Project REACH

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Calculus Tutor

CALCULUS
Derivatives

Let's Review!

- Constant Rule
- Constant Multiple Rule
- Power Rule
- Product Rule
- Quotient Rule
- Chain Rule
- Exponential Rule
- Logarithmic Rule
- Radical Rule
- Implicit Differentiation

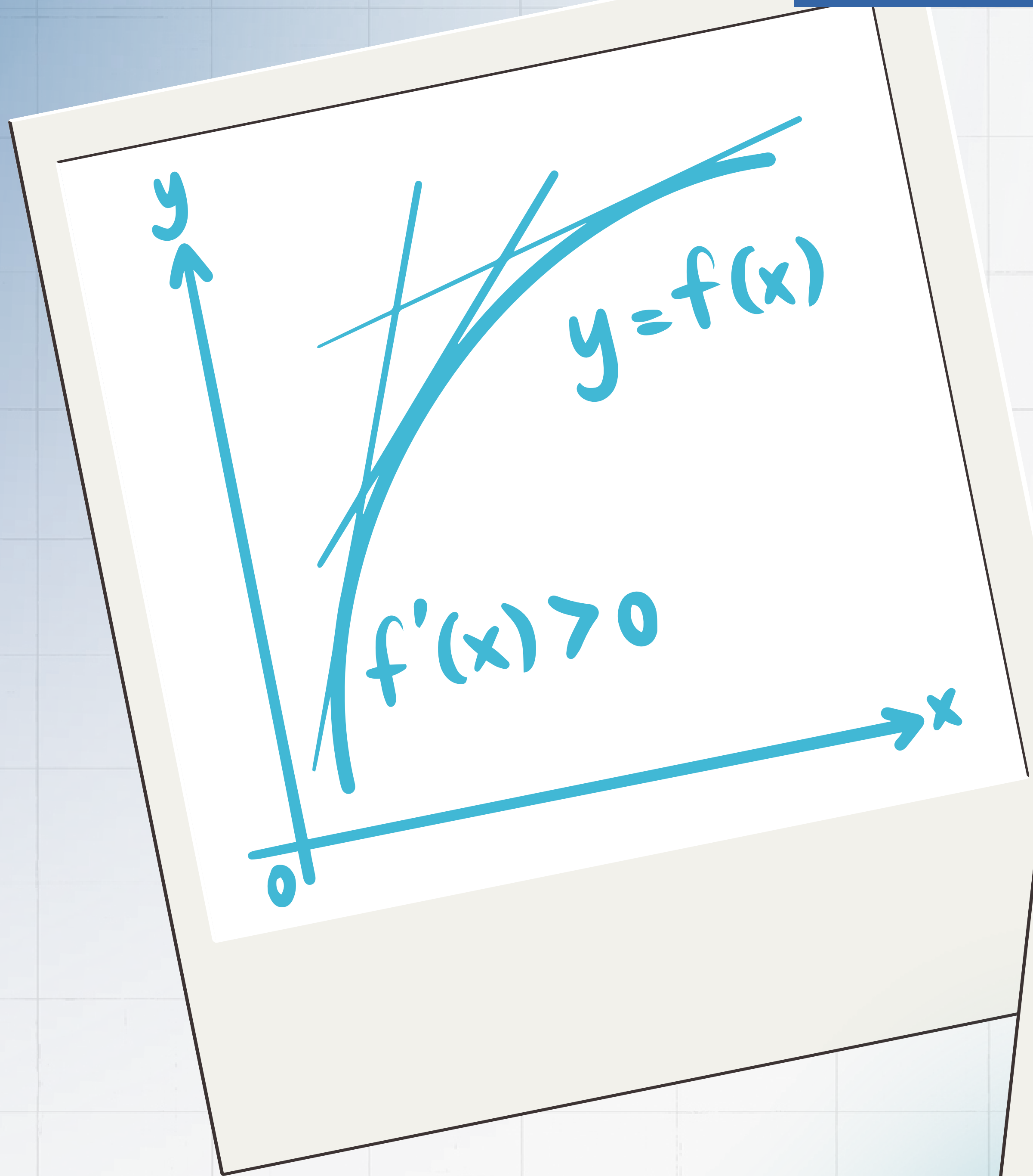
CALCULUS
Integrals

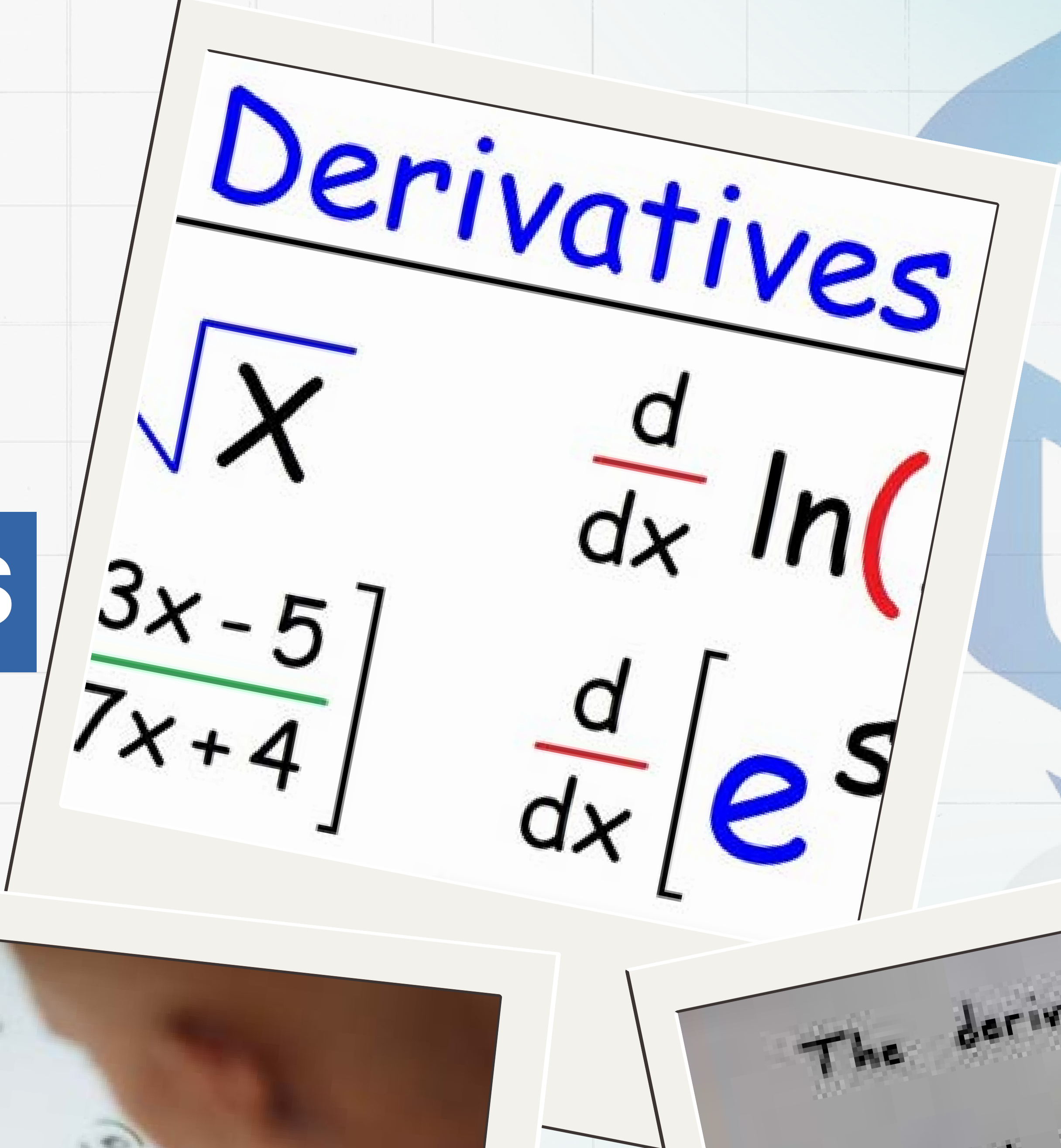
Let's Review!

