Digital Image Processing

COSC 6380/4393

Lecture – 6

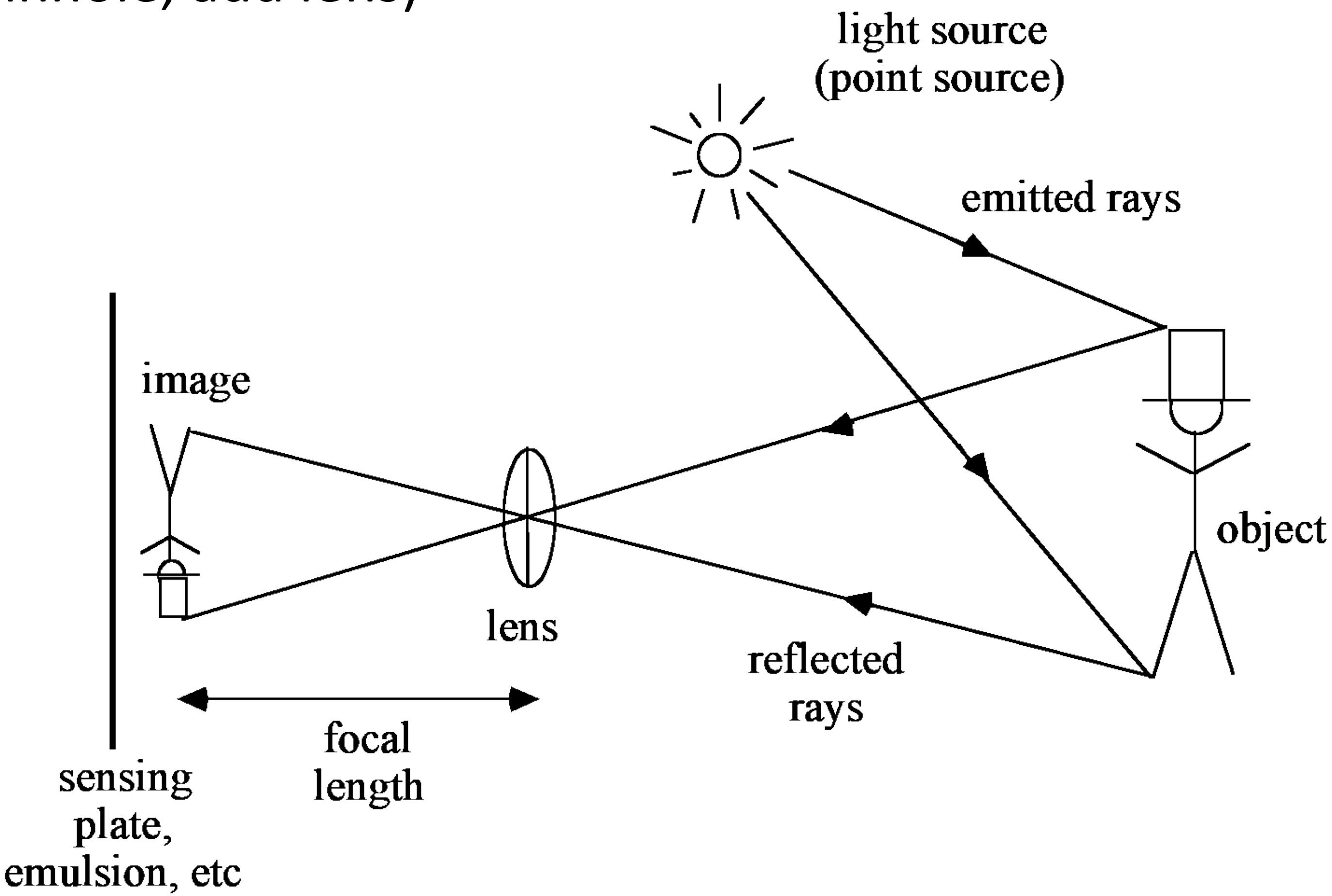
Feb. 2nd, 2023

Pranav Mantini

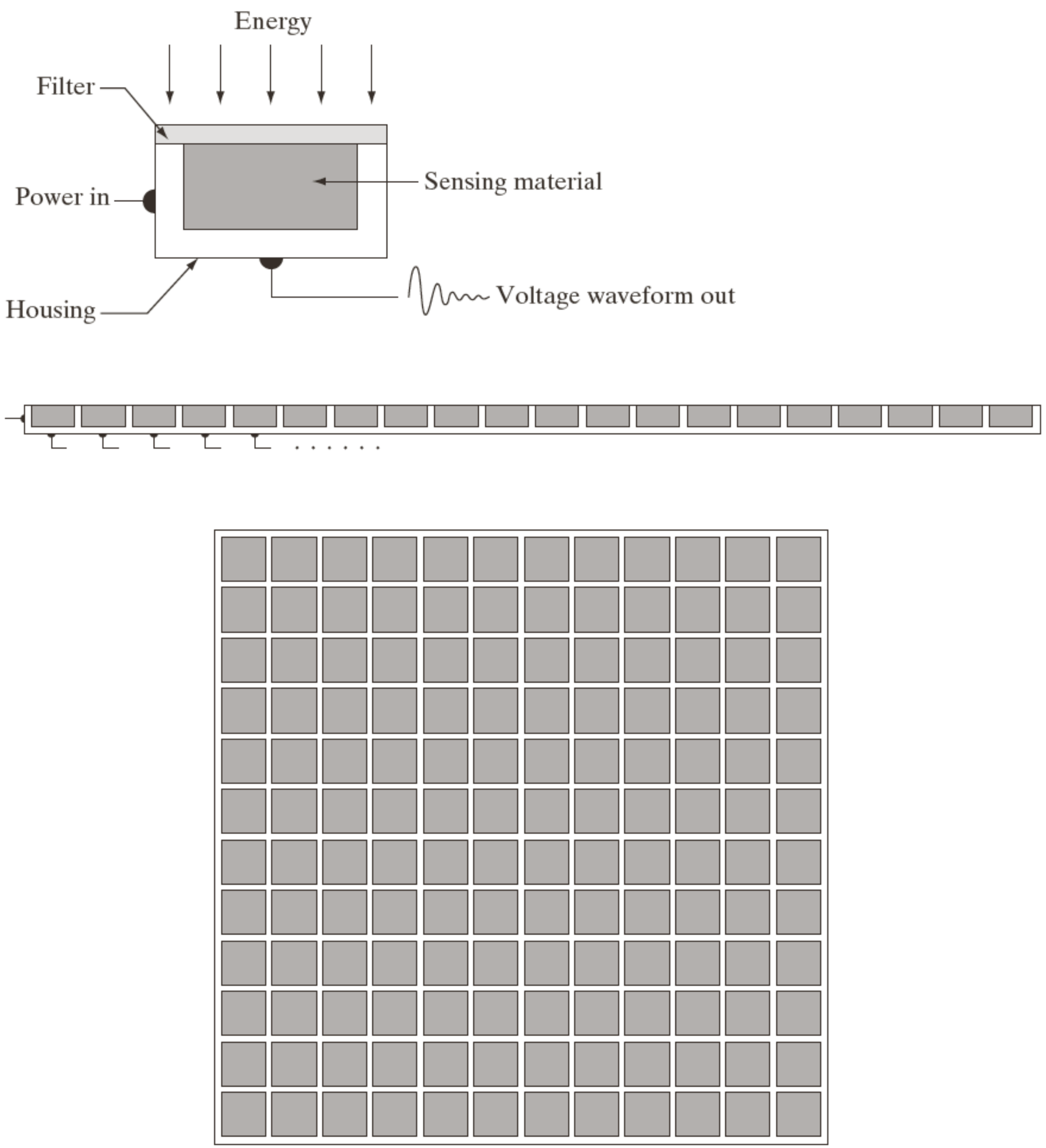
Slides from Dr. Shishir K Shah and Frank (Qingzhong) Liu

Review: IMAGING Formation

- Image formation (pinhole, add lens)
- Image acquisition



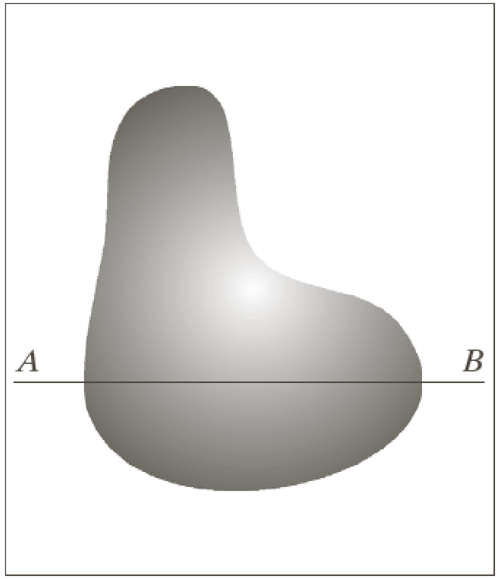
Review: Sensor Response Waveform

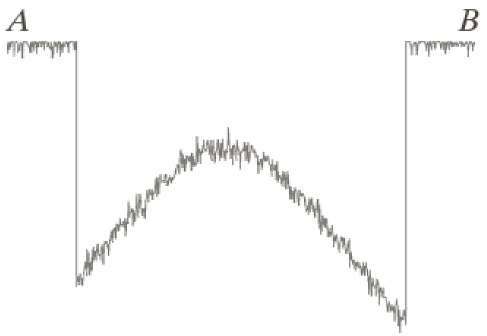


a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

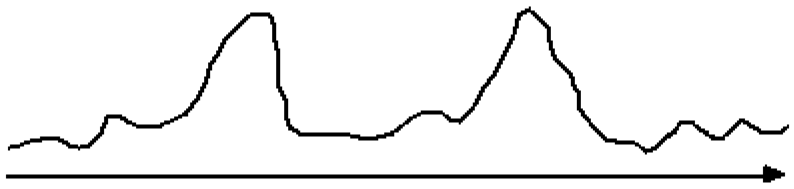
Review: Response from a raster scan





Review: **A / D CONVERSION**

- For computer processing, the analog image must undergo **ANALOG / DIGITAL (A/D) CONVERSION - **Consists of **sampling** and **quantization**



