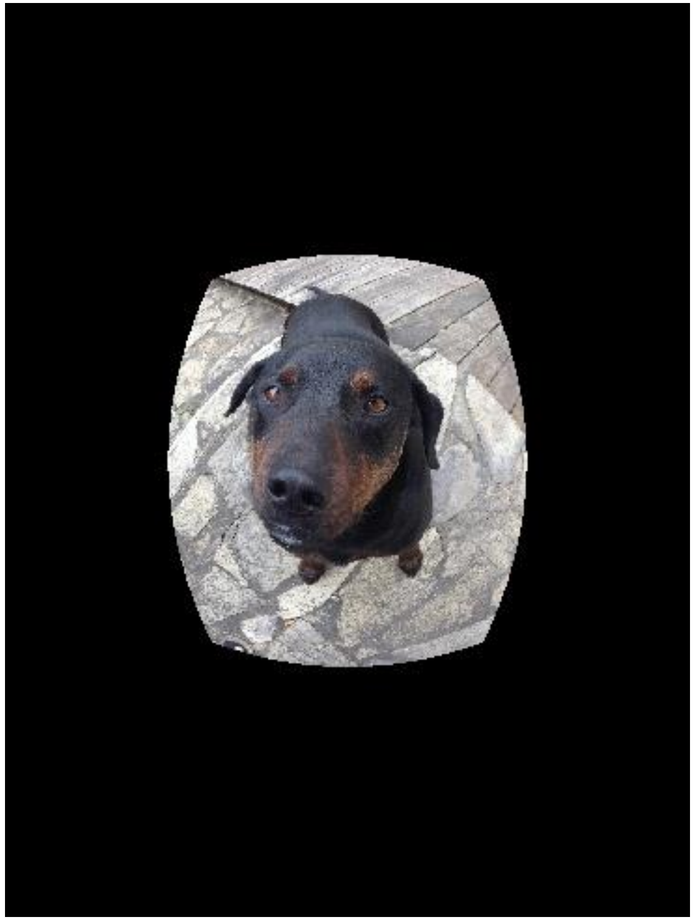
Corrected image (K = .0005)

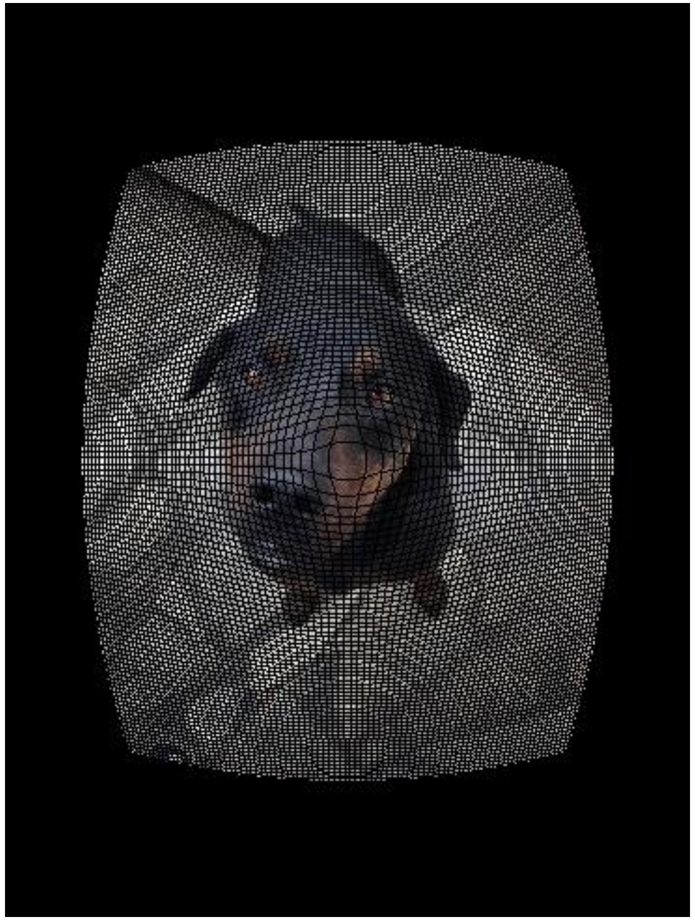




Corrected

Corrected image (K = .005)

