Digital Image Processing

COSC 6380/4393

Lecture – 4

Jan 26th, 2023

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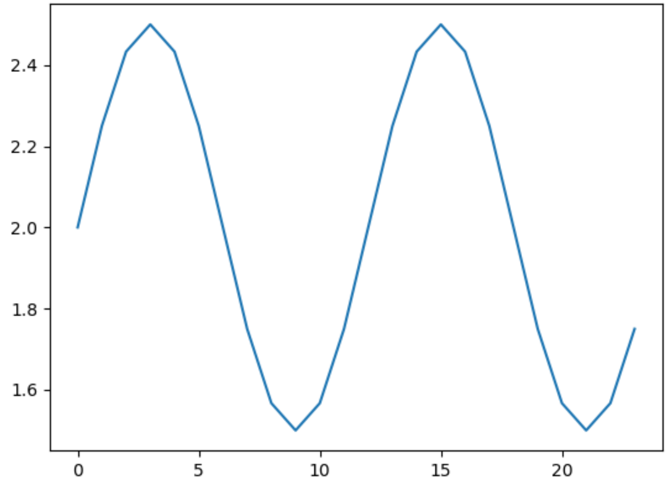
Review: Pre-Introduction

- Example: Measure depth of the water in meters at a certain pier
- Yet another representation
- **Image** as a mode/format to convey information usually for human consumption



Review: WHAT ARE DIGITAL IMAGES?

- **Images** are as variable as the **types of radiation** that exist and the ways in which radiation **interacts** with **matter**:

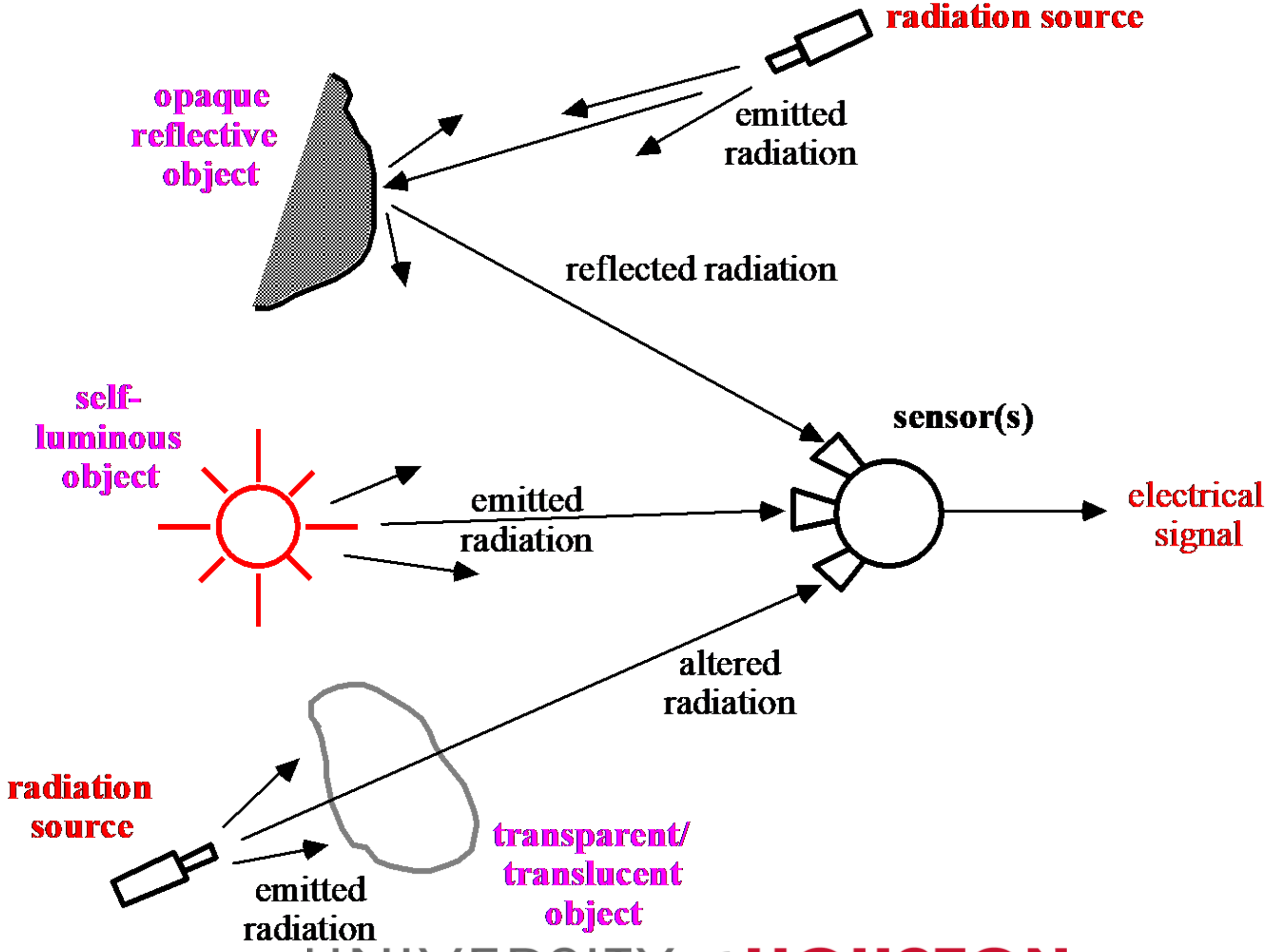
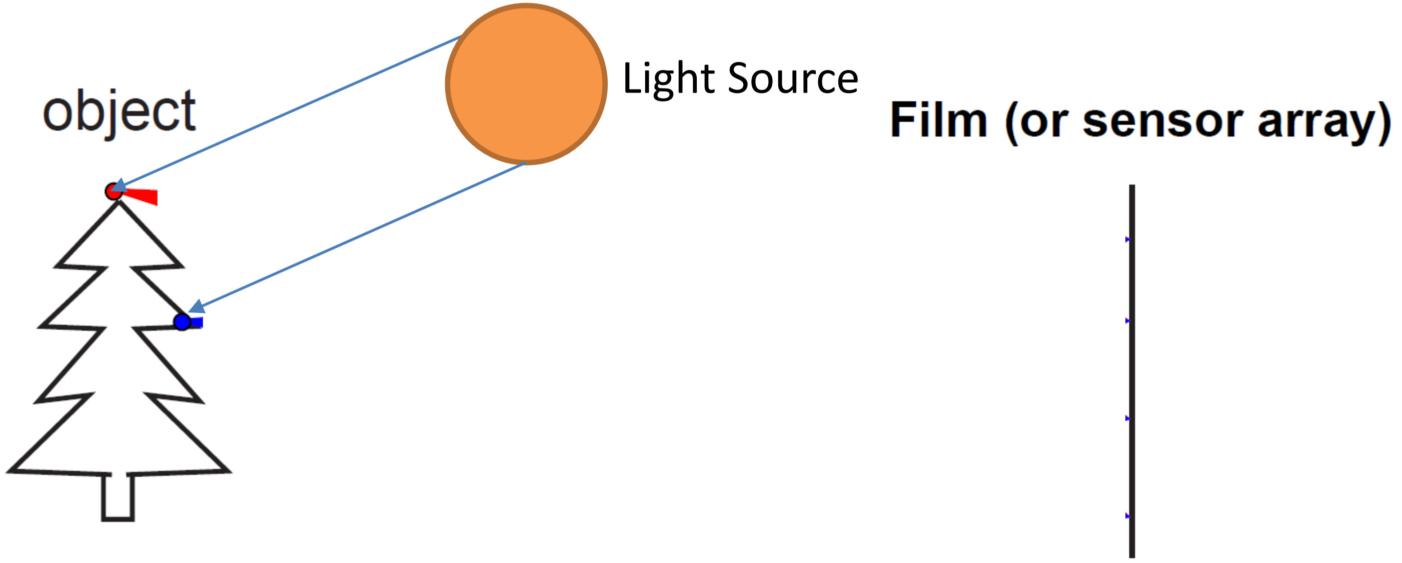


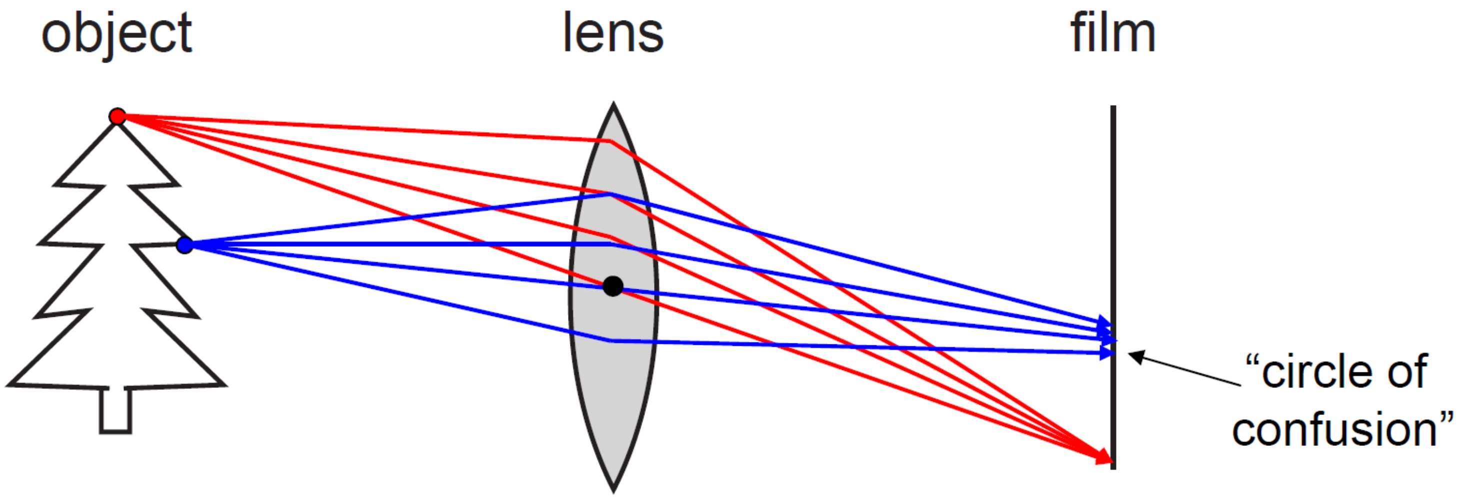
Image formation

- Let's design a method to capture reflection
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

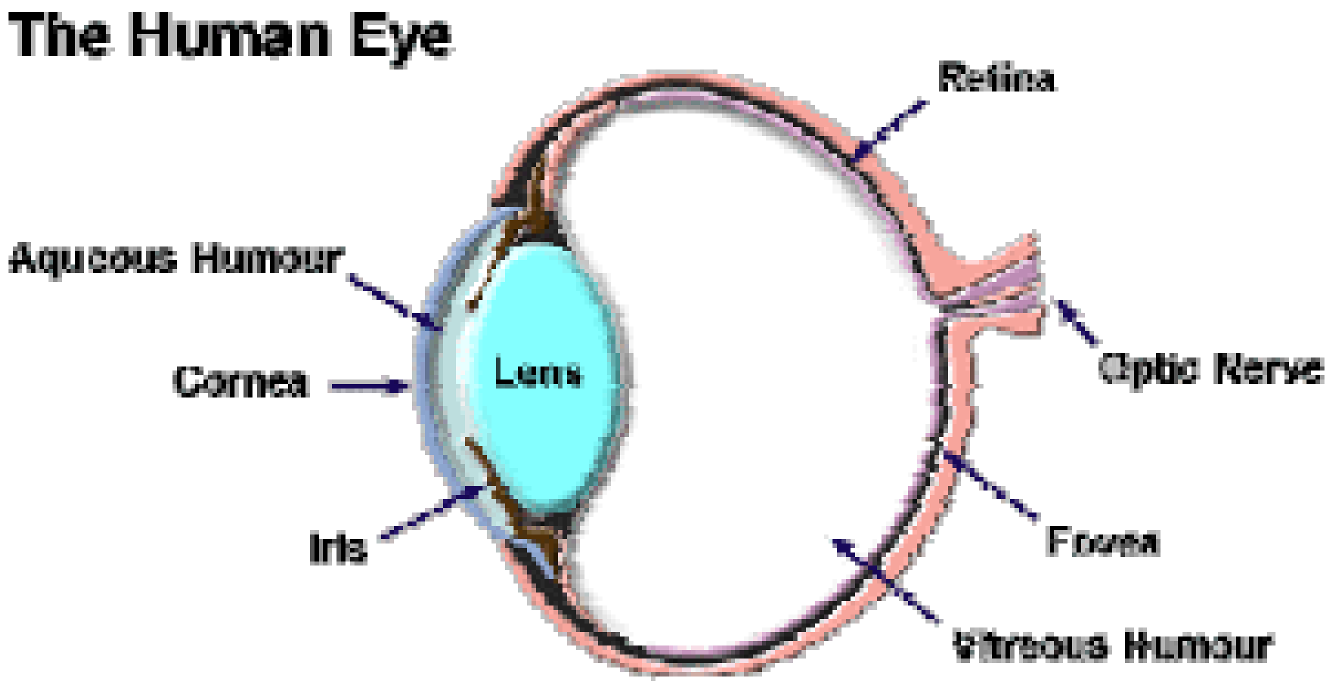


Review: Adding a lens

- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    - other points project to a "circle of confusion" in the image
  - Changing the shape of the lens changes this distance

