

**MATH 3339**  
**Statistics for the Sciences**  
**Live Lecture Help**

James West  
jdwest@uh.edu

University of Houston

Session 2

Office Hours: see schedule in the "Office Hours" channel on Teams  
Course webpage: [www.casa.uh.edu](http://www.casa.uh.edu)

When you email me you **MUST** include the following

- **MATH 3339 Section 20024** and a description of your issue in the **Subject Line**
- Your name and ID# in the **Body**
- Complete sentences, punctuation, and paragraph breaks
- Email messages to the class will be sent to your Exchange account (user@cougarnet.uh.edu)

# Correspondence Etiquette

Things to know about contacting your instructor:

- Teams chat messages are not appropriate unless told otherwise by your instructor
- In my case, Teams messages are only appropriate if I initiate there (for instance to schedule a meeting) or when following my directions regarding Office Hours
- Your instructor is Dr. Lastname if they have a PhD (I do) or Professor Lastname otherwise
- People like a greeting

# Using R and R-Studio

1. Download R from <https://cran.r-project.org/>
2. Download R-Studio from <https://www.rstudio.com/>

# Outline

- 1 Recap
- 2 Examples
- 3 Student submitted questions

# Assigning probabilities

- **Classical method** is used when all the experimental outcomes are equally likely. If  $n$  experimental outcomes are possible, a probability of  $1/n$  is assigned to each experimental outcome. Example: Drawing a card from a standard deck of 52 cards. Each card has a  $1/52$  probability of being selected.
- **Relative frequency method** is appropriate when data are available to estimate the proportion of the time the experimental outcome or collection of outcomes (event) will occur if the experiment is repeated a large number of times. That is for any event,  $E$ , probability of  $E$  is

$$P(E) = \frac{\text{number of times } E \text{ occurs}}{\text{total number of observations}} = \frac{\#(E)}{N}$$

- $P(E)$  is defined in the probability model for any event  $E$  that is a subset of  $\Omega$ .

# Permutations

If we wish to compute the number of outcomes when  $r$  objects are to be selected from a set of  $n$  objects where the order of selection is important, we call this the number of **permutations**. The number of permutations is given by

$${}_nP_r = P_r^n = \frac{n!}{(n-r)!}$$

- Where  $n! = n(n-1)(n-2) \cdots (2)(1)$
- Rcode:  $n! = \text{factorial}(n)$



# Combinations

If we wish to count the ways of selecting  $r$  objects from a (usually larger) set of  $n$  objects, this is the number of **combinations**. The number of combinations of  $n$  objects taken  $r$  unordered at a time is

$$nC_r = C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Rcode: `choose(n,r)`

## Example

1. Among 12 electrical components 4 are known not to function. If 6 components are randomly selected, how many ways can we have less than 2 of components not functioning?

12 total w/ 4 bad and 8 good, we choose 6

What is total # of ways to choose 6 out 12 components  $\binom{12}{6}$

We want # combinations w/ fewer than 2 bad.

This means we must account for 1 bad and 0 bad

$$\binom{4}{0}\binom{8}{6} + \binom{4}{1}\binom{8}{5}$$

> choose(4,0)\*choose(8,6)+choose(4,1)\*choose(8,5)  
[1] 252

## Counting Example

2. Suppose we select randomly 6 marbles drawn from a bag containing 9 white and 7 black marbles. What is the probability that at least 2 of the marbles drawn are white?

Let  $W$  denote # of white marbles,  $B$  # of black marbles

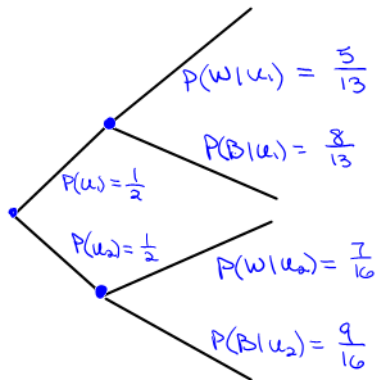
$$\begin{aligned}\text{Want } P(W \geq 2) &= P((W=2) \cup (W=3) \cup (W=4) \cup (W=5) \cup (W=6)) \\ &= P(W=2) + P(W=3) + P(W=4) + P(W=5) + P(W=6) \\ &= \sum_{k=2}^6 P(W=k)\end{aligned}$$