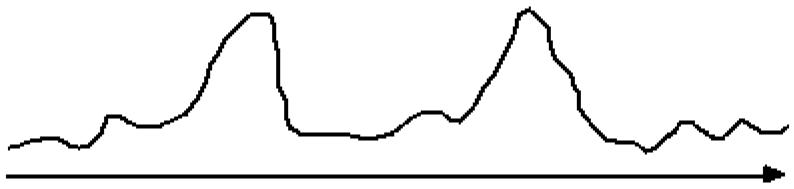
continuous electrical signal from one scanline

Review: **A / D CONVERSION**

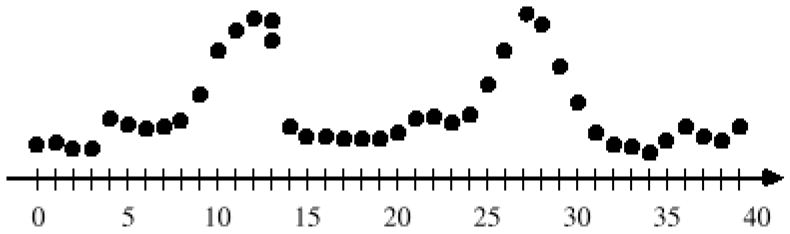
- For computer processing, the analog image must undergo **ANALOG / DIGITAL (A/D) CONVERSION** **-** Consists of **sampling** and **quantization**

Sampling

- Each video **raster** is converted from a **continuous voltage waveform** into a sequence of **voltage samples**:



continuous electrical signal from one scanline



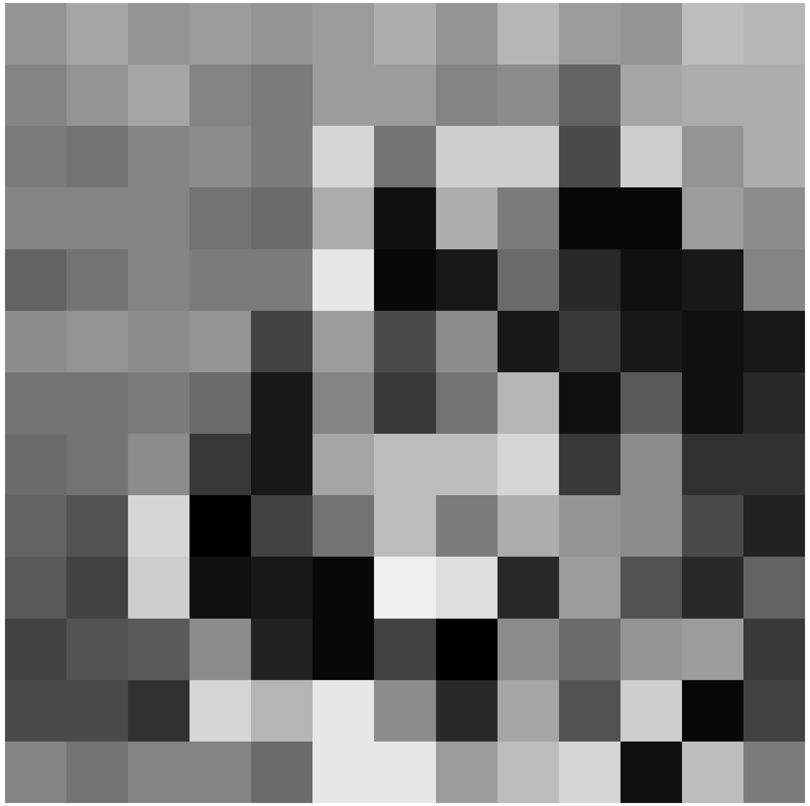
sampled electrical signal from one scanline indexed by discrete (integer) numbers

Spatial and Intensity Resolution

- Spatial resolution
  - — A measure of the smallest discernible detail in an image
  - — stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

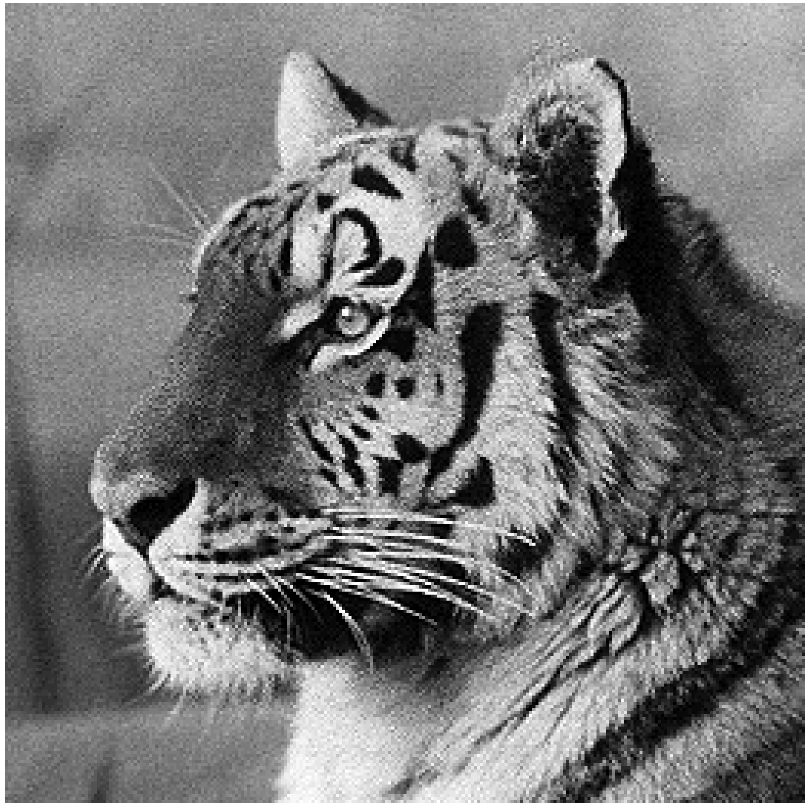
- Intensity resolution
  - — The smallest discernible change in intensity level
  - — stated with *8 bits, 12 bits, 16 bits, etc.*

Review: Sampling: Example



169 Samples

VS

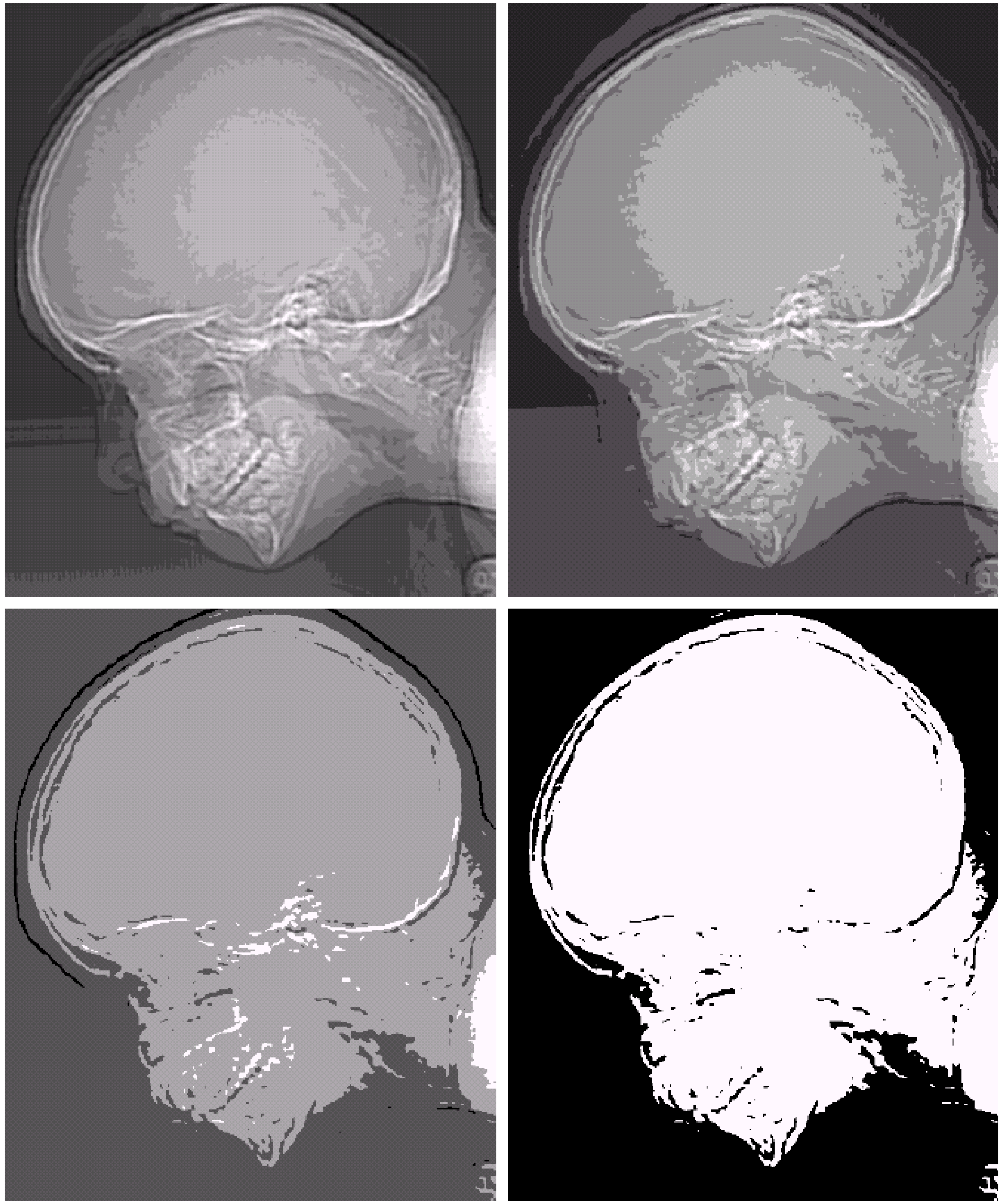


67,600 Samples

Review: Quantization

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Resampling

- Once the image is acquired.
- How to
  - Enlarge an image
  - Shrink an image
  - Zoom in
- Zooming Example:
  - Initial image size = 500 X 500
  - Required image size (= **X** 1.5) = 750 X 750

Geometric Transformation

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

Geometric Transformation

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

Geometric transformation for mapping pixels.

