# Digital Image Processing

COSC 6380/4393

Lecture – 12

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# HISTOGRAM SHAPING

- Apply point operation such that the intensity histogram has a desired shape (target shape)
- Often times, the transformation function is non-linear.
- We now describe methods for **histogram shaping**.
- Accomplished by point operations: object **shape** and **location** are **unchanged**.

# DEFINITION

- Define the **normalized histogram**:

$$\mathbf{p_I}(k) = \left(\frac{1}{N^2}\right) \mathbf{H_I}(k) \; ; \; k = 0, \ldots, K-1$$

- These values **sum to one**: $\sum_{k=0}^{K-1} \mathbf{p_I}(k) = 1$
- Here $\mathbf{p_I}(k)$ is the **probability** that gray-level k will occur (at any given pixel)
- $\mathbf{p_I}(k) \approx$ probability of gray-level k
- The **cumulative histogram** is

$$\mathbf{P_I}(r) = \sum_{k=0}^{r} \mathbf{p_I}(k) \; ; \; r = 0, \ldots, K-1$$

- $\mathbf{P_I}(r)$ is a nondecreasing function, and $\mathbf{P_I}(K-1) = 1$.

# INTERPRETATION

- With the probabilistic interpretation, at a point (i, j):
- $\mathbf{P}_I(r) = Pr\{I(i, j) \leq r\}$

# CONTINUOUS HISTOGRAMS

- Suppose **p**(x) and **P**(x) are **continuous**: they may be regarded as probability density (pdf) and cumulative distribution (cdf).
- $\mathbf{P}^{-1}(x)$ exists or can be defined **by convention**.

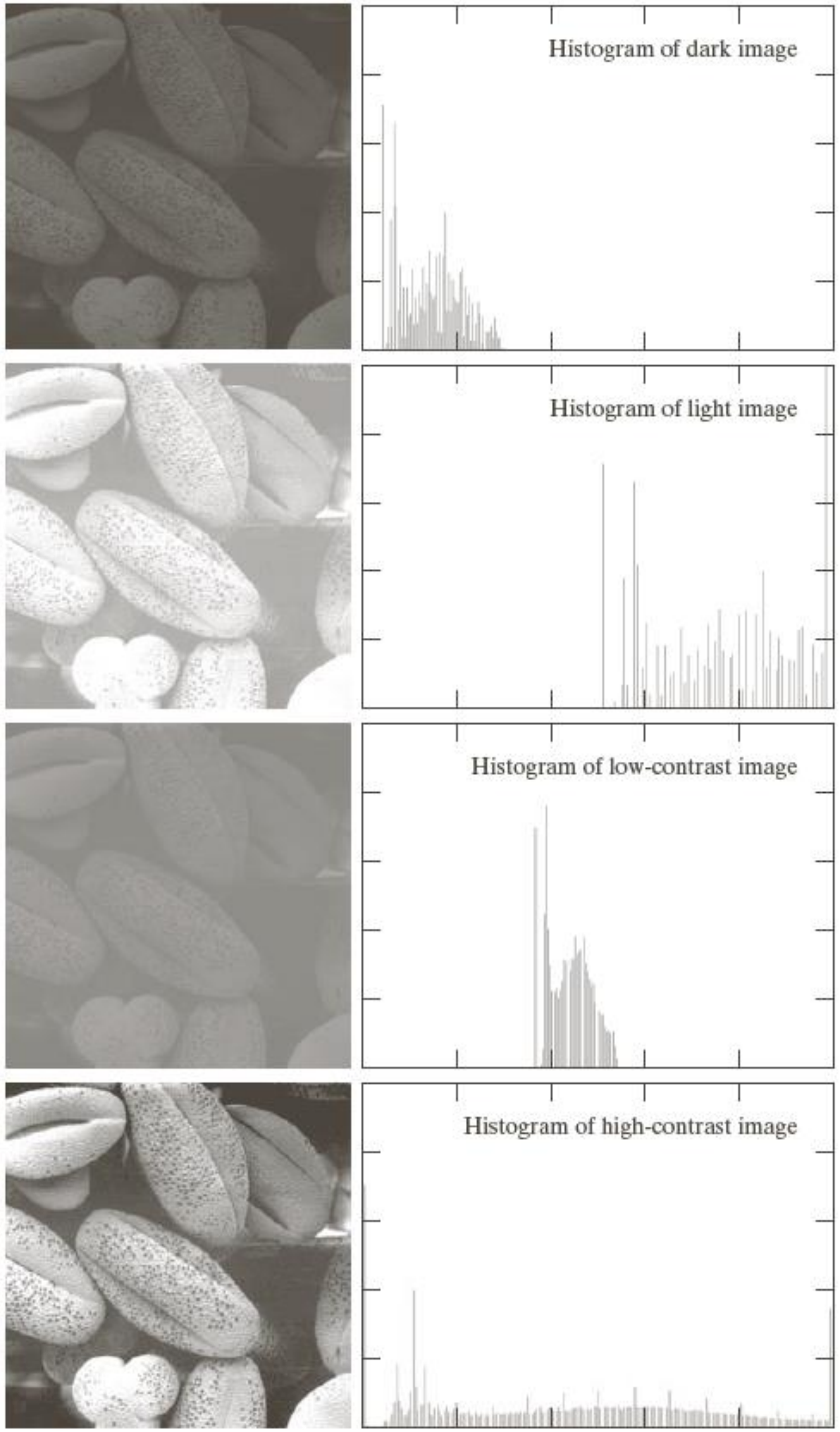
$$If\ Y\ =\ F(X),$$
$$F\ -\ a\ transformation\ function$$

