GENE638 - Homework 1

Daniel Osorio - dcosorioh@tamu.edu
Department of Veterinary Integrative Biosciences
Texas A&M University

1. Given

$$A = \begin{bmatrix} -1 & 7 & 9 & -2 & 3 \\ 3 & 13 & 10 & 2 & 6 \\ 11 & -9 & 0 & -3 & 2 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} C = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \underline{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) Calculate:

i. $\sum_{i=1}^{3} b_{i1}$

```
> sum(B[,1])
[1] 3
```

ii. $\sum_{i=1}^{3} b_{i2}$

```
> sum(B[,2])
[1] 0
```

iii. $\sum_{i=1}^{3} b_{i3}$

```
> sum(B[,3])
[1] 0
```

iv. $\sum_{i=1}^{j} b_{ij}$

```
> sum(B)
[1] 3
```

(b) Show that $1_3' B 1_3 = \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij}$

```
> t(rep(1,3)) %*% B %*% rep(1,3)
     [,1]
[1,]    3
> sum(B)
[1] 3
> all.equal(as.numeric(t(rep(1,3)) %*% B %*% rep(1,3)), sum(B))
[1] TRUE
```

(c) Find $B + C$

```
> B + C
     [,1] [,2] [,3]
[1,]    1   -1   -2
[2,]    0   -1   -1
[3,]    0    0    1
```

(d) Show that $(B + C)\underline{y} = B\underline{y} + C\underline{y}$

```
> (B + C) %*% underlineY
     [,1]
[1,]   -7
[2,]   -5
[3,]    3
> B %*% underlineY + C %*% underlineY
```

```
     [,1]
[1,]   -7
[2,]   -5
[3,]    3
```

```
> all.equal((B + C) %*% underlineY, B %*% underlineY + C %*% underlineY)
```

```
[1] TRUE
```

(e) Find $A'(B+C)\underline{y}$

```
> t(A) %*% (B + C) %*% underlineY
```

```
     [,1]
[1,]   25
[2,] -211
[3,] -113
[4,]   -5
[5,]  -45
```

2. In this comunication network, messages can be sent only in the direction of the arrows:

$$
\begin{tikzcd}
K_1 \arrow[r] \arrow[dr] & K_2 \arrow[r] \arrow[d] & K_4 \arrow[d] \\
 & K_3 \arrow[r, shift left] & K_5 \arrow[l, shift left]
\end{tikzcd}
$$

Message routes can be represented by a matrix $W = w_{ij}$, where $w_{ij} = 0$ except $w_{ij} = 1$ if a message can be sent from K_i to K_j.

$$
W = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}
$$

In the r^{th} power of this matrix $W^r = w_{ij}(r), w_{ij}(r)$ is the number of ways of getting a message from K_i to K_j in r steps.

(a) Find W^2. Identify the paths that a message can go from K_2 to K_5 in 2 steps

```
> W %*% W
```

```
     [,1] [,2] [,3] [,4] [,5]
[1,]    0    0    1    1    1
[2,]    0    0    0    0    2
[3,]    0    0    1    0    0
[4,]    0    0    1    0    0
[5,]    0    0    0    0    1
```

The paths are: $K_2 \to K_4 \to K_5$, and $K_2 \to K_3 \to K_5$

(b) Find W^3. Identify the paths that a message can go from K_2 to K_3 in 3 steps

```
> W %*% W %*% W
```

```
     [,1] [,2] [,3] [,4] [,5]
[1,]    0    0    1    0    2
[2,]    0    0    2    0    0
[3,]    0    0    0    0    1
[4,]    0    0    0    0    1
[5,]    0    0    1    0    0
```

The paths are $K_2 \to K_4 \to K_5 \to K_3$ and $K_2 \to K_3 \to K_5 \to K_3$

3. For $A\underline{x} = \underline{b}$ where $A = \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$

   (a) Find the rank of A

```
> as.numeric(Matrix::rankMatrix(A))
[1] 3
```

   (b) Show that $B = \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} = A^{-1}$

```
> solve(A)
     [,1] [,2] [,3]
[1,]   -1   -2    5
[2,]   -1   -1    3
[3,]   -1   -2    4
> all.equal(B, solve(A))
[1] TRUE
```

   (c) Solve for $\underline{x}$

```
> solve(A) %*% b
     [,1]
[1,]   13
[2,]    6
[3,]    9
```