Project REACH

GEOMETRY

Full Version!

Let's Review!

1. Angle Theorems
2. Triangle Postulates
3. Polygon Properties
4. Surface Area & Volume
5. Conic Sections

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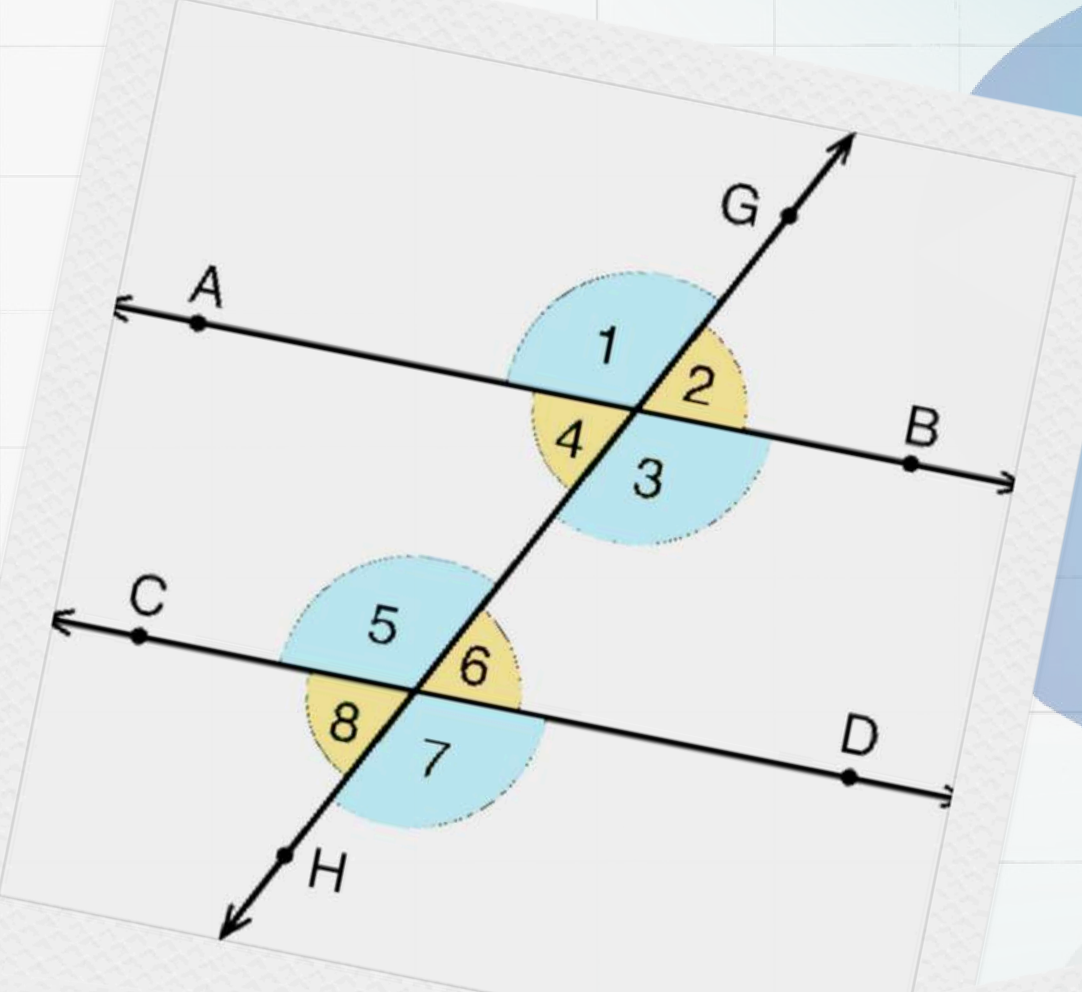
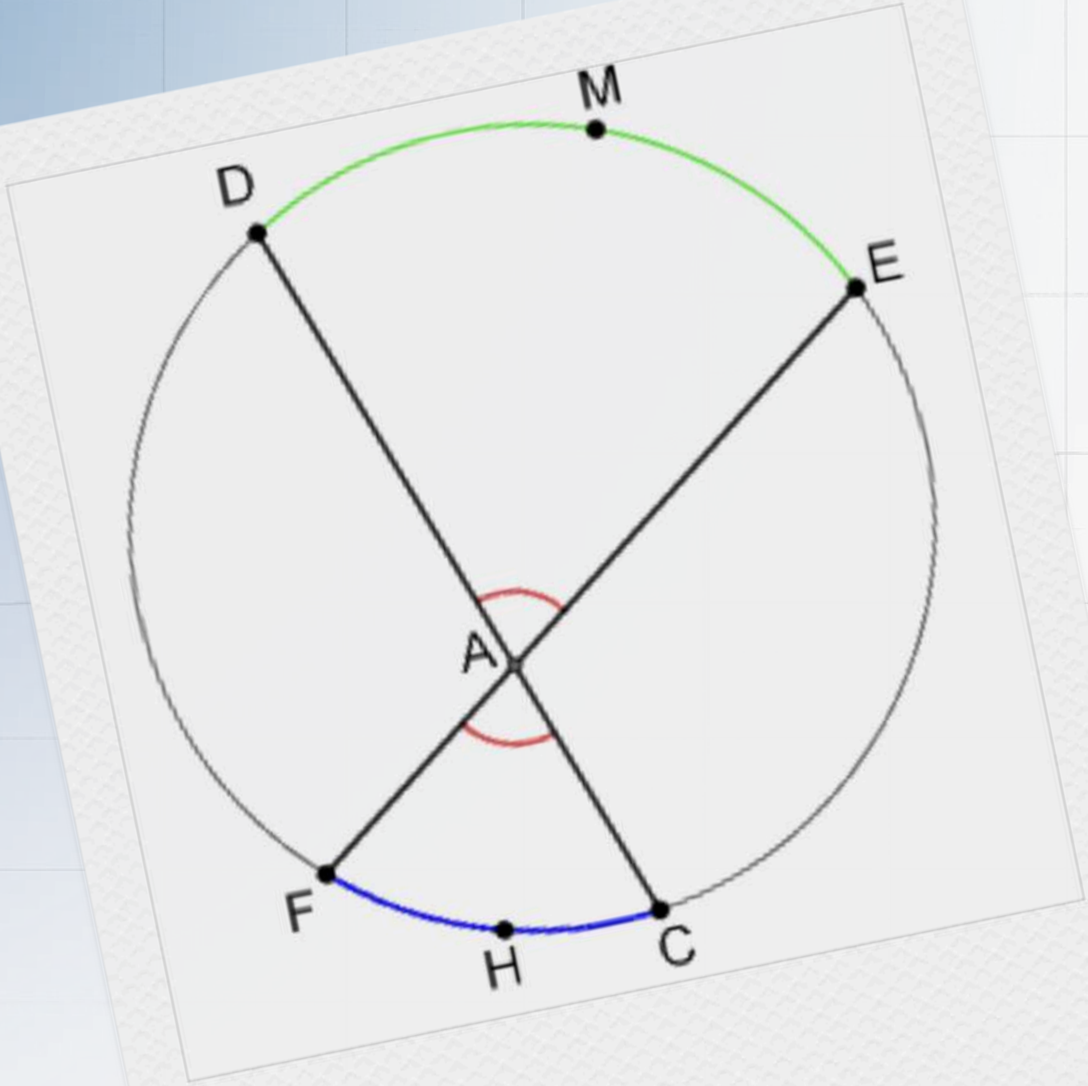
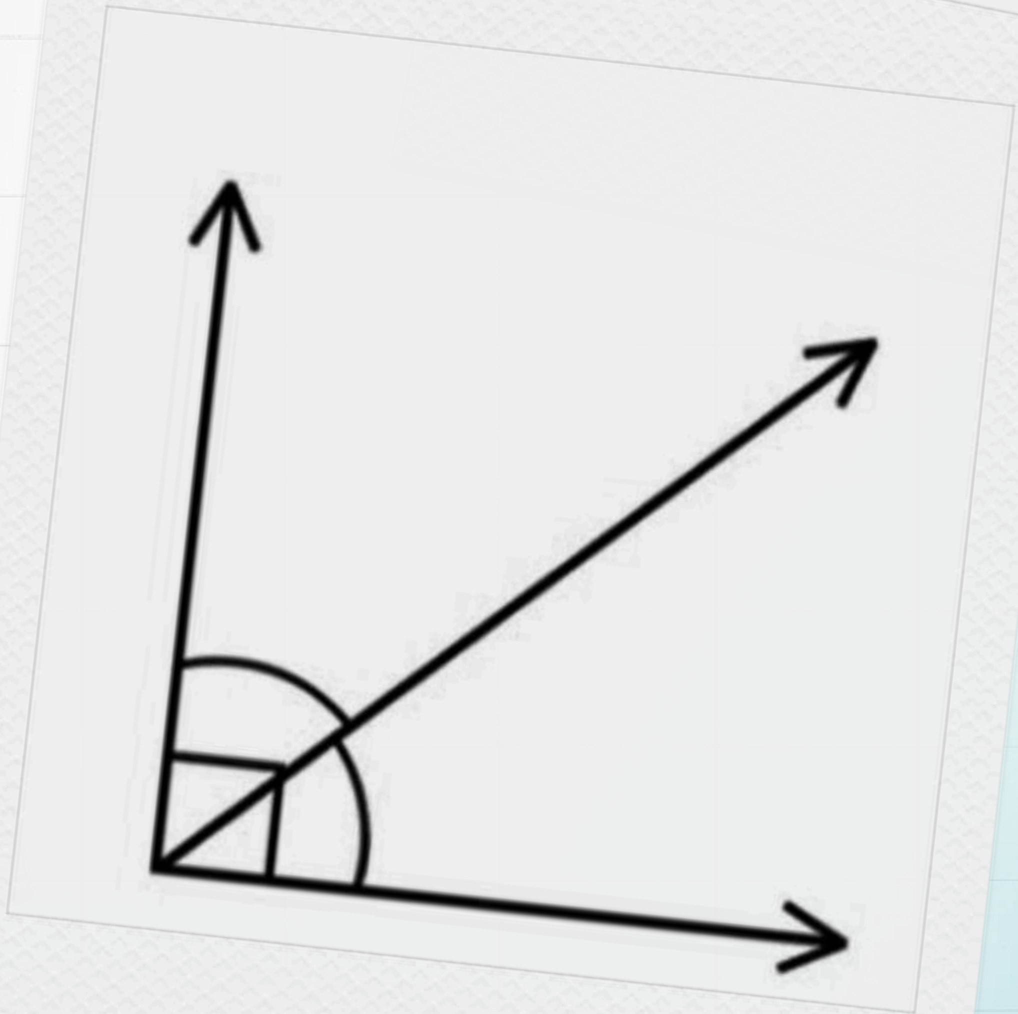
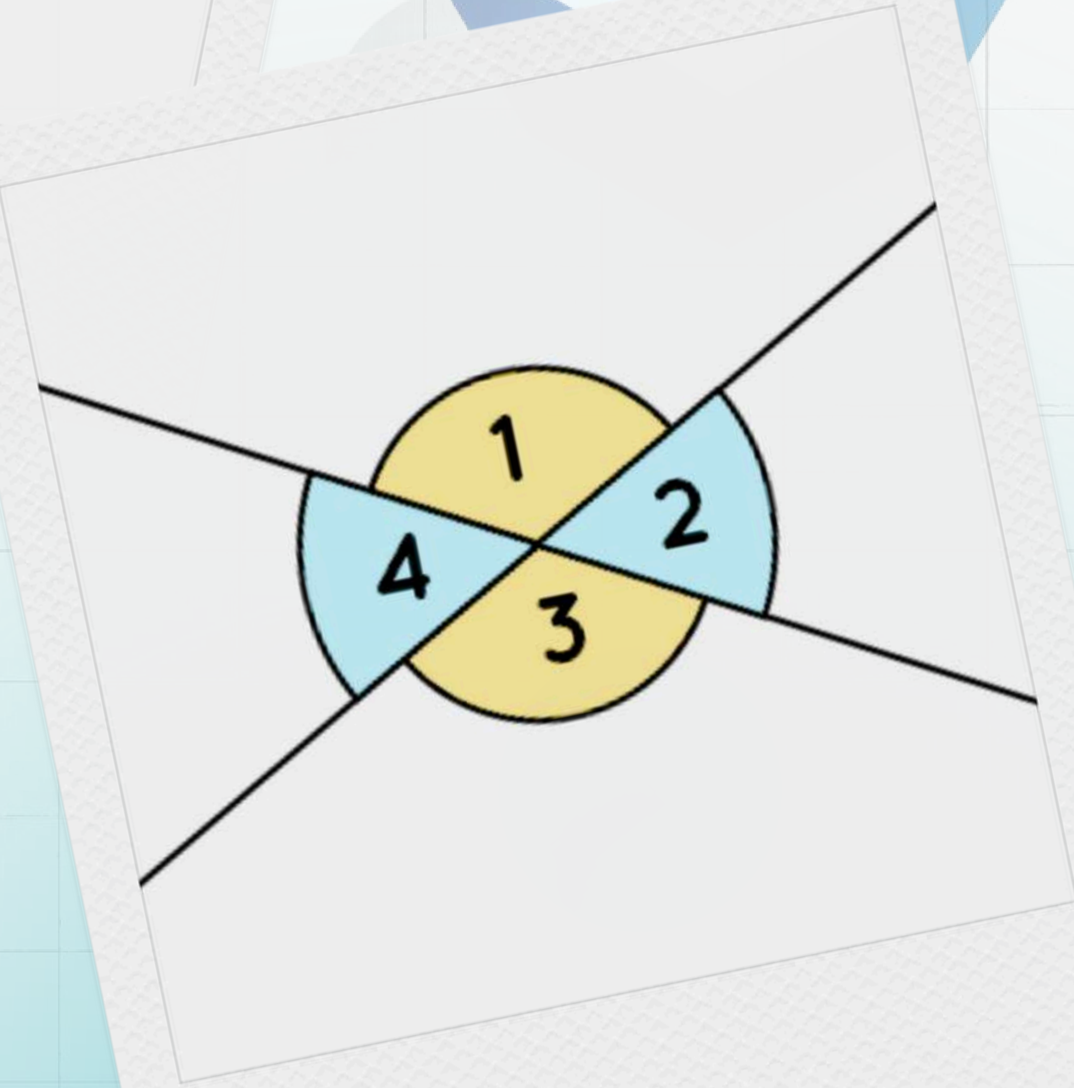
GEOMETRY

ANGLE THEOREMS



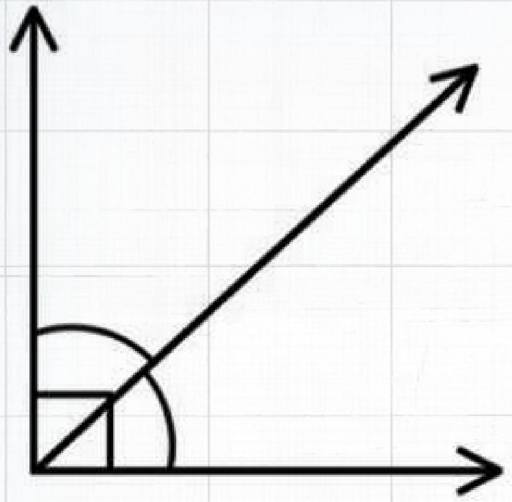






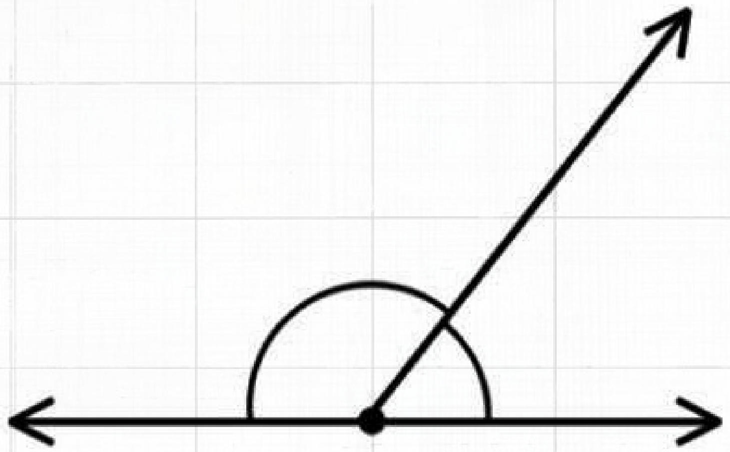
ANGLE THEOREMS

OVERVIEW



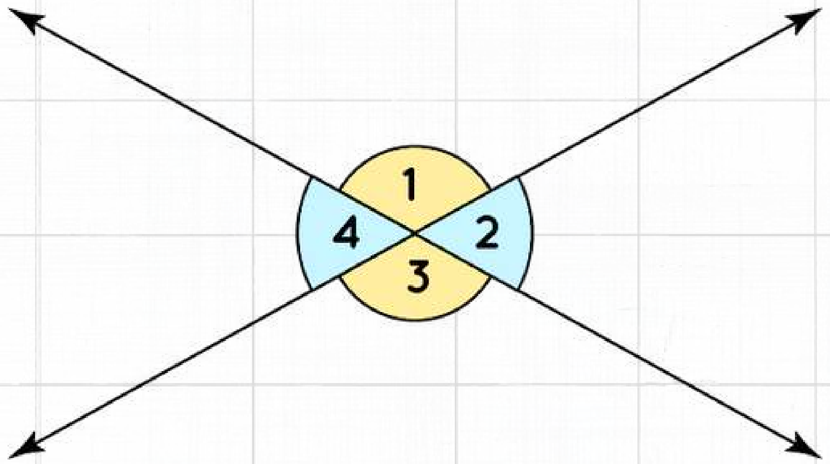
Complementary

Add up to 90°, forming right angles



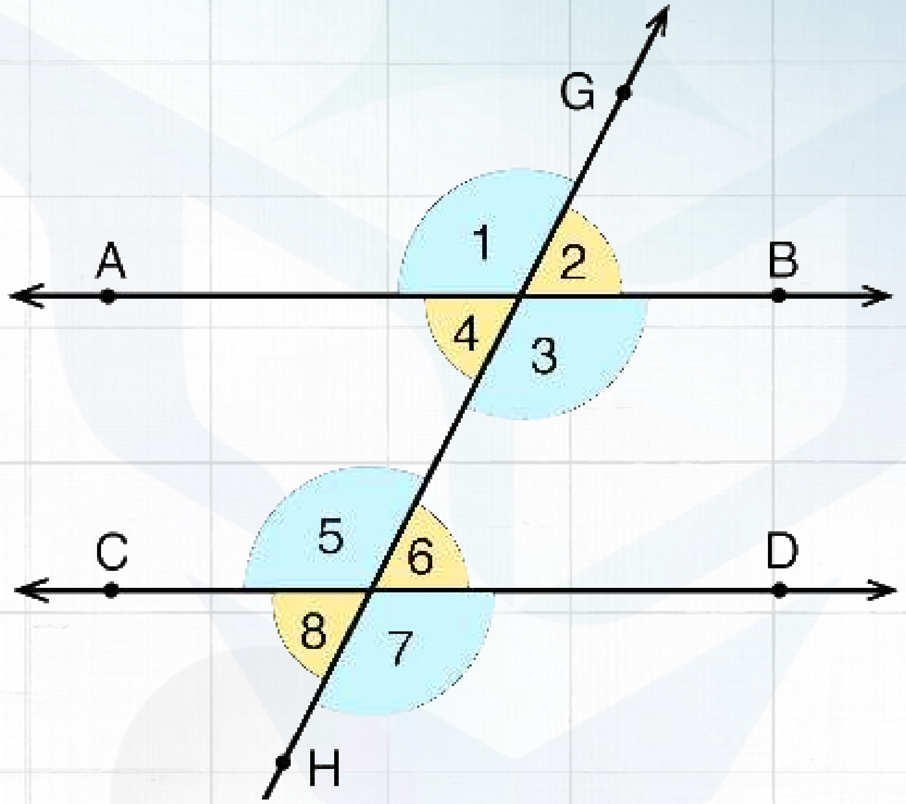
Supplementary

Add up to 180°, forming right angles



Vertical

Opposite angles are equal, adjacent are supplementary



Corresponding

Pairs of vertical angles found across a transversal

ANGLE THEOREMS

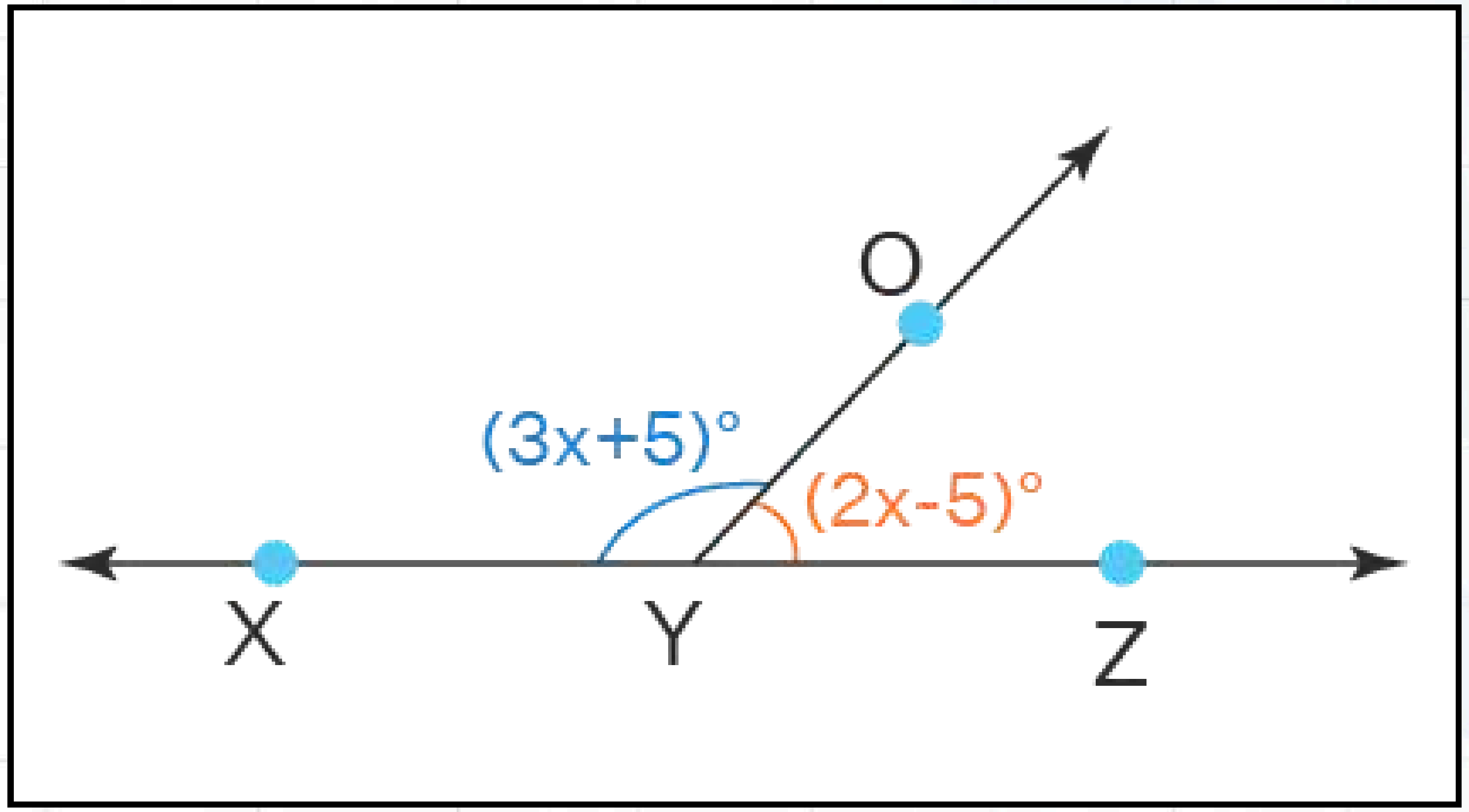
ANGLE ADDITION

We can add angles that share the same side.

