Exam 1 B - MATH 4322

Cathy Poliak

Spring 2022

Name: ______________________________________ **PSID**: ___________

Instructions

- Allow one sheet of notes front and back to be turned in for extra credit.
- Allow calculator.
- Total possible points 100.
- For multiple choice circle your answer on this test paper.
- For short answer questions answer fully on this test paper, partial credit will be given.
- Once completed leave at the desk, I will pick up your test.
- Data sets are coming from

UCI Machine Learning Repository

Problem 1

(32 possible points) We want to predict whether income exceeds $50K per year based on census data. The variables are: `Age`, `Education` (in years), `Gender` (1 for Female and 0 for Male), `Hours` (hours per week), and `Income` (0 for $\leq 50K$ and 1 for $> 50K$).

a. Is this a inference or prediction statistical learning problem?

This is a prediction learning problem

b. Is this a regression or classification problem?

This is a classification problem

c. Give the model formula for our problem. Use the variable names in the formula.

Answer

$$p(\text{Income} > 50K|X) = \frac{exp(\beta_0 + \beta_1 \times \text{age} + \beta_2 \times \text{Education} + \beta_3 \times \text{Gender} + \beta_4 \times \text{Hours})}{1 + exp(\beta_0 + \beta_1 \times \text{age} + \beta_2 \times \text{Education} + \beta_3 \times \text{Gender} + \beta_4 \times \text{Hours})}$$

d. Give the R code to predict the probability of income being greater than $50K.

glm.income = glm(Income   Age + Education + Gender + Hours, family = "binomial")

e. The following is the output from the data. Write out the equation with the estimates.

Predictor	Estimate	Std. Error	t value	P value
(Intercept)	-9.54	1.425	-6.69	0.0000
Age	0.04	0.013	2.87	0.0041
Education	0.45	0.083	5.44	0.0000
Gender Male	1.50	0.469	3.21	0.0013
Hours	0.02	0.014	1.52	0.1285

Answer

$$p(\hat{X}) = \begin{cases} \frac{exp(-8.49+0.04\text{Age}+0.45\text{Education}+0.02\text{Hours})}{1+exp(-8.49+0.04\text{Age}+0.45\text{Education}+0.02\text{Hours})} & \text{if Male} \\ \frac{exp(-9.54+0.04\text{Age}+0.45\text{Education}+0.02\text{Hours})}{1+exp(-9.54+0.04\text{Age}+0.45\text{Education}+0.02\text{Hours})} & \text{if Female} \end{cases}$$

f. Give the interpretation of the coefficient for the variable `Education`.

As the years of education increase, the probability of making more than $50,000 per year increases as well.

g. Are there any variables that are not needed in this model? Justify your answer.

Yes, the number of hours is not needed in the model

To test $H_0 : \beta_4 = 0$, given that the other terms are in the model we get a p-value of 0.1285, thus we would fail to reject the null hypothesis and state that there is no evidence that the number of hours are significant in predicting income, given that the other variables are in the model.

h. The following is the confusion matrix based on the removal of the variable. What is the error rate for this model?

		Predicted $> \$50K$	
		No	Yes
Actual	No	178	14
$> \$50K$	Yes	37	21

Answer

$$\text{Error Rate} = \frac{14+37}{178+14+37+21} = 0.204$$

Problem 2

(36 possible points) We want to be able to see the affect of student performance in secondary education by some predictors. The following are the variables used.

- age - student's age (numeric: from 15 to 22)
- internet - Internet access at home (binary: yes or no)
- absences - number of school absences (numeric: from 0 to 93)
- score - final grade (numeric: from 0 to 20, output target or response variable)

a. Is this a inference or prediction statistical learning problem?

This is an inference statistical learning problem

b. Is this a regression or classification problem?

This is a regression problem

c. Give the model formula for our problem. Use the variable names in the formula.

Answer

$$score = \beta_0 + \beta_1 \times \text{age} + \beta_2 \times \text{internet} + \beta_3 \times \text{absences} + \epsilon$$

d. The following is an output for predicting the final grade. Write out the equation with the estimates.

Predictor	Estimate	Std. Error	t value	P value
(Intercept)	17.81	2.559	6.96	0.0000
age	-0.37	0.155	-2.38	0.0179
internetyes	0.61	0.437	1.39	0.1664
absences	-0.09	0.042	-2.17	0.0311

Answer

$$\hat{score} = \begin{cases} 18.42 - 0.37\text{age} - 0.09\text{absences} & \text{If they have internent} \\ 17.81 - 0.37\text{age} - 0.09\text{absences} & \text{If they do not have internet} \end{cases}$$

e. Give the interpretation of the coefficient for the variable `Age`.

For each year increase in age, the final score will decrease on average by 0.37 points

f. Are there any variables that are not needed in this model? Justify your answer.