CDF:

$$\boldsymbol{if\ y = F(x) \Rightarrow y = P_Y{}^{-1}\left(P_X(x)\right)}$$

# HISTOGRAM SHAPING

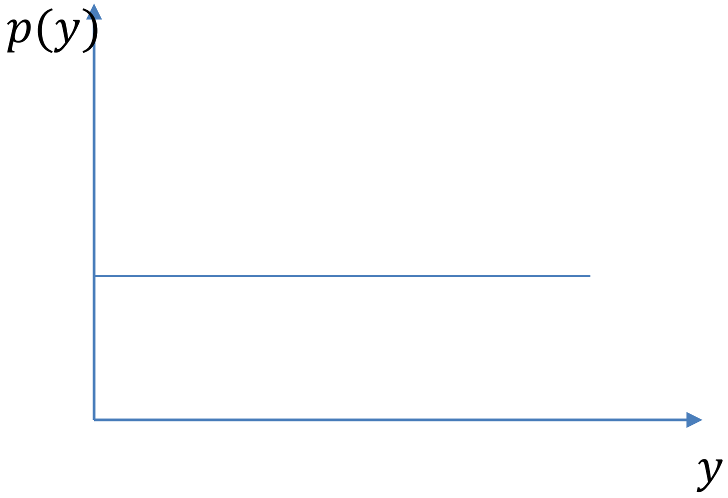
- We now describe methods for **histogram shaping**.
- Accomplished by point operations: object **shape** and **location** are **unchanged**.
- <u>**Histogram Flattening (Uniform)**</u>
- An image with a **flat** histogram makes rich use of the available gray-scale range. This might be an image with
  - Smooth gradations in gray scale covering many gray levels
  - Lots of texture covering many gray levels


Histogram of dark image
Histogram of light image
Histogram of low-contrast image
Histogram of high-contrast image

