

GENE638 - Homework 4

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COW	HERD	LACTATION	MILK FAT (lb)
1	1	1	600
1	1	2	680
2	1	1	500
3	2	1	800
3	2	2	895
4	2	1	775
5	2	1	600
5	2	2	715

Given $y_{ijk} = \mu + H_i + L_j + C_k + e_{ijk}$ where μ , herd (H_i) and lactation L_j are fixed effects; cows (C_k) and residuals (e_{ijk}) are random effects and $var \begin{bmatrix} \underline{c} \\ \underline{e} \end{bmatrix} = \begin{bmatrix} A\sigma_c^2 & 0 \\ 0 & I\sigma_e^2 \end{bmatrix}$ so the MME are: $\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + A^{-1}\lambda \end{bmatrix} \times \begin{bmatrix} \hat{\underline{\beta}} \\ \hat{\underline{u}} \end{bmatrix} = \begin{bmatrix} X'y \\ Z'y \end{bmatrix}$ with $\lambda = \frac{\sigma_e^2}{\sigma_c^2}$.

1. In the above model, indicate what each subscript indexes.

i = The herd to which the cow belong.

j = The lactation time in which the trait was measured.

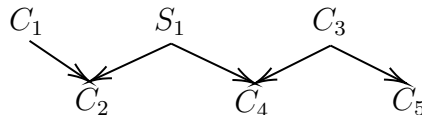
k = Each one of the cows.

2. What are the elements in $\hat{\underline{\beta}}$ and $\hat{\underline{u}}$

$$\hat{\underline{\beta}}' = [\mu \quad H_1 \quad H_2 \quad L_1 \quad L_2]$$

$$\hat{\underline{u}}' = [C_1 \quad S_1 \quad C_3 \quad C_2 \quad C_4 \quad C_5]$$

New pedigree:



3. Calculate A^{-1} using the Henderson's method for rapid inversion of A .

$$A^{-1} = \begin{bmatrix} 1.50 & 0.50 & 0.00 & -1.00 & 0.00 & 0.00 \\ 0.50 & 2.00 & 0.50 & -1.00 & -1.00 & 0.00 \\ 0.00 & 0.50 & 1.83 & 0.00 & -1.00 & -0.67 \\ -1.00 & -1.00 & 0.00 & 2.00 & 0.00 & 0.00 \\ 0.00 & -1.00 & -1.00 & 0.00 & 2.00 & 0.00 \\ 0.00 & 0.00 & -0.67 & 0.00 & 0.00 & 1.33 \end{bmatrix}$$

4. Write the observations in terms of the model $\underline{y} = X\underline{\beta} + Z\underline{u} + \underline{e}$

$$\begin{bmatrix} 600 \\ 680 \\ 500 \\ 800 \\ 895 \\ 775 \\ 600 \\ 715 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu \\ H_1 \\ H_2 \\ L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ S_1 \\ C_3 \\ C_2 \\ C_4 \\ C_5 \end{bmatrix} + \begin{bmatrix} e_{111} \\ e_{121} \\ e_{112} \\ e_{213} \\ e_{223} \\ e_{214} \\ e_{215} \\ e_{225} \end{bmatrix}$$