Complementary angles add up to 90° and form right angles.

Supplementary angles add up to 180° and form straight lines.

This is true for all adjacent angles, even if they don't add up to 90° or 180°



$$3x + 5° + 2x - 5° = 180°$$

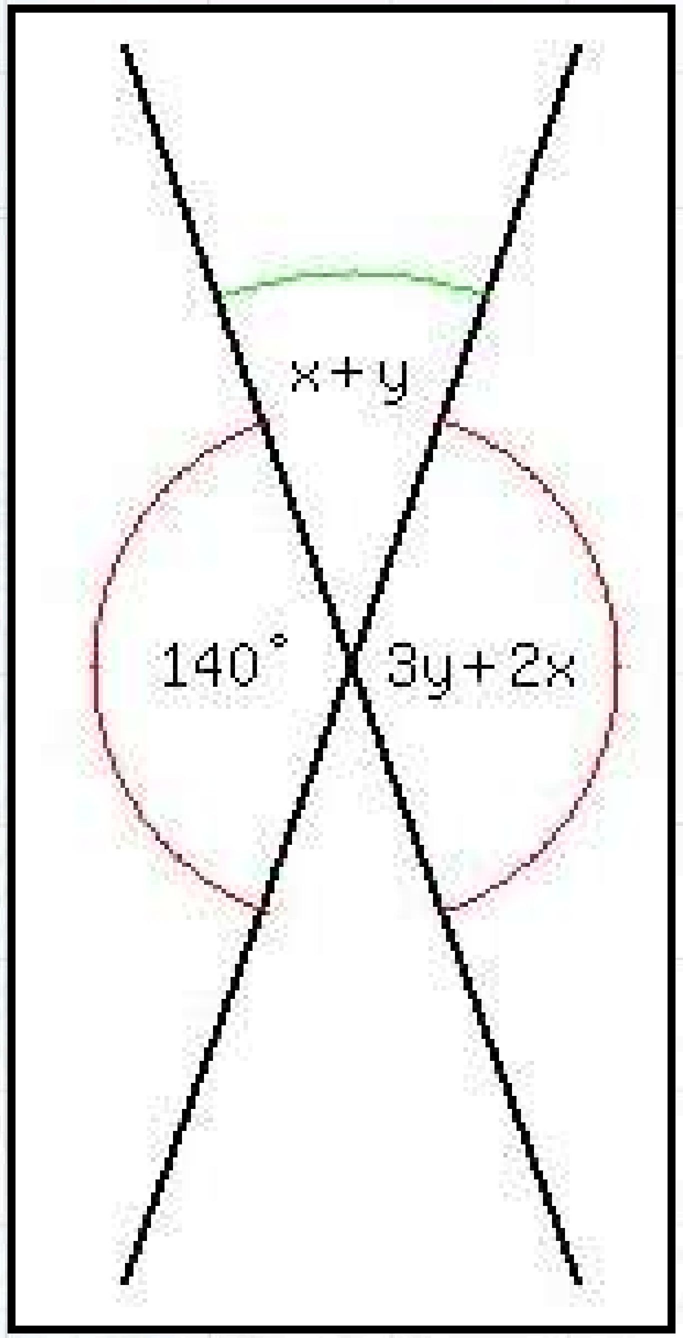
$$5x = 180°$$

$$x = 36°$$

ANGLE THEOREMS

ANGLE ADDITION

Solve for x and y.



Add the 1st pair of supplementary angles

$$x + y + 140° = 180°$$

$$x + y = 40°$$

Add the 2nd pair of supplementary angles

$$x + y + 3y + 2x = 180°$$

$$3x + 4y = 180°$$

Systems of equations

$$\begin{array}{r} 3x + 4y = 180° \\ -\ 3x - 3y = -\ 120° \\ \hline y = 60° \end{array}$$

$$x + y = 40° \rightarrow x + 60° = 40°$$

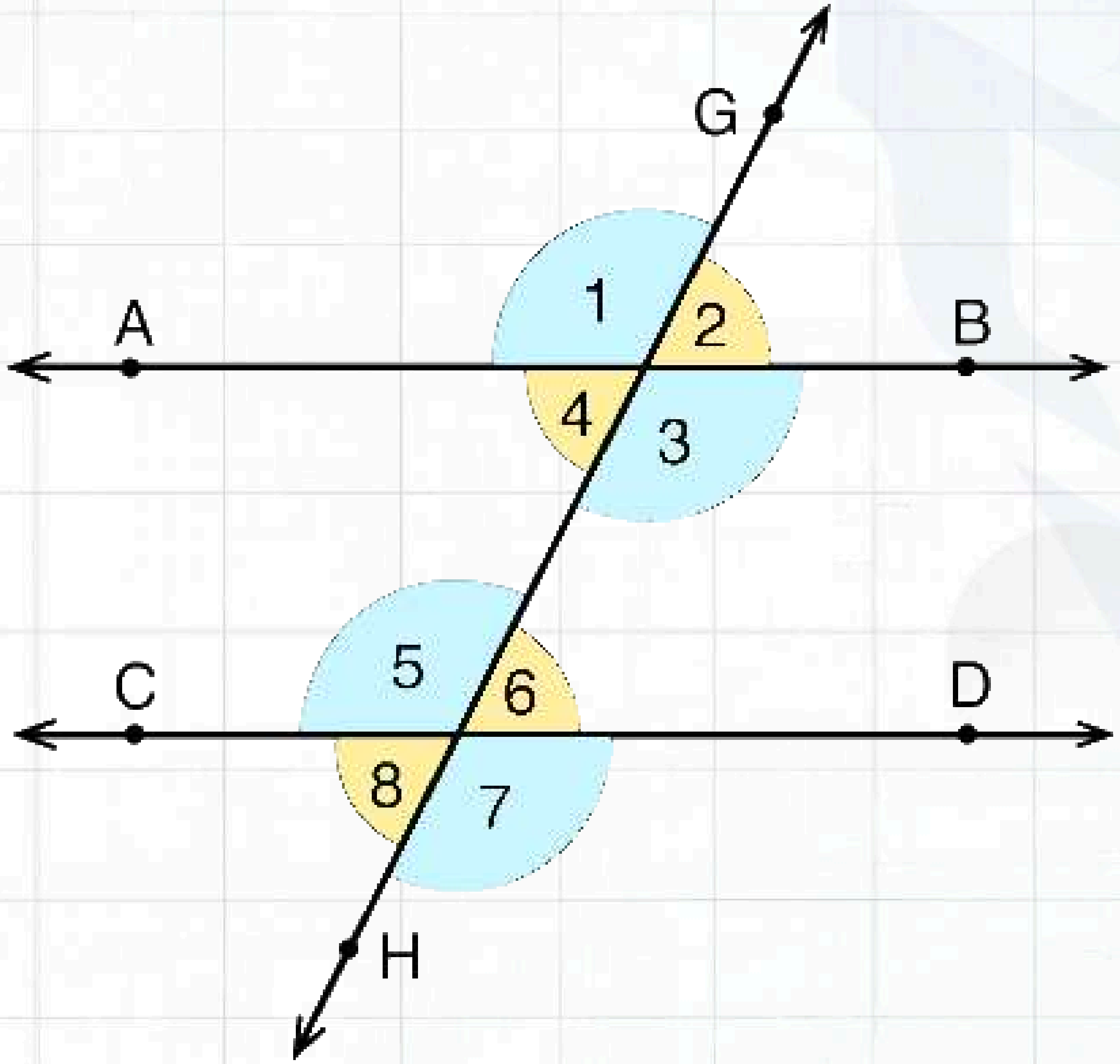
$$x = -\ 20°$$

ANGLE THEOREMS

ANGLE PAIRS ON TRANSVERSALS

In geometry, a **transversal** is a line that passes through two lines in the same plane at two distinct points.

In other words, it is a system of intersecting lines. We can form equivalent angle pairs if the **two lines being intersected are parallel**

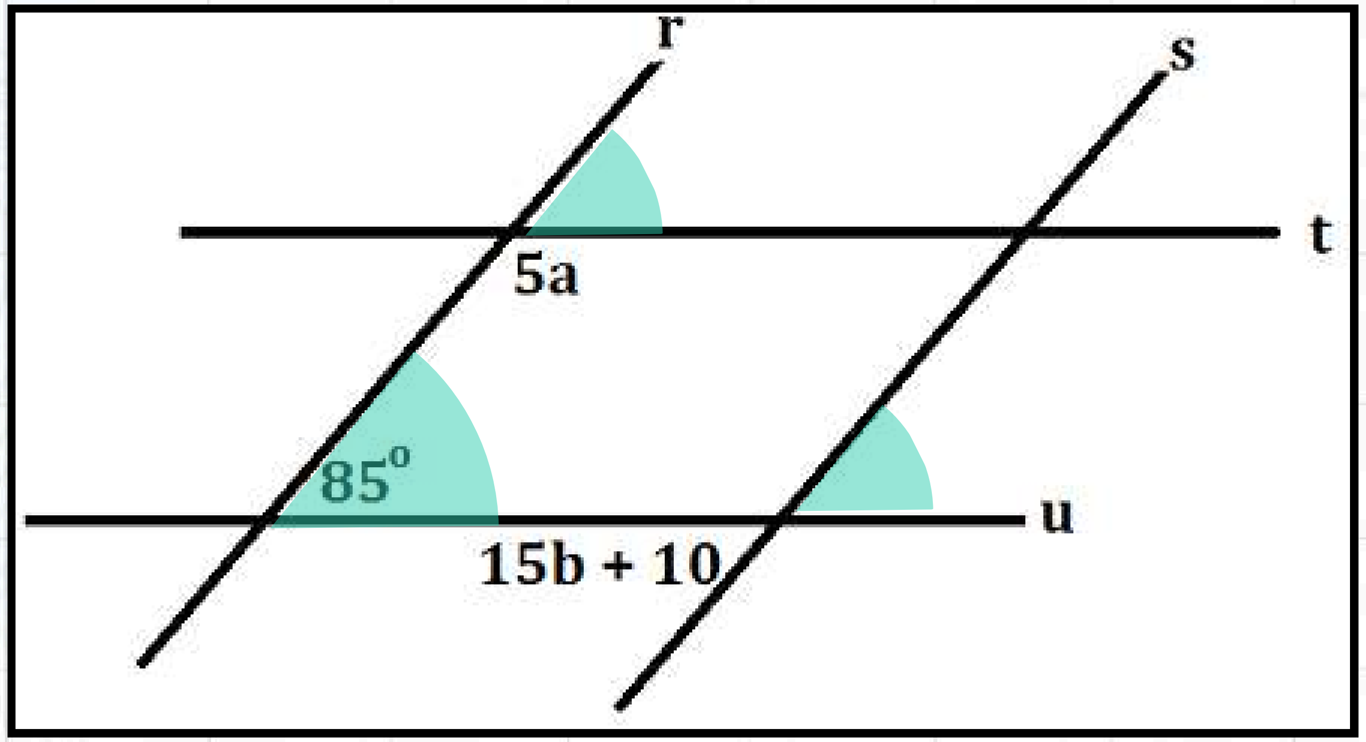


Angles, 1, 3, 5, and 7 are equal.
Angles 2, 4, 6, and 8 are equal.

ANGLE THEOREMS

ANGLE PAIRS ON TRANSVERSALS

Assuming r || s and t || u,
Solve for a and b.



Supplementary angles (Same-side interior)

$$5a + 85 = 180$$

$$5a = 95$$

$$a = 19$$

Equal angles (Alternate interior)

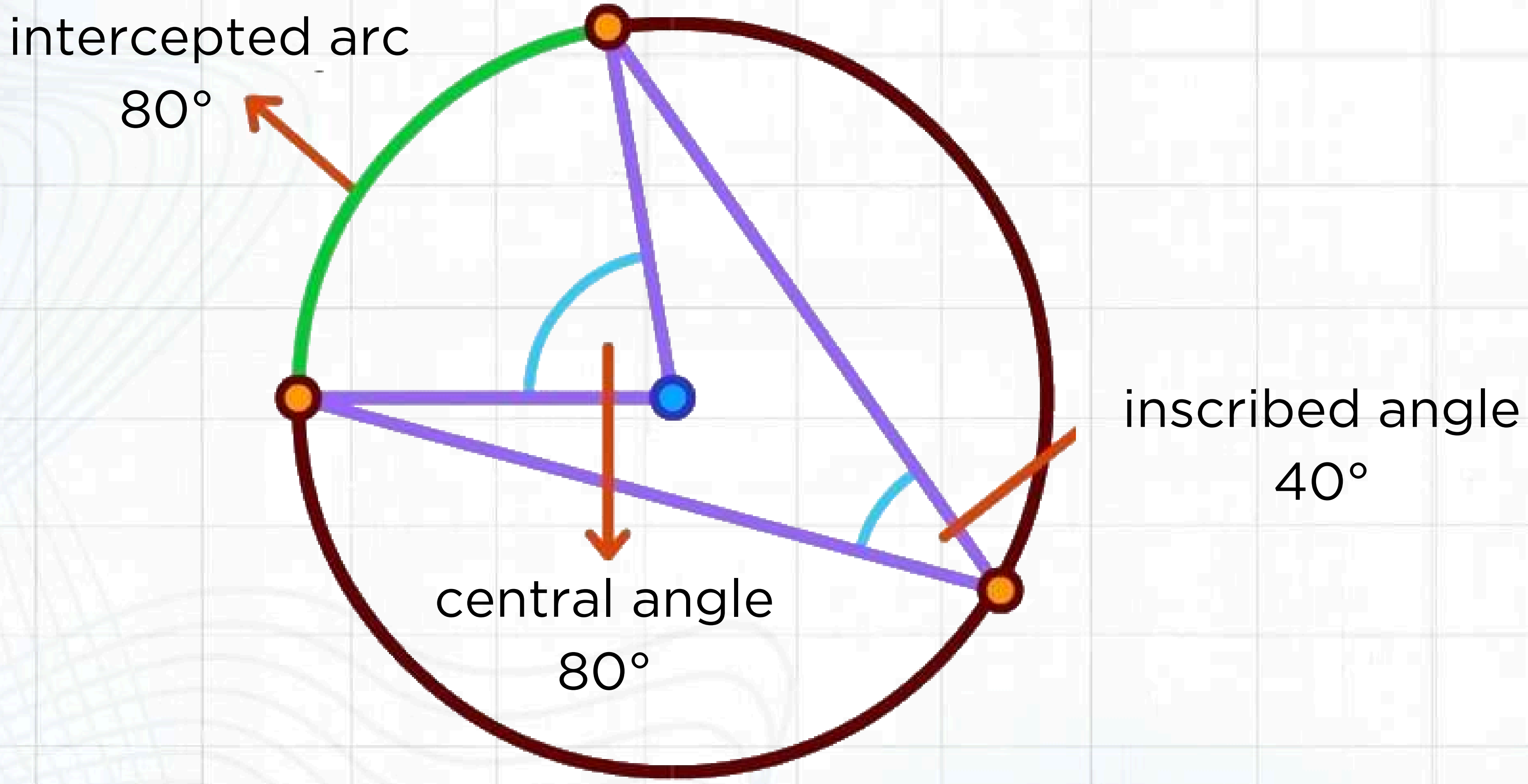
$$15b = 75$$

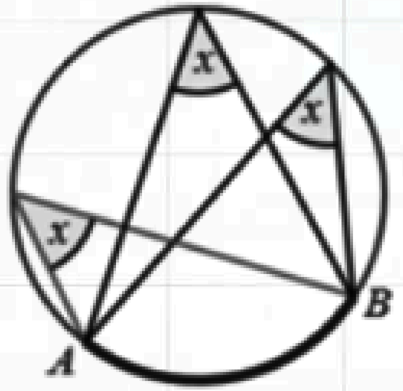
$$b = 5$$

ANGLE THEOREMS

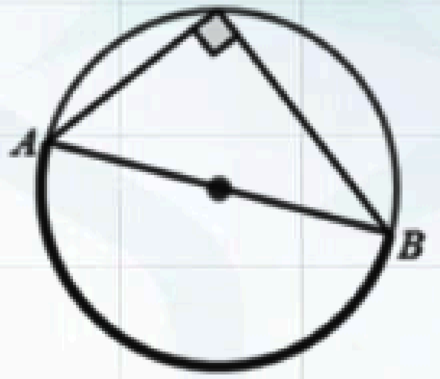
INSCRIBED ANGLE THEOREMS

An **inscribed angle** is always **one-half** the measure of either the central angle or the intercepted arc sharing endpoints of the inscribed angle's sides.

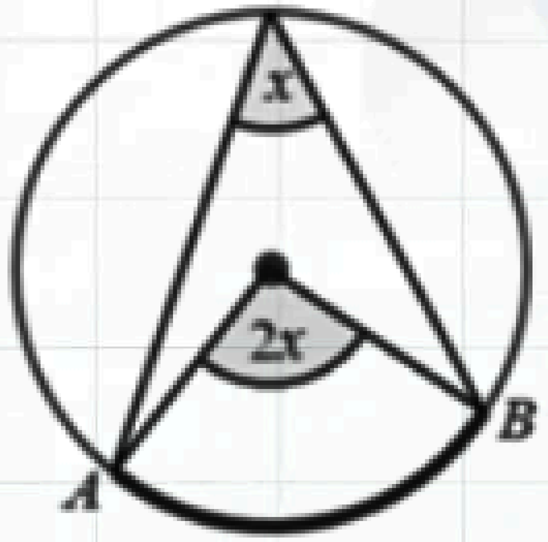




The inscribed angles subtended by the same arc are equal.



Inscribed angle in a semicircle is 90°.

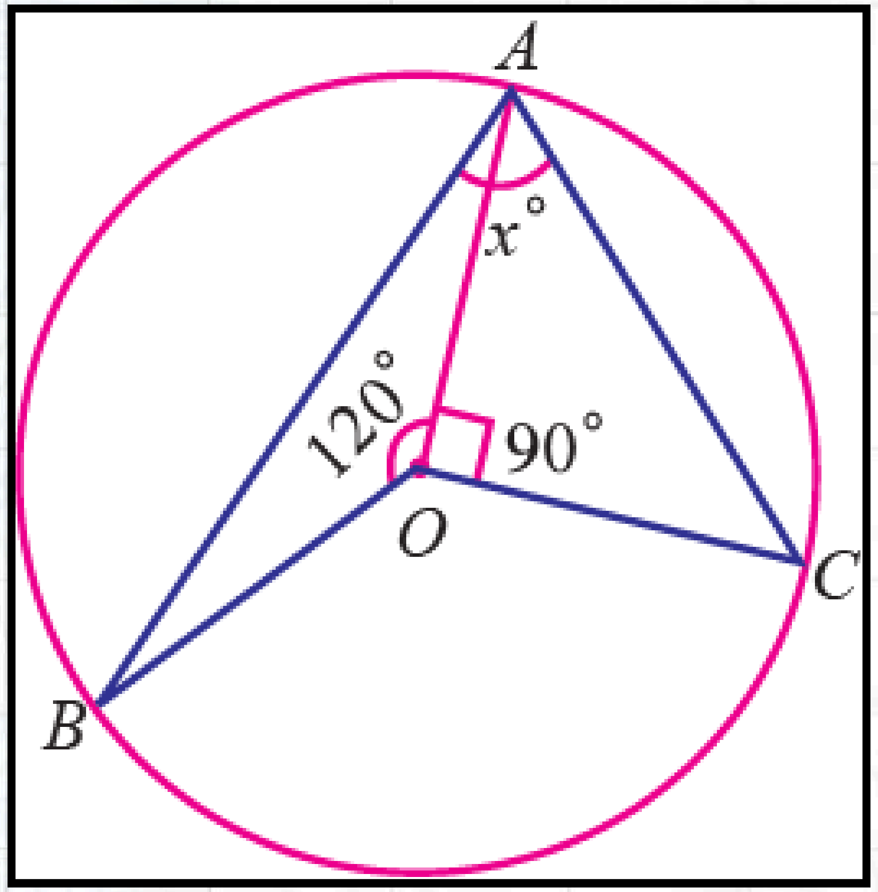


An inscribed angle is half of a central angle that subtends the same arc.

ANGLE THEOREMS

INSCRIBED ANGLES

Find the value of x based on the information of the circle centered on point O.



