Digital Image Processing

COSC 6380/4393

Lecture – 20

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Slides from Dr. Shishir K Shah, and Frank Liu

Filtering in the Frequency Domain

1. Introduce commonly used frequency filters.

Review: Two Smoothing Averaging Filter Masks

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

Review: Laplace Operator

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Demo Filters

Convolution Theorem

- Let f and h be two function
- Lets us consider the convolution

$$f(t) \otimes h(t) = \sum_{\tau=-\infty}^{\infty} f(\tau)h(t-\tau)$$

$$F[f(t) \otimes h(t)] = \sum_{t=-\infty}^{\infty} [\sum_{\tau=-\infty}^{\infty} f(\tau)h(t-\tau)]e^{-\sqrt{-1}\mu t}$$

$$= \sum_{\tau=-\infty}^{\infty} f(\tau)[\sum_{t=-\infty}^{\infty} h(t-\tau)\, e^{-\sqrt{-1}\mu t}]$$

$$= \sum_{\tau=-\infty}^{\infty} f(\tau)[\sum_{t=-\infty}^{\infty} h(t-\tau)\, e^{-\sqrt{-1}\mu(t-\tau)}]\, e^{-\sqrt{-1}\mu\tau}$$

$$= \sum_{\tau=-\infty}^{\infty} f(\tau)[H(\mu)]\, e^{-\sqrt{-1}\mu(\tau)} = H(\mu) \sum_{\tau=-\infty}^{\infty} f(\tau)\, e^{-\sqrt{-1}\mu(\tau)}$$

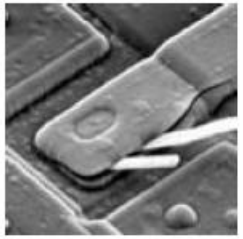
$$= H(\mu)F(\mu)$$

Convolution Theorem

- Fourier transform pairs

$$f(t) \otimes h(t) \Leftrightarrow H(\mu)F(\mu)$$

$$f(t)h(t) \Leftrightarrow H(\mu) \otimes F(\mu)$$



a	b	c
d	e	f
g	h	

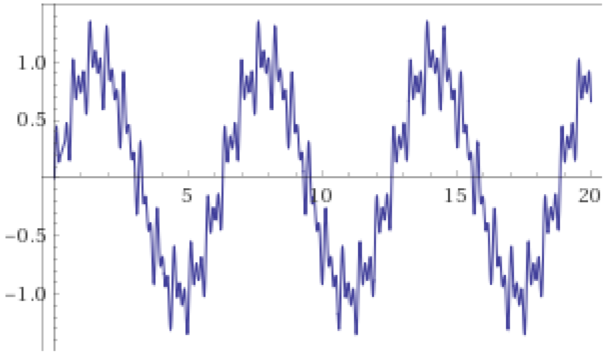
FIGURE 4.36
(a) An $M \times N$ image, f.
(b) Padded image, f_p of size $P \times Q$.
(c) Result of multiplying f_p by $(-1)^{x+y}$.
(d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$.
(f) Spectrum of the product HF_p.
(g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p.
(h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Filtering in Frequency Domain

- Smoothing
- Sharpening
- Band Pass and Band Reject

Example

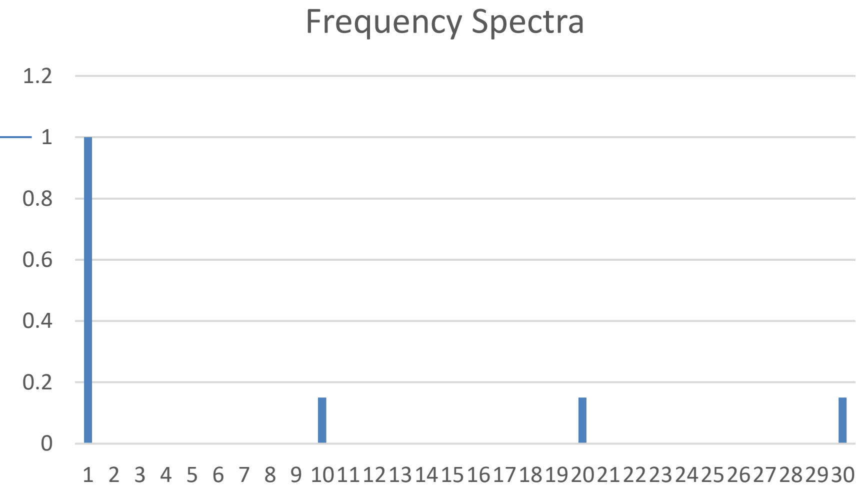
Filter: Remove Noise



FT

Coefficients of the sine and cosine waves

μ	0	1	…	10	…	20	….	….	30
cos	0	0	.	.	.	.	.	.	0
sin	0	1	0	0.15	0	0.15	.	0	0.15

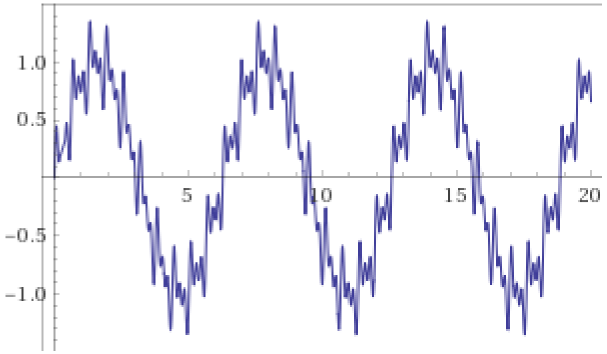


IFT

$$\mathrm{f}(t) = sin\ t + 0.15 sin\ 10t + 0.15 sin\ 20t + 0.15 sin\ 30t)$$

Example

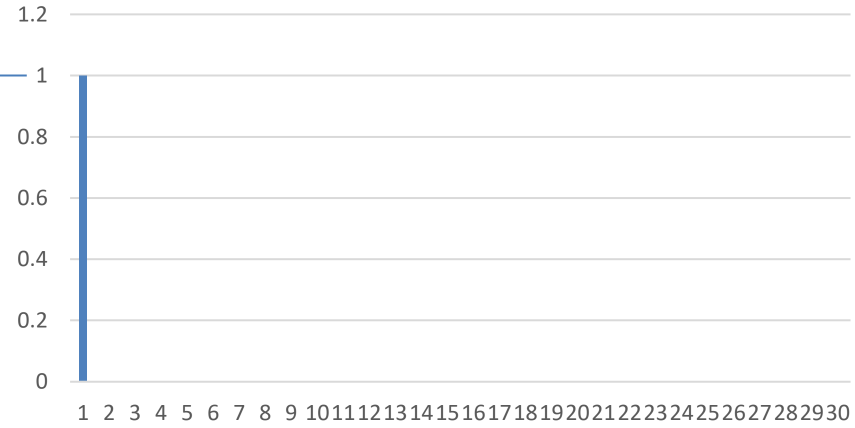
Filter: Remove Noise (Remove high frequencies)



