



MRC  
Biostatistics  
Unit



UNIVERSITY OF  
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# **Designing Phase I Single Agent Dose-Escalation Studies**

*Lecture 2: Model-based dose-escalation designs*

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# Single-Agent Dose-Finding Study

$k$  increasing doses :  $d_1 < d_2 < \dots < d_k$

Response:  $x = \begin{cases} 1 & \text{if a patient experienced a DLT} \\ 0 & \text{otherwise} \end{cases}$

Structure: treat successive cohorts of  $c$  subjects

Objective: find the “highest safe dose”

Based on the monotonicity assumption: **“the more the better”**:

Both toxicity and efficacy increase with the dose.

# Phase I in-patient trials

- Typically implemented as adaptive designs:

Dose (mg)	1	2.5	5	10	15	20	25
# patients	3	3	3				
# DLT	0	0	1				

- What dose should be selected for the next cohort?
  - More patients on the same dose (5mg)
  - More patients on a lower dose (2.5mg)?
  - Patients on a higher dose (10mg)?
- A good design will achieve
  - a high probability of the true MTD selection
  - most patients being treated on the MTD
  - few patients being overdosed

# Phase I designs for in-patient trials

- **Rule-based** designs

- ▶ No statistical inference
- ▶ Popular among clinicians
- ▶ Poor statistical properties (“operating characteristics”)

- **Model-based** designs

- ▶ Use a simple model for the dose-toxicity relationship
- ▶ Good operating characteristics
- ▶ Might lead to non-intuitive escalations/de-escalations

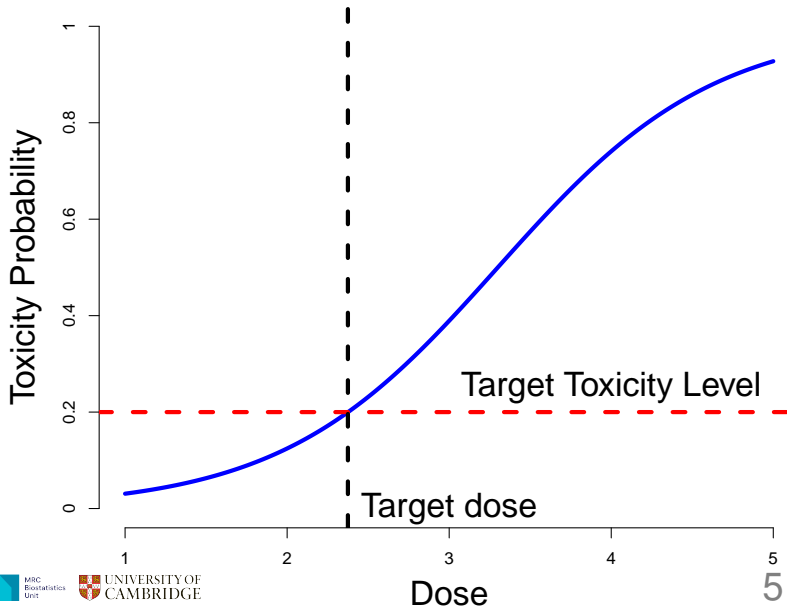
- **Model-assisted** designs

- ▶ Do not impose a particular “shape” on the dose-response
- ▶ Use a statistical model
- ▶ Still based on the monotonicity and a set of up-and-down rules

# Seeking a quantile

*MTD* — maximal dose acceptably tolerated by a particular patient population  
→ vague

*TD100 $\theta$*  — dose at which the probability of toxicity is  $\theta$   
(for  $0 < \theta < 1$ ), e.g. TD20  
→ more specific



# A Bayesian inference for Phase I trials

- Due to a small sample size, it is common to use Bayesian inference in early phase dose-escalation trials;
- Bayesian paradigm: Posterior  $\propto$  Data  $\times$  Prior
- Let  $\alpha$  be the parameter of interest with prior distribution  $f_0(\alpha)$ .
- The observations are  $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ ;
- The likelihood function  $L(\mathbf{x}|\alpha)$  is the distribution of  $\mathbf{x}$  conditional on specific values of  $\alpha$ .

Then:

$$f(\alpha|\mathbf{x}) = \frac{f(\mathbf{x}|\alpha)f_0(\alpha)}{f(\mathbf{x})}$$

# A Bayesian model-based design

## Planning a trial:

1. Choose doses  $d_1, \dots, d_k$ ;
2. Choose a form of dose-response relationship  $p(d_i, \alpha)$  where  $\alpha$  are model parameters;
3. Impose a prior distribution for  $\alpha$ ;
4. Choose a criterion to allocate patients;
5. Choose stopping rules



# A Bayesian model-based design

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## Conducting a trial:

1. Sequentially update estimates of  $\alpha$  ;
2. Select the dose for the next cohort using the criterion;
3. Stop if at least one of the stopping rules is met.