Central angles correspond to the intercepted arcs

$m\angle AOB = arc\ AB = 120^\circ$

$m\angle AOC = arc\ AC = 90^\circ$

Intercepted arc equals twice the inscribed angle

$m\angle BOC = 2x$

The arcs on a circle add up to 360°

$120^\circ + 90^\circ + 2x = 360^\circ$

$210^\circ + 2x = 360^\circ$

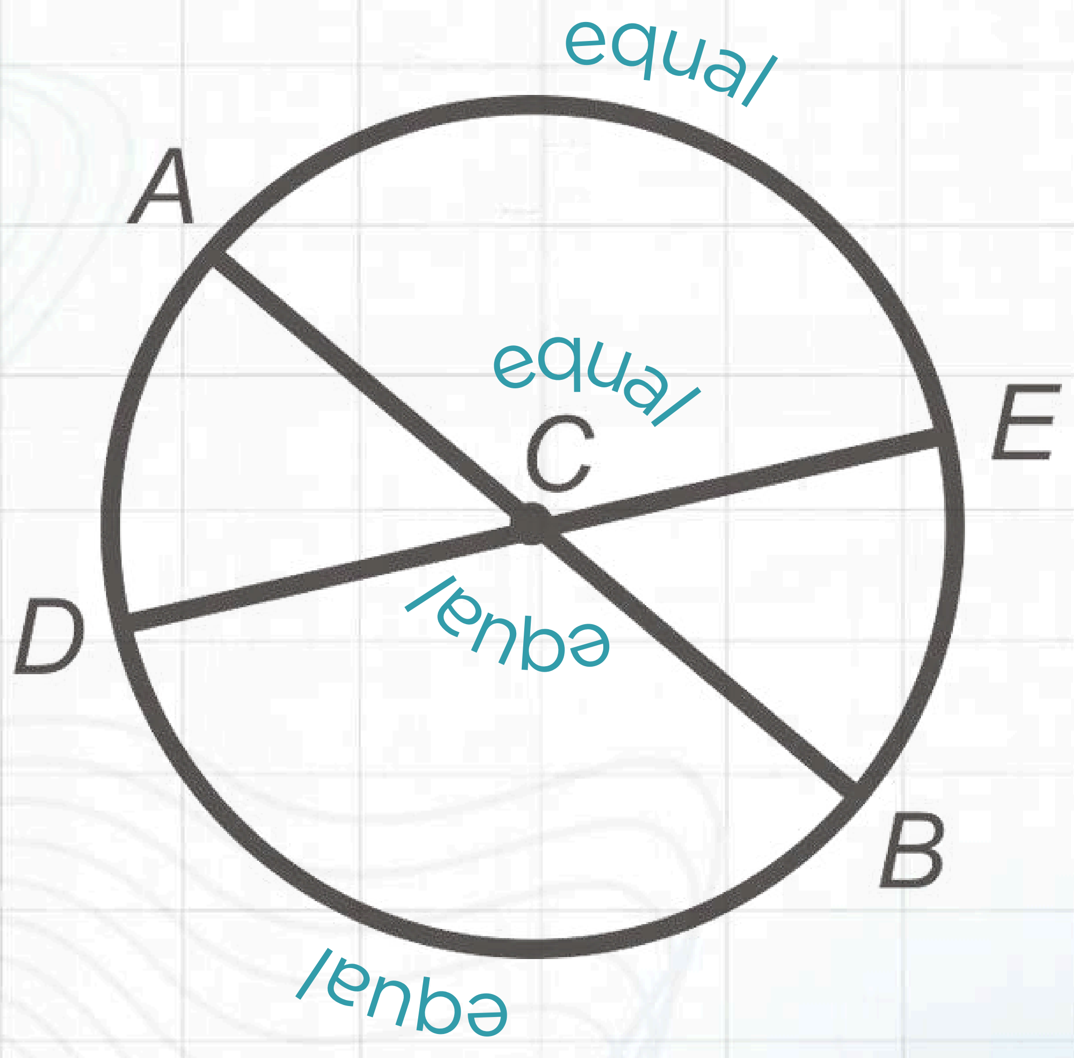
$2x = 150^\circ$

$x = 75^\circ$

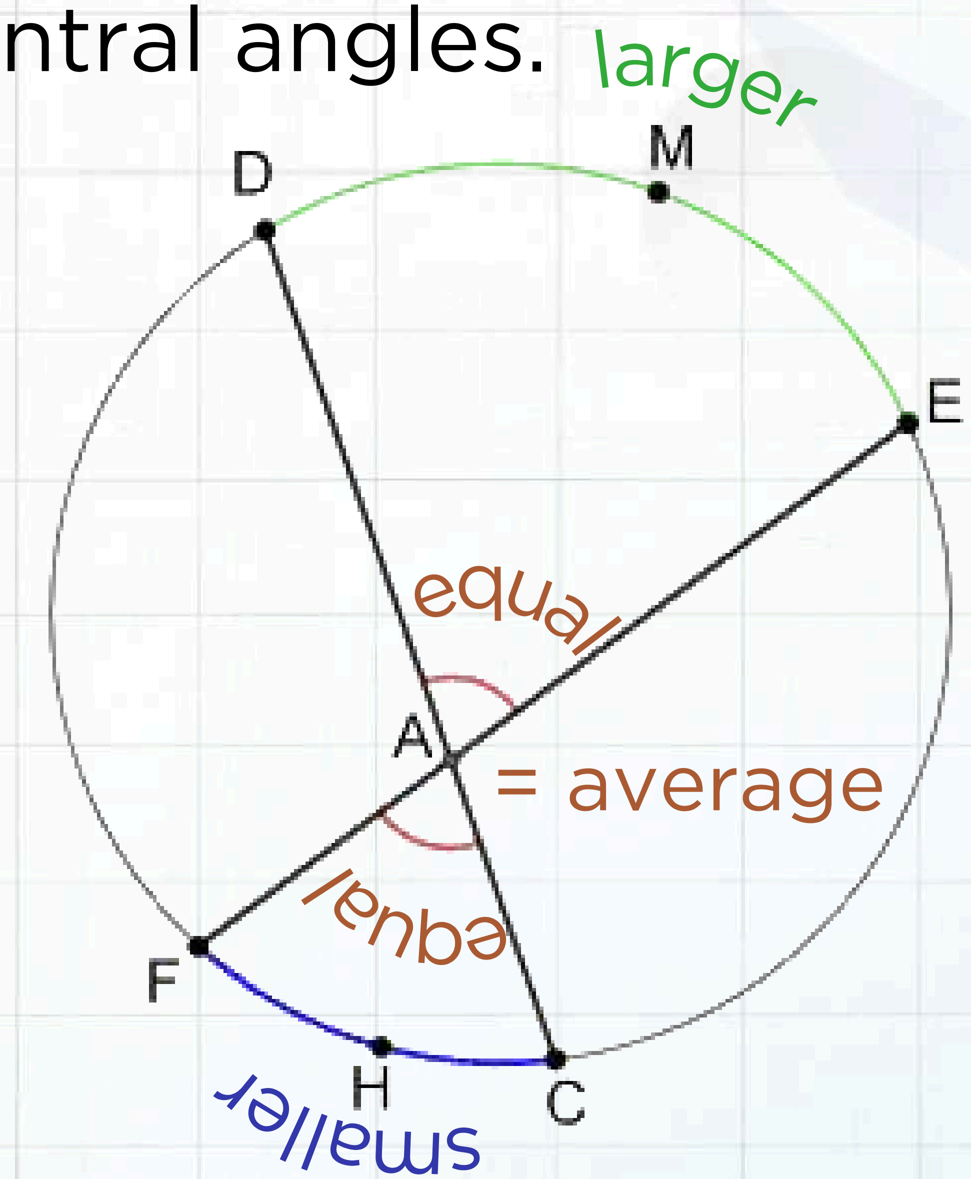
ANGLE THEOREMS

CENTRAL ANGLES & INTERCEPTED ARCS

If the chords pass through the center of the circle, we can say that the intercepted arcs and the central angles all have the same measure



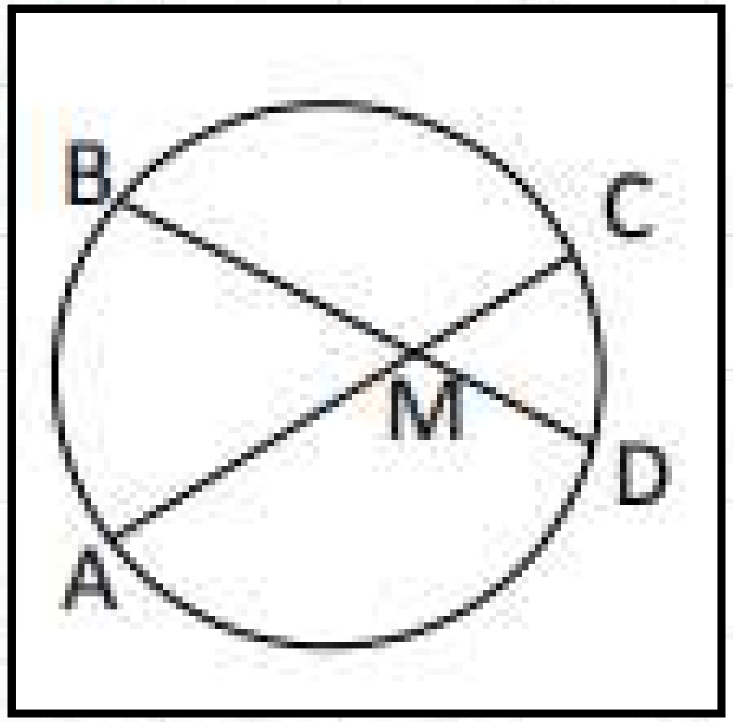
If the chords don't pass through the center of the circle, then the average of the intercepted arc's measures equals the measures of the central angles.



ANGLE THEOREMS

CENTRAL ANGLES & INTERCEPTED ARCS

Arc CD measures (x - 3)°, while arc AB measures (3x + 11)°. What algebraic expression represents the measure of angle CMD?



Average the intercepted arcs of the two central angles (not centered)

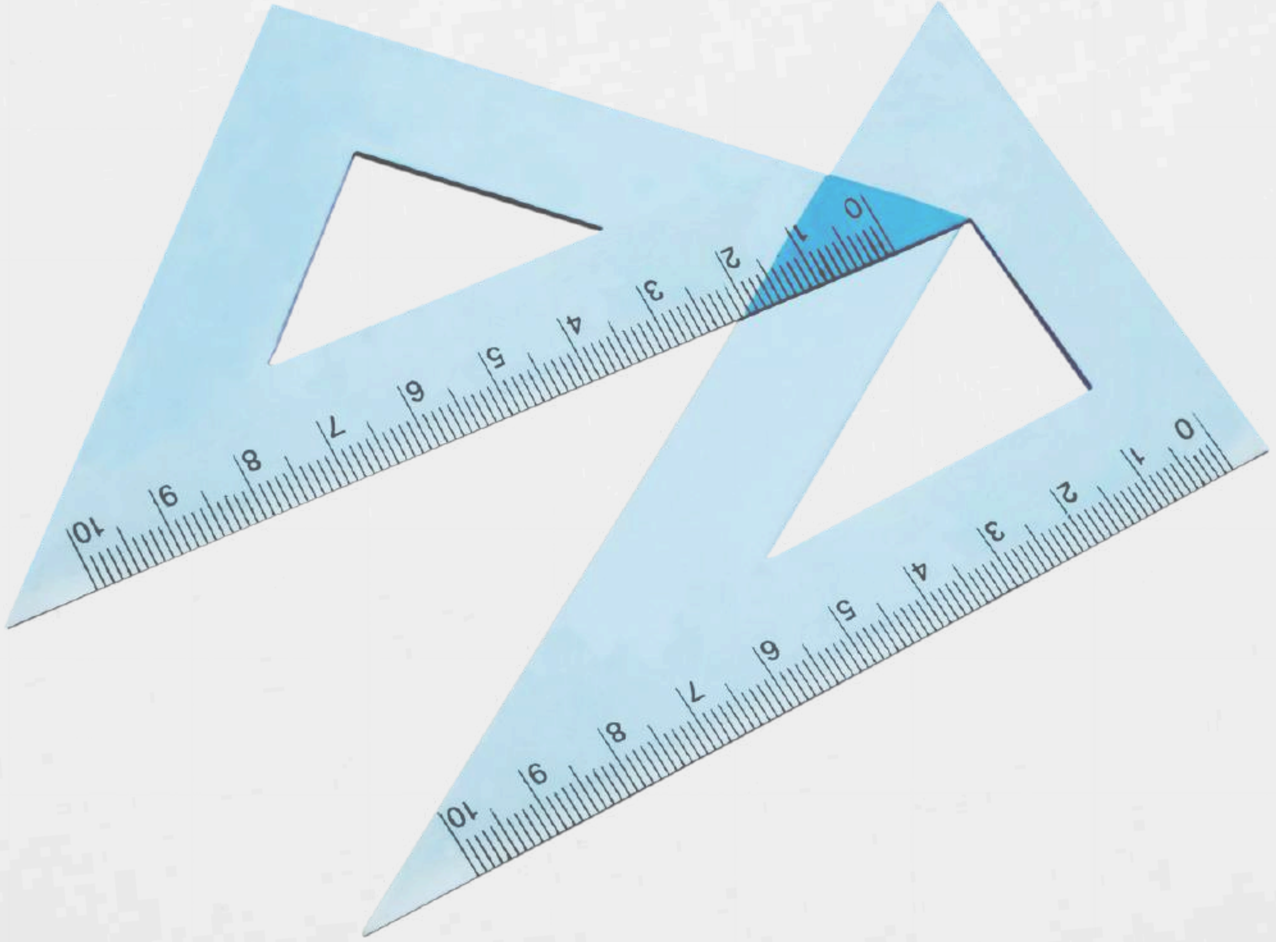
$$\angle CMD = \frac{1}{2}(3x + 11 + x - 3)$$

$$\angle CMD = \frac{1}{2}(4x + 8)$$

$$\angle CMD = 2x + 4$$

GEOMETRY

TRIANGLE POSTULATES



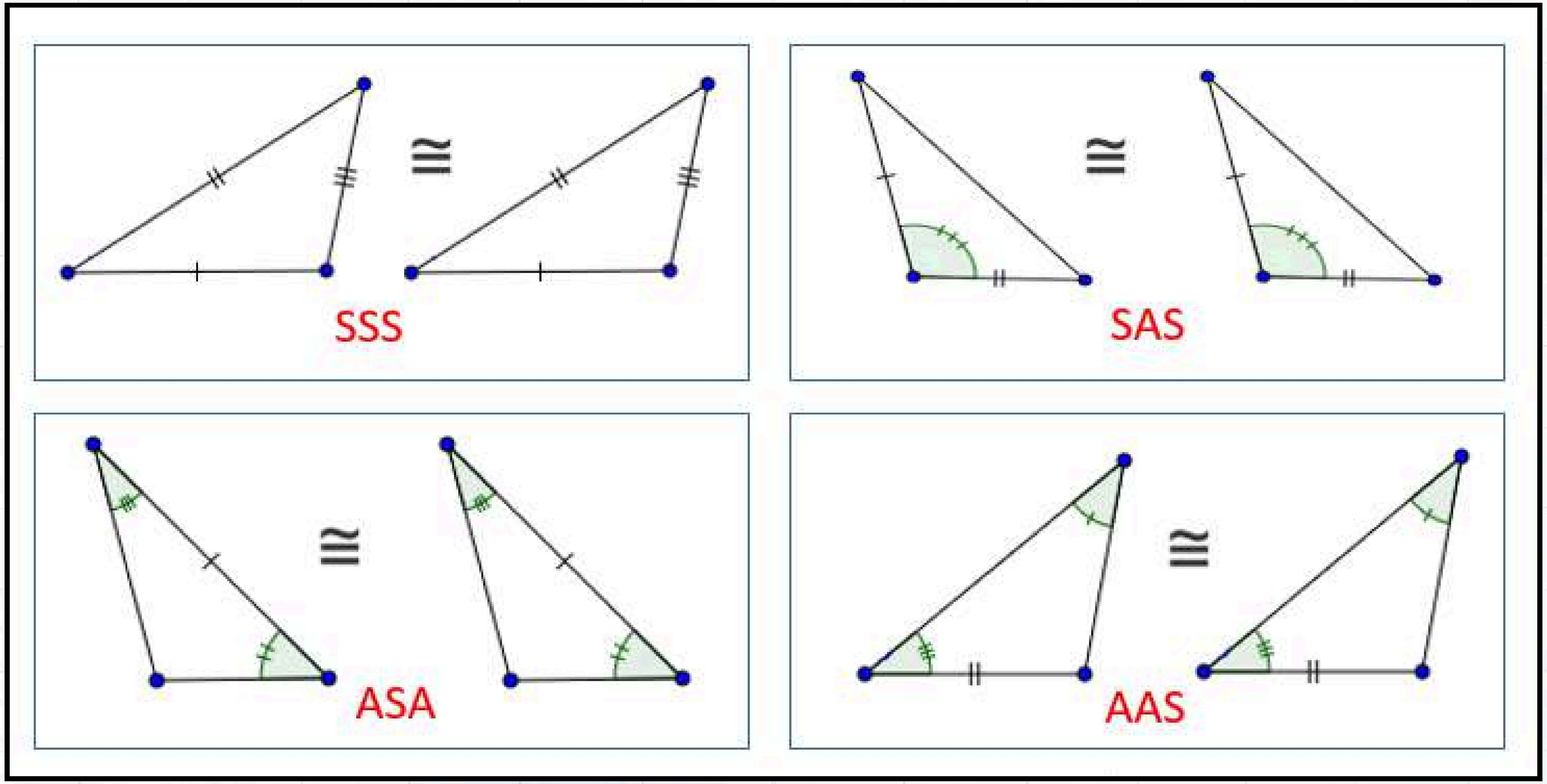
A **postulate** refers to a geometric statement that establishes conditions under which two triangles can be proven congruent or similar.

Congruence means two shapes are identical in size and shape, while **similarity** means two shapes are the same shape but different in size

TRIANGLE POSTULATES

CONGRUENCE

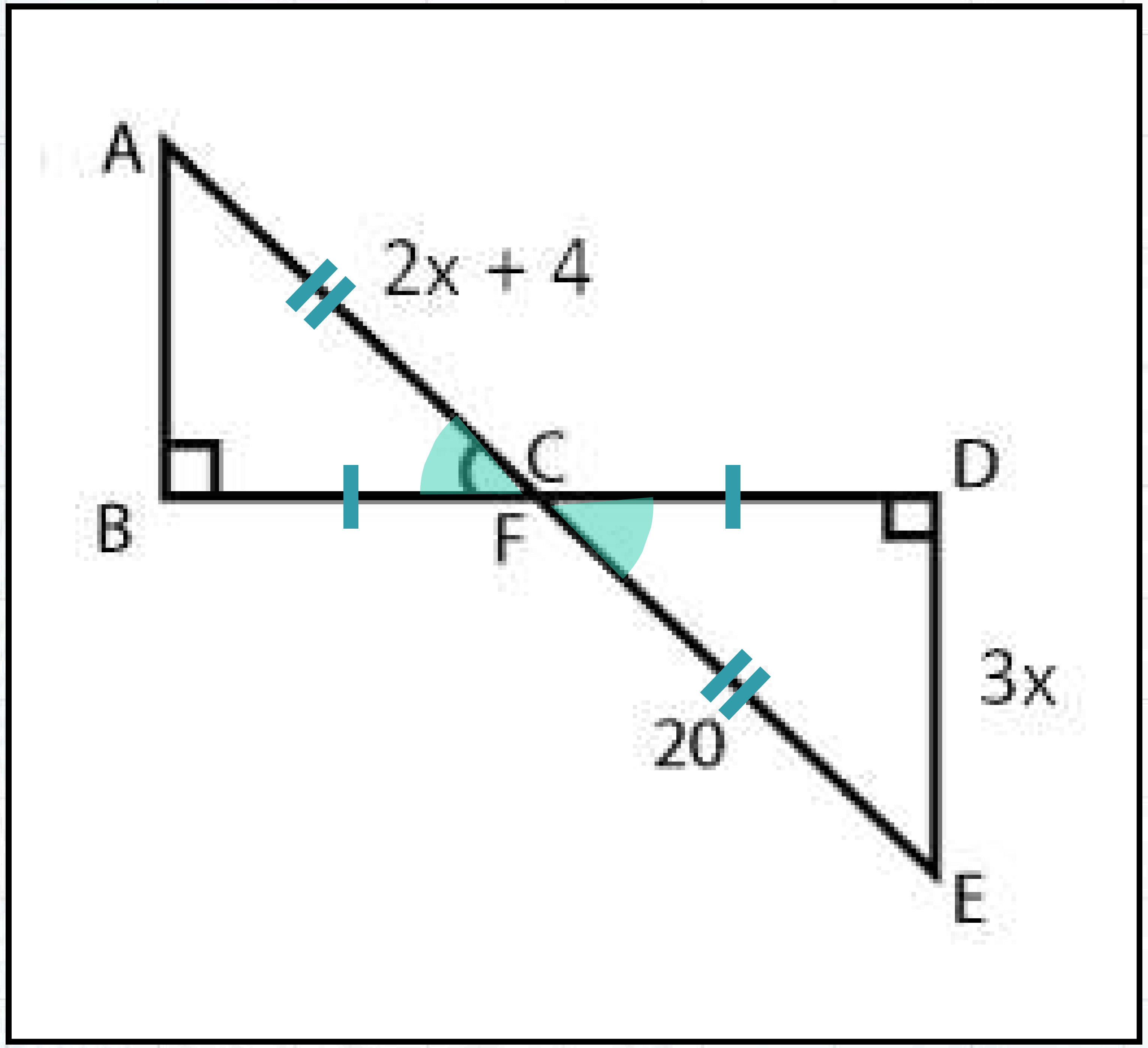
Congruence is a term used to describe when two objects or shapes have the same size and shape. We look at the **equal sides and angles** to determine this. Shapes can be congruent even when translated or rotated.



TRIANGLE POSTULATES

CONGRUENCE

Solve for the length of line segment DE.



$$AF \cong FE \text{ by SAS Congruence}$$

$$2x + 4 = 20$$

$$2x = 16$$

$$x = 8$$

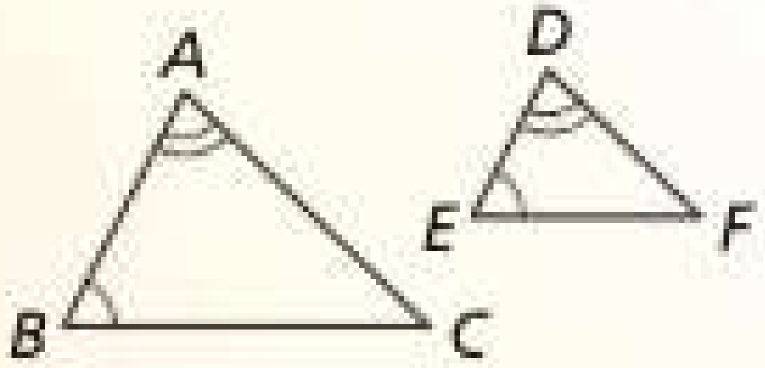
$$3x = 24$$

TRIANGLE POSTULATES

SIMILARITY

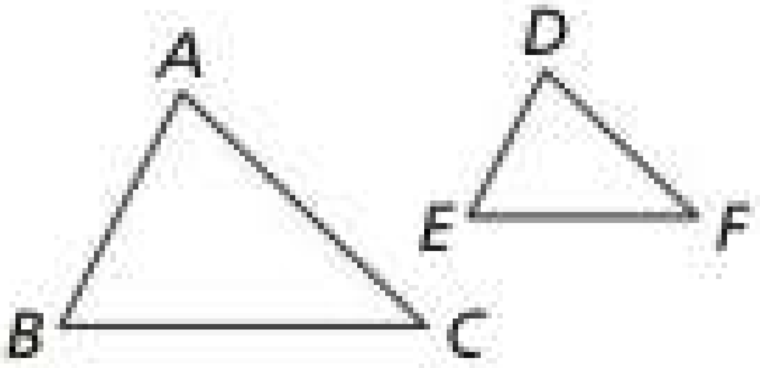
Two triangles are **similar** if they have the **same ratio** of corresponding sides and equal pair of corresponding angles. The idea is that the triangles are **proportional** or that they can scaled to different sizes.

AA Similarity Theorem



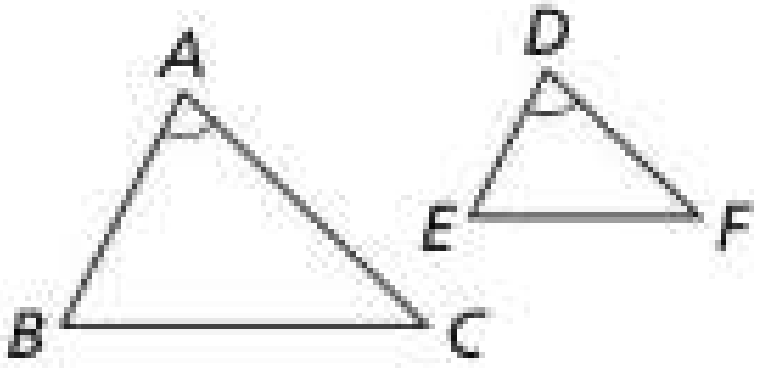
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

These are also called **Triangle Proportionality Theorems**.