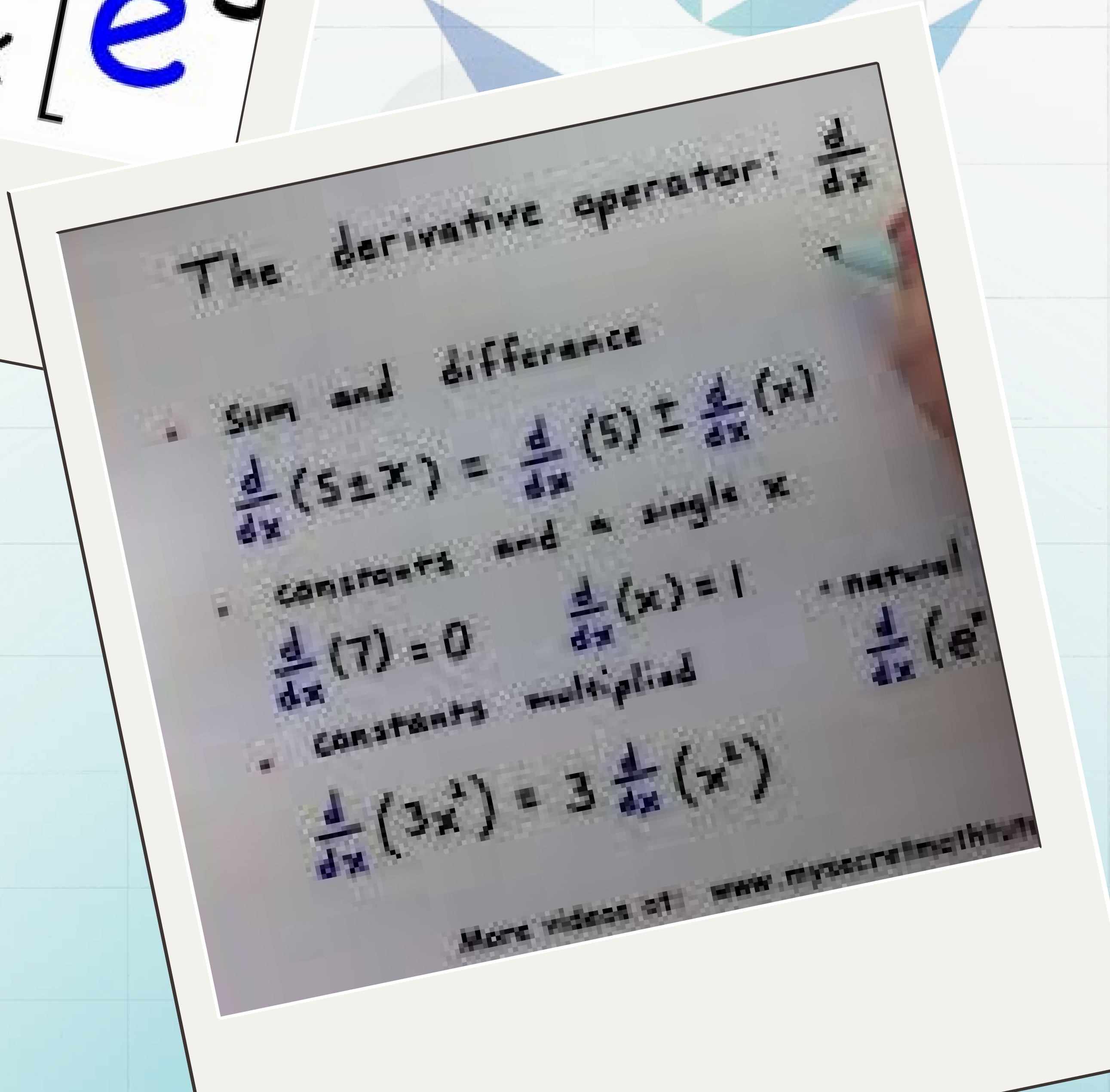
- Constant Rule
- Power Rule
- Exponential Rule
- Logarithmic Rule
- Trigonometric
- Integration by parts
- Definite Integral

CALCULUS

DERIVATIVES







CONSTANT RULE

- The derivative of a constant is always zero because constants do not change.

$$\frac{d}{dx}(C) = 0$$

NOTE:
"THE DERIVATIVE OF A CONSTANT IS ALWAYS **ZERO**!"

If a **function is just a constant (a real number)** and does not contain the variable you're differentiating with respect to, then its derivative is **zero**.

EXAMPLES

EXAMPLE #1

$$k(x) = 294,\ k'(x) = ?$$

$$k'(x) = \frac{dk}{dx} = \frac{d}{dx}(294) = 0$$

EXAMPLE #2

$$f(x) = \frac{68\pi^e(7+\ln(432+\pi)^\pi)}{4\ln(5)-\sqrt{e+79\pi-19.79}},\ f'(x) = ?$$

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}\left(\frac{68\pi^e\left(7+\ln(432+\pi)^\pi\right)}{4\ln(5)-\sqrt{e+79\pi-19.79}}\right) = 0$$

CONSTANT MULTIPLE RULE AND POWE RULE

- When a function is multiplied by a constant, the constant remains unchanged, and only the function is differentiated. **(CONSTANT MULTIPLE)**

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)] \quad \text{or} \quad \frac{d}{dx}(cu) = cu'$$

NOTE:
"**KEEP** THE <u>CONSTANT</u>, **DIFFERENTIATE** THE <u>FUNCTION</u>!"

- When differentiating a power of n, **bring down the exponent** and **subtract 1** from it. **(POWER RULE)**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

NOTE:
"BRING THE **EXPONENT DOWN**, **SUBTRACT ONE** FROM IT!"

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}(6x^4)$$

$$= 6 \cdot \frac{d}{dx}(x^4)$$

$$= 6 \cdot (4x^{4-1})$$

$$= 6 \cdot 4x^3$$

$$= \boxed{\mathbf{24x^3}}$$

EXAMPLE # 2

$$\frac{d}{dh}\left(\frac{9}{7\sqrt[5]{h^4}}\right)$$

$$= \frac{d}{dh}\left(\frac{9}{7}h^{-\frac{4}{5}}\right)$$

$$= \frac{9}{7} \cdot \frac{d}{dh}(h^{-\frac{4}{5}})$$

$$= \frac{9}{7} \cdot \left(-\frac{4}{5}h^{-\frac{4}{5}-1}\right)$$

$$= \frac{9}{7} \cdot \left(-\frac{4}{5}h^{-\frac{9}{5}}\right)$$

$$= -\frac{36}{35}h^{-\frac{9}{5}}$$

$$= \boxed{-\frac{\mathbf{36}}{\mathbf{35\sqrt[5]{h^9}}}}$$

PRODUCT RULE

- When differentiating the product of two or more functions, the **derivative is not simply the product** of their derivatives. It follows the:

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)] \quad \text{or} \quad \frac{d}{dx}(uv) = uv' + vu'$$

NOTE:
"REMEMBER (UV' + VU')."

Identify the **"f(x)"** and **"g(x)"** first or the **"u"** and **"v"**.
Find their **derivatives separately**, then **add** the results.

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}\left(2x^3\cos(3x)\right)$$

$$\text{Let } u = 2x^3, \quad v = \cos(3x)$$

$$\text{Then, } u' = \frac{d}{dx}(2x^3) = 2 \cdot 3x^{3-1} = 6x^2$$

$$v' = \frac{d}{dx}(\cos(3x)) = -\sin(3x) \cdot 3 = -3\sin(3x)$$

$$\text{Using the product rule:} \quad \frac{d}{dx}(uv) = uv' + vu'$$

$$= (2x^3)(-3\sin(3x)) + (\cos(3x))(6x^2)$$

$$= -6x^3\sin(3x) + 6x^2\cos(3x)$$

$$= \boxed{\mathbf{6x^2\cos(3x) - 6x^3\sin(3x)}}$$

EXAMPLES

EXAMPLE # 2

$$\frac{d}{dk}\left(e^k \ln(k)\cos(k)\right)$$

Let $u = e^k \ln(k), \quad v = \cos(k)$

Then, $u' = \dfrac{d}{dk}(e^k \ln(k))$

$$u_1 = e^k, \quad v_1 = \ln(k)$$

$$u_1' = e^k, \quad v_1' = \frac{1}{k}$$

$$u' = e^k \cdot \frac{1}{k} + \ln(k) \cdot e^k$$

$$u' = e^k \left(\frac{1}{k} + \ln(k)\right)$$

$$v' = \frac{d}{dk}(\cos(k)) = -\sin(k)$$

Now applying the product rule: $\dfrac{d}{dk}(uv) = uv' + vu$

$$= (e^k \ln(k))(-\sin(k)) + (\cos(k))(e^k(\frac{1}{k} + \ln(k)))$$

$$= -e^k \ln(k)\sin(k) + e^k \cos(k)\left(\frac{1}{k} + \ln(k)\right)$$

$$= \boxed{\mathbf{e^k \cos(k)\left(\frac{1}{k} + \ln(k)\right) - e^k \ln(k)\sin(k)}}$$

QUOTIENT RULE

- When differentiating the quotient of two functions, the **derivative is not simply the quotient** of their derivatives. Instead, it follows the quotient rule, which states:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} \quad \text{or} \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

NOTE:
"LOW **D(HIGH)** - HIGH **D(LOW)**, OVER LOW **SQUARED!**"

Identify the **"f(x)"** and **"g(x)"** first or the **"u"** and **"v"**.
Always **square the denominator** and **be careful** with the signs.

