Introduction to Applied Bayesian Analysis in Wildlife Ecology

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Bayesian linear models

General linear models

- Response variable (y) = deterministic part + stochastic part
  - Stochastic = random (unexplained information, error)
  - Deterministic = systematic
- Why are they called "linear" models?
  - Response is a function of explanatory variables whose effects are **additive**
  - "Linear in the parameters"
  - Can still model nonlinear relationships

Which of the following are linear models?

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

$$y_i = \beta_0 + \beta_1 x_i^2 + \epsilon_i$$

$$y_i = \beta_0 x_i^{\beta_1} + \epsilon_i$$

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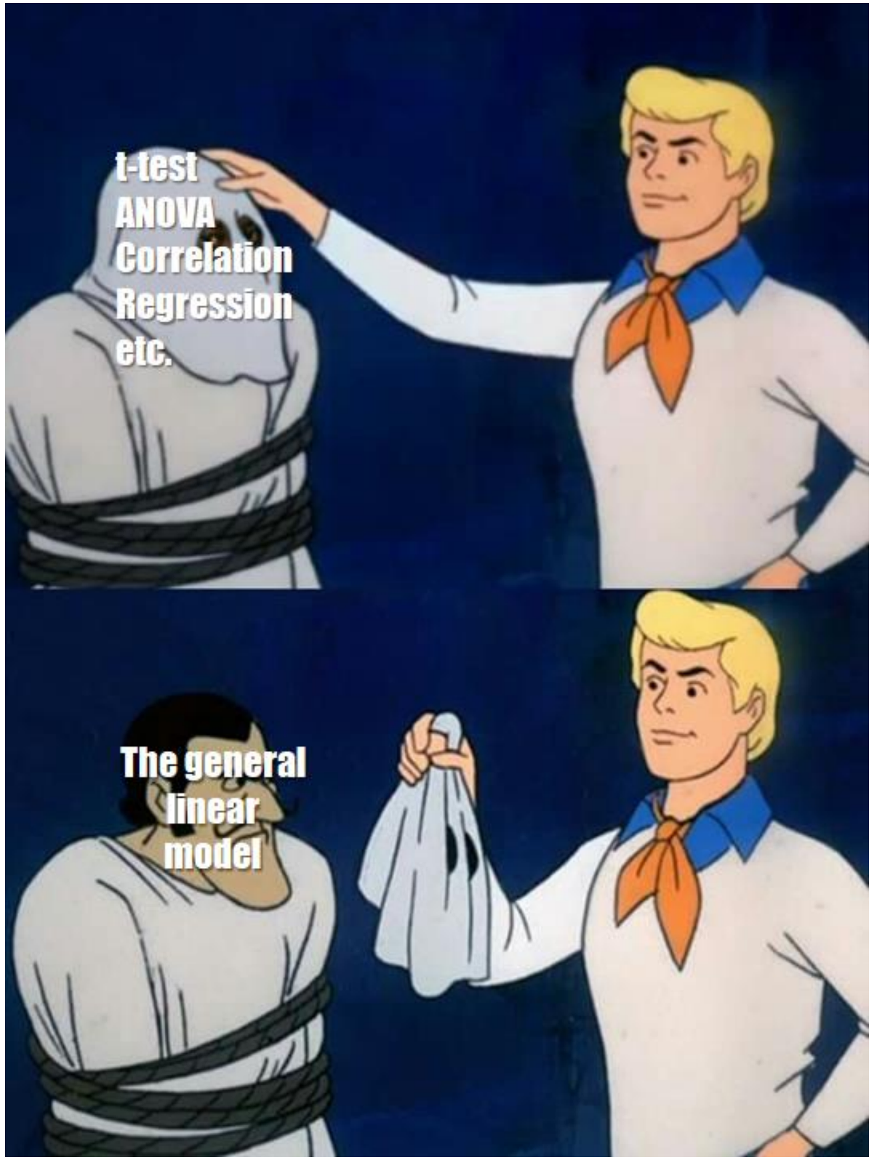
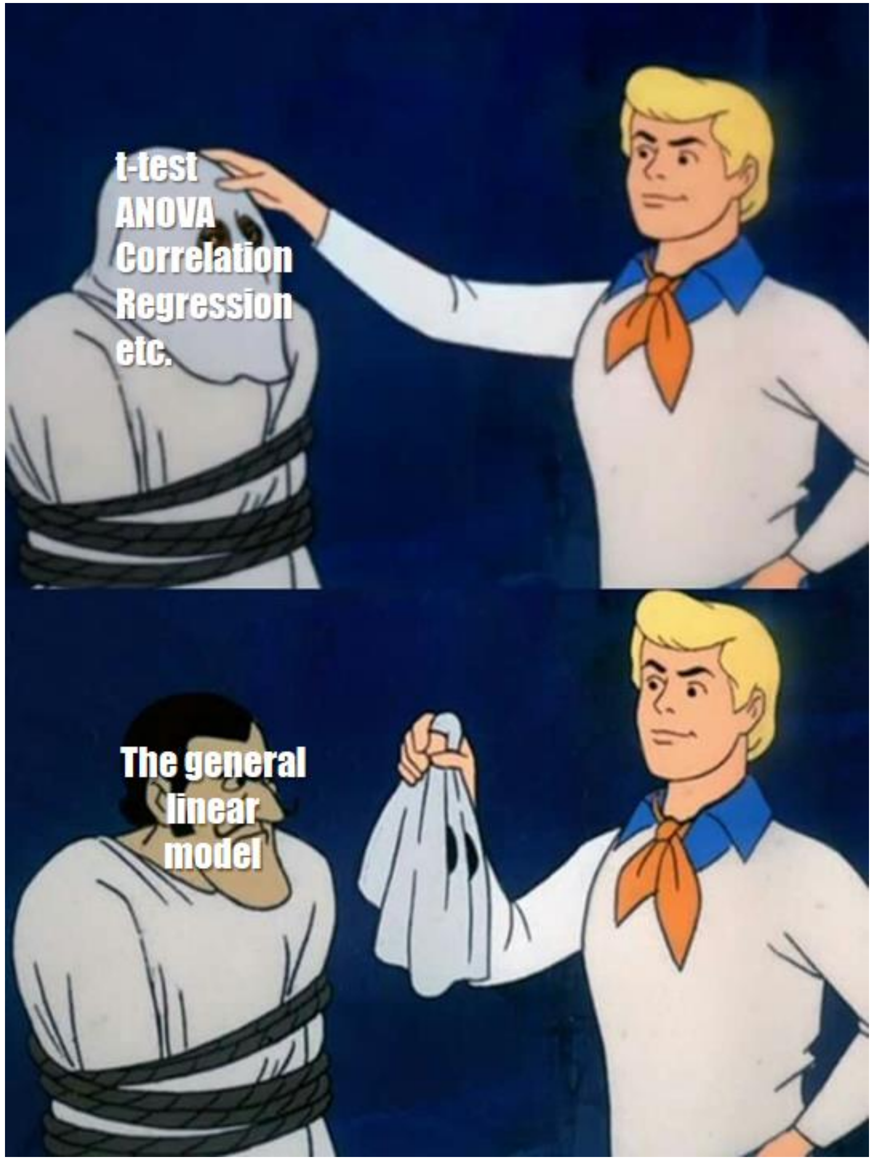
General linear models