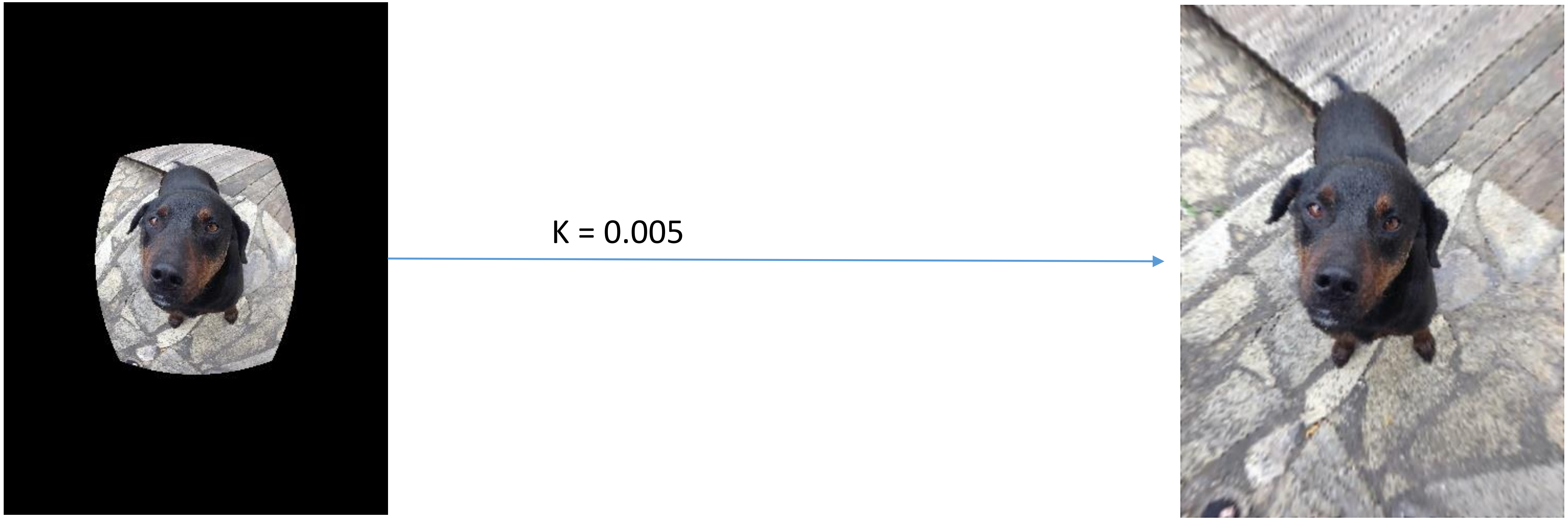




Corrected

Too many artifacts!