Yes, if they have internet or not

When testing $H_0 : \beta_2 = 0$ given that the other terms are in the model, we get a p-value = 0.1664. Thus we fail to reject the null hypothesis and state that there is no evidence that having access to the internet is significant in predicting the final score, given that the other variables are in the model.

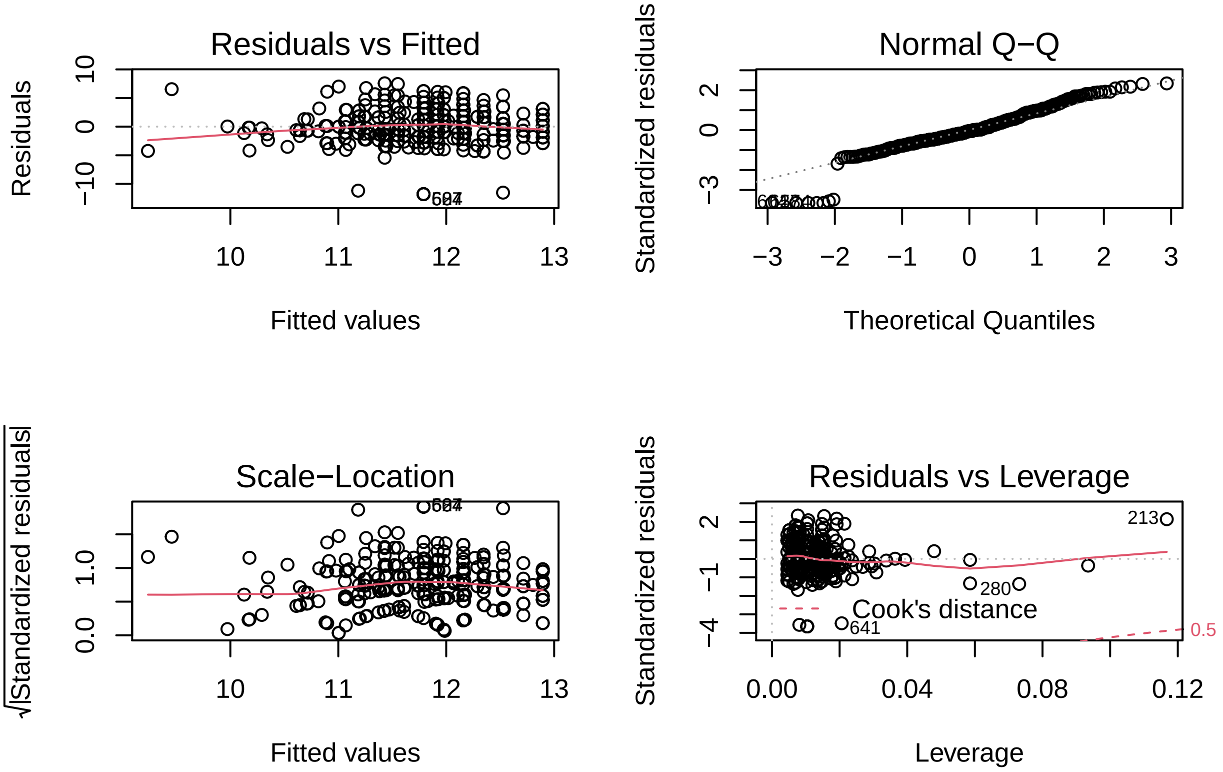
g. What is the predicted value of the final score, where the student is 17 years old, does not have internet, and has 2 absences?

predicted score = 17.81 -0.37(17) -0.09(2) = 11.34

h. What are the assumptions of this model?

Linear, Independent random sample, Normal distribution, and Equal variance among the residuals for each value of x.