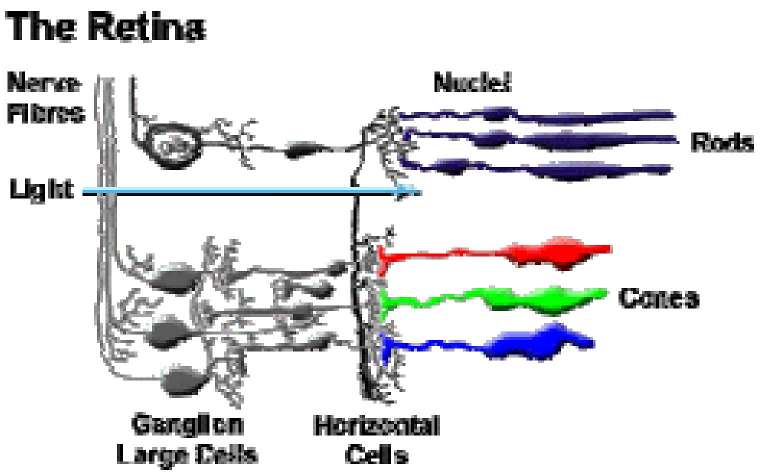


Review: OPTICS OF THE EYE



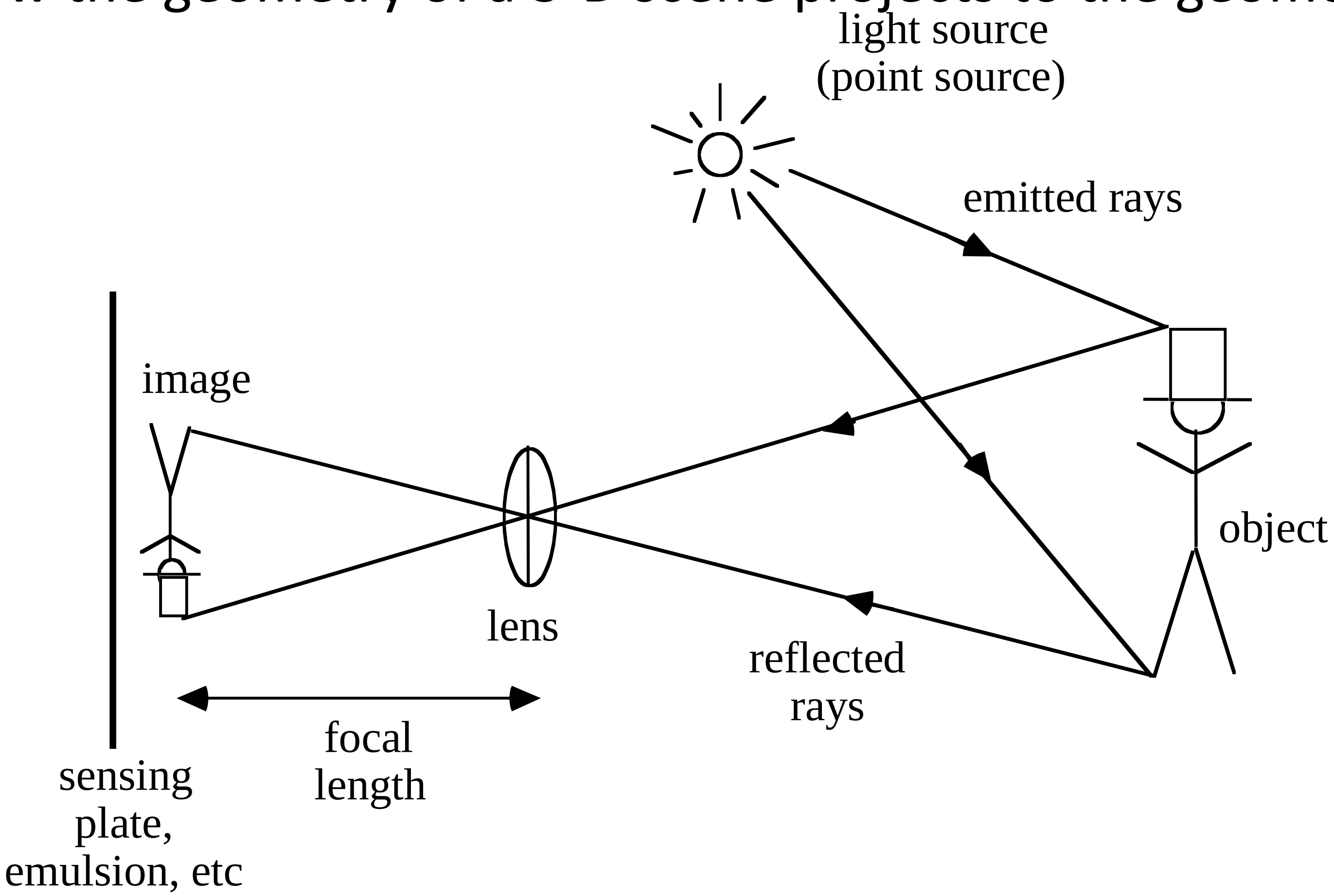
Review: PHOTORECEPTORS

- Rods are 1-2 microns in diameter; the cones are 2-3 microns in diameter in the fovea, but increase in diameter away from the fovea (No rods in the fovea)
- Cones are densely packed in the fovea and quickly decrease in density as a function of eccentricity
- Rods increase in density out to approximately 20 degree eccentricity, beyond which their density begins to decline



OPTICAL IMAGING GEOMETRY

- We will quantify how the geometry of a 3-D scene projects to the geometry of the image intensities:



PERSPECTIVE PROJECTION

- A reduction of dimensionality is projection - in this case perspective projection
- A precise geometric relationship between space (3-D) coordinates and image (2-D) coordinates exists under perspective projection
- We will require some coordinate systems

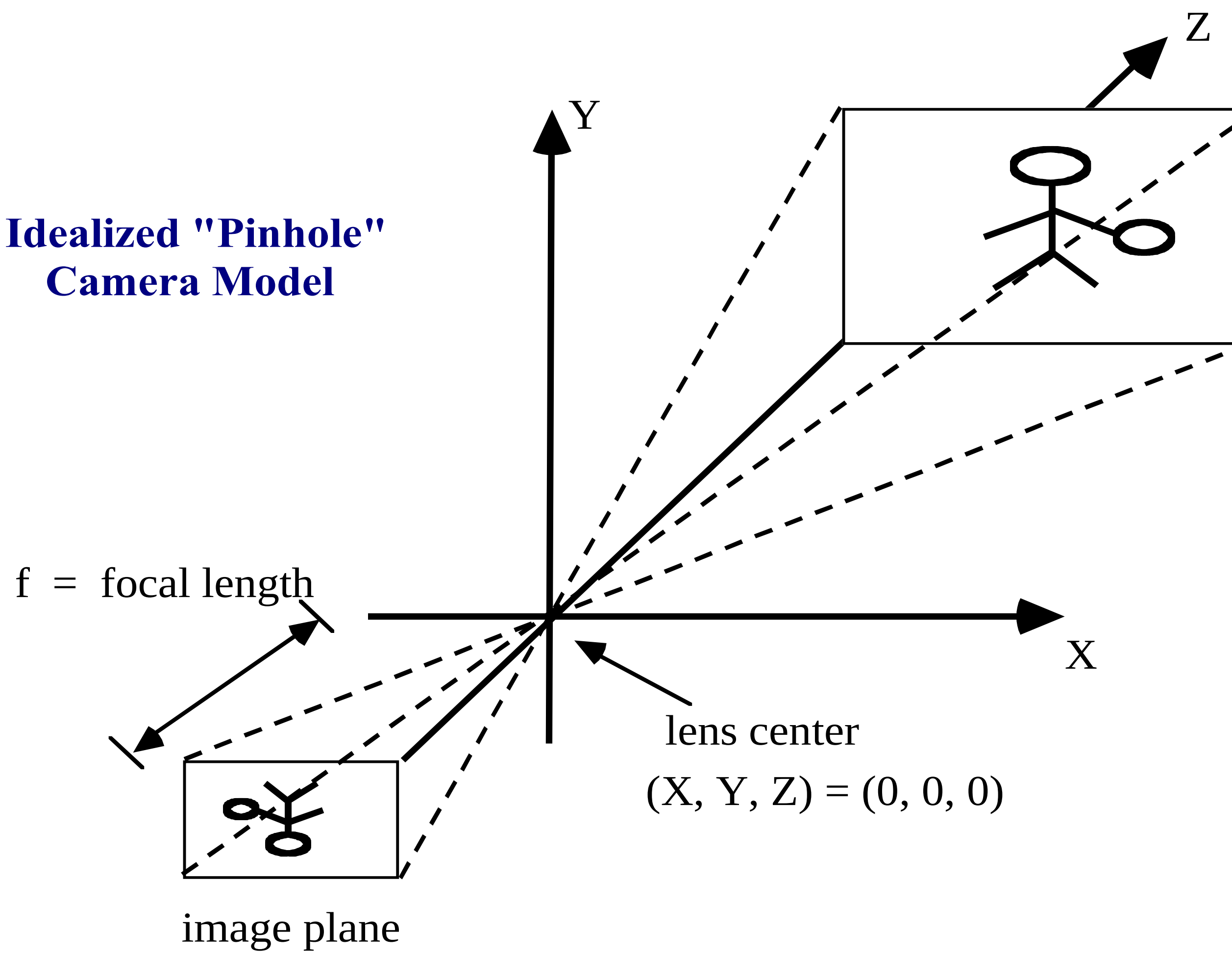
Real-World Coordinates

  - (X, Y, Z) denote points in 3-D space
  - The origin (X, Y, Z) = (0, 0, 0) is taken to be the lens center

Image Coordinates

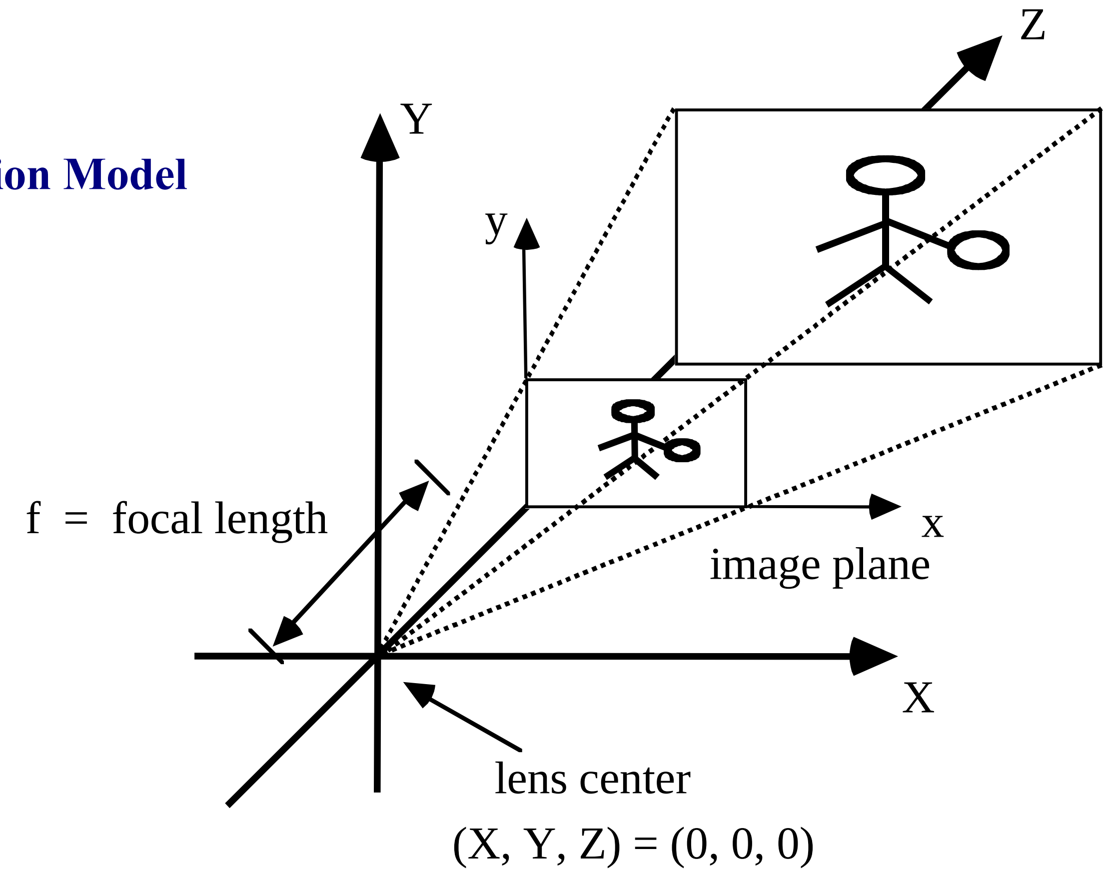
  - (x, y) denote points in the 2-D image
  - The x - y plane is chosen parallel to the X - Y plane
  - The optical axis passes through both origins