# Continual Reassessment Method (CRM)

Response:  $x = \begin{cases} 1 & \text{if a patient experienced a DLT} \\ 0 & \text{otherwise} \end{cases}$

Model:  $p(\pi_i, \alpha) = \pi_i^{\exp(\alpha)}$

Skeleton:  $\pi_i$  is a prior guess of the DLT risk

Prior on  $\alpha$ : Normal  $\alpha \sim \mathcal{N}(\mu, \sigma^2)$

Allocation Rule:  $\min |p(\pi_i, \hat{\alpha}) - \theta|$  where  $\hat{\alpha} = \mathbb{E}(\alpha)$

# Bayesian updating

1. Specify the prior distribution of  $\alpha$
2. Assign the first cohort to the starting dose (e.g. prior MTD)
3. Given the data, update the distribution of  $\alpha$

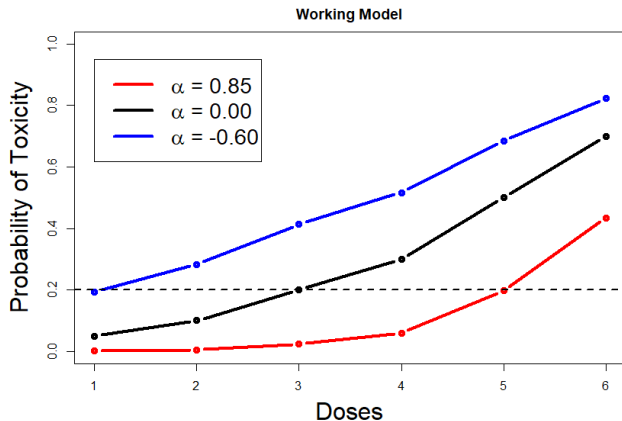
$$\text{Posterior} \propto \text{Prior} \times \text{Data}$$

4. Find the estimate of  $\alpha$ :  $\hat{\alpha}$
5. Find estimates of toxicity probabilities as  $\hat{p}_i = \pi_i^{\exp(\hat{\alpha})}$
6. Allocate the next cohort to the dose having the estimated toxicity closest to the target level  $\theta$ .
7. Repeat steps 3-6

# Representation of the model

Values for  $\pi_i$

$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$
0.05	0.10	0.20	0.30	0.50	0.70



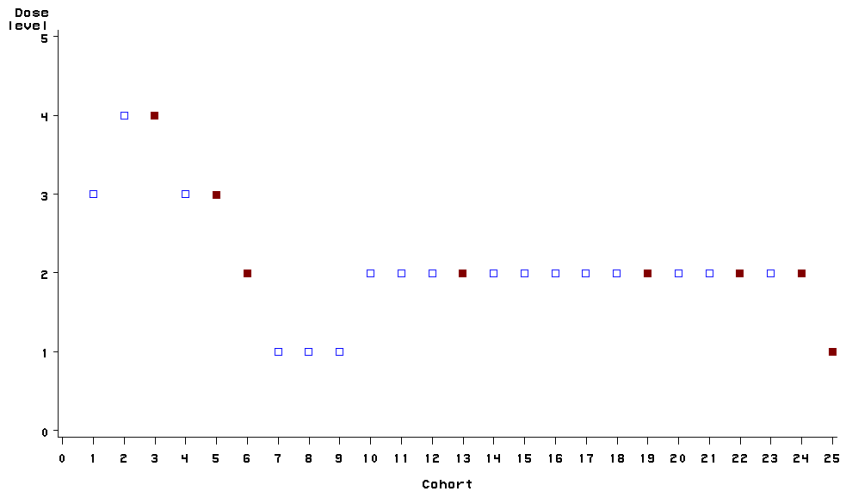
# Example

Assign values for  $\pi_i$

$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$
0.05	0.10	0.20	0.30	0.50	0.70

Thus first subject receives  $d_3$

# Simulated data



## **Criticisms:**

- Starting dose is usually high;
- Treats too many subjects on high doses (too aggressive);
- Doses can be skipped;
- No appropriate stopping rule.

## **Modifications:**

- Start at the lowest dose;
- Safety constraints;
- Alternative allocation rules;
- No dose skipping;
- Formal stopping rules.