3. Correction using Interpolation



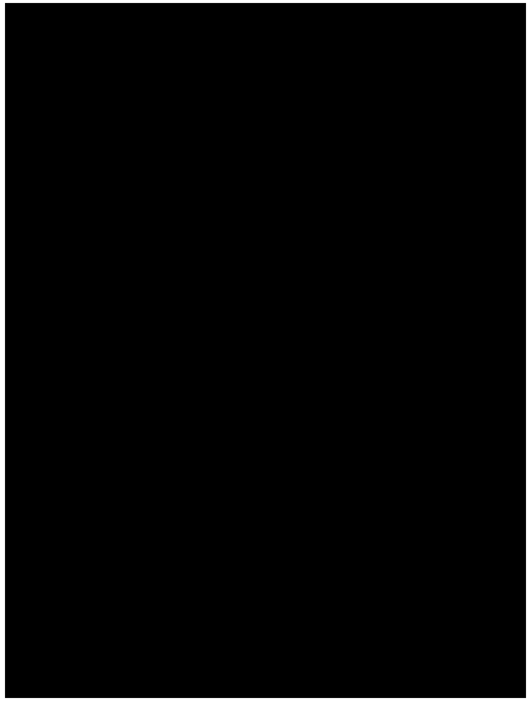
Corrected image

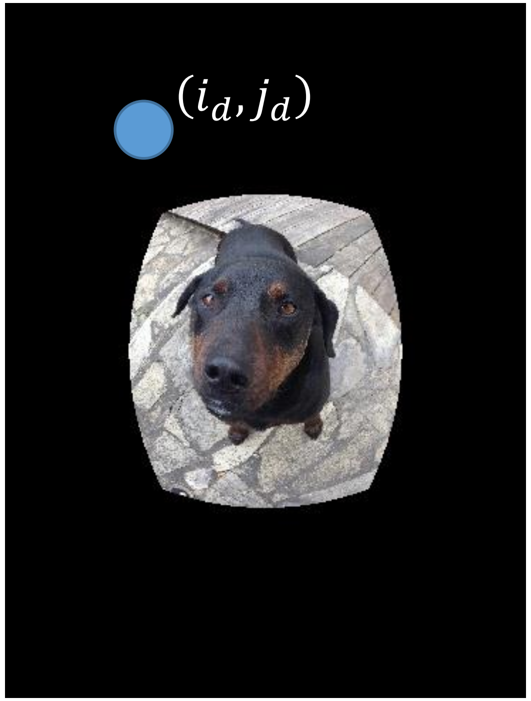
Inputs:
Distorted image
Distortion parameter (k)
Interpolation type (nearest neighbor)

Idea

- Create a corrected image with zeros
- For every pixel (i,j) in the corrected image apply distortion and calculate (i_d, j_d) co-ordinate it would map to.
- Now use neighbors around the point (i_d, j_d) to perform interpolation, and estimate the value for the location (i, j) in the corrected image.

I_c





Correction with Steps

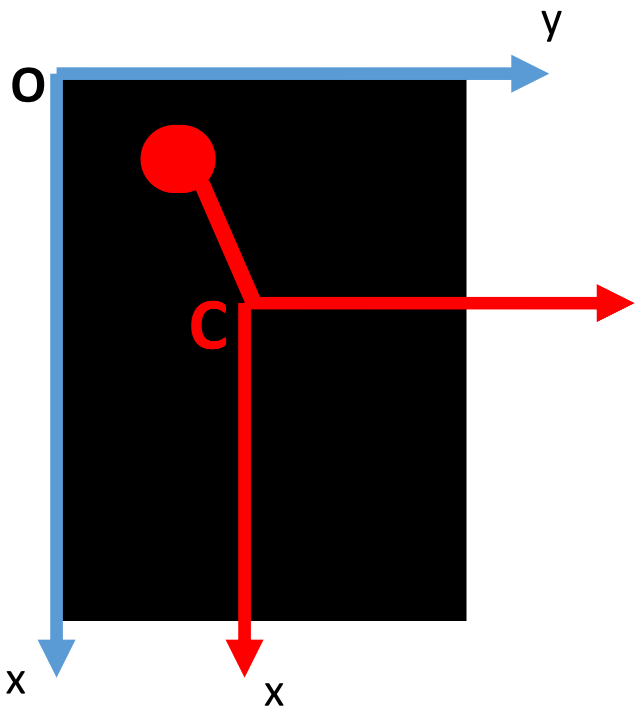
Let I_D be the distorted image, and the C $\left(c_x, c_y\right)$ its center.