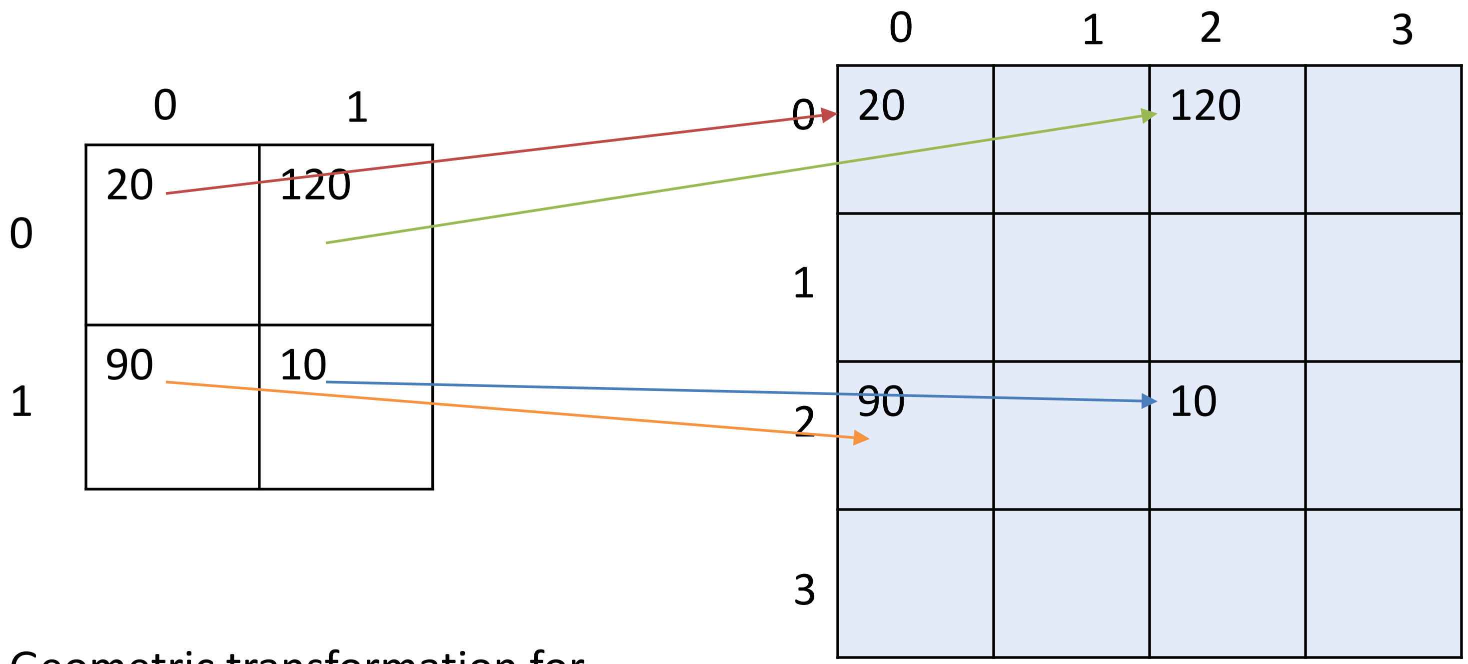
(0,0) X 2 → (0,0)

(0,1) X 2 → (0,2)

(1,0) X 2 → (2,0)

(1,1) X 2 → (2,2)

Geometric Transformation



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(0,0) X 2 → (0,0)

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Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

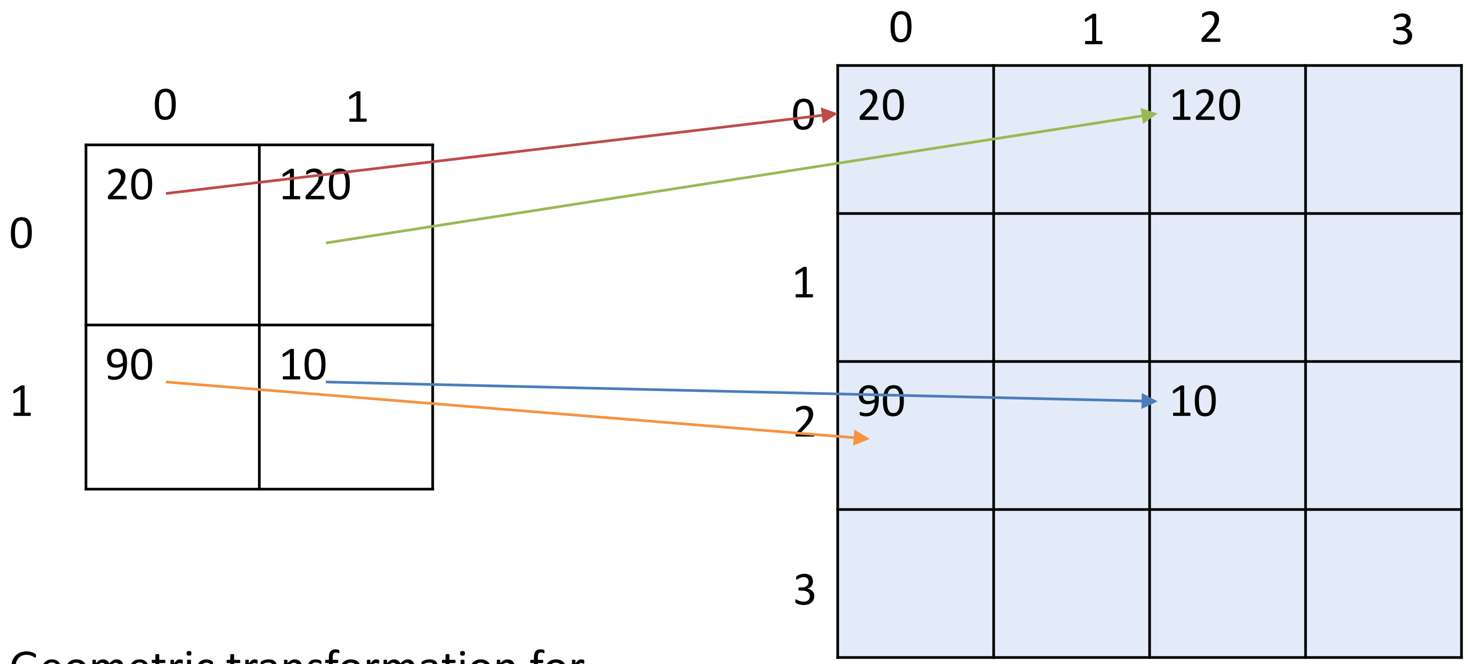
e.g., zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

http://www.dpreview.com/learn/?/key=interpolation

Geometric Transformation



Geometric transformation for mapping pixels.
(0,0) X 2 → (0,0)
(0,1) X 2 → (0,2)
(1,0) X 2 → (2,0)
(1,1) X 2 → (2,2)

It is difficult to interpolate and fill missing value when applying forward geometric transformation.

Geometric Transformation: Inverse lookup

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

1. Create an image of desired size

Geometric Transformation: Inverse lookup

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image

Geometric Transformation: Inverse lookup

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

Inverse mapping pixels.
(0,0) X 1/2 → (0,0)
(0,1) X 1/2 → (0,0.5)
(0,2) X 1/2 → (0,1)
(0,3) X 1/2 → (0,1.5)
…

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image.

Geometric Transformation: Inverse lookup

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0	20	120
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(0,1) X 1/2 → (0,0.5)
(0,2) X 1/2 → (0,1)
(0,3) X 1/2 → (0,1.5)
…

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image.
3. Use values from nearby pixel to guess missing values

Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  e.g., zooming, shrinking, rotating, and geometric correction

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  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

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Nearest Neighbor

	0	1
0	20	120
1	90	10

	0	1	2	3
0				
1				
2				
3				

Inverse mapping pixels.