5. Construct MME with $\lambda = 1.5$

$$\begin{bmatrix} 8.00 & 3.00 & 5.00 & 5.00 & 3.00 & 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 2.00 \\ 3.00 & 3.00 & 0.00 & 2.00 & 1.00 & 2.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 5.00 & 0.00 & 5.00 & 3.00 & 2.00 & 0.00 & 0.00 & 2.00 & 0.00 & 1.00 & 2.00 \\ 5.00 & 2.00 & 3.00 & 5.00 & 0.00 & 1.00 & 0.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 3.00 & 1.00 & 2.00 & 0.00 & 3.00 & 1.00 & 0.00 & 1.00 & 0.00 & 0.00 & 1.00 \\ 2.00 & 2.00 & 0.00 & 1.00 & 1.00 & 4.25 & 0.75 & 0.00 & -1.50 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.75 & 3.00 & 0.75 & -1.50 & -1.50 & 0.00 \\ 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 0.00 & 0.75 & 4.75 & 0.00 & -1.50 & -1.00 \\ 1.00 & 1.00 & 0.00 & 1.00 & 0.00 & -1.50 & -1.50 & 0.00 & 4.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 1.00 & 1.00 & 0.00 & 0.00 & -1.50 & -1.50 & 0.00 & 4.00 & 0.00 \\ 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 0.00 & 0.00 & -1.00 & 0.00 & 0.00 & 4.00 \end{bmatrix} \begin{bmatrix} \mu \\ H_1 \\ H_2 \\ L_1 \\ L_2 \\ C_1 \\ S_1 \\ C_3 \\ C_2 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 5565 \\ 1780 \\ 3785 \\ 3275 \\ 2290 \\ 1280 \\ 0 \\ 1695 \\ 500 \\ 775 \\ 1315 \end{bmatrix}$$

6. Show algebraically that $\lambda = \frac{1-h^2}{h^2}$

$$\begin{aligned} \frac{\sigma_e^2}{\sigma_a^2} &= \frac{1-h^2}{h^2} \\ \frac{\sigma_e^2}{\sigma_a^2} &= \frac{1 - \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}}{\frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}} \\ \frac{\sigma_e^2}{\sigma_a^2} &= \frac{\frac{\sigma_a^2 + \sigma_e^2}{\sigma_a^2 + \sigma_e^2} - \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}}{\frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}} \\ \frac{\sigma_e^2}{\sigma_a^2} &= \frac{\frac{\sigma_a^2 + \sigma_e^2 - \sigma_a^2}{\sigma_a^2 + \sigma_e^2}}{\frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}} \\ \frac{\sigma_e^2}{\sigma_a^2} &= \frac{\frac{\sigma_e^2}{\sigma_a^2 + \sigma_e^2}}{\frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}} \\ \frac{\sigma_e^2}{\sigma_a^2} &= \frac{\sigma_e^2}{\sigma_a^2} \end{aligned}$$

7. The row equation in the MME corresponding to \hat{S}_1 is:

$$\begin{aligned} 0.75\hat{C}_1 - 1.5\hat{C}_2 + 0.75\hat{C}_3 - 1.5\hat{C}_4 + 3\hat{S}_1 &= 0 \\ \hat{S}_1 &= -0.25\hat{C}_1 + 0.5\hat{C}_2 - 0.25\hat{C}_3 + 0.5\hat{C}_4 \\ \hat{S}_1 &= 0.5 \left(\hat{C}_2 - 0.5\hat{C}_1 \right) + 0.5 \left(\hat{C}_4 - 0.5\hat{C}_3 \right) \end{aligned}$$

- (a) Look at the pedigree above (and this prediction equation) and describe in words how \hat{S}_1 is being predicted here. S_1 is predicted as the average of their progeny after removing the expected proportion given by the cows used as genetic complement.
- (b) Show that $\mu = 818.87$; $\hat{H}_1 = -165.15$; $\hat{H}_2 = 0$; $\hat{L}_1 = -100.57$; $\hat{L}_2 = 0$; $\hat{C}_1 = 14.63$; $\hat{C}_2 = -9.27$; $\hat{C}_3 = 31.57$; $\hat{C}_4 = 24.55$; $\hat{C}_5 = -47.63$; and \hat{S}_1 (calculated as above) provides a solution to the system of equations.