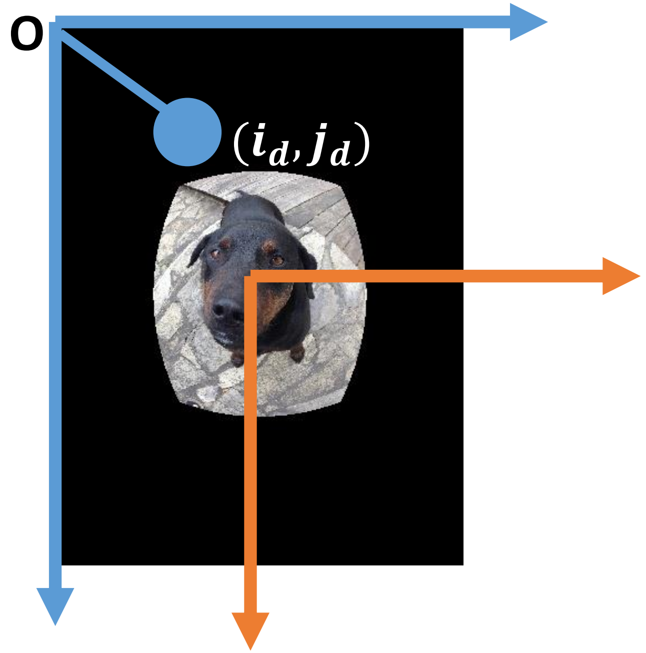
1. Initialize a zeros matrix for corrected image (I_c) with same dims as input image.
2. For each (i, j) in corrected image:
    1. Apply change in co-ordinate system, and find the point (i_c, j_c) with respect to center C.
    2. Apply the distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})
    3. Apply change in co-ordinate system again to get (i_d, j_d)
    4. If using nearest neighbour interpolation

       (i_{nn}, j_{nn})= $\left(round(i_d),\ \ round(j_d)\right)$

       Copy the pixel form the nearest neighbor

       $$\mathrm{I_C}(i, j) = I_D(i_{nn}, j_{nn})$$





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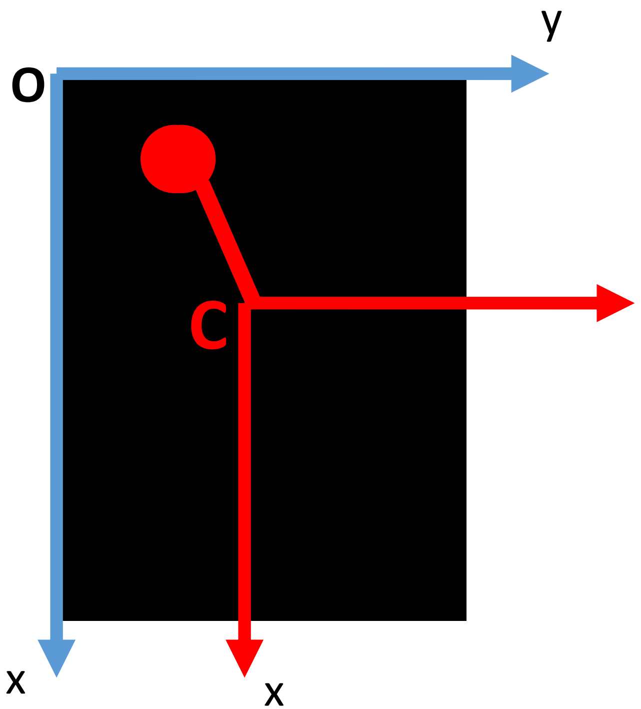
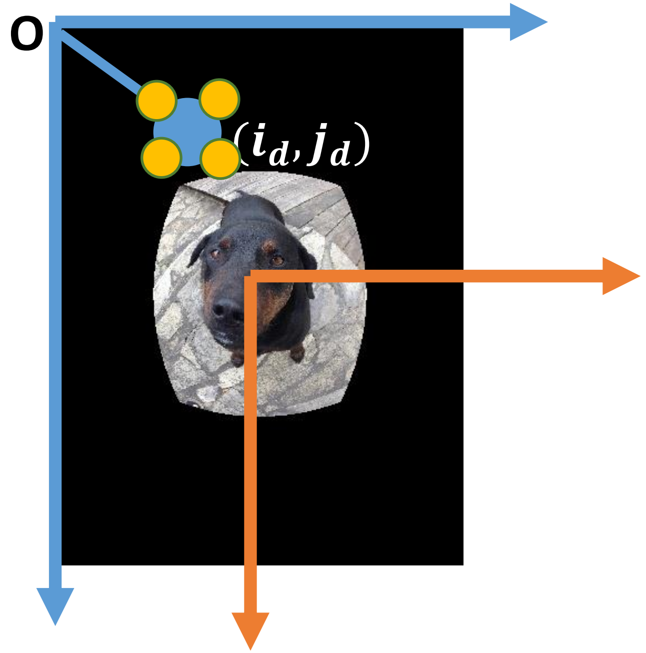
2. For each (i, j) in corrected image:
    1. Apply change in co-ordinate system, and find the point (i_c, j_c) with respect to center C.
    2. Apply the distortion function on (i_c, j_c) to get (i_{cd}, j_{cd})
    3. Apply change in co-ordinate system again to get (i_d, j_d)
    4. If using bilinear interpolation