# Choice of the model

- Pre-defined skeleton  $\pi_i, i = 1, \dots, k$ 
  - ▶ could be elicited from the clinicians on the trial;
  - ▶ could be chosen by statisticians (operational skeleton);
- For CRM, prior for  $\alpha \sim \mathcal{N}(0, 1.34^2)$  often leads to good OC;
- CRM is very quick to evaluate and run
- Although, the one-parameter model might not reflect the true dose-toxicity relationship...
- we are interested in the **single point**, the MTD, and the one-parameter model is rich enough for this goal
- If questions beyond the MTD identification to be considered - alternative models can provide a better answer.



# Bayesian Logistic Regression Model (BLRM)

## Model (Version 1):

- $d_{(1)} < \dots < d_{(k)}$  are doses;
- $D_{ref}$  is a reference dose;

$$p(d_{(j)}, \alpha_1, \alpha_2) = \frac{\exp\{\alpha_1 + \alpha_2 \log \frac{d_{(j)}}{D_{ref}}\}}{1 + \exp\{\alpha_1 + \alpha_2 \log \frac{d_{(j)}}{D_{ref}}\}}$$

- $(\alpha_1, \alpha_2)$  are parameters to be estimated
- requires **prior** distribution on  $(\alpha_1, \alpha_2)$

## Model (Version 2):

- $\tilde{d}_j$  are standardised doses (skeleton) obtained from the prior guesses of the DLT risk at each dose;

$$p(\tilde{d}_j, \alpha_1, \alpha_2) = \frac{\exp\{\alpha_1 + \alpha_2 \tilde{d}_j\}}{1 + \exp\{\alpha_1 + \alpha_2 \tilde{d}_j\}}$$

- $\hat{p}_j^{(0)}$  are prior estimates of the DLT probabilities associated with doses  $d_j$   $j = 0, \dots, m$

$$\tilde{d}_j = \frac{\text{logit}(\hat{p}_j^{(0)}) - \hat{\alpha}_1^{(0)}}{\hat{\alpha}_2^{(0)}}$$

- $\hat{\alpha}_1^{(0)}, \hat{\alpha}_2^{(0)}$  are prior point estimates of the model parameters;

## **Approach 1** by Neuenschwander *et al.* (2008)

- Specify two quantiles for toxicity probabilities at all doses
- Define prior distribution for  $(\alpha_1; \alpha_2)$  such that they are in close agreement with the above

## Ways to include prior information (II)

### Approach 3 by Whitehead & Williamson (1998)

- Specify two dose levels (the lowest and the highest)
- Illicit probability of toxicity at these levels from experts and determine how many patients this information is worth
- Include “pseudo-patients” in analysis based on above - no need in specifying the prior for  $(\alpha_1; \alpha_2)$

For example, the clinicians' believe that

- Toxicity risk at the lowest dose is 20%;
- Confidence in this response rate is equivalent to having 5 patients at the lowest dose;
- Toxicity risk at the highest dose is 40%;
- Confidence in this response rate is equivalent to having 1 patient at the highest dose;

# Allocation Criteria

- Escalation with Overdose Control (EWOC) and modifications:

$$\mathbb{E} \left( \nu(\theta - p_i)^+ + (1 - \nu)(p_i - \theta)^+ \right) \quad \text{e.g. } \nu = 0.25$$

- Convex Bounds Penalization (CIBP) Criterion:

$$\mathbb{E} \left( \frac{(p - \theta)^2}{p^\lambda (1 - p)^{2-\lambda}} \right), \quad \lambda - \text{asymmetry parameter} \quad (1)$$

- Loss function

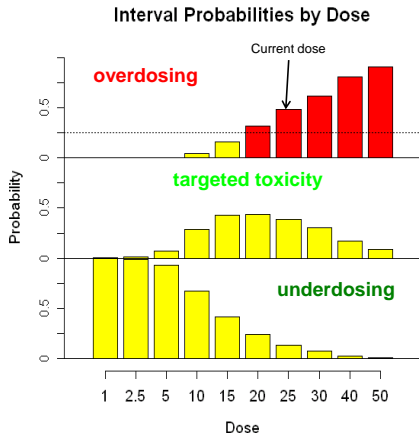
$$L = \begin{cases} 1 & \text{if } p \in (0.00, 0.20); & 0 & \text{if } p \in (0.20, 0.33) \\ 1 & \text{if } p \in (0.33, 0.50); & 2 & \text{if } p \in (0.50, 1.00) \end{cases}$$

- Maximising the probability of being in the target interval while safeguarding the patients

# Interval Probability Criterion

Recommend dose such that

- probability of overdosing  
 $P(p > 0.33 \mid d) < 0.25$
- probability of target toxicity  
 $P(p \in (0.20; 0.33) \mid d) \rightarrow \max$