- A broad class of models



General linear models

- A broad class of models
- Lots of fancy design names for different linear models and designs
  - t test, ANOVA, ANCOVA, randomized complete block design, split plot design, etc.



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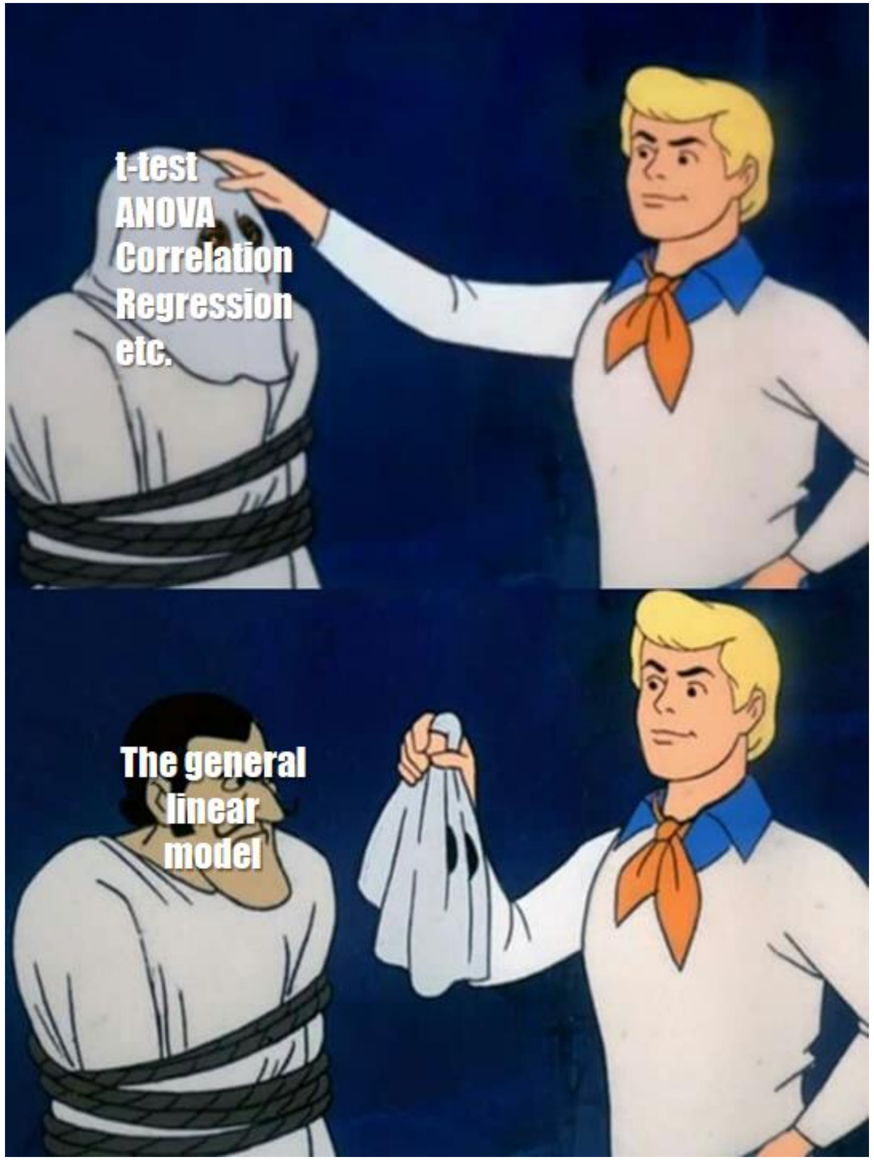

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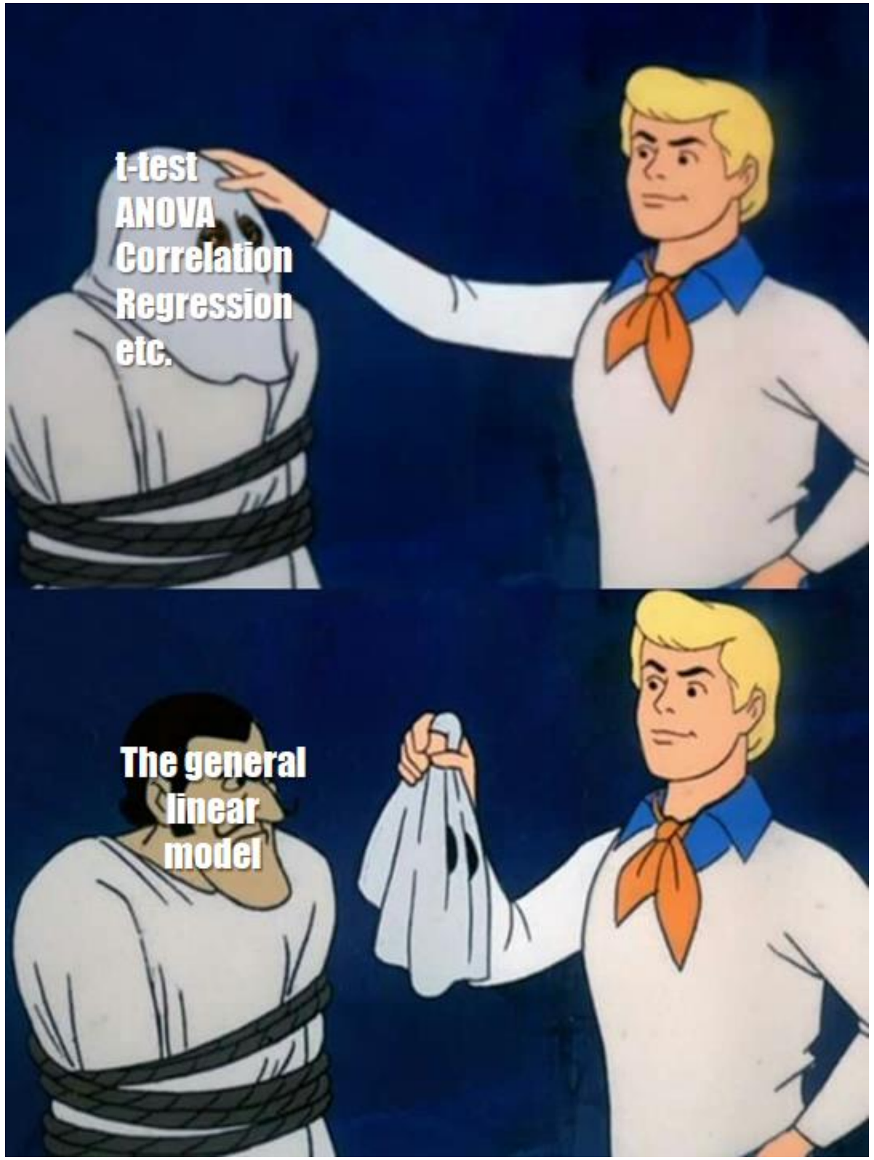
- Don't focus on design names!
- The Bayesian approach de-emphasizes these classical design names because the estimation approach is always the same!!