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}\left(\frac{2x}{x^2+1}\right)$$

Let $u = 2x, \quad v = x^2 + 1$

$$u' = \frac{d}{dx}(2x) = 2$$

$$v' = \frac{d}{dx}(x^2 + 1) = 2x$$

Applying the quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2 + 1)(2) - (2x)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2}$$

$$= \frac{-2x^2 + 2}{(x^2 + 1)^2}$$

$$= \boxed{\frac{\mathbf{2(1 - x^2)}}{\mathbf{(x^2 + 1)^2}}}$$

CHAIN RULE

- The derivative of a composite function is the **derivative of the outside function** times the **derivative of the inside function.**

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x) \quad \text{or} \quad \frac{d}{dx}[u^n] = nu^{n-1} \cdot u$$

NOTE:
"OUTSIDE **D(INSIDE)** TIMES **D(INSIDE)**!" (D = DERIVATIVE)

Find the "**inner**" and "**outer**" functions first.
Differentiate the **outer function** while keeping the **inner function unchanged**, then **multiply** by the **derivative** of the **inner function**.

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}\left((2x^3 - 5x)^5\right)$$

Let $u = 2x^3 - 5x$, then our function is u^5

Differentiate using the chain rule:

$$\frac{d}{dx}(u^5) = 5u^4 \cdot \frac{du}{dx}$$

Compute $\frac{du}{dx}$: $\quad \frac{d}{dx}(2x^3 - 5x) = 6x^2 - 5$

Substituting: $\quad 5(2x^3 - 5x)^4(6x^2 - 5)$

$$\boxed{\mathbf{(2x^3 - 5x)^4(30x^2 - 25)}}$$

TRIGONOMETRIC RULE

- The derivatives of trigonometric functions follow specific formulas:

$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$
$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

NOTE:

If a trigonometric function starts with the letter "**C**", its derivative is **negative**!

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}(\sin(x) + x^2)$$

Differentiate each term separately:

$$\frac{d}{dx}\sin(x) + \frac{d}{dx}x^2$$

$$\cos(x) + 2x$$

$$\boxed{\cos(\mathbf{x}) + \mathbf{2x}}$$

EXAMPLES

EXAMPLE # 2

$$\frac{d}{dx}(3\csc(x) + 9\tan(x) - 4\sec(x))$$

Differentiate each term separately:

$$3\frac{d}{dx}\csc(x) + 9\frac{d}{dx}\tan(x) - 4\frac{d}{dx}\sec(x)$$

$$3(-\csc(x)\cot(x)) + 9\sec^2(x) - 4(\sec(x)\tan(x))$$

$$-3\csc(x)\cot(x) + 9\sec^2(x) - 4\sec(x)\tan(x)$$

$$\boxed{-\mathbf{3}\csc(\mathbf{x})\cot(\mathbf{x}) + \mathbf{9}\sec^{\mathbf{2}}(\mathbf{x}) - \mathbf{4}\sec(\mathbf{x})\tan(\mathbf{x})}$$

EXPONENTIAL RULE

- When differentiating, the base remains the same, and you will only multiply by the **natural logarithm** if the base is not **e**.

Natural Exponential Function:

$$\frac{d}{dx}e^x = e^x$$

General Exponential Function:

$$\frac{d}{dx}a^x = a^x \ln(a)$$

NOTE:
"**E^X** *STAYS THE* **SAME**, BUT **A** *NEEDS* **LN(A)**!"

If there's a function inside, **multiply by the derivative** of that function.

EXPONENTIAL RULE

FORMULAS

Exponential Function with a Variable Exponent:

$$\frac{d}{dx}e^{f(x)} = e^{f(x)} \quad f'(x) \quad \text{or} \quad \frac{d}{dx}e^{u} = e^{u} \cdot u'$$

$$\frac{d}{dx}a^{f(x)} = a^{f(x)}\ln(a) \quad f'(x) \quad \text{or} \quad \frac{d}{dx}a^{u} = a^{u}\ln(a) \cdot u'$$

Exponential Function with a Negative Exponent:

$$\frac{d}{dx}\left(e^{-f(x)}\right) = -e^{-f(x)} \cdot f'(x)$$

EXPONENTIAL RULE

FORMULAS

Exponential Function with a Variable Exponent:

$$\frac{d}{dx}\left(g(x)\ \ e^{f(x)}\right) = g'(x)\ \ e^{f(x)} + g(x)\ \ e^{f(x)} \cdot f'(x)$$

Exponential Function with Fractional Exponent:

$$\frac{d}{dx}\left(e^{\frac{f(x)}{g(x)}}\right) = e^{\frac{f(x)}{g(x)}} \cdot \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Exponential Function with Function Power:

$$\frac{d}{dx}\left(\left(e^{f(x)}\right)^{g(x)}\right) = \left(e^{f(x)}\right)^{g(x)} \cdot \left(g'(x)\ln(e) \cdot e^{f(x)} + g(x)f'(x)\right)$$

EXAMPLES

EXAMPLE # 1.

$$\frac{d}{dx}e^{(6x+9)}$$

Using the rule: $\dfrac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$

$$e^{(6x+9)} \cdot \frac{d}{dx}(6x+9)$$

$$e^{(6x+9)} \cdot 6$$

$$\boxed{\mathbf{6e^{(6x+9)}}}$$

EXAMPLE # 2.

$$\frac{d}{dx}4^{\tan(x)}$$

Using the rule: $\dfrac{d}{dx}a^{f(x)} = a^{f(x)}\ln(a) \quad f'(x)$

$$4^{\tan(x)}\ln(4) \cdot \frac{d}{dx}\tan(x)$$

$$4^{\tan(x)}\ln(4) \cdot \sec^2(x)$$

$$\boxed{\mathbf{4^{\tan(x)}\ln(4)\sec^2(x)}}$$

LOGARITHMIC RULE

- Used when differentiating functions involving logarithmic, especially when the function inside the logarithm is complex.

Natural Logarithm:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

General Logarithm:

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

NOTE:
"**LN** IS **1 OVER X**, **LOG** JUST **ADDS LN(A)** UNDER!"

If there's a **function inside**, **multiply** by the **derivative** of that function.

LOGARITHMIC RULE

FORMULAS

Logarithm of a Function:

$$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)} \quad \text{or} \quad \frac{d}{dx}\ln u = \frac{u'}{u}$$

$$\frac{d}{dx}\log_a f(x) = \frac{f'(x)}{f(x)\ln(a)} \quad \text{or} \quad \frac{d}{dx}\log_a u = \frac{u'}{u\ln(a)}$$

Logarithmic Function Raised to a Power:

$$\frac{d}{dx}\left(\ln(f(x)^n)\right) = \frac{n \cdot f'(x)}{f(x)}$$

LOGARITHMIC RULE

FORMULAS

Logarithmic Differentiation (Product/Quotient Rule):

$$\frac{d}{dx}\left(\ln(u(x)\cdot v(x))\right) = \frac{d}{dx}\left(\ln(u(x)) + \ln(v(x))\right) = \frac{u'(x)}{u(x)} + \frac{v'(x)}{v(x)}$$

$$\frac{d}{dx}\left(\ln\left(\frac{u(x)}{v(x)}\right)\right) = \frac{u'(x)}{u(x)} - \frac{v'(x)}{v(x)}$$