# Example

- open-label, multicenter, dose-escalation cancer trial
- Find dose that has 30% risk of toxicity, the TD30.
- Use CRM but do not allow for skipping dose levels
- After 4 cohorts (4 dose levels) no DLTs
- Team decides to skip 2 dose levels
- Two DLTs in two patients

# Example

	Dose in mg									
	1	2.5	5	10	15	20	25	30	40	50
No. of patients	3	4	5	4	—	—	2	—	—	—
No. of DLTs	0	0	0	0	—	—	2	—	—	—
Posterior summaries:										
Mean	0.069	0.085	0.099	0.111	0.123	0.144	0.163	0.242	<b>0.330</b>	0.465
Std. dev.	0.055	0.062	0.068	0.072	0.076	0.082	0.087	0.101	0.109	0.108

Dose recommendation for next cohort:

- 3+3: Unclear
- CRM: 40mg
- BLRM: 15mg (from previous figure)



- BLRM is widely used in industry now;
- Specifying priors can be time consuming
- Running/Evaluating requires MCMC

# One-parameter or Two-parameter model

- It depends...
- One needs to decide what question they are trying to answer.
  - ▶ MTD estimation only?
  - ▶ Does one want to know more about the dose-toxicity shape?
  - ▶ Are there any further research questions to be answered?
- These all will affect the choice of the model.

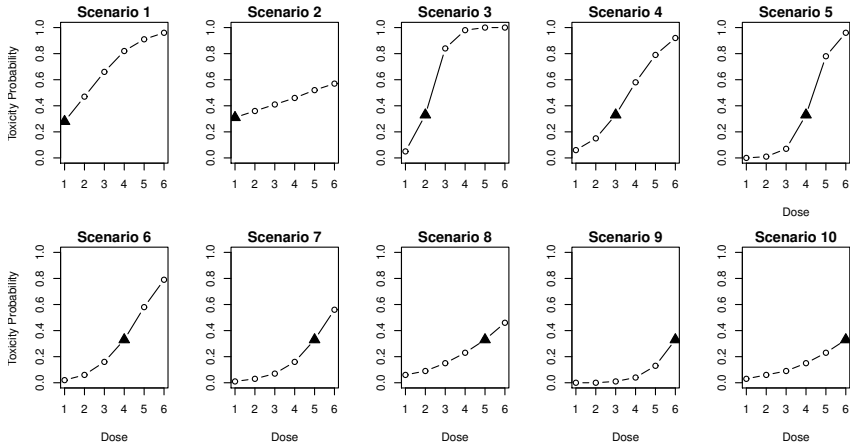
# Simulation study: setting

- 6 dose levels; target toxicity level of 33%;
- Compare various model-based designs:
  - ▶ Original CRM;
  - ▶ CRM with modified criteria (asymmetry = 0.5,0.25,0.10);
  - ▶ BLRM with loss function;
  - ▶ EWOC;
  - ▶ Modified EWOC (TDFB & TR);
- Assessed in terms of
  - ▶ Accuracy Index;

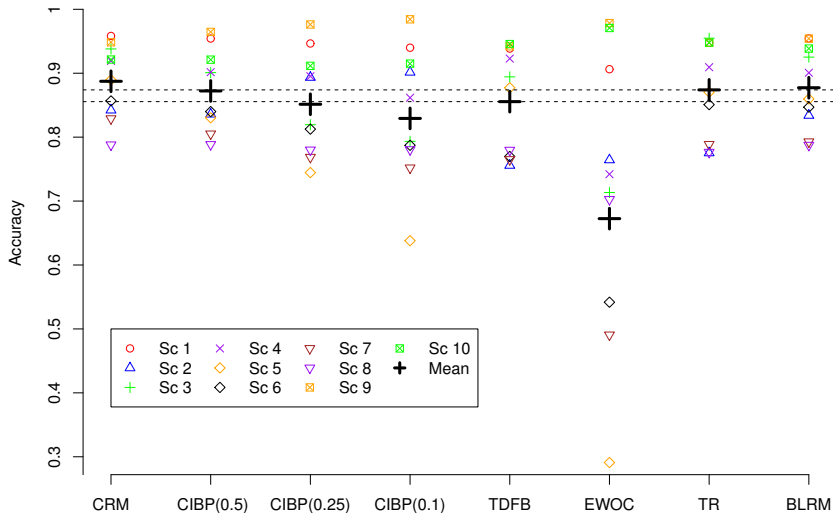
$$\mathcal{A} = 1 - m \frac{\sum_{i=1}^m (p_i - TTL)^2 \times \text{Selection Proportion}_i}{\sum_{i=1}^m (p_i - TTL)^2}$$

- ▶ Safety (average number of DLTs)

# Simulation study: scenarios



# Simulation study: Results. Accuracy



# Simulation study: Results. Safety