Linear models

Deterministic Component

$$\mu_i = \beta_0 + \beta_1 \cdot x_{1,i} + \beta_2 \cdot x_{2,i} + \cdots + \beta_p \cdot x_{p,i}$$

Linear models

Can include both continuous and categorical predictor variables

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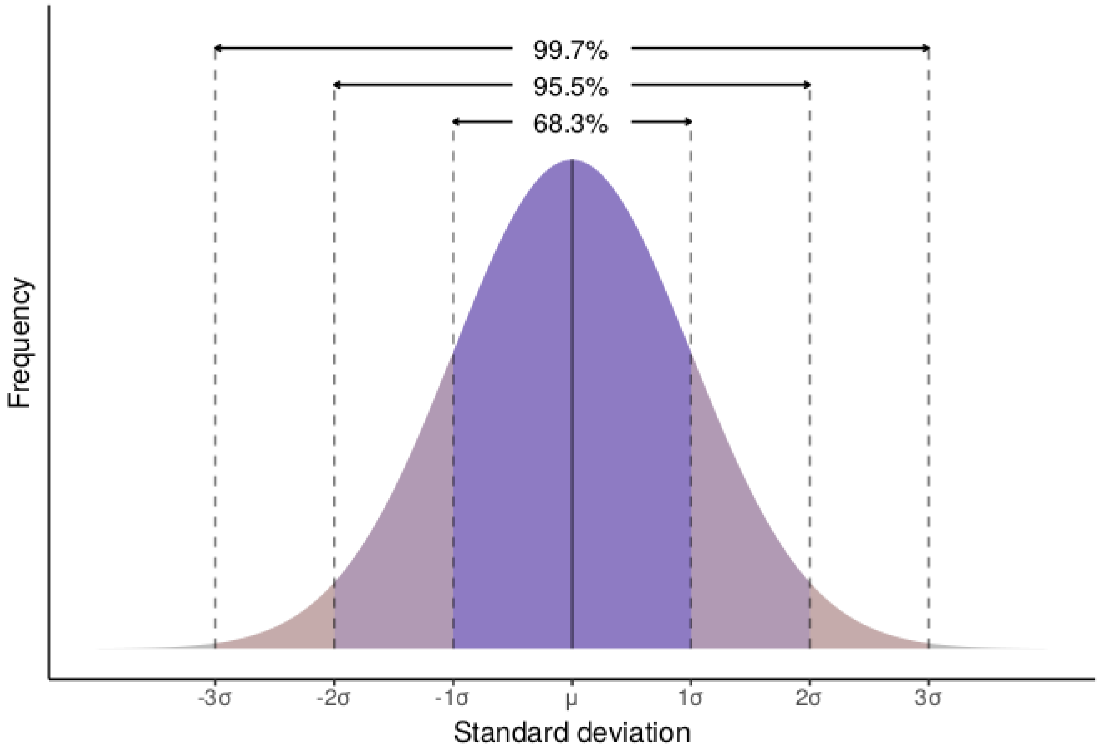
Priors in a Bayesian linear model

- We need priors for all regression coefficients (intercept and slopes) and *either* the residual variance or standard deviation
- Some software puts prior on variance, some software puts prior on standard deviation
- `brms` puts the prior on the residual standard deviation
- If you don't care about the priors much and just want them to be vague and weakly informative, then the default brms priors are a good option

Priors for regression coefficients

- Most common choice of prior distribution is a normal distribution with a mean of 0 and a very large variance



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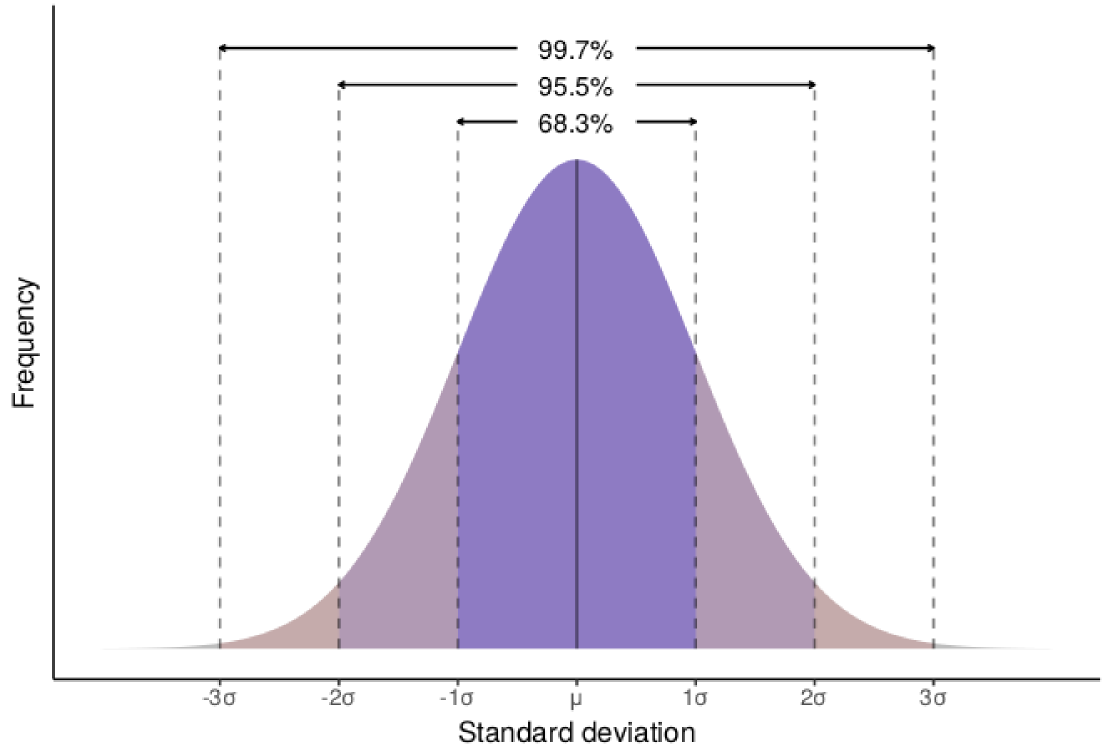
- "Flat prior": very common, gives equal probability to all possible values of the parameter