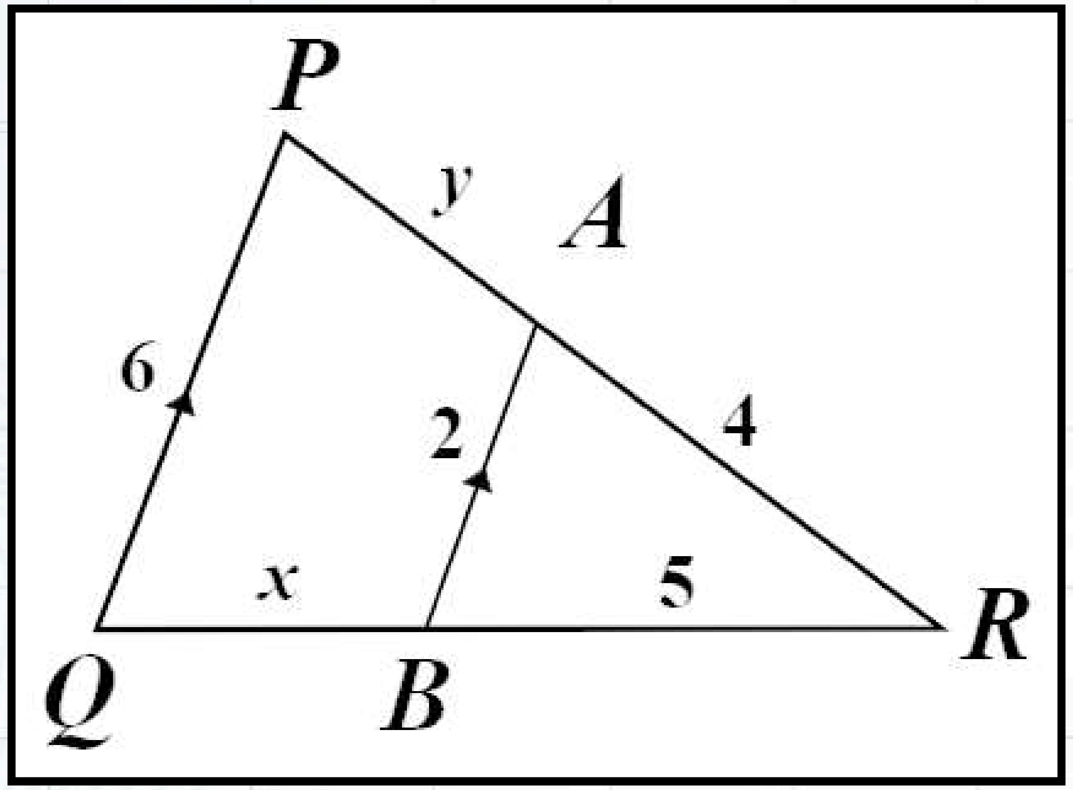
TRIANGLE POSTULATES

SIMILARITY

Triangle Proportionality Theorem

$$\frac{QR\ (big)}{BR\ (small)} = \frac{PR\ (big)}{AR\ (small)} = \frac{PQ\ (big)}{AB\ (small)}$$

Solve for x and y



$$\frac{x+5}{5} = \frac{6}{2}$$

$2(x + 5) = 5(6)$

$2x + 10 = 30$

$2x = 20$

$x = 10$

$$\frac{y+4}{4} = \frac{6}{2}$$

$2(y + 4) = 4(6)$

$2y + 8 = 24$

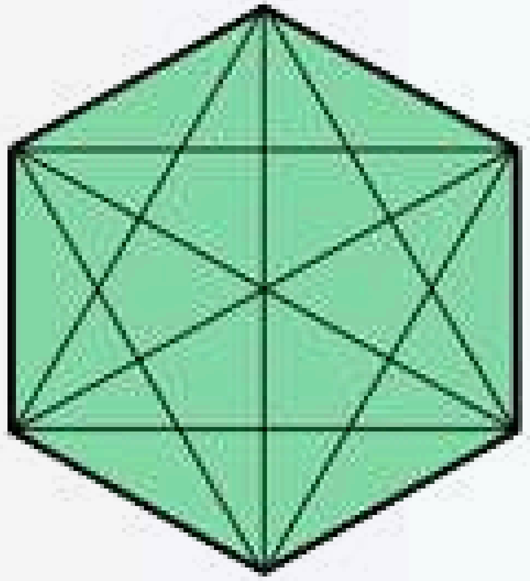
$2y = 16$

$y = 8$

Notice how the big triangle is 3 times bigger because 6/2 = 3.
This is also true for the sides, where 5 scales to 15 because x + 5 = 15
and 4 scales to 12 because y + 4 = 12

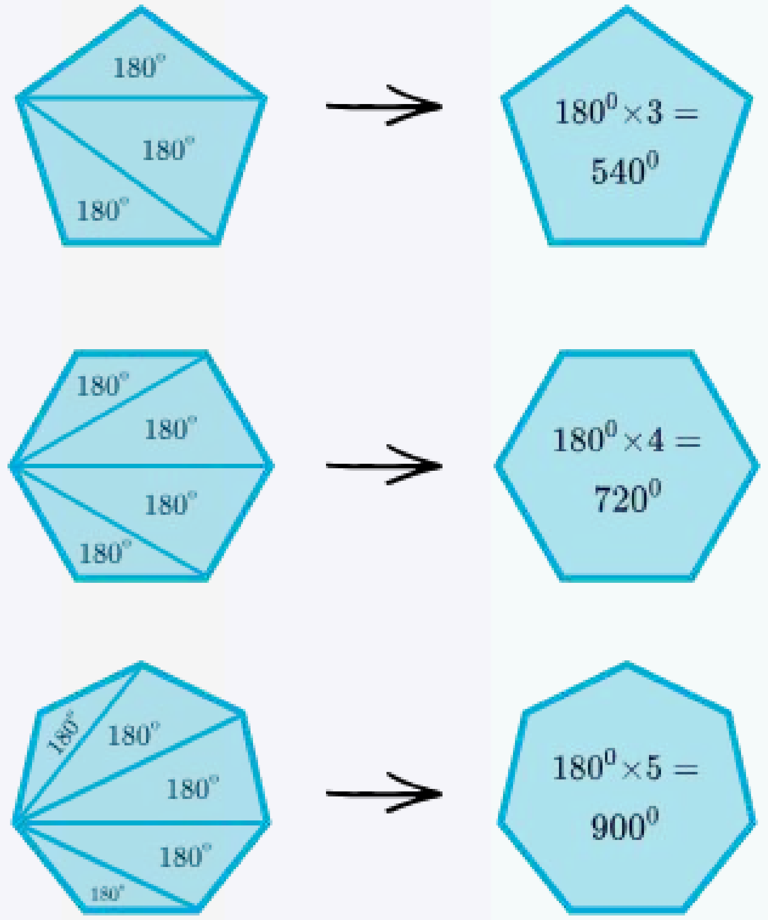
GEOMETRY

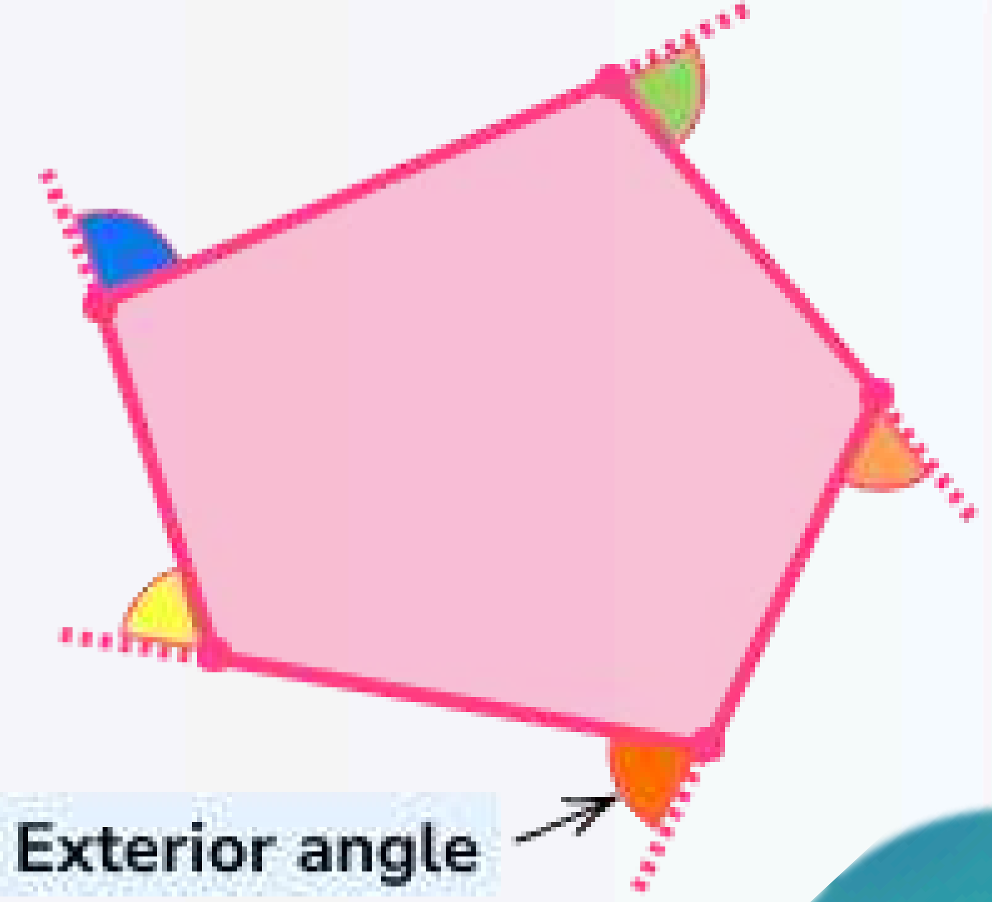
POLYGON PROPERTIES



Hexagon

Has nine diagonals





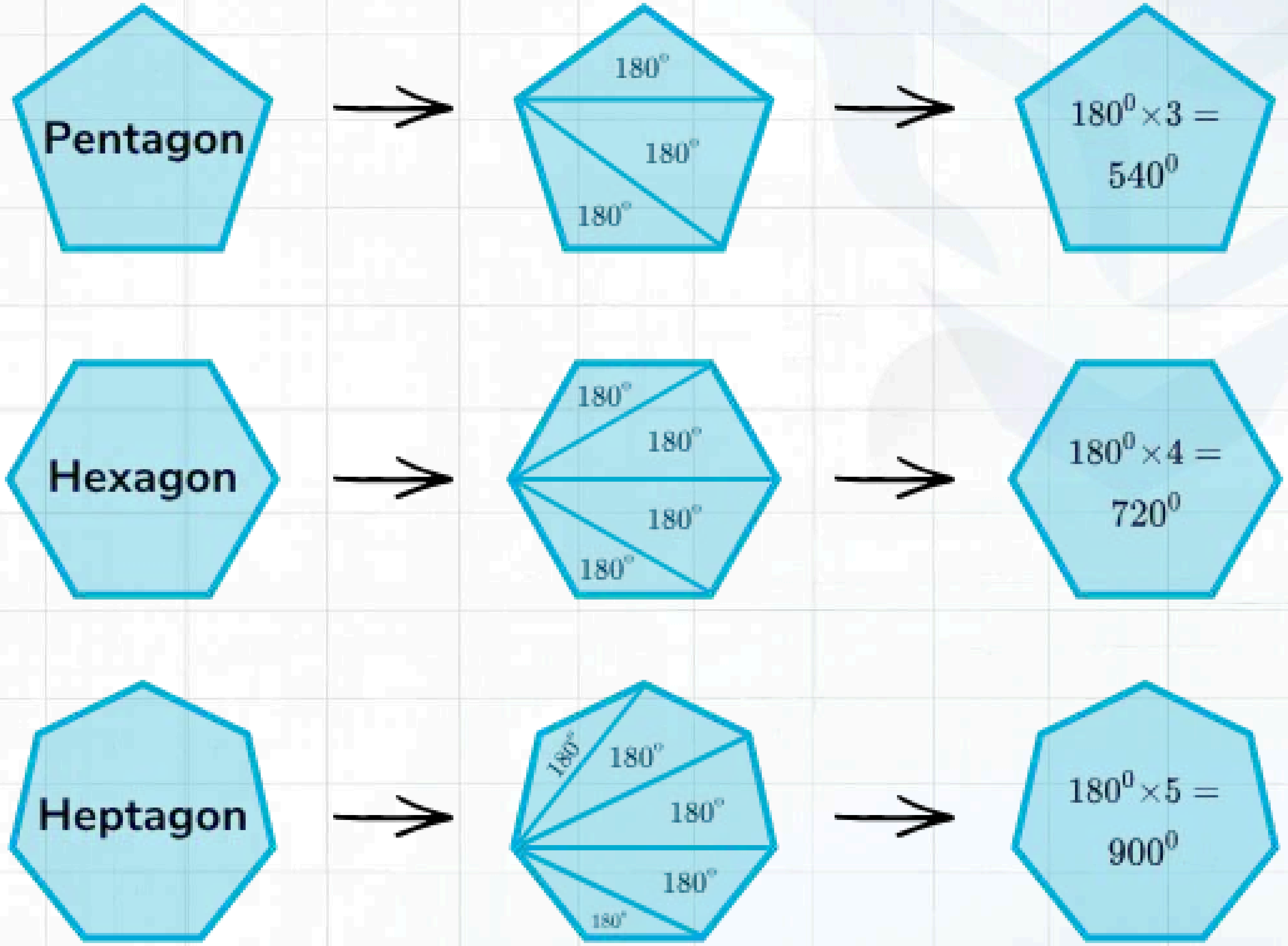
POLYGON PROPERTIES

INTERIOR ANGLES

The **interior angles** in a polygon adds up to:.

S = 180°(n - 2)
S: sum of interior angles
n: number of sides

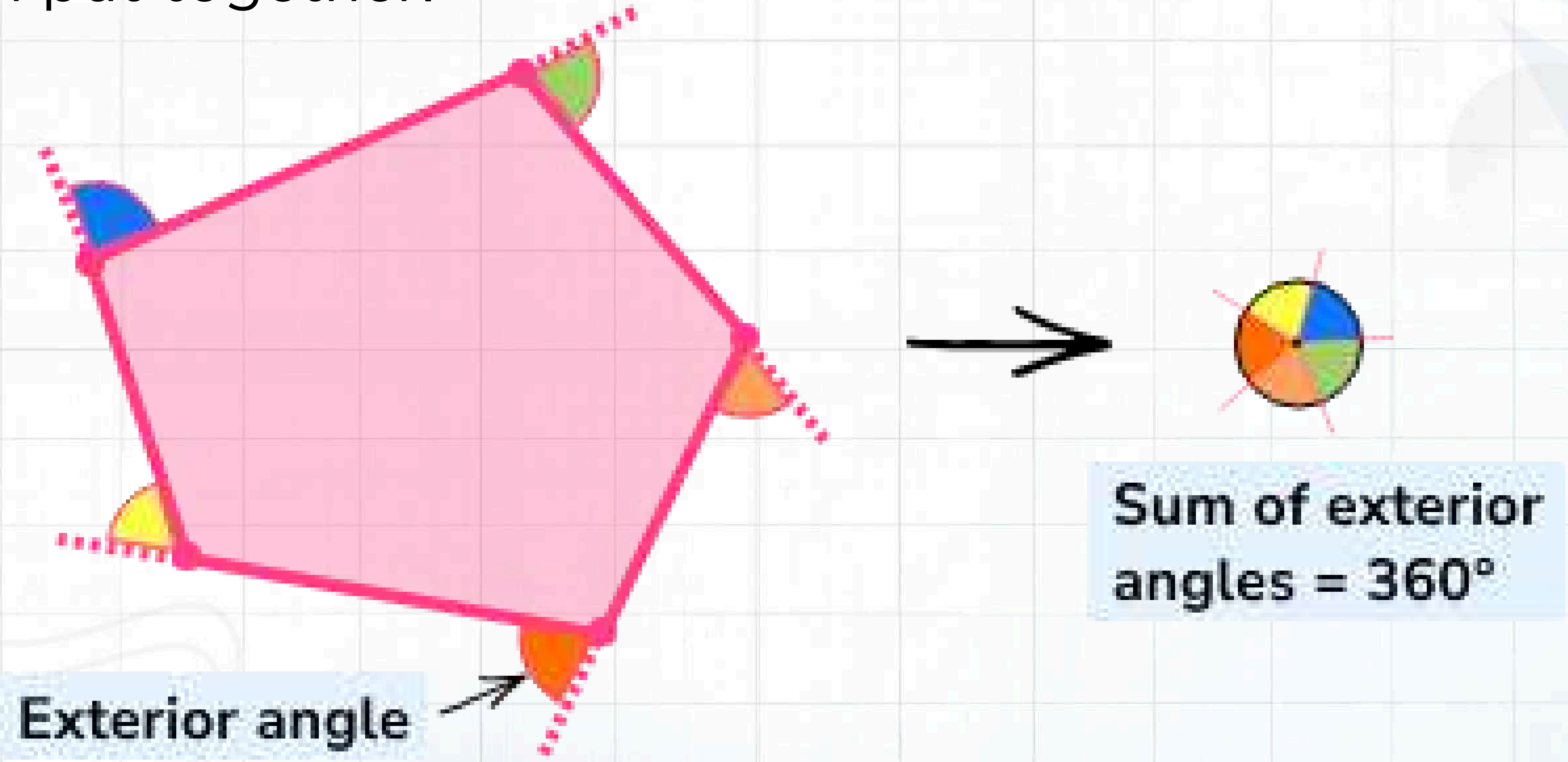
Triangles have 180°, quadrilaterals have 360°, pentagons have 540°, and so on. We can visualize this by **dividing the shape into smaller triangles**!



POLYGON PROPERTIES

EXTERIOR ANGLES

The **exterior angles** and interior angles are supplementary. You can find the exterior angle by drawing past the polygon as shown below. The **sum of exterior angles is always 360°** for convex polygons because they form a circle when put together.

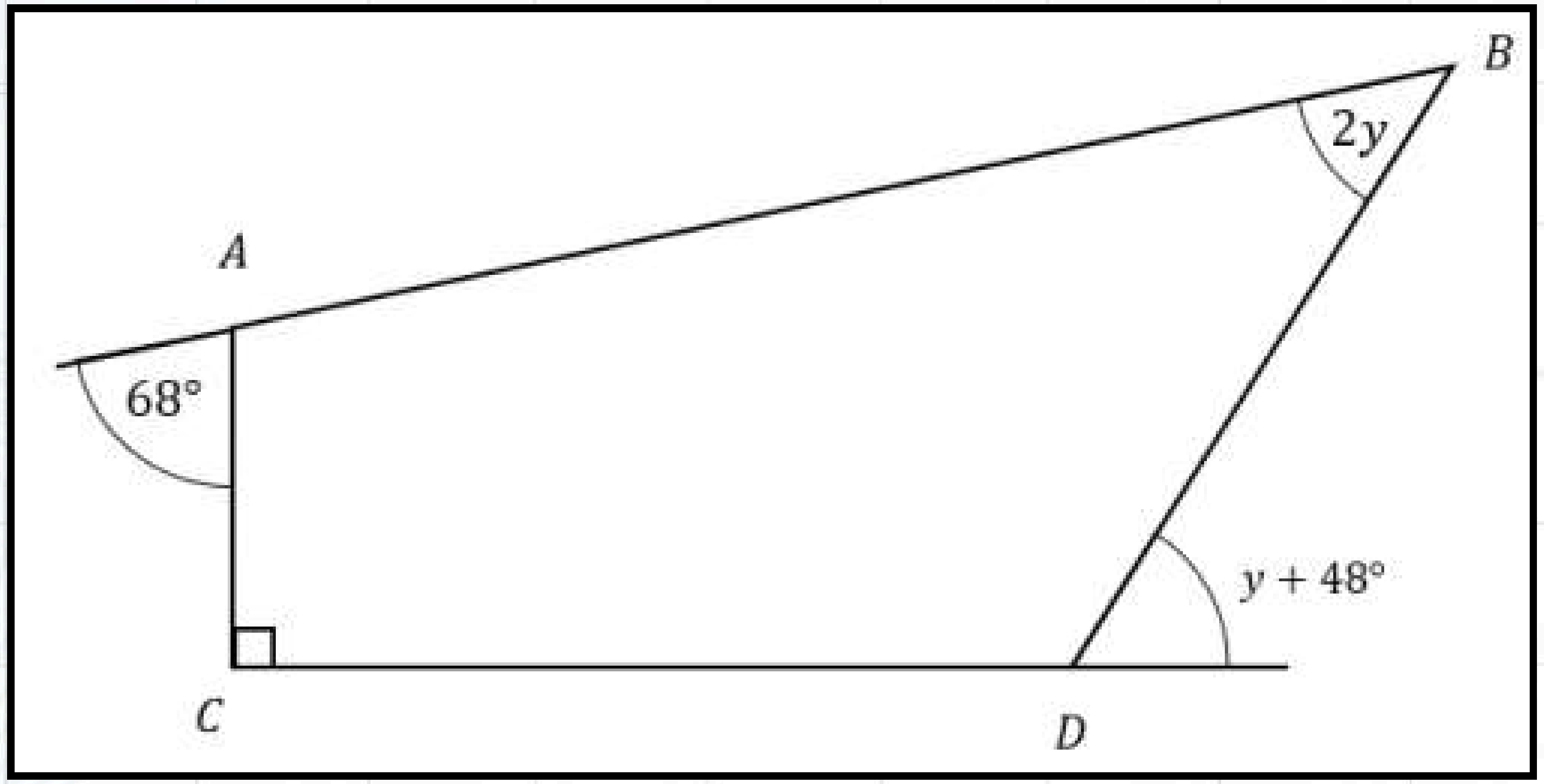


POLYGON PROPERTIES

INTERIOR & EXTERIOR ANGLES

Solve for angle B.