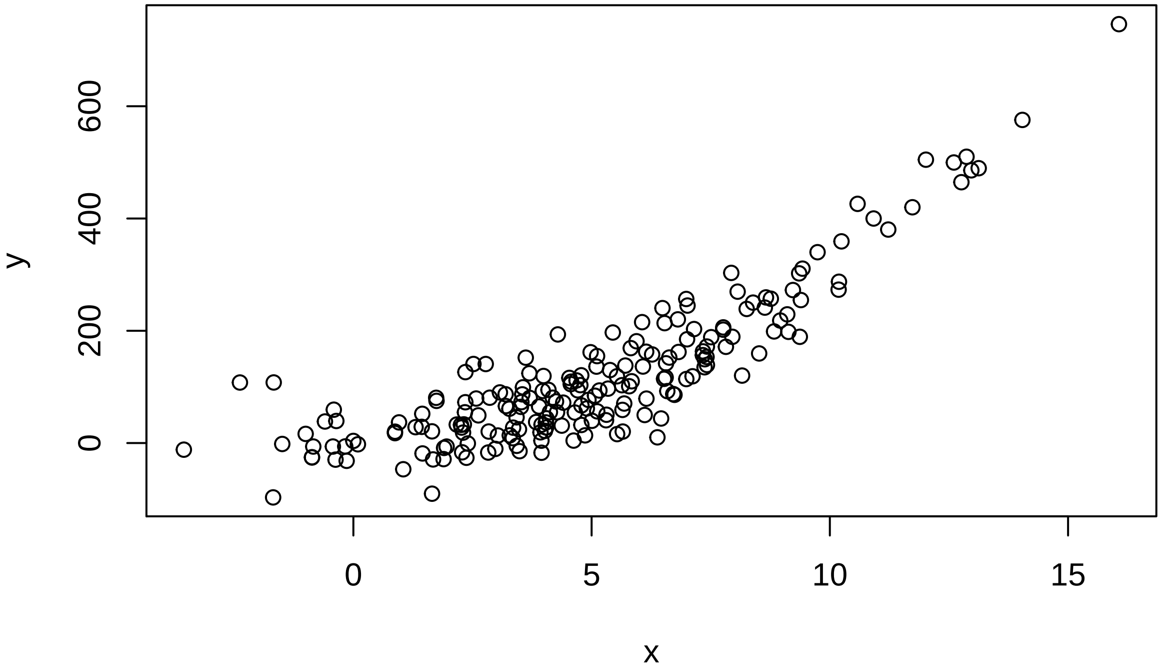
i. The plot below are the diagnostics plots. Are any of the assumptions violated with this model?



The only violation that appears is that there may be extreme values.

Problem 3

(8 possible points) a. Using the following plot below do we have a linear relationship?



No, this does not apear to be linear

b. The following is an output for a regression model with degree 1, 2, 3 and 4 respectively. Give the formula for the best model.

```
##            Adj.R2       Cp        BIC
## Degree 1 0.7347 187.6954 -255.8245
## Degree 2 0.8642   1.5024 -385.3815
## Degree 3 0.8638   3.0339 -380.5630
## Degree 4 0.8631   5.0000 -375.2995
```

Since the adjusted R^2 is large, and C_p and BIC are small for Degree 2, the following is the best equation

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$$

Problem 4

(8 possible points) Suppose we have a data set with five predictors, X_1 =GPA, X_2 = IQ, X_3 = Gender (1 for Female and 0 for Male), X_4 = Interaction between GPA and IQ, and X_5 = Interaction between GPA and Gender. The response is starting salary after graduation (in thousands of dollars). Suppose we use least squares to fit the model, and get the estimated model:

$$\hat{Salary} = 5.5 + 20X_1 + 0.05X_2 - 6.25X_3 + 0.03X4 + 30X_5$$

True or False: For a fixed value of IQ and GPA, females earn more on average than males provided the GPA for females is high. Justify your answer.

TRUE

Suppose IQ = 150 and GPA = 3.5
For Males: Salary = 5.5 + 20(3.5) +0.05(150) +0.03 (3.5)(150) = 98.75
For Females: Salary = 5.5 + 20(3.5) + 0.05(150) -6.25 +.03(3.5)(150) + 30(3.5) = 197.5

In this case the female had a higher salary. Actually because of the coefficient of the interaction term being so large, the female will always have a higher salary.

Problem 5

(4 points) Given the following ANOVA table, determine the AIC of this model. There are 200 observations.

```
## Analysis of Variance Table
##
## Response: y
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## x           1 2480746 2480746  552.22 < 2.2e-16 ***
## Residuals 198  889471    4492
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
```

a. 1950.43

b. 1684.01

c. 1889.15

d. 4492.28

e. 67.0244

## Problem 6

(4 points) Given the confusion matrix below, determine the sensitivity rate.

		Predicted $> \$50K$	
		No	Yes
Actual	No	178	14
$> \$50K$	Yes	37	21

a. 0.36

b. 0.6

c. 0.93

d. 0.83

e. 0.08

Problem 7

(4 points) Given the training data set, testing data set and MSE which statement is true?

a. The data sets most of the time will have the same vale of MSE.

b. If the testing data set has a larger MSE, this is called overfitting the data.

c. The training data set will have a MSE of zero (0).

d. The training data set most of the time will have the largest MSE.

e. The testing data set most of the time will have the largest MSE.

Problem 8

(4 points) Given a 95% confidence interval for the students final score in problem 2 below, which statement is correct?

[10.613, 12.131]

a. For one student, we predict the score to be between 10.613 and 12.131 with 95% confidence.

b. We predict the average score of the students to be between 10.613 and 12.131 with 95% confidence.

c. For one student, there is a 95% chance that the score is between 10.613 and 12.131.

d. There is a 95% chance that the average score of the students is between 10.613 and 12.131.

e. None of these are correct.