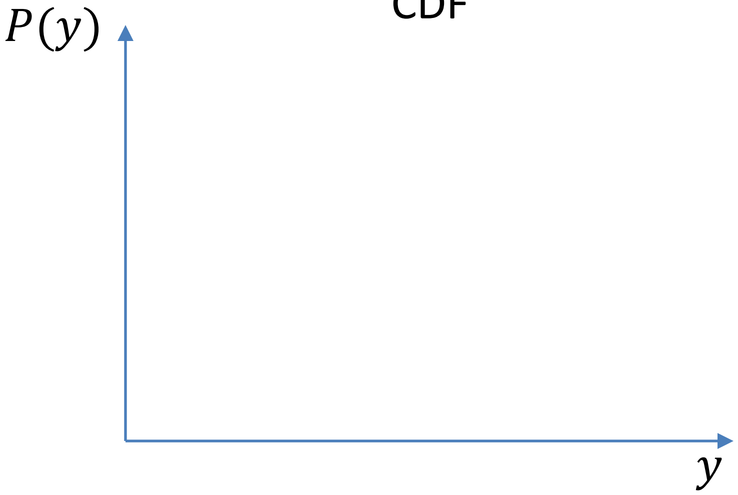
ON

# Uniform Distribution

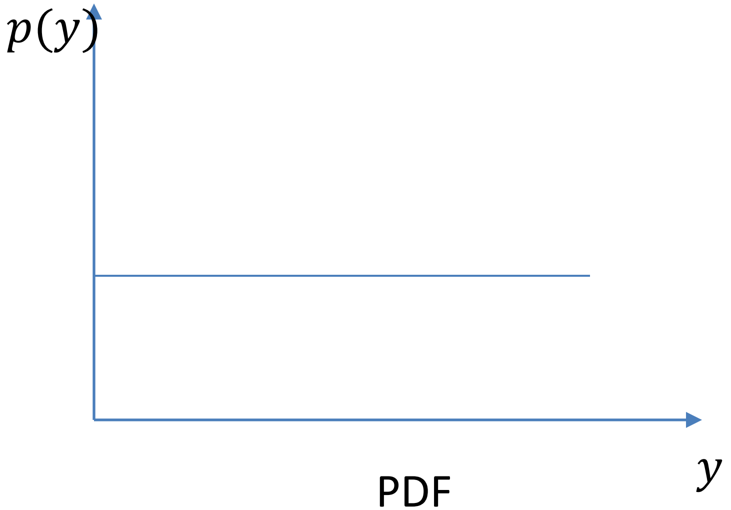
PDF



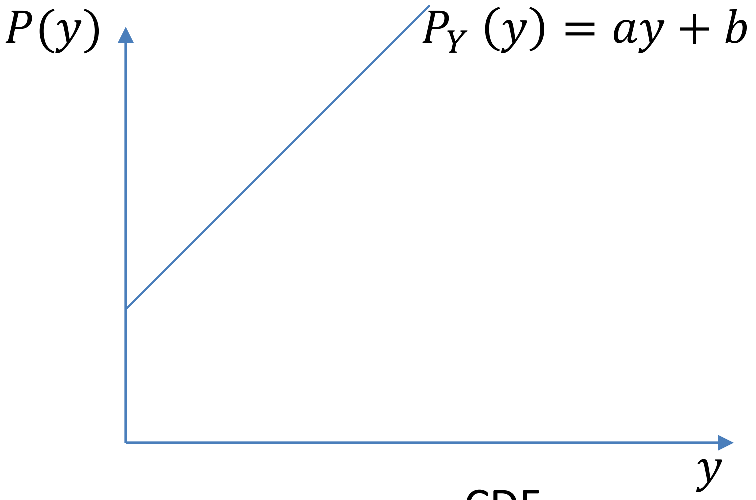

CDF



# Uniform Distribution




PDF




CDF

$$y \propto P_Y\,(y)$$

# Histogram Flattening

$$if\ y = F(x) \Rightarrow y = {P_Y}^{-1}\big(P_X(x)\big), P_Y(y) = P_X(x)$$
$$P_Y(y) = \mathrm{ay} + \mathrm{b},$$
$$P_X(x) = ay + b \Rightarrow y \propto P_X(x)$$

We can obtain an image **J** with an **approximately** flat histogram from an image **I** by the following procedure.