FT

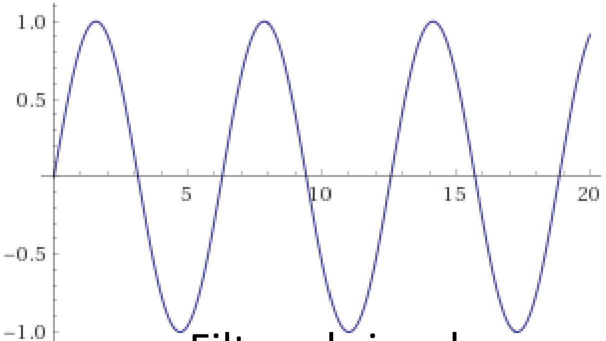
Coefficients of the sine and cosine waves

t	**0**	**1**	**…**	**10**	**…**	**20**	**….**	**….**	**30**
cos	0	0	.	.	.	.	.	.	0
sin	0	1	0	**0**	0	**0**	.	0	**0**

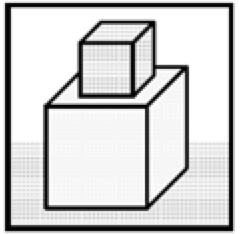


$$f(t) = \sin t + (0)\sin 10t + (0)\sin 20t + (0)\sin 30t) = \sin t$$

IFT



Filtered signal



Image

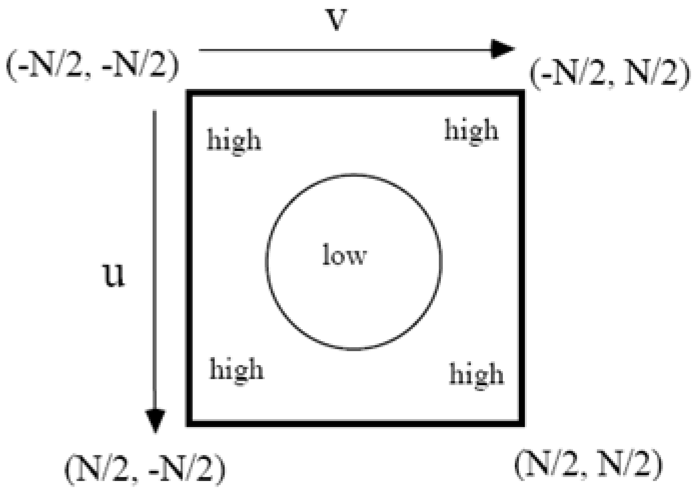


Image Smoothing Using Filter Domain Filters

1. Ideal Low Pass Filter
2. Butterworth Low Pass Filter
3. Gaussian Low Pass Filter

Image Smoothing Using Filter Domain Filters: ILPF

Ideal Lowpass Filters (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

Image Smoothing Using Filter Domain Filters: ILPF

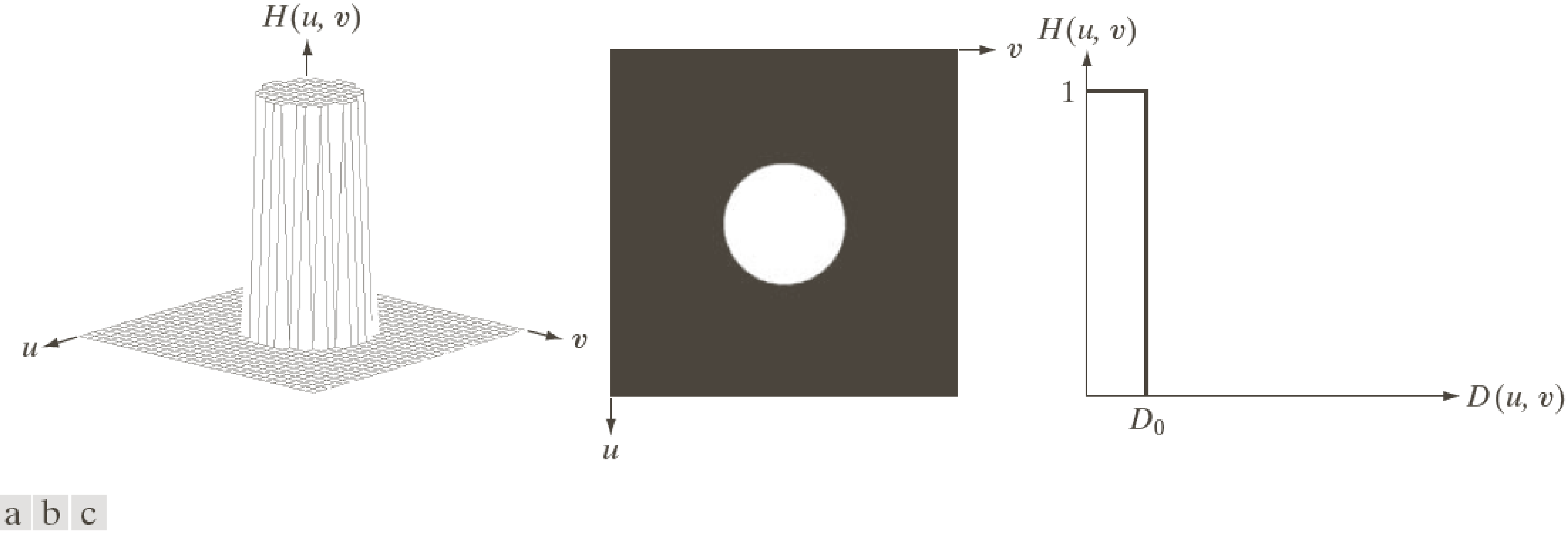
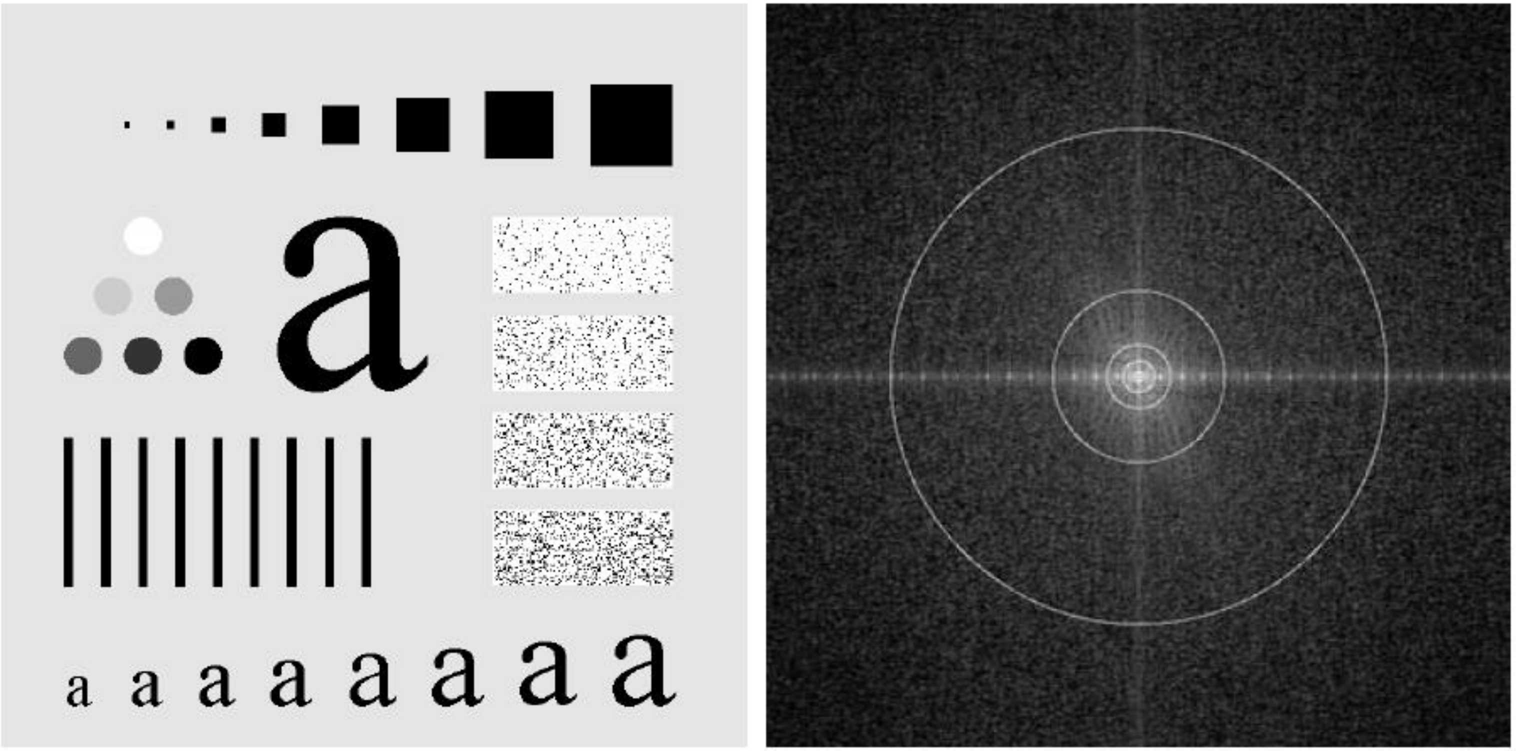