Logarithmic of Exponential Function:

$$\frac{d}{dx}\left(\ln(e^{f(x)})\right) = \frac{f'(x)}{1} = f'(x)$$

LOGARITHMIC RULE

FORMULAS

Natural Logarithm with a Constant:

$$\frac{d}{dx}(\ln(c \cdot f(x))) = \frac{f'(x)}{c \cdot f(x)}$$

Logarithmic Differentiation with Composite Functions:

$$\frac{d}{dx}(\ln(f(g(x)))) = \frac{f'(g(x)) \cdot g'(x)}{f(g(x))}$$

Logarithmic Function with a Power Inside:

$$\frac{d}{dx}(\ln(x^n)) = \frac{n}{x}$$

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}\ln(7x-5+x^3)$$

Using $\dfrac{d}{dx}\ln f(x)=\dfrac{f'(x)}{f(x)}$

Let $u=7x-5+x^3$, so $f'(x)=\dfrac{d}{dx}(7x-5+x^3)$

$$\frac{d}{dx}(7x-5+x^3)=7+3x^2$$

Applying the formula:

$$\frac{1}{7x-5+x^3}\cdot(7+3x^2)$$

$$\boxed{\mathbf{\frac{7+3x^2}{7x-5+x^3}}}$$

EXAMPLES

EXAMPLE # 2

$$\frac{d}{dx}\ln(\sin x)$$

Using $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

Let $u = \sin x$, so $f'(x) = \cos x$

$$\frac{1}{\sin x}\cdot\cos x$$

$$\boxed{\frac{\cos \mathbf{x}}{\sin \mathbf{x}} = \cot \mathbf{x}}$$

EXAMPLE # 3

$$\frac{d}{dx}\log_7(5-2x)$$

Using $\frac{d}{dx}\log_a f(x) = \frac{f'(x)}{f(x)\ln a}$

Let $u = 5-2x$, so $f'(x) = -2$

$$\frac{-2}{(5-2x)\ln 7}$$

$$\boxed{\frac{-\mathbf{2}}{(\mathbf{5}-\mathbf{2x})\ln \mathbf{7}}}$$

RADICAL RULE

- It involve roots, which can be rewritten in exponent form to apply differentiation rules easily.

Basic Rule:

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

General Logarithm:

$$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

NOTE:
"**REWRITE ROOTS** AS EXPONENTS, THEN **DIFFERENTIATE**!"

If it's a function inside, apply the **chain rule**

RADICAL RULE

FORMULAS

Nth Root Differentiation Rule:

$$\frac{d}{dx}\left(\sqrt[n]{x}\right) = \frac{1}{n \cdot \sqrt[n]{x^{n-1}}} \qquad \frac{d}{dx}\left(\sqrt[n]{f(x)}\right) = \frac{f'(x)}{n \cdot \sqrt[n]{f(x)^{n-1}}}$$

Coefficient-Modified Root Rule:

$$\frac{d}{dx}\left(c\sqrt[n]{f(x)}\right) = c \cdot \frac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$$

RADICAL RULE

FORMULAS

Radical with Powers Inside:

$$\frac{d}{dx}\left(\sqrt[n]{x^m}\right) = \frac{m \cdot x^{m-1}}{n \cdot \sqrt[n]{x^{n(m-1)}}}$$

Radical Function with a Logarithmic:

$$\frac{d}{dx}\left(\sqrt[n]{\ln(f(x))}\right) = \frac{1}{n \cdot \sqrt[n]{\ln(f(x))^{n-1}}} \cdot \frac{d}{dx}[\ln(f(x))]$$

EXAMPLES

EXAMPLE # 1

$$\frac{d}{dx}\sqrt[5]{x}$$

Using the rule: $\frac{d}{dx}\left(\sqrt[n]{x}\right) = \frac{1}{n\sqrt[n]{x^{n-1}}}$

For $n = 5$, we get:

$$\frac{1}{5\sqrt[5]{x^4}}$$

$$\boxed{\mathbf{\frac{1}{5\sqrt[5]{x^4}}}}$$

EXAMPLE # 2

$$\frac{d}{dx}\sqrt{x^3 - 5}$$

Using the rule: $\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{f'(x)}{2\sqrt{f(x)}}$

$f(x) = x^3 - 5, \quad f'(x) = 3x^2$

$$\frac{3x^2}{2\sqrt{x^3 - 5}}$$

$$\boxed{\mathbf{\frac{3x^2}{2\sqrt{x^3 - 5}}}}$$

EXAMPLES

EXAMPLE # 3

$$\frac{d}{dx}\left[7\sqrt[3]{5x+2}\right]$$

Using the rule: $\frac{d}{dx}\left(c \cdot \sqrt[n]{f(x)}\right) = c \cdot \frac{f'(x)}{n\sqrt[n]{f(x)^{n-1}}}$

Let $f(x) = 5x + 2, \quad f'(x) = 5, \quad c = 7, \quad n = 3$

$$7 \cdot \frac{5}{3\sqrt[3]{(5x+2)^2}}$$

$$\boxed{\frac{\mathbf{35}}{\mathbf{3\sqrt[3]{(5x+2)^2}}}}$$

IMPLICIT DIFFERENTIATION

- Implicit differentiation comes into play when you have an equation involving both x and y (where y is implicitly a function of x) and you need to find dy/dx, the derivative of y with respect to x.

Tips for Implicit Differentiation:

1. **Differentiate each term** on both sides of the equation. Apply the appropriate rules (product rule, chain rule, etc.). When differentiating terms involving y, remember to add $\frac{dy}{dx}$ because y is a function of x.
2. **Isolate terms with** $\frac{dy}{dx}$ on one side of the equation. Typically, $\frac{dy}{dx}$ is placed on the left side, but this is not a strict rule. Ensure that the terms involving x are on the other side of the equation.
3. **Factor out** $\frac{dy}{dx}$ from the isolated terms. Then, solve for $\frac{dy}{dx}$ by dividing both sides of the equation by the factor that's left behind. This gives you the value of $\frac{dy}{dx}$.

EXAMPLES

EXAMPLE # 1

$$x^5 + y^6 = 12$$

Differentiate both sides with respect to x :

$$\frac{d}{dx}(x^5) + \frac{d}{dx}(y^6) = \frac{d}{dx}(12)$$

$$5x^4 + 6y^5 \cdot \frac{dy}{dx} = 0$$

$$6y^5 \cdot \frac{dy}{dx} = -5x^4$$

$$\frac{dy}{dx} = \frac{-5x^4}{6y^5}$$

$$\boxed{\frac{-\mathbf{5x^4}}{\mathbf{6y^5}}}$$

EXAMPLES

EXAMPLE # 2.

$$x^2 \sin(y) + \cos(2x) + 5 = 9y^3$$

Differentiate both sides with respect to x :

$$\frac{d}{dx}(x^2 \sin(y)) + \frac{d}{dx}(\cos(2x)) + \frac{d}{dx}(5) = \frac{d}{dx}(9y^3)$$

Use the product rule for $x^2 \sin(y)$: $\frac{d}{dx}(x^2 \sin(y)) = x^2 \cos(y) \cdot \frac{dy}{dx} + \sin(y)2x$

$$\frac{d}{dx}(\cos(2x)) = -2\sin(2x)$$

$$\frac{d}{dx}(9y^3) = 27y^2 \cdot \frac{dy}{dx}$$

$$x^2 \cos(y) \cdot \frac{dy}{dx} + \sin(y)2x - 2\sin(2x) = 27y^2 \frac{dy}{dx}$$

$$x^2 \cos(y) \cdot \frac{dy}{dx} - 27y^2 \cdot \frac{dy}{dx} = -2x\sin(y) + 2\sin(2x)$$