       Find **four** nearest neighbors to (i_d, j_d)

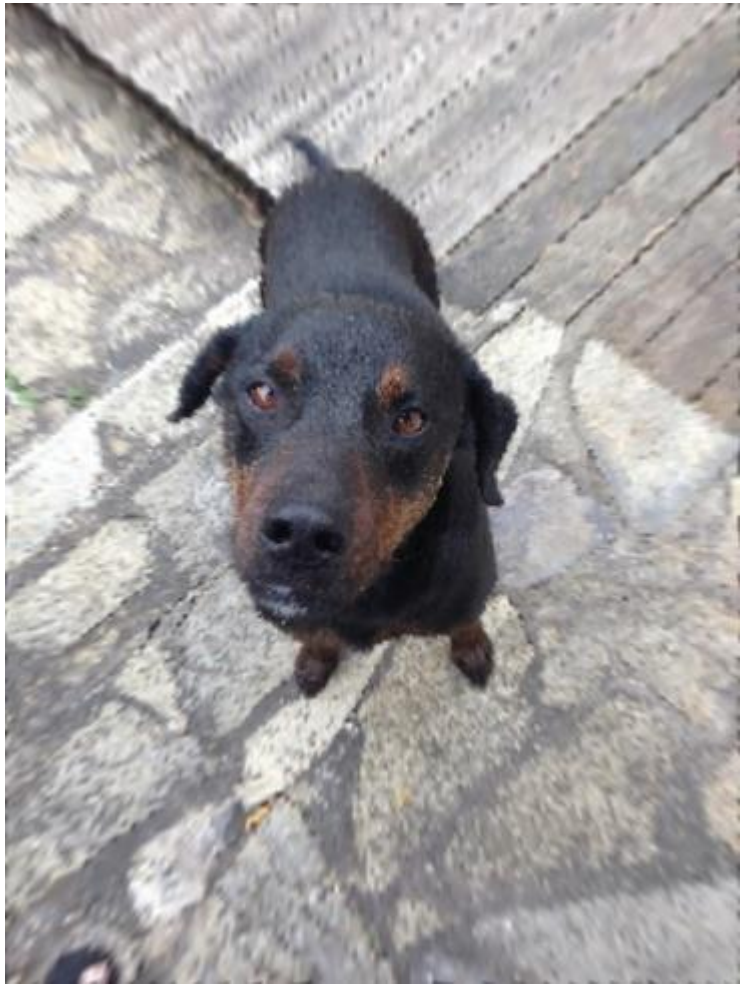
       Calculate bi-linear interpolated value (b)

