Digital Image Processing

COSC 6380/4393

Lecture – 11

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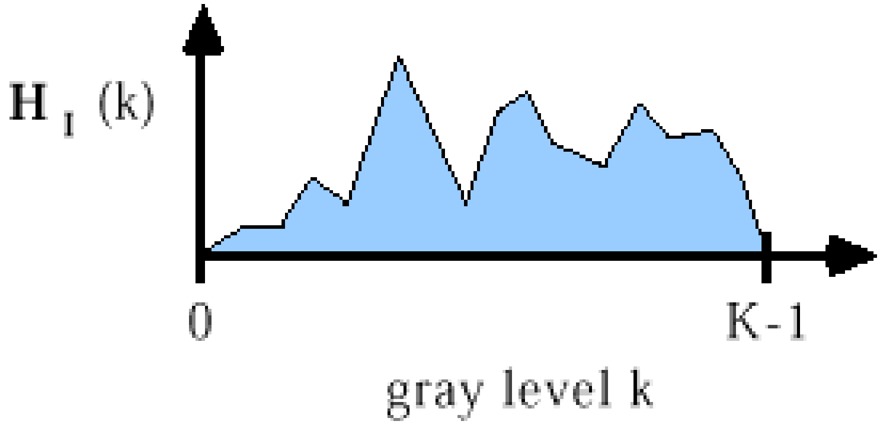
Point Operations

SIMPLE HISTOGRAM OPERATIONS

- Recall: the gray-level histogram $\mathbf{H_I}$ of an image **I** is a graph of the frequency of occurrence of each gray level in **I**
- $\mathbf{H_I}$ is a one-dimensional function with domain 0, ... , K-1 :
- $\mathbf{H_I}$ (k) = n if gray-level k occurs (exactly) n times in **I**, for each k = 0, ... K-1



SIMPLE HISTOGRAM OPERATIONS

- The histogram $\mathbf{H_I}$ contains **no spatial information** about **I** - only information about the relative frequency of intensities
- Nevertheless
  - Useful information can be obtained from the histogram
  - Image quality is effected (enhanced, modified) by altering the histogram

Average Optical Density

- A measure of the average intensity of the image $\mathbf{I}$:

$$\mathrm{AOD}(\mathbf{I}) = \left(\frac{1}{N^2}\right)\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} I(i,j) = \left(\frac{1}{N^2}\right)\sum_{j=0}^{N-1}\sum_{i=0}^{N-1} I(i,j)$$

- Can compute it from the histogram as well:

Average Optical Density

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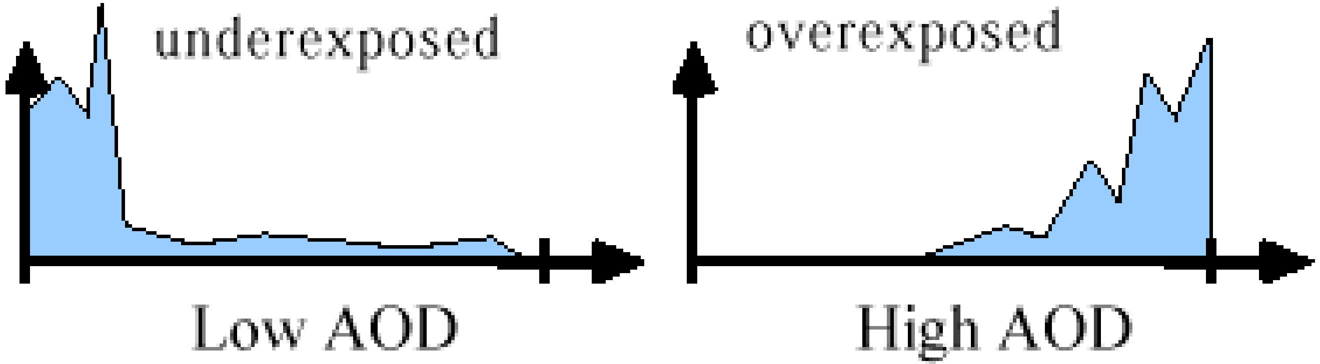
- Can compute it from the histogram as well:

$$\left(\frac{1}{N^2}\right)\sum_{k=0}^{K-1} k\mathbf{H}_{\mathbf{I}}(k)$$

- k^{th} term = (brightness level k) x (# occurrences of k)

