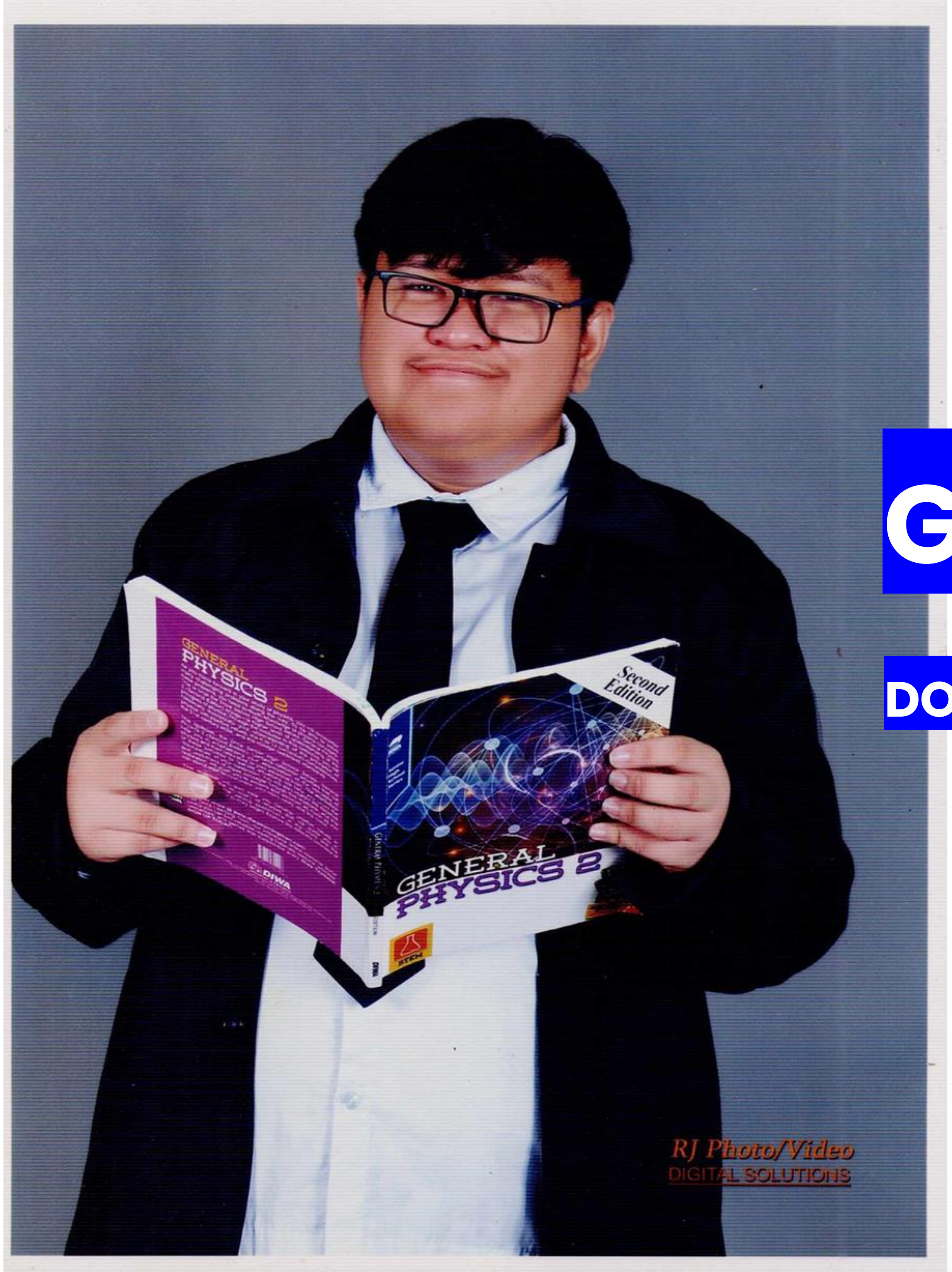
Project REACH

CALABARZON



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POLYNOMIAL

Factoring and Roots

Find all the real roots of the polynomial:

$$P(x) = x^3 - 6x^2 + 11x - 6$$

$$\boldsymbol{P(x) = x^3 - 6x^2 + 11x - 6}$$

Assume roots to be factors of -6.

Possible Roots:

$$\pm 1, \pm 2, \pm 3, \pm 6$$

Try $x = 1$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1x^2 & -5x & +6 & 0 \end{array}$$

= Remainder

x=1 is a root of P(x).

$x^2 - 5x + 6$

Find factors of 6 that will add up to -5.

1	+	6	=7
2	+	3	=5
-1	+	-6	=-7
-2	+	-3	=-5

$$x^2 - 5x + 6 = (x-2)(x-3)$$

$$x - 2 = 0 \qquad x - 3 = 0$$

$$x = 2 \qquad x = 3$$

The roots of P(x) are 1,2, and 3.