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

ILPF Filtering Example



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

ILPF Filtering Example

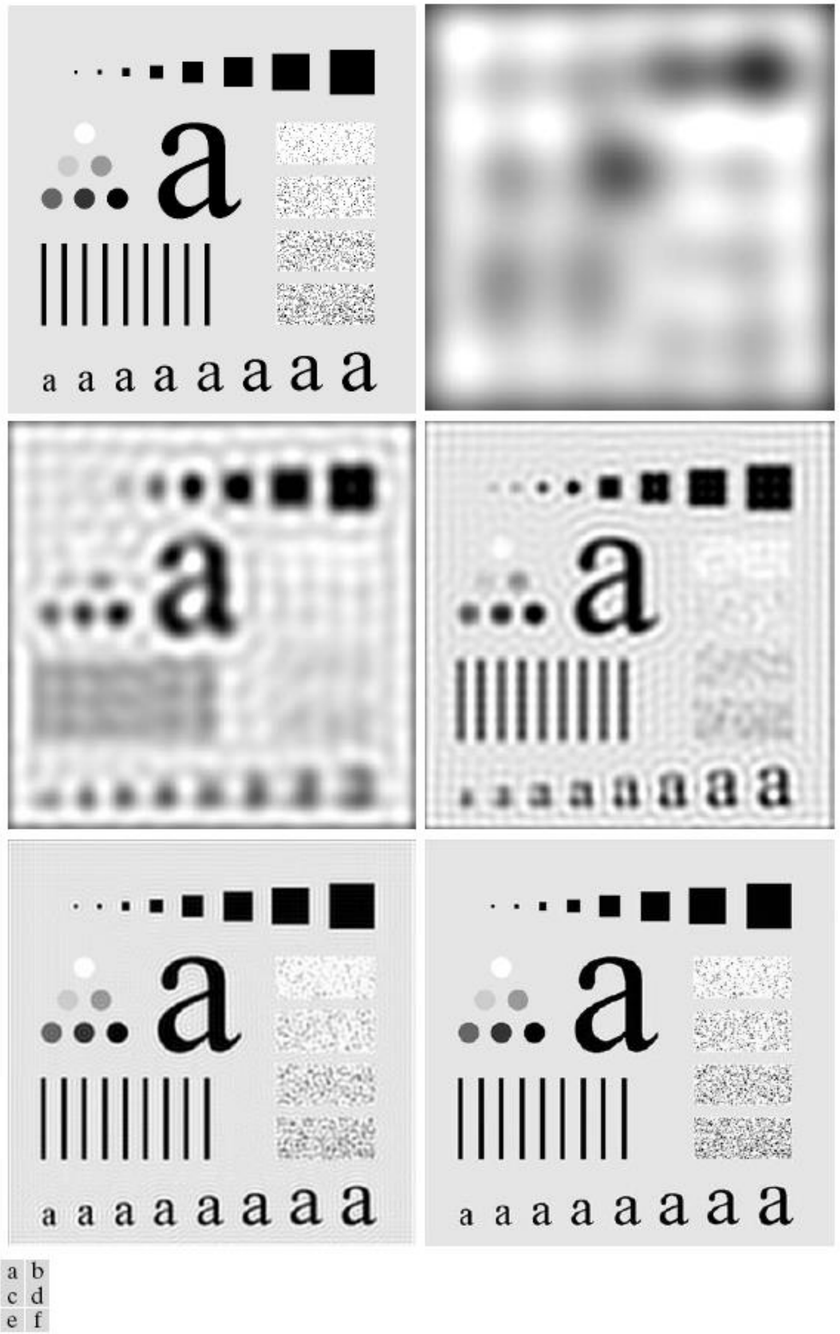
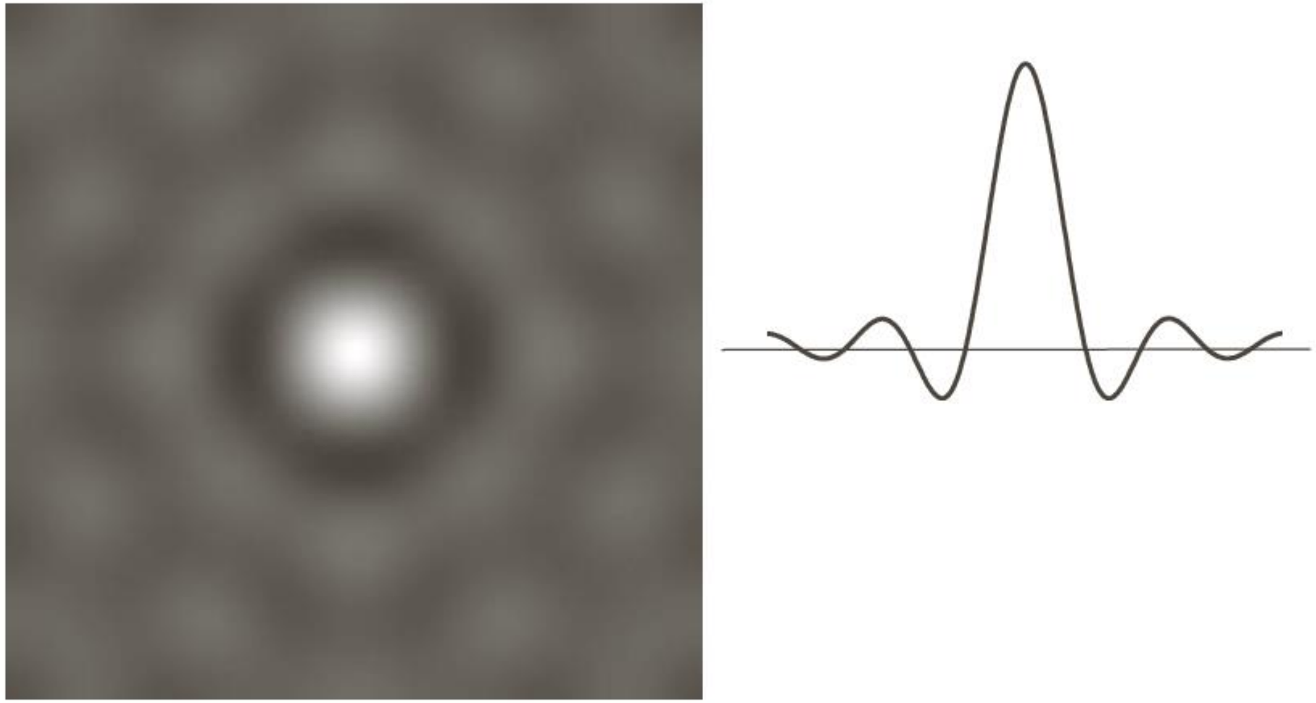