# HISTOGRAM FLATTENING

- Suppose we want to **flatten** the histogram of image **I**.
- Define the **cumulative histogram image** $\mathbf{J}_1 = \mathbf{P}(\mathbf{I})$:

$$J_1(i, j) = \mathbf{P_I}[I(i, j)] = \sum_{k=0}^{I(i, j)} \mathbf{p_I}(k)$$

- At each pixel, this is the cumulative histogram evaluated at the grey level of the pixel.
- Note that: $0 \leq J_1(i, j) \leq 1$
- The elements of the **cumulative probability** image $\mathbf{J}_1$ will be **approximately** linearly distributed between 0 and 1.
- Then scale $\mathbf{J}_1$ to cover the range 0, ..., K-1, produce the histogram-flattened image **J**:
- $J(i, j) = \text{INT}\left[\,(K-1)\cdot J_1(i, j) + 0.5\,\right]$
- This is best understood by an example:

# EXAMPLE

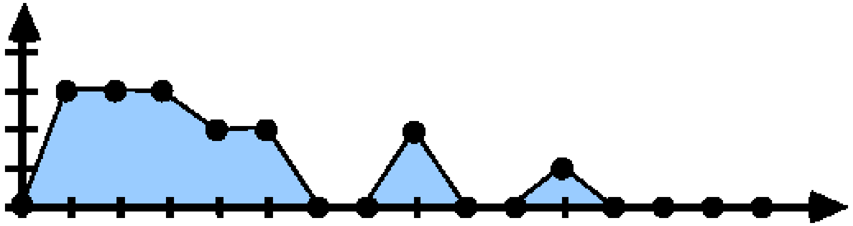
- Given a 4 x 4 image **I** with gray-level range {0, ..., 15} (K-1 = 15):

**I** =

| | | | |
|---|---|---|---|
| 1 | 1 | 3 | 4 |
| 2 | 5 | 3 | 2 |
| 8 | 1 | 8 | 2 |
| 4 | 5 | 3 | 11 |

- It's histogram is

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H(k) | 0 | 3 | 3 | 3 | 2 | 2 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |



# EXAMPLE

- The normalized histogram is

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p(k) | 0 | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{2}{16}$ | $\frac{2}{16}$ | 0 | 0 | $\frac{2}{16}$ | 0 | 0 | $\frac{1}{16}$ | 0 | 0 | 0 | 0 |

- From which we can compute the intermediate image **J1**

$$\mathbf{I} = \begin{array}{|c|c|c|c|}\hline 1 & 1 & 3 & 4 \\ \hline 2 & 5 & 3 & 2 \\ \hline 8 & 1 & 8 & 2 \\ \hline 4 & 5 & 3 & 11 \\ \hline \end{array}$$

$$\mathbf{J}_1 = \begin{array}{|c|c|c|c|}\hline 3/16 & 3/16 & 9/16 & 11/16 \\ \hline 6/16 & 13/16 & 9/16 & 6/16 \\ \hline 15/16 & 3/16 & 15/16 & 6/16 \\ \hline 11/16 & 13/16 & 9/16 & 16/16 \\ \hline \end{array}$$

# EXAMPLE

- The normalized histogram is

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p(k) | 0 | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{2}{16}$ | $\frac{2}{16}$ | 0 | 0 | $\frac{2}{16}$ | 0 | 0 | $\frac{1}{16}$ | 0 | 0 | 0 | 0 |

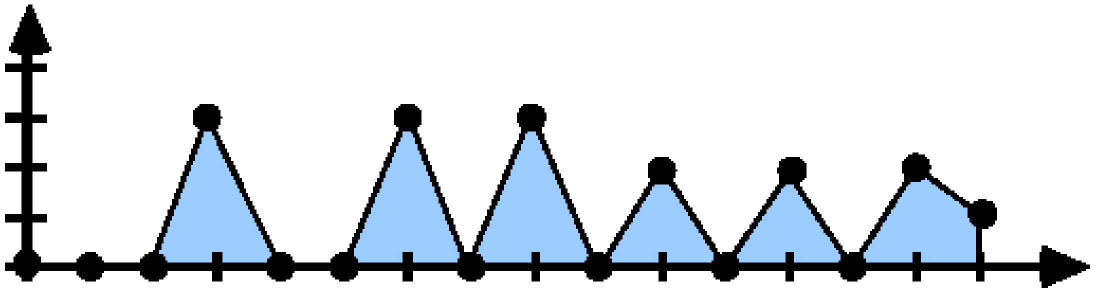
- From which we can compute the intermediate image **J1** and finally the "flattened" image **J**:

$$
\mathbf{J}_1 = \begin{array}{|c|c|c|c|}\hline 3/16 & 3/16 & 9/16 & 11/16 \\ \hline 6/16 & 13/16 & 9/16 & 6/16 \\ \hline 15/16 & 3/16 & 15/16 & 6/16 \\ \hline 11/16 & 13/16 & 9/16 & 16/16 \\ \hline \end{array}
\qquad
\mathbf{J} = \begin{array}{|c|c|c|c|}\hline 3 & 3 & 8 & 10 \\ \hline 6 & 12 & 8 & 6 \\ \hline 14 & 3 & 14 & 6 \\ \hline 10 & 12 & 8 & 15 \\ \hline \end{array}
$$

# EXAMPLE

- The new, **flattened** histogram looks like this:

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H(k) | 0 | 0 | 0 | 3 | 0 | 0 | 3 | 0 | 3 | 0 | 2 | 0 | 2 | 0 | 2 | 1 |



- The heights H(k) cannot be reduced, just moved - or stacked, so:
- **Digital** histogram flattening doesn't really "flatten" the histogram - it just makes it "flatter" by **spreading out** the histogram.
- The **spaces** that appear are highly characteristic of a "flattened" histogram - especially when the original histogram is highly compressed.

# Histogram Shaping

$$if\ y = F(x) \Rightarrow P_Y\ (y) = P_X(x)$$
$$P_Y(y) = P_X(x)$$
$$y = P_Y^{-1}(P_X(x))$$

# HISTOGRAM SHAPING

- Can create a modified image **J** with an approximate **specified** histogram shape, such as a triangle or bell-shaped curve.
- Let $\mathbf{H_J}(k)$ be the desired histogram shape, with corresponding normalized values (probabilities) $\mathbf{p_J}(k)$.
- Define the cumulative histogram image as before

$$J_1(i, j) = \mathbf{P_I}[I(i, j)] = \sum_{k=0}^{I(i, j)} \mathbf{p_I}(k)$$

- We also define the cumulative probabilities:

$$\mathbf{P_J}(n) = \sum_{k=0}^{n} \mathbf{p_J}(k)$$

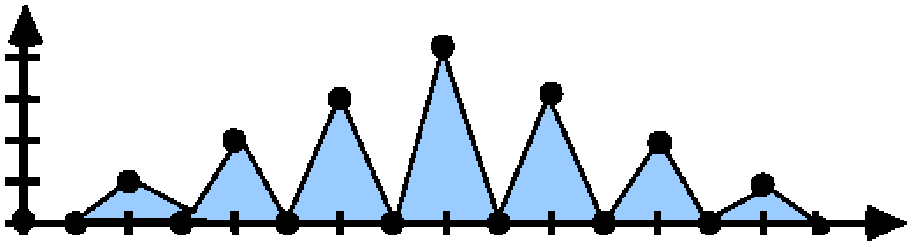
# EXAMPLE

- Consider the same image as in the last example. We had

$$\mathbf{I} = \begin{bmatrix} 1 & 1 & 3 & 4 \\ 2 & 5 & 3 & 2 \\ 8 & 1 & 8 & 2 \\ 4 & 5 & 3 & 11 \end{bmatrix} \qquad \mathbf{J}_1 = \begin{bmatrix} 3/16 & 3/16 & 9/16 & 11/16 \\ 6/16 & 13/16 & 9/16 & 6/16 \\ 15/16 & 3/16 & 15/16 & 6/16 \\ 11/16 & 13/16 & 9/16 & 16/16 \end{bmatrix}$$