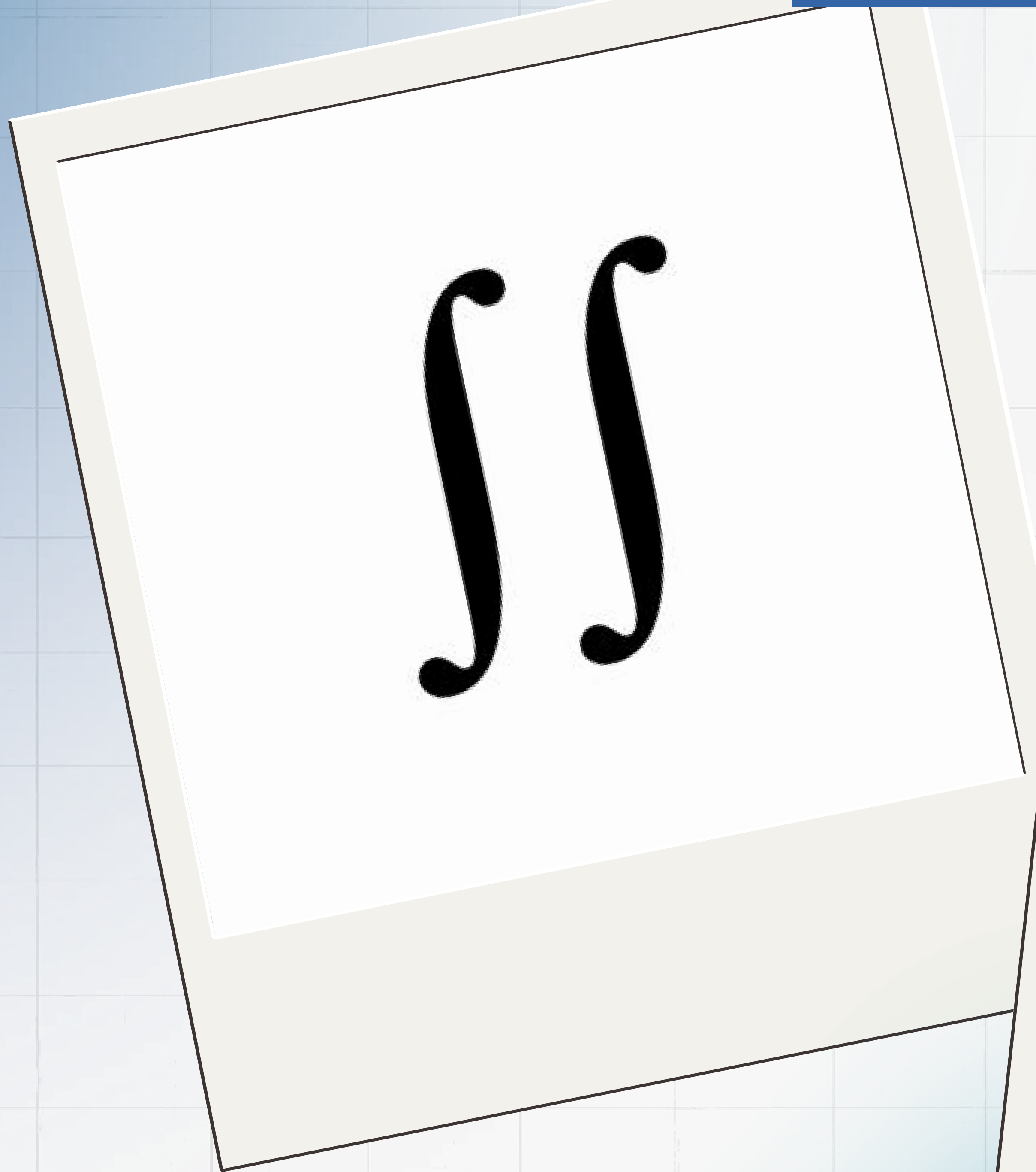
$$\frac{dy}{dx}(x^2 \cos(y) - 27y^2) = -2x\sin(y) + 2\sin(2x)$$

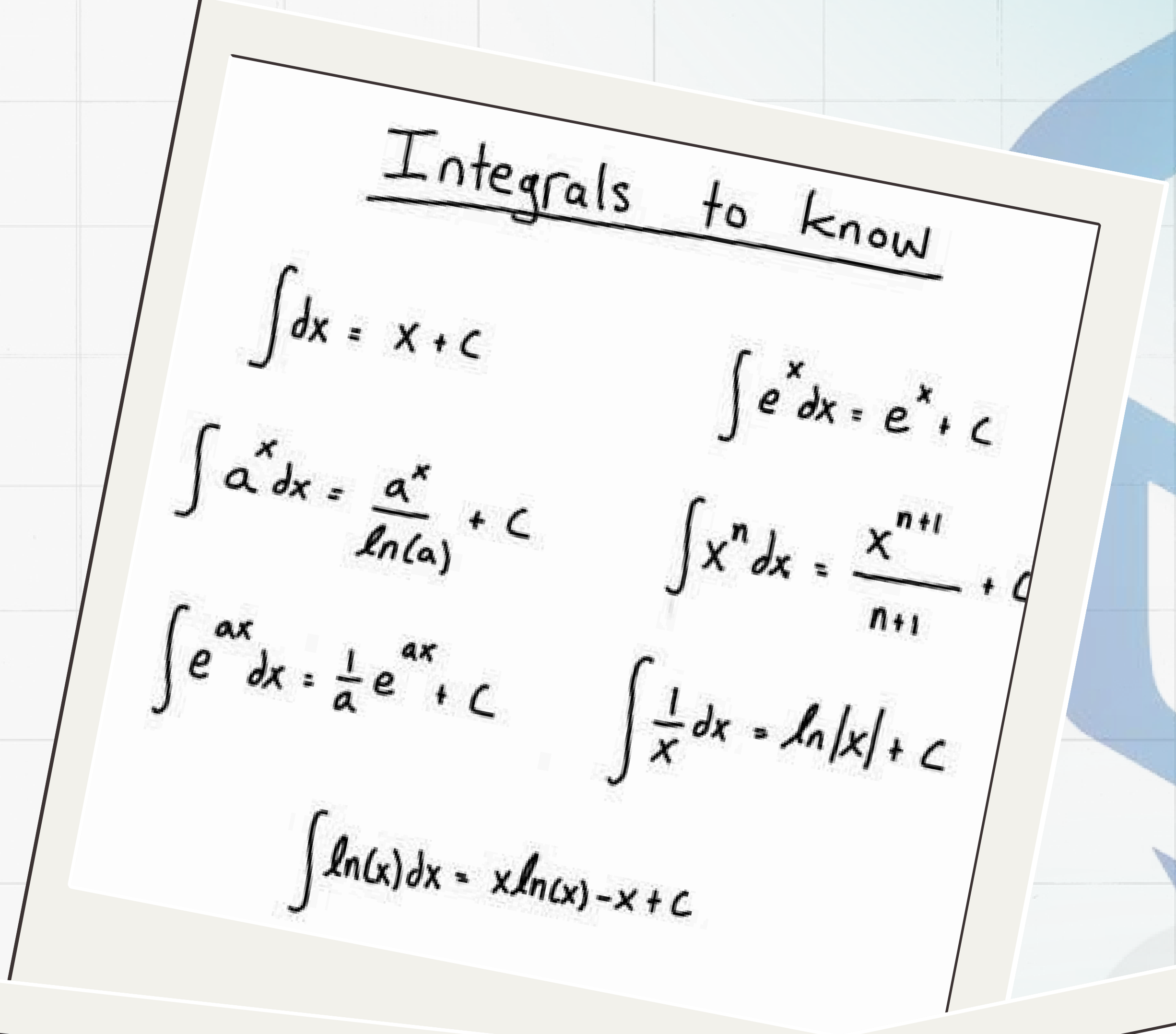
$$\frac{dy}{dx} = \frac{-2x\sin(y) + 2\sin(2x)}{x^2\cos(y) - 27y^2}$$

