Project REACH

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STATISTICS TUTOR

TOPIC OUTLINE

PROBABILITY

- Sets and Probability
- Sequence and Series
- Permutation and Combination

TOPIC OUTLINE

STATISTICS

- Central Modes of Tendency
- Range
- Variance
- Standard Deviation
- Coefficient of Variation

PROBABILITY

SETS

AND

PROBABILITY

Sets

Set is a collection of different things; these things are called elements or members of the set and are typically mathematical objects of any kind: numbers, symbols, points in space, lines, other geometrical shapes, variables, or even other sets.

EXAMPLE

In a class of 60 students, 45 like science and 30 like math. How many students only likes science?

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Let $Science = S, Math = M$

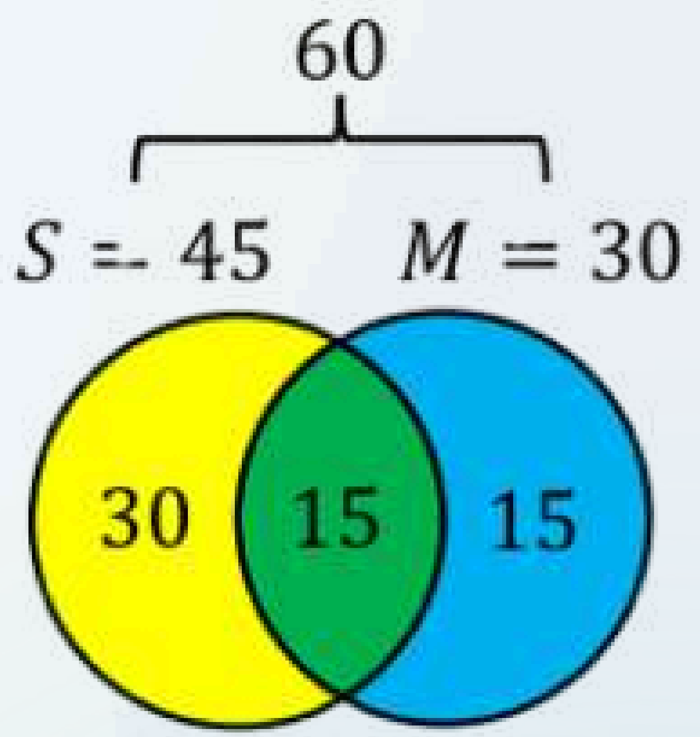
$$n(S \cup M) = n(S) + n(M) - n(S \cap M)$$

$$60 = 45 + 30 - n(S \cap M) \quad \rightarrow \quad n(S \cap M) = 15$$

Determining the number of students who only likes science:

$$n(S) = n(S - M) + n(S \cap M) \quad \rightarrow \quad 45 = n(S - M) + 15$$

$$n(S - M) = \boxed{30}$$



EXAMPLE

In a group of 500 people, 350 people can speak English, and 400 people can speak Hindi. Find how many people can speak both languages?

EXAMPLE

If a set $A = \{1,2,3\}$ and a set $B = \{3,4,5\}$, what is $A \cap B$?

EXAMPLE

In a group of 100 students, 60 students like Math and 70 students like Science. Among them, 40 students like both Math and Science. How many students do not like either Math or Science?

EXAMPLE

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Result of I^{st} dice

Result of II^{nd} dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

EXAMPLE

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Intuition: In a deck of cards, there are 12 faces i.e., 3 faces for each kind (*King*, *Queen*, Jack) multiplied by 4 (*Clover*, *Heart*, *Spade*, *Diamond*)

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$$Probability = \frac{Favorable\ Outcomes}{Total\ Outcomes} = \frac{x}{n}$$

$$\frac{Favorable\ Outcomes}{Total\ Outcomes} \rightarrow \frac{12}{52} = \boxed{\frac{3}{13}}$$

TIP:

Be familiar with the common configurations used in probability problems like coin flips, card draws, and permutation and combination.