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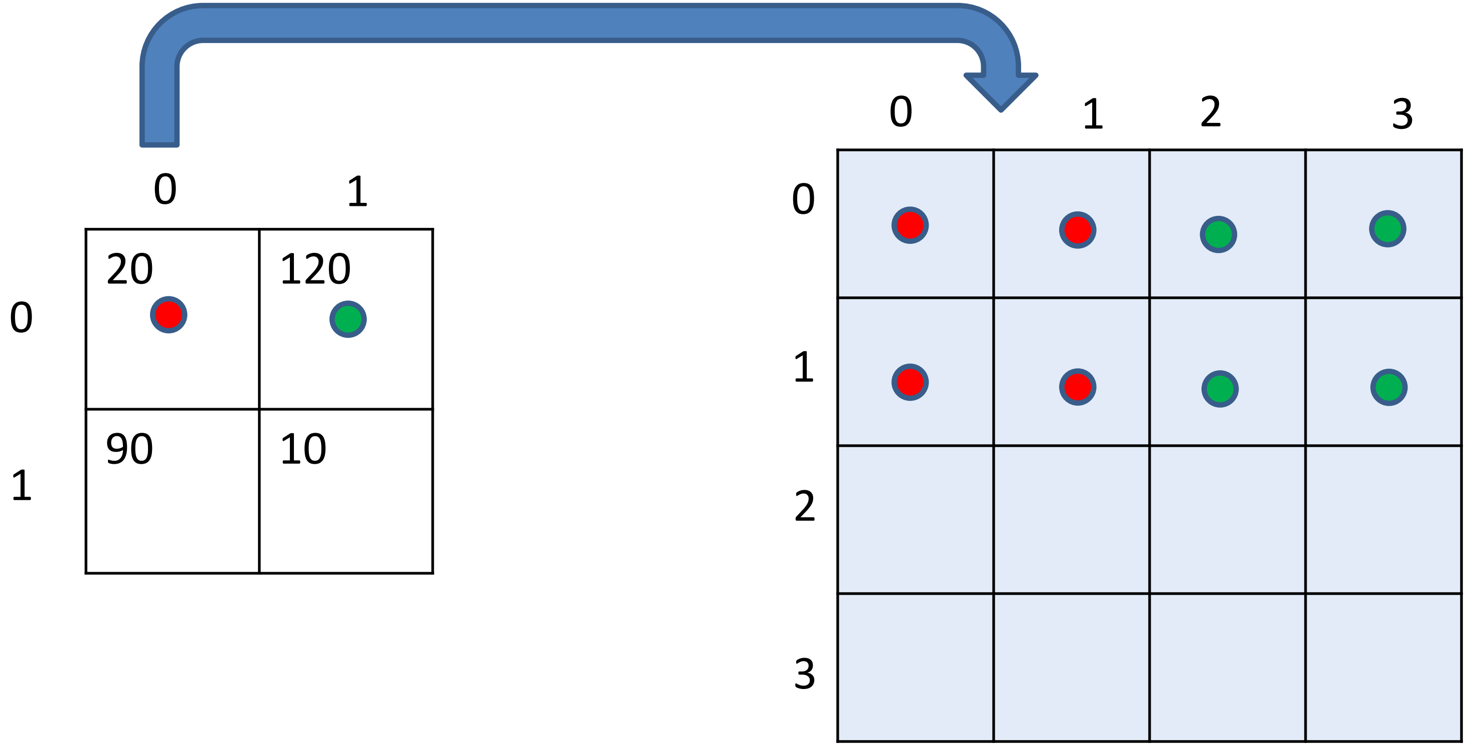
(0,1) X 1/2 → (0,0.5)

(0,2) X 1/2 → (0,1)

(0,3) X 1/2 → (0,1.5)

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image.
3. Use nearest pixel values

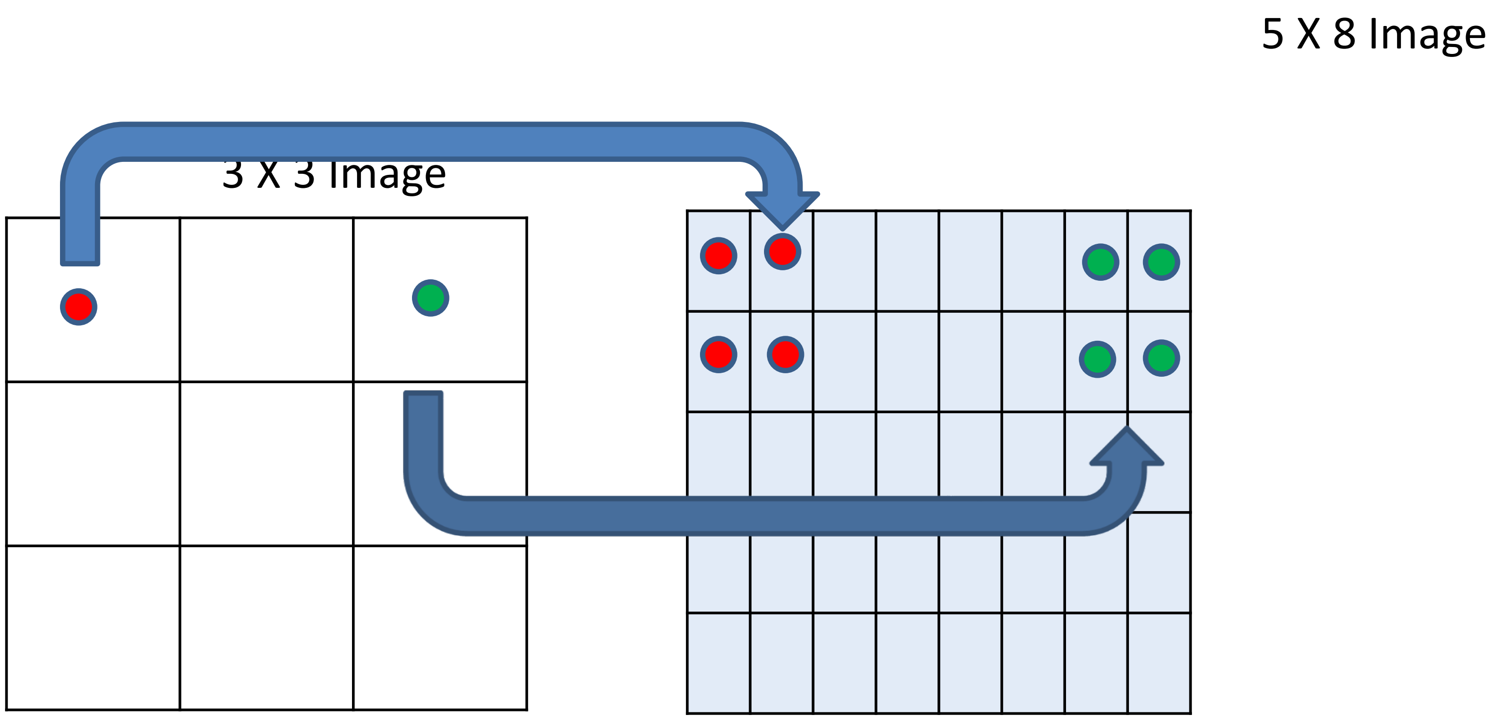
Nearest Neighbor



Inverse mapping pixels.
(0,0) X 1/2 → (0,0)
(0,1) X 1/2 → (0,0.5)
(0,2) X 1/2 → (0,1)
(0,3) X 1/2 → (0,1.5)

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image.
3. Use **nearest pixel** values for missing values

