Introduction to Applied Bayesian Analysis in Wildlife Ecology

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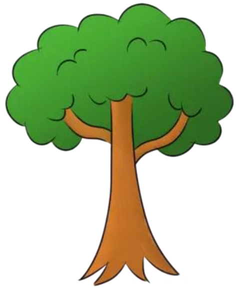
May 11, 2024

Course Website

https://doserjef.github.io/TWS24-Bayesian-Workshop/

A bit about me

Bayesian statistics



Wildlife and natural resources conservation

Software development

Course Overview

Model building

Frequentist vs. Bayesian

Bayesian basics

Bayesian linear models

Bayesian GLMs

Hierarchical Bayesian models

Course Learning Objectives

1. Understand foundational differences between frequentist and Bayesian approaches
2. Obtain a basic understanding of Bayesian analysis (and associated jargon) to impress your colleagues (and understand methods sections of papers)
3. Fit key statistical models such as linear models, generalized linear models, and mixed models in a Bayesian framework in R
4. Generate a solid Bayesian toolbox that you can build upon for your own work

Model building and modes of inference

What is a model?

An explanation of an observed pattern

Simplified version of nature

Statements that explain *why* the observations have occurred

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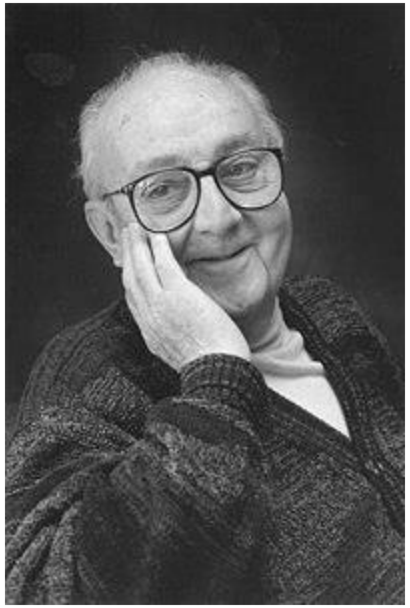
- George Box



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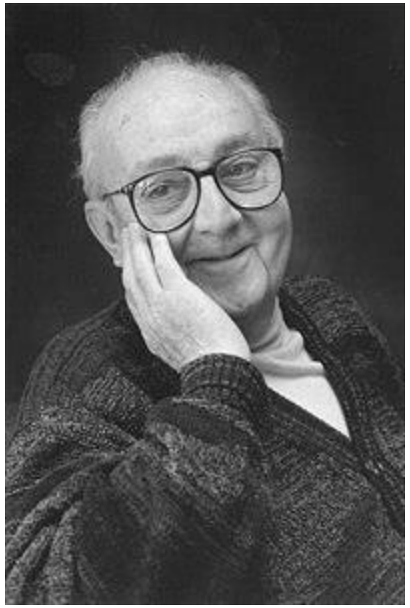

Nature is too complex. We need to simplify the complexity

Model objectives

The type of model we use depends on the objective of the model

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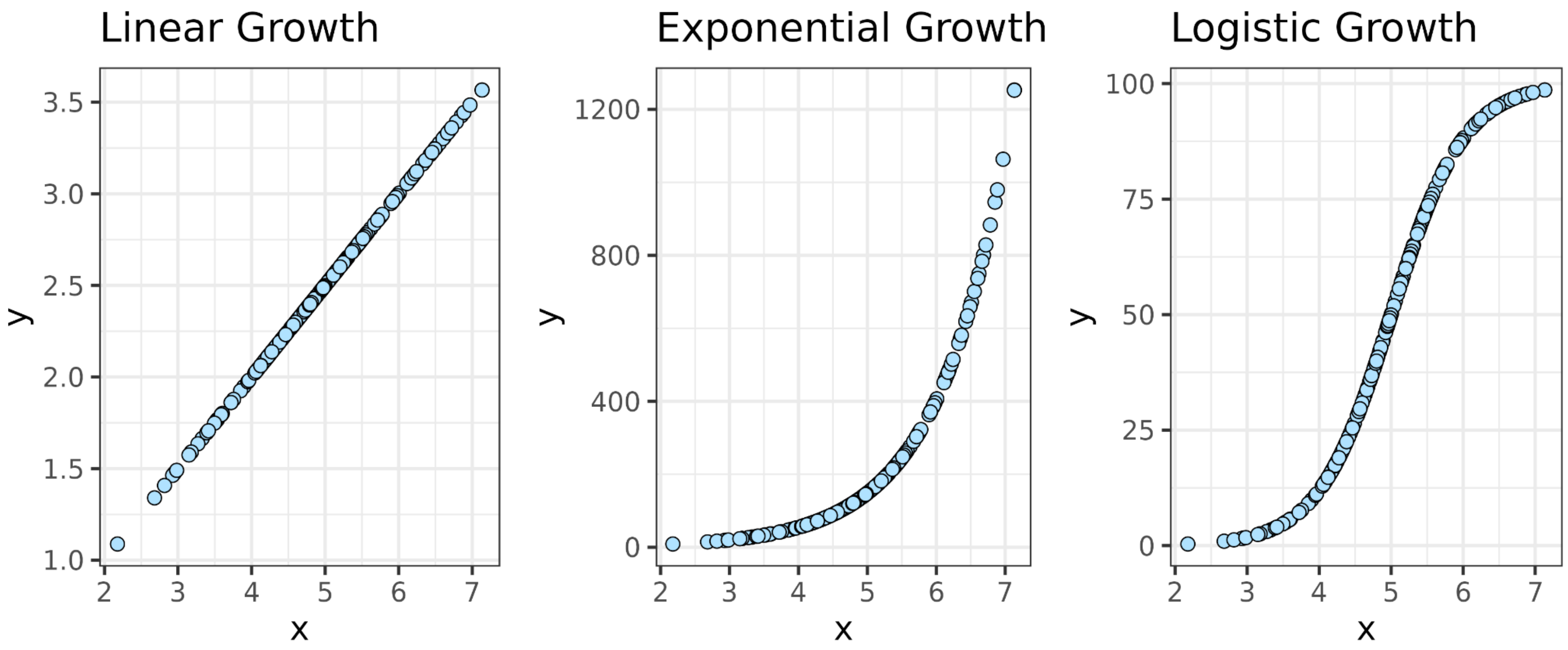
Inference:
understand
mechanisms

Prediction:
description

Mathematical models



$$y = \beta_0 + \beta_1 x$$

$$\frac{dN}{dt} = rN$$

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

Mathematical notation

Are there any differences between these equations?

$$y = \beta_0 + \beta_1 x$$

$$y = mx + b$$

Statistical Models

- A description of a system composed of variables but where one or more **random variables** are related to other variables
- Explicitly acknowledge stochasticity (uncertainty/error) in systems
- Response = systematic component + random component
- Example: simple linear regression

