GENE613 - Homework 7

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1. If an animal has available its own performance ($X_1 = 50$), the average performance of 8 paternal half-sibs ($X_2 = 65$), the average performance of 10 progeny ($X_3 = 40$) and the performance of its sire ($X_4 = 75$), all this information can be used to calculate the animal's EBV using the selection index approach, solving for

$$P\underline{b} = \underline{r}$$

Assume the phenotypic and additice genetic variances are 2250 and 860, respectively.

a) Construct the P matrix as the variances and covariances for the sources of information.

$$P = \begin{bmatrix} \sigma^2_P & 0{,}25\sigma^2_A & 0{,}5\sigma^2_A & 0{,}5\sigma^2_A \\ 0{,}25\sigma^2_A & \sigma^2_P & 0{,}125\sigma^2_A & 0{,}5\sigma^2_A \\ 0{,}5\sigma^2_A & 0{,}125\sigma^2_A & \sigma^2_P & 0{,}25\sigma^2_A \\ 0{,}5\sigma^2_A & 0{,}5\sigma^2_A & 0{,}25\sigma^2_A & \sigma^2_P\sigma^2_A \end{bmatrix} = \begin{bmatrix} 2250 & 215 & 430 & 430 \\ 215 & 2250 & 107{,}5 & 430 \\ 430 & 107{,}5 & 2250 & 215 \\ 430 & 430 & 215 & 2250 \end{bmatrix}$$

b) Construct the $\underline{r}$ vector as the covariates of the animal's true BV with the sources of information.

$$\underline{r} = \begin{bmatrix} \sigma^2_A \\ 0{,}25\sigma^2_A \\ 0{,}5\sigma^2_A \\ 0{,}5\sigma^2_A \end{bmatrix} = \begin{bmatrix} 860 \\ 215 \\ 430 \\ 430 \end{bmatrix}$$

c) Invert the P matrix

$$P^{-1} = \begin{bmatrix} 0{,}000478 & -0{,}000027 & -0{,}000083 & -0{,}000078 \\ -0{,}000027 & 0{,}000463 & -0{,}000009 & -0{,}000083 \\ -0{,}000083 & -0{,}000009 & 0{,}000463 & -0{,}000027 \\ -0{,}000078 & -0{,}000083 & -0{,}000027 & 0{,}000478 \end{bmatrix}$$

d) Pre-multiply the P^{-1} matrix to the $\underline{r}$ vector and show the resulting $\hat{\underline{b}}$ values.

$$\hat{\underline{b}} = P^{-1} \times \underline{r}$$

$$\hat{\underline{b}} = \begin{bmatrix} 0{,}000478 & -0{,}000027 & -0{,}000083 & -0{,}000078 \\ -0{,}000027 & 0{,}000463 & -0{,}000009 & -0{,}000083 \\ -0{,}000083 & -0{,}000009 & 0{,}000463 & -0{,}000027 \\ -0{,}000078 & -0{,}000083 & -0{,}000027 & 0{,}000478 \end{bmatrix} \times \begin{bmatrix} 860 \\ 215 \\ 430 \\ 430 \end{bmatrix} = \begin{bmatrix} 0{,}336 \\ 0{,}037 \\ 0{,}115 \\ 0{,}109 \end{bmatrix}$$

e) Calculate the EBV as $I =$ index on the animal.

$$I = \begin{bmatrix} 50 & 65 & 40 & 75 \end{bmatrix} \times \begin{bmatrix} 0{,}336 \\ 0{,}037 \\ 0{,}115 \\ 0{,}109 \end{bmatrix} = 31{,}966$$

f) Calculate the ACC value associated with the EBV.

$$ACC = \sqrt{\left(\frac{r}{\sigma_A^2}\right)' \times \underline{\hat{b}}} = \sqrt{\begin{bmatrix}1 & 0{,}25 & 0{,}5 & 0{,}5\end{bmatrix} \times \begin{bmatrix}0{,}336\\0{,}037\\0{,}115\\0{,}109\end{bmatrix}} = 0{,}676$$