Method 1: Interior Angles



Method 1: Internal Angles

Find the internal angle for each external angle

$180° - 68° = 112°$

$180° - (y + 48°) = 132° - y$

Interior angles in a quadrilateral add up to 360°

$112° + 2y + 132° - y + 90° = 360°$

$y + 334° = 360$

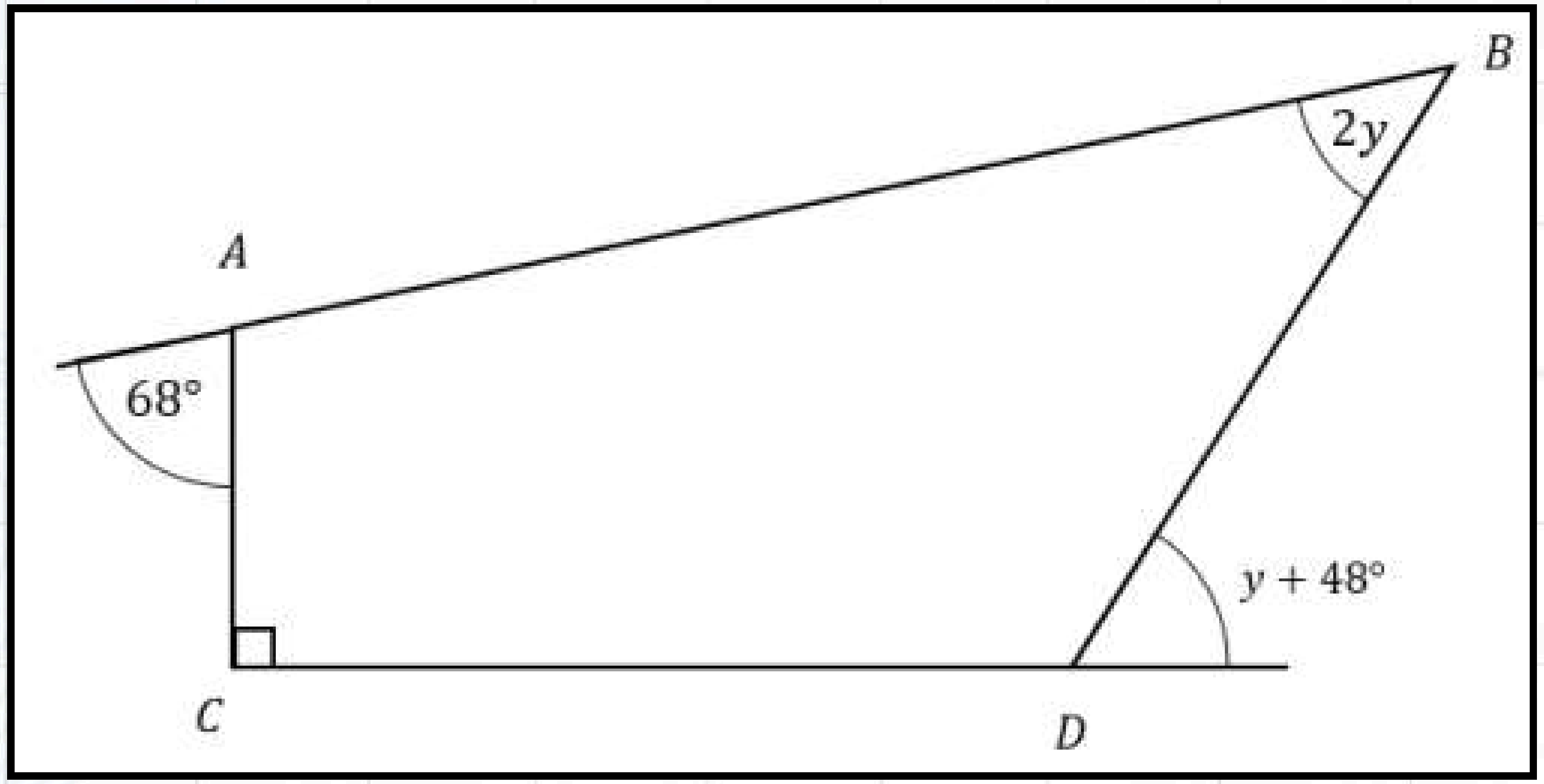
$y = 26°$

POLYGON PROPERTIES

INTERIOR & EXTERIOR ANGLES

Solve for angle B.

Method 2: Exterior Angles



Method 2: External Angles

Find the external angle for each internal angle

$180° - 90° = 90°$

$180° - 2y = 180° - 2y$

Exterior angles of a polygon add up to 360°

$68° + 180° - 2y + y + 48° + 90 = 360°$

$-y + 386° = 360$

$-y = -26°$

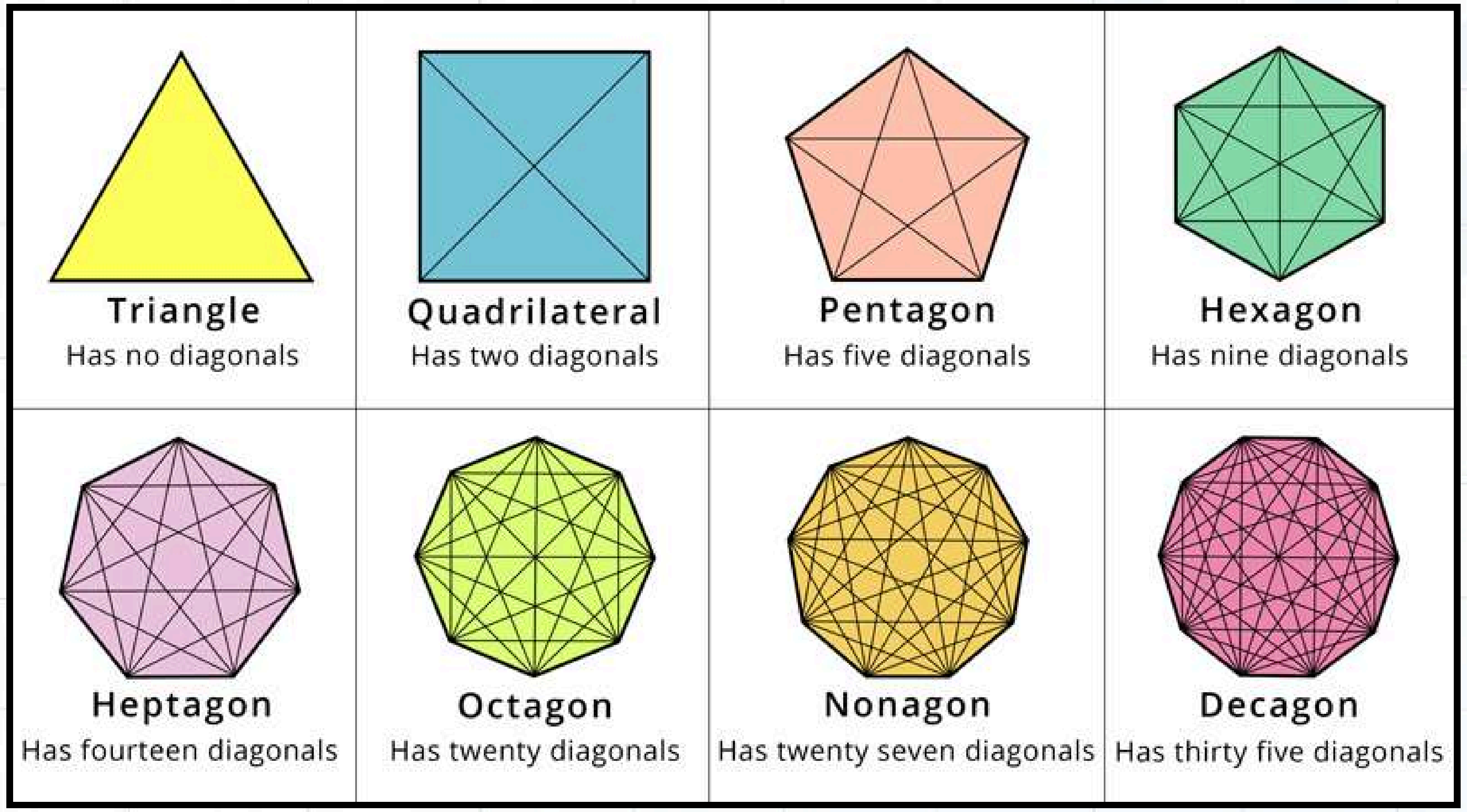
$y = 26°$

POLYGON PROPERTIES

NUMBER OF DIAGONALS

The number of diagonals depends on the number of sides a polygon has.

$$d = \frac{n(n-3)}{2}$$

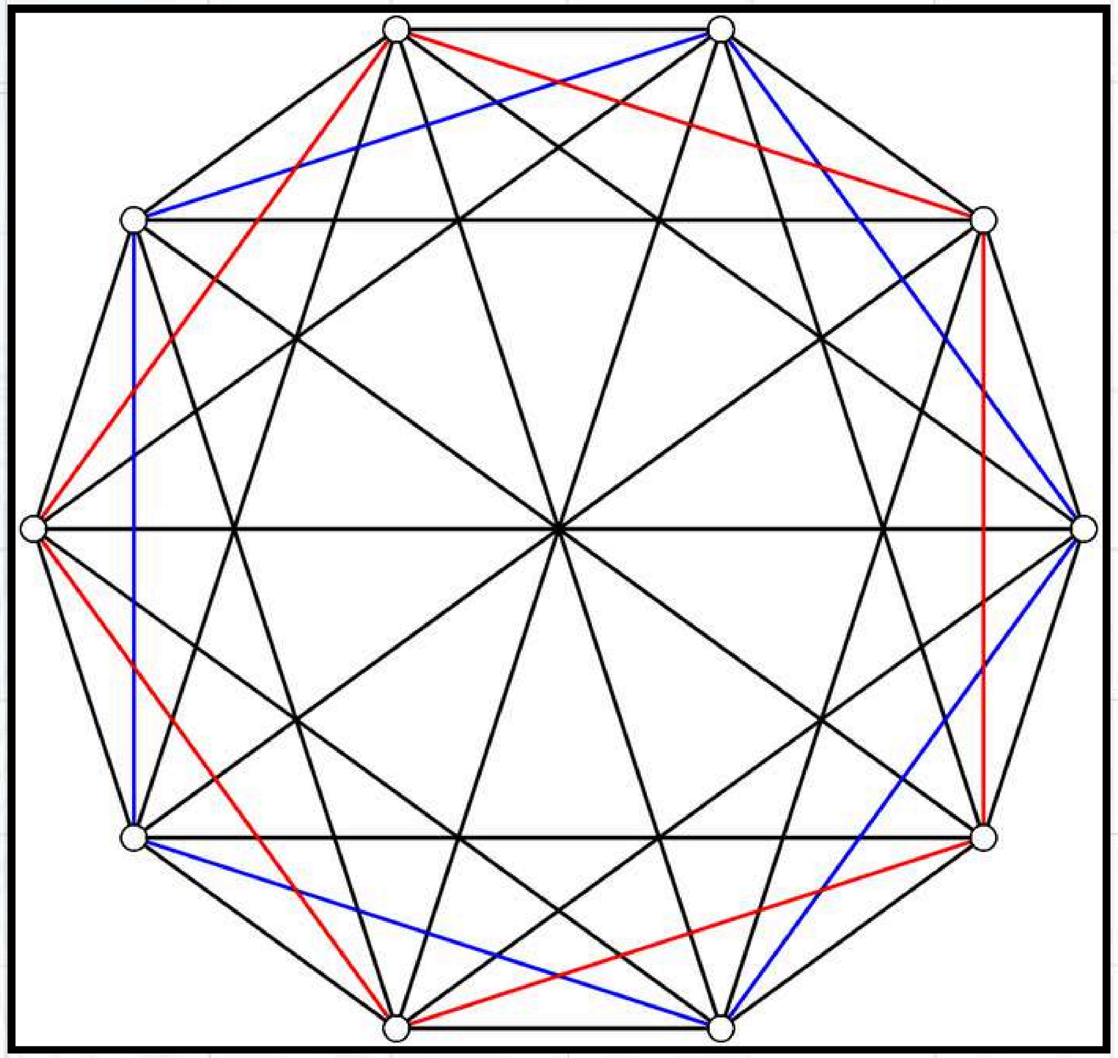


Imagine yourself as a vertex. You cannot draw a diagonal to yourself or the two vertices beside you, so that is why there is a -3. We then divide by 2 because we need a "partner" on the other end of the line.

POLYGON PROPERTIES

NUMBER OF DIAGONALS

How many diagonals does this polygon have?



$$d = \frac{n(n-3)}{2}$$

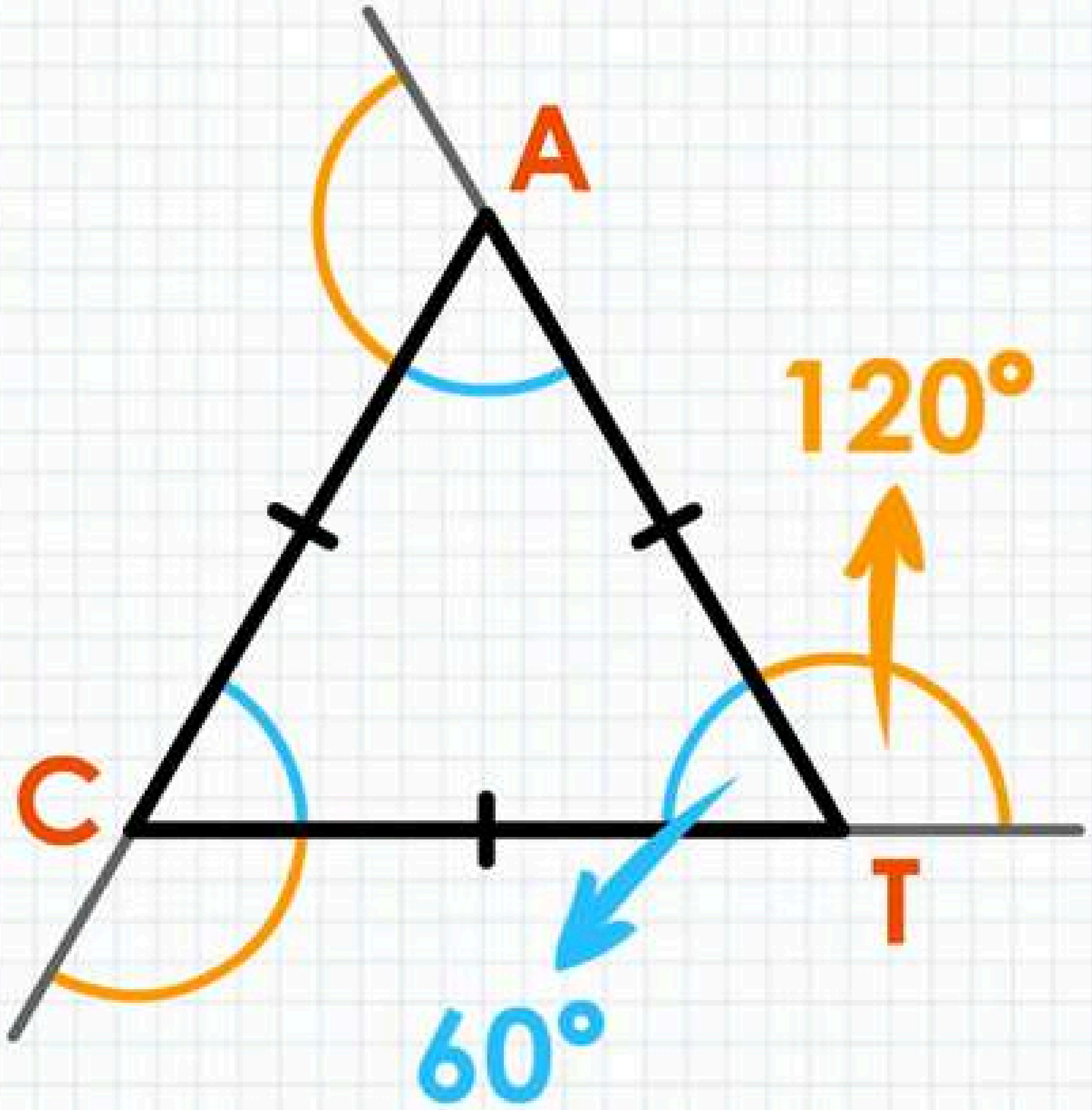
$$d = \frac{10(10-3)}{2}$$

$$d = 5(7)$$

$$d = 35$$

POLYGON PROPERTIES
REGULAR POLYGONS

Regular polygons have congruent sides and angles.

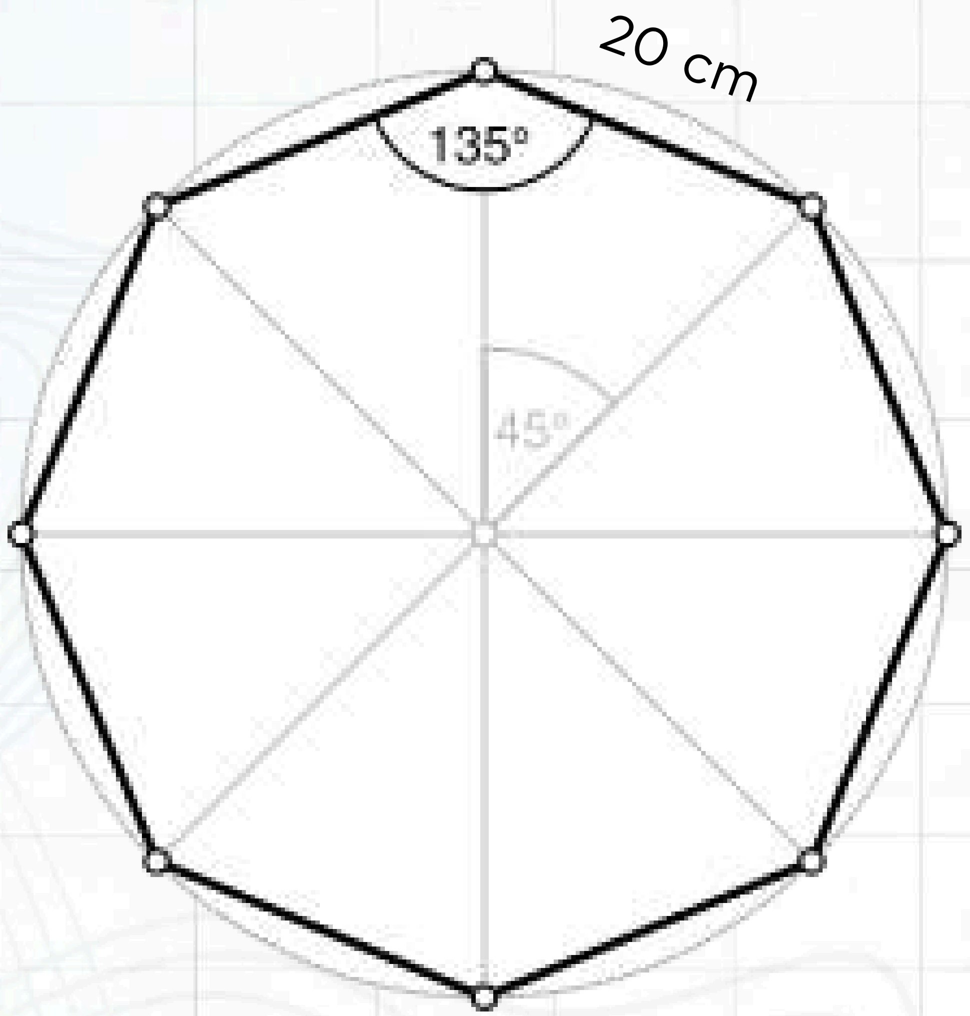


- ✓ Is it two-dimensional?
- ✓ Is it made with straight sides?
- ✓ Are its sides congruent?
- ✓ Does it close in a space, creating an interior and exterior?
- ✓ Does it have equal interior angles?
- ✓ Does it have equal exterior angles?

POLYGON PROPERTIES

REGULAR POLYGONS

The interior angle of a regular polygon measures 135°. If its side length is 20cm, what is its perimeter?



Use the sum of and the measure of the interior angle(s) to find the number of sides

$$\frac{180°(n-2)}{n} = 135°$$

$$180°n - 360 = 135°n$$

$$45°n = 360°$$

$$n = 8$$

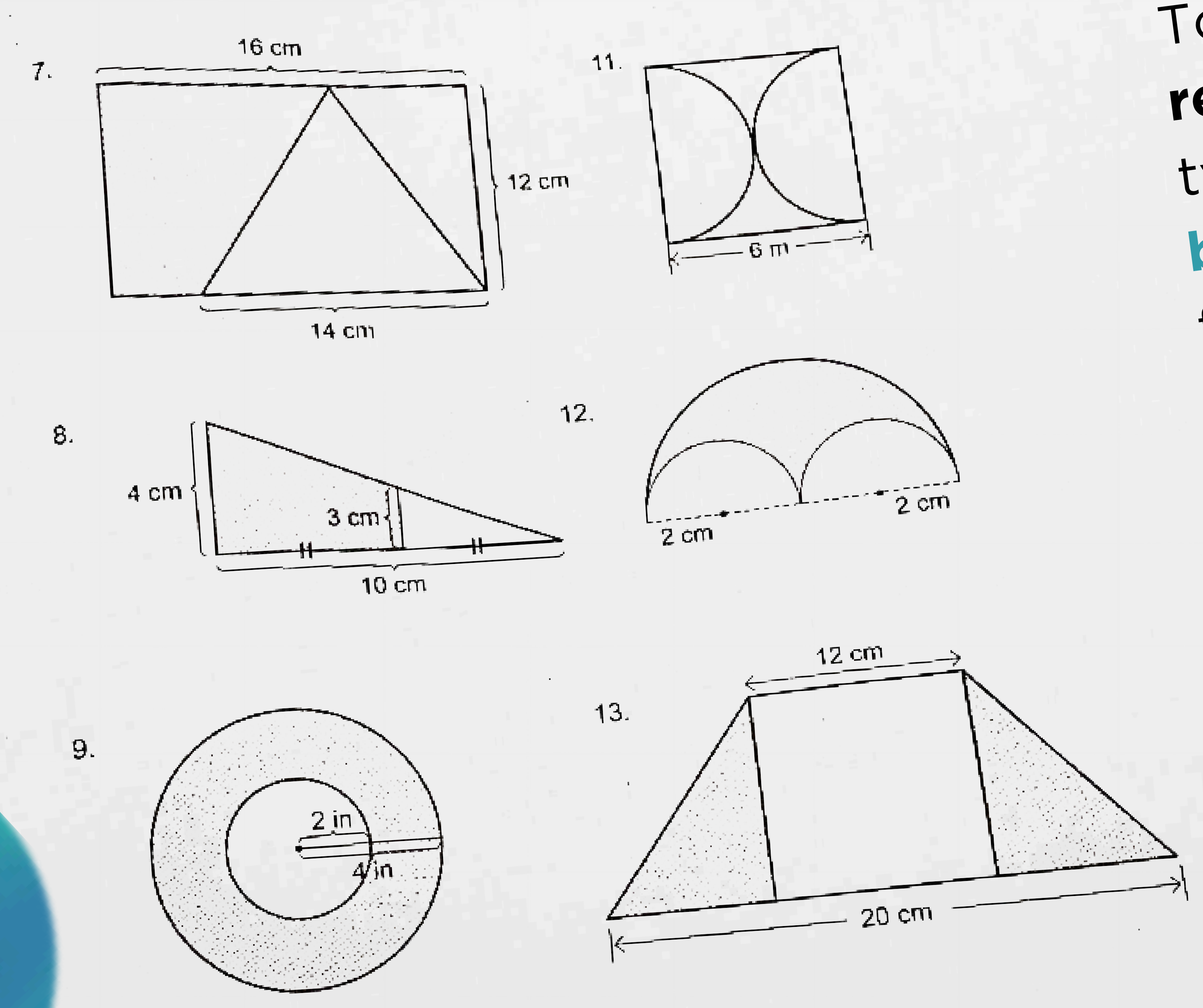
Solve for the perimeter

$$P = 20n\ cm$$

$$P = 20(8)\ cm = 160cm$$

GEOMETRY

AREA OF A SHADED REGION

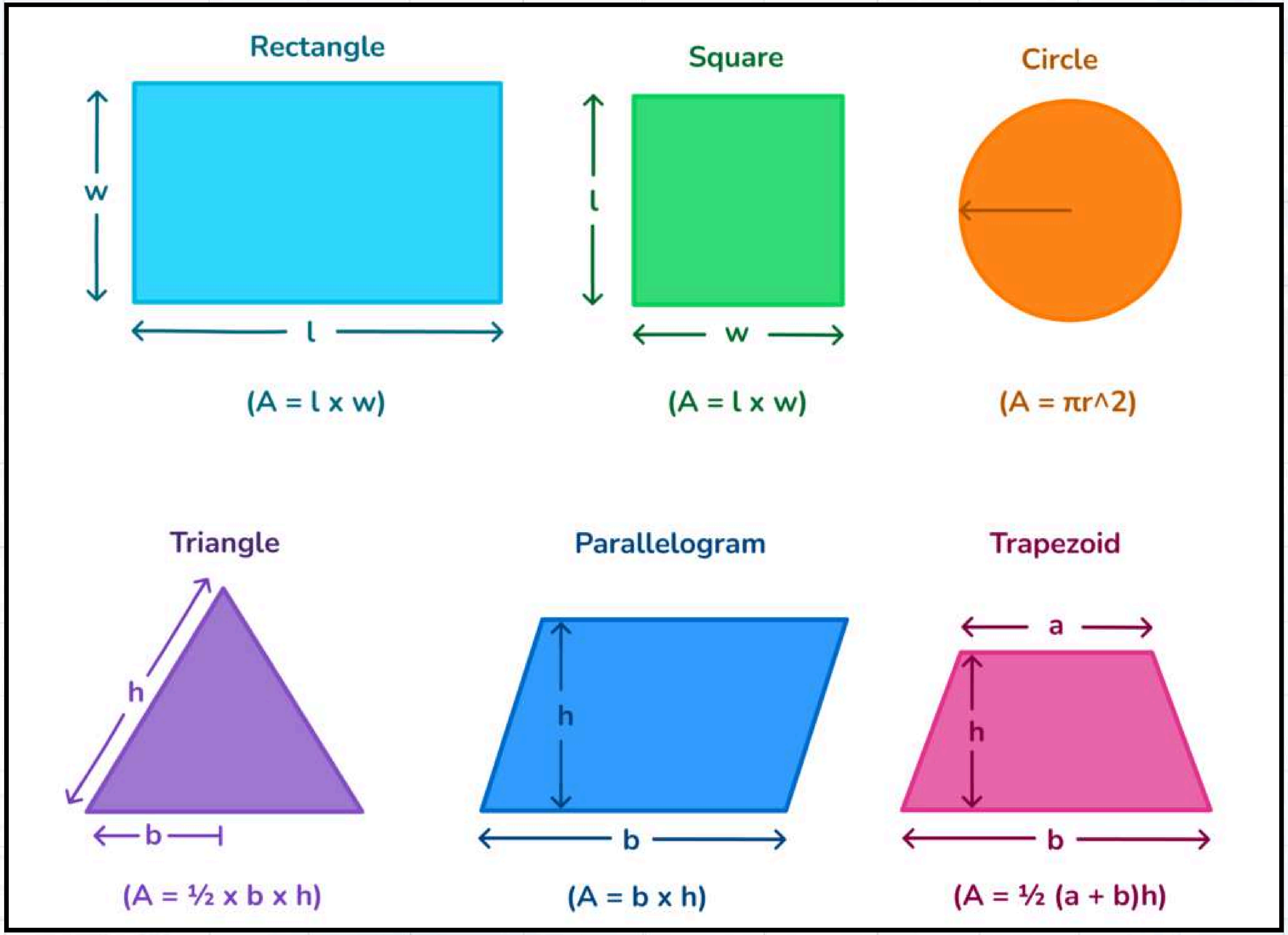


To find the area of a **shaded region** in geometry, you typically calculate the **difference between the area** of the whole figure and the area of the unshaded part.