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

The Spatial Representation of ILPF



a b

FIGURE 4.43
(a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000.
(b) Intensity profile of a horizontal line passing through the center of the image.

Image Smoothing Using Filter Domain Filters: BLPF

Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

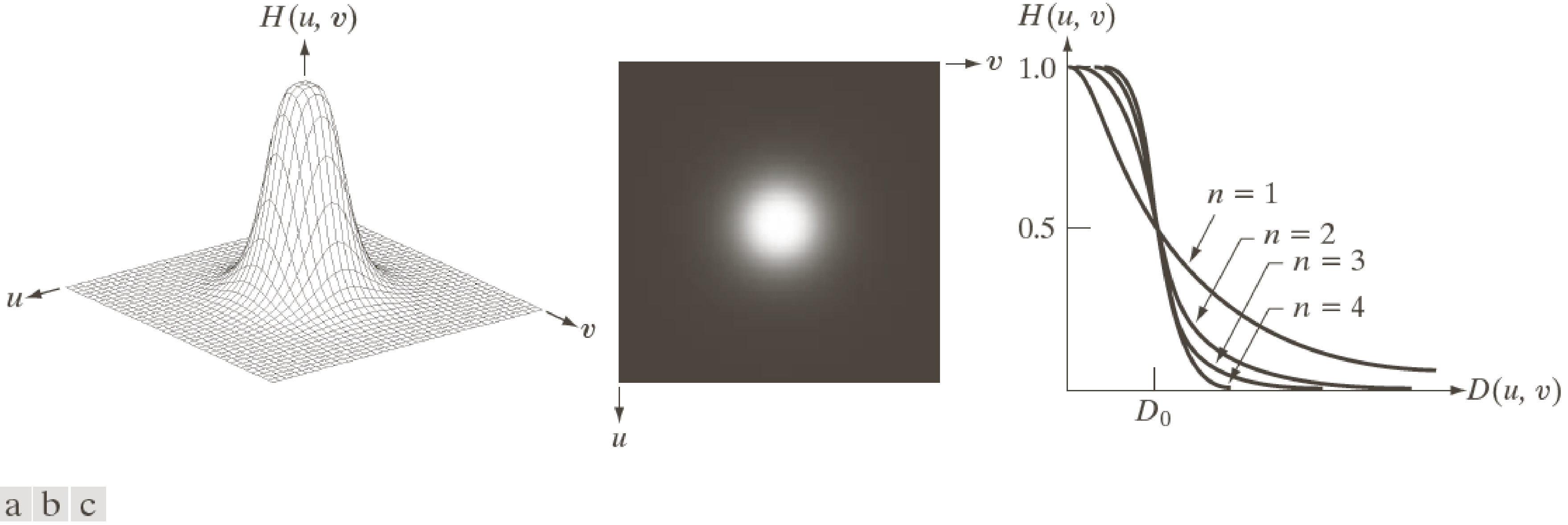
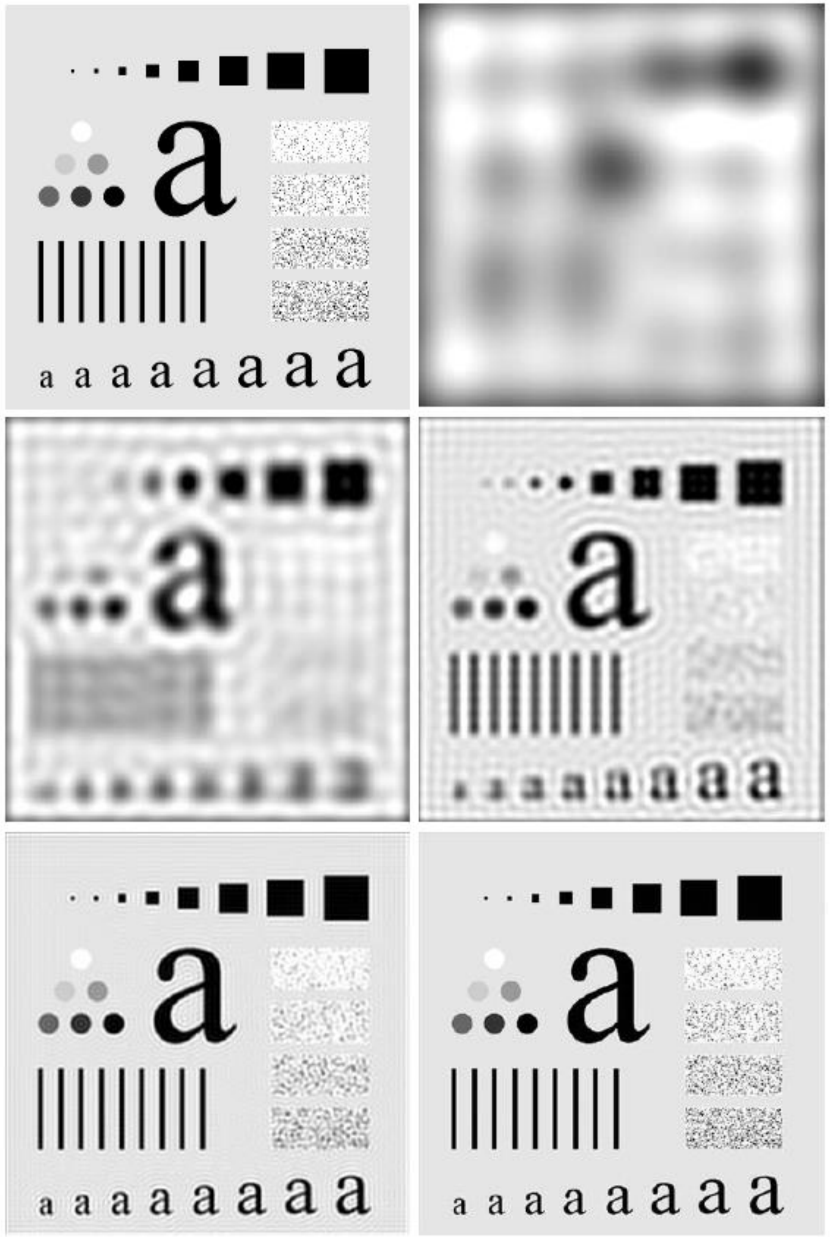
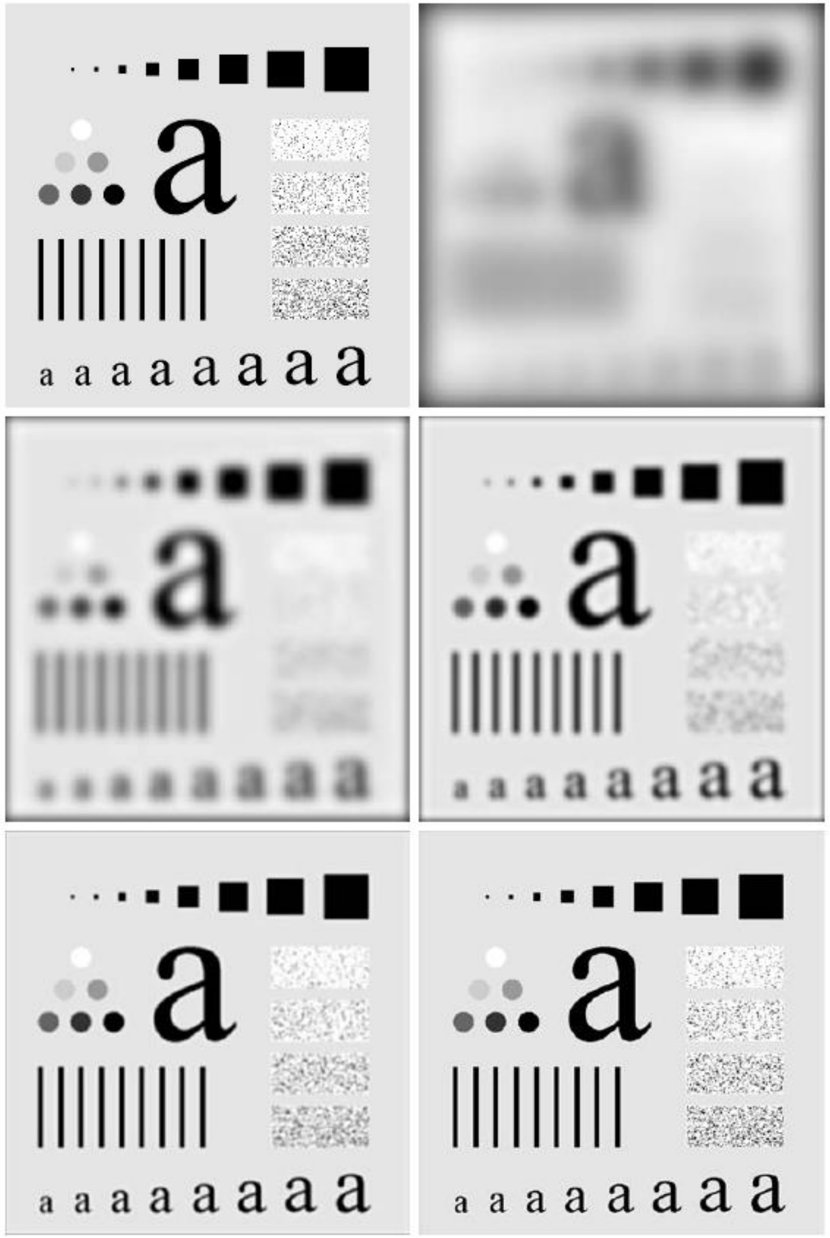