PIN-HOLE PROJECTION GEOMETRY



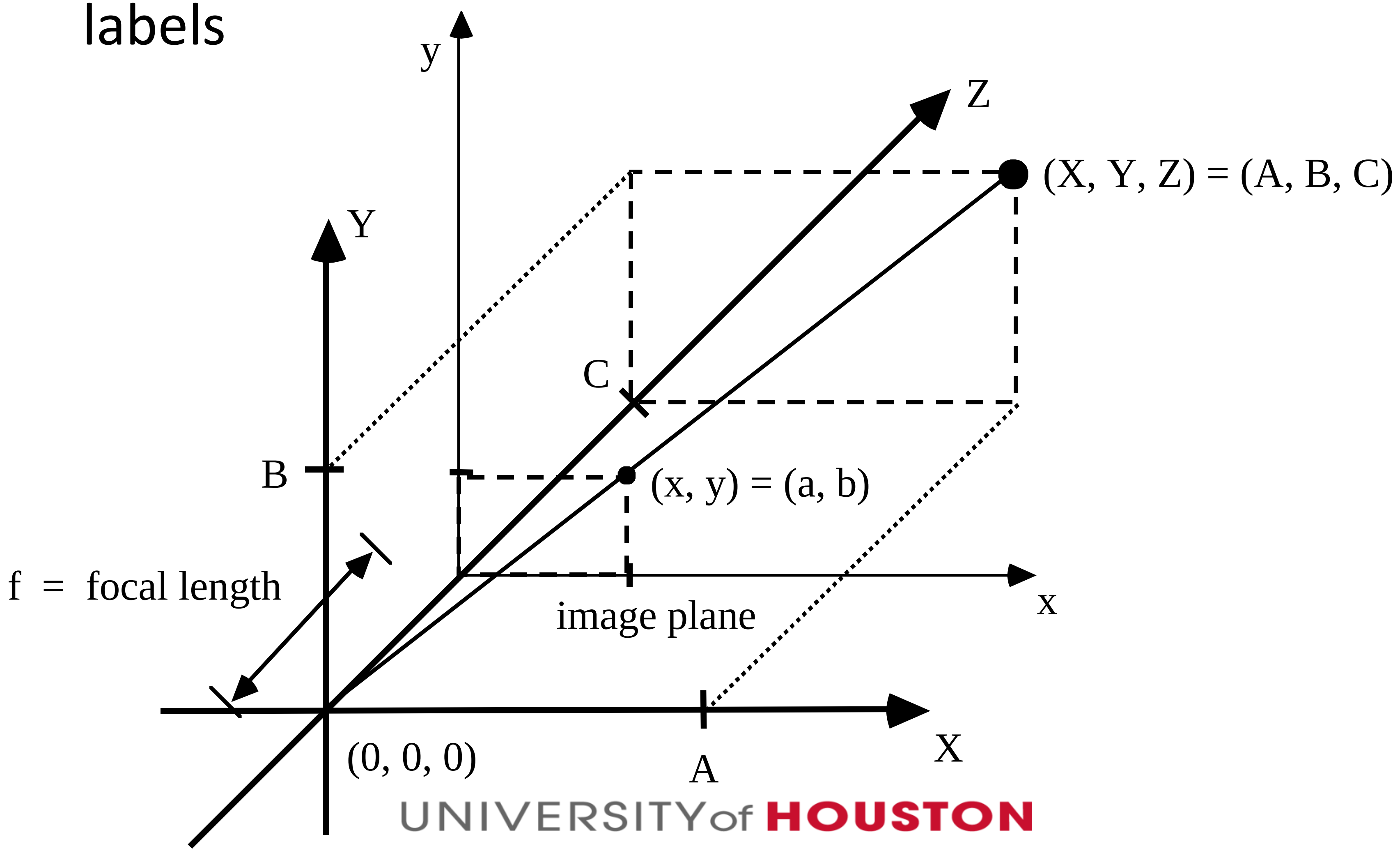
UPRIGHT PROJECTION GEOMETRY

Upright Projection Model



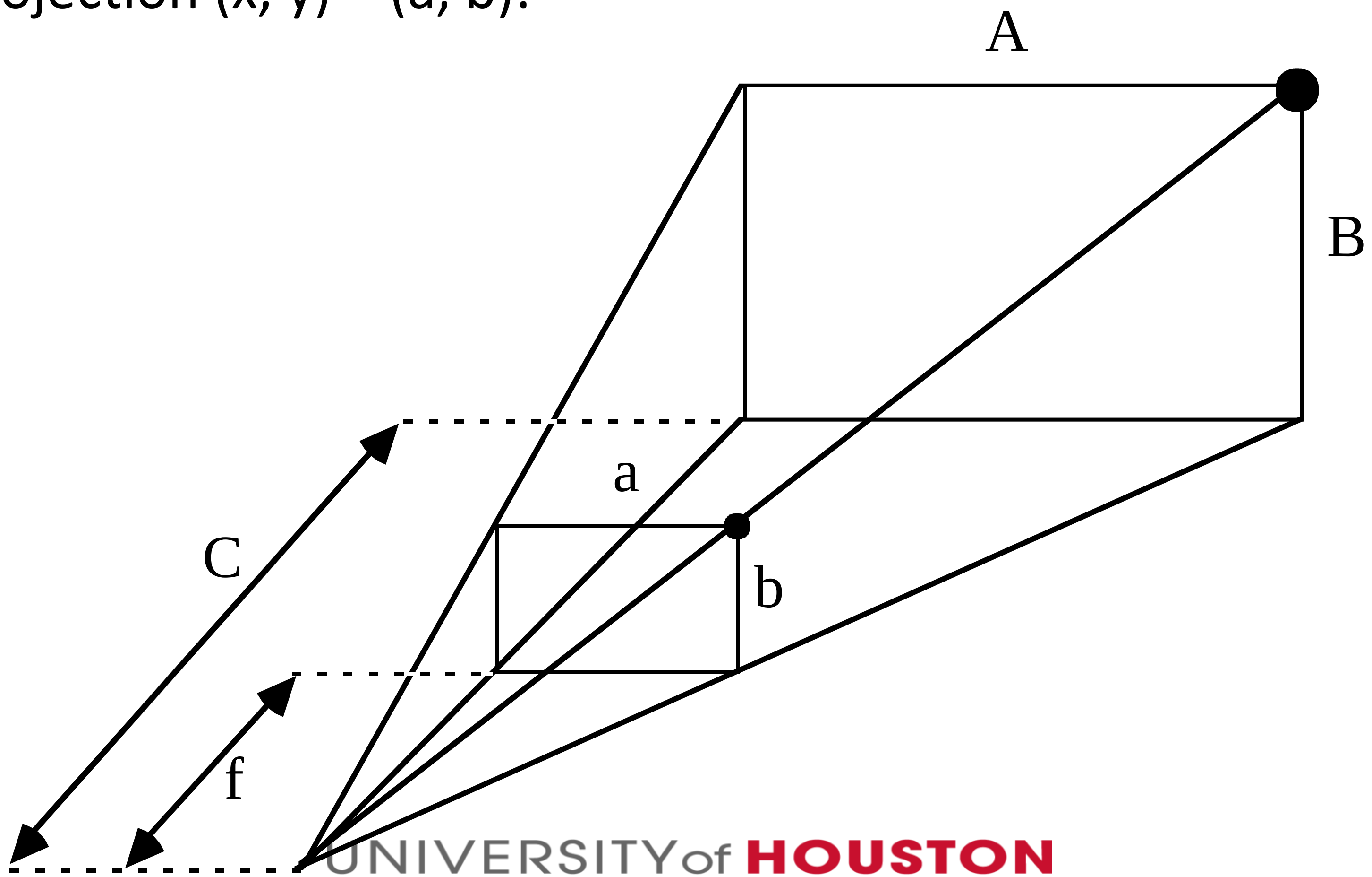
PROJECTION

- This diagram shows all of the coordinate axes and labels



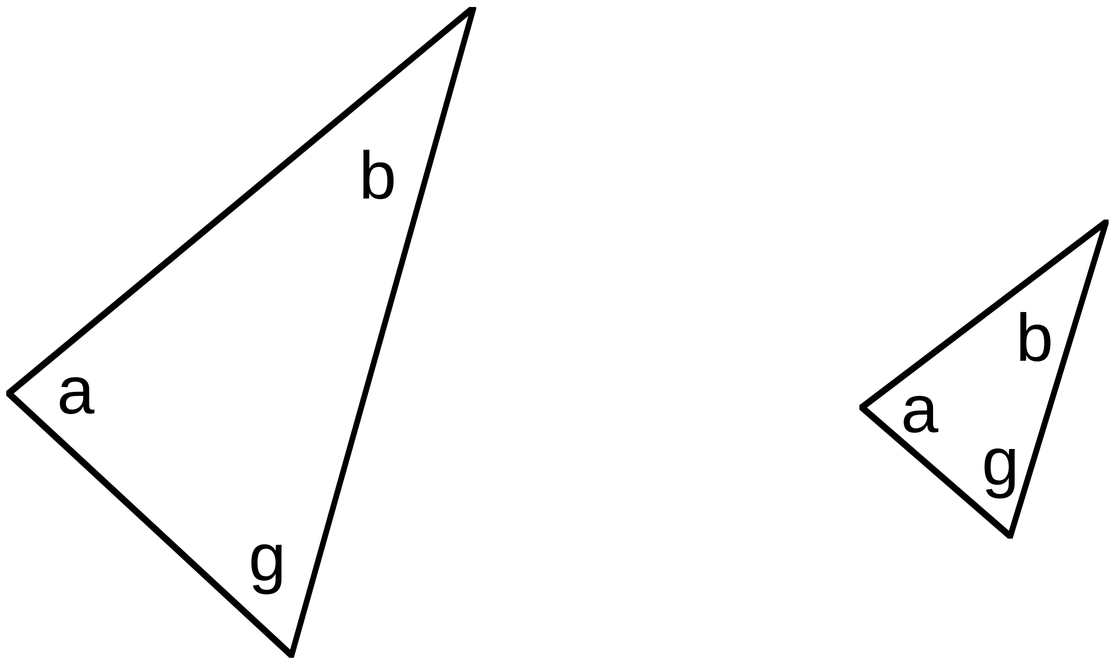
PROJECTION (contd.)

- This equivalent simplified diagram shows only the relevant data relating (X, Y, Z) = (A, B, C) to its projection (x, y) = (a, b):



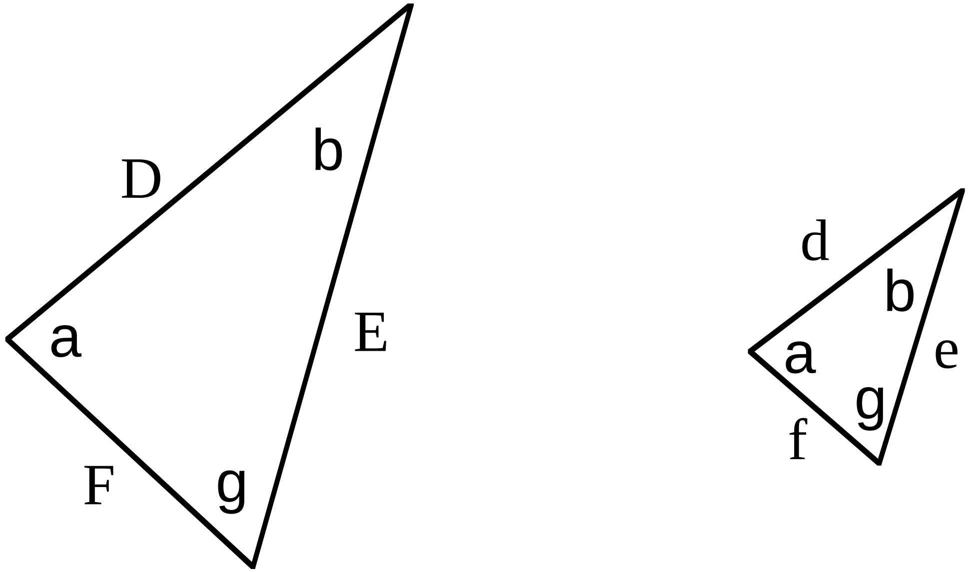
SIMILAR TRIANGLES

- Triangles are similar if their corresponding angles are equal:



SIMILAR TRIANGLES

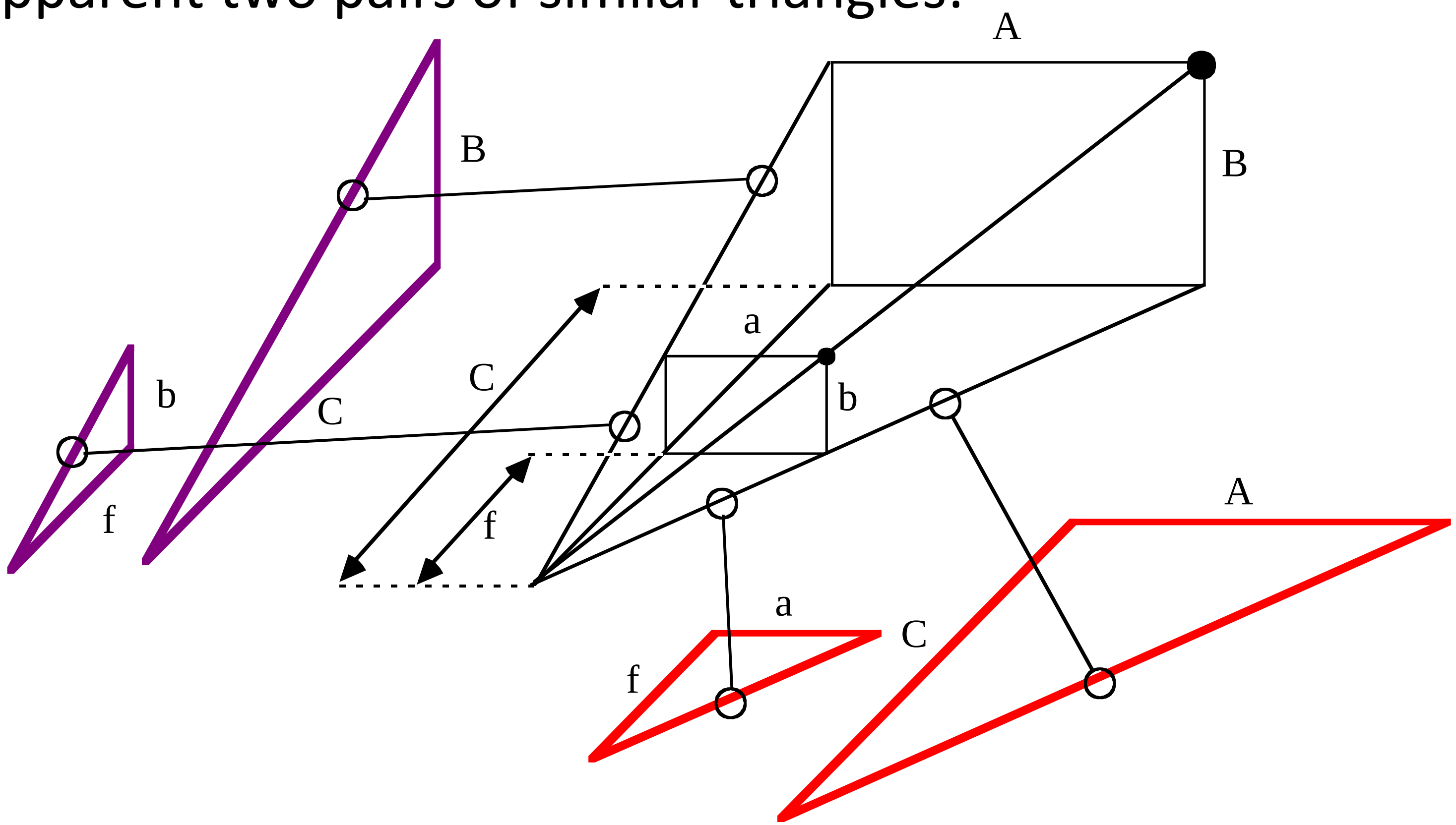
- Similar Triangles Theorem - Similar triangles have their side lengths in the same proportions.