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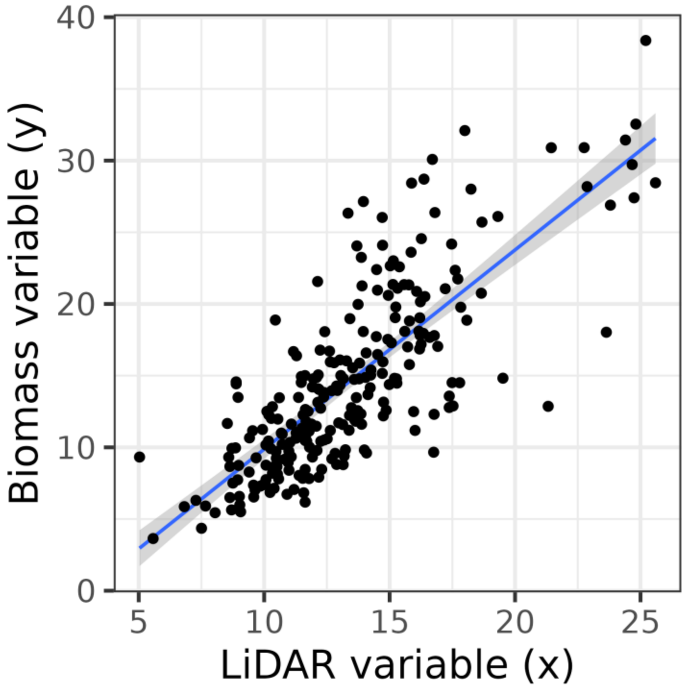

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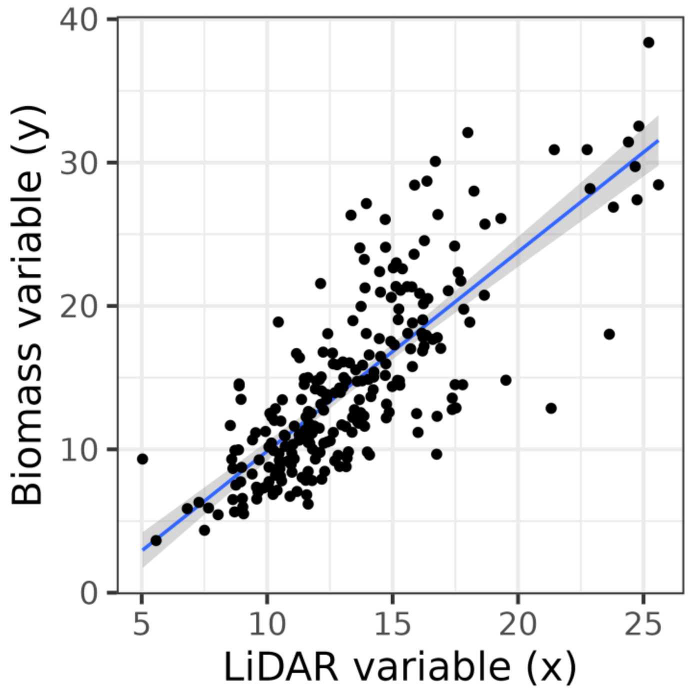

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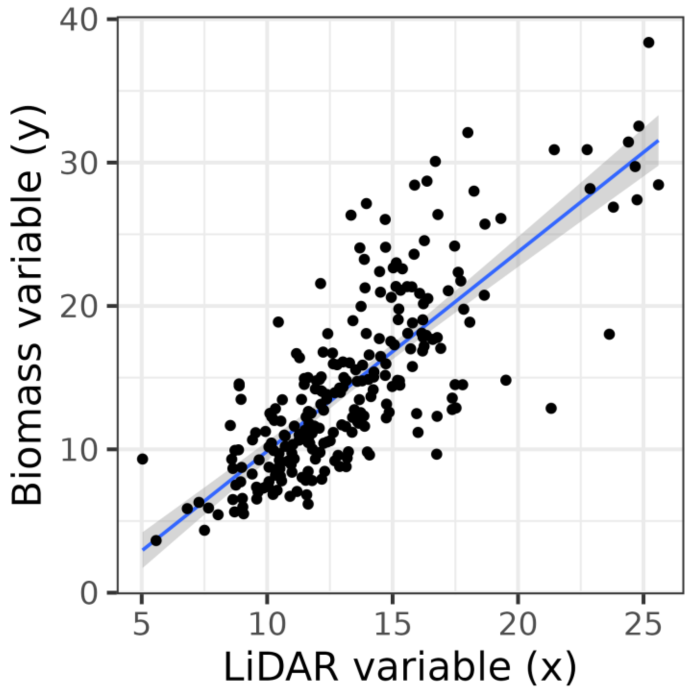
Random error component



Why do we need statistical models in wildlife ecology?

Three essential kinds of random variability

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Three essential kinds of random variability

1. **Measurement error**: variability imposed by our imperfect observation of the world. It is often modeled by adding normally distribution variability around a mean value
   - **Sampling error**: a type of measurement error that results from us drawing conclusions about a *population* using a smaller *sample* of that population

Why do we need statistical models in wildlife ecology?

Three essential kinds of random variability

2. **Demographic Stochasticity**: the innate variability in outcomes due to random processes. For example, flipping a coin 20 times, you might get 10, 9, or 11 heads, even though you're flipping the coin the same way each time.

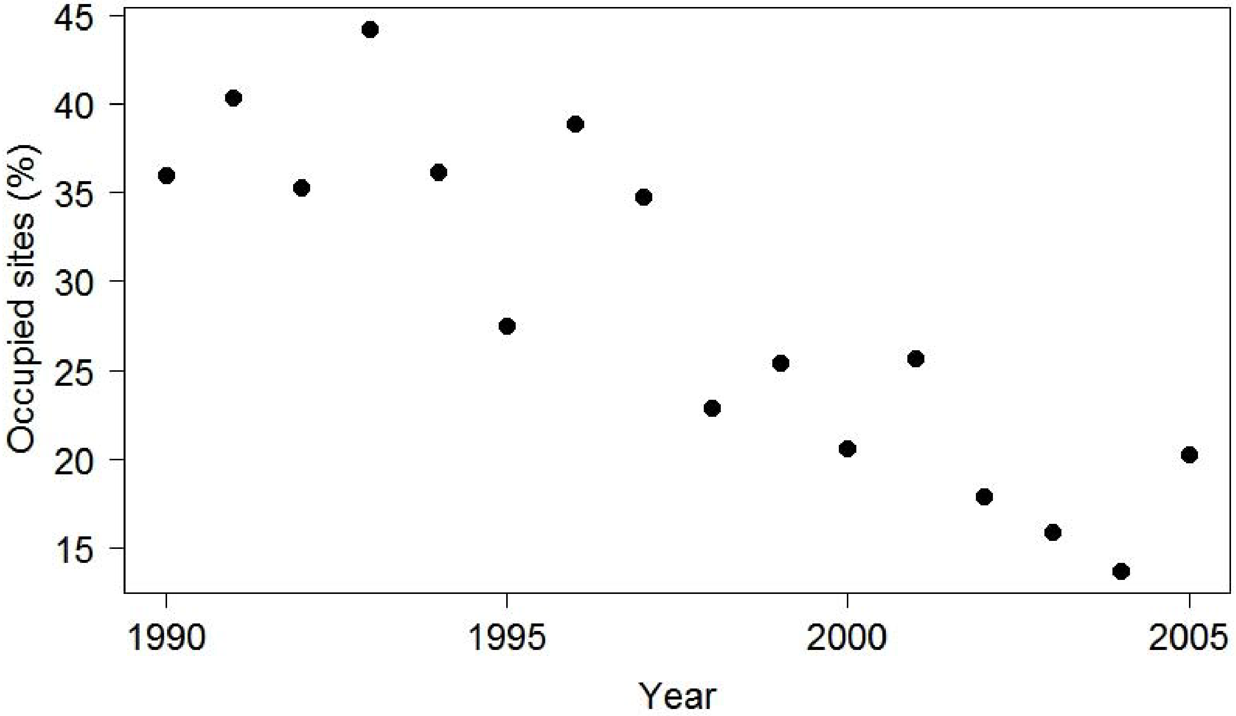
Why do we need statistical models in wildlife ecology?