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

The Spatial Representation of BLPF

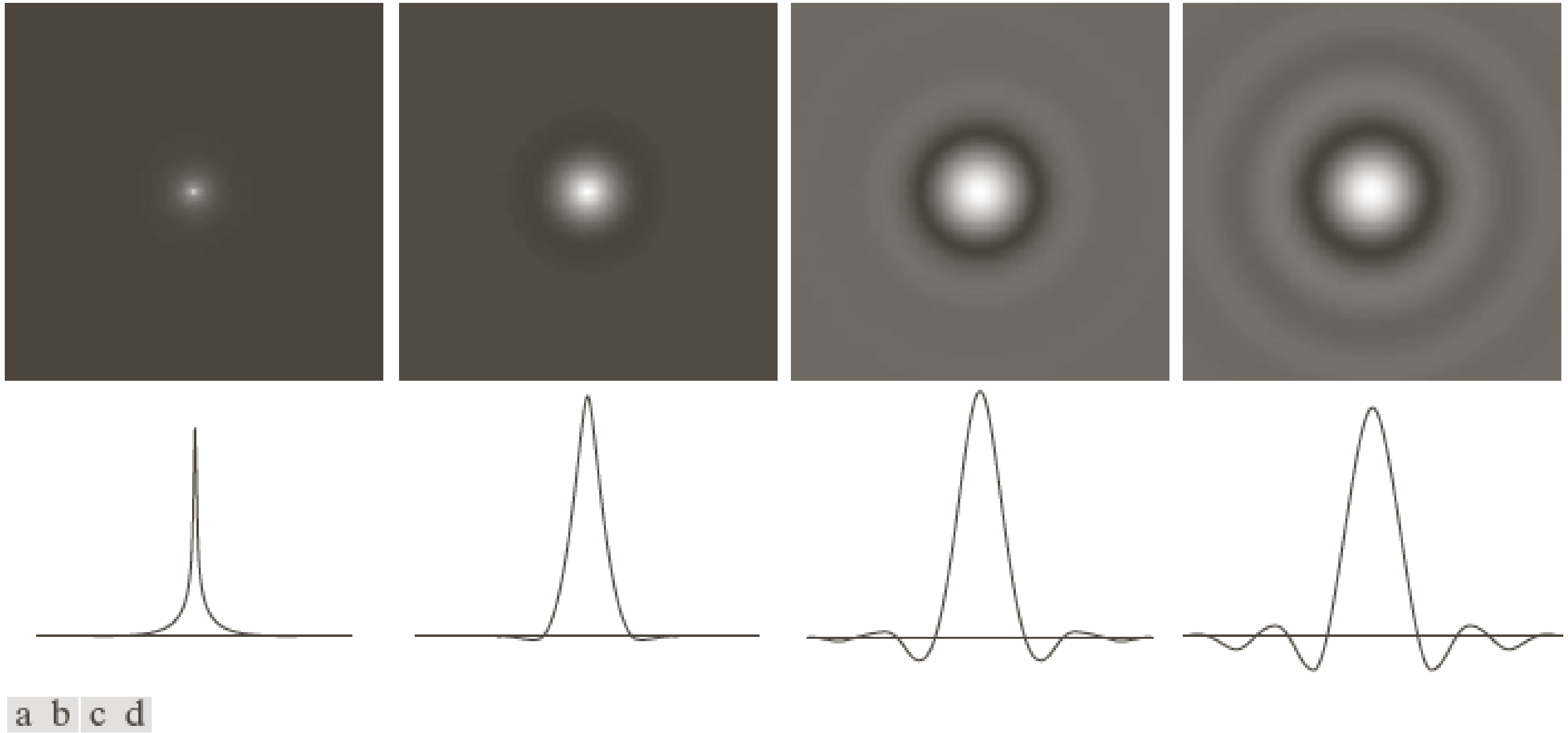


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Image Smoothing Using Filter Domain Filters: GLPF