$$\frac{D}{E} = \frac{d}{e} \qquad \frac{E}{F} = \frac{e}{f} \qquad \frac{F}{D} = \frac{f}{d}$$

SOLVING PERSPECTIVE PROJECTION

- Using similar triangles we can solve for the relationship between 3-D coordinates in space and 2-D image coordinates
- Redraw the imaging geometry once more, this time making apparent two pairs of similar triangles:



SOLVING PERSPECTIVE PROJECTION

- By the Similar Triangles Theorem, we conclude that

$$\frac{a}{f} = \frac{A}{C} \quad \text{and} \quad \frac{b}{f} = \frac{B}{C}$$

OR

$$(a, b) = \frac{f}{C} \cdot (A, B) = (fA/C, fB/C)$$

PERSPECTIVE PROJECTION EQUATION

- Thus the following relationship holds between 3-D space coordinates (X, Y, Z) and 2-D image coordinates (x, y) :

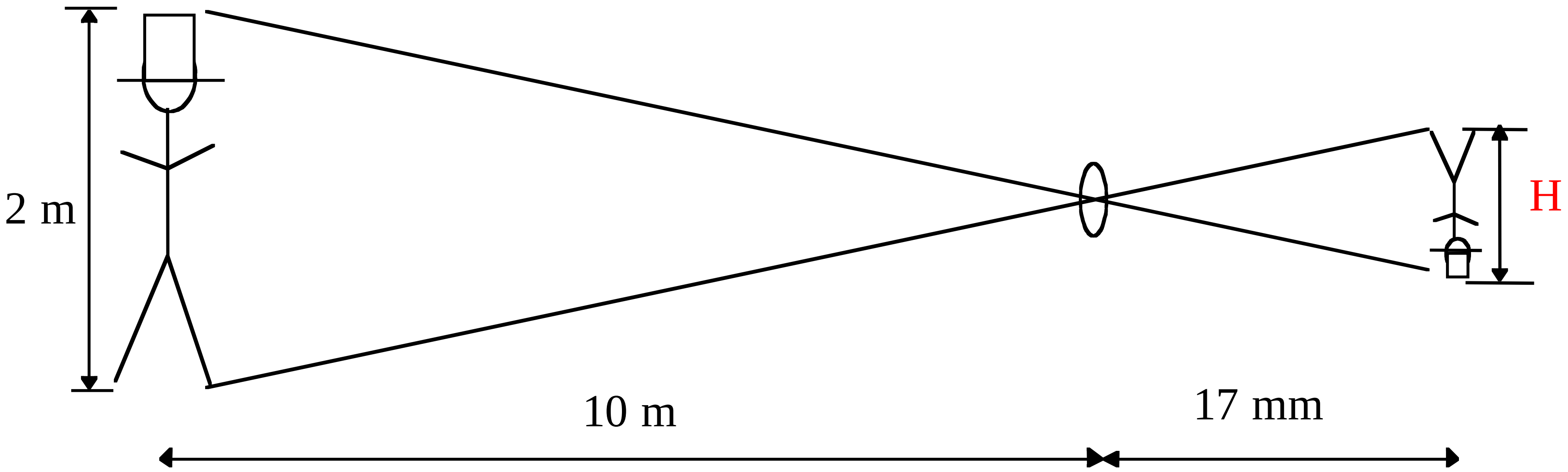
$$(x, y) = \frac{f}{Z} \cdot (X, Y)$$

  where f = focal length.

- The ratio f/Z is the magnification factor, which varies with the range Z from the lens center to the object plane.

EXAMPLE

- There is a man standing 10 meters (m) in front of you
- He is 2 m tall
- The focal length of your eye is about 17 mm
- Question: What is the height H of his image on your retina?



ANSWER

- By similar triangles,

$$\frac{2\text{ m}}{10\text{ m}} = \frac{H}{17\text{ mm}}$$

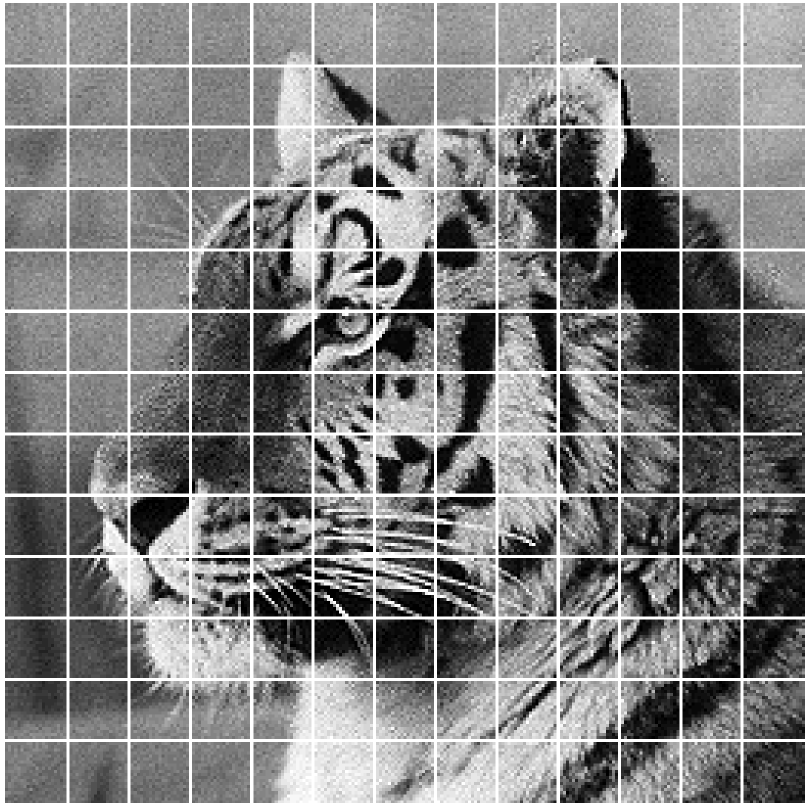
$$\underline{H = 3.4\text{ mm}}$$

How Do We Generate A Digital Image?

- Start with a picture of something

Slide by K. R. Castleman

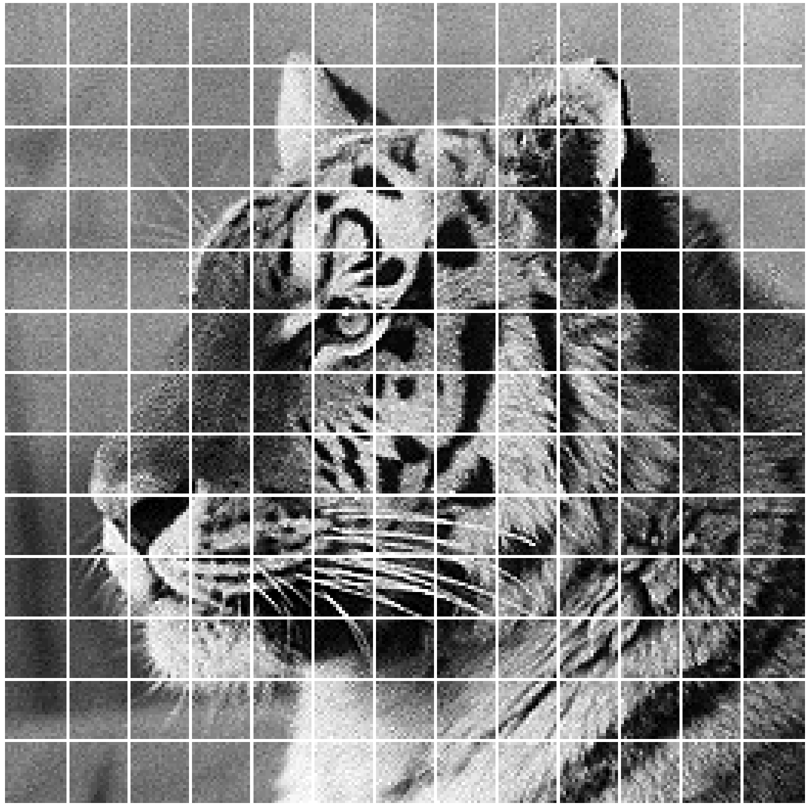
How Do We Generate A Digital Image?



- Start with a picture of something
- Lay a grid over the picture

Slide by K. R. Castleman

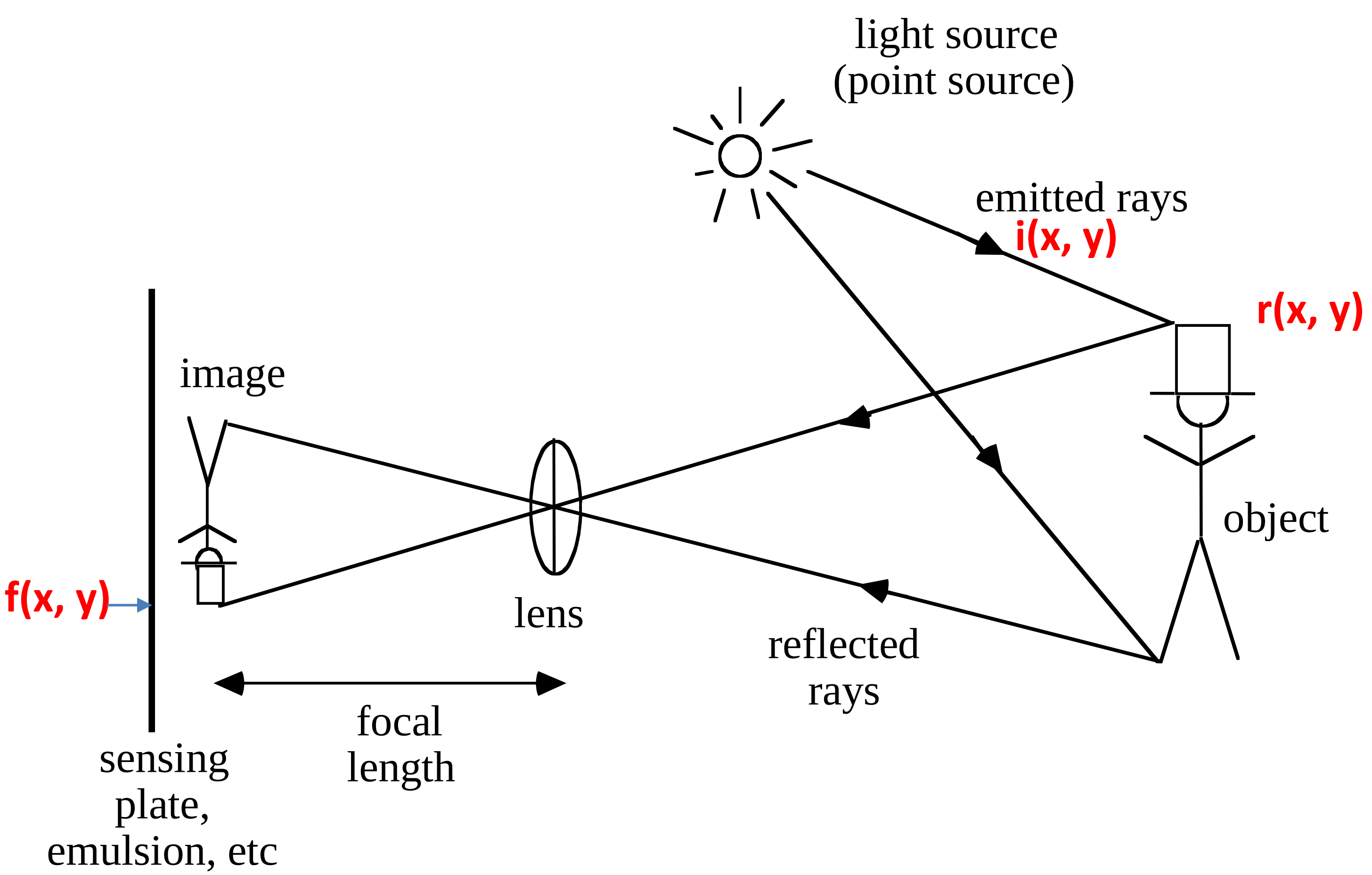
How Do We Generate A Digital Image?



