

# STAT636 - Homework 1

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1. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$$

Without using a computer:

- a) Find the eigenvalues and normalized eigenvectors of  $\mathbf{A}$ .

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = 0$$

- b) Write the spectral decomposition of  $\mathbf{A}$ .

- c) Verify that the determinant of  $\mathbf{A}$  equals the product of its eigenvalues.

- d) The trace of a square matrix equals the sum of its diagonal elements. Verify that the trace of  $\mathbf{A}$  equals the sum of its eigenvalues.

- e) Is  $\mathbf{A}$  orthogonal? Why or why not?

- f) Is  $\mathbf{A}$  positive definite? Why or why not?

- g) Find  $\mathbf{A}^{-1}$  and determine its eigenvalues and normalized eigenvectors.

2. Consider the matrices These matrices are identical except for a small difference in the (2, 2) position. Also, the columns of A and B are nearly linearly dependent. Show that  $\mathbf{A}^{-1} \approx (-3)\mathbf{B}^{-1}$ . So, small changes - perhaps due to rounding - can result in substantially different inverses.