Priors for regression coefficients

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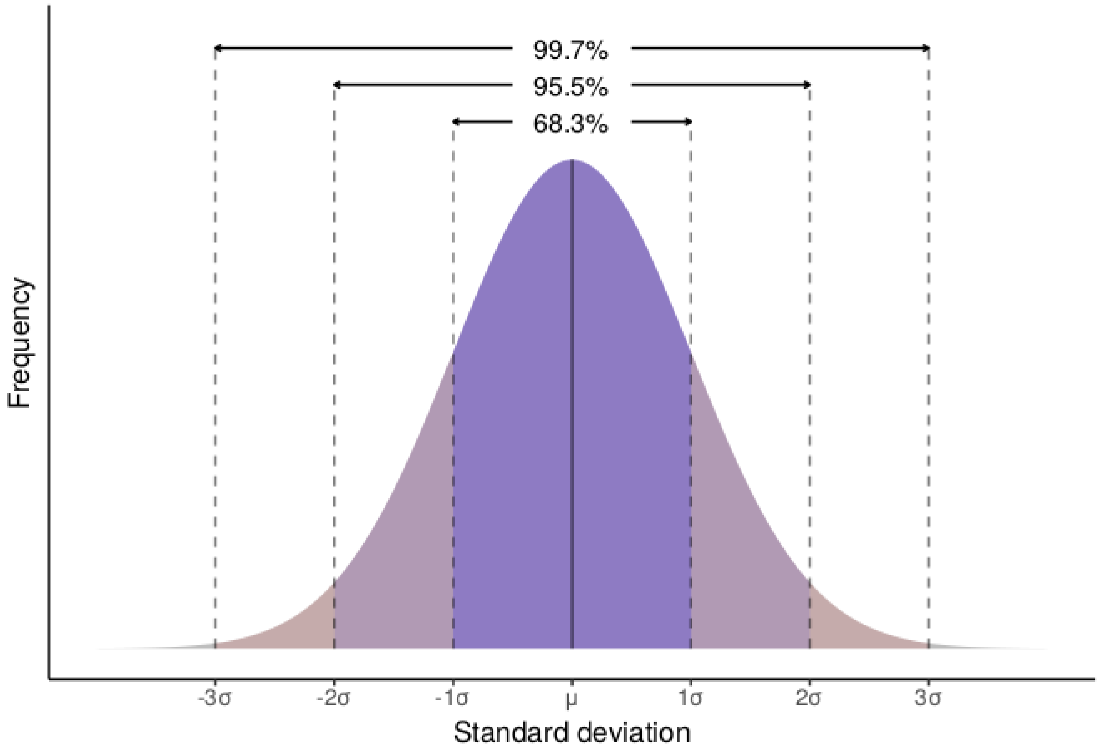
- "Flat prior": very common, gives equal probability to all possible values of the parameter
- Other options: Student's t distribution
- `brms` defaults:
  - Intercept: Student's t
  - Slope terms: flat prior



Priors for the standard deviation

- Remember: the standard deviation only takes positive values
- Common choices:
  - Gamma
  - Inverse-gamma
  - Half Student's t
  - Half Cauchy
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A key benefit to modern Bayesian software like brms is the default priors are suitable for many applications

Example: Bird richness across an elevation gradient

How does bird richness vary across an elevation gradient and in relation to landowner type (private, state, federal)?

`3a-linear-model-brms.R`

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3. Model selection (model comparison)

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Model checking (GoF assessment)

- Does your model adequately reflect the data that you have?
- Does your data meet the assumptions of the given model?
- A good model should generate "replicate" data that closely resemble the true data you have collected
- If our model cannot generate new data similar to the data we collected, it may not adequately represent the true process we are interested in