- Start with a picture of something
- Lay a grid over the picture
- Measure the brightness/intensity in each of the squares

Slide by K. R. Castleman

A Simple Image Formation Model



A Simple Image Formation Model

$$f(x,y) = i(x,y)r(x,y)$$

$f(x, y)$: intensity at the point (x, y)

$i(x, y)$: illumination at the point (x, y)

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point (x, y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

Some Typical Ranges of Reflectance

- Illumination - i(x, y)

  Lumen — A unit of light flow or luminous flux

  Lumen per square meter (lm/m2) — The metric unit of measure for illuminance of a surface

  - 90,000 lm/m^2 clear day
  - 10,000 lm/m^2 cloudy day
  - 1,000 lm/m^2 Indoor Office
  - 0.1 lm/m^2 clear evening

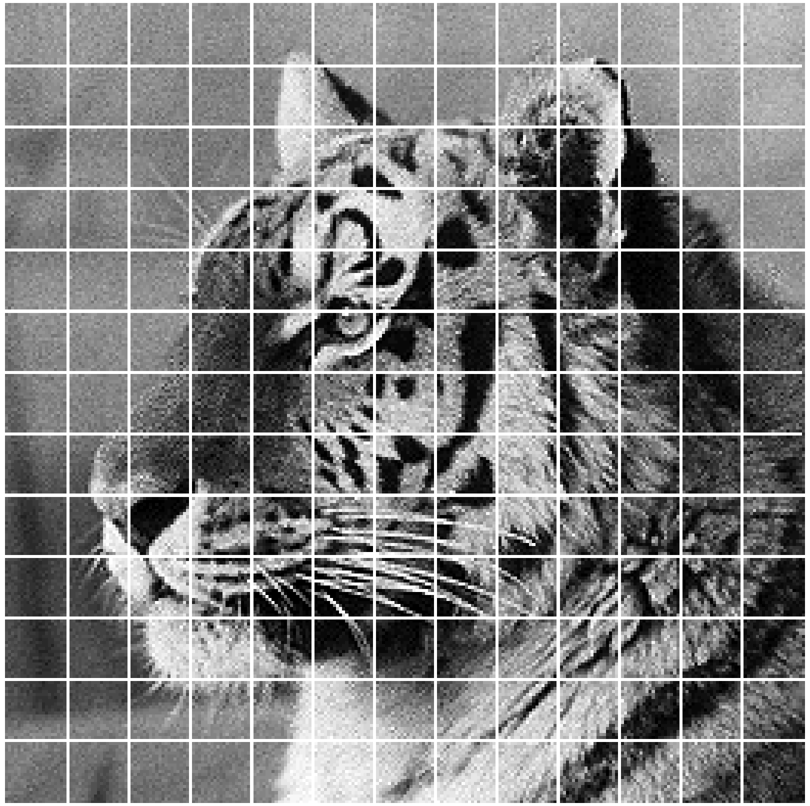
- Reflectance - r(x, y)

  - 0.01 for black velvet
  - 0.65 for stainless steel
  - 0.80 for flat-white wall paint
  - 0.90 for silver-plated metal
  - 0.93 for snow

Representation of intensity

- If $l = f(x, y)$
- Let $Lmin \leq l \leq Lmax$
- Using previous intensities,
  - We may expect, $Lmin \cong 10$ & $Lmax \cong 1000$, for Indoor
- $[Lmin, Lmax] \rightarrow grey\ scale$

How Do We Generate A Digital Image?



- Start with a picture of something
- Lay a grid over the picture
- Measure the brightness in each of the squares

Slide by K. R. Castleman

How Do We Generate A Digital Image?

$$\begin{bmatrix} 8 & 7 & 9 & 11 & 12 & \cdots \\ 9 & 12 & 10 & 9 & & \\ 7 & 9 & 11 & & & \\ 10 & 8 & & \ddots & & \\ 9 & & & & & \\ \vdots & & & & & \end{bmatrix}$$

- Start with a picture of something
- Lay a grid over the picture
- Measure the brightness in each of the squares
- The resulting array of numbers(digits) is the digital image

Slide by K. R. Castleman

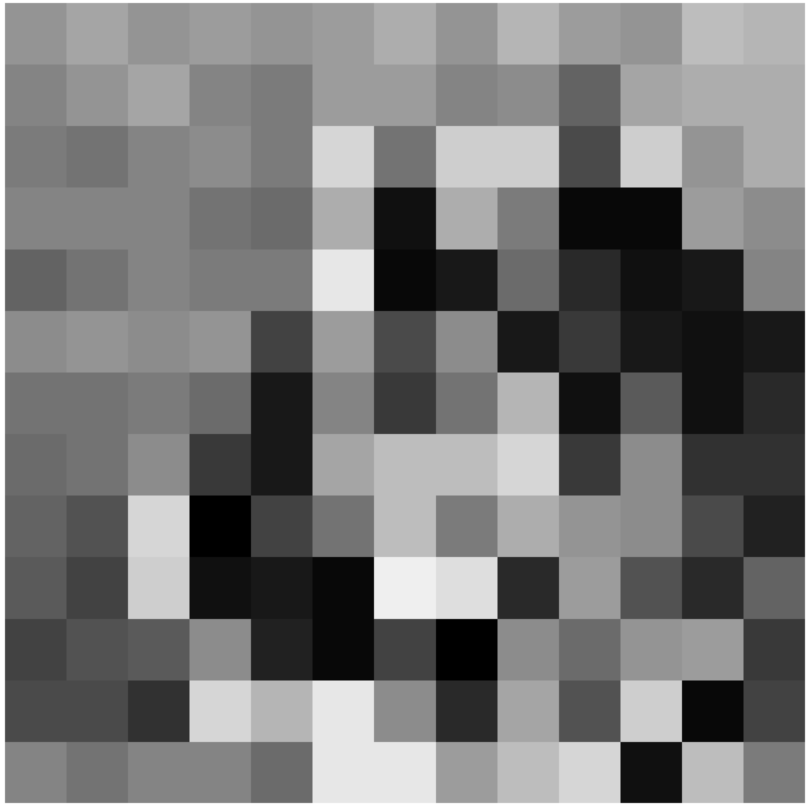
How Do We Generate A Digital Image?

$$
\begin{bmatrix}
8 & 7 & 9 & 11 & 12 & \cdots \\
9 & 12 & 10 & 9 & & \\
7 & 9 & 11 & & & \\
10 & 8 & & \ddots & & \\
9 & & & & & \\
\vdots & & & & &
\end{bmatrix}
$$

- Each number represents the brightness (0 – Max) at the corresponding position in the image
- Each number is the “gray level” or “pixel value” of the corresponding pixel.

Slide by K. R. Castleman

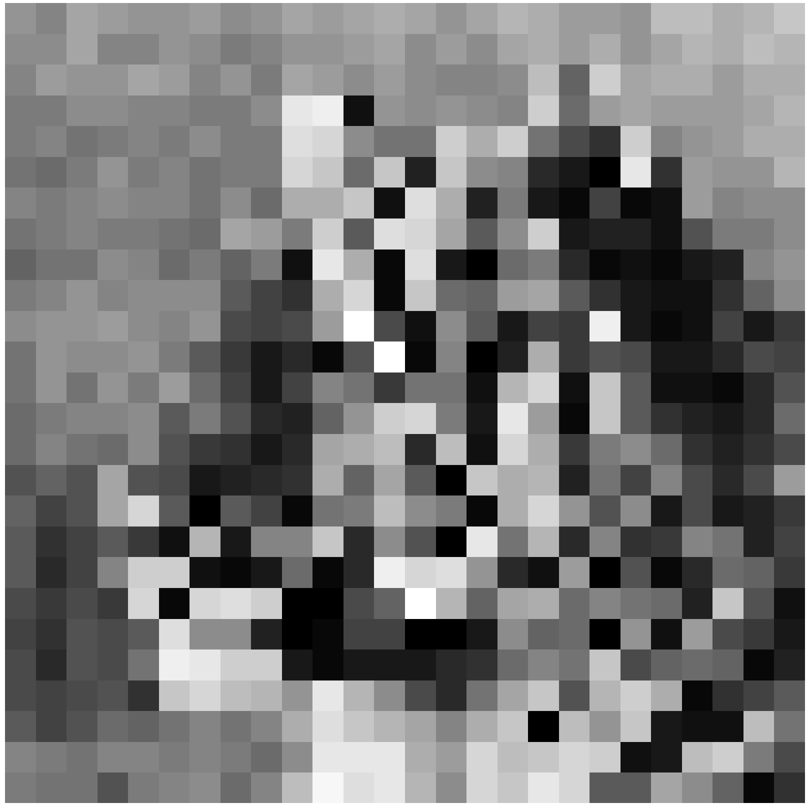
What are the Pixels?



- Every pixel has a location in the image.
- A pixel's location is specified by it's row number and column number (x,y address).
- Every pixel has a gray level value.

Slide by K. R. Castleman

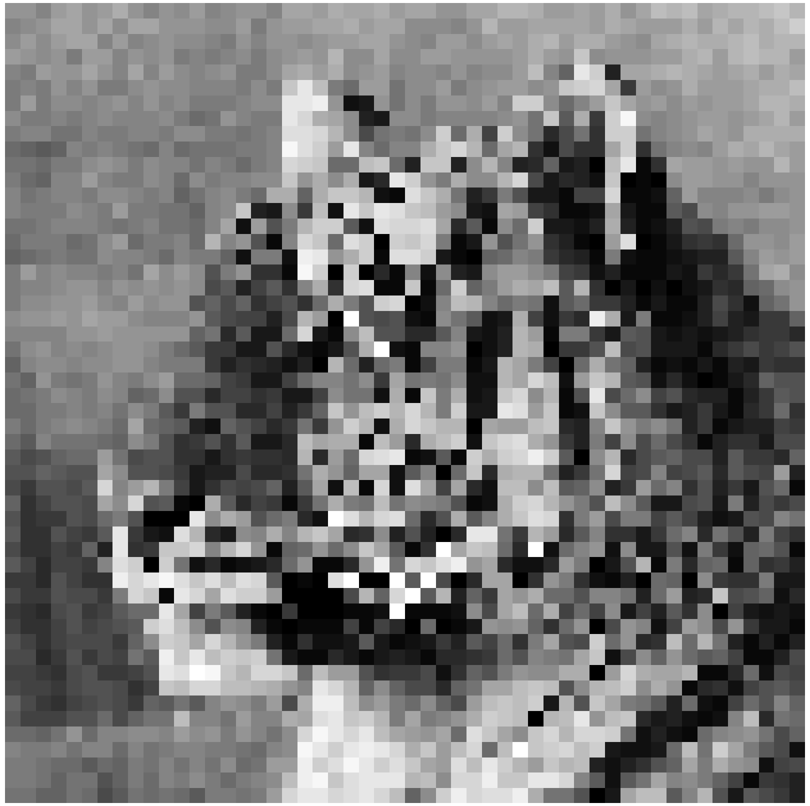
What Happened To Our Tiger?



- We only used 169 pixels (not enough).
- Increase to 26 X 26 pixels
- Here he is with 676 pixels.

Slide by K. R. Castleman

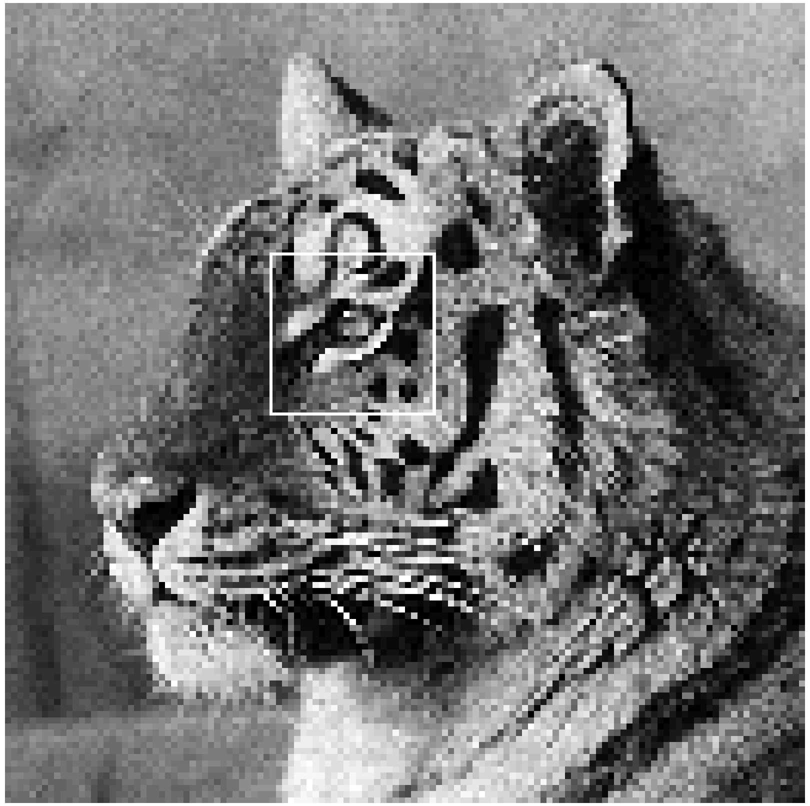
What Happened To Our Tiger?



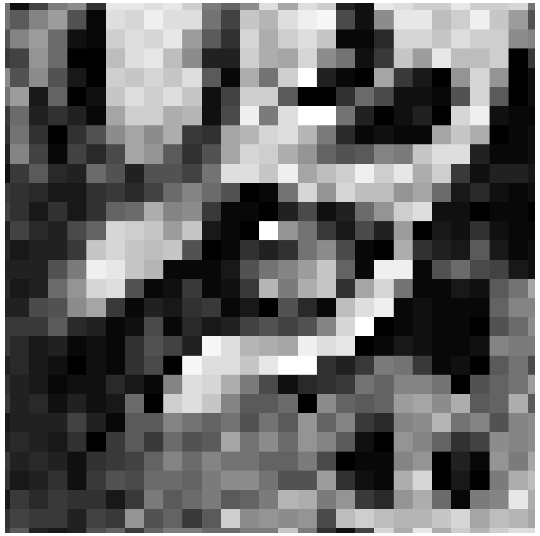
- We only used 169 pixels (not enough).
- 52 X 52
- Here he is with 2704 pixels.

Slide by K. R. Castleman

What Happened To Our Tiger?



- We only used 169 pixels (not enough).
- 130 X 130
- Here he is with 16,900.



Slide by K. R. Castleman

What Happened To Our Tiger?