Gaussian Lowpass Filters (GLPF) in two dimensions is given

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Image Smoothing Using Filter Domain Filters: GLPF

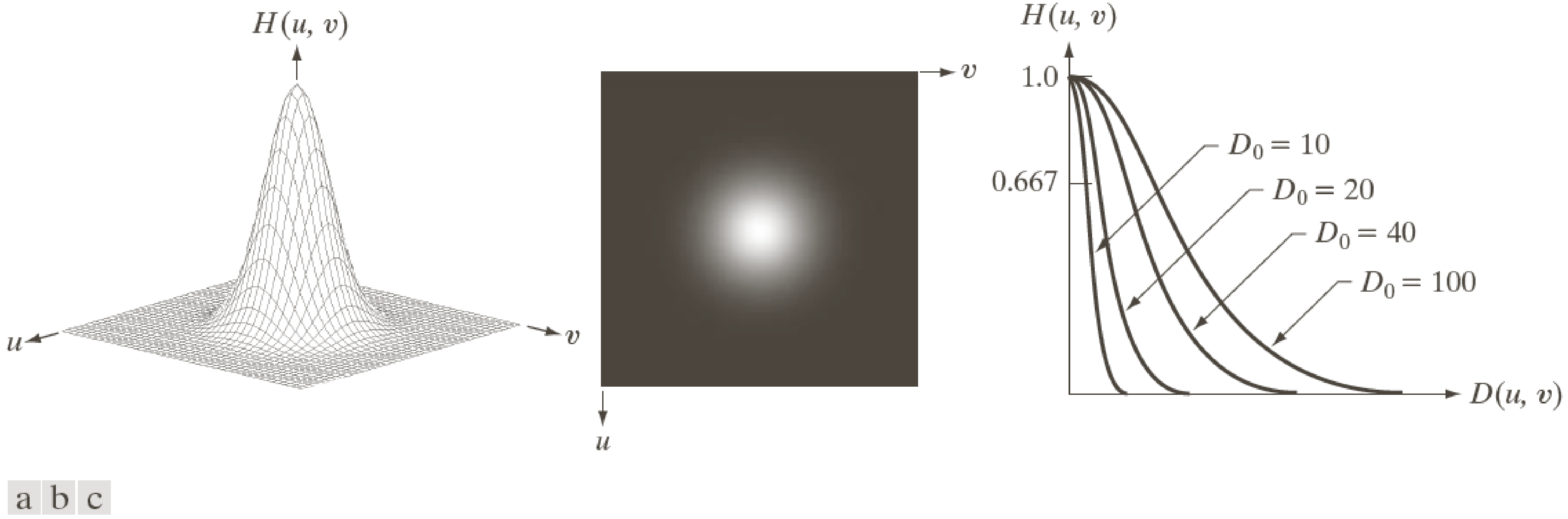
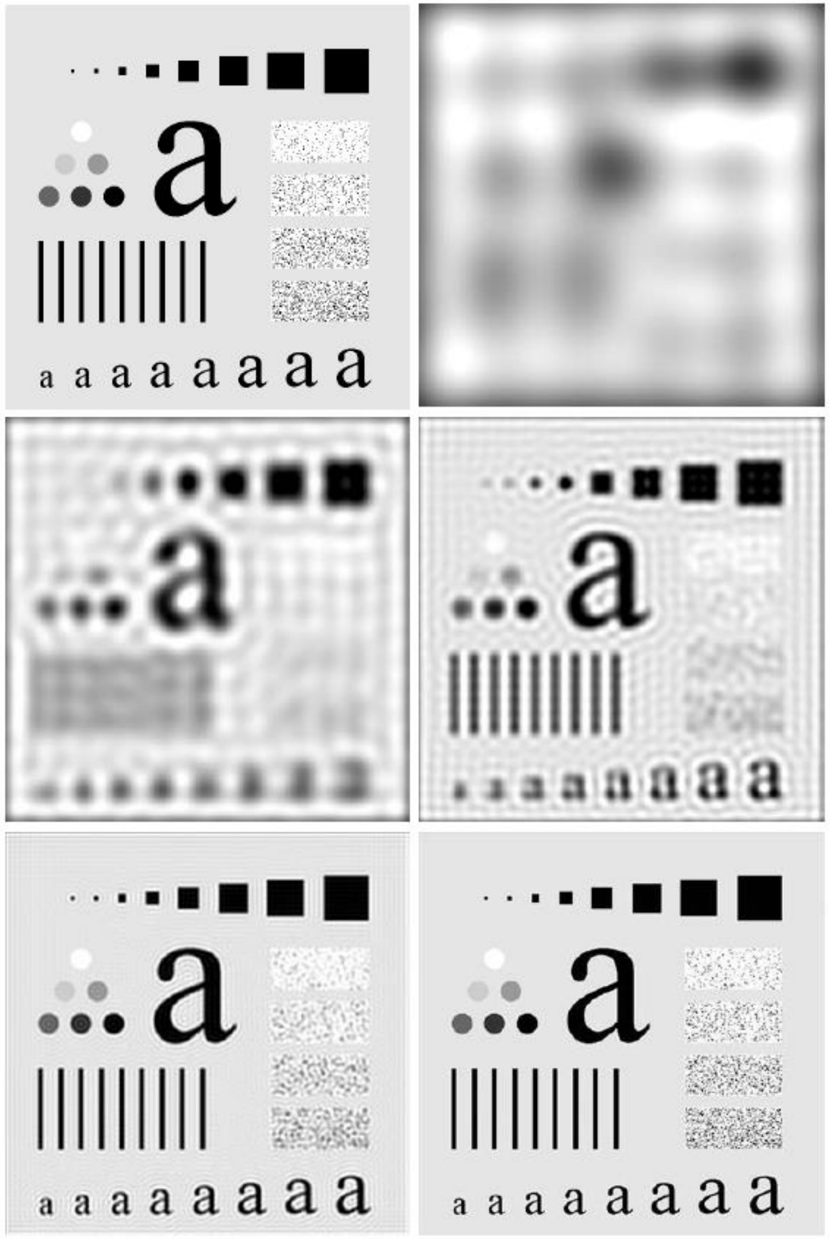
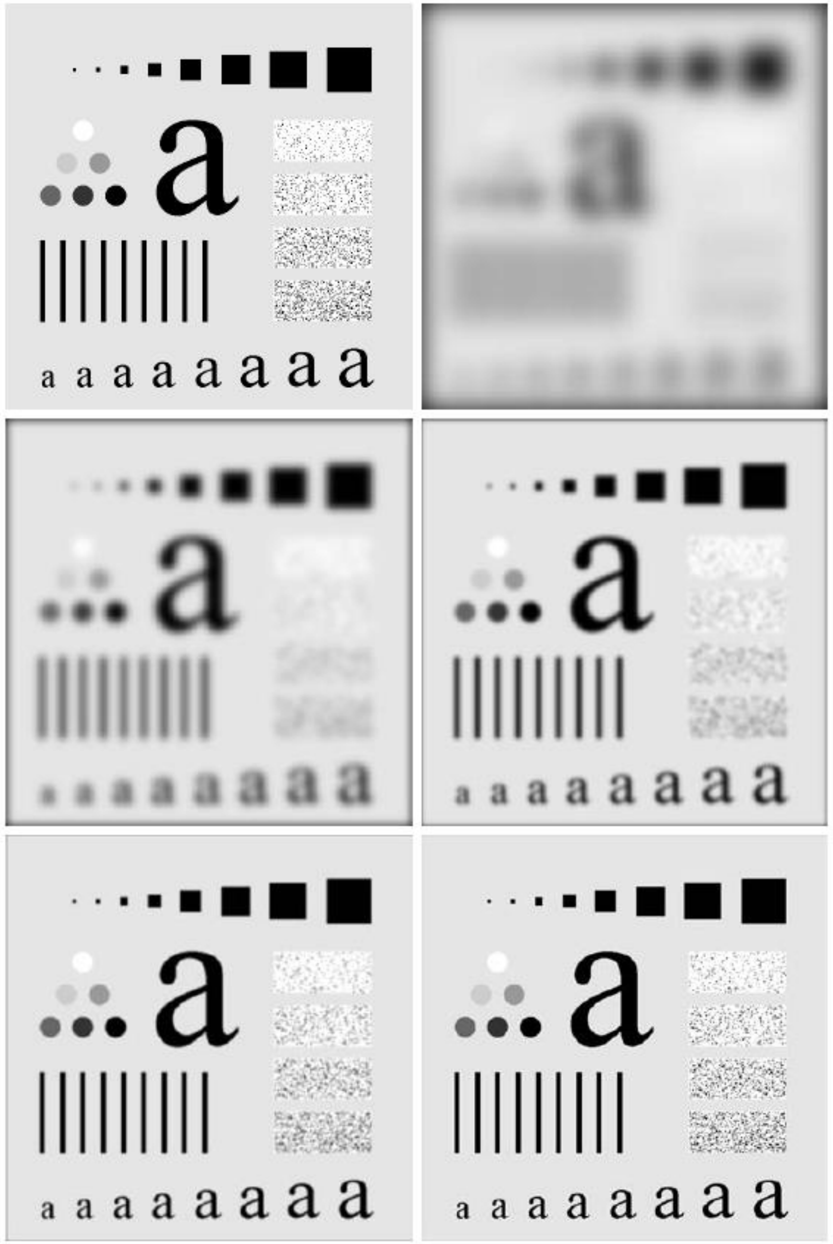