Three essential kinds of random variability

3. Environmental Stochasticity: variability imposed from "outside" the ecological system, such as climatic, seasonal, or topographic variation

A simple example of a statistical model

Spotted Salamander



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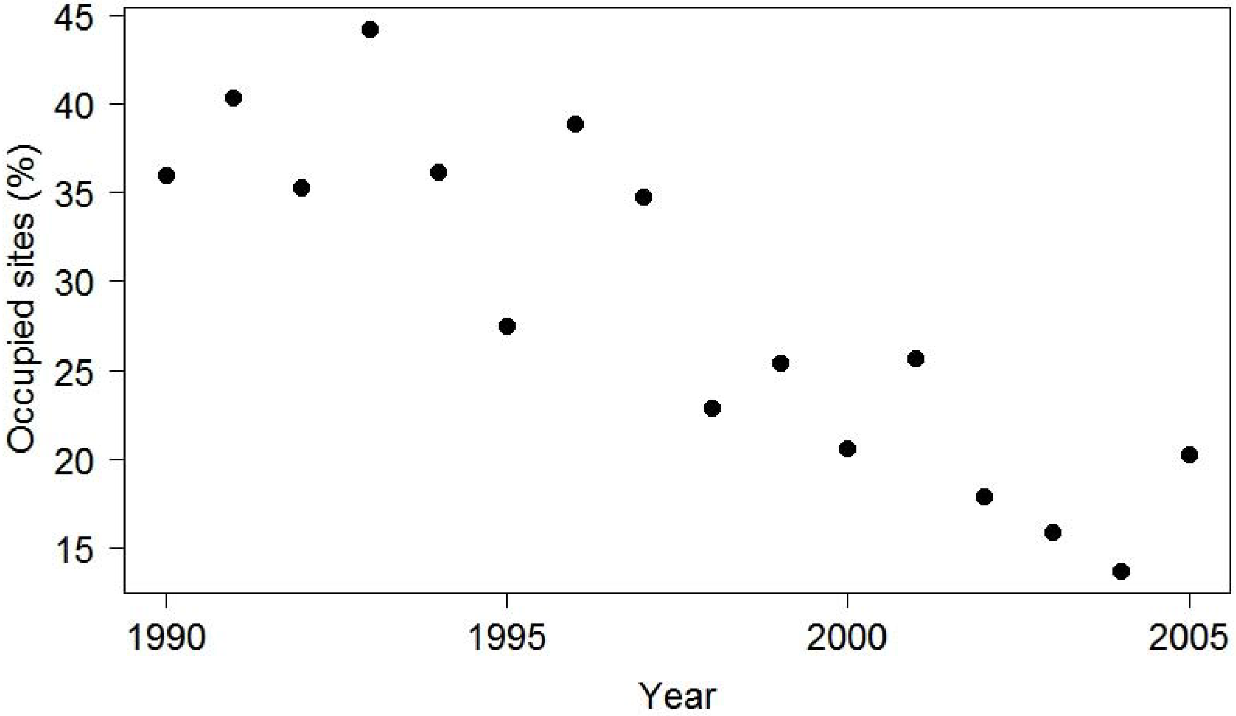

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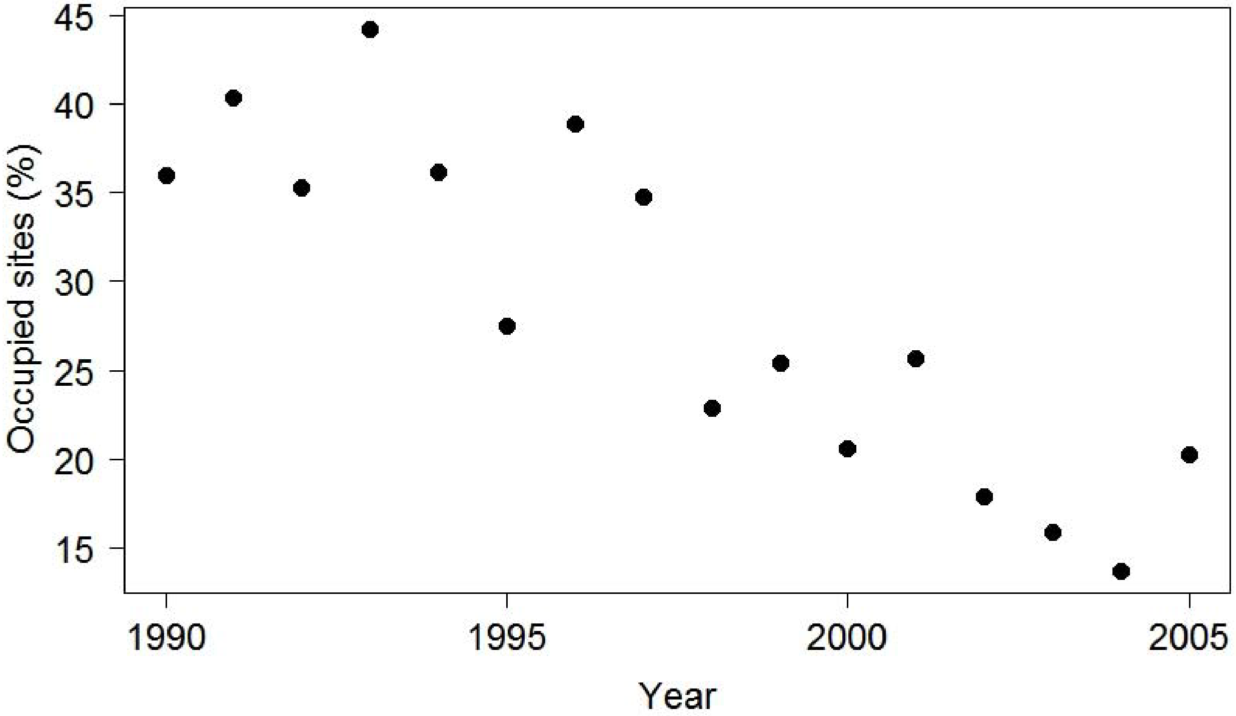

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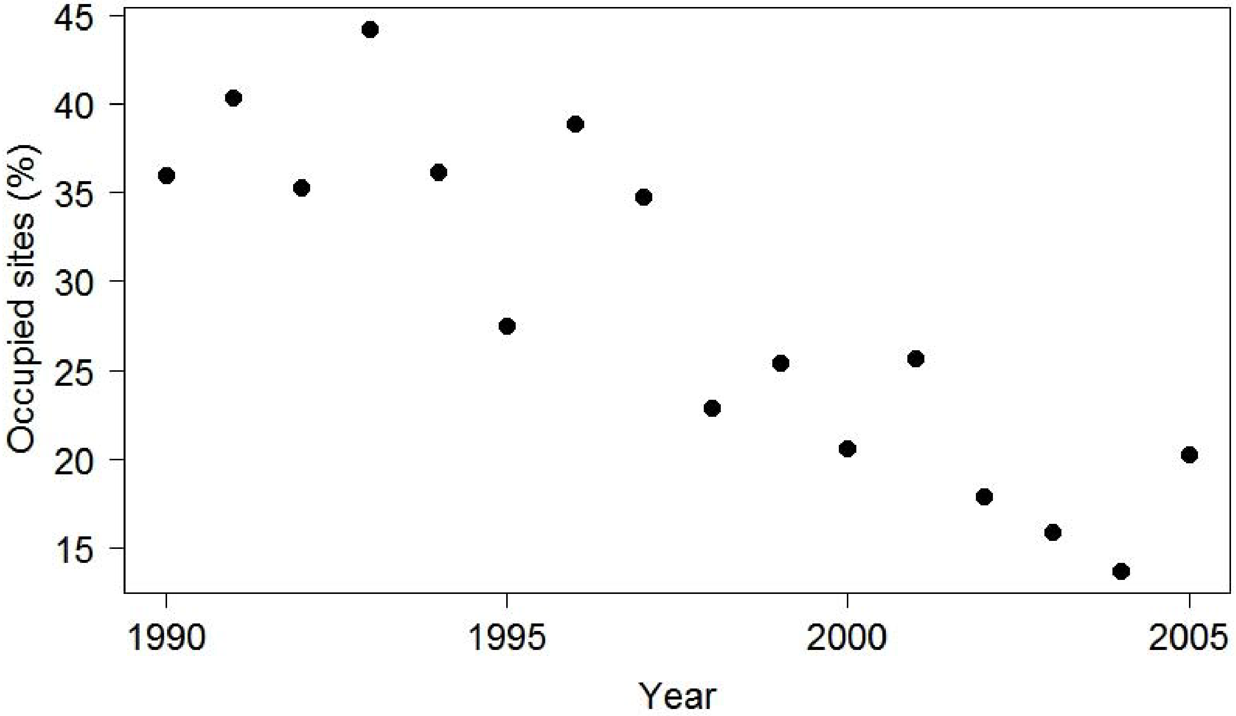