Polynomial Division

Divide $P(x) = 3x^4 - 5x^3 + 2x^2 - 8x + 4$ by $D(x) = x^2 - x + 1$ using long division or synthetic division.

$$P(x) = 3x^4 - 5x^3 + 2x^2 - 8x + 4$$
$$D(x) = x^2 - x + 1$$

$$
\begin{array}{r|l}
 & \quad 3x^2 \quad -2x \quad -3 \\
\hline
x^2 - x + 1 & 3x^4 - 5x^3 + 2x^2 - 8x + 4 \\
 & -(3x^4 - 3x^3 + 3x^2) \\
\hline
 & \qquad -2x^3 - \quad x^2 - 8x \\
 & \quad -(-2x^3 + 2x^2 - 2x) \\
\hline
 & \qquad\qquad -3x^2 - 6x + 4 \\
 & \qquad -(-3x^2 + 3x - 3) \\
\hline
 & \qquad\qquad\qquad 9x + 7 \ = \text{Remainder}
\end{array}
$$

The Quotient is $3x^2 - 2x - 3 + \dfrac{9x + 7}{x^2 - x + 1}$.

Remainder Theorem

If $P(x) = x^4 - 3x^3 + 5x^2 - 3x + k$ is divided by $x - 2$, the remainder is 7. Find k.

$P(x) = x^4 - 3x^3 + 5x^2 - 3x + k$

Since $x - 2$ is our divisor.

$x - 2 = 0$
$x = 2$

By Remainder Theorem,
$P(2) = 7$

$P(2) = 2^4 - 3(2^3) + 5(2^2) - 3(2) + k$
$7 = 16 - 3(8) + 5(4) - 6 + k$
$7 = 16 - 24 + 20 - 6 + k$
$7 = 8 + k$
$k = 7 - 8$
$k = -1$

SERIES AND SEQUENCES

Find the nth Term of an Arithmetic Sequence

The first term of an arithmetic sequence is $a_1 = 5$, and the common difference is $d = 3$. Find the 15^{th} term.

$$a_n = a_1 + (n-1)d$$

$$a_1 = 5$$

$$n = 15$$

$$d = 3$$

$$a_{15} = 5 + (15-1)(3)$$

$$a_{15} = 5 + (14)(3)$$

$$a_{15} = 5 + 42$$

$$a_{15} = 47$$

Sum of the First n Terms of an Arithmetic Series

Find the sum of the first 20 terms of the arithmetic series:

3, 7, 11, 15, ...

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$a_1 = 3$$

$$n = 20$$

$$d = 7 - 3$$

$$d = 4$$

$$S_{20} = \frac{20}{2}(2(3) + (20-1)(4))$$

$$S_{20} = 10\,(6 + (19)(4))$$

$$S_{20} = 10\,(6 + 76)$$

$$S_{20} = 10\,(82)$$

$$S_{20} = 820$$

Find the nth Term of a Geometric Sequence

The first term of a geometric sequence is $a_1 = 3$, and the common ratio is $r = 3$. Find the 6^{th} term.

$$a_n = a_1 \times r^{n-1}$$

$$a_1 = 3$$

$$n = 6$$

r= 2

$$a_6 = 3 \times 2^{6-1}$$

$$a_6 = 3 \times 2^5$$

$$a_6 = 3 \times 32$$

$$a_6 = 96$$

Find the Sum of a Geometric Series

Find the sum of the first 5 terms of the geometric series:

2, 6, 18, 54, ...

$$S_n = \frac{a_1(r^n - 1)}{(r - 1)}$$

$$a_1 = 2$$

$$n = 5$$

$$r = \frac{6}{2}$$

$$r = 3$$

$$S_5 = \frac{(2)\,(3^5 - 1)}{(3 - 1)}$$

$$S_5 = \frac{\cancel{(2)}\,(243 - 1)}{\cancel{(2)}}$$

$$S_5 = 242$$

Find the Number of Terms in an Arithmetic Sequence

How many terms are in the arithmetic sequence 2, 5, 8, ..., 50 ?

$$a_n = a_1 + (n-1)d$$
$$a_1 = 2$$
$$a_n = 50$$
$$d = 5 - 2$$
$$d = 3$$
$$50 = 2 + (n-1)(3)$$
$$50 - 2 = (n-1)(3)$$
$$48 = (n-1)(3)$$
$$\frac{48}{3} = (n-1)$$
$$16 = (n-1)$$
$$n = 16 + 1$$
$$n = 17$$