$$\begin{aligned}\text{or } P(W \geq 2) &= 1 - P(W \leq 1) = 1 - P(W=0) - P(W=1) \\ &= 1 - \frac{\binom{9}{0}\binom{7}{6}}{\binom{16}{6}} - \frac{\binom{9}{1}\binom{7}{5}}{\binom{16}{6}}\end{aligned}$$

```
> 1-(choose(9,0)*choose(7,6)+choose(9,1)*choose(7,5))/choose(16,6)
[1] 0.9755245
```

## Example of Tree Diagram

Urn 1 contains 5 white and 8 blue marbles. Urn 2 contains 7 white and 9 blue marbles. One of the two urns is chosen at random with one as likely to be chosen as the other. An urn is selected at random and then a marble is drawn from the chosen urn. Draw a probability tree diagram to show all the outcomes the experiment. In LaTeX, "`\cup`" = "`U`" and "`\cap`" = "`\cap`"



$$P(U_1 \cap W) = P(U_1) \cdot P(W|U_1) \\ = \frac{1}{2} \cdot \frac{5}{13} = \frac{5}{26}$$

$$P(U_1 \cap B) = \frac{1}{2} \cdot \frac{8}{13} = \frac{8}{26}$$

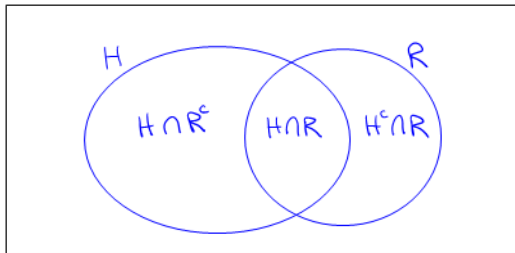
$$P(U_2 \cap W) = \frac{1}{2} \cdot \frac{7}{16} = \frac{7}{32}$$

$$P(U_2 \cap B) = \frac{1}{2} \cdot \frac{9}{16} = \frac{9}{32}$$

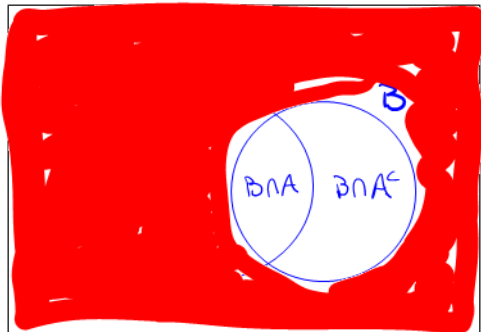
The probability that a randomly selected person has high blood pressure (the event  $H$ ) is  $P(H) = 0.5$  and the probability that a randomly selected person is a runner (the event  $R$ ) is  $P(R) = 0.4$ . The probability that a randomly selected person has high blood pressure and is a runner is 0.2. Find the probability that a randomly selected person either has high blood pressure or is a runner or both.

$$P(H) = 0.5, P(R) = 0.4, P(H \cap R) = 0.2$$

$$\begin{aligned} P(H \cup R) &= P(H) + P(R) - P(H \cap R) \\ &= P(H \cap R^c) + P(H \cap R) + P(H^c \cap R) \end{aligned}$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



can you explain what a proper set is, a subset, and an element?

$$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\text{Let } A = \{2, 4, 7, 9, 10\}$$

$$\text{Let } B = \{2, 7, 10\}$$

Is  $7 \in A$ ? yes

Is  $B \subseteq A$ ? yes since if  $b \in B$  then  $b \in A$

Is  $B \subset A$ ? yes since if  $b \in B$  then  $b \in A$  but  $9 \in A$  and  $9 \notin B$



An experimenter is randomly sampling 4 objects in order from among 61 objects. What is the total number of samples in the sample space?

1 7 57 60  
60 7 57 1

$$\sum_{k=2}^4 P(W=k)$$

What does this mean?

$\sum$  means we are adding

What will we add?  $P(W=k)$

beginning w/  $k=2$  and  $k$  counts up to 4

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