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.



a b
c d
e f

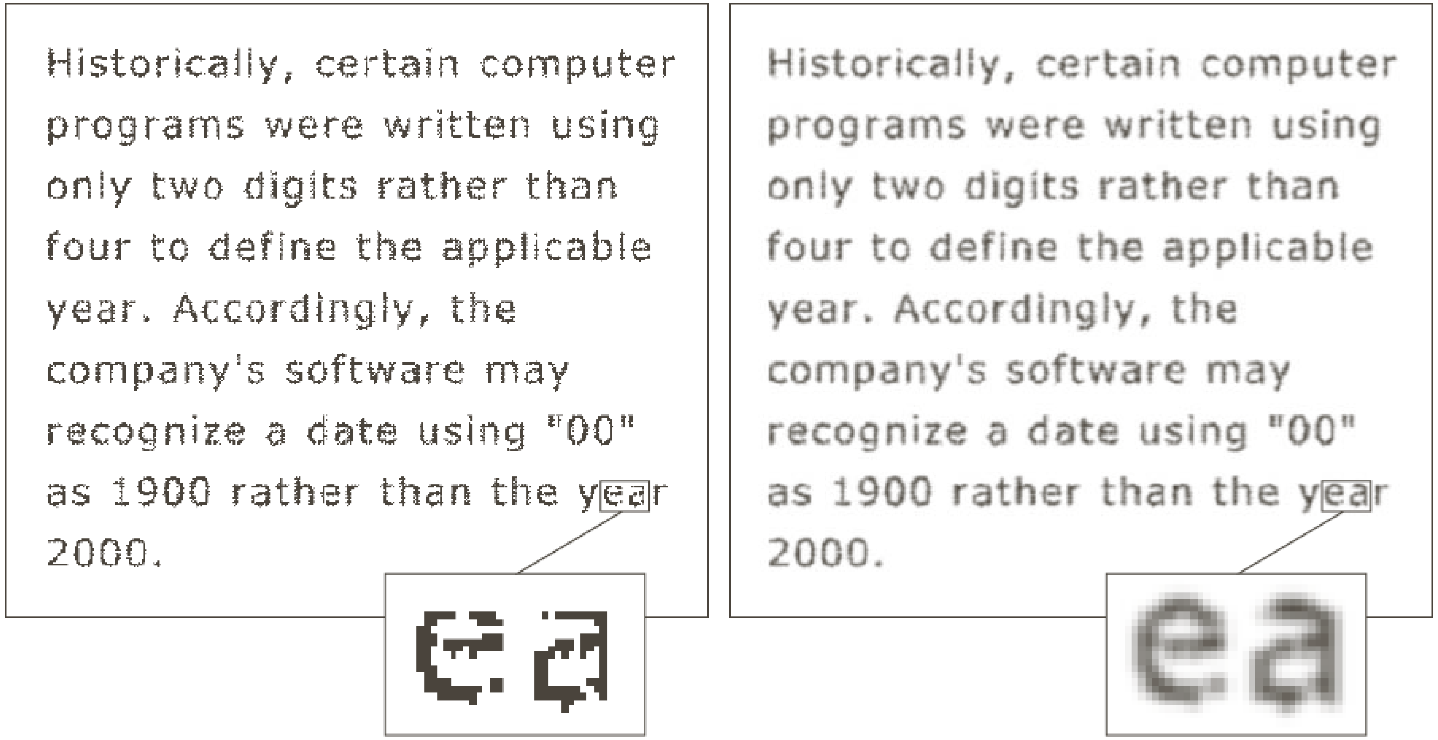
FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Examples of smoothing by GLPF (1)



a b

FIGURE 4.49
(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Examples of smoothing by GLPF (2)

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Image Sharpenning Using Filter Domain Filters

1. Ideal High Pass Filter
2. Butterworth High Pass Filter
3. Gaussian High Pass Filter

Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Image Sharpening Using Frequency Domain Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