- Fit this t

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_{\mathbf{J}}(k)$ | 0 | 0 | 1 | 0 | 2 | 0 | 3 | 0 | 4 | 0 | 3 | 0 | 2 | 0 | 1 | 0 |
| $p_{\mathbf{J}}(k)$ | 0 | 0 | $\frac{1}{16}$ | 0 | $\frac{2}{16}$ | 0 | $\frac{3}{16}$ | 0 | $\frac{4}{16}$ | 0 | $\frac{3}{16}$ | 0 | $\frac{2}{16}$ | 0 | $\frac{1}{16}$ | 0 |



# EXAMPLE

- Here's the cumulative (summed) probabilities associated with it:

| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathbf{P}_{\mathbf{J}}(n)$ | 0 | 0 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{6}{16}$ | $\frac{6}{16}$ | $\frac{10}{16}$ | $\frac{10}{16}$ | $\frac{13}{16}$ | $\frac{13}{16}$ | $\frac{15}{16}$ | $\frac{15}{16}$ | $\frac{16}{16}$ | $\frac{16}{16}$ |

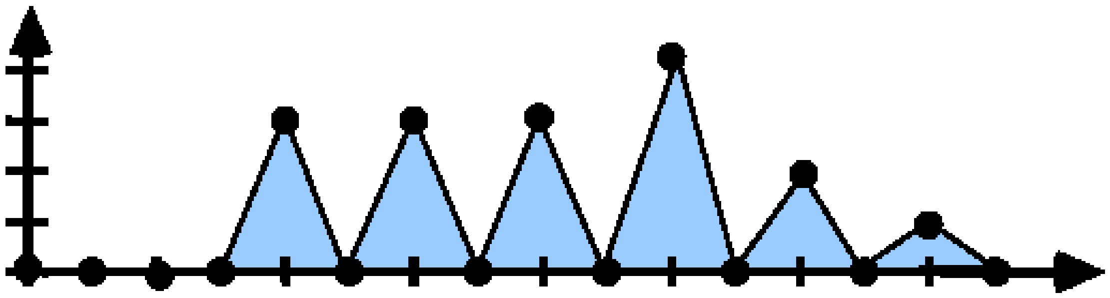
- **Careful** visual inspection of $\mathbf{J}_1$ let's us form the new image:

$$\mathbf{J} = \begin{array}{|c|c|c|c|} \hline 4 & 4 & 8 & 10 \\ \hline 6 & 10 & 8 & 6 \\ \hline 12 & 4 & 12 & 6 \\ \hline 10 & 10 & 8 & 14 \\ \hline \end{array}$$

# EXAMPLE

- Here's the new histogram:

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H_J(k)$ | 0 | 0 | 0 | 0 | 3 | 0 | 3 | 0 | 3 | 0 | 4 | 0 | 2 | 0 | 1 | 0 |



# HISTOGRAM MATCHING

- Just a special case of histogram shaping.
- Difference: the histogram of the original image **I** is matched to that of another image **I**´ .
- Otherwise the procedure is identical, once the cumulative probabilities are computed for the model image **I**´ .
- Useful application: **Comparing** similar images of the same scene obtained under different conditions (e.g., lighting, time of day). Extends the concept of equalizing AOD described earlier.

# BASIC ALGEBRAIC IMAGE OPERATIONS

- Algebraic image operations (between images) are quite simple
- Suppose we have two N x N images $\mathbf{I}_1$ and $\mathbf{I}_2$. The four basic algebraic operations (like the ones on your calculator) are:
- **Pointwise Matrix Addition**
- $\mathbf{J} = \mathbf{I}_1 + \mathbf{I}_2$ if $J(i, j) = \mathbf{I}_1(i, j) + \mathbf{I}_2(i, j)$ for $0 \le i, j \le N-1$
- **Pointwise Matrix Subtraction**
- $\mathbf{J} = \mathbf{I}_1 - \mathbf{I}_2$ if $J(i, j) = \mathbf{I}_1(i, j) - \mathbf{I}_2(i, j)$ for $0 \le i, j \le N-1$
- **Pointwise Matrix Multiplication**
- $\mathbf{J} = \mathbf{I}_1 .* \mathbf{I}_2$ if $J(i, j) = \mathbf{I}_1(i, j) \times \mathbf{I}_2(i, j)$ for $0 \le i, j \le N-1$
- **Pointwise Matrix Division**
- $\mathbf{J} = \mathbf{I}_1 ./ \mathbf{I}_2$ if $J(i, j) = \mathbf{I}_1(i, j) / \mathbf{I}_2(i, j)$ for $0 \le i, j \le N-1$