$$\mathrm{I_C}(i, j) = b$$

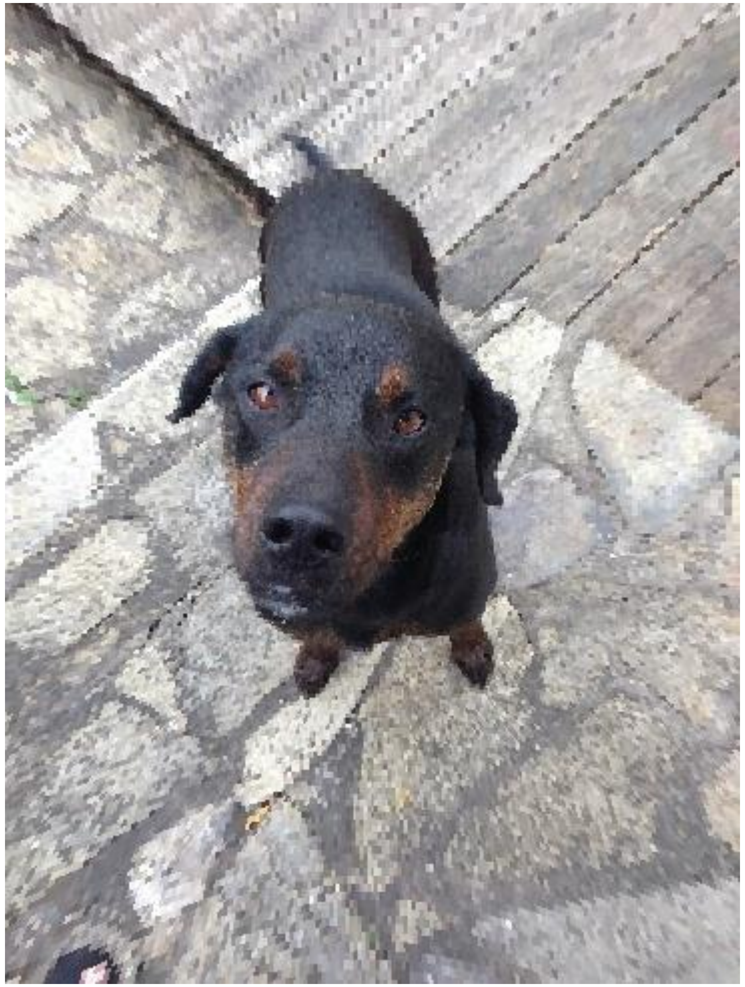




Corrected Image (K=0.005)