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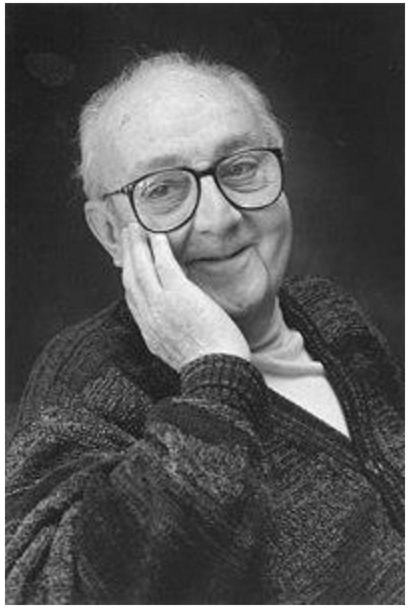
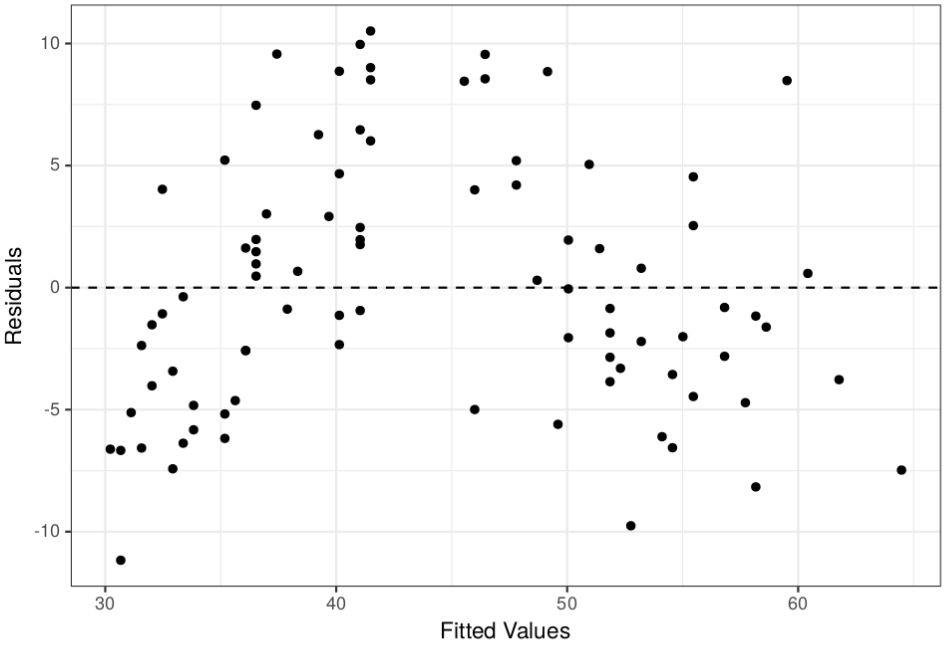
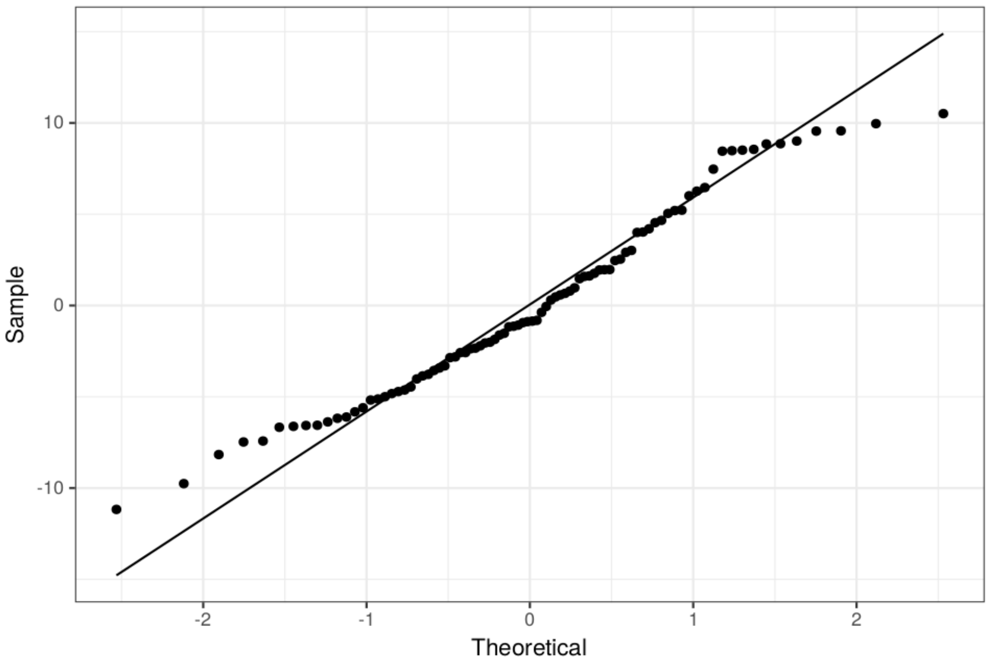
"All models are wrong, but some are useful"
- George Box



Residual diagnostics are a form of model checking

- Standard residual diagnostics can be used for Bayesian linear models
- Plot residuals vs. Model fitted (predicted) values: should show no clear and obvious patterns
- Normal probability plot (qq plot): should fall along a diagonal line





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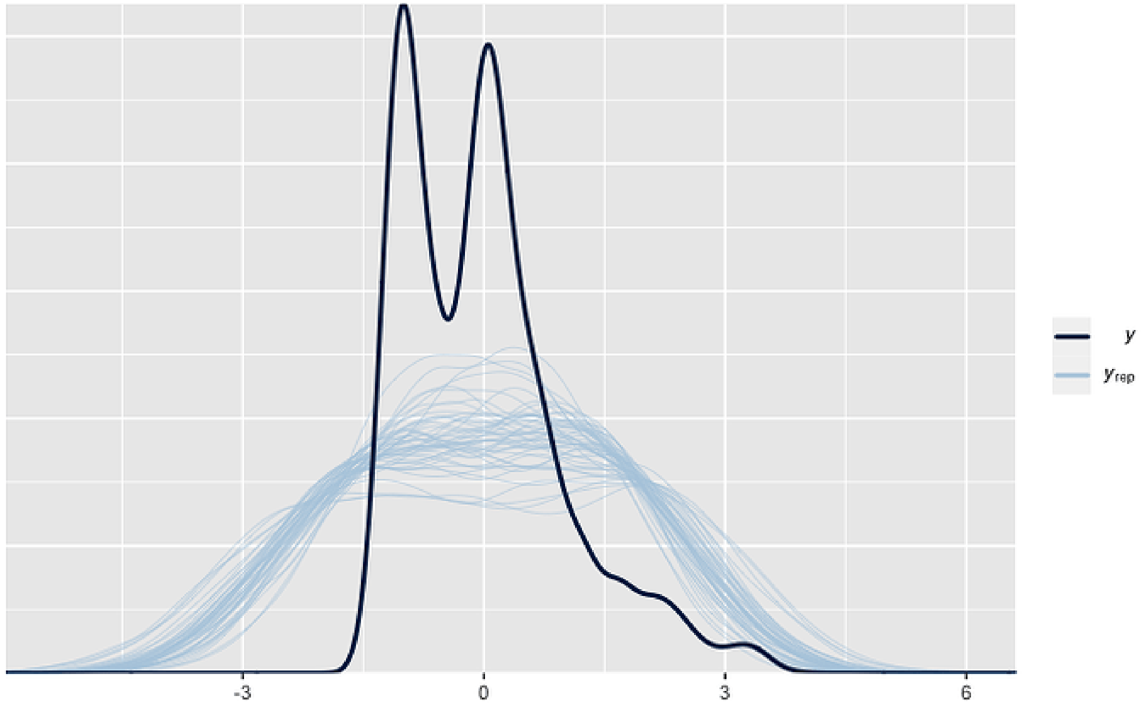
Model checking: posterior predictive checks