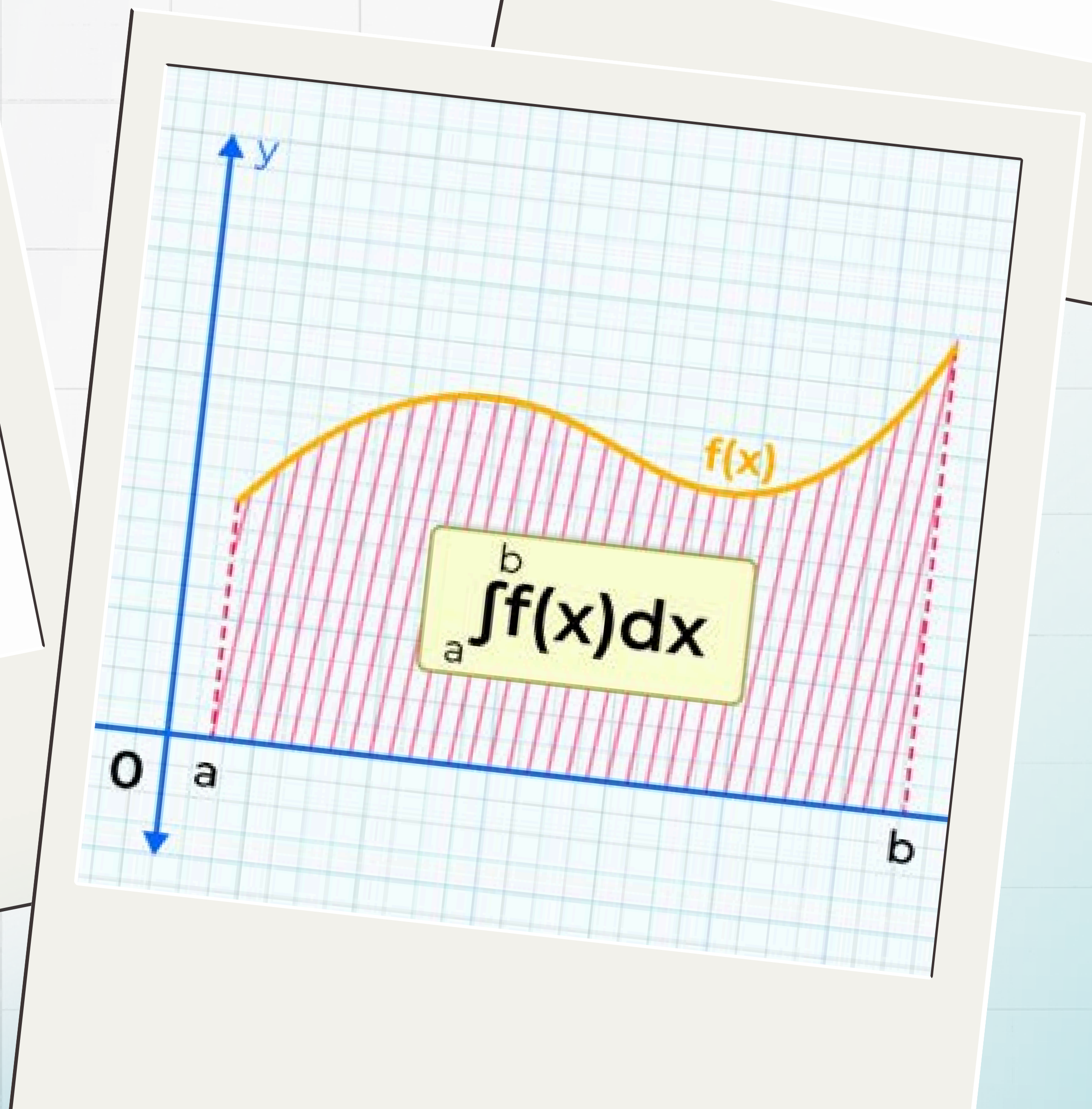
$$\boxed{\frac{\mathbf{-2x}\sin(\mathbf{y}) + \mathbf{2}\sin(\mathbf{2x})}{\mathbf{x^2}\cos(\mathbf{y}) - \mathbf{27y^2}}}$$

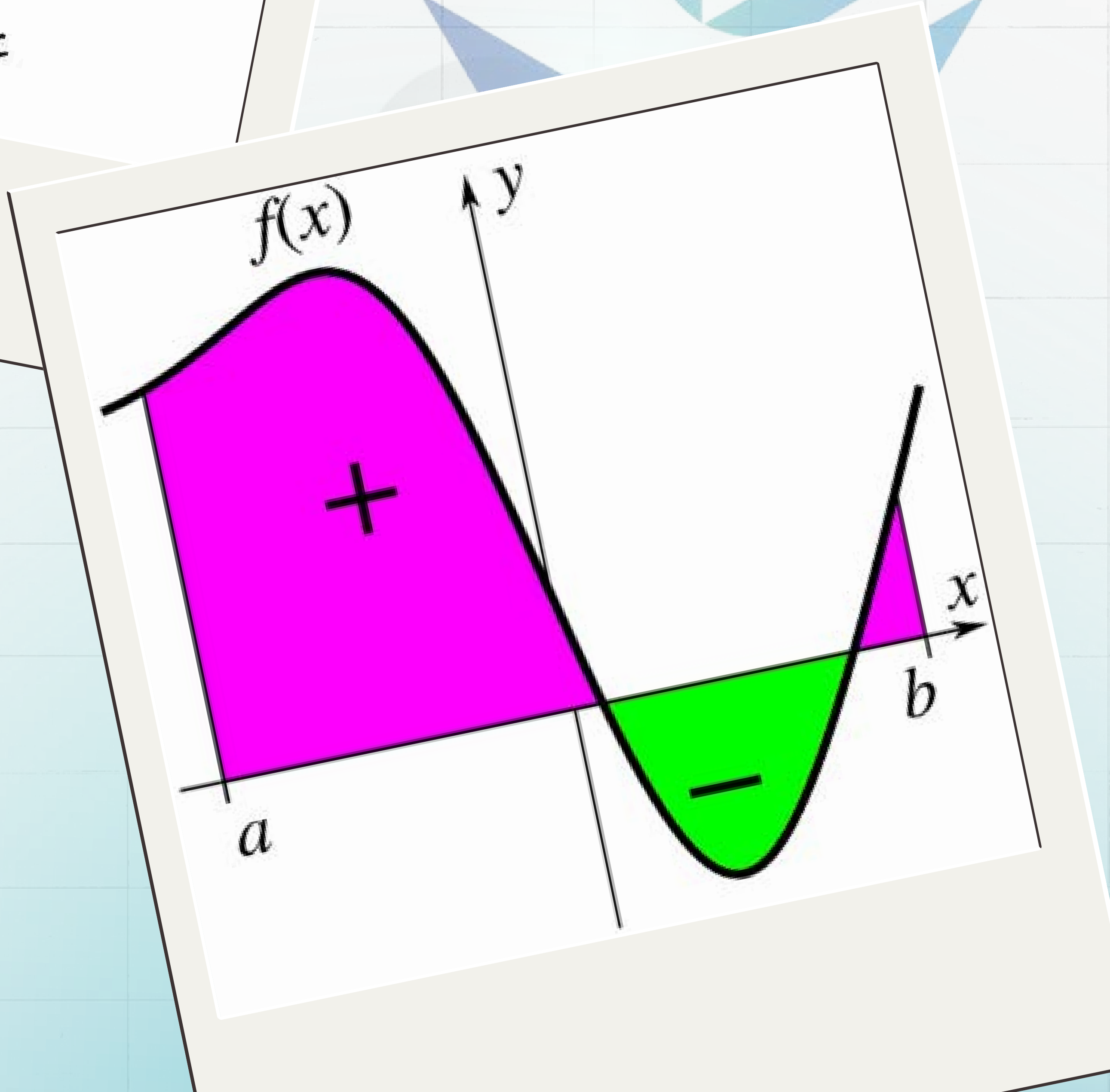
CALCULUS

INTEGRALS









INTEGRALS

Integration the process of getting the integral of a derived function, also known as **“anti-differentiation”**

$$\int f(x)\,dx = F(x) + C$$

Where:

- **f(x)** is the ***integrand*** (the function being integrated).
- **F(x)** is the ***antiderivative*** of f(x) (also called the indefinite integral).
- **C** is the ***arbitrary constant***, which accounts for all possible antiderivatives.

Why Do We Add **C**?

When we integrate a function, we are essentially **reversing differentiation**. However, differentiation removes constants because the derivative of any constant is zero. This means that when we integrate, we must include for all possible original functions by adding a constant C.

$$\frac{d}{dx}(F(x) + C) = F'(x) + 0 = f(x)$$

$$\int 3\,dx = 3x + C$$

$$\frac{d}{dx}(3x + C) = 3 + 0 = 3$$

$$\frac{d}{dx}(3x + 100) = 3$$

$$\frac{d}{dx}(3x - 5) = 3$$

CONSTANT RULE

- It states that when integrating a constant, the result is the constant multiplied by the variable of integration.

$$\int c\,dx = cx + C$$

EXAMPLES

$$\int 5\,dx = 5x + C$$

$$\int e\,dx = ex + C$$

$$\int 3\pi^2\,dy = 3\pi^2 y + C$$

POWER RULE

- Used to integrate functions of the form x^2, where n ≠ -1

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad \text{for } n \neq -1$$

EXAMPLE # 1

$$\int x\,dx$$

Here, $n = 1$

$$\int x^1\,dx = \frac{x^{1+1}}{1+1} + C$$

$$= \frac{x^2}{2} + C$$

$$\boxed{\frac{\mathbf{x^2}}{\mathbf{2}} + \mathbf{C}}$$

EXAMPLES

EXAMPLE # 2

$$\int \frac{x^3}{\sqrt[4]{x}}\,dx$$

Rewrite the denominator using exponents: $\sqrt[4]{x} = x^{\frac{1}{4}}$

$$\frac{x^3}{\sqrt[4]{x}} = x^3 \div x^{\frac{1}{4}}$$

Use exponent division rule: $x^a \div x^b = x^{a-b}$

$$x^3 \div x^{\frac{1}{4}} = x^{3-\frac{1}{4}}$$

$$= x^{\frac{12}{4}-\frac{1}{4}} = x^{\frac{11}{4}}$$

Now integrate: $\int x^{\frac{11}{4}}\,dx$

Use power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Here, $n = \frac{11}{4}$

$$\int x^{\frac{11}{4}}\,dx = \frac{x^{\frac{11}{4}+1}}{\frac{11}{4}+1} + C$$

$$= \frac{x^{\frac{15}{4}}}{\frac{15}{4}} + C$$

$$= \frac{4}{15}x^{\frac{15}{4}} + C$$

$$\boxed{\mathbf{\frac{4}{15}x^{\frac{15}{4}} + C}}$$

EXPONENTIAL RULE

FORMULAS

- Used when integrating exponential functions, particularly those with base ***e*** or any other constant base.