- We only used 169 pixels (not enough).
- 260 X 260
- Here he is with 67,600 pixels.

Slide by K. R. Castleman

Image Acquisition

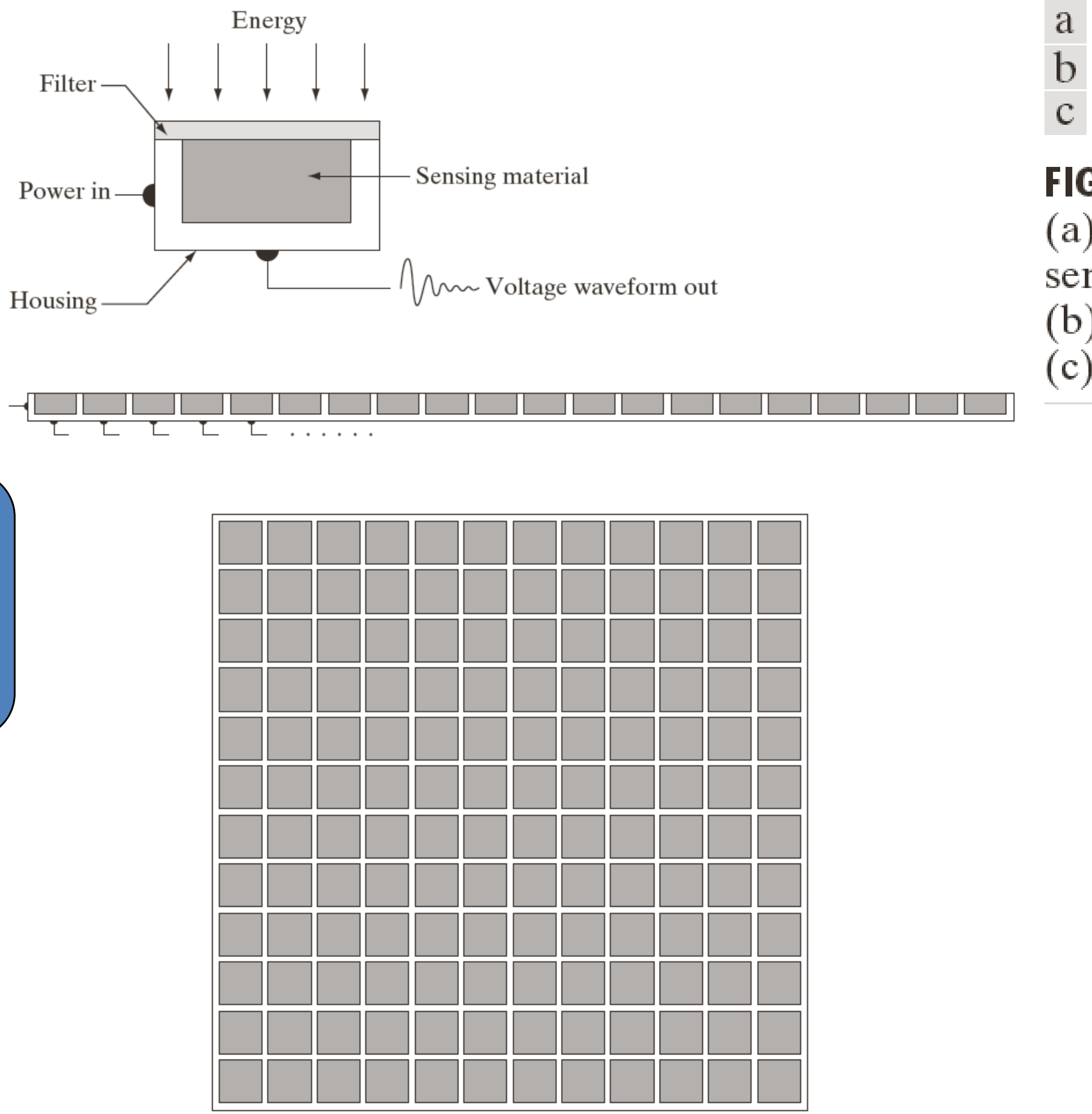


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Transform illumination energy into digital images

Image Acquisition Using a Single Sensor

FIGURE 2.13
Combining a single sensor with motion to generate a 2-D image.