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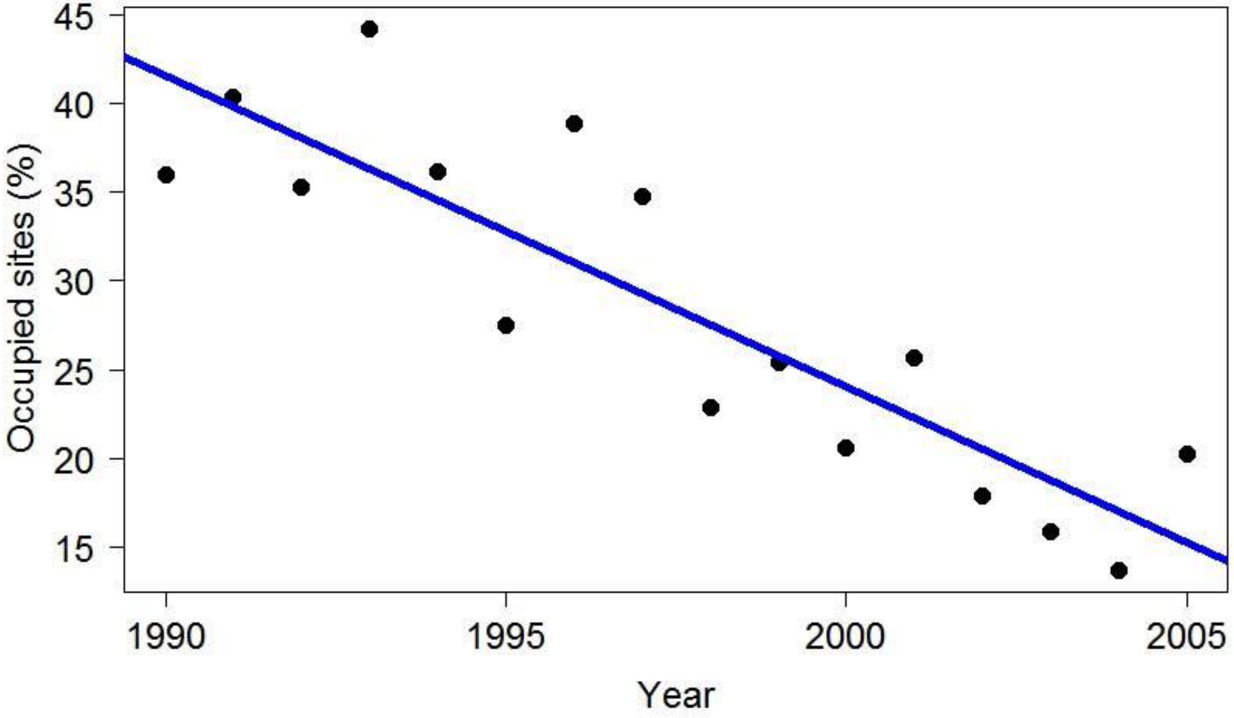
Trend Estimate



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An alternative, but equivalent model description

$$y \sim \text{Normal}(\beta_0 + \beta_1 x, \sigma^2)$$

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- **Non-parametric Statistical Models**: do not assume the data belong to a particular distribution (but make other assumptions)
- **Generalized Linear Models (GLMs)**: quintessential parametric statistical model
    - A majority of the statistical models used in wildlife ecology can be viewed as an extension of a GLM

Statistical models

Two frequently used GLMs in wildlife ecology

- **Normal response**
  - Random component $y \sim \text{Normal}(\text{mean} = \mu, \text{variance} = \sigma^2)$
  - Systematic component $\mu = \beta_0 + \beta_1 x$

Statistical models

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  - Systematic component $\mu = \beta_0 + \beta_1 x$
- **Binomial response**
  - Random component: $y \sim \text{Binomial}(\text{number of trials} = N, \text{success probability} = p)$
  - Systematic component: $\text{logit}(p) = \beta_0 + \beta_1 x$

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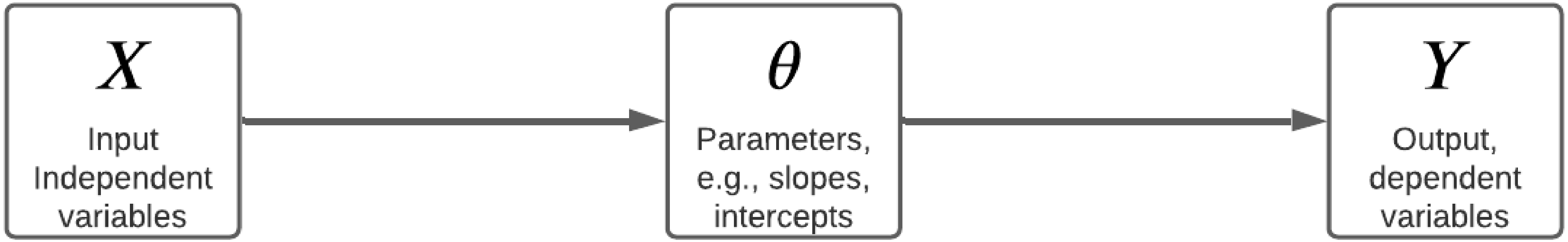
- Examples
    - Average biomass of trees in a forest (mean)
    - Total number of mammals in a management unit (total)
    - Relationship between number of spring growing degree days and monarch abundance (slope)
    - Survival probability of breeding birds over the non-breeding season (probability)

Analysis of a statistical model



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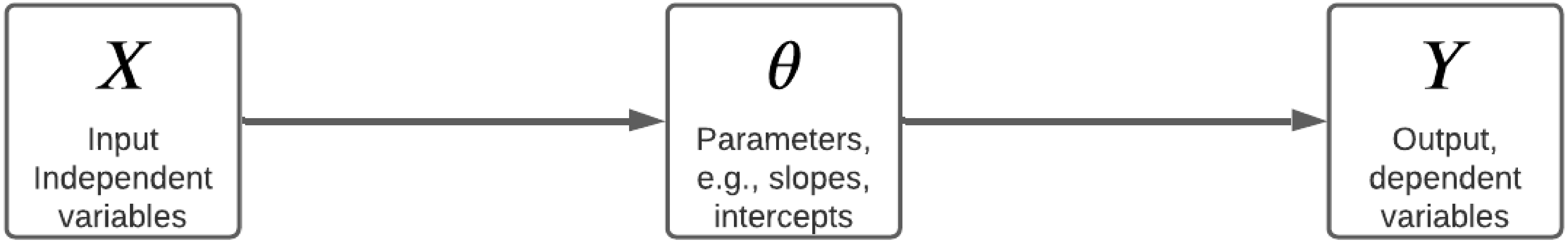

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- How can we guess at values of θ?