Alternatively, you can split up a composite figure into **smaller, basic shapes**. This approach lets you solve each part individually so you can sum it up later.

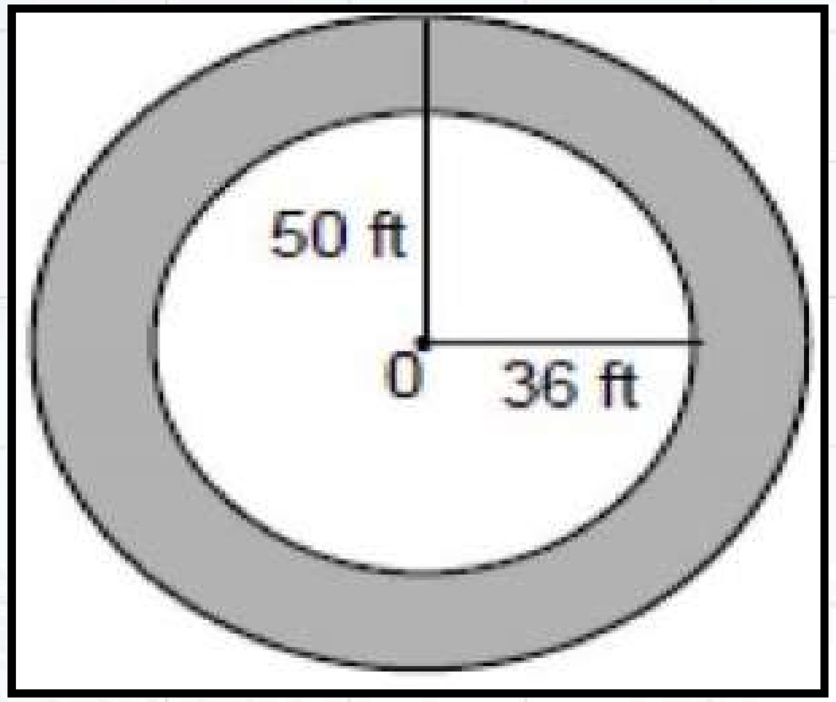
AREA OF A SHADED REGION

BASIC FORMULAS



AREA OF A SHADED REGION

HOLE IN A SHAPE



Subtract the small shape from the big shape

$$A = A_{big} - A_{small}$$

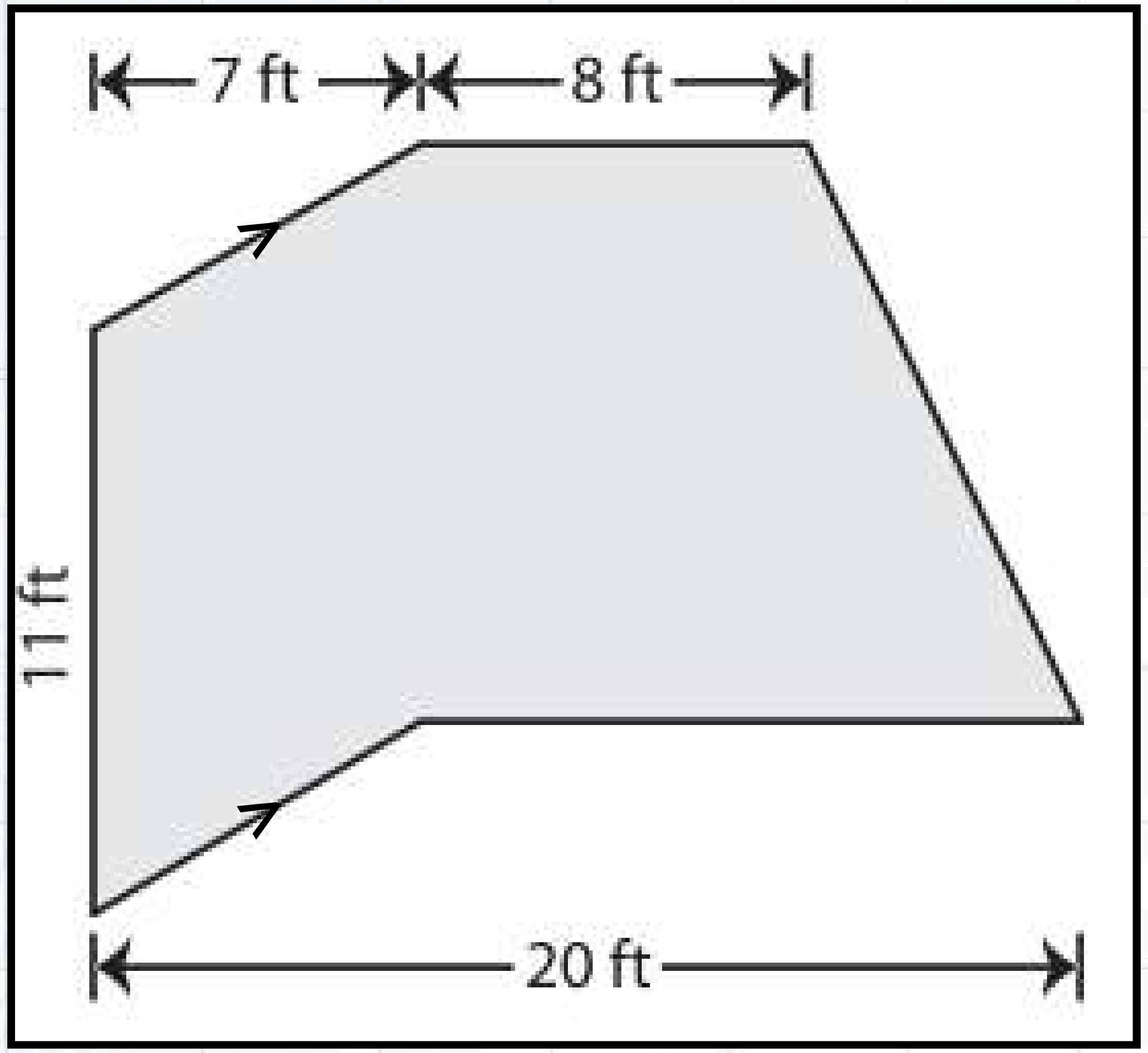
$$A = \pi(50\ ft)^2 - \pi(34\ ft)^2$$

$$A = 2500\pi\ ft^2 - 1156\pi\ ft^2$$

$$A = 1204\pi\ ft^2$$

AREA OF A SHADED REGION

COMPOSITE SHAPES



Split the figure into three basic shapes

$$A = A_{parallelogram} + A_{rectangle} + A_{triangle}$$

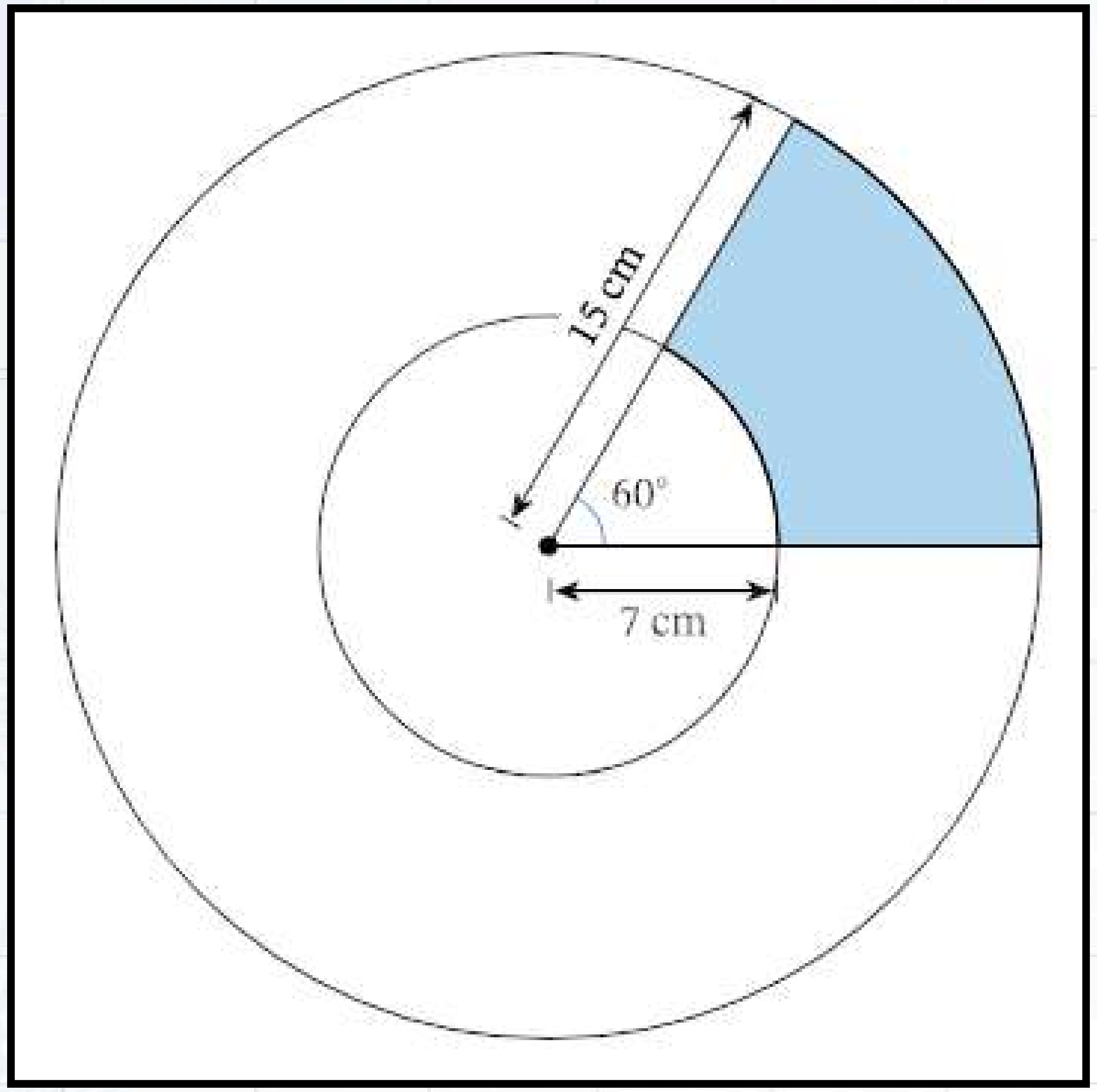
$$A = [\,7(11) + 8(11) + \frac{1}{2}(5)(11)\,]\,ft^2$$

$$A = [\,77 + 88 + 27.5]\,ft^2$$

$$A = 192.5\,ft^2$$

AREA OF A SHADED REGION

SECTOR OF A CIRCLE



Find the area of the ring

$$A_{ring} = A_{big} - A_{small}$$

$$A_{ring} = \pi(15cm)^2 - \pi(7cm)^2$$

$$A_{ring} = 225\pi\ cm^2 - 49\pi\ cm^2$$

$$A_{ring} = 176\pi\ cm^2$$

The sector equals 60/360 of the ring

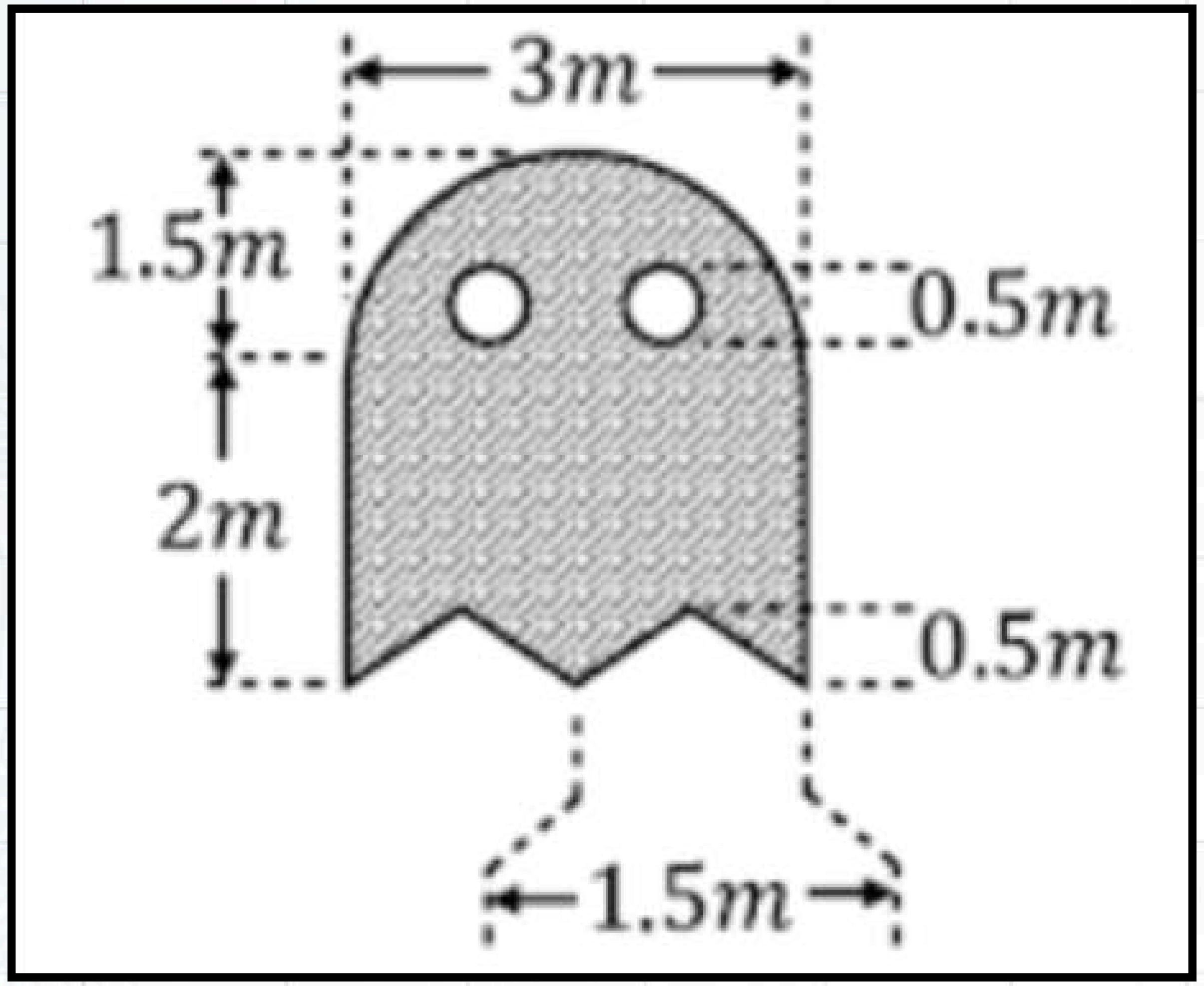
$$A_{sector} = \frac{60°}{360°} A_{ring}$$