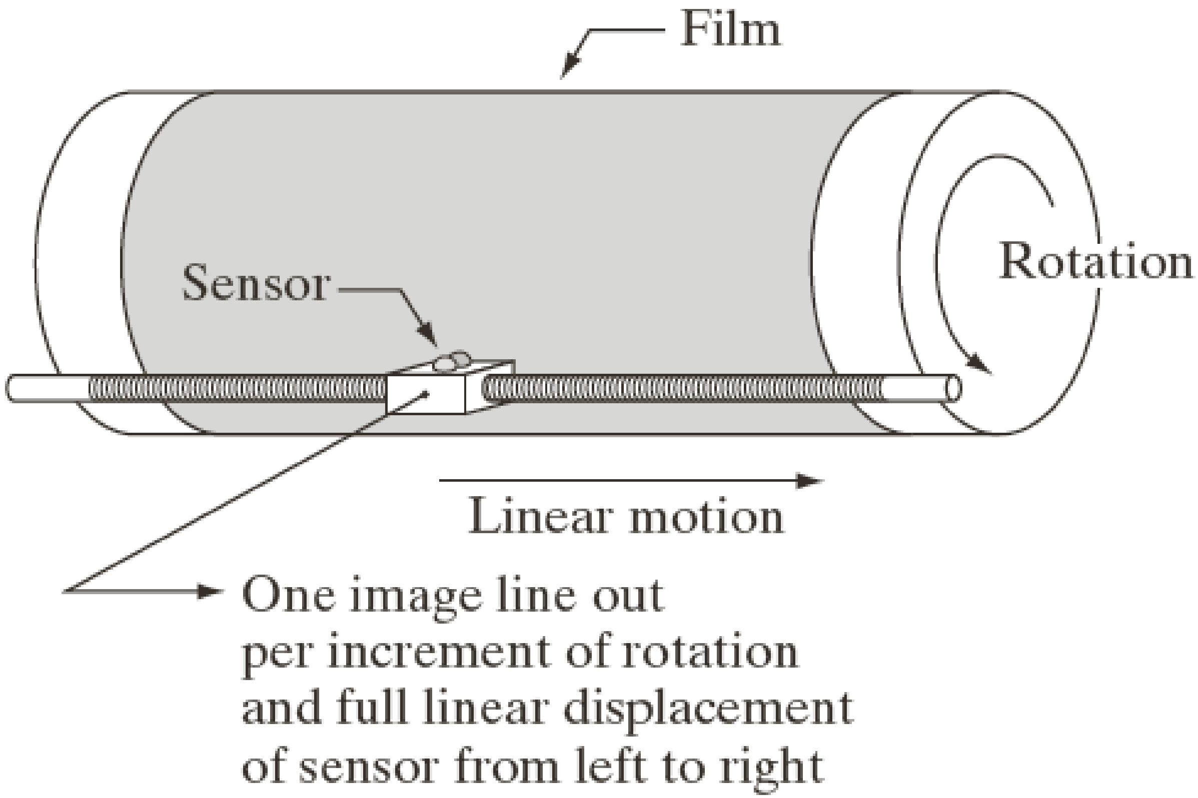


Image Acquisition Using Sensor Strips

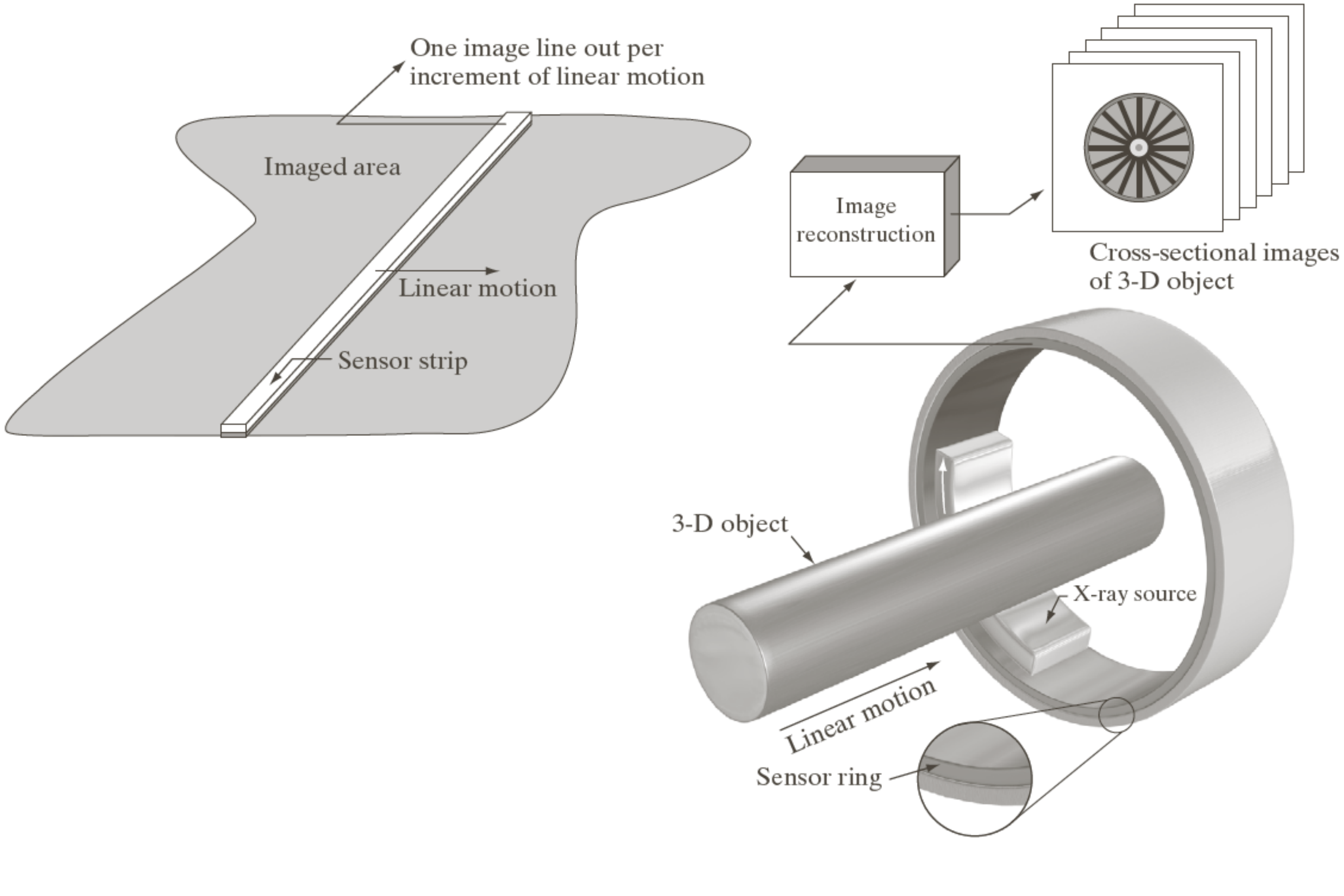


FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image Acquisition Process

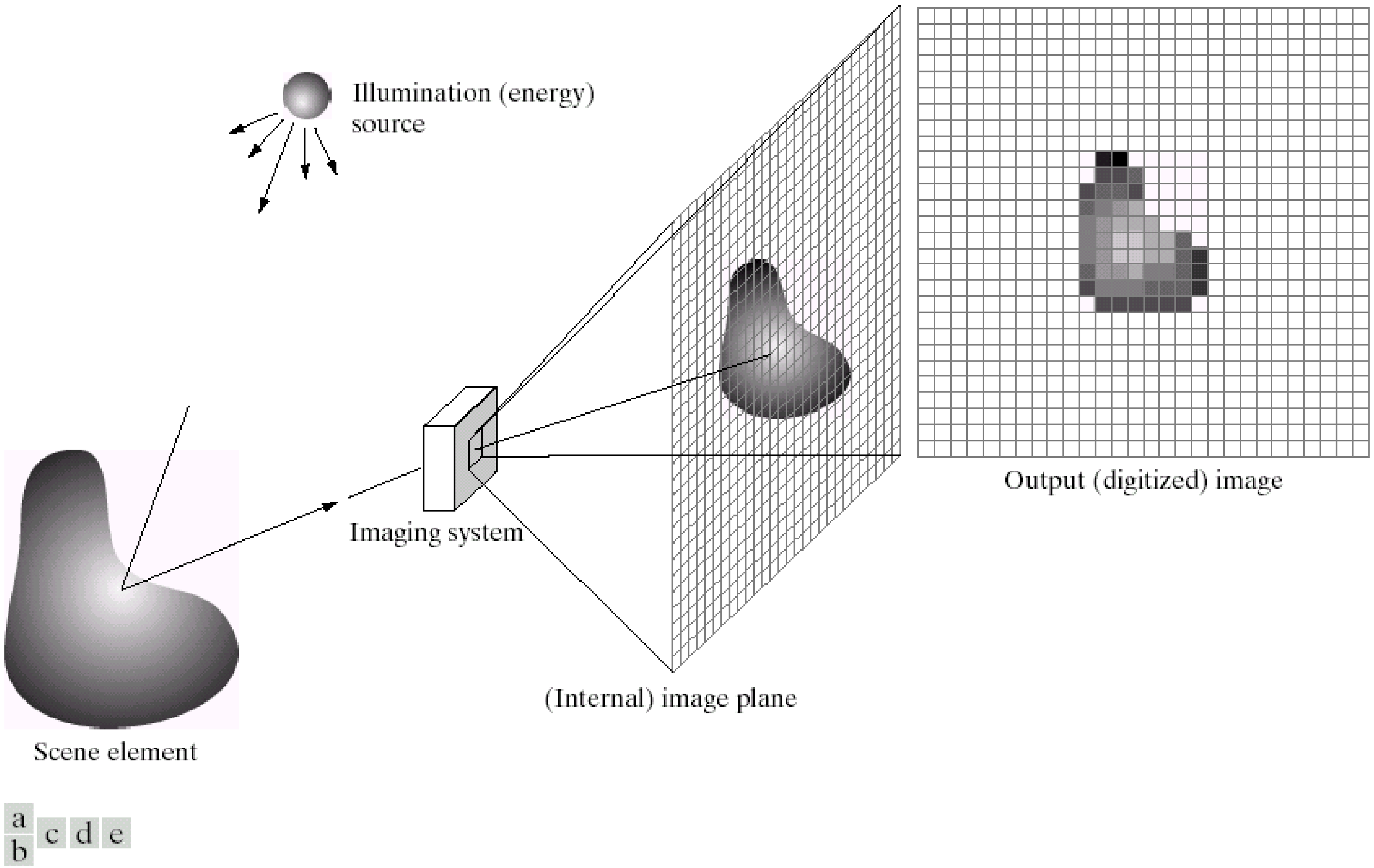


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Sensor Response Waveform

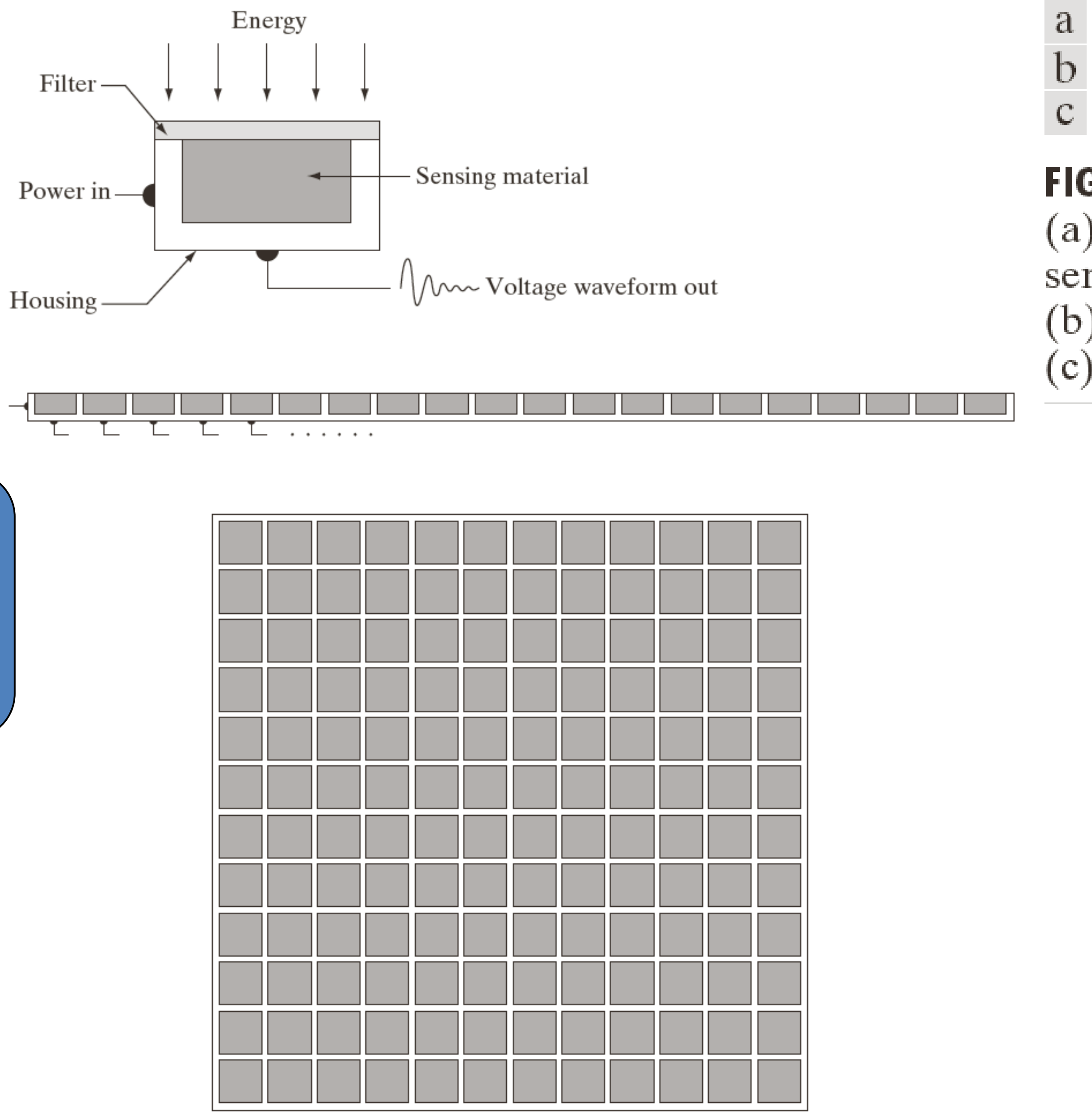
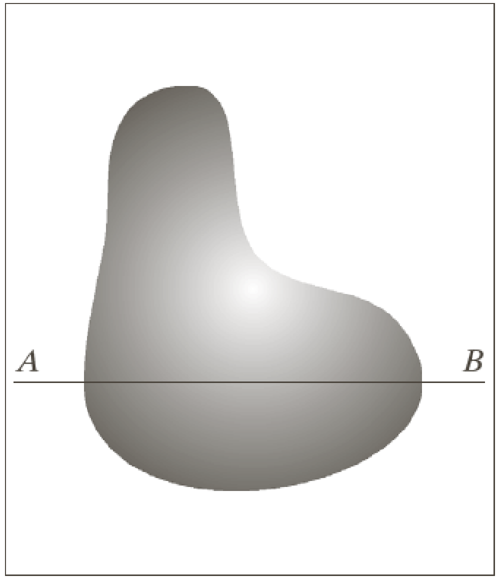
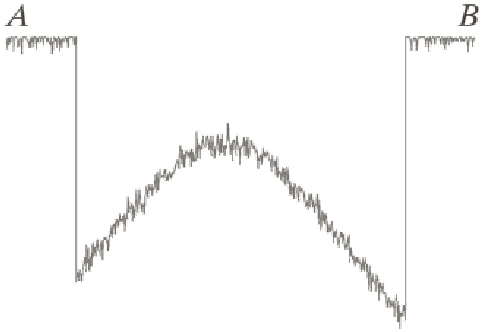


FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Transform illumination energy into digital images

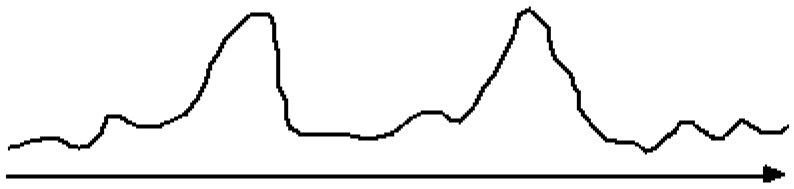
Response from a raster scan





A / D CONVERSION

- For computer processing, the analog image must undergo **ANALOG / DIGITAL (A/D) CONVERSION** **-** Consists of **sampling** and **quantization**



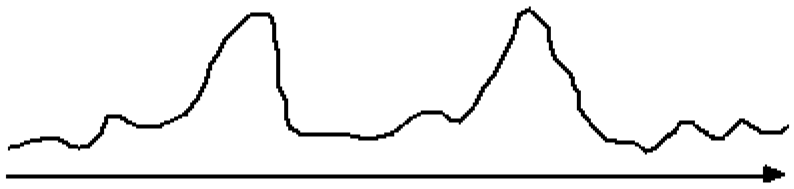
continuous electrical signal from one scanline

A / D CONVERSION

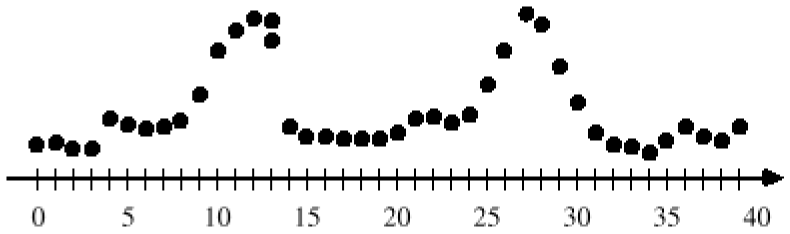
- For computer processing, the analog image must undergo **ANALOG / DIGITAL (A/D) CONVERSION** - Consists of **sampling** and **quantization**

Sampling

- Each video **raster** is converted from a **continuous voltage waveform** into a sequence of **voltage samples**:

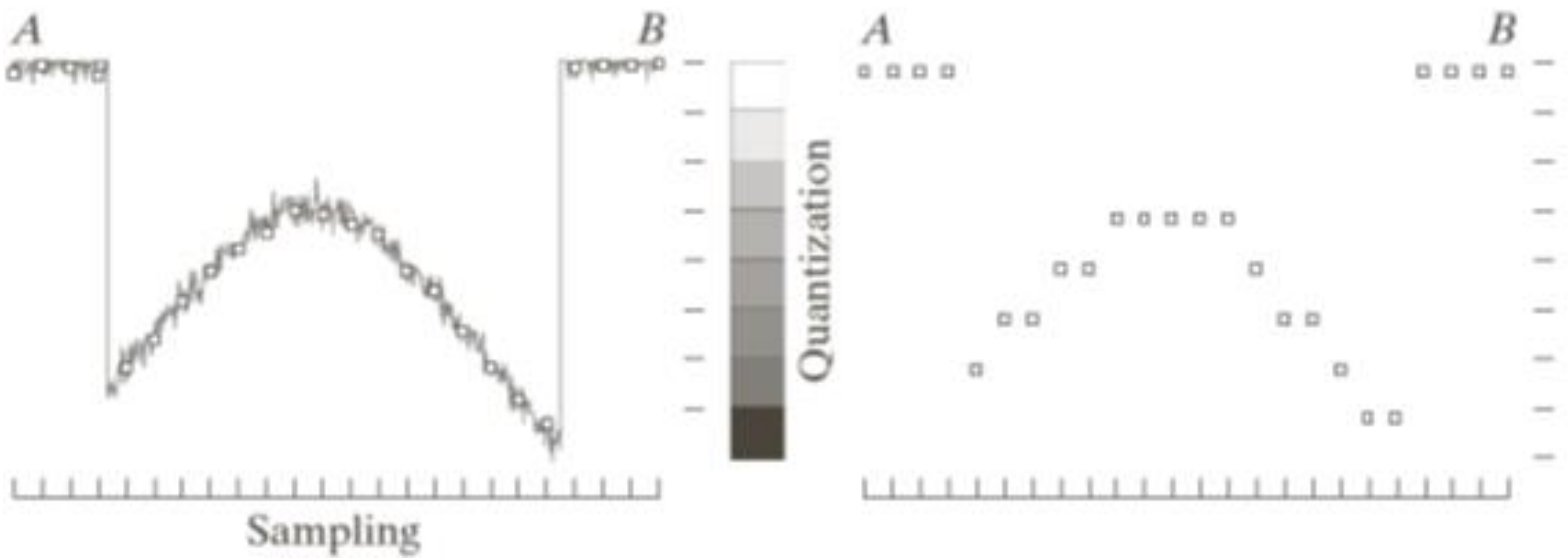


continuous electrical signal from one scanline



sampled electrical signal from one scanline indexed by discrete (integer) numbers

A / D CONVERSION

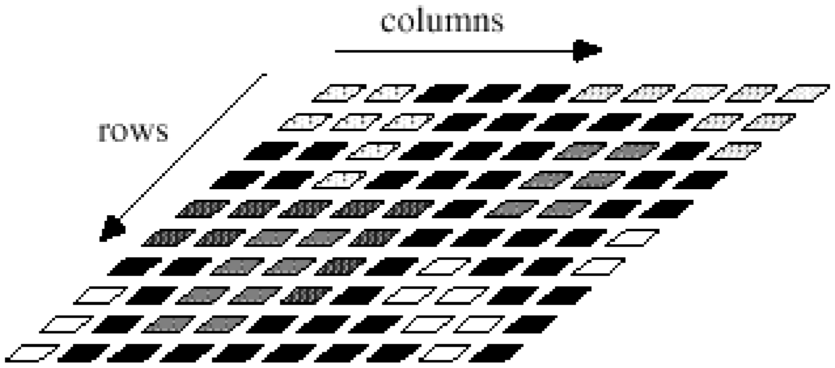


A / D CONVERSION (contd.)

- **Video digitizer board** interfaces with the video camera
- Some new "all-digital cameras" include A/D **inside** the camera

Sampled Image

- A **sampled image** is an array of numbers representing the sampled (row, column) image intensities
- Each of these **picture elements** is called a **pixel**



depiction of 10 x 10 image array

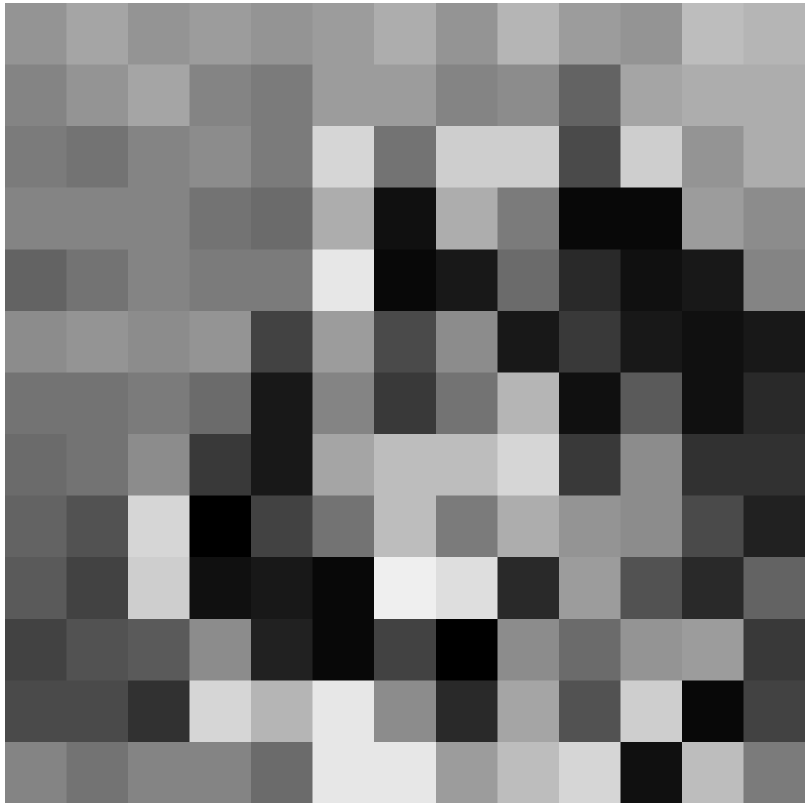
A / D CONVERSION (contd.)