Bad model fit



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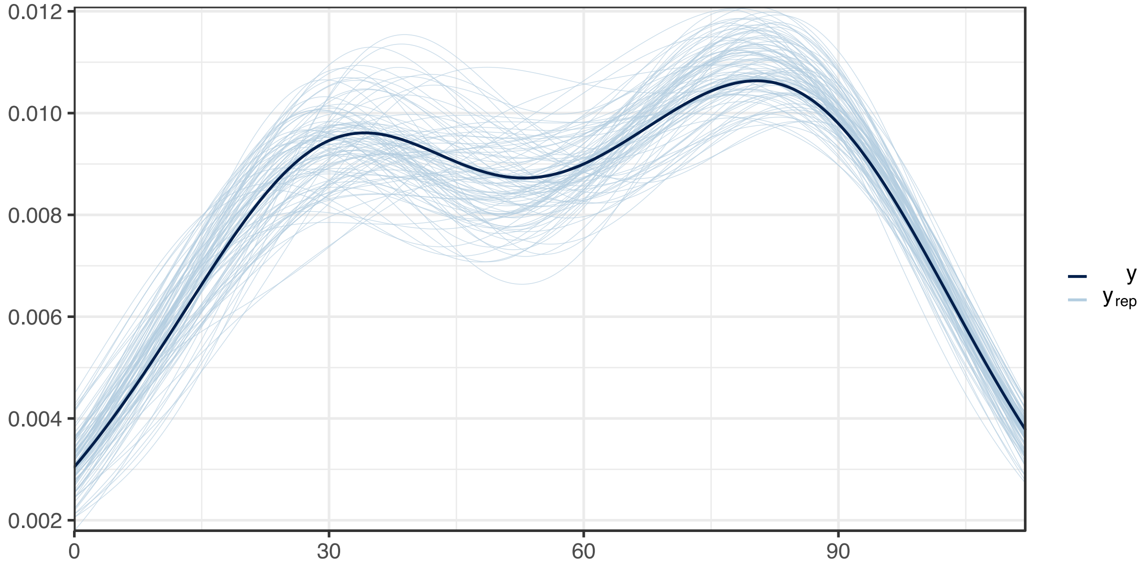

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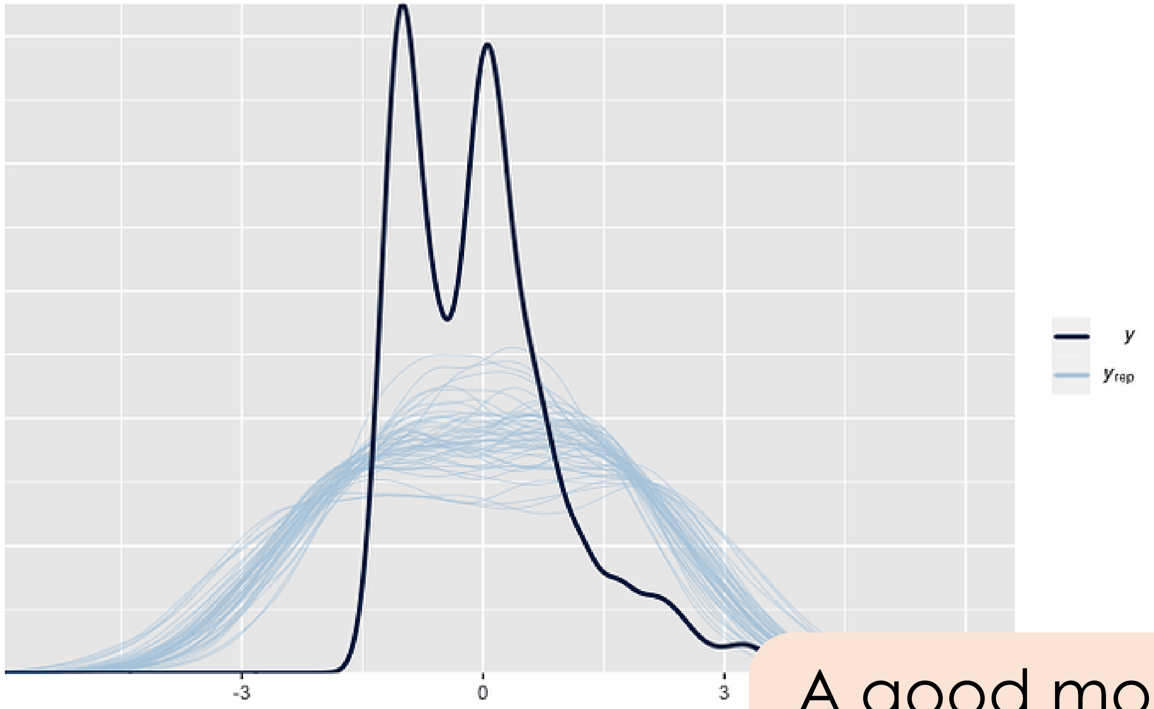