Interpolation: Nearest Neighbor



Fill in values preserving
spatial relationship

Interpolation: Nearest neighbor

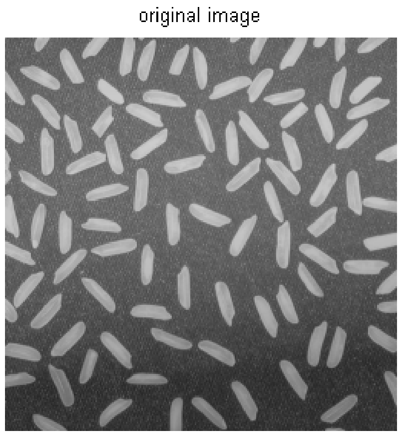
Original Image

Original

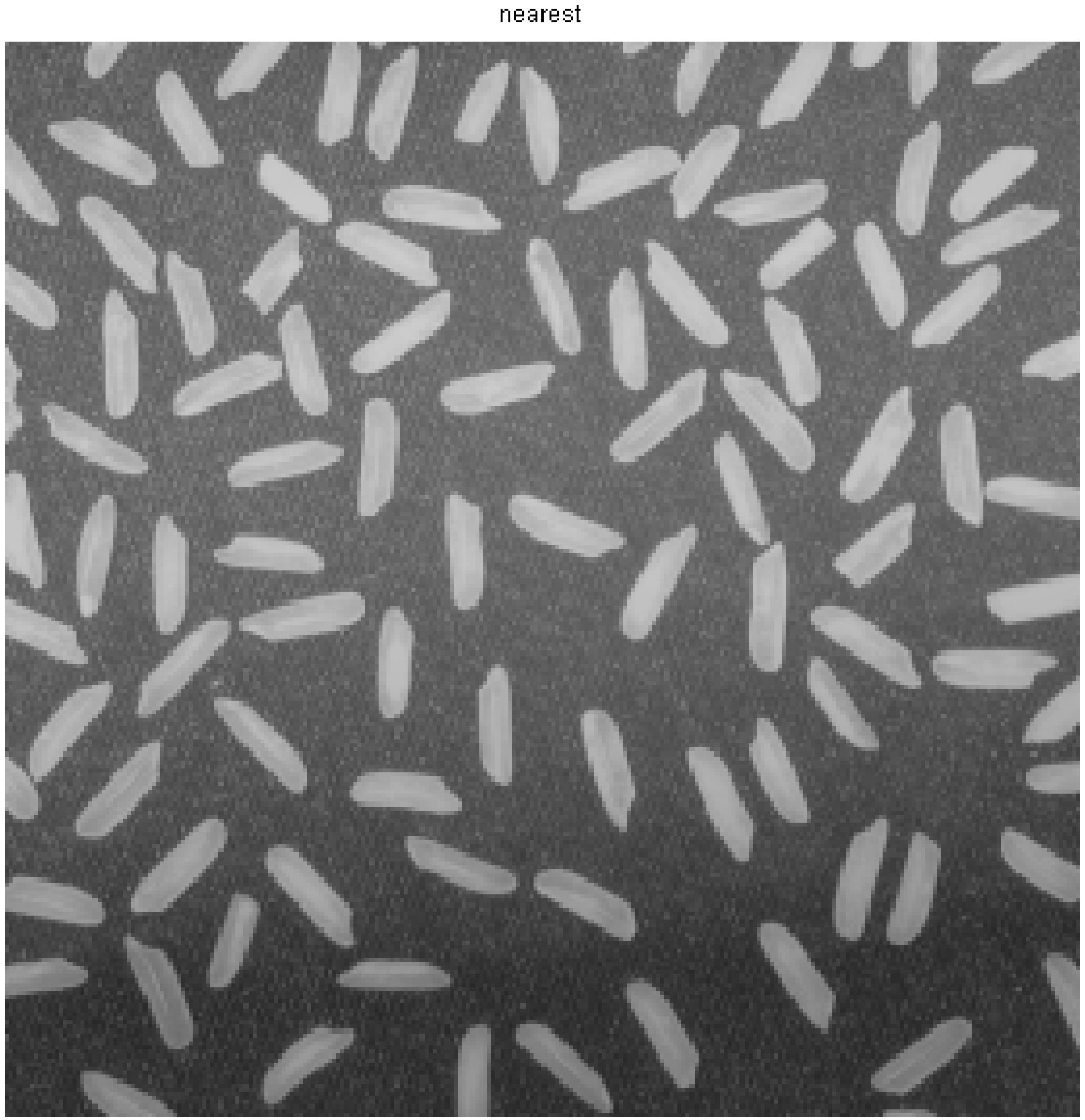
Nearest Neighbor Interpolation

Zoom

Interpolation: Nearest neighbor



Original



Zoom

Interpolation (1D)

- Known points $x_1 \, and \, x_2$ with values
- $function \; f \to \mathbb{R}$
- $f(x_1) = I_1$and $f(x_2) = I_2$
- How to find the value I at point x

Linear Interpolation

- Underlying assumption: f is linear

Linear Interpolation

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$$f(z) = az + b$$

Linear Interpolation

- Underlying assumption: f is linear

$$f(z) = az + b$$

$$f(x_1) = ax_1 + b$$

$$f(x_2) = ax_2 + b$$

Linear Interpolation

- Underlying assumption: f is linear

$$f(z) = az + b$$

$$f(x_1) = ax_1 + b$$

$$f(x_2) = ax_2 + b$$

$$f(x_2) - f(x_1) = (ax_2 + b) - (ax_1 + b)$$

$$I_2 - I_1 = a(x_2 - x_1)$$

$$\Rightarrow I_2 - I_1 \propto (x_2 - x_1)$$

Linear Interpolation

$$I_2 - I_1 \propto (x_2 - x_1)$$

$$I - I_1 \propto ?$$

Linear Interpolation

$$I_2 - I_1 \propto (x_2 - x_1)$$
$$I - I_1 \propto (x - x_1)$$

Linear Interpolation

$$I_2 - I_1 \propto (x_2 - x_1)$$
$$I - I_1 \propto (x - x_1)$$

Dividing them,

$$\frac{I_2 - I_1}{I - I_1} = \frac{(x_2 - x_1)}{(x - x_1)}$$

Linear Interpolation

Solve for I

$$\frac{I_2 - I_1}{I - I_1} = \frac{(x_2 - x_1)}{(x - x_1)}$$

$$(I_2 - I_1)\left(\frac{(x - x_1)}{(x_2 - x_1)}\right) = I - I_1$$

$$I = I_1 + (I_2 - I_1)\left(\frac{(x - x_1)}{(x_2 - x_1)}\right)$$

Linear Interpolation

Solve for I

$$\frac{I_2 - I_1}{I - I_1} = \frac{(x_2 - x_1)}{(x - x_1)}$$

$$(I_2 - I_1)\left(\frac{(x - x_1)}{(x_2 - x_1)}\right) = I - I_1$$

$$I = I_1 + \boxed{(I_2 - I_1)\left(\frac{(x - x_1)}{(x_2 - x_1)}\right)}$$

Linear Interpolation

Solve for I

$$I = \frac{I_1(x_2 - x_1) + (I_2 - I_1)(x - x_1)}{(x_2 - x_1)}$$

$$I = \frac{I_1(x_2 - x) + I_2(x - x_1)}{(x_2 - x_1)}$$

$$I = \frac{I_1(x_2 - x)}{(x_2 - x_1)} + \frac{I_2(x - x_1)}{(x_2 - x_1)}$$

Example: Linear Interpolation

Solve for I

Example: Linear Interpolation

Solve for I

$$I = \frac{I_1(x_2 - x)}{(x_2 - x_1)} + \frac{I_2(x - x_1)}{(x_2 - x_1)}$$

Example: Linear Interpolation

Solve for I

$$I = \frac{I_1(x_2 - x)}{(x_2 - x_1)} + \frac{I_2(x - x_1)}{(x_2 - x_1)}$$

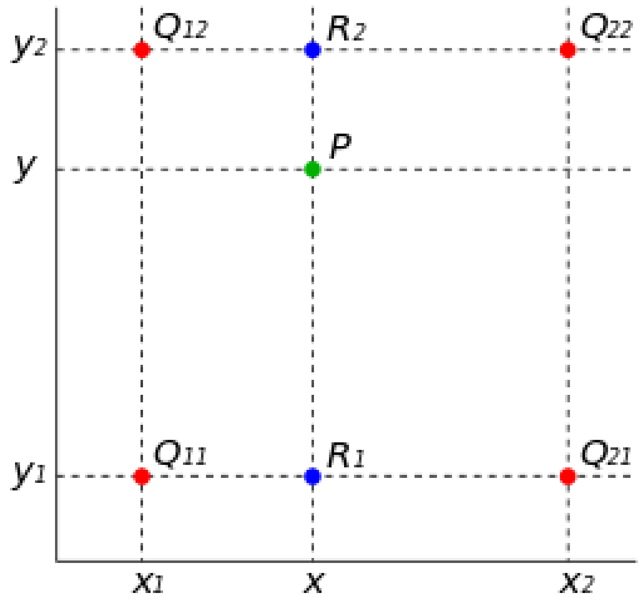
$$I = \frac{10(1 - 0.3)}{(1 - 0)} + \frac{15(0.3 - 0)}{(1 - 0)}$$

$$I = 7 + 4.5 = 11.5$$

Bi-Linear Interpolation(2D)

$Q_{11} = (x_1, y_1)$,
$Q_{12} = (x_1, y_2)$,
$Q_{21} = (x_2, y_1)$,
and $Q_{22} = (x_2, y_2)$
$f(Q_i) \rightarrow intensity\ at\ Q_i$

Find the value at P



https://en.wikipedia.org/wiki/Bilinear_interpolation

Bi-Linear Interpolation(2D)

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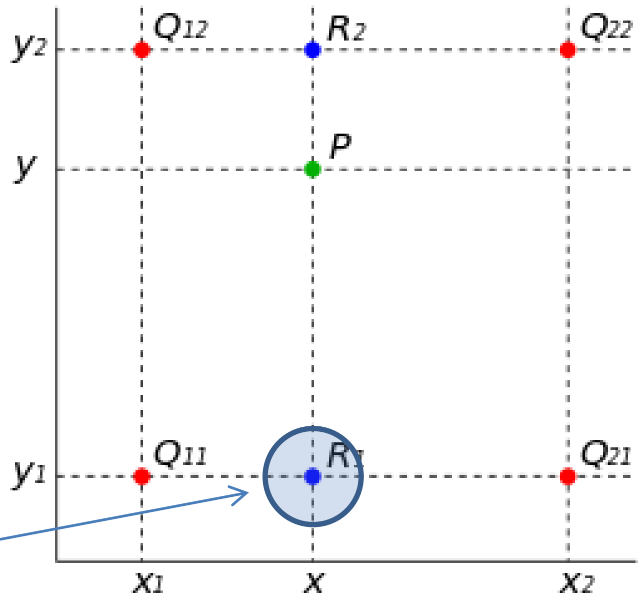

$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$

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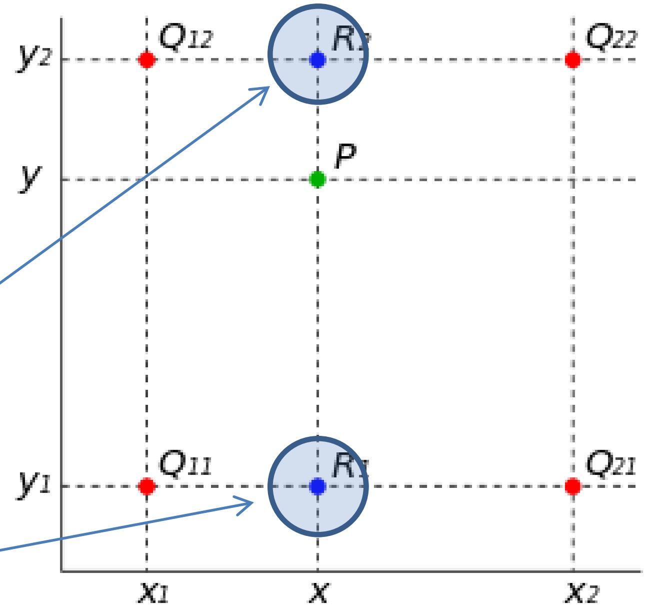
Find the value at P



$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$
$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$

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Bi-Linear Interpolation(2D)

