MATH 3339
Statistics for the Sciences
Live Lecture Help

James West
jdwest@uh.edu

University of Houston

Session 11

Office Hours: see schedule in the "Office Hours" channel on Teams
Course webpage: www.casa.uh.edu

Email policy

When you email me you **MUST** include the following

- MATH 3339 Section 20024 and a description of your issue in the **Subject Line**
- Your name and ID# in the **Body**
- Complete sentences, punctuation, and paragraph breaks
- Email messages to the class will be sent to your Exchange account (user@cougarnet.uh.edu)

Using R and RStudio

1. Download R from https://cran.r-project.org/
2. Download RStudio from https://www.rstudio.com/

Outline

1. Updates and Announcements
2. Recap
3. Student submitted questions

Updates and Announcements

- Test 2 Scheduling begins tonight at midnight!
- Lecture 21 is available and popper closes 11/06.
- I will use Final Exam to replace one lower test grade.

Null Hypothesis of significant tests

- The statement that is assumed to be true. We assume “no effect” or “no difference” for the parameter tested.
- Abbreviate the null hypothesis by H_0.
- From mean body temperature example, $H_0 : \mu = 98.6°$F.
- For a significant test of the mean, the null hypothesis is always equal to some value of what we assume the mean to be.
- The null hypothesis is always $H_0 : \mu = \mu_0$, where μ_0 is some value that is assumed to be the true mean.

Possible values for the Alternative Hypothesis

There are three possible ways that we would want to test against the null hypothesis.

1. Test to prove that the mean is really lower than what is assumed. This is called a **left-tailed test**. $H_a : \mu < \mu_0$
2. Test to prove that the mean is greater than what is assumed. This is called a **right-tailed test**. $H_a : \mu > \mu_0$
3. Test to prove that the mean is not equal (either higher or lower) than what is assumed. This is called a **two-tailed test**. $H_a : \mu \neq \mu_0$

Decision

- Since there are only two hypotheses, there are only two possible decisions.
- **Reject** the null hypothesis in favor of the alternative hypothesis. (RH_0)
- **Fail to** reject the null hypothesis. ($FTRH_0$)
- We will **never** say that we accept the null hypothesis.

Type I and Type II Errors

- If we reject H_0 when it is true this is a **Type I error**.
- If we do not reject H_0 when it is false, this is a **Type II error**.
- In the example of the mean body temperature:
  - Type I error: We would conclude that the mean body temperature is less than 98.6 degrees, when in fact it truly is 98.6.
  - Type II error: We would conclude that the mean body temperature is not less than 98.6 degrees, when in fact the mean body temperature is less than 98.6.

Test Statistic

- A value calculated based on sample data and the type of distribution.
- Test Statistic is used to measure the difference between the data and what is expected based on the null hypothesis.
- For a hypothesis about the population mean if σ is known.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

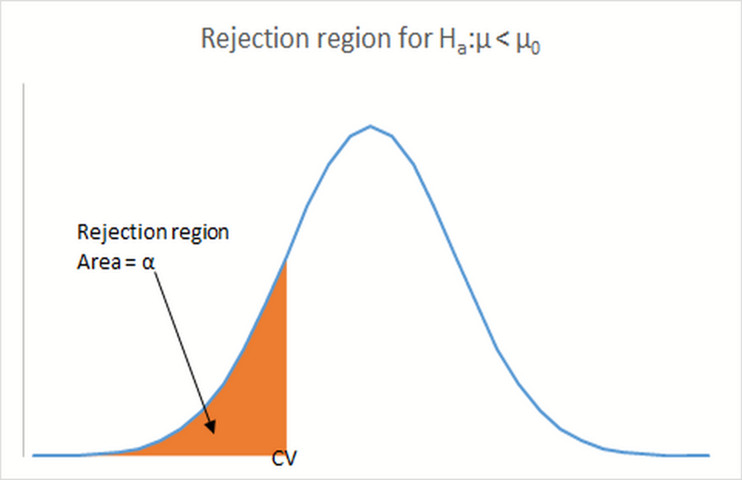
- For a hypothesis about the population mean if σ is not known.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

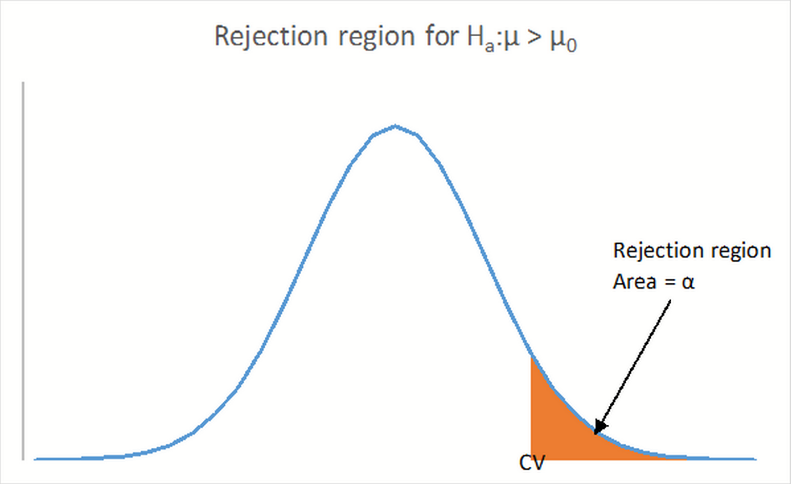
Rejection Region

- A **rejection region** is the set of values for which the test statistic leads to a rejection of the null hypothesis.
- The critical value is the boundary of the rejection region, based on the alternative hypothesis and the level of significance, α.