SEQUENCE
AND
SERIES

Sequence

A sequence is a set of numbers arranged in a specific order. The numbers in a sequence follow a certain pattern, but there is no requirement for them to follow a specific mathematical rule.

Series

A series is the sum of the terms of a sequence. Instead of simply listing the numbers in a particular order, as in a sequence, a series involves adding those numbers together to obtain a cumulative total.

A series can be either finite, where only a limited number of terms are summed, or infinite, where the summation continues indefinitely.

FORMULAS

Arithmetic Sequences

nth Term: $a_n = a_1 + (n-1)d$

Sum: $S_n = \frac{n}{2}(a_1 + a_n)$ or $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

Geometric Sequences

nth Term: $a_n = a_1 r^{n-1}$

Sum: $S_n = \frac{a_1(1-r^n)}{1-r}$ $\quad S_\infty = \frac{a_1}{1-r}$

EXAMPLE

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$$a_n = a_1 + d(n-1) \rightarrow a_n - a_1 = d(n-1)$$

$$\frac{a_n - a_1}{d} = n - 1 \rightarrow \frac{a_n - a_1}{d} + 1 = n$$

$$\frac{78-3}{5} + 1 = \boxed{16th\ term}$$

3 8 13 18

V V V

8-3=5 | 13-8=5 | 18-13=5

Therefore, $d = 5$

EXAMPLE

The first 5 terms of a series are -6, -3, 0, 3, 6. Find the sum of the first 20 terms of this sequence.

EXAMPLE

Find the 7th term of the sequence: 2, 5, 8, 11, ...

EXAMPLE

Evaluate the sum of: $1 + 4 + 7 + 10 + \ldots + 52$.

PERMUTATION AND COMBINATION

Permutation

A permutation refers to an arrangement of objects where the order in which they are placed is important. This means that if the order of selection changes, the overall arrangement is considered different.

Combination

A combination refers to a selection of objects where the order in which they are chosen does not matter. Unlike permutations, where arranging objects in different sequences creates distinct outcomes, combinations consider only the elements selected, regardless of their order.

FORMULAS

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

n: number of objects in set, r: number of chosen objects in set

NOTE:

In **permutation,** the arrangement matters.

In **combination.** the order or the arrangement doesn’t matter.

EXAMPLE

There are 5 students in a table. Two of these students are Romina and Daniela, who don’t get along very well. In how many ways can the teacher arrange the students in a row, so that Romina and Daniela are not together?

EXAMPLE

Formula: $nPr = \frac{n!}{(n-r)!}$

Total number of arrangements of the 10 students:

$$5P5 \rightarrow \frac{5!}{(5-5)!} \rightarrow \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{0!} = 120$$

Total number of arrangements in which Romina and Daniela (R&D) are together:

$$4P4 \rightarrow \frac{4!}{(4-4)!} \rightarrow \frac{4\cdot 3\cdot 2\cdot 1}{0!} = 24$$

Total number of arrangements in which Daniela and Romina (D&R) are together:

$$4P4 \rightarrow \frac{4!}{(4-4)!} \rightarrow \frac{4\cdot 3\cdot 2\cdot 1}{0!} = 24$$

Total number of arrangements in which Romina and Daniela are not together:

$$P = 5P5 - 4P4 - 4P4 \rightarrow P = 120 - 24 - 24 = \boxed{72}$$

EXAMPLE

Out of a group of 5 people, a pair needs to be formed. Find the number of possible outcomes.

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Solution:

Given: $n = 5, r = 2$ (*Pair*) | Find: Number of possible combinations

Formula: $nCr = \frac{n!}{r!(n-r)!}$

$$nCr = \frac{n!}{r!\,(n-r)!} \rightarrow 5C2 \rightarrow \frac{5!}{2!\,(5 - 2)!} \rightarrow \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1(3\cdot2\cdot1)} = \frac{120}{12} = \boxed{10}$$

EXAMPLE

A 6 member DOST Scholar's Regional Council is to be established by selecting members form 10 active DOST Scholars. In how many ways can the members of the council be selected?