$Q_{11} = (x_1, y_1)$,
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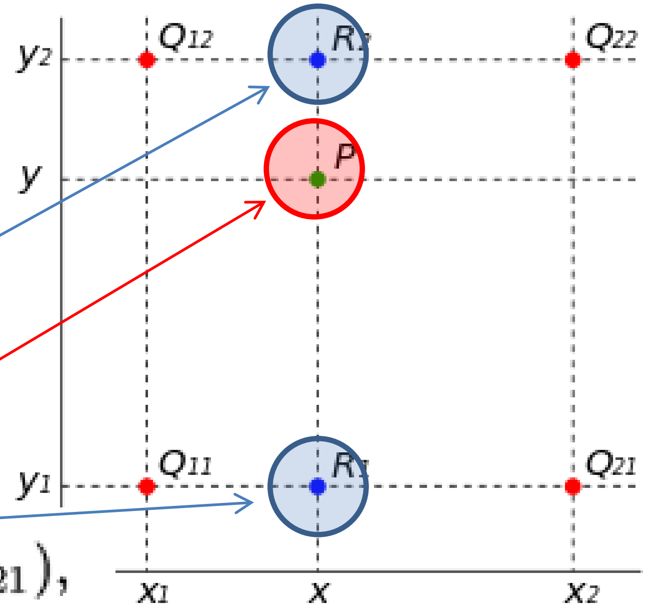
Find the value at P



$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$

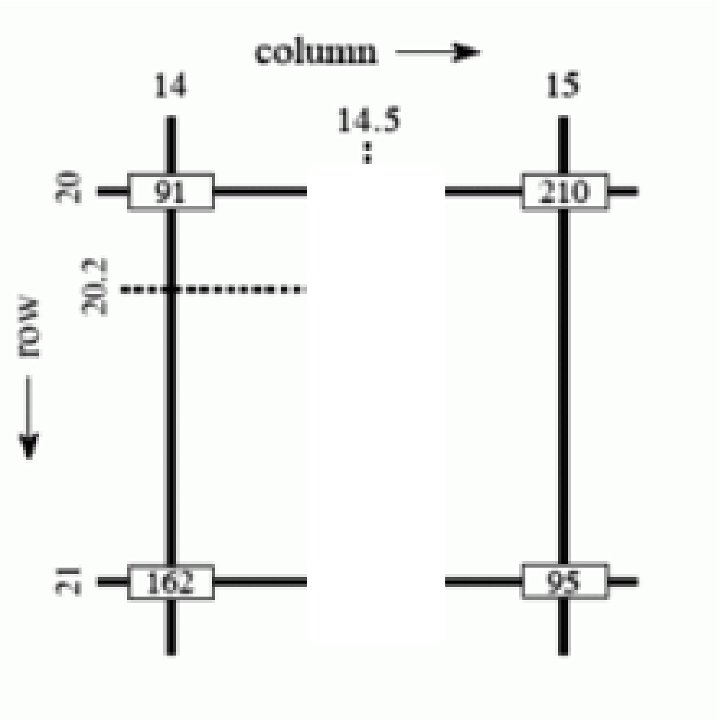
$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2)$$

https://en.wikipedia.org/wiki/Bilinear_interpolation

Example

$I(21,14) = 162,$
$I(21,15) = 95,$
$I(20,14) = 91,$
$I(20,15) = 210$
$I(20.2, 14.5) = ?$



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$I(21,14) = 162,$

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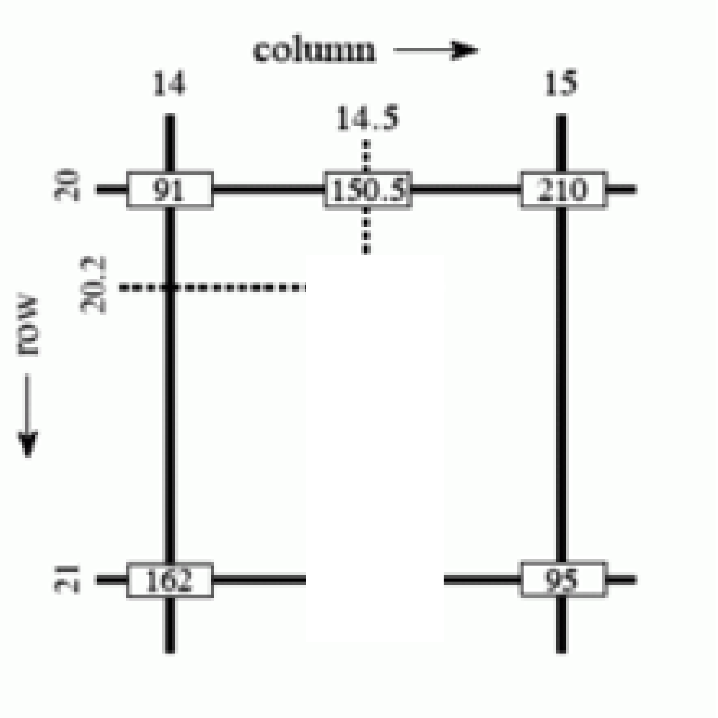
$I(20.2, 14.5) = ?$

$$I_{20,14.5} = \frac{15-14.5}{15-14} \cdot 91 + \frac{14.5-14}{15-14} \cdot 210 = 150.5,$$



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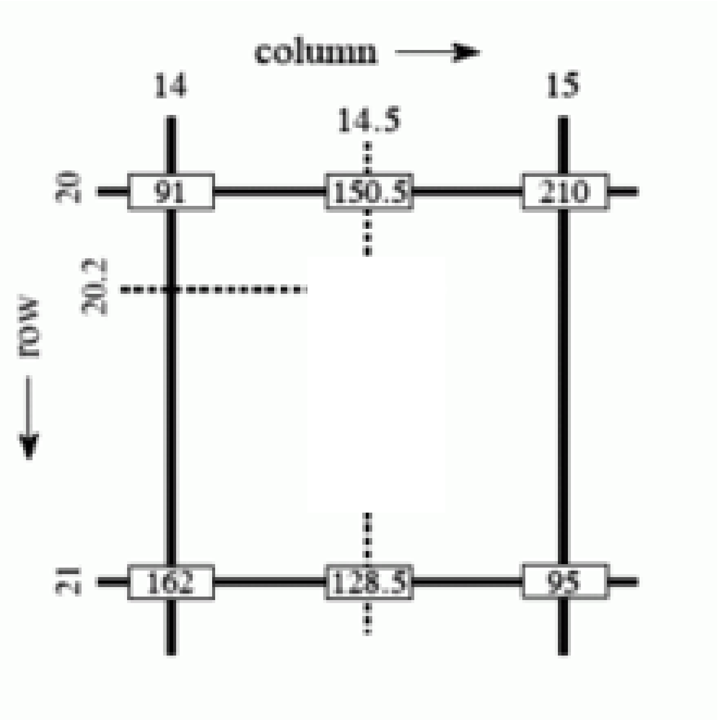
$$I_{20,14.5} = \frac{15-14.5}{15-14} \cdot 91 + \frac{14.5-14}{15-14} \cdot 210 = 150.5,$$
$$I_{21,14.5} = \frac{15-14.5}{15-14} \cdot 162 + \frac{14.5-14}{15-14} \cdot 95 = 128.5,$$



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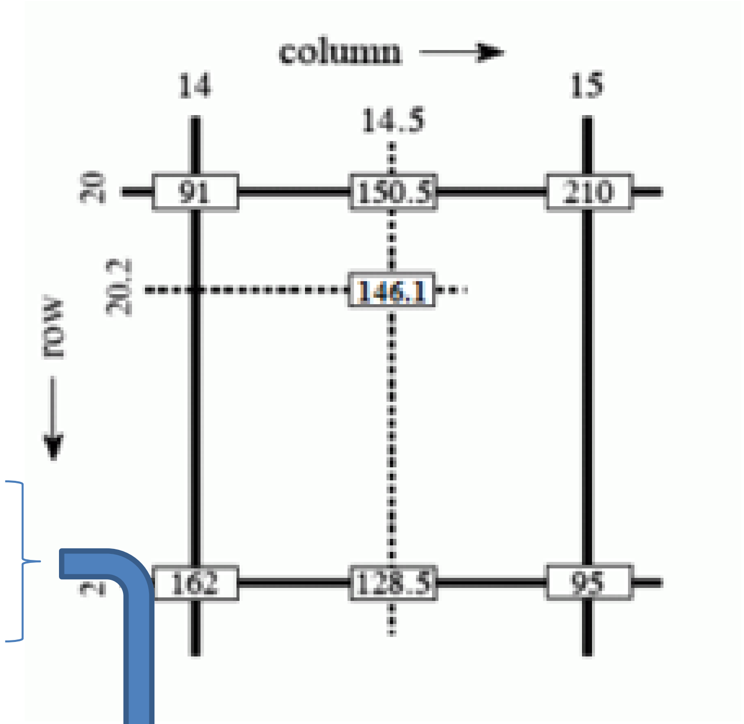
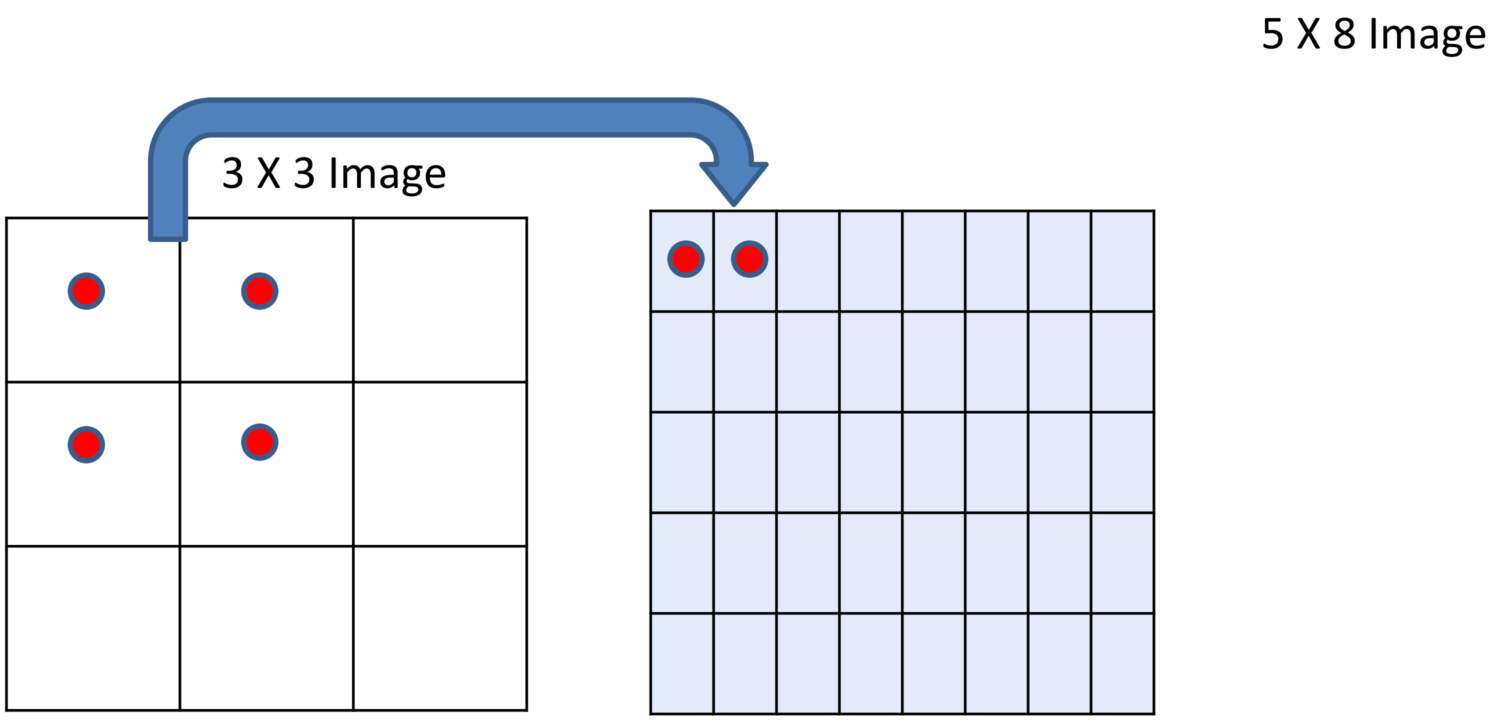