# APPLICATIONS OF ALGEBRAIC OPERATIONS

- Although simple, algebraic operations form the backbone of most of digital image processing
- We will look at two simple but **important** applications of algebraic operations on images:
  - Frame averaging for noise reduction
  - Motion detection

# Frame Averaging for Noise Reduction

- An image **J** is often corrupted by **additive noise**:
  - Surface radiation scatter
  - Noise in the camera
  - Thermal noise in a computer circuit
  - Channel transmission noise
- We can model such a **noisy image** as the sum an an **original, uncorrupted image I** and a **noise image N**:

$$\mathbf{J} = \mathbf{I} + \mathbf{N},$$

- where the elements N(i, j) of **N** are **random variables**

# Frame Averaging

- We will assume that the noise is **zero mean** (ergodic), which means that the **sample mean** of M noise matrices tends towards zero as M grows large:

$$\left(\frac{1}{M}\right)\sum_{i=1}^{M}\mathbf{N}_i = \left(\frac{1}{M}\right)\left[\,\mathbf{N}_1 + \cdots + \mathbf{N}_M\,\right] \sim= \mathbf{0}\ \ (\textbf{matrix of zeros})$$

- Averaging together large many zero-mean noise samples produces a value near zero

# Frame Averaging

- Suppose that we obtain M images $J_1, \ldots, \mathbf{J}_M$ of the **same scene**
- \- In rapid succession, so that there is **no motion** between frames
- \- Or there is **no motion** in the scene.
- However, the frames are **noisy:**

$$\mathbf{J}_i = \mathbf{I}_i + \mathbf{N}_i \text{ for } i = 1, \ldots, M.$$

- Suppose that we **average** the frames together:

$$\mathbf{J} = \left(\frac{1}{M}\right)\sum_{i=1}^{M} \mathbf{J}_i = \left(\frac{1}{M}\right)\sum_{i=1}^{M} \left[\, \mathbf{I}_i + \mathbf{N}_i \,\right] = \left(\frac{1}{M}\right)\sum_{i=1}^{M} \mathbf{I}_i + \left(\frac{1}{M}\right)\sum_{i=1}^{M} \mathbf{N}_i$$

# Frame Averaging

- However, since $\mathbf{I}_1 = \mathbf{I}_2 = \cdots = \mathbf{I}_M = \mathbf{I}$, then

$$\left(\frac{1}{M}\right)\sum_{i=1}^{M} \mathbf{I}_i = \left(\frac{1}{M}\right)\left[\,\mathbf{I} + \mathbf{I} + \cdots + \mathbf{I}\,\right] = \left(\frac{1}{M}\right) \cdot M \cdot \mathbf{I} = \mathbf{I}$$

- and from before

$$\left(\frac{1}{M}\right)\sum_{i=1}^{M} \mathbf{N}_i \sim= \mathbf{0}$$

- Hence we can expect that

$$\mathbf{J} \sim= \mathbf{I} + \mathbf{0} \sim= \mathbf{I}$$

- if enough frames (M) are averaged together

# Motion Detection

- Often it is of interest to detect **object motion** between frames
- **Applications**: video compression, target recognition and tracking, security cameras, surveillance, automated inspection, etc.
- Here is a **simple** approach:
- Let $\mathbf{I}_1$ , $\mathbf{I}_2$ be consecutive frames taken in close time proximity, e.g., from a video camera
- Form the absolute difference image

$$\mathbf{J} = |\mathbf{I}_1 - \mathbf{I}_2|$$

- Applying a **full-scale contrast stretch** to **J** will give a more visually dramatic result