Integral of an Exponential Function:

$$\int e^x \, dx = e^x + C$$

Integral of an Exponential Function with a Coefficient:

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} + C, \quad (\text{for } a \neq 0)$$

EXPONENTIAL RULE
FORMULAS

Integral of a General Exponential Function:

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \quad (\text{for } a > 0, a \neq 1)$$

Integral of a General Exponential Function with a Coefficient:

$$\int a^{bx} \, dx = \frac{a^{bx}}{b \ln a} + C, \quad (\text{for } a > 0, a \neq 1, b \neq 0)$$

EXAMPLES

EXAMPLE # 1

$$\int e^{3x}dx$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$$

$$\int e^{3x}dx$$

$$= \frac{e^{3x}}{3} + C$$

$$\boxed{\frac{e^{3x}}{3} + C}$$

EXAMPLE # 2

$$\int xe^{x^2}dx$$

Let $u = x^2, \quad du = 2xdx$

$$\frac{du}{2} = xdx$$

$$\int xe^{x^2}dx = \int e^u \cdot \frac{du}{2}$$

$$= \frac{1}{2}\int e^u du$$

$$= \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{x^2} + C$$

$$\boxed{\frac{1}{2}e^{x^2} + C}$$

LOGARITHMIC RULE

FORMULAS

- Used when integrating functions of the form **1/x** and more generally functions where the denominator involves **x.**

Basic Log Integral:

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Linear Function in the Denominator:

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

LOGARITHMIC RULE

FORMULAS

More General Form:

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

Integral of a Fraction with a Root Denominator:

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + C$$

Integral of Logarithm Itself (By Parts):

$$\int \ln x\,dx = x\ln x - x + C$$

EXAMPLES

EXAMPLE # 1

$$\int \frac{1}{3x-5}\,dx$$

$$\text{Let } u = 3x-5, \quad du = 3dx \Rightarrow dx = \frac{du}{3}$$

$$\int \frac{1}{u}\cdot\frac{du}{3}$$

$$= \frac{1}{3}\int \frac{du}{u}$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln|3x-5| + C$$

$$\boxed{\mathbf{\frac{1}{3}\ln|3x-5| + C}}$$

EXAMPLE # 2

$$\int \frac{x^2}{x^3+5}\,dx$$

$$\text{Let } u = x^3+5, \quad du = 3x^2dx$$

$$\frac{du}{3} = x^2dx$$

$$\int \frac{x^2}{x^3+5}\,dx = \int \frac{du}{3u}$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln|x^3+5| + C$$

$$\boxed{\mathbf{\frac{1}{3}\ln|x^3+5| + C}}$$

TRIGONOMETIC

FORMULAS

- Trigonometric integrals involve integrating the six basic trigonometric functions. Here are the standard integral formulas for them:

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \cot(x)\,dx = \ln|\sin(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$$

EXAMPLES

EXAMPLE # 1

$\int \sin^5(x)\cos^2(x)\,dx$

Using $\sin^5(x) = \sin^4(x)\sin(x)$,
and $\sin^4(x) = (1 - \cos^2(x))^2$,

$\sin^2(x) = 1 - \cos^2(x)$

$$\int (1-\cos^2(x))^2 \sin(x)\cos^2(x)\,dx$$

Let $u = \cos(x), \quad du = -\sin(x)dx$

$$= \int (1-u^2)^2 u^2(-du)$$

$$= -\int (1-2u^2+u^4)u^2\,du$$

$$= -\int (u^2-2u^4+u^6)\,du$$

$$= -\left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + C$$

$$= -\left(\frac{\cos^3(x)}{3} - \frac{2\cos^5(x)}{5} + \frac{\cos^7(x)}{7}\right) + C$$

$$\boxed{-\left(\frac{\cos^3(\mathbf{x})}{\mathbf{3}} - \frac{\mathbf{2}\cos^5(\mathbf{x})}{\mathbf{5}} + \frac{\cos^7(\mathbf{x})}{\mathbf{7}}\right) + \mathbf{C}}$$

INTEGRATION BY PARTS

- A technique used to integrate the product of two functions. It is derived from the product rule for differentiation:

$$\frac{d}{dx}[uv] = u'v + uv'$$

Rearranging and **integrating both sides**:

$$\int u\,dv = uv - \int v\,du$$

INTEGRATION BY PARTS

Choosing u and dv (LIATE Rule)

A helpful rule for choosing u is **LIATE**, prioritizing functions in this order:

1. Logarithmic ($\ln x, \log_a x$)
2. Inverse Trigonometric ($\tan^{-1} x, \sin^{-1} x$)
3. Algebraic ($x, x^2, x^3, \ldots$)
4. Trigonometric ($\sin x, \cos x, \tan x$)
5. Exponential (e^x, a^x)

Always pick u from **higher priority** functions in LIATE.

EXAMPLES

EXAMPLE # 1

$$\int xe^x \, dx$$

Let $u = x, \quad dv = e^x \, dx$

$du = 1 \, dx, \quad v = e^x$

$$\int u \, dv = uv - \int v \, du$$

$$= xe^x - \int e^x \, dx$$

$$= xe^x - e^x + C$$

$$= e^x(x - 1) + C$$

$$\boxed{\mathbf{e^x(x - 1) + C}}$$

EXAMPLES

EXAMPLE # 1

$$\int x^2 \sin(x)\, dx$$

Using **Integration by Parts** twice:

First,

$$\text{Let } u = x^2, \quad dv = \sin(x)\, dx$$
$$du = 2x\, dx, \quad v = -\cos(x)$$
$$\int u\, dv = uv - \int v\, du$$
$$= -x^2 \cos(x) + \int 2x \cos(x)\, dx$$

Apply **Integration by Parts** again on $\int 2x \cos(x)\, dx$:

$$u = 2x, \quad dv = \cos(x)\, dx$$
$$du = 2\, dx, \quad v = \sin(x)$$
$$= -x^2 \cos(x) + \left(2x \sin(x) - \int 2 \sin(x)\, dx\right)$$
$$= -x^2 \cos(x) + 2x \sin(x) + 2\cos(x) + C$$
$$\boxed{\mathbf{-x^2 \cos(x) + 2x \sin(x) + 2\cos(x) + C}}$$

DEFINITE INTEGRALS

- A definite integral calculates the net area under a curve between two given limits **a** and **b**. Unlike an indefinite integral, which includes a constant of integration C, a definite integral results in a specific numerical value.

  It is represented as:

$$\int_a^b f(x)\,dx$$

Using the **Fundamental Theorem of Calculus**, the definite integral is evaluated as:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

EXAMPLES

EXAMPLE # 1

$\int_{-1}^{2}(x^3+1)^2\,dx$

$$\text{Expand:}\quad (x^3+1)^2 = x^6+2x^3+1$$

$$\int (x^6+2x^3+1)\,dx = \frac{x^7}{7}+\frac{2x^4}{4}+x$$

$$= \frac{x^7}{7}+\frac{x^4}{2}+x\Big|_{-1}^{2}$$

$$\text{Evaluate at } x=2:$$

$$F(2) = \frac{2^7}{7}+\frac{2^4}{2}+2 = \frac{128}{7}+\frac{16}{2}+2 = \frac{128}{7}+8+2 = \frac{128}{7}+10 = \frac{198}{7}$$

EXAMPLES

Evaluate at $x = -1$:

$$F(-1) = \frac{(-1)^7}{7} + \frac{(-1)^4}{2} + (-1) = \frac{-1}{7} + \frac{1}{2} - 1$$

$$= \frac{-1}{7} + \frac{1}{2} - \frac{2}{2} = \frac{-1}{7} + \frac{-1}{2} = \frac{-2-7}{14} = \frac{-9}{14}$$