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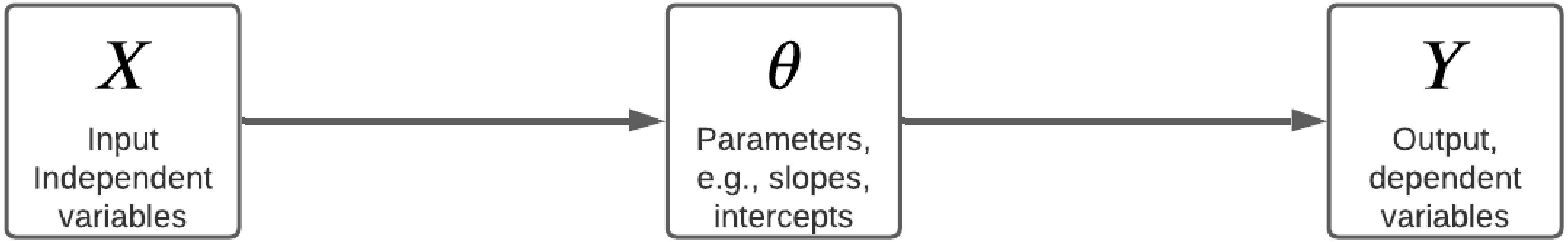

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  - Ordinary-least squares (OLS)
  - Maximum likelihood
  - Bayesian analysis

Frequentist analysis

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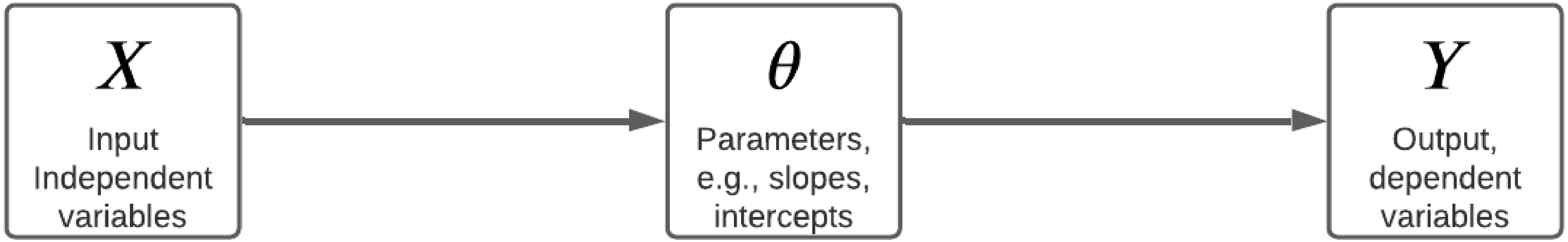

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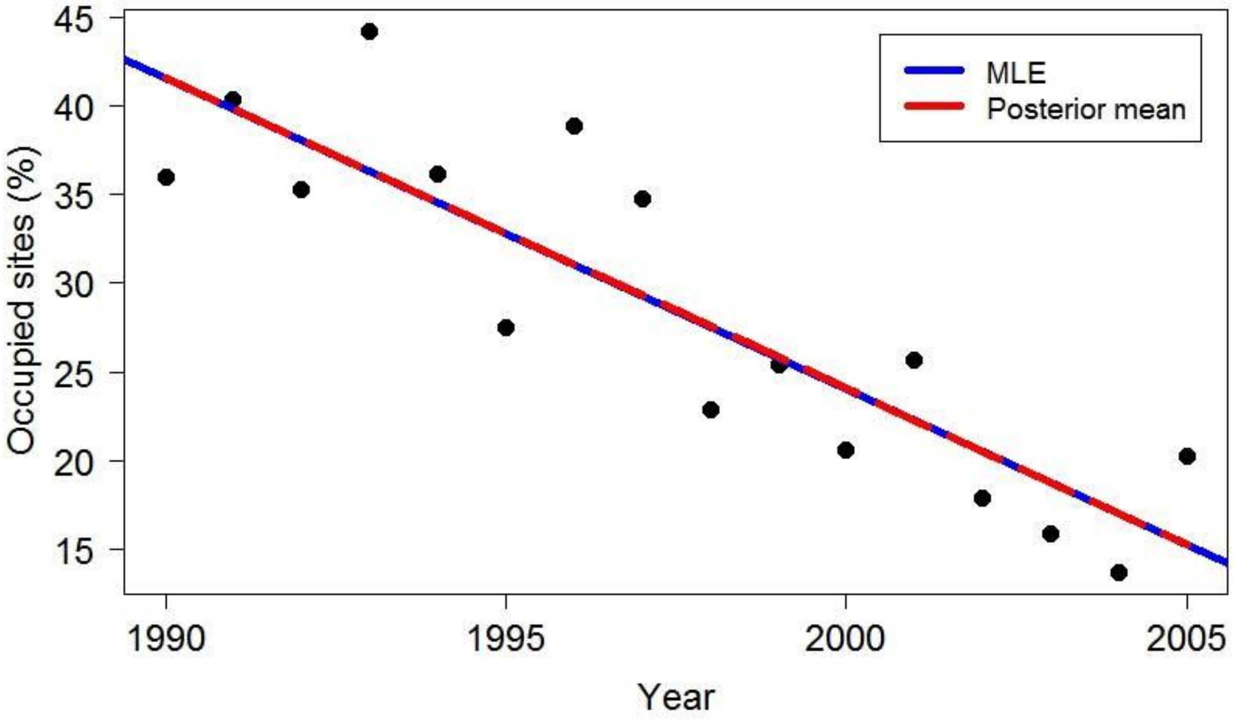

Maximum Likelihood Estimation

Frequentist analysis

Example: estimate the detection probability of spring peepers

- Originally released n = 50 in an artificial pond
- Recaptured y = 20

Maximum likelihood analysis

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- Likelihood is probability in reverse

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- 0.4 is the most likely detection probability

Maximum likelihood analysis

- One way to estimate θ: **Maximum Likelihood Estimation**
- **Sampling distribution**: p(y | θ)
  - Read as "probability of observing data y, given a fixed parameter value θ
- Note the probability statement is about the data, not about θ
- Probability is defined as the long-run frequency in hypothetical replicate data sets
- Example: binomial sampling distribution (y ~ Binomial(θ, N))

$$p(y \mid \theta) = \frac{N!}{y!(N-y)!}\theta^y(1-\theta)^{N-y}$$

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Maximum likelihood: fundamental idea

- For a given statistical model, maximum likelihood finds a set of parameters that makes the observed data *most likely* to occur
- A good choice of θ is that which maximizes the function value of the sampling distribution of the data set
- Answer the question: "what parameter value is most likely to have generated the data I observed?"

Maximum likelihood: fundamental idea

- Probability describes a function of the outcome given a fixed parameter value
  - If a coin is flipped 10 times and it is a fair coin, what is the probability of it landing heads every time?
- Likelihood is used when describing a function of a parameter given an outcome
  - If a coin is flipped 10 times and it has landed heads 10 times, what is the likelihood that the coin is fair?