2. Estimate breeding values on the 6 individuals below using BLUP and the MME. Weights are already adjusted for sex differences. Assume $\sigma_P^2 = 2500$ and $\sigma_A^2 = 900$

ID	SIRE	DAM	GC	WEIGHT
1	0	0	1	930
2	0	0	1	880
3	1	0	2	965
4	1	2	2	945
5	3	0	3	970
6	4	0	3	950

a) Construct the MME

$$\begin{bmatrix}X'X & X'Z\\ Z'X & Z'Z + A^{-1}\lambda\end{bmatrix} \times \begin{bmatrix}\hat{\beta}\\ \underline{\hat{u}}\end{bmatrix} = \begin{bmatrix}X'\underline{y}\\ Z'\underline{y}\end{bmatrix}$$

$$\begin{bmatrix}
6 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1\\
2 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
2 & 0 & 2 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\
2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 1\\
1 & 1 & 0 & 0 & 4{,}26 & 0{,}89 & -1{,}19 & -1{,}78 & 0 & 0\\
1 & 1 & 0 & 0 & 0{,}89 & 3{,}67 & 0 & -1{,}78 & 0 & 0\\
1 & 0 & 1 & 0 & -1{,}19 & 0 & 3{,}96 & 0 & -1{,}19 & 0\\
1 & 0 & 1 & 0 & -1{,}78 & -1{,}78 & 0 & 5{,}15 & 0 & -1{,}19\\
1 & 0 & 0 & 1 & 0 & 0 & -1{,}19 & 0 & 3{,}37 & 0\\
1 & 0 & 0 & 1 & 0 & 0 & 0 & -1{,}19 & 0 & 3{,}37
\end{bmatrix} \times \begin{bmatrix}\mu\\ \beta_1\\ \beta_2\\ \beta_3\\ u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\end{bmatrix} = \begin{bmatrix}5740\\1810\\1910\\1920\\930\\880\\965\\945\\970\\950\end{bmatrix}$$

b) After deleting the equation for μ, show the solutions vector $(\hat{\beta}, \hat{u})$ containing contemporary group effects and estimated breeding values.

$$\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\end{bmatrix} = \begin{bmatrix}
0{,}77 & 0{,}21 & 0{,}11 & -0{,}28 & -0{,}27 & -0{,}16 & -0{,}26 & -0{,}09 & -0{,}12\\
0{,}21 & 0{,}85 & 0{,}17 & -0{,}26 & -0{,}17 & -0{,}34 & -0{,}35 & -0{,}17 & -0{,}18\\
0{,}11 & 0{,}17 & 0{,}8 & -0{,}13 & -0{,}08 & -0{,}17 & -0{,}18 & -0{,}3 & -0{,}3\\
-0{,}28 & -0{,}26 & -0{,}13 & 0{,}46 & 0{,}1 & 0{,}24 & 0{,}27 & 0{,}12 & 0{,}14\\
-0{,}27 & -0{,}17 & -0{,}08 & 0{,}1 & 0{,}44 & 0{,}09 & 0{,}24 & 0{,}06 & 0{,}11\\
-0{,}16 & -0{,}34 & -0{,}17 & 0{,}24 & 0{,}09 & 0{,}48 & 0{,}21 & 0{,}22 & 0{,}13\\
-0{,}26 & -0{,}35 & -0{,}18 & 0{,}27 & 0{,}24 & 0{,}21 & 0{,}49 & 0{,}13 & 0{,}23\\
-0{,}09 & -0{,}17 & -0{,}3 & 0{,}12 & 0{,}06 & 0{,}22 & 0{,}13 & 0{,}46 & 0{,}13\\
-0{,}12 & -0{,}18 & -0{,}3 & 0{,}14 & 0{,}11 & 0{,}13 & 0{,}23 & 0{,}13 & 0{,}47
\end{bmatrix} \times \begin{bmatrix}1810\\1910\\1920\\930\\880\\965\\945\\970\\950\end{bmatrix}$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} 906{,}01 \\ 952{,}8 \\ 958{,}9 \\ 8{,}67 \\ -10{,}68 \\ 7{,}47 \\ -3{,}07 \\ 5{,}93 \\ -3{,}72 \end{bmatrix}$$