$$A_{sector} = \frac{1}{6}(176\pi\ cm^2)$$

$$A_{sector} = 29.33\pi\ cm^2$$

AREA OF A SHADED REGION

ADDITIVE & SUBTRACTIVE COMPONENTS



Break down the figure into additive and subtractive components

$$A_{semicircle} + A_{rectangle} - A_{2\ circles} - A_{2\ triangles}$$

Solve each component separately

$$A_{semicircle} = \frac{1}{2}(\pi)(1.5m)^2 = 1.125\pi\ m^2$$

$$A_{rectangle} = (2m)(3m) = 6\ m^2$$

$$A_{2\ circles} = 2(\pi)(0.25m)^2 = 0.125\pi\ m^2$$

$$A_{2\ triangles} = 2(\tfrac{1}{2})(1.5m)(0.5m) = 0.75\ m^2$$

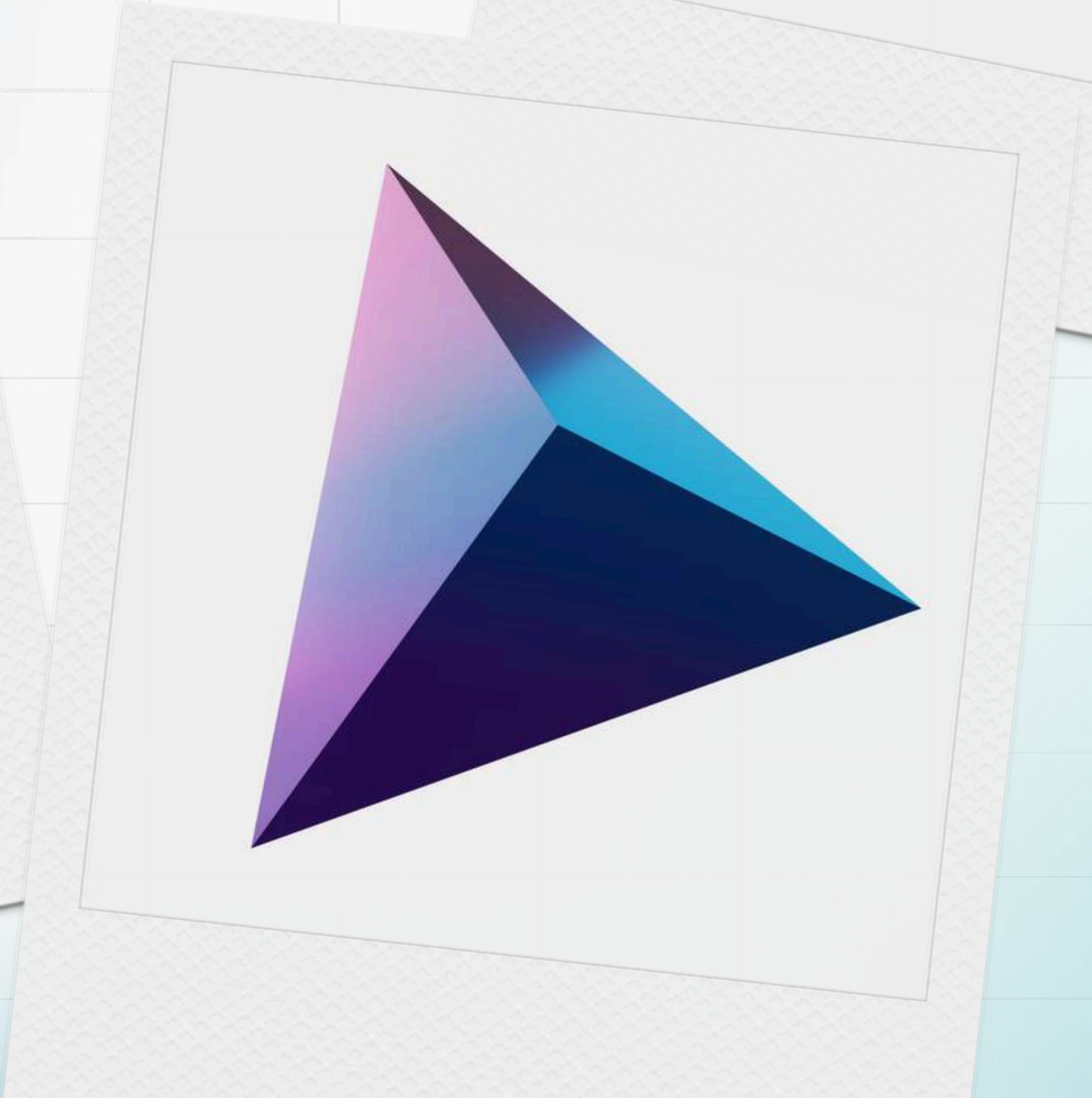
Compute the area of the shaded region

$$(1.125\pi + 6 - 0.125\pi - 0.75)\ m^2$$

$$(1\pi + 5.25)\ m^2$$

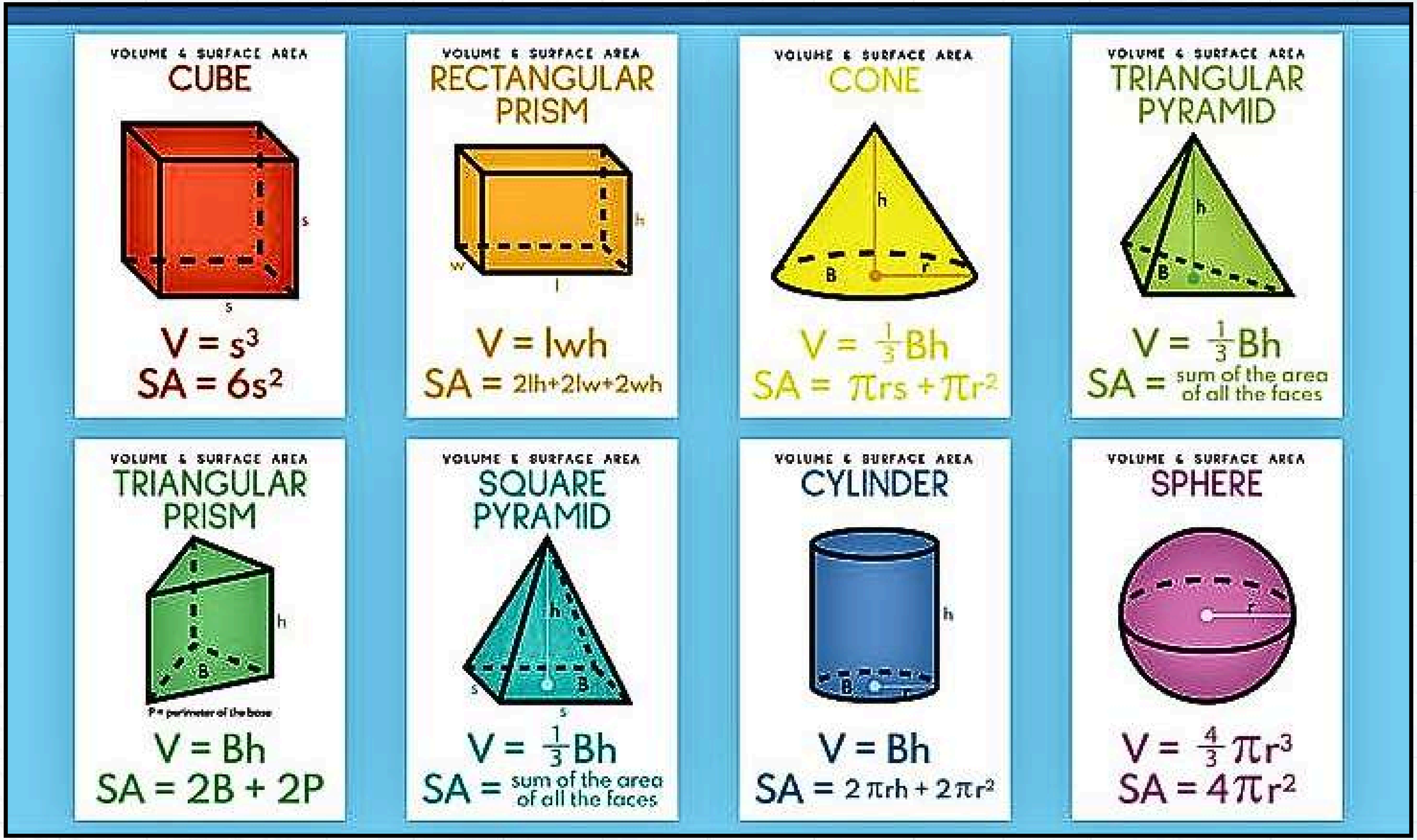
GEOMETRY

SURFACE AREA & VOLUME



SURFACE AREA & VOLUME

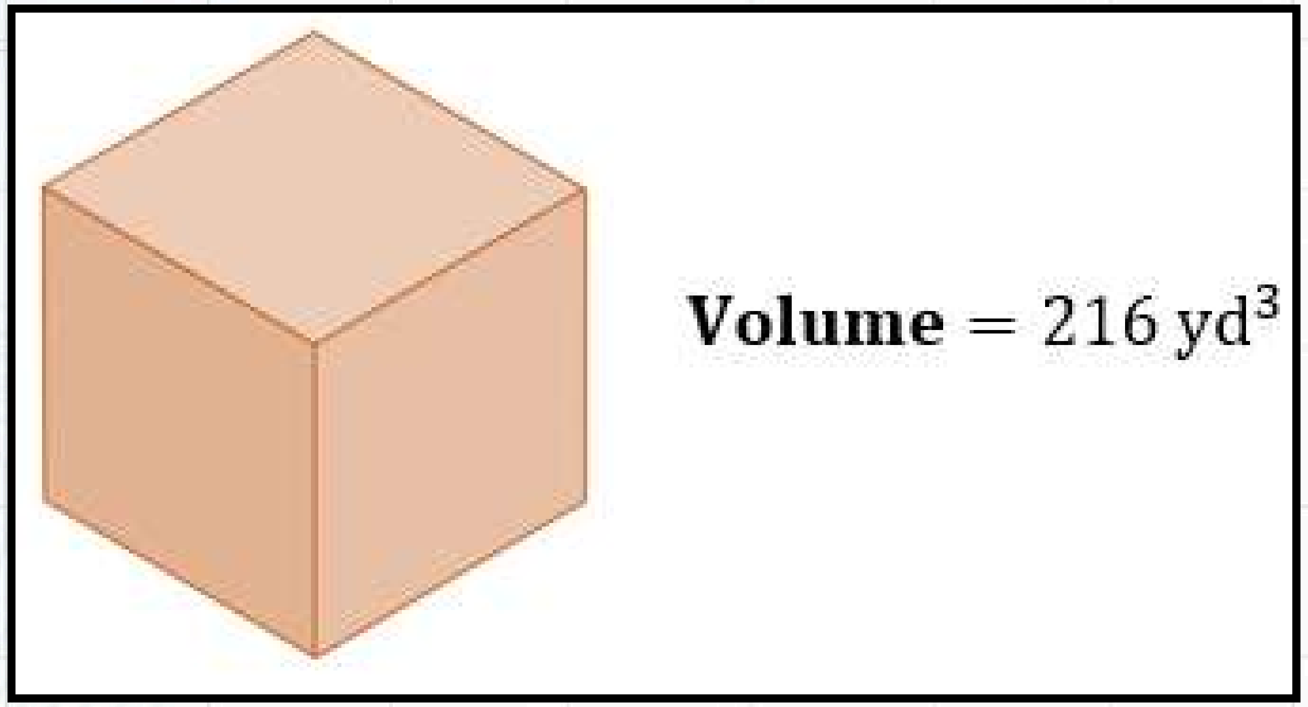
FORMULAS



SURFACE AREA & VOLUME

CUBE

What is the surface area of a cube if its volume is 216 yd^3?



What will happen to the volume if the side length is doubled?

The volume of a cube is the edge cubed, so the cube's edge is the cube root of the volume

$$V = e^3, \quad e = \sqrt[3]{V}$$

$$e = \sqrt[3]{216\ yd^3}$$

$$e = 6\ yd$$

A cube's surface area is made of six square faces

$$SA = 6e^2$$

$$SA = 6(6\ yd)^2$$

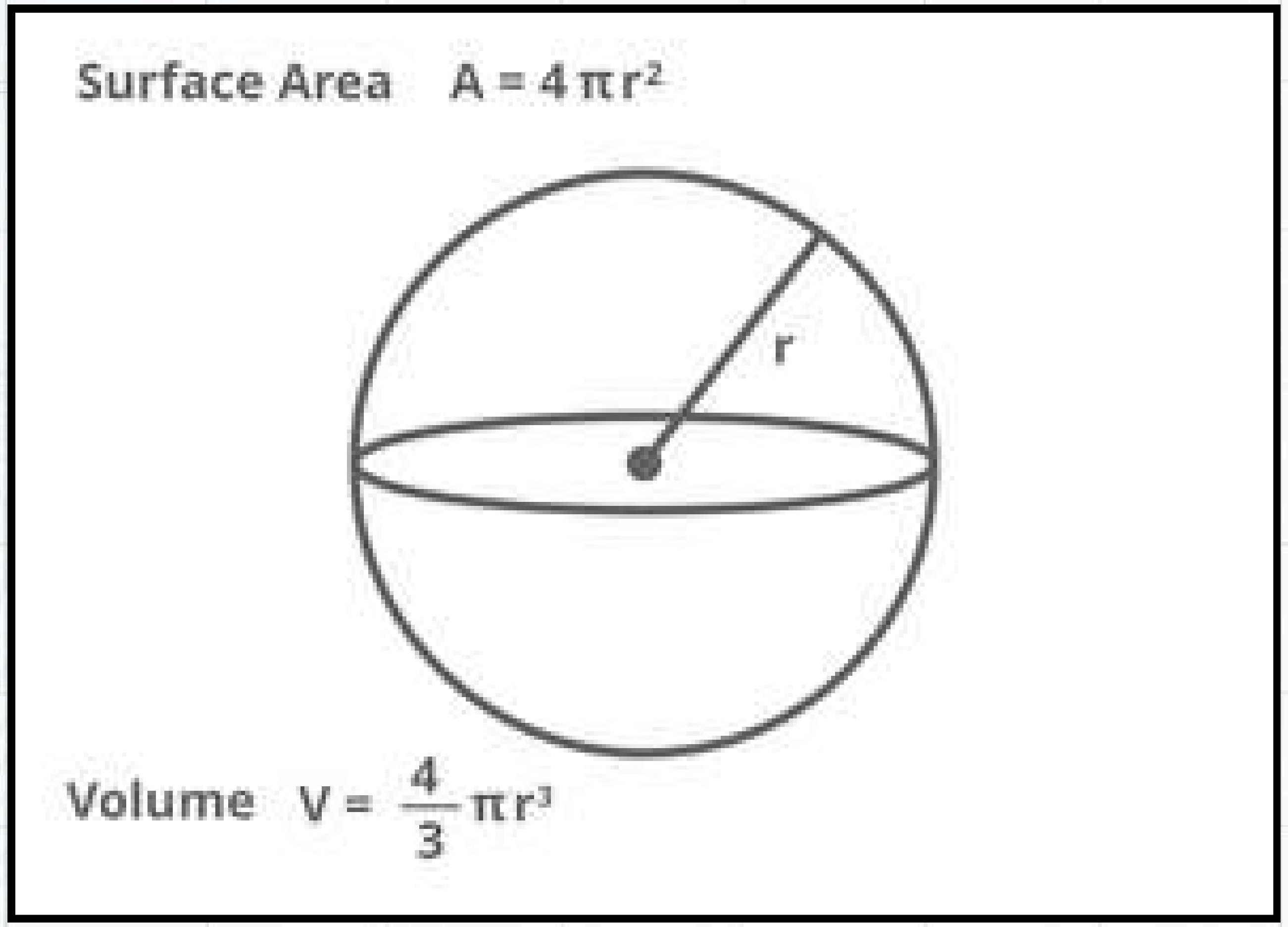
$$SA = 6\left(36\ yd^2\right)$$

$$SA = 216\ yd^2$$

SURFACE AREA & VOLUME

SPHERE

The surface area of a sphere is 100π in^2. What is its volume?



Solve for the radius from the surface area

$$SA = 4\pi r^2$$

$$4\pi r^2 = 100\pi\ in^2$$

$$r^2 = 25\ in^2$$

$$r = 5\ in$$

Solve for the volume

$$V = \pi r^3$$

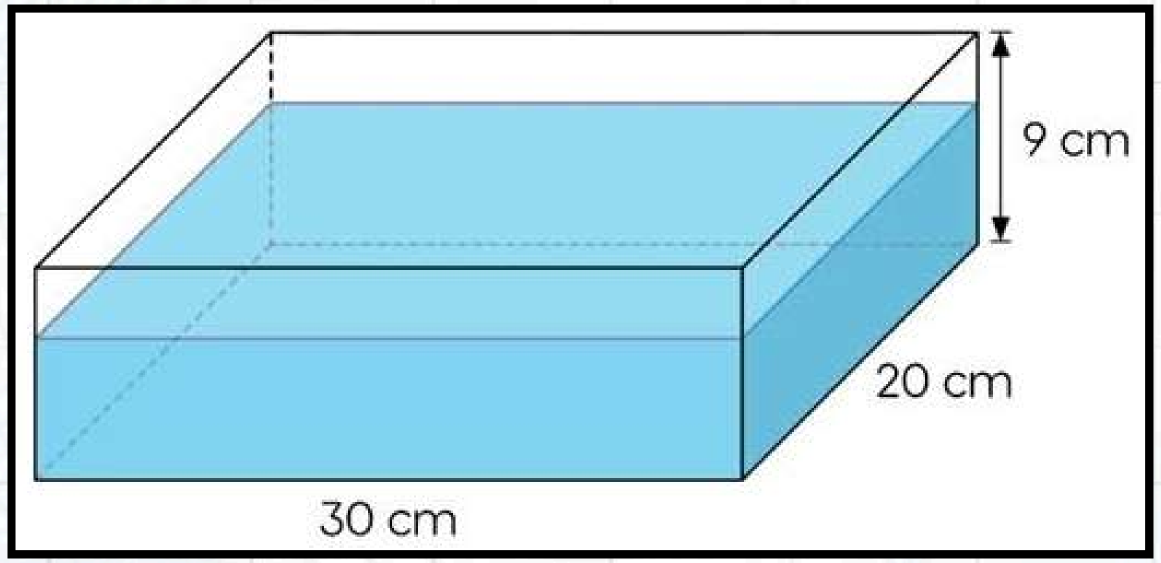
$$V = \pi(5\ in)^3$$

$$V = 125\pi\ in^3$$

SURFACE AREA & VOLUME

RECTANGULAR PRISM

The rectangular fish tank shown below contains 3600 cm^3 of water. If half of the remaining space will be filled with sand, what will be the new volume of the contents in the tank?



*The volume is length * width * height*

$$V_{total} = lwh$$

$$V_{total} = (30\ cm)(20\ cm)(9\ cm)$$

$$V_{total} = 5400\ cm^3$$

Solve for the remaining space in the tank

$$V_{remaining} = V_{total} - V_{water}$$

$$V_{remaining} = 5400\ cm^3 - 3600\ cm^3$$

$$V_{remaining} = 1800\ cm^3$$

Solve for the volume of sand

$$V_{sand} = \frac{1}{2}V_{remaining}$$

$$V_{sand} = \frac{1}{2}(1800\ cm^3)$$

$$V_{sand} = 900\ cm^3$$

Solve for the occupied volume

$$V_{occupied} = V_{water} + V_{sand}$$

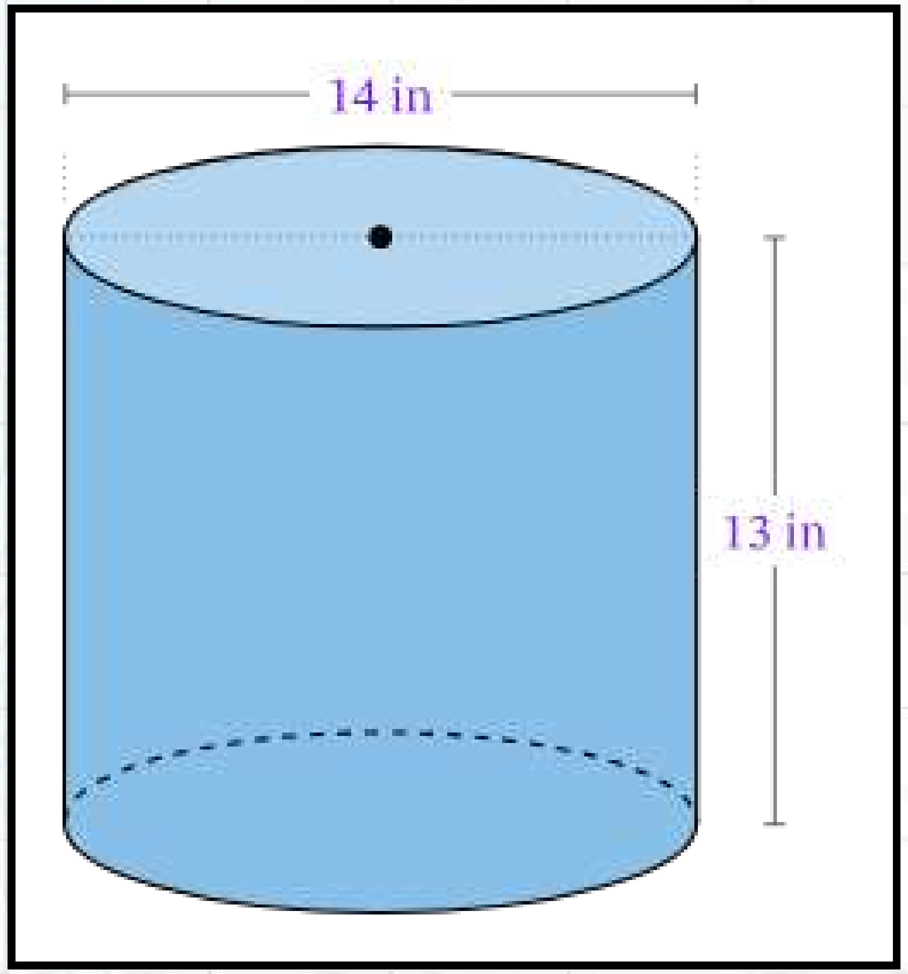
$$V_{occupied} = 1800\ cm^3 + 900\ cm^3$$

$$V_{occupied} = 2700\ cm^3$$

SURFACE AREA & VOLUME

CYLINDER

What is the volume and surface area of the cylinder?



*Volume of a cylinder = base * height*

$$V = \pi r^2 h$$

$$V = \pi(7\ in)^2(13\ in)$$

$$V = \pi(49\ in^2)(13\ in)$$

$$V = 637\pi\ in^3$$

Surface area of a cylinder = 2 bases + body

$$SA = 2\pi r^2 + 2rh$$

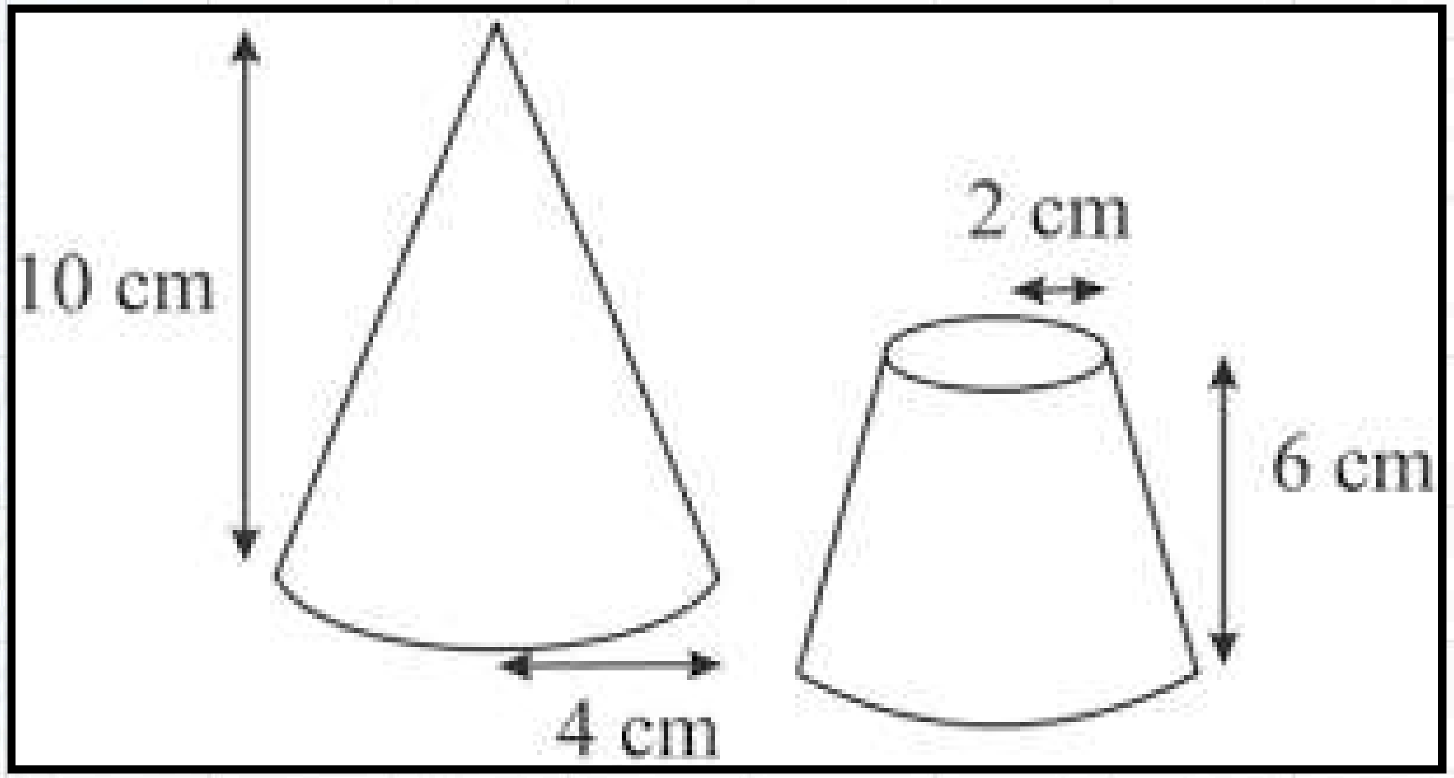
$$SA = 2\pi(7\ in)^2 + 2(7\ in)(13\ in)$$

$$SA = 49\pi\ in^2 + 182\ in^2$$

SURFACE AREA & VOLUME

CONE

The top of the cone is removed to form a truncated cone as shown below. What is the volume of the remaining part?