STATISTICS

CENTRAL MODES OF TENDENCY

EXAMPLE

Determine the Mean, Median, and Mode of the data set.

14	36	40	14	21

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Calculating Mean: $\frac{\sum Data}{\# \; of \; Data}$

$$\frac{14 + 36 + 40 + 14 + 21}{5} = \frac{125}{5} = \boxed{25}$$

EXAMPLE

Determine the Mean, Median, and Mode of the data set.

14	36	40	14	21

Calculating the Median: Middle value of the data set

14	36	40	14	21

Rearranging to ascending order ↓

14	14	<u>21</u>	36	40

$\boxed{21}$ is the middle value or median

EXAMPLE

Determine the Mean, Median, and Mode of the data set.

14	36	40	14	21

Calculating the Mode: Most commonly occurring number in the data set

14	14	21	36	40

$\boxed{14}$ is the most commonly occurring number, appearing twice

EXAMPLE

The ages of students in a class are recorded as follows: 16, 12, 15, 18, 11, 21, 19. Identify the median age of the class

RANGE

EXAMPLE

Determine the Range of the data set.

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14	36	40	14	21

$\underline{14}$	14	21	36	$\underline{40}$

14 and 40 are the lowest and highest data respectively.

$$Range = Highest - Lowest \quad \rightarrow \quad 40 - 14 = \boxed{26}$$

POPULATION VARIANCE

EXAMPLE

Determine the Population Variance of the data set

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$\bar{x} = 25$

EXAMPLE

Formula: $Variance\ \sigma = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$

i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	$14 - 25 = -11$	$(-11)^2 = 121$
2	$14 - 25 = -11$	$(-11)^2 = 121$
3	$21 - 25 = -4$	$(-4)^2 = 16$
4	$36 - 25 = 11$	$(11)^2 = 121$
5	$40 - 25 = 15$	$(15)^2 = 225$

$$\sum (x_i - \bar{x})^2 = 604$$

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$$\sum (x_i - \bar{x})^2 = 604$$

$$Variance\ \sigma = \frac{\Sigma(x_i - \bar{x})^2}{n-1} \rightarrow \frac{604}{5-1} = \frac{604}{4} = \boxed{151}$$

STANDARD DEVIATION

EXAMPLE

Determine the Standard Deviation of the data set.

14	36	40	14	21

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Formula: $Standard\ Deviation = \sqrt{\sigma}$

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$\sigma = 151$

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$\sigma = 151$

$$Standard\ Deviation = \sqrt{\sigma} \quad \rightarrow \quad \boxed{\sqrt{151}}$$

COEFFICIENT OF VARIATION

EXAMPLE

Determine the Coefficient of Variation of the data set.

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Formula: $Coefficient\ of\ Variation = \frac{Standard\ Deviation}{Mean}$

Given: $Mean = 25$

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Formula: $Coefficient\ of\ Variation = \frac{Standard\ Deviation}{Mean}$

Given: $Mean = 25$ $Standard\ Deviation = \sqrt{151}$

EXAMPLE

Determine the Coefficient of Variation of the data set.

14	36	40	14	21

Formula: $Coefficient\ of\ Variation = \frac{Standard\ Deviation}{Mean}$

Given: $Mean = 25$ $\quad$ $Standard\ Deviation = \sqrt{151}$

$$Coefficient\ of\ Variation = \frac{Standard\ Deviation}{Mean} \rightarrow \frac{\sqrt{151}}{25}$$

EXAMPLE

What is the difference between the median and the mode in the following set of data?

72, 44, 58, 32, 34, 68, 94, 22, 67, 45, 58

ALL THE BEST, ISKOLARS!

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