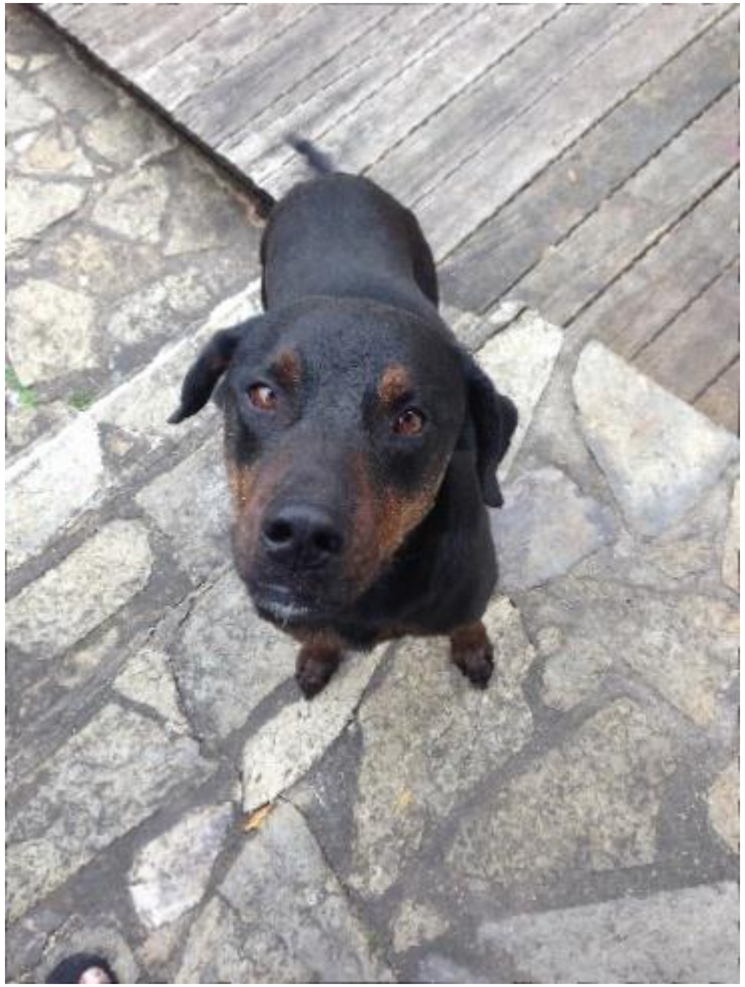


Bilinear

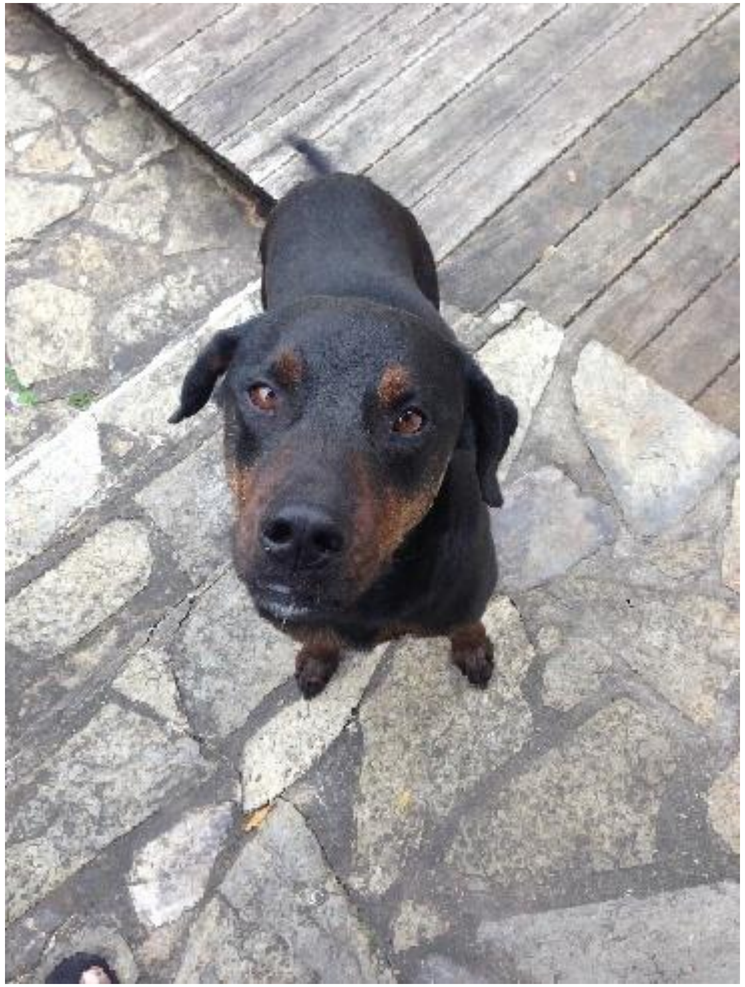


Nearest Neighbor

Corrected Image (K=0.0005)



Bilinear



Nearest Neighbor

Assignment - 1

1. Distortion (20 Pts.)
2. Naïve Correction (20 Pts)
3. Correction with interpolation (35 Pts)

Total: 75 Pts.

Due Date: Feb 21st , 11:59 PM

Submission Instructions

- Must use the **starter code** available in **Github**
- Submission allowed only through **Github**
- You will receive an email with invitation to join **Github** classroom
- Start by reading the **readme.md** file.
- Instructions are available here
- Github will **automatically** save the **last commit as a submission** before the deadline