Final Result: $F(2) - F(-1) = \frac{198}{7} - \left(\frac{-9}{14}\right)$

$$= \frac{396}{14} + \frac{9}{14} = \frac{405}{14}$$

$$\boxed{\frac{405}{14}}$$

EXAMPLES

EXAMPLE # 2

$$\int_0^2 \int_1^3 xy^2 \, dy \, dx$$

$$\text{Inner Integral:} \quad \int_1^3 xy^2 \, dy$$

$$x \int_1^3 y^2 \, dy = x \left[\frac{y^3}{3} \right]_1^3$$

$$= x \left(\frac{3^3}{3} - \frac{1^3}{3} \right) = x \left(\frac{27}{3} - \frac{1}{3} \right) = x \left(\frac{26}{3} \right) = \frac{26x}{3}$$

EXAMPLES

$$\text{Outer Integral:} \quad \int_0^2 \frac{26x}{3}\,dx$$

$$= \frac{26}{3}\int_0^2 x\,dx = \frac{26}{3}\left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{26}{3}\left(\frac{2^2}{2} - \frac{0^2}{2}\right) = \frac{26}{3}\left(\frac{4}{2} - 0\right)$$

$$= \frac{26}{3} \times 2 = \frac{52}{3}$$

$$\boxed{\mathbf{\frac{52}{3}}}$$

DIFFERENTIAL CALCULUS

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2 u)u'$
29. $\frac{d}{dx}[\text{sech } u] = -(\text{sech } u \tanh u)u'$
30. $\frac{d}{dx}[\text{csch } u] = -(\text{csch } u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\text{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\text{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

INTEGRAL CALCULUS

1. $\int kf(u)\,du = k\int f(u)\,du$

2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$

3. $\int du = u + C$

4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

5. $\int \frac{du}{u} = \ln|u| + C$

6. $\int e^u\,du = e^u + C$

7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$

8. $\int \sin u\,du = -\cos u + C$

9. $\int \cos u\,du = \sin u + C$

10. $\int \tan u\,du = -\ln|\cos u| + C$

11. $\int \cot u\,du = \ln|\sin u| + C$

12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

14. $\int \sec^2 u\,du = \tan u + C$

15. $\int \csc^2 u\,du = -\cot u + C$

16. $\int \sec u \tan u\,du = \sec u + C$

17. $\int \csc u \cot u\,du = -\csc u + C$

18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

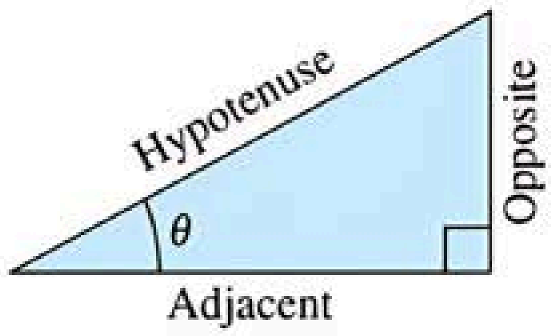
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

TRIGONOMETRIC IDENTITIES

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

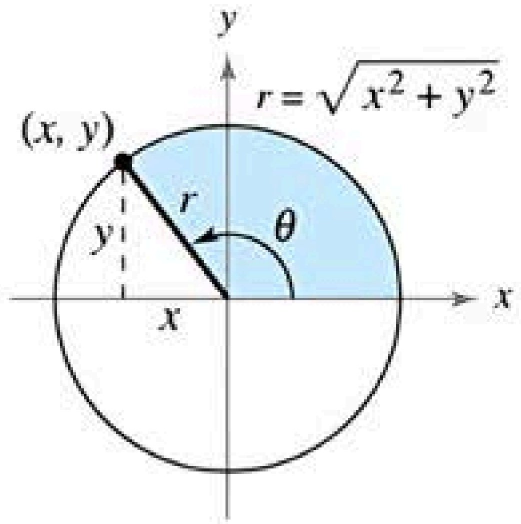


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

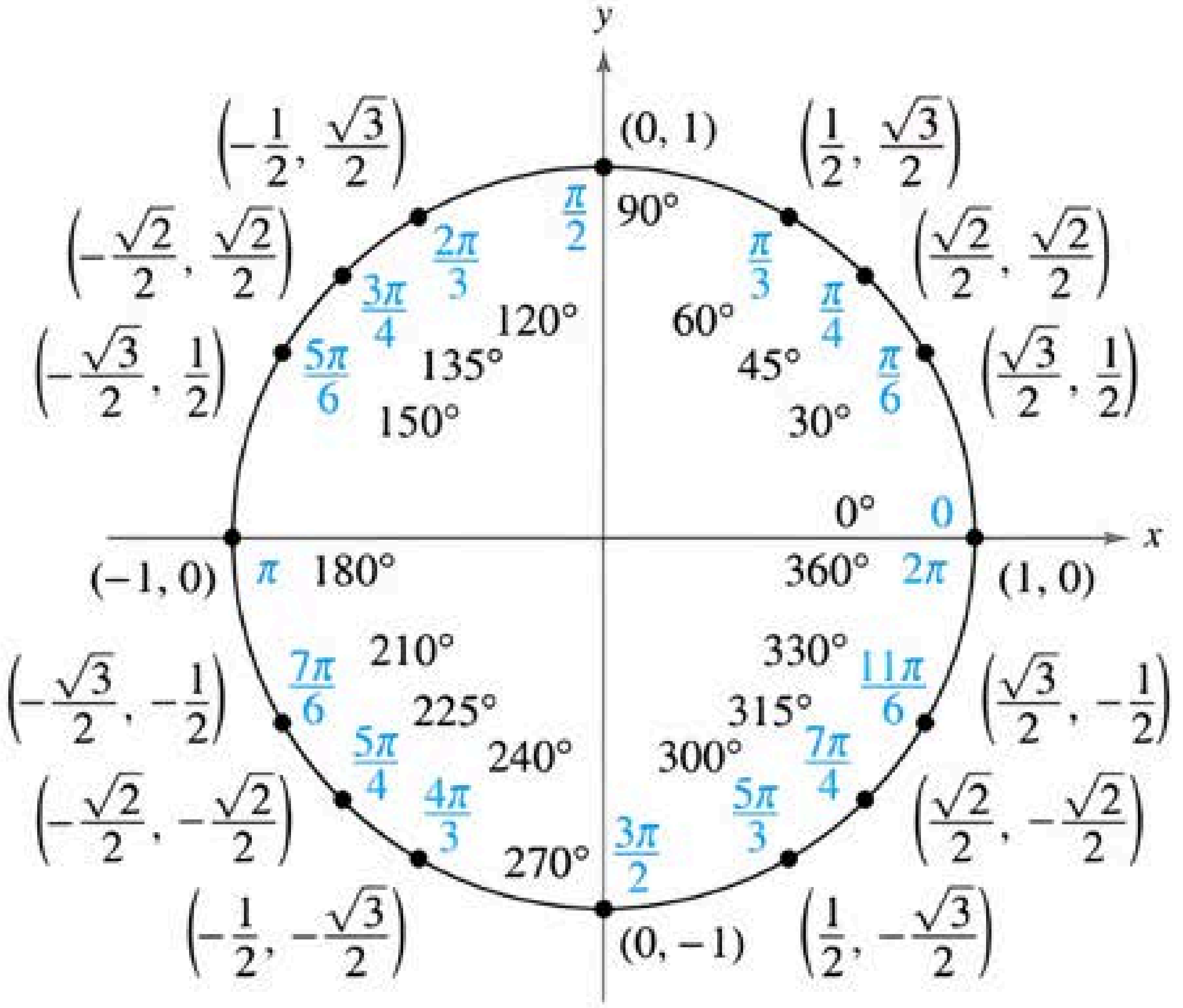
Circular function definitions, where θ *is any angle.*



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Double-Angle Formulas

$$\sin 2u = 2\sin u\cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

TRIGONOMETRIC IDENTITIES

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\csc\left(\dfrac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

Even/Odd Identities

$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$

$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$

$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

Sum-to-Product Formulas

$\sin u + \sin v = 2 \sin\left(\dfrac{u + v}{2}\right) \cos\left(\dfrac{u - v}{2}\right)$

$\sin u - \sin v = 2 \cos\left(\dfrac{u + v}{2}\right) \sin\left(\dfrac{u - v}{2}\right)$

$\cos u + \cos v = 2 \cos\left(\dfrac{u + v}{2}\right) \cos\left(\dfrac{u - v}{2}\right)$

$\cos u - \cos v = -2 \sin\left(\dfrac{u + v}{2}\right) \sin\left(\dfrac{u - v}{2}\right)$

Product-to-Sum Formulas

$\sin u \sin v = \dfrac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$

THANK YOU!