Maximum likelihood: fundamental idea

- **Likelihood Function**: read the sampling distribution "in reverse" as a function of θ.

$$p(y \mid \theta) = L(\theta \mid y)$$

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- Call the value of θ that maximizes L(θ | y) the **Maximum Likelihood Estimate (MLE)**

Maximum likelihood: how to find the MLE?

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Analytically

- Take the derivative of the likelihood and find maximum values
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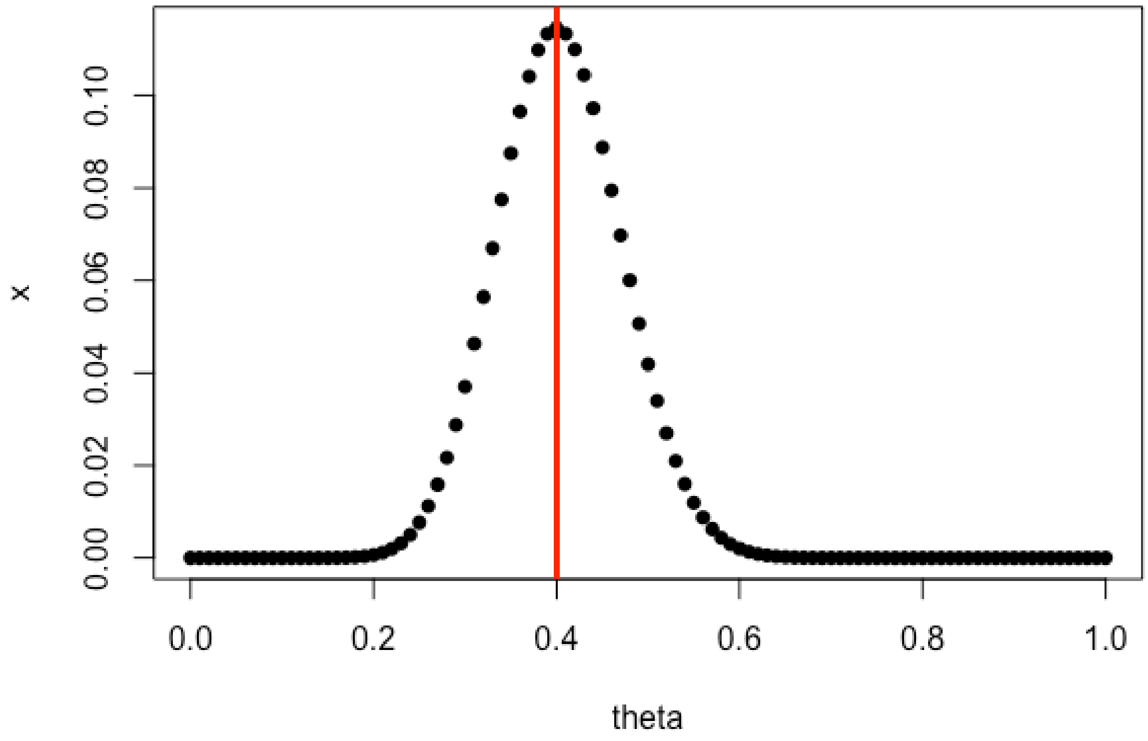
Numerically

- Estimation by brute force
- Try out and plot a large number of values for θ
- Find the value that maximizes the log likelihood (or minimizes the negative log likelihood)

Maximum likelihood: brute force numerical estimation



$$L(\theta \mid 20) = \frac{50!}{20!(50-20)!}\theta^{20}(1-\theta)^{50-20}$$

Maximum likelihood: brute force numerical estimation

- Numerical estimation by function minimization
  - `optim()` function in R (broad)
  - Specialized functions in R (e.g., `nlm()`, `glm()`)
- Basic approach
  - Specify the likelihood function
  - Take the derivative through numerical approximation
  - The MLE is the value that minimizes the negative log-likelihood
- Minimizing the negative log-likelihood is equivalent to maximizing the likelihood, but more computationally efficient

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- Lots of (easy to use) software packages

Drawbacks of the frequentist approach

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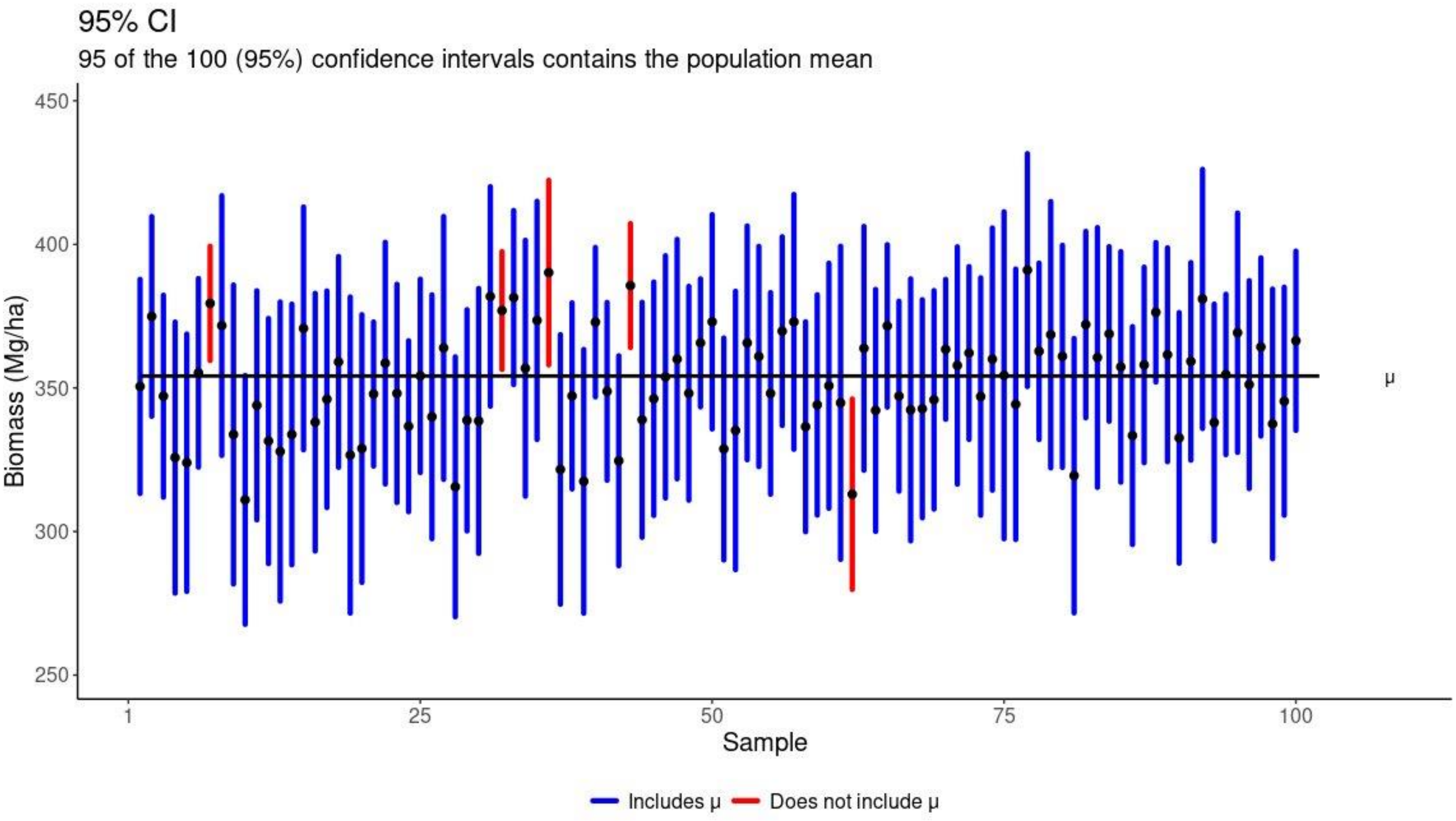
  - Frequentist statistics cannot assign a probability to a hypothesis (e.g., no probability associated with the value of a parameter)

Drawbacks: confidence interval interpretation



Questions?