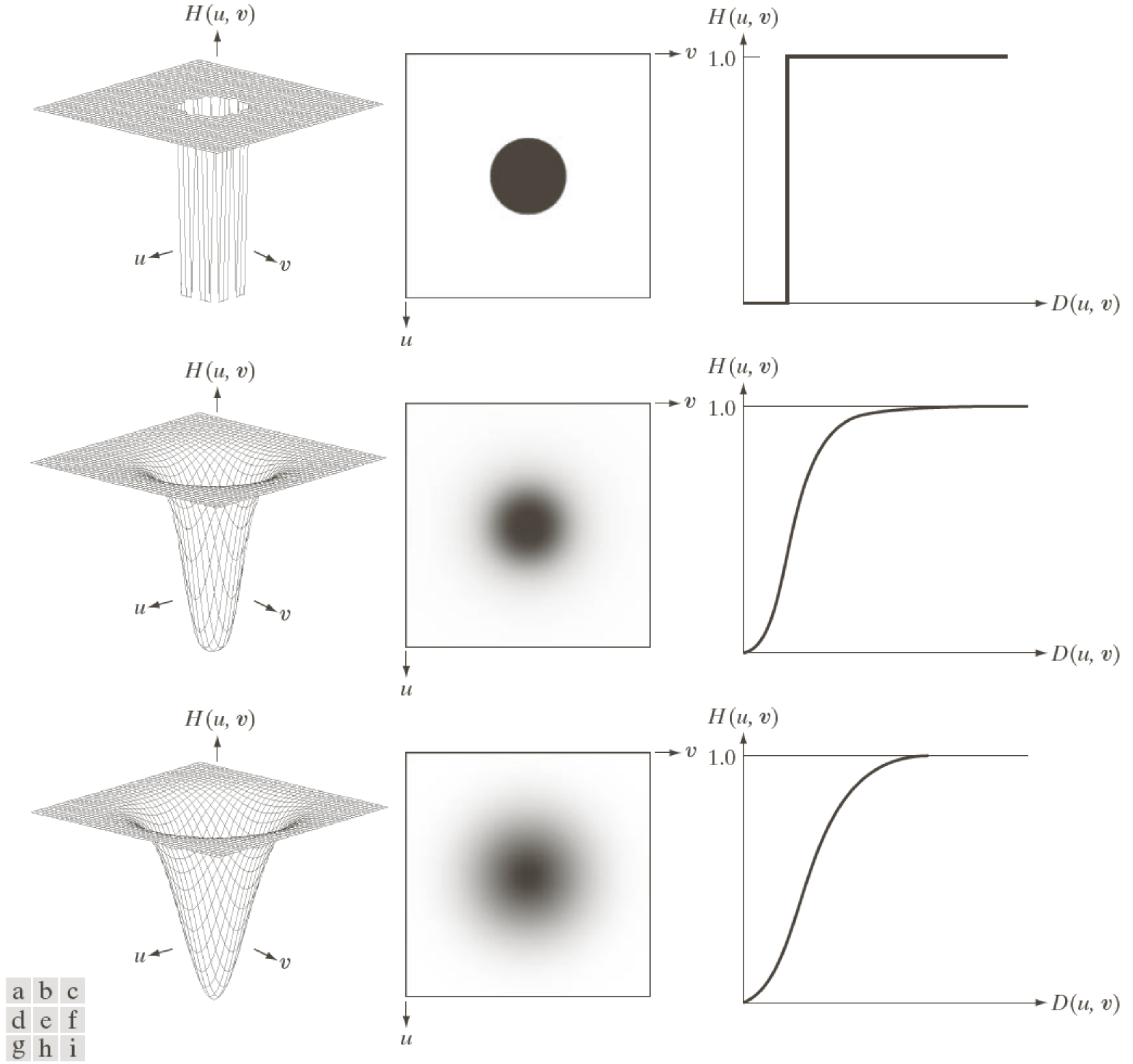


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

The Spatial Representation of Highpass Filters

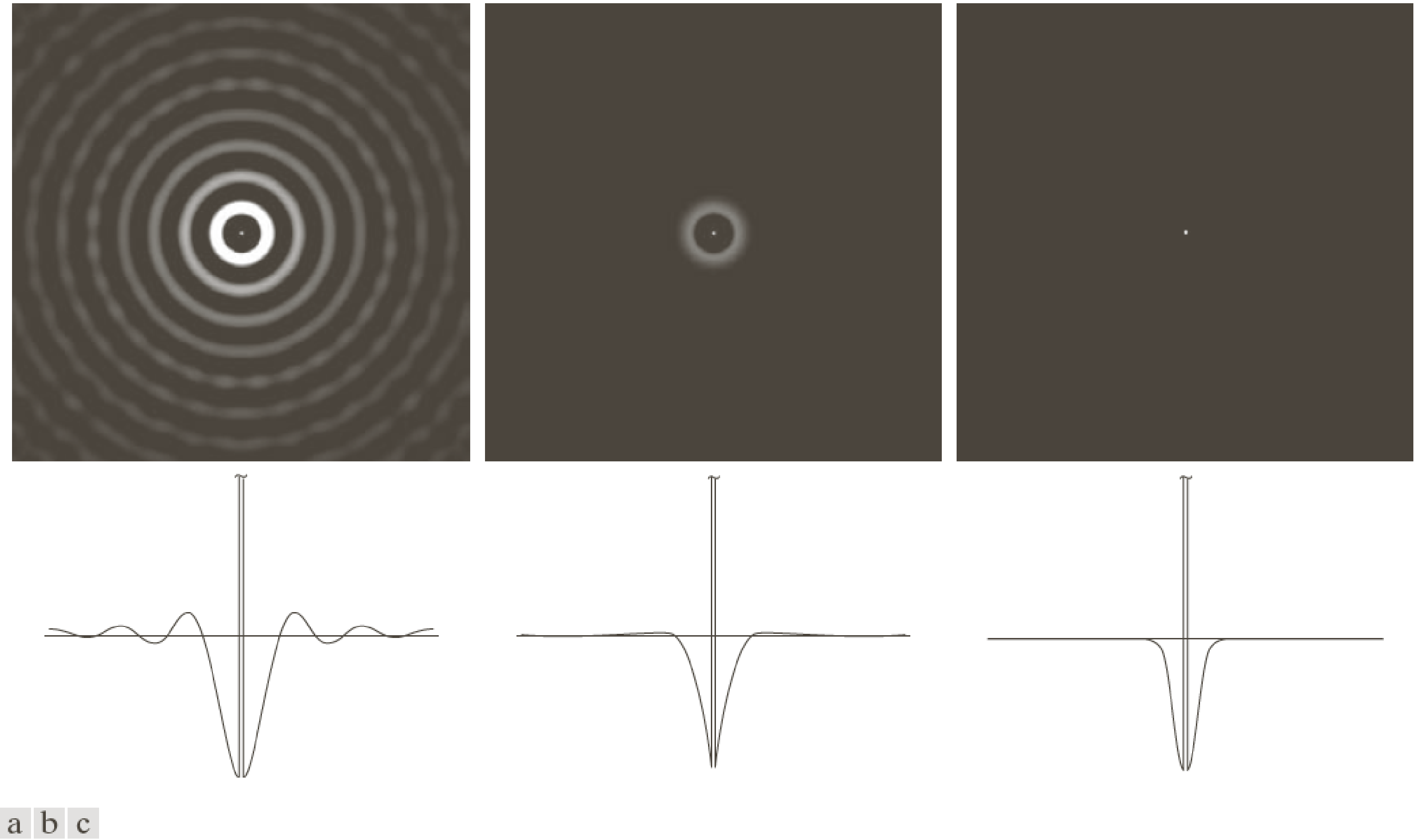


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Filtering Results by IHPF

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

Filtering Results by BHPF

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Filtering Results by GHPF

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.