

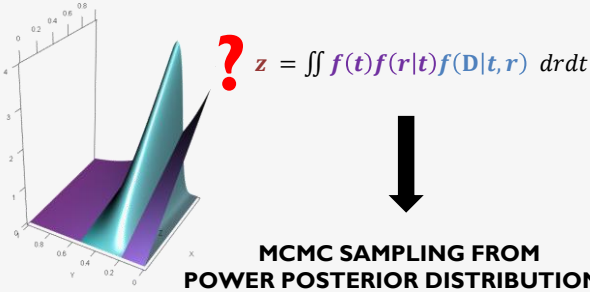


BACKGROUND

MARGINAL LIKELIHOOD ESTIMATION

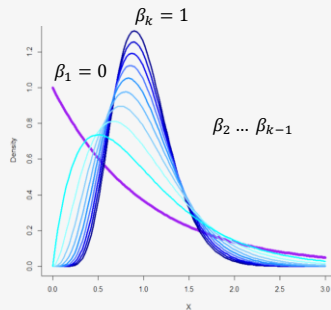
$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{(\text{marginal likelihood})}$$

$$f(t, r | D) = \frac{1}{Z} f(r | t) f(t) f(D | t, r)$$



$$f_{\beta}(t, r | D) \propto f(r | t) f(t) f(D | t, r)^{\beta}$$

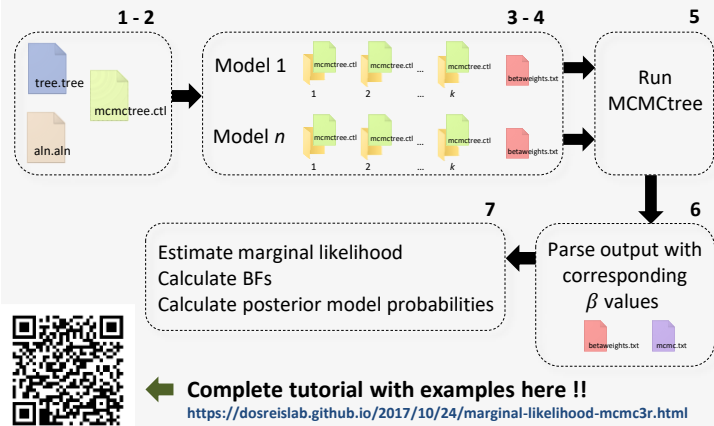
- Select k β values
- $0 \leq \beta \leq 1$
- Sample $f(D | t, r)^{\beta}$
- Estimate Z
 - TI^{1,2,3} (Thermodynamic integration)
 - SS⁴ (Steppingstone)
- Calculate Bayes factors (BFs)
- Select model



IMPLEMENTATION

mcmc3r⁵ (R package) & MCMCtree (PAML^{6,7}, current v. 4.9h)

- Prepare **sequence alignment** (aln.aln) and **tree** (tree.tree) files in MCMCtree format for each model to evaluate.
- Prepare MCMCtree **control files** (mcmctree.ctl).
- R** Find appropriate k β values (betaweights.txt) to calculate the marginal likelihood using TI or SS approach.
- R** Create k **directories** with the corresponding mcmctree.ctl file with the appropriate β value appended.
- Run MCMCtree within **each directory created by mcmc3r** having the mcmctree.ctl with the appropriate β value.
- R** Parse MCMCtree's output to calculate the **marginal likelihood estimates and standard errors**.
- R** Compute BFs and posterior model probabilities.



RESULTS OF A BAYESIAN MODEL SELECTION ANALYSIS

Table 1. Bayesian selection of relaxed-clock model for 10 randomly sampled mammal genes [8]. Three different methods to calculate the Bayes factors and posterior probabilities are used.

Gene ⁽¹⁾	Thermodynamic integration			Steppingstone			Harmonic mean (bad method)		
	P(M _{ILN} D)	P(M _{GBM} D)	BF _{ILN,GBM}	P(M _{ILN} D)	P(M _{GBM} D)	BF _{ILN,GBM}	P(M _{ILN} D)	P(M _{GBM} D)	BF _{ILN,GBM}
cds_ENSG00000178691 (SUZ12, q1q1)	1.000	0.000	25.034	1.000	0.000	25.261	1.000	0.000	11.678
cds_ENSG00000164066 (INTU, q1q2)	0.000	1.000	-20.389	0.000	1.000	-20.393	0.474	0.526	-0.104
cds_ENSG00000108296 (CWC25, q2g1)	0.000	1.000	-10.999	0.000	1.000	10.985	1.000	0.000	8.567
cds_ENSG00000164169 (PRMT9, q2g2)	0.883	0.117	2.022	0.884	0.116	2.031	1.000	0.000	7.680
cds_ENSG00000182504 (CEP97, q3g1)	1.000	0.000	11.223	1.000	0.000	10.750	0.998	0.002	6.392
cds_ENSG00000188266 (HYKK, q3g2)	0.002	0.998	-6.327	0.002	0.998	-6.231	0.999	0.001	6.762
cds_ENSG00000164099 (PRSS12, q4g1)	0.000	1.000	-13.644	0.000	1.000	-13.924	0.975	0.025	3.672
cds_ENSG00000094963 (FMO2, q4g2)	0.864	0.136	1.850	0.887	0.112	2.067	0.933	0.067	2.630
cds_ENSG00000170456 (DENND5B, q5g1)	1.000	0.000	11.879	1.000	0.000	11.775	1.000	0.000	9.107
cds_ENSG00000196943 (NOP9, q5g2)	0.000	1.000	-8.308	0.000	1.000	-7.805	0.995	0.005	5.322

⁽¹⁾ Ensembl identifiers for the genes studied in [8]. Next to them, we have added the gene name and a tag, which indicates the order of the gene sampled and the quintile from where it was sampled. gX: gene number X; qY: quintile Y.

- Note: ILN: Independent-rates model under log-normal distribution, GBM: Autocorrelated-rates model under geometric Brownian motion. The R package psych was used to calculate the harmonic mean estimator.

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