- **Typically** the image array is square (N x N) with dimensions that are a power of 2: N = 2^{M} (for simple computer addressing)

M = 7	128 x 128 (2^{14} ~ 16,000 pixels)
M = 8	256 x 256 (2^{16} ~ 65,500 pixels)
M = 9	512 x 512 (2^{18} ~ 262,000 pixels)
M = 10	1024 x 1024 (2^{20} ~ 1,000,000 pixels)

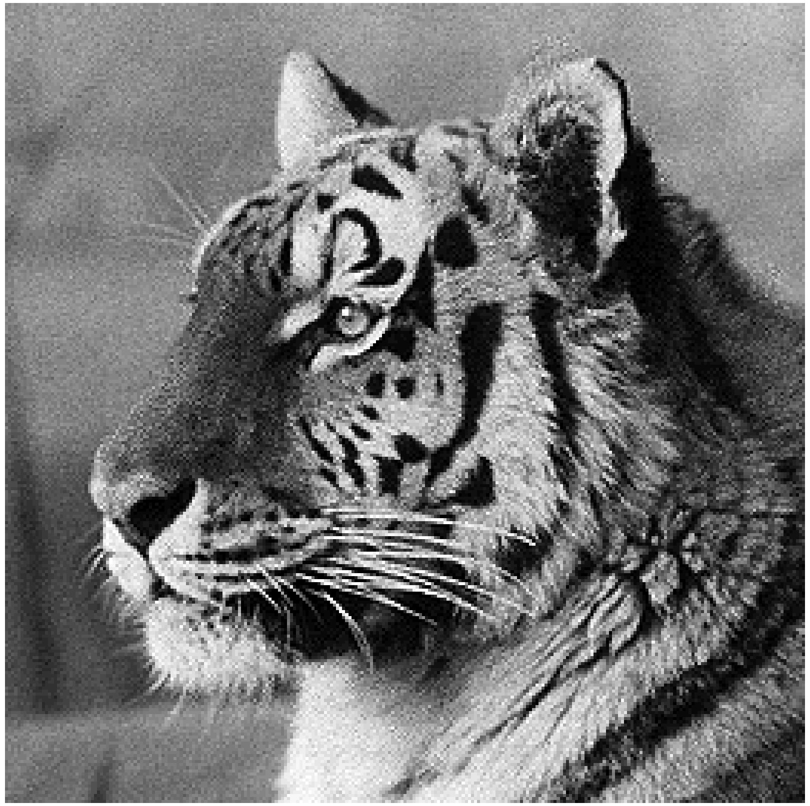
- Important that the **image be sampled sufficiently densely**
- Otherwise the image quality will be severely degraded
- This can be expressed mathematically (The Sampling Theorem) but the effects are very **visually obvious**

Sampling: Example



169 Samples

VS



67,600 Samples

Review: Representation of intensity

- If $l = f(x, y)$
- Let $Lmin \leq l \leq Lmax$
- Using previous intensities,
  - We may expect, $Lmin \cong 10$ & $Lmax \cong 1000$
- $[Lmin, Lmax] \rightarrow grey\ scale$

QUANTIZATION

- Each pixel **gray level** is quantized: assigned one of a finite set of numbers (generally integers indexed from 0 to K-1
- Typically there $K = 2^{B}$ possible gray levels:
- **Each pixel is represented by B bits**, where usually $1 \leq B \leq 8$
- The pixel intensities or gray levels must be quantized sufficiently densely so that excessive information is not lost
- This is **hard** to express mathematically, but again, quantization effects are **visually obvious**

a pixel

8-bit representation

DIGITAL IMAGE REPRESENTATION

- Once an image is **digitized** (A/D) and stored it is an array of **voltage or magnetic potentials**
- Not easy to work with from an algorithmic point of view
- The **representation** that is easiest to work with from an **algorithmic perspective** is that of a **matrix of integers**

Matrix Image Representation

- Denote a (square) image matrix $\mathbf{I} = [I(i, j);\ 0 < i, j < N-1]$ where
- $(i, j) = (\text{row}, \text{column})$
- $I(i, j)$ = image value at coordinate or pixel (i, j)

DIGITAL IMAGE REPRESENTATION (contd.)

- **Example** - Matrix notation

$$\mathbf{I} = \begin{bmatrix} I(0,0) & I(0,1) & \cdots & I(0,N-1) \\ I(1,0) & I(1,1) & \cdots & I(1,N-1) \\ \vdots & \vdots & & \vdots \\ I(N-1,0) & I(N-1,1) & \cdots & I(N-1,N-1) \end{bmatrix}$$

- **Example** - Pixel notation - an N x N image

What's the minimum number of bits/pixel allocated?

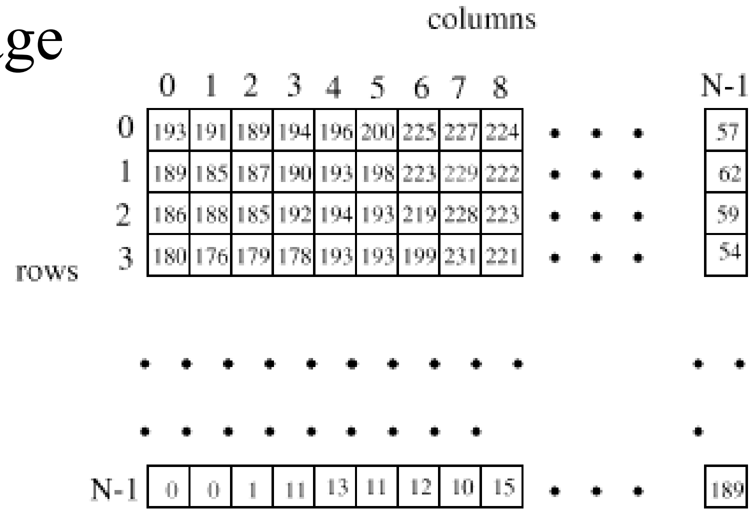


Image **I**

DIGITAL IMAGE REPRESENTATION (contd.)

- **Example** - Binary Image (2-valued, usually BLACK and WHITE)

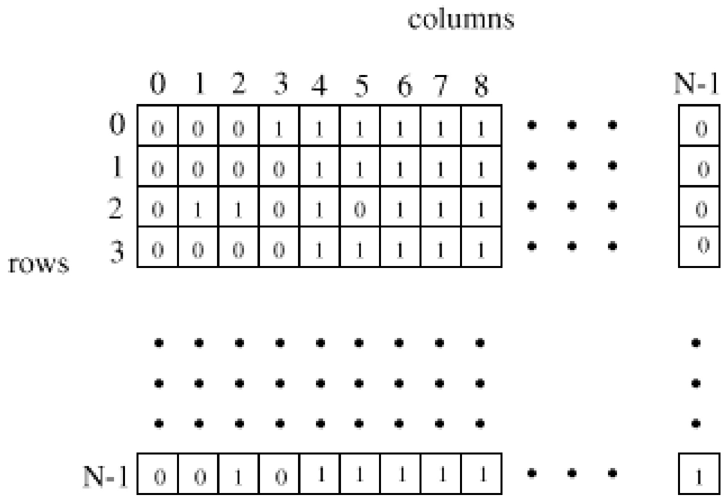


Image **I**

- Another way of depicting the image:

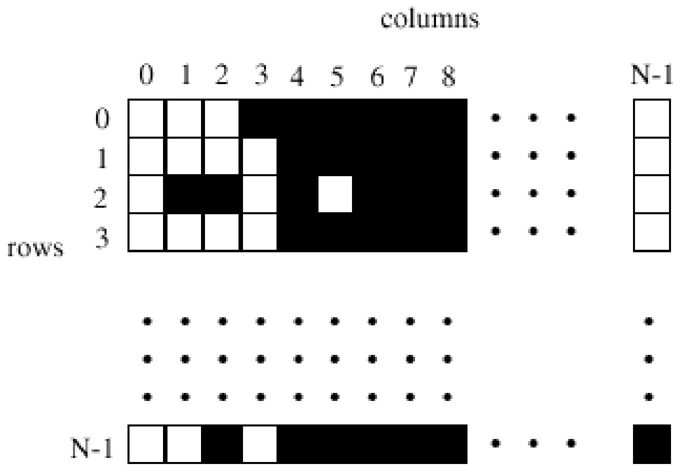


Image **I**

Representing Digital Images

- Discrete intensity interval [0, L-1], $L=2^k$
- Aka. Dynamic Range
- The number b of bits required to store a $M \times N$ digitized image

Representing Digital Images

- Discrete intensity interval $[0, L-1]$, $L=2^k$

- The number b of bits required to store a $M \times N$ digitized image

$$\text{total bits} = M \times N \times k$$

Representing Digital Images

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Intensity Resolution

- Spatial resolution
  - A measure of the smallest discernible detail in an image
  - stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

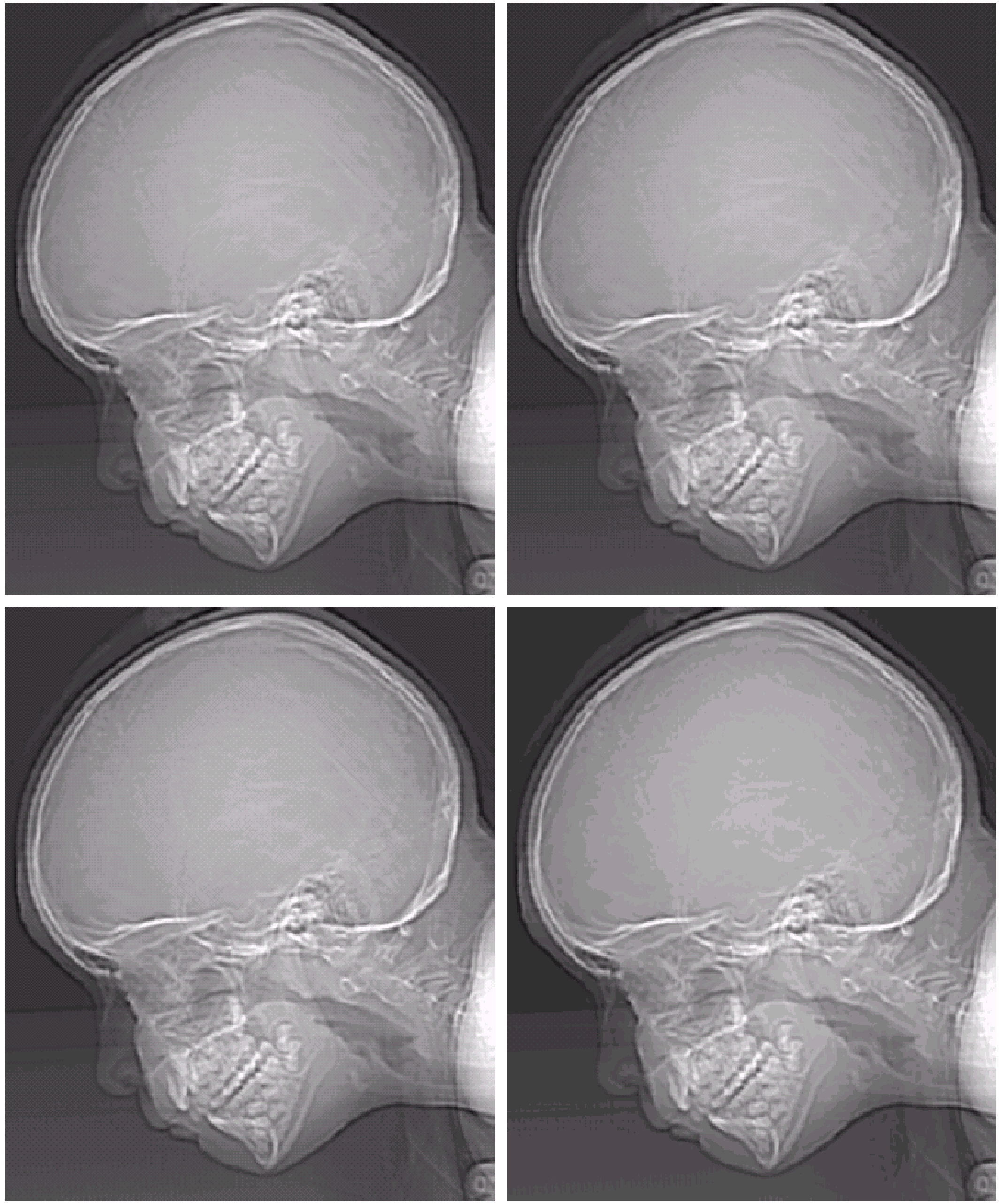
- Intensity resolution
  - The smallest discernible change in intensity level
  - stated with *8 bits, 12 bits, 16 bits, etc.*

Spatial Resolution

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Intensity Resolution



a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Spatial and Intensity Resolution

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)