# Basic Geometric Image Operations

- Geometric image operations are the **opposite** of point operations: they modify **spatial positions** of pixels but not **gray levels**
- A geometric operation generally requires **two steps:**
- (1) A **spatial mapping of image coordinates** giving a new image function **J**: J(i, j) = I(i´ , j´ ) = I[a(i, j), b(i, j)]
- The coordinates a(i, j) and b(i, j) **are not generally integers !**
- For example: a(i, j) = i/3.5, b(i, j) = j/4.5
- Then J(i, j) = I(i/3.5, j/4.5), which has undefined coordinates!
- Which element of **I** do we define J(i, j) to be?

# INTERPOLATION

- Thus implies the need for a second operation:
- (2) **Interpolate** non-integer coordinates a(i, j) and b(i, j) to integer values, so that **J** can be expressed in **row-column format**

**Nearest Neighbor Interpolation**

- Simple-minded
- The geometrically transformed coordinates are mapped to the **nearest integer coordinates**:

  J(i, j) = I **{** INT[a(i, j)+0.5], INT[b(i, j)+0.5]**}**
- Serious drawback: Sudden intensity changes lead to the "jagged edge" effect

# The Basic Geometric Transformations

- The most basic geometric transformations are
- \- Translation
- \- Rotation
- \- Zooming

**Translation**

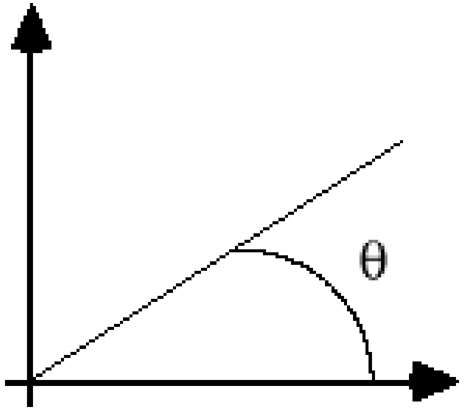
- Translation is the simplest geometric operation and requires no interpolation
- Let $a(i, j) = i - i_0$ , $b(i, j) = j - j_0$ where $(i_0 , j_0 )$ are **constants**
- In this case $J(i, j) = I(i - i_0 , j - j_0 )$; a **shift** or translation of the image by an amount $i_0$ in the vertical (row) direction and an amount $j_0$ in the horizontal direction

# Rotation

- Rotation of an image by an angle **q** relative to the x-axis is accomplished by the following transformation:

$$a(i, j) = i \cos(\theta) - j \sin(\theta)$$

$$b(i, j) = i \sin(\theta) + j \cos(\theta)$$




- Simplest cases:

$$\theta = 90° : [a(i, j), b(i, j)] = (-j, i)$$

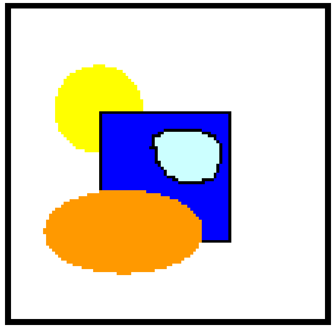
$$\theta = 180° : [a(i, j), b(i, j)] = (-i, -j)$$

$$\theta = -90° : [a(i, j), b(i, j)] = (j, -i)$$

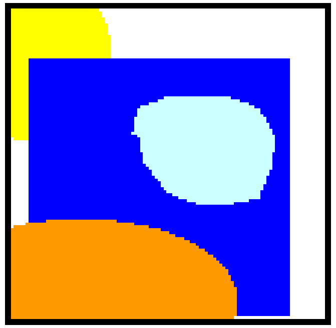
# Zooming

- Zooming **magnifies** an image by the mapping functions

  a(i, j) = i / c and b(i, j) = j / d
- where c >= 1 and d >= 1



original



2x zoomed

- For large magnifications, the zoomed image will look "blotchy" if a simple nearest neighbor interpolation is used