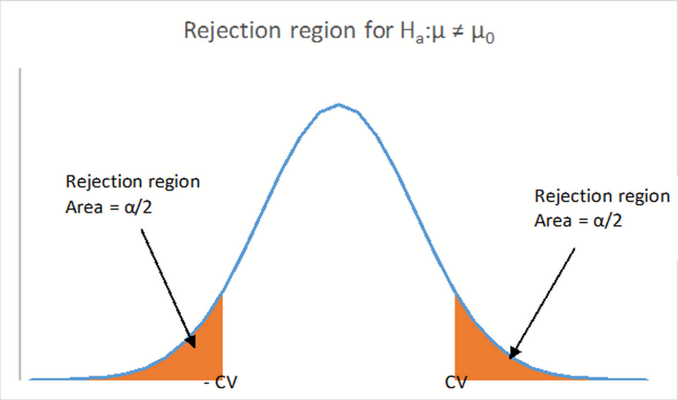
Rejection Region for $H_a : \mu < \mu_0$



Rejection Region for $H_a : \mu > \mu_0$



Rejection Region for $H_a : \mu \neq \mu_0$



Decision of the test using p-values

- One approach is to announce how much evidence **against** H_0 we will require to reject H_0. We compare the P-value with a level that says "this evidence is strong enough." This decisive level is called the **significance level** denoted by α. This significance level is given. If not we will assume $\alpha = 0.05$. When we compare the P-value to α, we have to choose from these two decisions.
- **Reject** H_0 if the P-value is as small or smaller than α. Thus we say that the data are statistically significant at level α.
- **Do not reject** H_0 if the P-value is larger than α.

p-value

- The probability (assuming that H_0 is true) that the test test statistic would take a value as extreme or more extreme (in the way of H_a) than that actually observed.
- The smaller the P-value, the stronger the evidence **against** H_0 provided by the data.
- To calculate the P-value for the mean we will use the sampling distribution of the means which by the central limit theorem is the Normal distribution.
- Hence, this probability is the same as the area under a normal curve depending on the alternative hypothesis.

Determining p-values

- If $H_a : \mu < \mu_0$, then p-value = P(Z $<$ test statistic).
- If $H_a : \mu > \mu_0$, then p-value = P(Z $>$ test statistic).
- If $H_a : \mu \neq \mu_0$, then

$$\begin{aligned} \text{p-value} &= P(Z < -|\text{test statistic}| \text{ or } Z > +|\text{test statistic}|) \\ &= P(Z < -|\text{test statistic}|) + P(Z > +|\text{test statistic}|) \\ &= 2P(Z > |\text{test statistic}|). \end{aligned}$$

- If we use the t as the test statistic replace Z with T.

Strenght of evidence based on p-value

If the P-value for testing H_0 is less than:

- 0.1 we have **some evidence** that H_0 is false.
- 0.05 we have **strong evidence** that H_0 is false.
- 0.01 we have **very strong evidence** that H_0 is false.
- 0.001 we have **extremely strong evidence** that H_0 is false.

If the P-value is greater than 0.1, we **do not have any evidence** that H_0 is false.

Inference for a Population Proportion

- For these inferences, p_0 represents the given population proportion and the hypothesis will be
  - $H_0 : p = p_0$
  - $H_a : p \neq p_0$ or $p < p_0$ or $p > p_0$
- Conditions:
  1. The sample must be a SRS from the population of interest.
  2. The population must be at least 10 times the size of the sample.
  3. The number of successes and the number of failures must each be at least 10 (both $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$).
- Recall, the statistic used for proportions is: $\hat{p} = \frac{\text{\# of successes}}{\text{\# of observations}} = \frac{x}{n}$.
- For tests involving proportions that meet the above conditions, we will use the z-test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

1. Let p denote the proportion of all Math 3339 students who are women. On some random class day, count the number of students attending your class and the number of them who are women. At a significance level of $\alpha = 0.05$, test the null hypothesis $H_0 : p = 1/2$ against the alternative $H_1 : p < 1/2$. Assume that the students attending your class are a random sample of Math 3339 students.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Suppose $n = 500$, $x = 215$

$$\hat{p} = \frac{215}{500} = \frac{43}{100} = 0.43$$

$$z = \frac{0.43 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{500}}} \approx -3.13$$

$$\text{p-value} = P(Z \leq -3.13)$$

```
[1] -3.130495
> pnorm(.Last.value)
[1] 0.0008725593
```

Using R and RStudio

1. Download R from https://cran.r-project.org/
2. Download RStudio from https://www.rstudio.com/