Average Optical Density

- Examining the histogram can reveal possible errors in the imaging process:



- Methods for correcting such errors utilize the histogram
- The histogram will arise throughout this lecture

POINT OPERATIONS

- A point operation on an image **I** is a **function** f that maps **I** to another image **J** by operating on **individual pixels** in **I**:

$$J(i, j) = f[I(i, j)],\ 0 <= i, j <= N-1$$

- The same function f is applied at every image coordinate
- This is different from **local operation**s such as OPEN, CLOSE, etc., since those are functions of both I(i, j) **and its neighbors**

LINEAR POINT OPERATIONS

- Point operations **do not** modify **spatial relationships** between pixels
- They **do** modify the **image histogram**, and therefore the overall appearance of the image
- **Linear point operations** are the simplest class of point operations

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$$F(X) = P.X + L$$

Image Offset

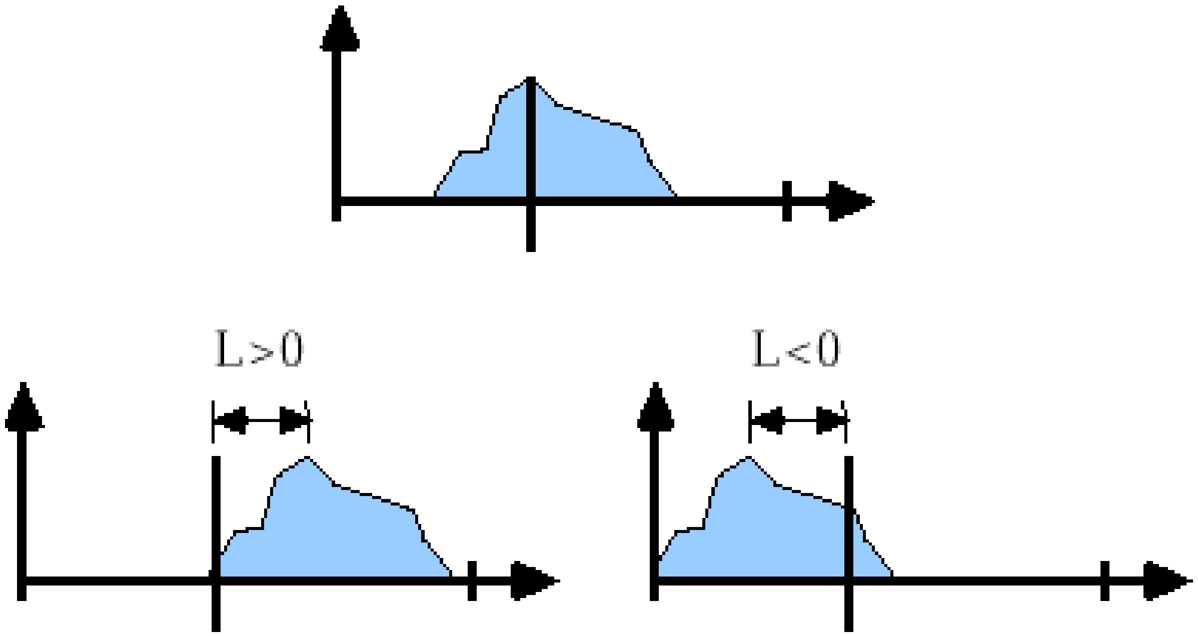
- Suppose L falls in the range -(K-1) <= L <= K-1 (± the nominal gray scale)
- An **additive image offset** is defined by the function

  $$J(i, j) = I(i, j) + L, \text{ for } 0 \le i, j \le N-1$$

- Thus, the same constant L is added to every image pixel value
- If L > 0, **J** will be a **brightened** version of the image **I**
- Otherwise its appearance will be essentially the same