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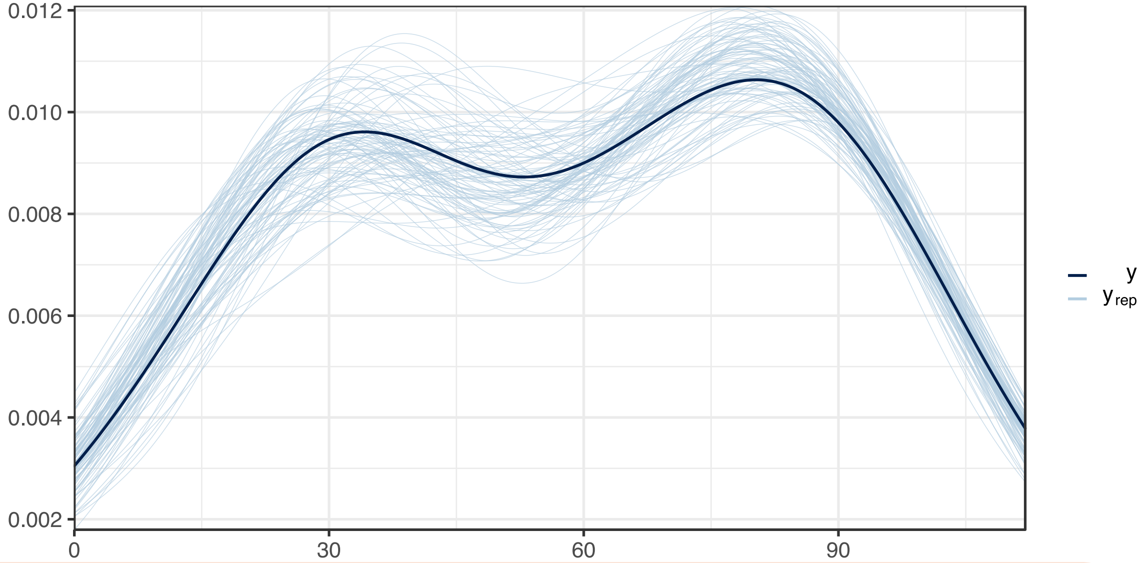
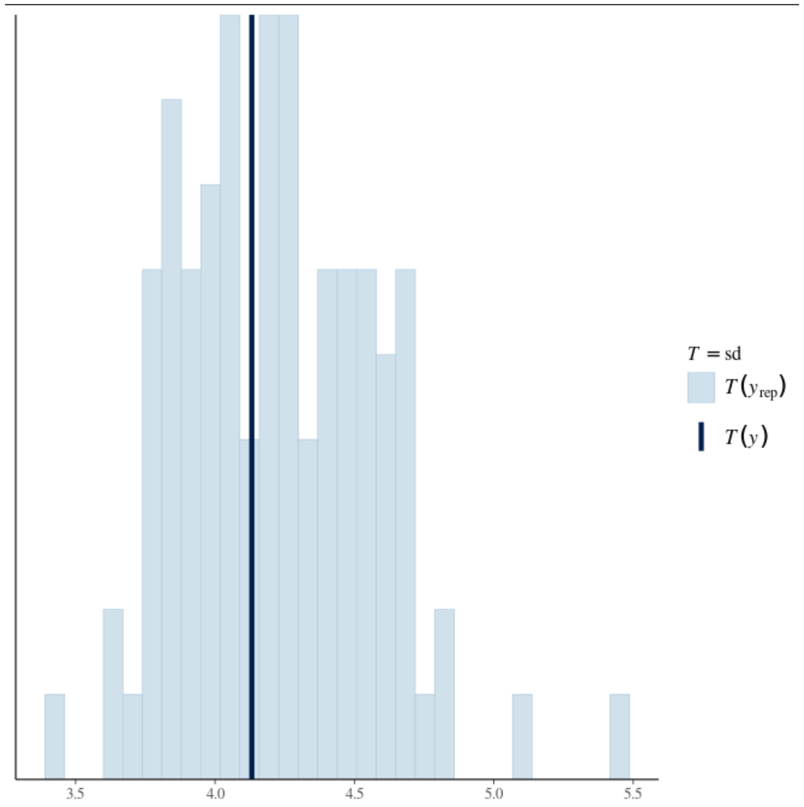


A good model fit is one where the curve for the true data could easily be one of the curves from the model fit

Model checking: posterior predictive checks

- Posterior predictive checks can be summarized with what is called a Bayesian p-value



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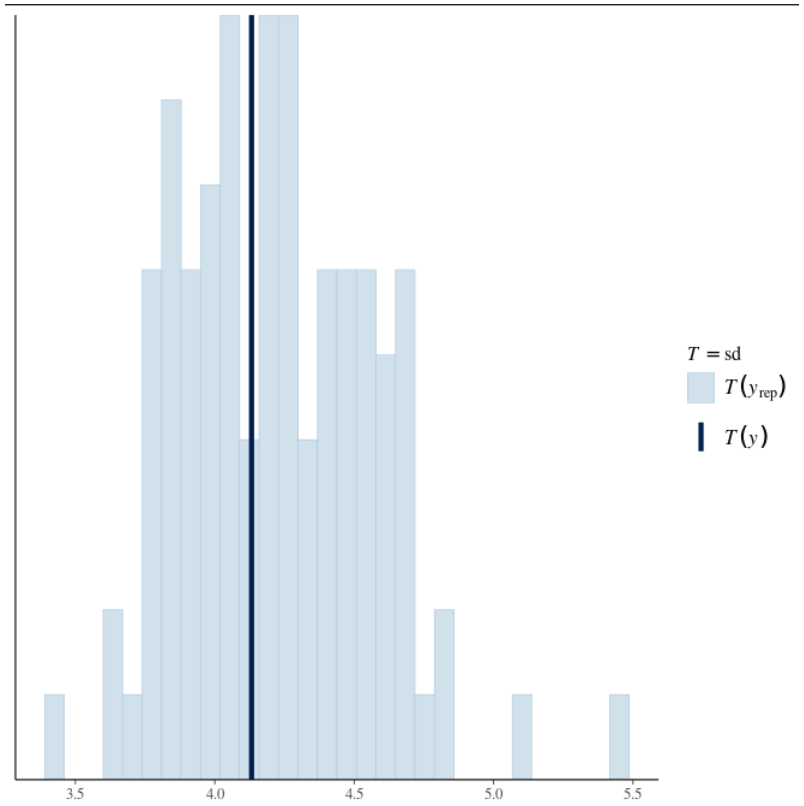

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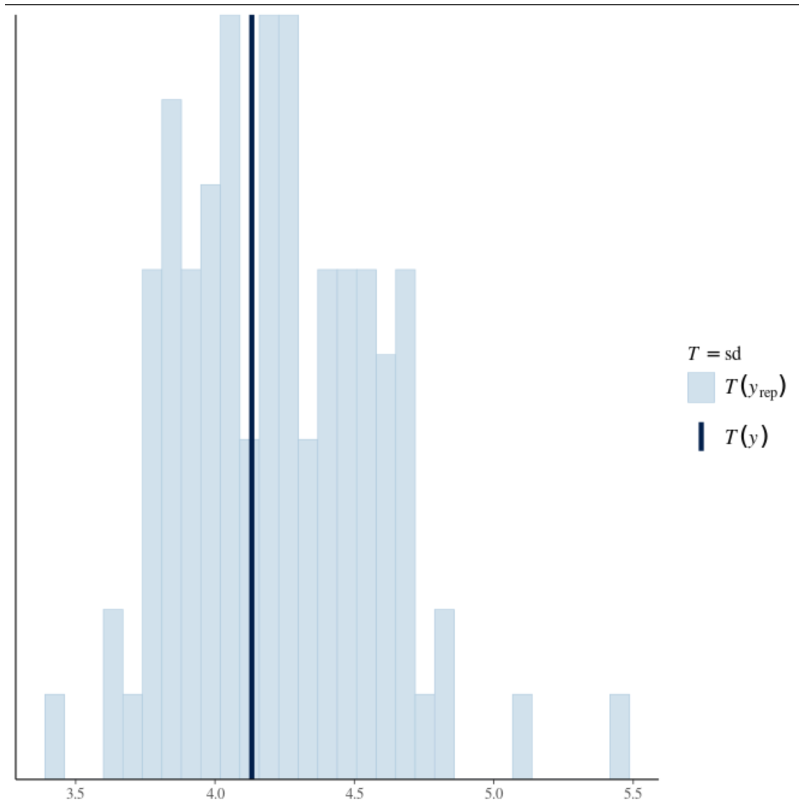