$$I_{20,14.5} = \frac{15-14.5}{15-14} \cdot 91 + \frac{14.5-14}{15-14} \cdot 210 = 150.5,$$
$$I_{21,14.5} = \frac{15-14.5}{15-14} \cdot 162 + \frac{14.5-14}{15-14} \cdot 95 = 128.5,$$

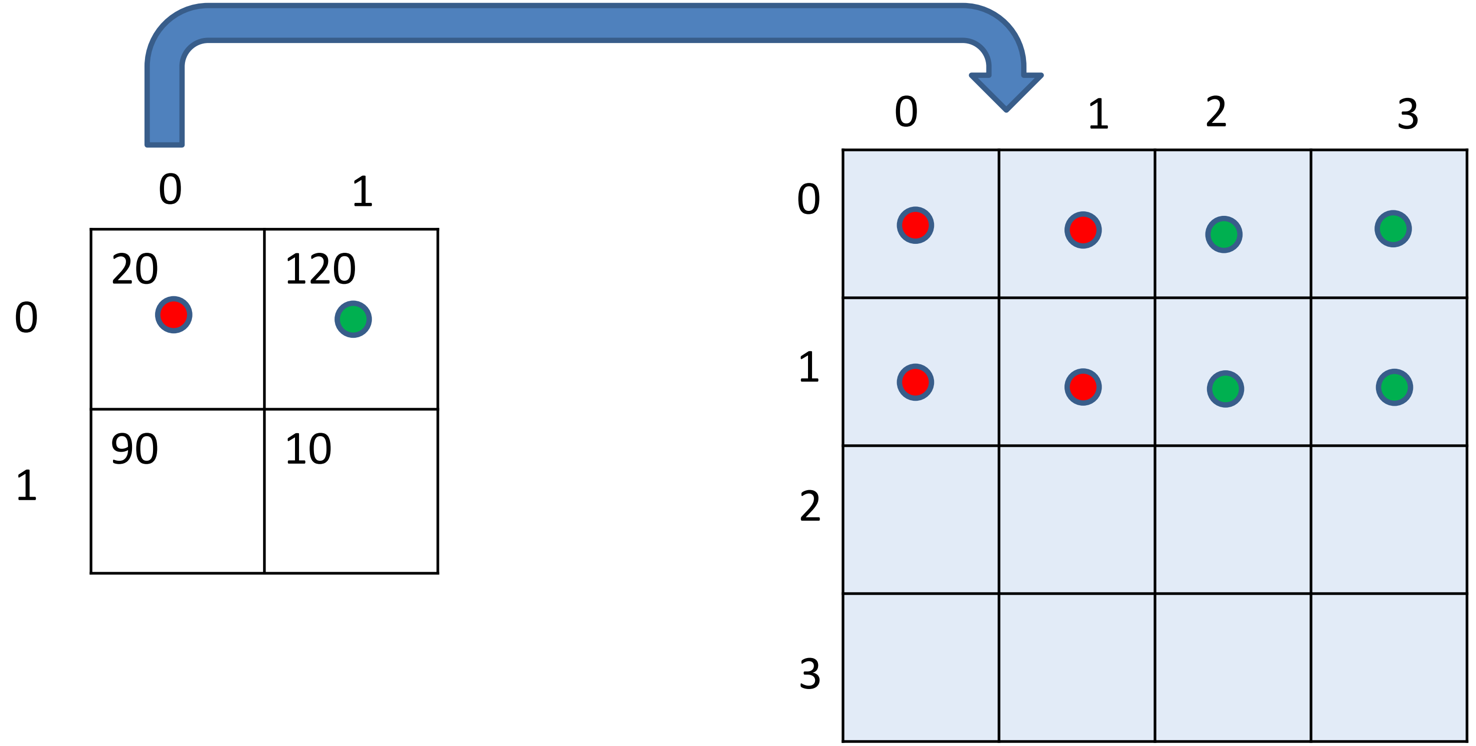
$$I_{20.2,14.5} = \frac{21-20.2}{21-20} \cdot 150.5 + \frac{20.2-20}{21-20} \cdot 128.5 = 146.1.$$

https://en.wikipedia.org/wiki/Bilinear_interpolation

Bilinear Interpolation

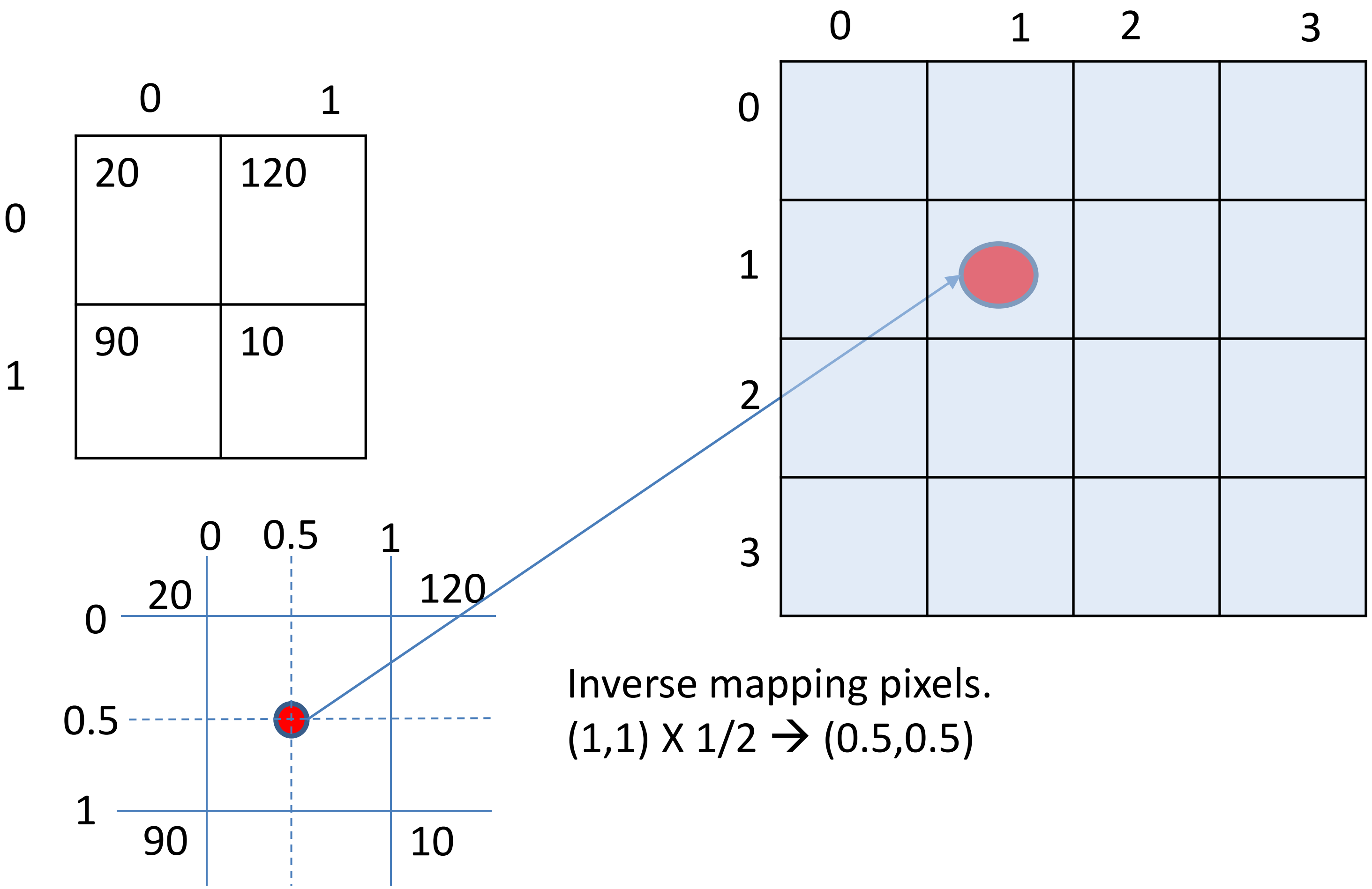


Bilinear Interpolation



Inverse mapping pixels.
(0,0) X 1/2 → (0,0)
(0,1) X 1/2 → (0,0.5)
(0,2) X 1/2 → (0,1)
(0,3) X 1/2 → (0,1.5)
…
…