$$\begin{bmatrix}
 8.00 & 3.00 & 5.00 & 5.00 & 3.00 & 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 2.00 \\
 3.00 & 3.00 & 0.00 & 2.00 & 1.00 & 2.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\
 5.00 & 0.00 & 5.00 & 3.00 & 2.00 & 0.00 & 0.00 & 2.00 & 0.00 & 1.00 & 2.00 \\
 5.00 & 2.00 & 3.00 & 5.00 & 0.00 & 1.00 & 0.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
 3.00 & 1.00 & 2.00 & 0.00 & 3.00 & 1.00 & 0.00 & 1.00 & 0.00 & 0.00 & 1.00 \\
 2.00 & 2.00 & 0.00 & 1.00 & 1.00 & 4.25 & 0.75 & 0.00 & -1.50 & 0.00 & 0.00 \\
 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.75 & 3.00 & 0.75 & -1.50 & -1.50 & 0.00 \\
 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 0.00 & 0.75 & 4.75 & 0.00 & -1.50 & -1.00 \\
 1.00 & 1.00 & 0.00 & 1.00 & 0.00 & -1.50 & -1.50 & 0.00 & 4.00 & 0.00 & 0.00 \\
 1.00 & 0.00 & 1.00 & 1.00 & 0.00 & 0.00 & -1.50 & -1.50 & 0.00 & 4.00 & 0.00 \\
 2.00 & 0.00 & 2.00 & 1.00 & 1.00 & 0.00 & 0.00 & -1.00 & 0.00 & 0.00 & 4.00
 \end{bmatrix}
 \begin{bmatrix}
 818.87 \\
 -165.15 \\
 0.00 \\
 -100.57 \\
 0.00 \\
 14.63 \\
 -3.91 \\
 31.57 \\
 -9.27 \\
 24.55 \\
 -47.65
 \end{bmatrix}
 =
 \begin{bmatrix}
 5565 \\
 1780 \\
 3785 \\
 3275 \\
 2290 \\
 1280 \\
 0 \\
 1695 \\
 500 \\
 775 \\
 1315
 \end{bmatrix}$$

8. What do \hat{H}_1 and \hat{L}_1 estimate? $H_1 = H_1 - H_2$ and $L_1 = L_1 - L_2$
9. Show that $1'A^{-1}\hat{u} = 0$. What does this mean? As cows and sire were treated as random effects they are assumed to be normally distributed with mean 0. What $1'A^{-1}\hat{u}$ do is to sum the standardized values of \hat{u} across generations

```
> t(rep(1,6)) %*% Ainv %*% uHat
```

```
      [,1]
```

```
[1,] 1.847411e-13
```

10. What are the predicted phenotypes $\hat{p} = \underline{y} - X\hat{\beta}$

```
> y - (X %*% betaHat)
```

```
      [,1]
```

```
[1,] 46.85338
```

```
[2,] 26.27948
```

```
[3,] -53.14662
```

```
[4,] 81.70794
```

```
[5,] 76.13404
```

```
[6,] 56.70794
```

```
[7,] -118.29206
```

```
[8,] -103.86596
```

11. Find $\hat{e}'\hat{e}$ and compare results to those in Homework 3.

```
> yHat <- X %*% betaHat + Z %*% uHat
```

```
> eHat <- (y - yHat)
```

```
> t(eHat) %*% eHat
```

```
      [,1]
```

```
[1,] 16785.19
```

Comparing the $\hat{e}'\hat{e} = 10719.24$ from the Homework 3 when cows were treated as random effects but unrelated and the actual $\hat{e}'\hat{e} = 16785.19$ taking into account the relatedness of the individuals seems that using that information improves the accuracy of the computed breeding value for the individuals in the model.

12. Do the predicted cow breeding values rank the same as they did in Homework 3 when all cows were treated as unrelated? Explain why or why not.

```
> oldBV <- c(21.5, -21.5, 43.8, 21, -64.8)
> newBV <- c(14.63, -9.27, 31.57, 24.55, -47.63)
> order(oldBV, decreasing = TRUE)
```

```
[1] 3 1 4 2 5
```

```
> order(newBV, decreasing = TRUE)
```

```
[1] 3 4 1 2 5
```

No, they don't rank the same as they did in Homework 3 when all cows were treated as unrelated, because relatedness information affects the breeding values calculation and if the data is correct improve the prediction the genetic and environmental trend