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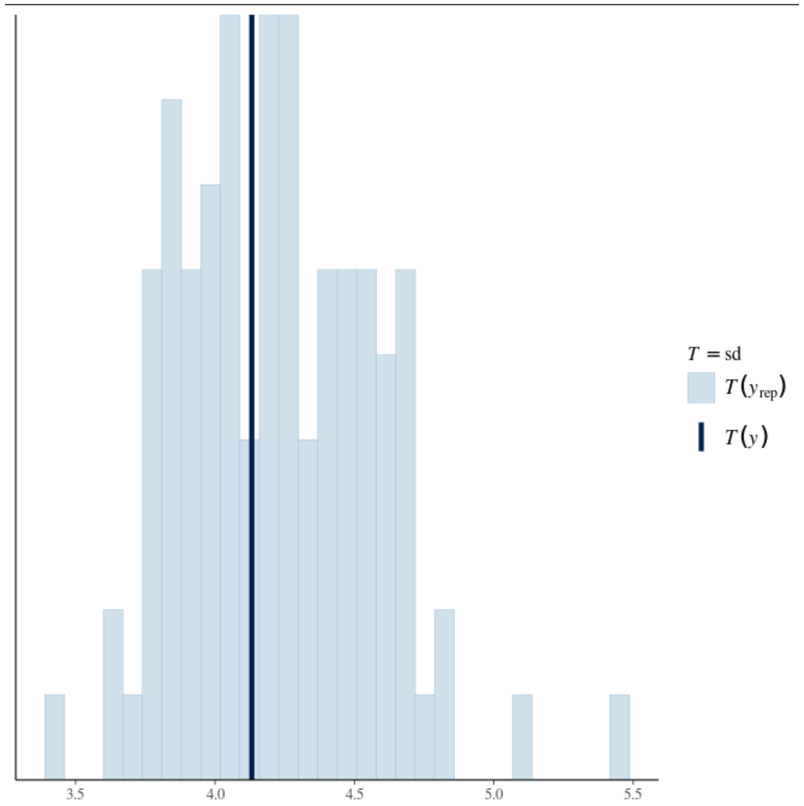

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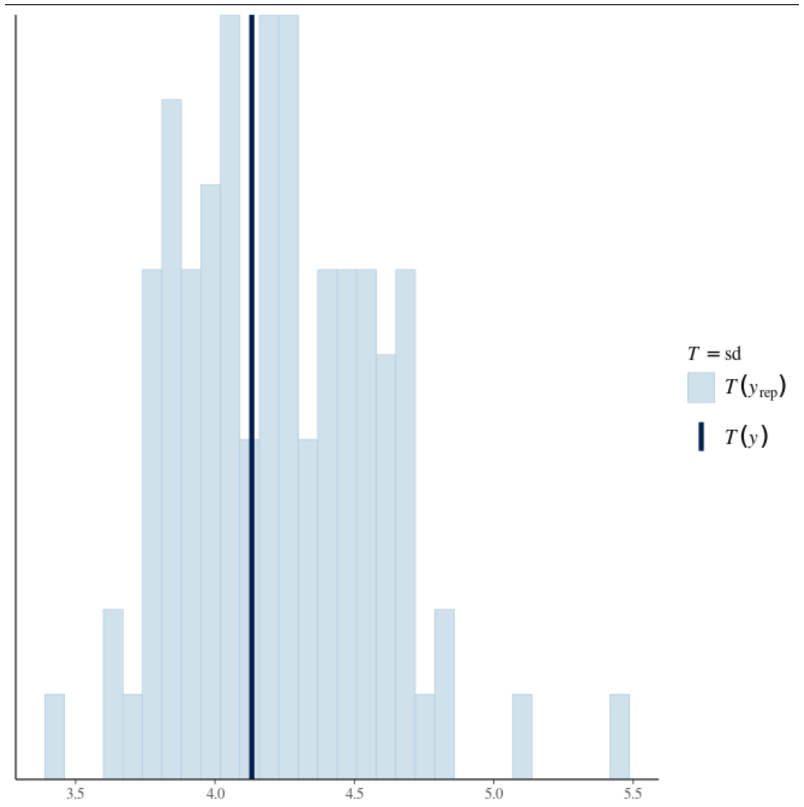
The Bayesian p-value is the proportion of bars to the right of the black line



Use caution with Bayesian p-values

- Can be extremely sensitive to outliers
- A Bayesian p-value of 0.5 does not necessarily indicate you have a good model

Bayesian p-values are useful, but you should always use graphical methods to assess PPCs as well

More on Bayesian model checking

ECOLOGICAL MONOGRAPHS
ECOLOGICAL SOCIETY OF AMERICA

Concepts & Synthesis | Full Access

A guide to Bayesian model checking for ecologists

Paul B. Conn, Devin S. Johnson, Perry J. Williams, Sharon R. Melin, Mevin B. Hooten

What do you do after you fit a Bayesian model?

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Model selection/comparison

- How can we compare different models?
- Which model among a set of candidate models is the best?
- In frequentist methods, we can use AIC (the most common type of information criterion).
- Bayesian methods have a variety of information criteria as well.
- Two common approaches: WAIC and LOO

WAIC and LOO

- Widely Applicable Information Criterion or Watanabe-Akaike Information Criterion
- Leave-one-out cross validation
- Can be used in the same manner as AIC
- Very applicable for a variety of complex models, unlike other information criteria (DIC) for Bayesian models
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Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC

Published: 30 August 2016

Volume 27, pages 1413–1432, (2017) Cite this article

Aki Vehtari, Andrew Gelman & Jonah Gabry

Example: Bird richness across an elevation gradient

How does bird richness vary across an elevation gradient and in relation to landowner type (private, state, federal)?

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Questions?