1. Create an image of desired size
2. For each pixel in the new image calculate which pixel it corresponds to in the original image.
3. Use **four nearest pixel** to perform bi-linear interpolation



Inverse mapping pixels.
(1,1) X 1/2 → (0.5,0.5)

Nearest neighbor Interpolation

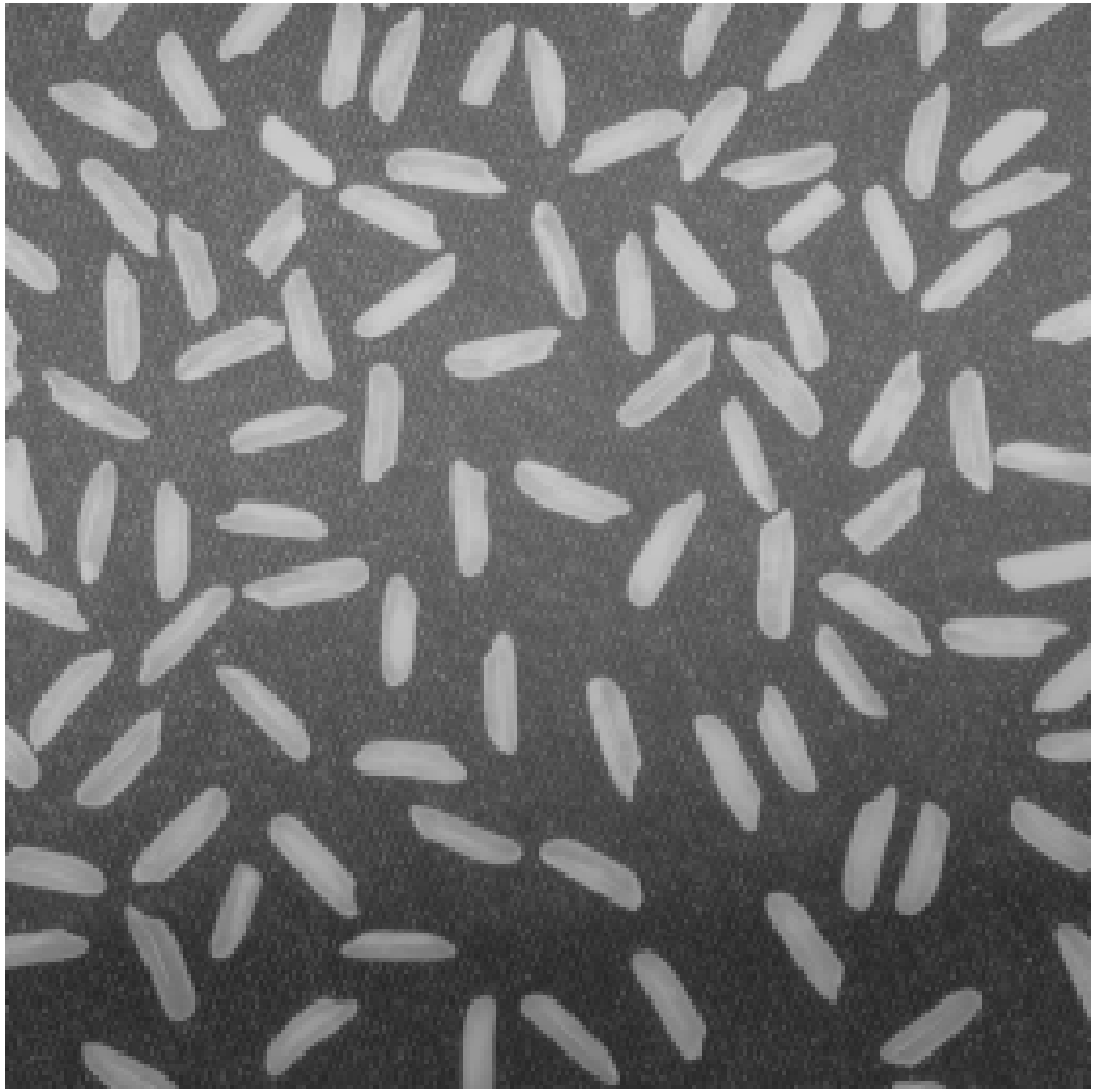
Nearest Neighbor Interpolation

Bilinear Interpolation

Bilinear Interpolation

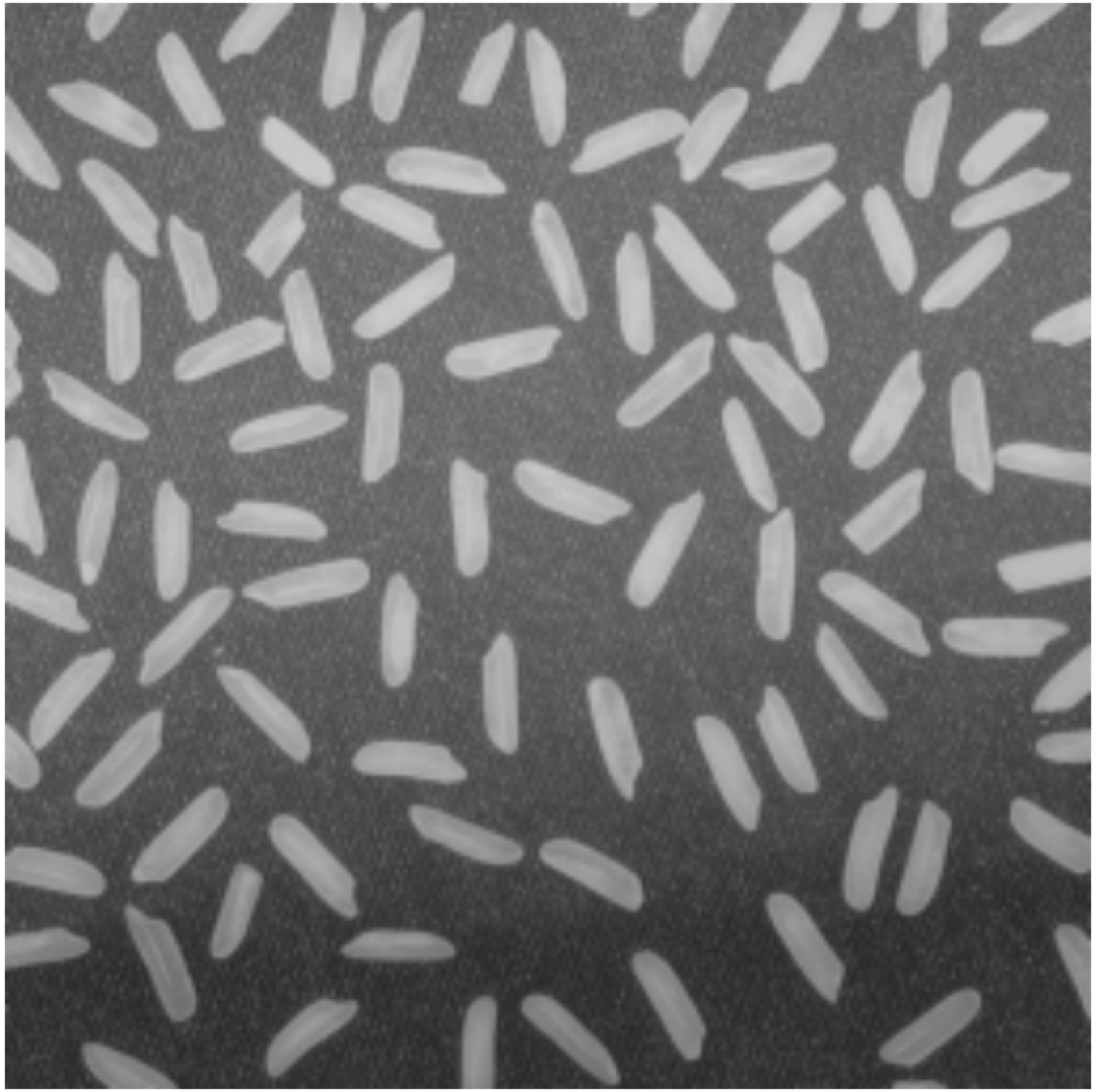
Nearest neighbor Interpolation

nearest



Bilinear Interpolation

bilinear



Bilinear: Alternative algorithm

- An alternative way to write the solution to the interpolation problem is

$$f(x, y) \approx a_0 + a_1 x + a_2 y + a_3 xy,$$

$$\begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ 1 & x_2 & y_1 & x_2 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f(Q_{11}) \\ f(Q_{12}) \\ f(Q_{21}) \\ f(Q_{22}) \end{bmatrix}.$$

- Not linear but quadratic

Image Interpolation:

Bicubic Interpolation

- The intensity value assigned to point (x,y) is obtained by the following equation

$$f_3(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}x^i y^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

Bilinear Interpolation

Bilinear Interpolation

Bicubic Interpolation

Bicubic Interpolation