Image Offset

- If $L < 0$, **J** will be a **dimmed** version of the image **I**
- Adding offset L **shifts** the histogram by amount L to left or right:



Histograms of additive image offsets

- The input and output histograms are related by:

$$\mathbf{H_J}(k) = \mathbf{H_I}(k-L)$$

Image Offset Example

- Suppose it is desired to compare multiple images I_1, $\mathbf{I}_2$,..., $\mathbf{I}_n$ of the same scene
- However, the images were taken with a variety of different exposures or lighting conditions
- One solution: **equalize** the AOD's of the images
- If the gray-scale range of the images is 0 ,..., K-1, a reasonable AOD is K/2
- Let L_m = AOD(I_m), for m = 1 ,..., n
- Then define "AOD-equalized" images J_1, $\mathbf{J}_2$,..., $\mathbf{J}_n$ according to

  $J_m(i, j) = I_m(i, j) - L_m + K/2$, for $0 <= i, j <= N-1$

Image Offset Example

- The effect:

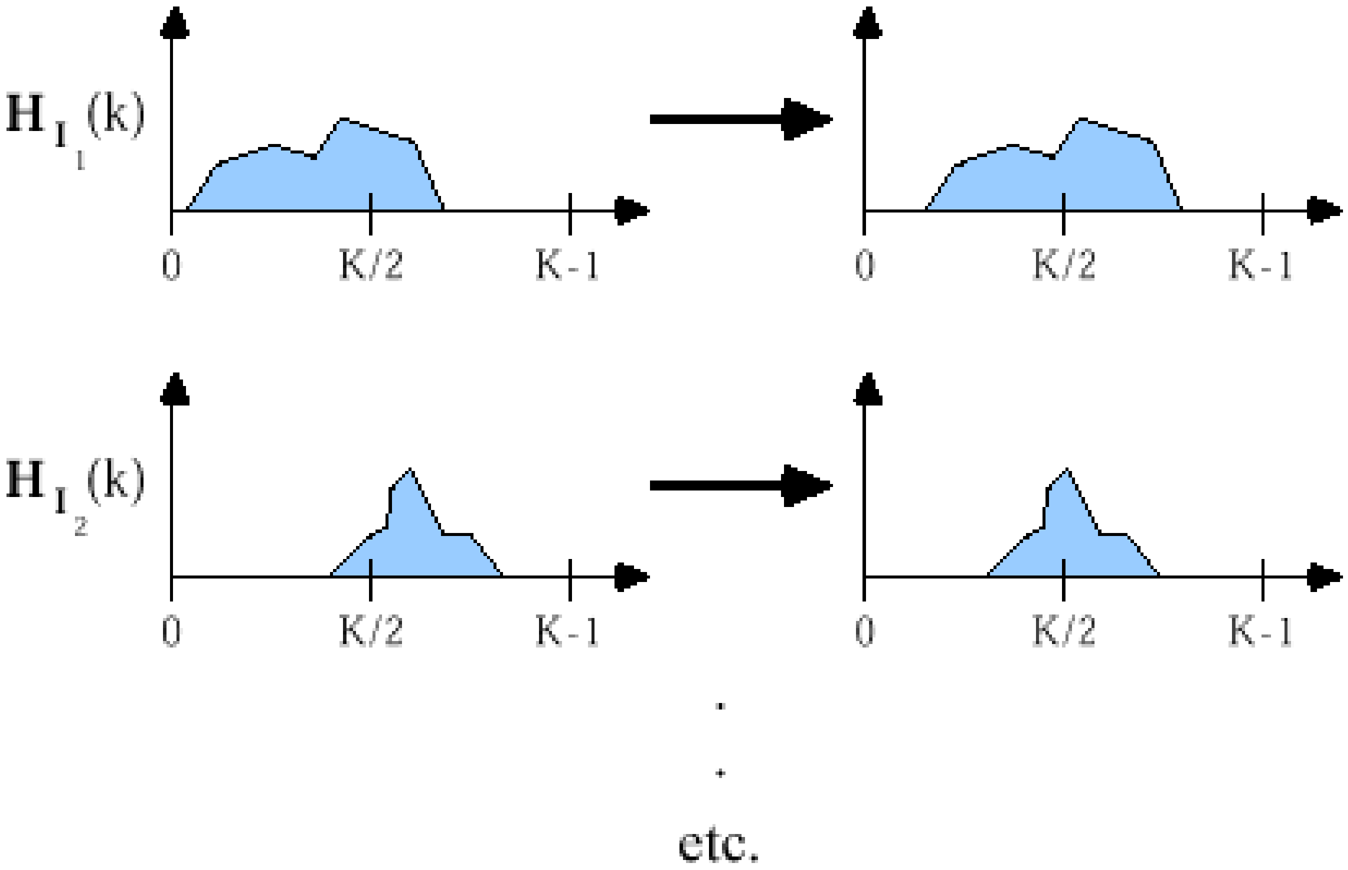


Image Scaling

- Suppose $P > 0$ (not necessarily an integer)
- **Image scaling** is defined by the function

$$J(i, j) = P \cdot I(i, j) \text{ , for } 0 <= i, j <= N-1$$

- Thus, P multiplies every image pixel value
- In practice:

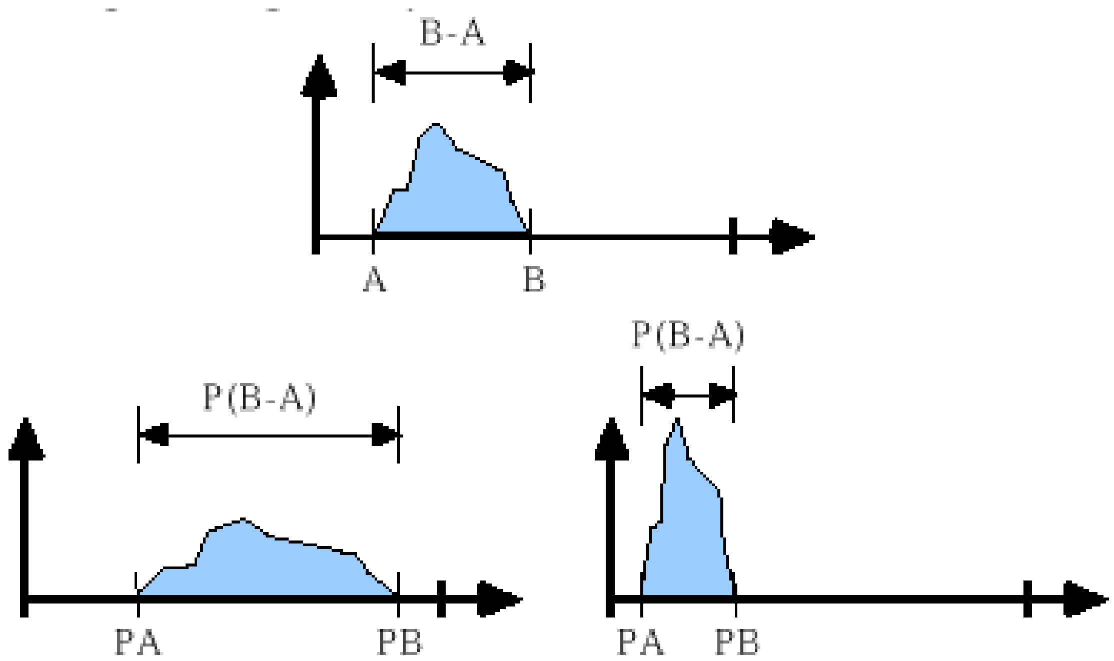
$$J(i, j) = \mathrm{INT}[\ P \cdot I(i, j) + 0.5\ ] \text{ , for } 0 <= i, j <= N-1$$