Find the volume of the whole cone

$$V = \frac{1}{3}\pi r^2 h$$

$$V_{whole} = \frac{1}{3}\pi(4\ cm)^2(10\ cm)$$

$$V_{whole} = \frac{1}{3}\pi(16\ cm^2)(10\ cm)$$

$$V_{whole} = \frac{160\pi}{3}\ cm^3$$

Find the volume of the top cone

$$V_{top} = \frac{1}{3}\pi(2\ cm)^2(10cm - 6\ cm)$$

$$V_{top} = \frac{1}{3}\pi(4\ cm^2)(4\ cm)$$

$$V_{top} = \frac{16\pi}{3}\ cm^3$$

A truncated cone equals the whole minus the top

$$V_{bottom} = V_{whole} - V_{top}$$

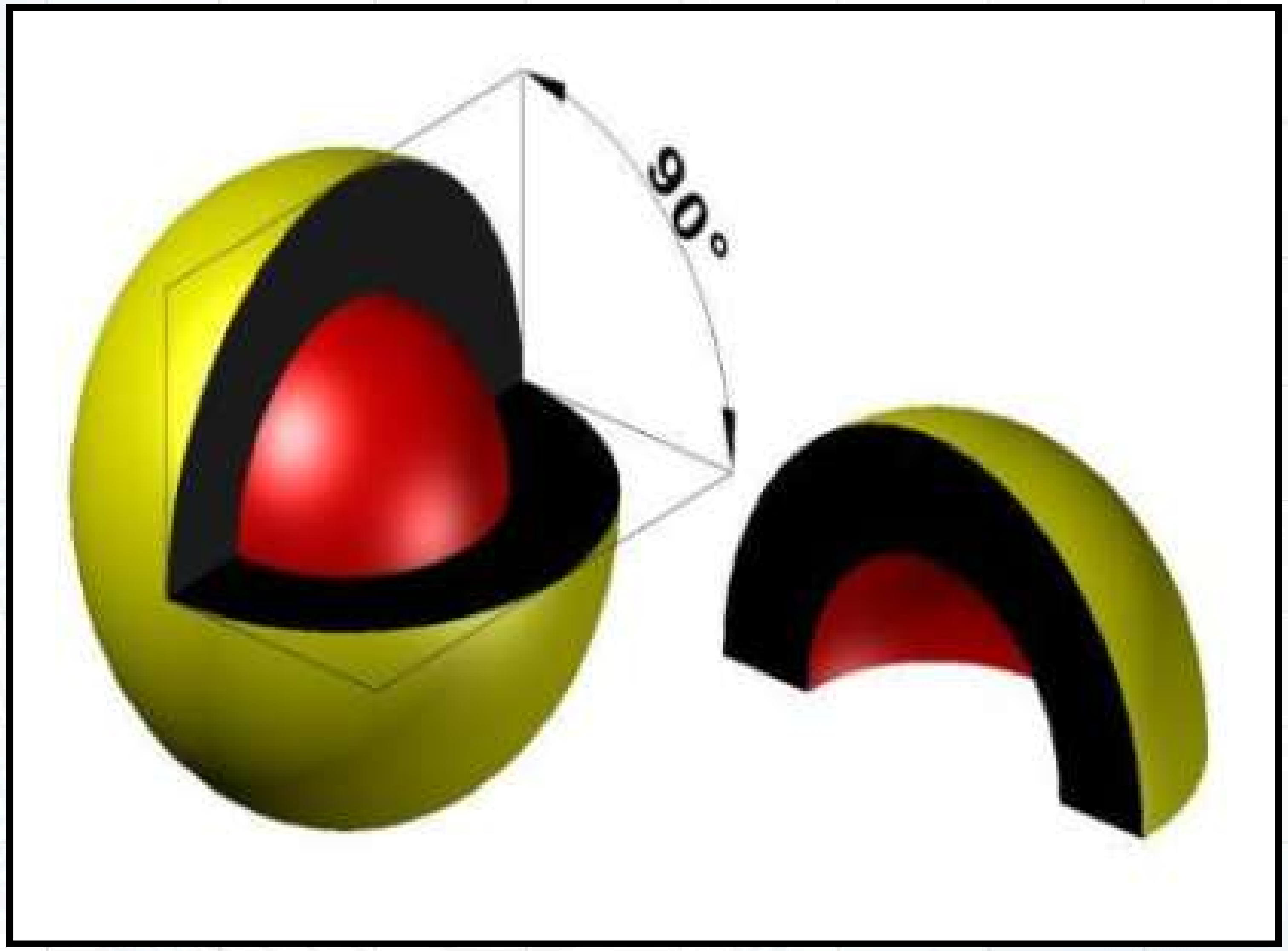
$$V_{bottom} = \frac{160\pi}{3}\ cm^3 - \frac{16\pi}{3}\ cm^3$$

$$V_{bottom} = \frac{144\pi}{3}\ cm^3 = 48\pi\ cm^3$$

SURFACE AREA & VOLUME

SECTOR OF A SPHERE

Determine the volume of the figure on the right if the radius of the red and yellow spheres are 3m and 5m respectively.



Find the volume of the hollow sphere and multiply it by 90°/360°, similar to the sector of a circle in the prev. section

$$V_{sector} = \frac{90°}{360°}\left(V_{yellow} - V_{red}\right)$$

$$V_{hollow} = \frac{1}{4}\left(\frac{4}{3}\pi r^3_{yellow} - \frac{4}{3}\pi r^3_{red}\right)$$

$$V_{hollow} = \frac{1}{4}\left(\frac{4}{3}\pi(5m)^3 - \frac{4}{3}\pi(3m)^3\right)$$

$$V_{hollow} = \frac{1}{4}\left(\frac{4}{3}125\pi\, m^3 - \frac{4}{3}27\pi m^3\right)$$

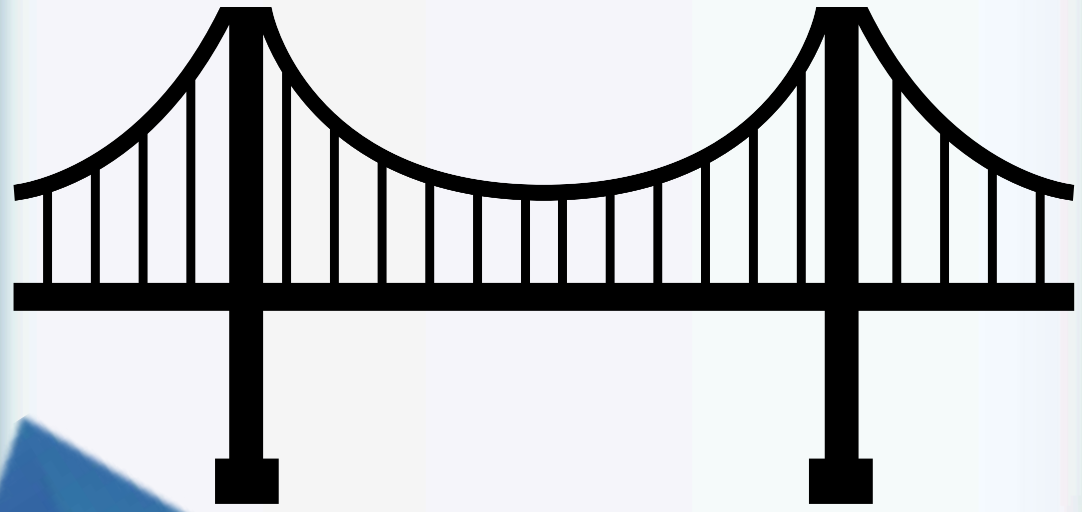
$$V_{hollow} = \frac{1}{4}\left(\frac{4}{3}98\pi\, m^3\right)$$

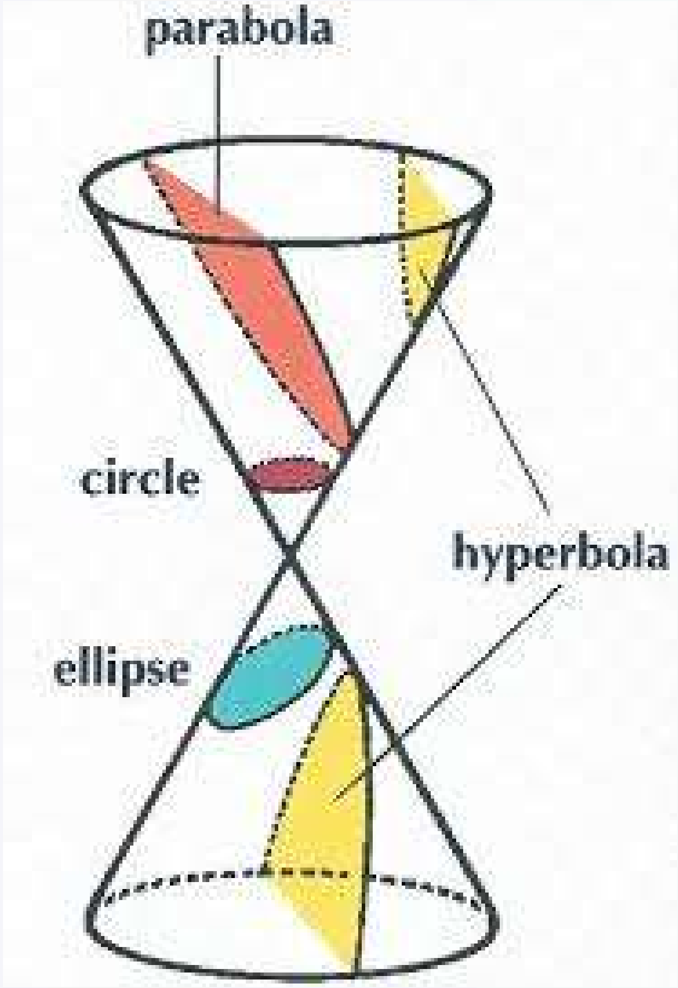
$$V_{hollow} = \frac{1}{3}\left(98\pi\, m^3\right)$$

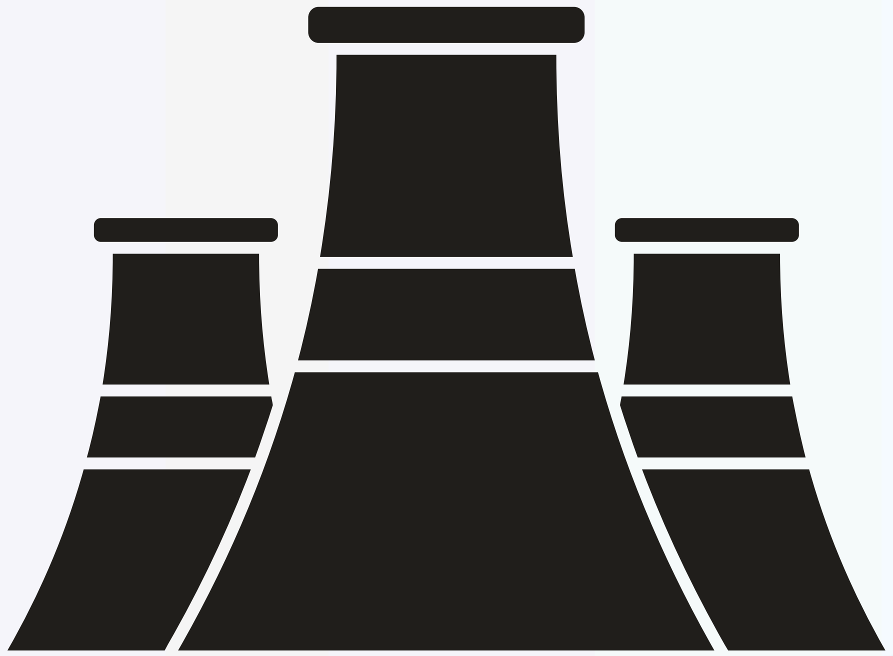
$$V_{hollow} = \frac{98\pi}{3}\, m^3$$

GEOMETRY

CONIC SECTIONS







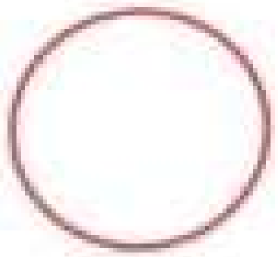
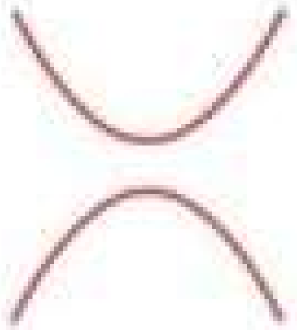
CONIC SECTIONS

FORMULAS

The four conic sections have **similar variables**:

x: x-value of a given point
y: y-value of a given point
h: x-value of the vertex/center
k: y-value of the vertex/center
a: length of semi-major axis
b: length of major axis

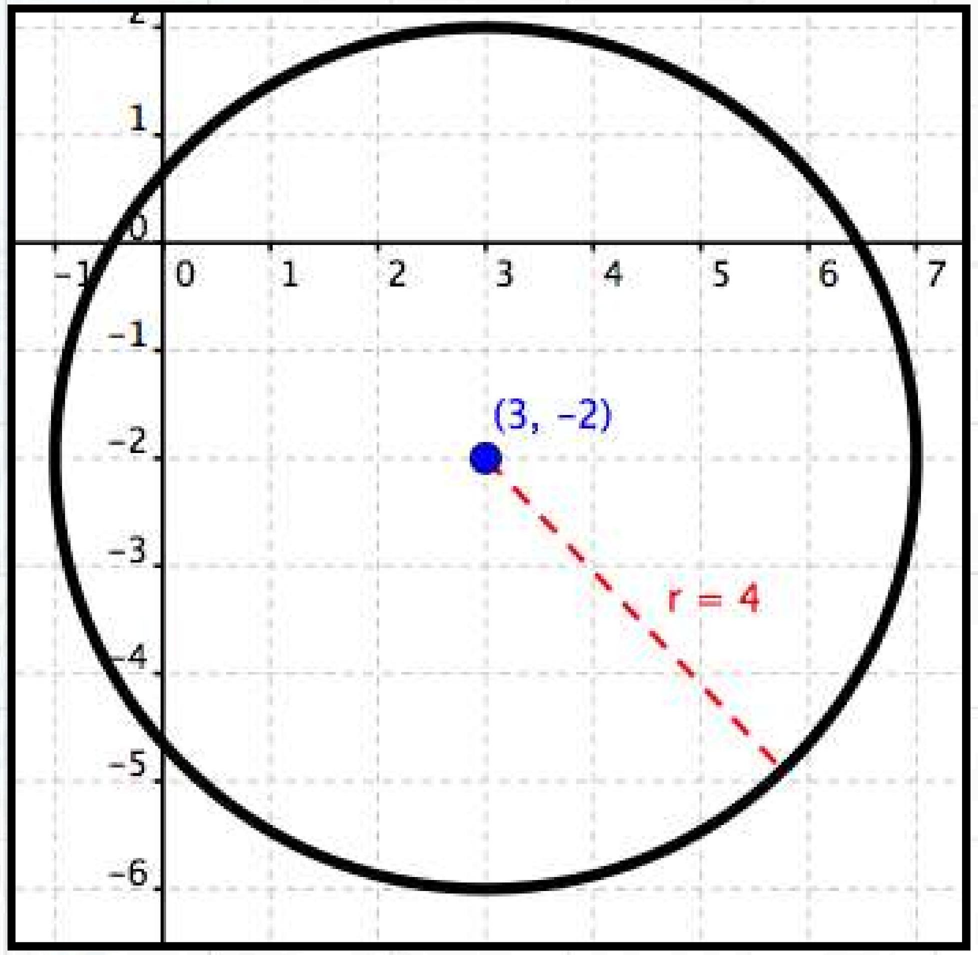
There are a lot of properties for each conic section, but let's focus on using their equations for today.

Conic section	Conic equation
Circle 	$(x-h)^2+(y-k)^2=r^2$
Ellipse 	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
Parabola 	$y-k=a(x-h)^2$ $x-h=a(y-k)^2$
Hyperbola 	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

CONIC SECTIONS

CIRCLE

Find the general equation of the circle shown below:



Identify the variables

Center: $(h, k) = (3, -2)$

Radius: $r = 4$

Conic formula of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 3)^2 + (y - (-2))^2 = 4^2$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

Convert to general formula

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 16$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 - 16 = 0$$

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

Where is the center of this ellipse located?

$9x^2 - 4y^2 - 36x + 40y + 100 = 0$

Use completing the square

$$9x^2 - 4y - 36x + 40y + 100 = 0$$

$$(9x^2 - 36x) + (-4y + 40y) + 100 = 0$$

$$9(x^2 - 4x) - 4(y - 10y) + 100 = 0$$

$$9(x^2 - 4x + 4) - 9(4) = 9(x - 2)^2 - 36$$

$$4(y - 10y + 25) - 4(25) = 4(y - 5)^2 - 100$$

CONIC SECTIONS

ELLIPSE

Where is the center of this ellipse located?

$9x^2 - 4y^2 - 36x + 40y + 100 = 0$

Convert into conic formula

$$9(x - 2)^2 - 36 - 4(y - 5)^2 - 100 + 100 = 0$$

$$9(x - 2)^2 - 4(y - 5)^2 - 36 = 0$$

$$9(x - 2)^2 - 4(y - 5)^2 = 36$$

$$\frac{(x-2)^2}{4} - \frac{(y-5)^2}{9} = 1$$

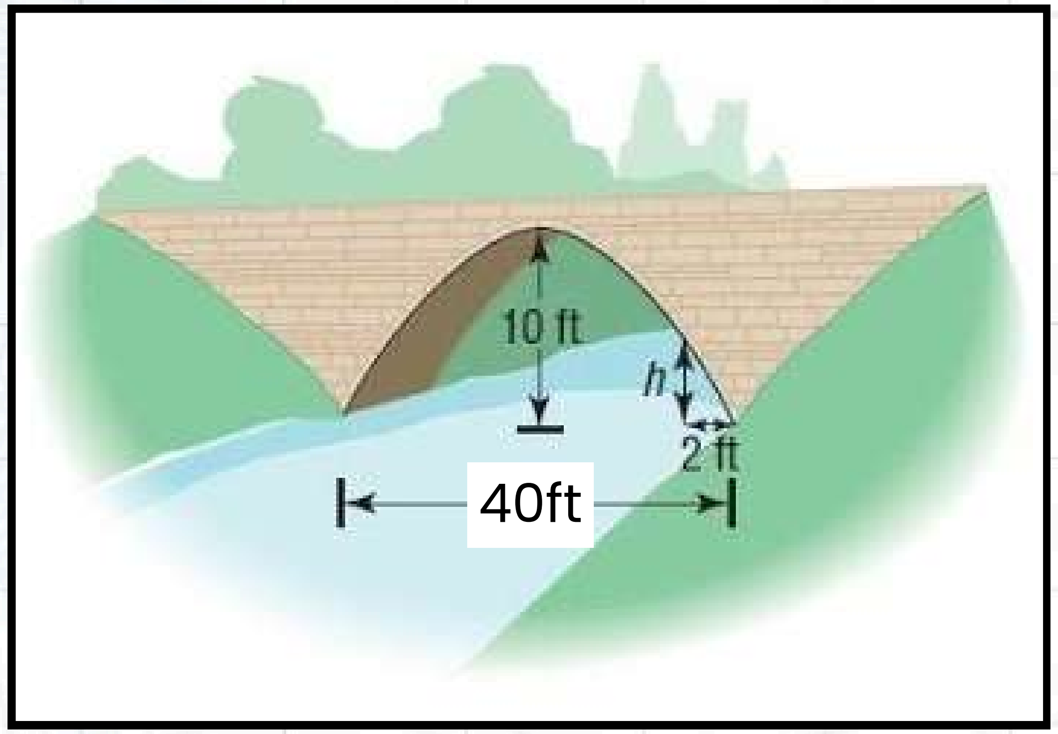
Identify the variables

$$(h, k) = (2, 5)$$

CONIC SECTIONS

PARABOLA

This bridge is in the shape of a parabolic arch. What is the height h of the arch 2 feet from shore?

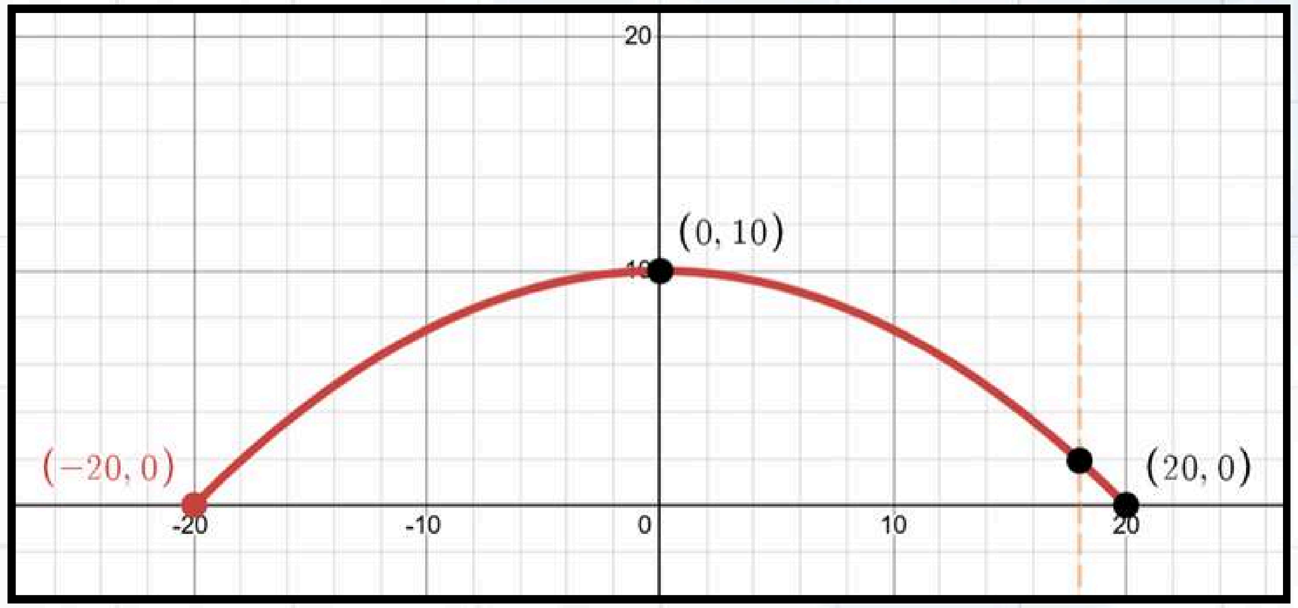


Identify the variables to find the "a" variable

$$y = a(x - h)^2 + k$$

$$\text{Center } (h, k) = (0, 10)$$

$$\text{Point } (x, y) = (20, 0)$$



$$0 = a(20 - 0)^2 + 10$$

$$0 = 400a + 10$$

$$-400a = 10$$

$$a = -\frac{1}{40}$$

Use the formula to solve for y when x = 18

$$y = -\frac{1}{40}x^2 + 10$$

$$y = -\frac{1}{40}(18^2) + 10$$

$$y = -\frac{1}{40}(324) + 10$$

$$y = -8.1 + 10$$

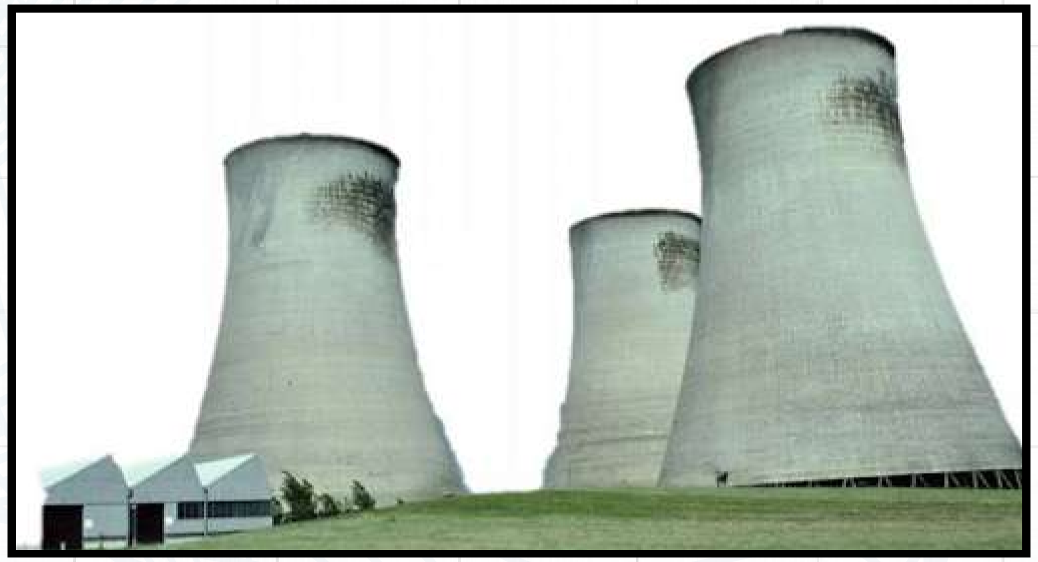
$$y = 1.9$$

CONIC SECTIONS

HYPERBOLA

The hyperbola shape of a cooling tower is modeled by the equation below. How wide is the narrowest point of the tower?

$$225x^2 - 16y^2 = 3600$$



Convert to conic formula

$$225x^2 - 16y^2 = 3600$$

$$\frac{x^2}{16} - \frac{y^2}{225} = 1$$

$$\frac{x^2}{4^2} - \frac{y^2}{15^2} = 1$$

The narrowest part is between the two vertices. The gap is 2a in width.

$$a^2 = 4^2$$

$$a = 4$$

$$2a = 8$$