Bayesian Analysis

Recall: fundamental idea of maximum likelihood

- A good choice of θ is that which maximizes the likelihood function
- Answers the question: "What parameter value is most likely to have generated the data I observed?"
- Probability statement is about the data, **not the parameters**

Bayesian analysis

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Bayesian analysis

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Fundamental difference between frequentist and Bayesian analysis:

- **Frequentist**: parameters are unknown, but fixed values
- **Bayesian**: parameters are unknown, random quantities

Bayesian analysis

- Bayesian approach: in the face of uncertainty about θ, use conditional probability, $p(\theta \mid y)$
  - "Probability of the parameter, given the observed data"
- Make probability statements about hypotheses (parameters)

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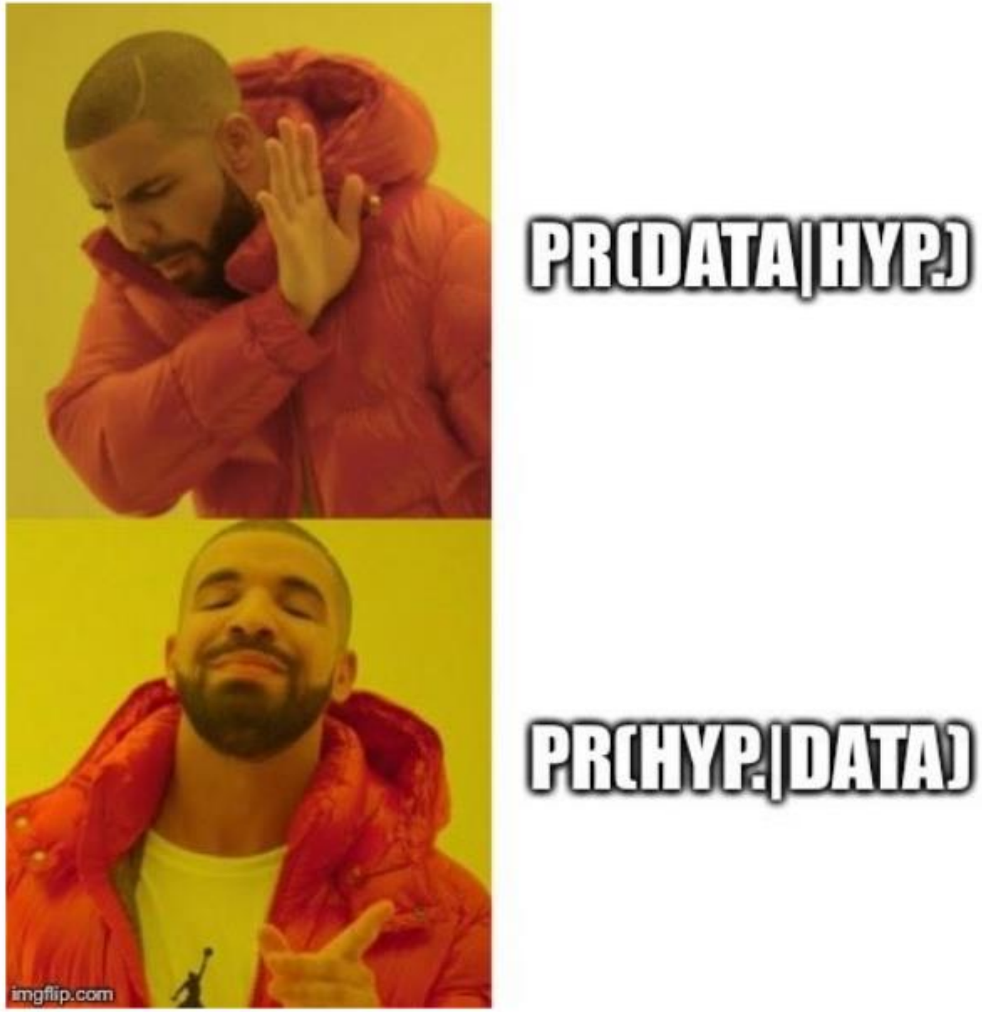

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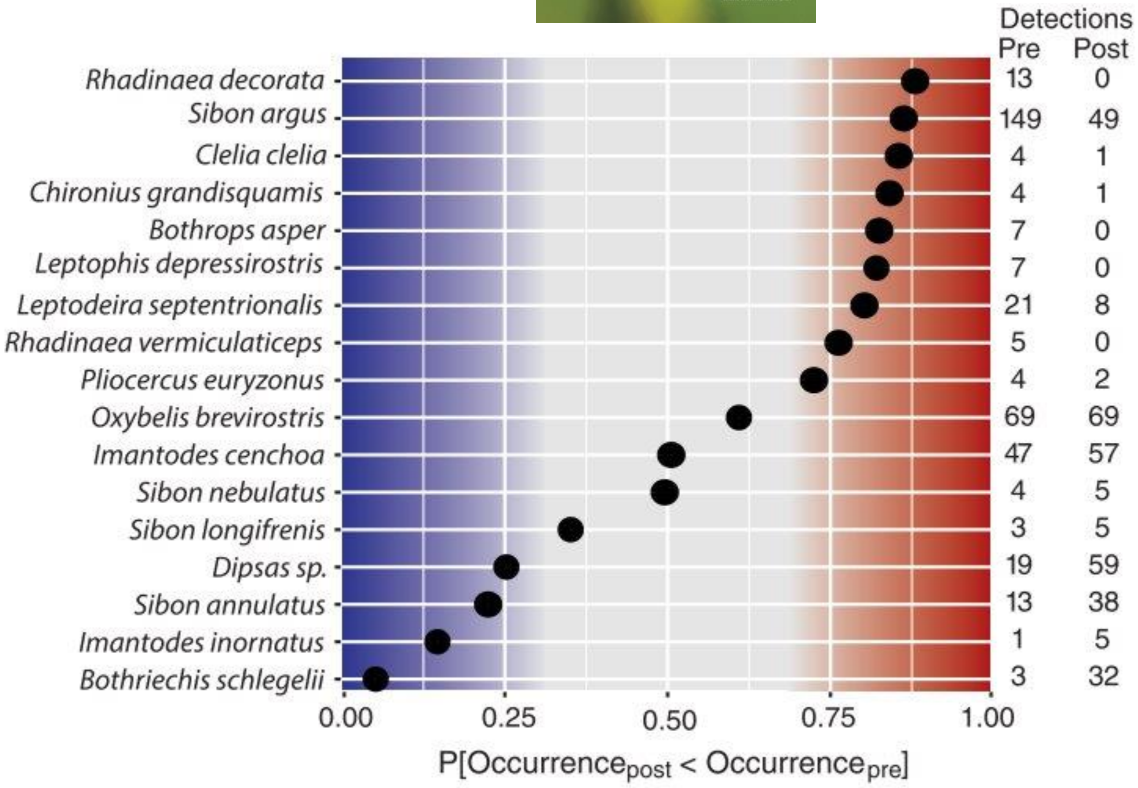
3. What to do?
   - Frequentist: Use the likelihood $L(\theta \mid y)$ to calculate the MLE
     - Probability statement about the data, not parameters
   - Bayesian: Calculate the posterior distribution $p(\theta \mid y)$
     - Probability statement about the parameters, not data

"Probability statement about the parameters"

In a Bayesian framework, you can say things like:

- There is a 97.3% probability the treatment had a positive effect on the outcome
- There is an 88.5% probability the population is declining



Foundation of Bayesian stats: Bayes' Theorem

$$p(A \mid B) = \frac{p(B|A)p(A)}{p(B)} = \frac{p(A,B)}{p(B)}$$

- Mathematical fact of probability
- Thomas Bayes (1702-1761)
- Applied the rule to parameters for parameter estimation
- But why did Bayesian inference only recently become popular?