LOGARITHMIC AND EXPONENTIAL FUNCTIONS

Solve for x in an Exponential Equation

Solve for x in the equation:

$$5^{x+2} = 125$$

$$5^{x+2} = 125$$

$$5^{x+2} = 5^3$$

$$x + 2 = 3$$

$$x = 3 - 2$$

$$x = 1$$

Solve for x in a Logarithmic Equation

Solve for x:

$$\log_3 x = 4$$

$$\log_3 x = 4$$

Since $\log_a b = y$ is $a^y = b$,

$$3^4 = x$$

$$x = 81$$

Evaluate a Logarithmic Expression

Simplify:

$$\log_2 16 + \log_3 27$$

$\log_2 16 + \log_3 27$

Evaluate each term

$\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$x = 4$

$\log_2 16 = 4$

$\log_3 27 = y$

$3^y = 27$

$3^y = 3^3$

$y = 3$

$\log_3 27 = 3$

$\log_2 16 + \log_3 27 = 4 + 3$

$\log_2 16 + \log_3 27 = 7$

Solve for x in a Logarithmic Equation Using Properties

Solve for x:

$$\log_5(x-1)+\log_5(x+3)=1$$

$\log_5(x-1) + \log_5(x+3) = 1$

Since $\log_a b + \log_a c = \log_a bc$

$\log_5(x-1) + \log_5(x+3) = \log_5((x-1)(x+3))$

$1 = \log_5(x^2 + 3x - x - 3)$

$\log_5(x^2 + 2x - 3) = 1$

$5^1 = x^2 + 2x - 3$

$5 = x^2 + 2x + 1 - 3 - 1$

$(x+1)^2 - 4 = 5$

$(x+1)^2 = 5 + 4$

$(x+1)^2 = 5+4$

$(x+1)^2 = 9$

$x+1 = \pm\sqrt{9}$

$x+1 = \pm 3$

$x = \pm 3 - 1$

$x = 3 - 1$ $\qquad$ $x = -3 - 1$

$x = 2$ $\qquad$ $x = -4$

$x = 2$ $x = -4$

Check for restrictions.

$\log_5(2-1) + \log_5(2+3) = 1$

$\log_5(1) + \log_5(5) = 1$

$\log_5(1 \times 5) = 1$

$\log_5(5) = 1$

$1 = 1$

$x = 2$ is valid.

$\log_5(-4-1) + \log_5(-4+3) = 1$

$\log_5(-5) + \log_5(-1) = 1$

$\log_5(-5)$ and $\log_5(-1)$ are undefined.

$x = -4$ is invalid.

Therefore, $x = 2$.

SIMULTANEOUS EQUATIONS

Solve Using Substitution Method

Solve the system of equations using the substitution method:

$$y = 2x + 3$$

$$3x + 2y = 12$$

$y = 2x + 3$

$3x + 2y = 12$

Substitute y to the second equation.

$3x + 2(2x + 3) = 12$

$3x + 4x + 6 = 12$

$7x = 12 - 6$

$7x = 6$

$x = \frac{6}{7}$

$$x = \frac{6}{7}$$

Substitute x to either equation.

$$y = 2\left(\frac{6}{7}\right) + 3$$

$$y = \frac{12}{7} + 3$$

$$y = \frac{12}{7} + \frac{21}{7}$$

$$y = \frac{33}{7}$$

$$3\left(\frac{6}{7}\right) + 2y = 12$$

$$\frac{18}{7} + 2y = 12$$

$$2y = 12 - \frac{18}{7}$$

$$2y = \frac{84}{7} - \frac{18}{7}$$

$$2y = \frac{66}{7}$$

$$y = \frac{66}{7} \times \frac{1}{2}$$

$$y = \frac{66}{14}$$

$$y = \frac{33}{7}$$

Solve Using Elimination Method

Solve the system of equations using the elimination method:

$$2x + 3y = 12$$

$$4x - 5y = -2$$

$$2x + 3y = 12$$

$$4x - 5y = -2$$

$$5(2x + 3y) = 5(12)$$

$$3(4x - 5y) = 3(-2)$$

$$10x + 15y = 60$$

$$12x - 15y = -6$$

$$\begin{array}{r} 10x + 15y = 60 \\ 12x - 15y = -6 \\ \hline 22x \qquad\quad = 54 \end{array}$$

$$x = \frac{54}{22}$$

$$x = \frac{27}{11}$$

$$2x + 3y = 12$$

$$4x - 5y = -2$$

$$2(2x + 3y) = 2(12)$$

$$-(4x - 5y) = -(-2)$$

$$4x + 6y = 24$$

$$-4x + 5y = 2$$

$$\begin{array}{r} 4x + 6y = 24 \\ -4x + 5y = 2 \\ \hline 11y = 26 \end{array}$$

$$y = \frac{26}{11}$$

Solve a System with Fractions

Solve the system of equations:

$$\frac{x}{2} + \frac{y}{3} = 4$$
$$\frac{x}{4} - \frac{y}{5} = 1$$

$$\frac{x}{2} + \frac{y}{3} = 4$$
$$\frac{x}{4} - \frac{y}{5} = 1$$

$$6\left(\frac{x}{2} + \frac{y}{3}\right) = 6(4)$$
$$20\left(\frac{x}{4} - \frac{y}{5}\right) = 20(1)$$

$$3x + 2y = 24$$

$$5x - 4y = 20$$

$$2(3x + 2y) = 2(24)$$

$$5x - 4y = 20$$

$$\begin{aligned} 6x + 4y &= 48 \\ 5x - 4y &= 20 \\ \hline 11x \qquad &= 68 \end{aligned}$$

$$x = \frac{68}{11}$$

$$x = \frac{68}{11}$$

$$\frac{x}{2} + \frac{y}{3} = 4$$

$$\frac{\frac{68}{11}}{2} + \frac{y}{3} = 4$$

$$\frac{68}{2 \times 11} + \frac{y}{3} = 4$$

$$\frac{68}{22} + \frac{y}{3} = 4$$

$$\frac{34}{11} + \frac{y}{3} = 4$$

$$\frac{y}{3} = 4 - \frac{34}{11}$$

$$\frac{y}{3} = \frac{44}{11} - \frac{34}{11}$$

$$\frac{y}{3} = \frac{44}{11} - \frac{34}{11}$$

$$\frac{y}{3} = \frac{10}{11}$$

$$y = \frac{10}{11} \times 3$$

$$y = \frac{30}{11}$$

SLOPES

Find the Slope of a Given Line

Find the slope of the line that passes through the points (3, 5) and (7, 11).

$$Point\ A = (3,5)$$
$$Point\ B = (7,11)$$

$$Point\ A = (x_1, y_1)$$
$$Point\ B = (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11 - 5}{7 - 3}$$

$$m = \frac{6}{4}$$

$$m = \frac{3}{2}$$

Find a Parallel Line Equation

Find the equation of the line parallel to $y = 4x - 2$ that passes through the point (2, 3).

$y = 4x - 2$

$point\ (2, 3)$ $point\ (x, y)$

Parallel lines have the same slope with different y-intercepts.

$y = mx + b$

$m = 4 \quad b = ?$

$y = 4x + b$

$3 = 4(2) + b$

$3 = 8 + b$

$b = 3 - 8$

$b = -5$

$y = 4x - 5$

Therefore, $y = 4x - 5$ is the equation parallel to the given.

Find a Perpendicular Line Equation

Find the equation of the line perpendicular to $y = -\frac{1}{2}x + 6$ that passes through the point (-4, 7).

$y = -\frac{1}{2}x + 6$

$point\ (-4, 7)$ $point\ (x, y)$

Perpendicular line is the negative reciprocal of slope with different y-intercepts.

$y = mx + b$

$m = -\frac{1}{2}$

$m_{\perp} = 2 \quad b = ?$

$y = 2x + b$

$7 = 2(-4) + b$

$7 = -8 + b$

$b = 7 + 8$

$b = 15$

$y = 2x + 15$

Therefore, $y = 2x + 15$ is the equation perpendicular to the given.

OPERATIONS WITH FRACTIONS

Addition and Subtraction of Algebraic Fractions

Simplify:

$$\frac{x}{4}+\frac{3x}{6}-\frac{2x}{3}$$

$$\frac{x}{4} + \frac{3x}{6} - \frac{2x}{3}$$

$$\frac{6x + 12x - 16x}{24}$$

$$\frac{2x}{24}$$

$$\frac{x}{12}$$

Multiplication of Algebraic Fractions

Simplify:

$$\left(\frac{2x}{5}\right) \times \left(\frac{10}{x^2}\right)$$

$$\left(\frac{2x}{5}\right) \times \left(\frac{10}{x^2}\right)$$

$$\frac{20x}{5x^2}$$

$$\frac{4}{x}$$

$$\left(\frac{2\cancel{x}}{_1\cancel{5}}\right) \times \left(\frac{\cancel{10}^{\,2}}{x\cancel{^2}}\right)_1$$

$$\frac{4}{x}$$

Division of Algebraic Fractions

Simplify:

$$\left(\frac{x^2 - 4}{x + 2}\right) \div \left(\frac{x - 2}{x + 2}\right)$$

$$\left(\frac{x^2-4}{x+2}\right) \div \left(\frac{x-2}{x+2}\right)$$

$$KCC\ (Keep, Change, Change)\ Rule$$

$$\left(\frac{x^2-4}{x+2}\right) \times \left(\frac{x+2}{x-2}\right)$$

$$\left(\frac{(x+2)\cancel{(x-2)}}{\cancel{x+2}}\right) \times \left(\frac{\cancel{x+2}}{\cancel{x-2}}\right)$$

$$x+2$$