where INT[R] = nearest integer that is <= R

- If $P > 1$, **J** will have a **broader grey level range** than image **I**

Image Scaling

- If $P < 1$, **J** will have a **narrower grey-level range** than **I**
- Multiplying by a constant P **stretches** or **compresses** the "width" of the image histogram by a factor P:



Comments

- An image with a compressed gray level range generally has a **reduced visual contrast**
- Such an image may have a **washed-out** appearance
- An image with a wide range of gray levels generally has an **increased visual contrast**
- Such an image may have a more striking, viewable appearance

Linear Point Operations: Offset & Scaling

- Suppose L and P are real numbers (not necessarily integers)
- A **linear point operation** on **I** is defined by the function

  $J(i, j) = P \cdot I(i, j) + L$, for $0 <= i, j <= N-1$

- In practice:

  $J(i, j) = INT[\, P \cdot I(i, j) + L + 0.5 \,]$, for $0 <= i, j <= N-1$

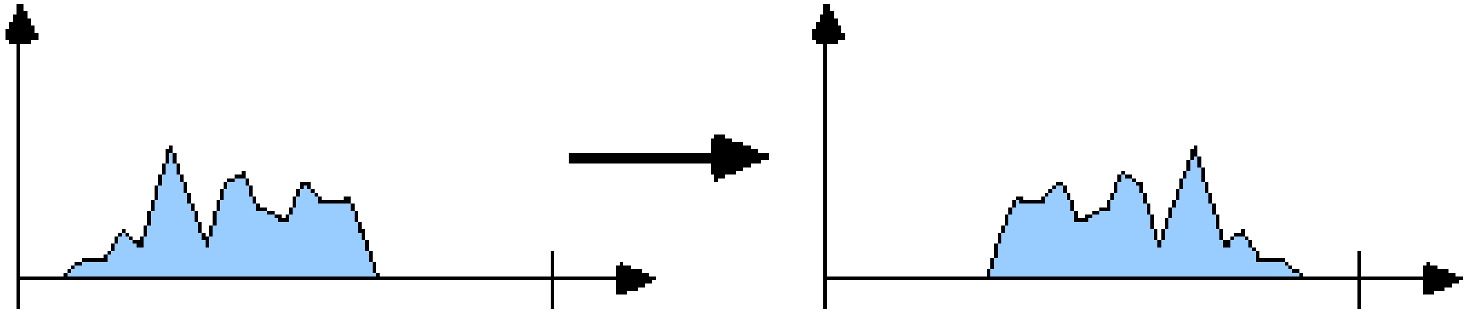
- The image **J** is a version of **I** that has been scaled and given an additive offset

Linear Point Operations: Offset & Scaling

- If $P < 0$, the histogram is **reversed**, creating a **negative** image
- By far the most common use is $P = -1$ and $L = K-1$:

  $J(i, j) = (K-1) - I(i, j)$, for $0 <= i, j <= N-1$

- Hereafter w



Histogram of "**digital negative image**"

Caveat

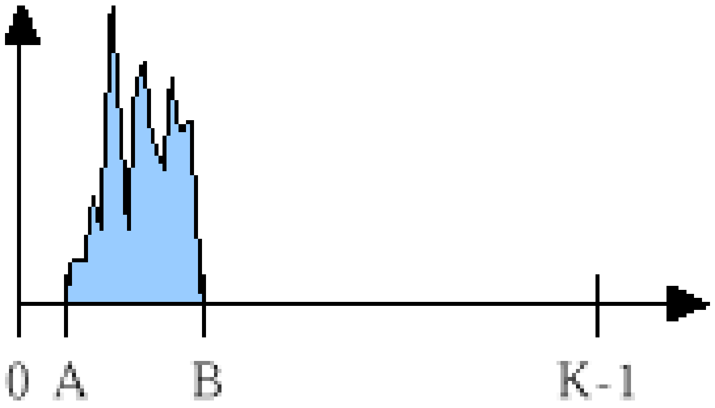
- Generally, the available gray-scale of the transformed image **J** is the same as that of the original image **I**: {0 ,..., K-1}
- When making the transformation

