## GENE638 - Homework 1

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## 1. Given

$$A = \begin{bmatrix} -1 & 7 & 9 & -2 & 3 \\ 3 & 13 & 10 & 2 & 6 \\ 11 & -9 & 0 & -3 & 2 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} C = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \underline{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

## (a) Calculate:

i. 
$$\sum_{i=1}^{3} b_{i1}$$
 iii.  $\sum_{i=1}^{3} b_{i3}$  >  $sum(B[,3])$  [1] 3 [1] 0 iv.  $\sum_{i=1}^{j} b_{ij}$  >  $sum(B[,2])$  [1] 0 [1] 3

(b) Show that 
$$1_3'B1_3 = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij}$$

[1] 3

> all.equal(as.numeric(t(rep(1,3)) %\*% B %\*% rep(1,3)), sum(B))

[1] TRUE

(c) Find 
$$B + C$$

(d) Show that  $(B+C)\underline{y} = B\underline{y} + C\underline{y}$ 

> B %\*% underlineY + C %\*% underlineY

[2,] -5

[3,] 3

> all.equal((B + C) %\*% underlineY, B %\*% underlineY + C %\*% underlineY)

[1] TRUE

(e) Find  $A'(B+C)\underline{y}$ 

[1,] 25

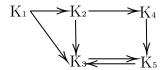
[2,] -211

[3,] -113

[4,] -5

[5,] -45

2. In this comunication network, messages can be sent only in the direction of the arrows:



Message routes can be represented by a matrix  $W = w_{ij}$ , where  $w_{ij} = 0$  except  $w_{ij} = 1$  if a message can be sent from  $K_i$  to  $K_j$ .

$$W = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

In the  $r^{th}$  power of this matrix  $W^r = w_{ij}(r), w_{ij}(r)$  is the number of ways of getting a message from  $K_i$  to  $K_j$  in r steps.

(a) Find  $W^2$ . Identify the paths that a message can go from  $K_2$  to  $K_5$  in 2 steps

The paths are:  $K_2 \to K_4 \to K_5$ , and  $K_2 \to K_3 \to K_5$ 

(b) Find  $W^3$ . Identify the paths that a message can go from  $K_2$  to  $K_3$  in 3 steps

The paths are  $K_2 \to K_4 \to K_5 \to K_3$  and  $K_2 \to K_3 \to K_5 \to K_3$ 

3. For 
$$A\underline{x} = \underline{b}$$
 where  $A = \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$  and  $\underline{b} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$ 

- (a) Find the rank of A
  - > as.numeric(Matrix::rankMatrix(A))

[1] 3

(b) Show that 
$$B = \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} = A^{-1}$$

> solve(A)

$$\begin{bmatrix} 1, \\ -1 \\ -2 \\ \end{bmatrix}$$
  $\begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$   $\begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$ 

$$[3,]$$
 -1 -2 4

- > all.equal(B, solve(A))
- [1] TRUE
- (c) Solve for  $\underline{x}$ 
  - > solve(A) %\*% b