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Probability of A and B

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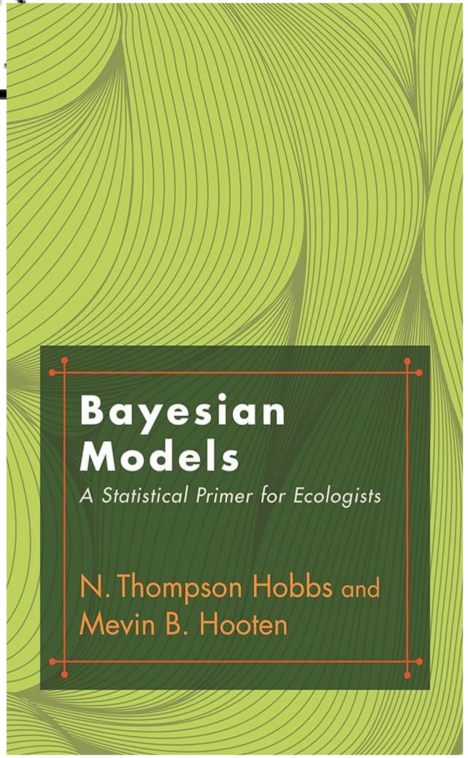
Probability event A occurs "given" event B occurs



Bayes' Theorem

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- Use probability to express imperfect knowledge

Bayes' Theorem for statistical inference

$$p(\theta \mid y) = \frac{p(y \mid \theta)p(\theta)}{p(y)} = \frac{p(\theta, y)}{p(y)}$$

Bayes' Theorem for statistical inference

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Posterior distribution

Margin
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Some numeric constant that we don't need to think about

Bayes' Theorem for statistical inference

Likelihood

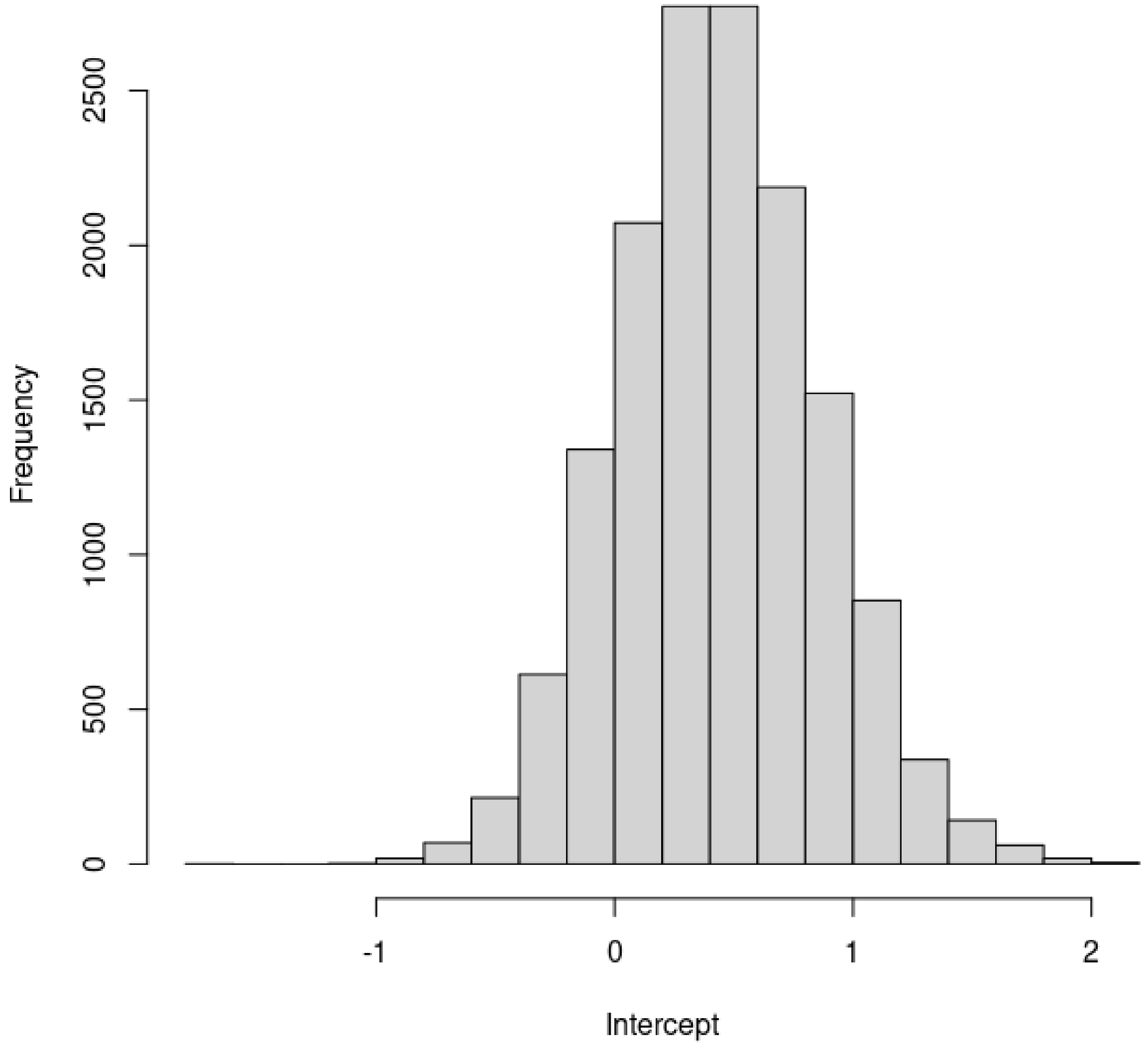
Prior distribution

$$p(\theta \mid y) \propto p(y \mid \theta) p(\theta)$$

Posterior distribution

The posterior distribution

- The posterior distribution is what we use to summarize results from a Bayesian analysis
- For each parameter, we get a distribution of possible values.
- We summarize that distribution in different ways (e.g., mean, median, mode)



Bayesian analysis: Summary of major points

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- All statistical inference is a simple probability calculation
- Express prior information via $p(\theta)$
- Update your prior information using the likelihood, $p(y \mid \theta)$, to form the posterior $p(\theta \mid y)$
- **Final result is a distribution of possible values for the unknown parameters, unlike maximum likelihood estimation, where our result is a single value**

Benefits of the Bayesian approach

1. Natural concept of probability
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    - Our final result is the combination of experience (prior) and new information (likelihood)
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3. Much more flexible than frequentist approaches to account for complex data types
    - Multiple species
    - Multiple data sources
    - Lots of different random effect structures
    - Complex interactions across parameters

Drawbacks of the Bayesian approach

1. Results are always dependent on the prior distribution
   - Can specify weakly informative (vague) priors
   - Be transparent
   - Justify your choice

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   - LOTS of canned software in active development
     - Ex: `brms`, `spOccupancy`, `oSCR`, `marked`

Frequentist

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Questions?