  $J(i, j) = P \cdot I(i, j) + L$, for $0 <= i, j <= N-1$
- care must be taken that the maximum and minimum values J_{max} and J_{min} satisfy

  $J_{max} <= K-1$ and $J_{min} >= 0$
- **At best**, values outside these ranges will be "clipped"
- At worst, an overflow or sign-error condition may occur
- In that instance, the gray-scale value assigned to an error pixel will be highly unpredictable

Full-Scale Contrast Stretch

- The **most common** linear point operation. Suppose **I** has a compressed histogram:



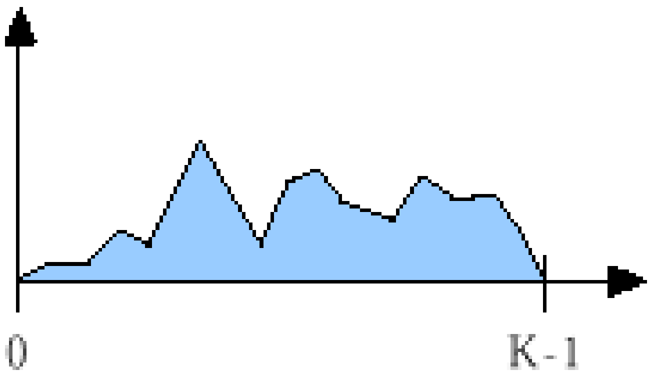
- Let A and B be the min and max gray levels in **I**
- Define

$$J(i, j) = P \cdot I(i, j) + L$$

- such that $P \cdot A + L = 0$ and $P \cdot B + L = (K-1)$

Full-Scale Contrast Stretch

- The result of solving these **2 equations in 2 unknowns** (P, L) is an image **J** with a full-range histogram:



- The solution to the above equations is

$$P = \left(\frac{K-1}{B-A}\right) \quad \text{and} \quad L = -A\left(\frac{K-1}{B-A}\right)$$

or

$$J(i, j) = \left(\frac{K-1}{B-A}\right)\left[I(i, j) - A\right]$$

NONLINEAR POINT OPERATIONS

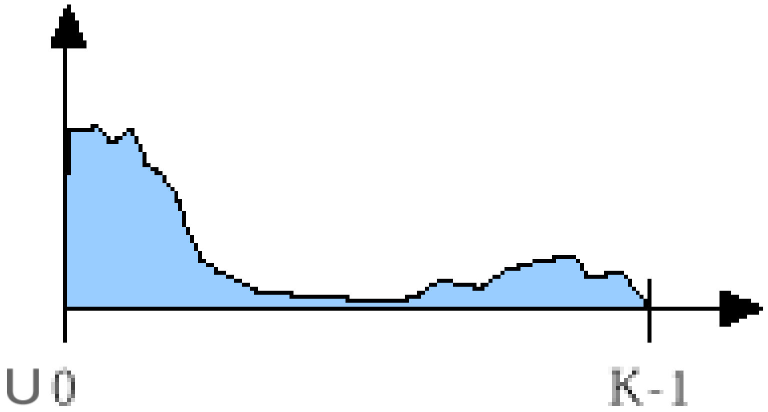
- A **nonlinear point operation** on **I** is a **pointwise function f** mapping **I** to **J**:

  $J(i, j) = f[I(i, j)]$ for $0 <= i, j <= N-1$

- where **f** is a **nonlinear function**.
- This is of course a very broad class of functions
- However, only a few are used much:
  - $J(i, j) = |I(i, j)|$ (absolute value or magnitude)
  - $J(i, j) = [I(i, j)]^2$ (square-law)
  - $J(i, j) = I(i, j)^{1/2}$ (square root)
  - $J(i, j) = \log[1+I(i, j)]$ (logarithm)
  - $J(i, j) = \exp[I(i, j)] = e\ I(i,j)$ (exponential)

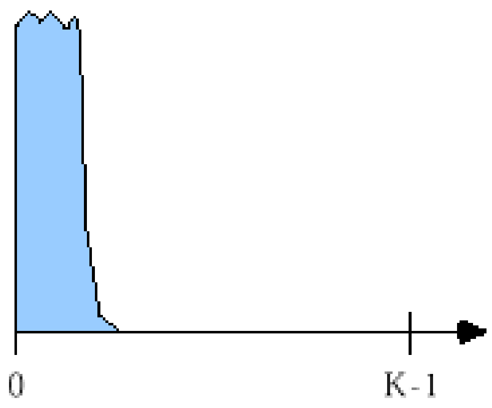
Logarithmic Range Compression

- **Motivation**: An image may contain information-rich, smoothly-changing low intensities - and small very bright regions
- Useful for detecting faint **objects**
- The bright pixels will dominate our visual perception of the image
- A typical histogram:

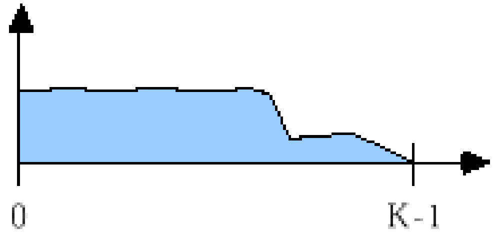


Logarithmic Range Compression

- **Logarithmic transformation** J(i, j) = log[1+I(i, j)] nonlinearly **compresses** and **equalizes** the gray-scales
- Bright intensities are compressed much more heavily - thus **faint details** emerge
- A full-scale contrast stretch then utilizes the full gray-scale range:



logarithmic transformation



stretched contrast

HISTOGRAM SHAPING

- Apply point operation such that the intensity histogram has a desired shape (target shape)
- Often times, the transformation function is non-linear.
- We now describe methods for **histogram shaping**.
- Accomplished by point operations: object **shape** and **location** are **unchanged**.

DEFINITION

- Define the **normalized histogram**:

$$p_I(k) = \left(\frac{1}{N^2}\right) H_I(k) \; ; \; k = 0, \ldots, K-1$$

- These values **sum to one**: $\sum_{k=0}^{K-1} p_I(k) = 1$
- Here $\mathbf{p_I}$(k) is the **probability** that gray-level k will occur (at any given pixel)
- $\mathbf{p_I}$(k) ≈ probability of gray-level k
- The **cumulative histogram** is

$$P_I(r) = \sum_{k=0}^{r} p_I(k) \; ; \; r = 0, \ldots, K-1$$

- $\mathbf{P_I}$(r) is a nondecreasing function, and $\mathbf{P_I}$(K-1) = 1.

INTERPRETATION

- With the probabilistic interpretation, at a point (i, j):
- $\mathbf{P}_I(r) = Pr\{I(i, j) \leq r\}$

CONTINUOUS HISTOGRAMS

- Suppose **p**(x) and **P**(x) are **continuous**: they may be regarded as probability density (pdf) and cumulative distribution (cdf).
- **P**$^{-1}$(x) exists or can be defined **by convention**.

$$If\ Y\ =\ F(X),$$
$$F\ -\ a\ transformation\ function$$

CDF:

$$\boldsymbol{if\ y = F(x) \Rightarrow y = {P_Y}^{-1}\big(P_X(x)\big)}$$